# CBSE Class 10 Mathematics Standard Sample Paper - 01 (2020-21)

## Solution

#### Part-A

1.  $(2 \times 5)^n = 2^n \times 5^n$ 

= 10<sup>n</sup>

If n = 0 then  $10^0 = 1$ 

If n > 0 then  $10^n$  will end with 0

If n < 0 then 10n ends with 1 (e.g. 0.1, 0.01, 0.001)

Hence for all values of n,  $2^n \times 5^n$  an never end with 5.

### OR

 $45470971 = 7 \times 7 \times 13 \times 13 \times 17 \times 17 \times 19$  $= 7^{2} \times 13^{2} \times 17^{2} \times 19$ 

2. We have equation

f(x) =  $x^2 + 2x + 4 = 0$ By comparing with  $ax^2 + bx + c = 0$ , we get, a = 1, b = 2 and c = 4 ∴ D =  $b^2 - 4ac$ =  $(2)^2 - 4 \times 1 \times 4$ = 4 - 16 = -12

So the discriminant of the equation is -12

3. The given system of linear equations is :

ax + by = c.....(1) lx + my = n.....(2) Here, a<sub>1</sub> = a, b<sub>1</sub> = b, c<sub>1</sub> = -c a<sub>2</sub> = l, b<sub>2</sub> = m, c<sub>2</sub> = -n If the given system of linear equations has a unique solution, then  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ 

$$\Rightarrow \quad \frac{a}{l} \neq \frac{b}{m} \\ \Rightarrow \quad am \neq bl$$

This is the required solution.

Consider C is an external point as shown in figure.

According to theorem, from an external point only two tangents can be drawn to a circle. So, value of k = 2

5. 
$$an = a + (n - 1)d$$
  
 $\Rightarrow a_n = 3.5 + (105 - 1)0$   
 $\Rightarrow a_n = 3.5$ 

OR

$$T_n = 3^{n+1}$$
  

$$\Rightarrow T_1 = 3^{1+1} = 9$$
  

$$T_2 = 3^{2+1} = 27,$$
  

$$T_3 = 3^{3+1} = 81,$$
  

$$T_4 = 3^{4+1} = 243$$

1st four terms are 9, 27, 81 and 243.

- 6. We have  $a_1 = -1$ ,  $a_2 = -1$ ,  $a_3 = -1$  and  $a_4 = -1$ 
  - a<sub>2</sub> a<sub>1</sub> = 0
  - a<sub>3</sub> a<sub>2</sub> = 0
  - a<sub>4</sub> a<sub>3</sub> = 0

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

7.  $3x^2 - 2x + 8 = 0$ 

Comparing it with  $ax^2 + bx + c = 0$ , we get a = 3, b = -2 and c = 8  $\therefore D = b^2 - 4ac$   $= [(-2)^2 - 4(3)(8)]$  = (4 - 96)= -92

OR

 $(x - 1)^2 + 2(x + 1) = 0$  $x^2 - 2x + 1 + 2x + 2 = 0$  $x^2 + 3 = 0$ 

No, since the equation is simplified to  $x^2 + 3 = 0$  whose discriminant is less than zero.

8. Two parallel tangents can exist at the two ends of the diameter of the circle. Therefore, the distance between the two parallel tangents will be equal to the diameter of the circle. In the problem the radius of the circle is given as 5 cm. Therefore,

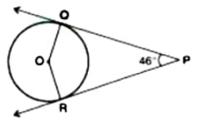
Diameter =  $5 \times 2$ 

Diameter = 10 cm

Hence, the distance between the two parallel tangents is 10 cm.

9. Here,  $\angle APB = 50^{\circ}$   $\angle PAB = \angle PBA = \frac{180^{\circ} - 50^{\circ}}{2} = 65^{\circ}$   $\angle OAB = 90^{\circ} - \angle PAB$  $= 90^{\circ} - 65^{\circ} = 25^{\circ}$ 

