CBSE
Class XII Mathematics

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises 4 questions of one mark each, Section B comprises 8 questions of two marks each, Section C comprises 11 questions of four marks each and Section D comprises 6 questions of six marks each.
3. Use of a calculator is not permitted.

SECTION A

1. Is '*' defined on the set {1, 2, 3, 4, 5} by a * b = L.C.M. (a, b) a binary operation?

2. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find (A + 2B)′.

OR

If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'

3. Find the projection of $\vec{a} = \hat{i} - 3\hat{k}$ on $\vec{b} = 3\hat{i} + \hat{j} - 4\hat{k}$.

4. Find the equation of a line parallel to the x-axis and passing through the origin.

SECTION B

5. Find the value of x.

$$\sin\left\{\sin^{-1}\frac{3}{5} + \cos^{-1} x\right\} = 1$$

6. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

7. The income (I) of a doctor is given by

$I = x^3 - 3x^2 + 5x.$

Can an insurance agent ensure the growth of his income?

8. Find the distance of the plane $3x - 4y + 12z = 3$ from the origin.

OR

Find the shortest distance between the lines

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k})\mu\ (2\hat{i} + 4\hat{j} - 5\hat{k})$

9. Without expanding, find the value of the determinant

$$\begin{vmatrix} 0 & q-r & r-s \\ r-q & 0 & p-q \\ s-r & q-r & 0 \end{vmatrix}$$

10. Evaluate: $\int \frac{1 + \cot x}{x + \log \sin x} dx$

OR

Evaluate:

$$\int \frac{\cos\ 2x + 2\ \sin^2 x}{\cos^2 x}\ dx$$

11. Write the direction cosines of the vectors $-2\hat{i} + \hat{j} - 5k.$

12. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, where a is a non-zero real number, then without actually evaluating adj A, find the value of |adj A|.

OR

Given A = $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$

SECTION C

13. Show that the function f: N → N defined by $f(n) = n - (-1)^n$ for all (n ∈ N) is a bijection.

OR

Show that relation R defined by (a, b) R (c, d) ⇒ a + d = b + c on the set N x N is an equivalence relation.

14. Find the value of $\tan\left[\frac{1}{2}\left\{\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}\right]$, $|x|<1$, $y > 0$, $xy < 1$.

15. Using the properties of determinants, prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

OR

Find the equation of the line joining points A(1, 3) and B(0, 0) using determinants and find k if D(k, 0) is a point such that the area of ΔABD is 3 square units.

16. If sin y = x sin(a + y), prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

OR

Differentiate w.r.t. x

$y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$

17. Discuss the continuity of the function f(x) at x = 1.

$$\text{Given } \begin{cases} \frac{3}{2} - x & , \quad \frac{1}{2} \le x \le 1 \\ \frac{3}{2} & , \quad x = 1 \\ \frac{3}{2} + x & , \quad 1 < x \le 2 \end{cases}$$

18. Evaluate: $\int_{-1}^{2}(7x-5)dx$, as a limit of sums.

19. Evaluate: $\int_2^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\,dx$

20. A company has two plants to manufacturing scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30%. At plant I, 30% of the scooters are rated standard quality, and at plant II, 90% of the scooters are rated standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it is manufactured by plant II.

21. Find the co-ordinates of the points on the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$, which are at a distance of 3 units from the point (1, −2, 3).

22. Give the intervals in which the function f(x) = $\frac{4\sin x-2x-x\cos x}{2+\cos x}$ is increasing or decreasing.

23. If $u=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $-1 < x < 1$, then write the value of $\frac{du}{dv}$.

SECTION D

24. Given two matrices, $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, verify that BA = 6I. Use the result to solve the system x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7.

OR

Find the inverse of $A=\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}$ by elementary row transformation.

25. Solve the differential equation

$(x^2 - y^2)\,dx + 2xy\,dy = 0$

Given that y = 1 when x = 1.

26. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30, respectively. The company makes a profit of Rs. 8000 on each piece of Model A and Rs. 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

OR

A factory manufactures two types of screws A and B. Each type of screw requires the use of two machines, one automatic and the other hand-operated. It takes 4 minutes on the automatic machine and 6 minutes on the hand-operated machine to manufacture a packet of screws of A, while it takes 6 minutes on the automatic machine and 3 minutes on the hand-operated machine to manufacture a packet of screws of B. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws A at a profit of 70 paise and screws B at a profit of 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximise his profit? Formulate the above LPP, solve it graphically and find the maximum profit.

27. Find the volume of the largest cylinder which can be inscribed in a sphere of radius r.

28. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bonded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

29. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum numbers of times must he fire so that the probability of hitting the target at least once is more than 0.99?

OR

A black and red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.