CBSE Class 12 Mathematics
Sample Paper 01 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \longrightarrow P$ given by $(A, B) \longrightarrow A \cup B$ and $\cap : P \times P \longrightarrow P$ given by $(A, B) \longrightarrow A \cap B$ are binary operations on the set P.

OR

State with reason whether the function has inverse:

$f: \{1, 2, 3, 4\} \to \{10\}$ with $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

2. Define a function. What do you mean by the domain and range of a function? Give examples.

OR

Determine whether the relation is reflexive, symmetric and transitive:

Relation R in the set A of human beings in a town at a particular time given by

R = {(x, y) : x and y live in the same locality}

3. If $f: R \to R$ is defined by $f(x) = x^2$, write $f^{-1}(25)$.

4. If $A = \begin{bmatrix} -2 & 3 \\ 4 & 5 \\ 1 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ -7 & 3 \\ 6 & 4 \end{bmatrix}$ find a matrix C such that A + B - C = 0.

5. Consider the matrix $A = \begin{bmatrix} 3 & -2 & 5 \\ 6 & 9 & 1 \end{bmatrix}$.

then write the values of a11, a12, a13, a21, a22 and a23.

OR

If $A = \begin{bmatrix} 5 & 4 & -2 \\ 6 & -1 & 7 \end{bmatrix}$, Find 3A

6. If $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$, $X = \begin{bmatrix} n \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$ and AX = B, then find n.

7. Write a value of $\int \frac{(\tan^{-1} x)^3}{1+x^2} dx$

OR

Evaluate: $\int \frac{x}{(x^4 - x^2 + 1)} dx$

8. Find the value of c for which the area of figure bounded by the curve y = 3, the straight lines x=1 and x=c and the x-axis is equal to $\frac{16}{3}$

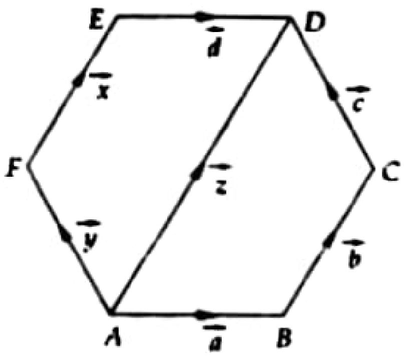
9. Find the general solution for differential equation: $\frac{dy}{dx} = (1 + x)(1 + y^2)$

OR

Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$$

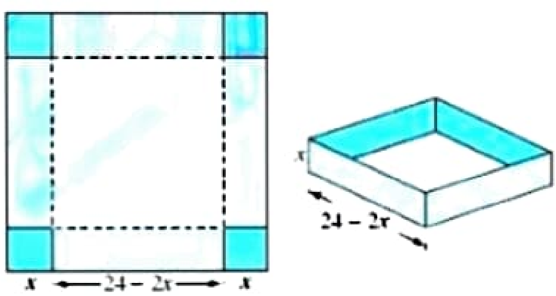
10. Is the measure of 5 seconds vector or scalar?
11. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.
12. In fig. ABCD is a regular hexagon, which vector is Coinitial?



13. Find a normal vector to the plane 2x - y + 2z = 5. Also, find a unit vector normal to the plane.
14. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.
15. The probability that a student selected at random from a class will pass in Mathematics is 4/5, and the probability that he/she passes in Mathematics and Computer Science is 1/2. What is the probability that he/she will pass in Computer Science if it is known that he/she has passed in Mathematics?
16. A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of getting at least one success.

Section - II

17. A man has an expensive square shape piece of golden board of size 24 cm is to be made into a box without top by cutting from each corner and folding the flaps to form a box.



i. Volume of open box formed by folding up the flap:

a. $4(x^3 - 24x^2 + 144x)$

b. $4(x^3 - 34x^2 + 244x)$

c. $x^3 - 24x^2 + 144x$

d. $4x^3 - 24x^2 + 144x$

ii. In the first derivative test, if $\frac{dy}{dx}$ changes its sign from positive to negative as x increases through c_1, then function attains a:

a. Local maxima at $x = c_1$

b. Local minima at $x = c_1$

c. Neither maxima nor minima at $x = c_1$

d. None of these

iii. What should be the side of the square piece to be cut from each corner of the board to be hold the maximum volume?