OR



Since,  $OQ \perp OP$  and  $OR \perp RP$ 

 $\angle QOR + \angle QPR + \angle PRQ + \angle QOR = 360^{\circ}$ 

or,  $\angle QOR + 46^{\circ} = 180^{\circ}$ 

or,  $\angle QOR = 180^{\circ} - 46^{\circ} = 134^{\circ}$ 

10. In  $\triangle ABC$ , DE ||BC3 cm 6 cm E x cm 6 cm Let EC = x cm. then  $\frac{AD}{DB} = \frac{AE}{EC}$  [By basic proportionality theorem]  $\Rightarrow \frac{3}{4} = \frac{9}{x}$  $\Rightarrow 3x = 6 \times 4$  $\Rightarrow x = \frac{4 \times 6}{3}$  $\Rightarrow$  x = 8cm .:. EC = 8 cm 11. We have to find the 18th term of the sequence defined by  $a_n = \frac{n(n-3)}{n+4}$ . We have,  $a_n = \frac{n(n-3)}{n+4}$ Putting n = 18, we get  $a_{18} = \frac{18 \times (18 - 3)}{18 + 4} = \frac{18 \times 15}{22} = \frac{135}{11}$ 12. negative integer = -1 positive integer = +2

their difference = -1-(+2) = -1-2 = -3.

13. It is given that, cosec A = sec B

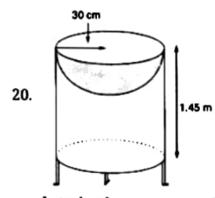
or, cosec A = cosec (90° - B)[ $:: cosec(90^{\circ} - \theta) = sec\theta$ ]

or, A = 90° - B

Therefore,A + B = 90°

14. Let n be the number of cones that will be needed to store the water, and R and H be the radius and height of the cylindrical vessel and cone. Volume of the cylindrical vessel = n × Volume of each cone

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\Rightarrow \pi R^2 H = n \times \frac{1}{3} \pi R^2 H
     \Rightarrow 1 = n \times \frac{1}{3}
     \Rightarrow n = 3
15. 3y - 1, 3y + 5 and 5y + 1 in A.P.
     (3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)
     or, 3y + 5 - 3y + 1 = 5y + 1 - 3y - 5
     or, 6 = 2y - 4
     or, 2y = 6 + 4
     or, 2y = 10
     or, y = \frac{10}{2}
     y = 5
16. Defective bubs = 14
      Good one = 98
      Total bulbs = 14 + 98 = 112
     P(\text{good one}) = \frac{\text{Number of good one}}{\text{Total bulbs}} = \frac{98}{112} = \frac{7}{8}
     : the probability that the bulb taken out is a good one will be \frac{7}{8}.
17. i (c) (\frac{15}{2}, \frac{33}{2})
      ii. (a) 4
     iii. (c) 16
     iv. (d) (2.0, 8.5)
      v. (b) x - 13 = 0
18. i. (b) 100 ft
      ii. (a) 2400 ft<sup>2</sup>
     iii. (c) 25 ft
      iv. (a) 100 ft
      v. (b) 150 ft<sup>2</sup>
19. i. (c) 43
      ii. (c) 60
     iii. (b) Median
      iv. (c) 80
      v. (c) 31
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Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

i. (d) Curved surface area of the hemisphere =  $2\pi r^2$ 

 $= 2 \times 3.14 \times 30^2 = 0.56 \text{ m}^2$ 

ii. (d) Curved surface area of the cylinder =  $2\pi rh$  =  $2 \times \pi \times 0.3 \times 1.45$  =  $0.87\pi$  m<sup>2</sup>

iii. (b) Let S be the total surface area of the bird-bath.

S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$$\Rightarrow \quad S = 2\pi r h + 2\pi r^2 = 2\pi r (h+r)$$

$$\Rightarrow$$
  $S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000 \text{ cm}^2 = 3.3 \text{ m}^2$ 

iv. (c)  $2\pi imes r imes h + 2\pi r^2$ 

v. (a) remain unaltered

#### Part-B

21. Let us assume that  $\sqrt{2} + \sqrt{3}$  is a rational number

Let  $\sqrt{2} + \sqrt{3} = rac{a}{b}$  Where a and b are co-prime positive integers

On squaring both sides, we get

$$(\sqrt{2} + \sqrt{3})^2 = \frac{a^2}{b^2}$$

$$2 + 3 + 2\sqrt{6} = \frac{a^2}{b^2}$$

$$5 + 2\sqrt{6} = \frac{a^2}{b^2}$$

$$2\sqrt{6} = \frac{a^2 - 5b^2}{b^2}$$

$$2\sqrt{6} = \frac{a^2 - 5b^2}{b^2}$$

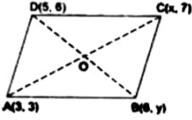
$$\sqrt{6} = \frac{a^2 - 5b^2}{2b^2}$$
Now  $\frac{a^2 - 5b^2}{2b^2}$  is a rational number.  
This shows that  $\sqrt{6}$  is a rational number.

