PRE-BOARD EXAMINATION (2018-19)
MATHEMATICS (041)
CLASS XII
SET – I

Time Allowed: 3 HOURS. ***Max. Marks: 100***
Date : 17 - 12 - 2018

General Instructions:

(i) *All questions are compulsory.*
(ii) *This question paper contains **29 questions.***
(iii) ***Question 1 - 4** in Section A are very short answer type questions carrying **1mark** each.*
(iv) ***Question 5 - 12** in Section B are short answer type questions carrying **2 marks** each.*
(v) ***Question 13 - 23** in Section C are long answer-I type questions carrying **4 marks** each.*
(vi) ***Question 24 - 29** in Section D are long answer - II type questions carrying **6 marks** Each.*

SECTION A

1. A square matrix A of order 3, has $|A| = 5$, find $|A \text{ adj} A|$

2. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1 + x) = 0$

3. If $f(x) = |\cos x - \sin x|$, then find $f'\left(\frac{\pi}{3}\right)$

4. The foot of the perpendicular drawn from the origin to the plane is (2,5,7). Find the equation of the plane.

OR

The x- coordinate of a point P on the line joining the points Q(2, 2,1) and R(5,1,-2) is 4. Find its z-coordinate.

SECTION B

5. If functions f and g are given by f = { (1,2), (3,5), (4,1), (2,6) } and g = { (2,6), (5,4), (1,3), (6,1)}, find the range of f and g and write down the functions fog and gof.

6. If the vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \mu\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$ are coplanar, then find the value of μ.

OR

Find the projection vector of $2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\hat{\imath} - 2\hat{\jmath} + \hat{k}$

7. Form the differential equation of all circles which pass through origin and whose centres lie on y- axis.

8. If A and B are square matrices of the same order, find out whether (AB' – BA') is symmetric or skew symmetric

9. A and B are independent events. The probability that both A and B occur is $\frac{1}{8}$ and the probability that neither occurs is $\frac{3}{8}$. Find P(A)' and P(B)'.

OR

A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6 Find the probability that B is selected.

10. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A. If 2 or 3 turns up, a ball is picked up from bag B. If 4,5, or 6 turns up, a ball is picked up from bag C. Bag A contains 3 red and 2 white balls; bag B contains 3 red and 4 white balls; bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked and a ball is drawn. Find the probability that a red ball is drawn.

11. Find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin|x| - \cos|x|)dx$

OR

Find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$

12. Find $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}} dx$

SECTION C

13. Let f: [0,∞)→ R be a function defined by f(x) = $9x^2 + 6x - 5$. Prove that f is not invertible. Modify only the codomain of f to make f invertible and then find the inverse.

OR

Determine whether the relation R defined in the set R of all real numbers as

R = { (a,b) : a,b∈ $R, a - b + \sqrt{5} \in S, S$ *is the set of irrational numbers*} is an equivalence relation or not.

14. Examine the differentiability of f, where f is defined by f(x) = $\begin{cases} x[x], & if\ 0 \le x < 2 \\ (x-1)x, & if\ 2 \le x < 3 \end{cases}$ at x = 2

15. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a^3(x^3-y^3)$, prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

OR

If cosy = xcos(a+y), with cos a ≠ ±1, then prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$. Hence show that

$$\sin a\frac{d^2y}{dx^2}+\sin 2(a+y)\frac{dy}{dx}=0$$

16. If a ≠ p, b ≠ q, r ≠ c and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix}=0$, find the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$

17. Find $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$

18. Solve the differential equation : $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

OR

Find the particular solution of the differential equation $(\tan^{-1}y-x)\,dy=(1+y^2)\,dx$; given when y = 0, x = 1

19. Find $\int \sin^{-1}\left(\sqrt{\frac{x}{a+x}}\right)dx$

20. Find the equations of the line which intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and passes through the point (1, 1, 1)

21. Solve the equation $\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=\frac{-\pi}{2}$

22. Determine the intervals in which the function $f(x)=x^4-8x^3+22x^2-24x+21$ is strictly increasing or strictly decreasing.

23. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

SECTION D

24. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, verify that BA = 6I. Use this result to solve $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$

OR

Use elementary transformations to find the inverse of $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

25. Find the sides of a rectangle of greatest area that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

26. Find the equation of the plane which contains the line of intersection of the plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and whose x- intercept is twice its z- intercept. Also write the vector equation of a plane passing through the point (2, 3,-1) parallel to the plane obtained above.

OR

Find the direction ratios of the normal to the plane, which passes through the points (1,0,0) and (0,1,0) and makes an angle $\frac{\pi}{4}$ with the plane $x + y = 3$

27. Using integration, find the area of that part of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

OR

Using integration, find the area bounded by the curves $y = |x-1|$ and $y = 3 - |x|$

28. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

29. A toy company manufactures two types of dolls, A and B, Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further the production levels of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

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