a. 14 cm

b. 12 cm

c. 4 cm

d. 5 cm

iv. What should be the maximum volume of open box?

a. 1034 cm^3

b. 1024 cm^3

c. 1204 cm^3

d. 4021 cm^3

v. The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in $[0, 9]$ is:

a. 126

b. 0

c. 135

d. 160

18. A shopkeeper sells three types of flower seeds A_1, A_2, and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%.

Based on the above information answer the following questions:

i. The probability of a randomly chosen seed to germinate:

a. 0.69

b. 0.39

c. 0.49

d. 0.59

ii. The probability that the seed will not germinate given that the seed is of type A_3:

a. $\frac{15}{100}$

b. $\frac{65}{100}$

c. $\frac{75}{100}$

d. $\frac{55}{100}$

iii. The probability that the seed is of the type A_2 given that a randomly chosen seed does not germinate.

a. $\frac{22}{51}$

b. $\frac{55}{51}$

c. $\frac{51}{16}$

d. $\frac{16}{51}$

iv. Calculate the probability that it is of the type A_1 given that a randomly chosen seed does not germinate.

a. $\frac{51}{22}$

b. $\frac{22}{51}$
c. $\frac{16}{51}$
d. $\frac{7}{51}$

v. The probability that it will not germinate given that the seed is of type A_1:

a. $\frac{55}{100}$
b. $\frac{65}{100}$
c. $\frac{35}{100}$
d. $\frac{45}{100}$

Part - B Section - III

19. Prove that: $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\} = \frac{x}{2}, 0 < x < \frac{\pi}{2}$

20. Solve the matrix equation $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$, where X is a 2 × 2 matrix.

OR

Solve $\begin{bmatrix} 3 & -4 \\ 9 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$ for x and y.

21. Find $\frac{dy}{dx}$ if $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

22. Find the point at which the tangent to the curve y = $\sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$.

23. Evaluate: $\int$ x log (1 + x) dx

OR

Evaluate: $\int \frac{\log\left(\tan\frac{x}{2}\right)}{\sin x} dx$

24. Find the area of the region common to the parabolas $4y^2 = 9x$ and $3x^2 = 16y$.

25. Find the general solution of the differential equation: $\frac{dy}{dx}$ + (sec x) y = tan x

26. Find a vector of magnitude 49, which is perpendicular to both the vectors $2\hat{i} + 3\hat{j} + 6\hat{k}$ and $3\hat{i} - 6\hat{j} + 2\hat{k}$

27. The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ Write its vector form.

28. A die is rolled. If the outcome is an even number, what is the probability that it is a number greater than 2?

OR

Two dice were thrown and it is known that the numbers which come up were different.

Find the probability that the sum of the two numbers was 5.

Section - IV

29. Let $f : N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$ for all $n \in N$. State whether the function f is bijective. Justify your answer.

30. Differentiate the function with respect to x: $\sin^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}, -1 < x < 1.$

31. Find the points of discontinuity, if any, of the function: $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ 2x+3, & x \geq 0 \end{cases}$

OR

Find $\frac{dy}{dx}$ when $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$

32. Find the intervals in which the function $f(x) = (x - 1)^3 (x - 2)^2$ is (i) increasing. (ii) decreasing.

33. Evaluate: $\int \frac{(x^2+1)}{(x^4+x^2+1)}dx$

34. Find the area of the region bounded by the parabola $y^2 = 2x + 1$ and the line $x - y - 1 = 0$.

OR

Find the area enclosed by the curves $3x^2 + 5y = 32$ and $y = |x - 2|$.

35. In the differential equation show that it is homogeneous and solve it: $(x - y)\frac{dy}{dx} = x + 3y$.

Section - V

36. If A = $\begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix}$ and B = $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.

OR

Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = 0$ and hence find A^{-1}.

37. Find the equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, -7) and show that the line $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ also lies in the same plane.

OR

Find the vector equation of the plane passing through the intersection of planes

$\bar{r}\cdot(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=5$ and the point (1,1,1).

38. A company manufactures three kinds of calculators: A, Band C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is Rs 12000 and of factory II is Rs 15000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically.

OR

An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps. D, E and F whose requirements are 45000 L, 3000 L and 3500 L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km.)		
From/To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Rs.1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?