But this contradicts the fact that  $\sqrt{6}$  is an irrational number.

This contradiction has raised because we assume that  $(\sqrt{2} + \sqrt{3})$  is a rational number. Hence, our assumption is wrong and  $(\sqrt{2} + \sqrt{3})$  is an irrational number.

22. Let A(3, 3), B(6, y), C(x, 7) and D(5, 6) be the vertices of a parallelogram ABCD. Join Ac and BD, interesting each other at the point O.

We know that the diagonals of parallelogram bisect each other.



Therefore, O is the midpoint of AC as well as that of BD.

: midpoint of AC is  $\left(\frac{x+3}{2}, \frac{7+3}{2}\right)$ , i.e.  $\left(\frac{x+3}{2}, 5\right)$ and midpoint of BD is  $\left(\frac{5+6}{2}, \frac{6+y}{2}\right)$ , i.e.  $\left(\frac{11}{2}, \frac{6+y}{2}\right)$ 

But, these points coincide at the point O.

$$\therefore \quad \frac{x+3}{2} = \frac{11}{2} \text{ and } \frac{6+y}{2} = 5$$
  

$$\Rightarrow x+3 = 11 \text{ and } 6+y=10$$
  

$$\Rightarrow x = 8 \text{ and } y = 4$$
  
Hence, x = 8 and y = 4.

OR

$$PQ = 10$$

$$PQ^{2} = 10^{2} = 100$$

$$\Rightarrow (10 - 2)^{2} + \{y - (-3)\}^{2} = 100$$

$$\Rightarrow (8)^{2} + (y + 3)^{2} = 100$$

$$\Rightarrow 64 + y^{2} + 6y + 9 = 100$$

$$\Rightarrow y^{2} + 6y - 27 = 0$$

$$\Rightarrow y^{2} + 9y - 3y - 27 = 0$$

$$\Rightarrow y(y + 9) - 3(y + 9) = 0$$

$$\Rightarrow (y + 9) (y - 3) = 0$$

 $\Rightarrow y + 9 = 0 \text{ or } y - 3 = 0$   $\Rightarrow y = -9 \text{ or } y = 3$   $\Rightarrow y = -9, 3$ Hence, the required value of y is -9 or 3. 23. Let  $p(x) = 4x^2 - 4x - 3$ For zeros of p(x), we put p(x) = 0, then,  $4x^2 - 4x - 3 = 0$   $4x^2 + 2x - 6x - 3 = 0$  2x(2x+1) - 3(2x+1) = 0 2x(2x+1) - 3(2x+1) = 0Putting (2x - 3) = 0 and (2x + 1) = 0, we get, zeros of p(x) are  $\frac{3}{2}, -\frac{1}{2}$ Now, sum of zeros  $= \frac{3}{2} - \frac{1}{2} = 1 = -\frac{-4}{4} = -\frac{coeff. of x}{coeff. of x^2}$ and product of zeros  $= \frac{3}{2} \times -\frac{1}{2} = -\frac{3}{4} = \frac{constant term}{coeff. of x^2}$ 

24.

Steps of construction:

i. Draw a circle with centre O and radius 5 cm.

ii. Draw any radius OT.

iii. Construct.  $\angle TOT' = 180^{\circ} - 60^{\circ} = 120^{\circ}$ 

iv. Draw and  $TP \perp OT T'P \perp OT'$ . Then PT' and PT are the two required tangents such that.  $\angle TPT' = 60^{\circ}$  Here, PT = PT'.

25. To prove: 
$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2 \sin^2 A - 1}$$
$$LHS = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$$
$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\sin^2 A - \cos^2 A}$$
$$= \frac{\frac{\sin^2 A + \cos^2 A + 2\sin A \cos A + \sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1+1}{\sin^2 A - \cos^2 A} (:: \sin^2 A + \cos^2 A = 1)$$
  
=  $\frac{2}{\sin^2 A - 1 + \sin^2 A} = \frac{2}{2 \sin^2 A - 1} (:: \cos^2 A = 1 - \sin^2 A)$   
= RHS

OR

L.H.S = 
$$\sec^4\theta - \sec^2\theta$$
  
=  $\sec^2\theta$  ( $\sec^2\theta - 1$ )  
=  $\sec^2\theta$  ( $\tan^2\theta$ ) [ $\cdot$ : 1 +  $\tan^2\theta$  =  $\sec^2\theta$  or  $\tan^2\theta = \sec^2\theta - 1$ ]  
= (1+  $\tan^2\theta$ )  $\tan^2\theta$  =  $\tan^2\theta + \tan^4\theta$  = R.H.S  
Hence proved.

26. In Figure, common tangents AB and CD to the two circles with centres  $O_1$  and  $O_2$  intersect at E.We have to prove that AB = CD.

EA and EC are tangents from point E to the circle with centre O1

EA = EC .....(i)

EB and ED are tangents from point E to circle with centre O 2

EB = ED.....(ii)

Eq. (i) +Eq(ii),we get,

EA + EB = EC + ED

AB = CD

27. We can prove  $7\sqrt{5}$  irrational by contradiction.

Let us suppose that  $7\sqrt{5}$  is rational.

It means we have some co-prime integers a and b ( $b \neq 0$ )

such that

R.H.S of (1) is rational but we know that  $\sqrt{5}$  is irrational.

It is not possible which means our supposition is wrong.

Therefore,  $7\sqrt{5}$  cannot be rational.

Hence, it is irrational.

28. i. 
$$\sqrt{6x+7} - (2x-7) = 0$$
  
 $\sqrt{6x+7} = 2x - 7$ 

Squaring both sides of the equation  

$$\Rightarrow (\sqrt{6x+7})^2 = (2x-7)^2$$

$$\Rightarrow 6x+7 = (2x)^2 - 2 \times 2x \times 7 + (7)^2$$

$$\Rightarrow 6x+7 = 4x^2 - 28x + 49$$

$$\Rightarrow 4x^2 - 34x + 42 = 0$$

$$\Rightarrow 2x^2 - 17x + 21 = 0$$

$$\Rightarrow 2x^2 - 14x - 3x + 21 = 0$$

$$\Rightarrow 2x(x-7) - 3(x-7) = 0$$

$$\Rightarrow (2x-3)(x-7) = 0$$

$$\Rightarrow 2x - 3 = 0 \text{ and } x - 7 = 0$$

$$\therefore x = \frac{3}{2} \text{ and } x = 7$$
ii.  $\sqrt{2x+9} + x = 13$   
 $\sqrt{2x+9} + x = 13$   
 $\sqrt{2x+9} = 13 - x$   
Squaring both sides of the equation  

$$\Rightarrow (\sqrt{2x+9})^2 = (13-x)^2$$

$$\Rightarrow 2x + 9 = 169 + x^2 - 26x$$

$$\Rightarrow x^2 - 28x + 160 = 0$$

$$\Rightarrow x(x-20) - 8(x-20) = 0$$

: x = 8 and x = 20

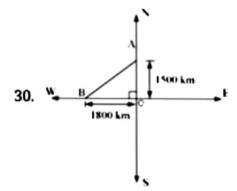
#### OR

We have,  $ax^2 + 7x + b = 0$ Since  $x = \frac{2}{3}$ , -3 are the solutions of the given equation Substitute  $x = \frac{2}{3}$  in the given equation, we get  $a(\frac{2}{3})^2 + 7(\frac{2}{3}) + b = 0$  $\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0$ Multiplying the equation by 9, we get 4a + 9b + 42 = 0 .....(i) Now, substitute x = -3 in the given equation, we get  $a(-3)^2 + 7(-3) + b = 0$  9a + b - 21 = 0 ......(ii) Multiplying the equation by 9, we get 81a + 9b - 189 = 0 ......(ii) subtracting(ii) from (i), we get -77a = -231 or a = 3Substitue a = 3 in (i), we get 4(3) + 9b + 42 = 0  $\Rightarrow 9b + 54 = 0$  or b = -6So, a = 3 and b = -6

29. 
$$f(x) = x^3 - 5x^2 - 16x + 80$$

Let  $\alpha$ ,  $\beta$ ,  $\gamma$  be the zeroes of polynomial f(x) such that  $\alpha + \beta = 0$ . Then, Sum of the zeroes =  $-\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$  $\Rightarrow \quad \alpha + \beta + \gamma = -\left(-\frac{5}{1}\right)$  $\Rightarrow 0+\gamma=5(:\alpha+\beta=0)$  $\Rightarrow \gamma = 5$ Product of the zeroes =  $-\frac{\text{Constant term}}{\text{Coefficient of } x^3}$  $\Rightarrow \alpha\beta\gamma = -\frac{80}{1}$  $\Rightarrow 5\alpha\beta = -80$  $\Rightarrow \alpha\beta = -16$  $\Rightarrow -\alpha^2 = -16$  $\Rightarrow \alpha = \pm 4$ **Case I:** When  $\alpha = 4$ : In this case,  $\alpha + \beta = 0$  $\Rightarrow 4 + \beta = 0$  $\Rightarrow \beta = -4$ So, the zeroes are  $\alpha = 4, \beta = -4$  and  $\gamma = 5$ **Case II:** When  $\alpha = -4$ : In this case,  $\alpha + \beta = 0$  $\Rightarrow -4 + \beta = 0$  $\Rightarrow \beta = 4$ So, the zeroes are  $\alpha = -4$ ,  $\beta = 4$  and  $\gamma = 5$ 

Hence, in either case the zeroes are (4, -4 and 5) and (-4, 4, 5).



Distance traveled by the plane flying toward north in  $1\frac{1}{2}$  hrs

 $= 1000 \times 1\frac{1}{2} = 1500 \text{ km}$ 

Similarly, distance traveled by the plane flying towards west in  $1\frac{1}{2}$  hrs

 $=1,200 \times 1\frac{1}{2} = 1,800 km$ 

Let these distances are represented by OA and OB respectively.

Now applying Pythagoras theorem

Distance between these planes after  $1\frac{1}{2}$  hrs  $AB = \sqrt{OA^2 + OB^2}$ =  $\sqrt{(1,500)^2 + (1,800)^2} = \sqrt{2250000 + 3240000}$ =  $\sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$ So, distance between these planes will be  $300\sqrt{61}$  km, after  $1\frac{1}{2}$  hrs.

OR

Let AB denoted the vertical pole of length 6m.BC is the shadow of the pole on the ground BC = 4m.

Let DE denote the tower.

EF is shadow of the tower on the ground.

EF = 28 m.

Let the height of the tower be h m.



In  $\triangle$  ABC and  $\triangle$  DEF,

 $\angle$  B =  $\angle$ E ......[Each equal to 90° because pole and tower are standing vertical to the ground]

 $\angle C = \angle F \dots$  [Same elevation]

 $\angle A = \angle D$ . shadows are cast at the same time

 $\therefore \triangle ABC$  and  $\triangle DEF$ ,

 $\angle$  B=  $\angle$  E .....[Each equal to 90° because pole and tower are standing vertical to the ground.]

 $\angle A = \angle D$  ( :: shadows are cast at the same time)

∴ △ ABC ~ △DEF ......(AA similarity criterion)

 $\therefore \frac{AB}{DE} = \frac{BC}{EF}$  ............[:: corresponding sides of two similar triangles are proportional]  $\Rightarrow \frac{6}{h} = \frac{4}{28}$  $\Rightarrow h = \frac{6 \times 28}{4} \Rightarrow h = 42$ 

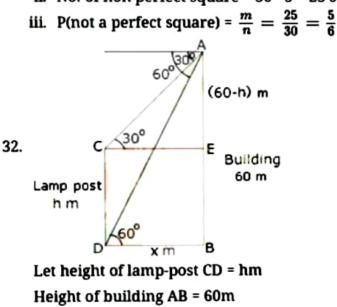
Hence, the height of the tower is 42 m

31. No. of possible outcomes = 30

i. P(prime no. > 7) = 11,13,17,19,23,29 so m=6 P(E<sub>1</sub>) =  $\frac{m}{n} = \frac{6}{30} = \frac{1}{5}$ 

No. of perfect square are 1,4,9,16,25, = 5

ii. No. of non perfect square = 30 - 5 = 25 so m = 25



Let distance BD = x m

i. In  $\triangle ABD$ 

$$\tan 60^{\circ} = \frac{AB}{BD}$$
  

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$
  

$$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
  

$$\Rightarrow x = \frac{60\sqrt{3}}{3} = 20\sqrt{3} = 20 \times 1.732 = 34.64m$$
  
ii. In  $\triangle AEC$   

$$\tan 30^{\circ} = \frac{AE}{CE}$$
  

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$$
  

$$\Rightarrow x = 60\sqrt{3} - h\sqrt{3}$$

$$\Rightarrow 20\sqrt{3} = 60\sqrt{3} - h\sqrt{3}$$
$$\Rightarrow 20 = 60 - h$$

$$\Rightarrow h = 60 - 20 = 40m$$

iii. Difference between the heights of building and lamp-post

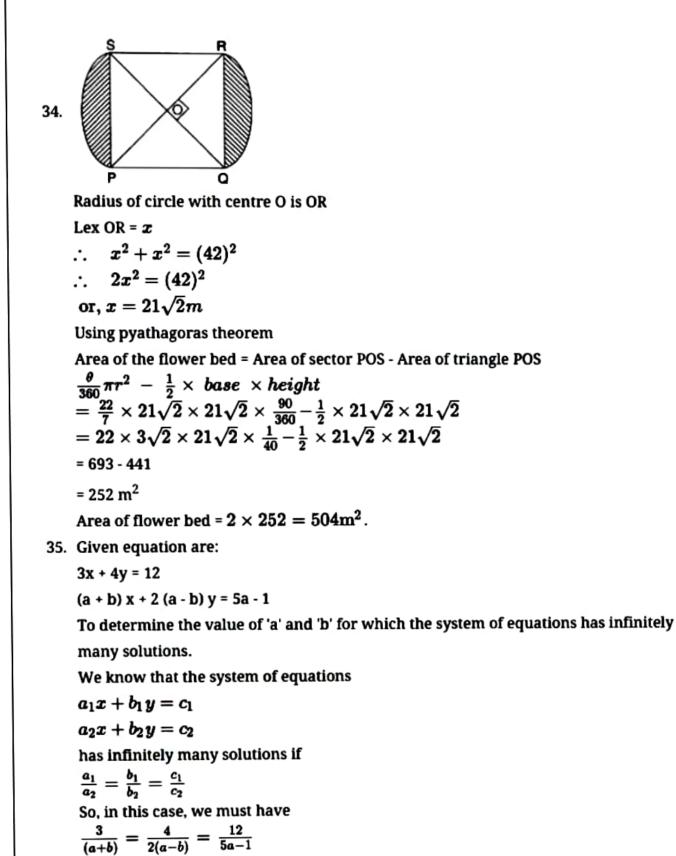
- = 60 40
- = 20m

33.

Daily expenditure on milk (in Rs)	0 - 30	30 - 60	60 - 90	90 - 120	120 - 150
Number of households	5	f <sub>o</sub> = 6	f <sub>1</sub> = 9	f <sub>2</sub> =6	4

Here, maximum frequency = 9, hence modal class is 60 - 90

Mode = 
$$l_1 + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$
  
Here,  $l_1 = 60$ ,  $f_0 = 6$ ,  $f_1 = 9$ ,  $f_2 = 6$  and  $h = 30$ .  
 $\therefore$  Mode =  $60 + 30\left(\frac{9 - 6}{2 \times 9 - 6 - 6}\right)$   
=  $60 + 30\left(\frac{3}{18 - 12}\right)$   
=  $60 + \frac{30 \times 3}{6}$   
=  $60 + \frac{90}{6}$   
=  $60 + 15$   
= 75  
Mode of given data is 75.



First, consider the following

 $\frac{4}{2(a-b)} = \frac{12}{5a-1}$ 

24a - 24b = 20a - 4 a - 6b = -1.....(1)Again, consider  $\frac{3}{(a+b)} = \frac{12}{5a-1}$  12a + 12b = 15a - 3 3a - 12b = 3 a - 4b = 1.....(2)Subtracting eq. (1) from eq. (2), we get 2b = 2  $\Rightarrow b = 1$ Substituting the value of 'b' in eq. (2) we get  $a - 4 \times 1 = 1$  $\Rightarrow a = 5$ 

Hence for a = 5 and b = 1 the system of the equation has infinitely many solutions.

36. Let OA be the horizontal ground, and let K be the position of the kite at a height h above the ground. Then, AK = h.

It is given that OK = 100 metres and  $\angle AOK = 60^{\circ}$ .

Thus, in  $\triangle OAK$ , we have hypotenuse OK = 100 m and  $\angle AOK = 60^{\circ}$  and we wish to find the perpendicular AK. So, we use the trigonometric ratio involving perpendicular and hypotenuse.

100 m  
h  
h  
h  

$$\int 60^{\circ}$$
  
 $\int 60^{\circ}$   
 $\int A$   
In  $\triangle AOK$ , we have  
 $\sin 60^{\circ} = \frac{AK}{OK}$   
 $\Rightarrow \sin 60^{\circ} = \frac{h}{100}$   
 $\Rightarrow h = 100 \sin 60^{\circ}$   
 $\Rightarrow h = 100 \frac{\sqrt{3}}{2} = 50\sqrt{3} = 86.60$  meters.  
Hence, the height of the kite is 86.60 metres.