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SYLLABUS

JEE MAIN

SECTION A (80% weightage)

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Physics, technology and society, SI units, Fundamental and derived units. Least count, accuracy and precision of measuring instruments, Errors in measurement, Significant figures.

Dimensions of Physical quantities, dimensional analysis and its applications.

UNIT II Kinematics

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UNIT III Laws of Motion

Force and Inertia, Newton's First Law of motion; Momentum, Newton's Second Law of motion; Impulse; Newton's Third Law of motion. Law of conservation of linear momentum and its applications, Equilibrium of concurrent forces. Static and Kinetic friction, laws of friction, rolling friction.

Dynamics of uniform circular motion: Centripetal force and its applications.

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UNIT IX Kinetic Theory of Gases

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UNIT XI Electrostatics

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Electric flux, Gauss's law and its applications to find field due to infinitely long, uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Electric potential and its calculation for a point charge, electric dipole and system of charges; Equipotential surfaces, Electrical potential energy of a system of two point charges in an electrostatic field.

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UNIT XIV Electromagnetic Induction and Alternating Currents

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UNIT XVI Optics

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UNIT XVIII Atoms and Nuclei

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UNIT XX Communication Systems

Propagation of electromagnetic waves in the atmosphere; Sky and space wave propagation, Need for modulation, Amplitude and Frequency Modulation, Bandwidth of signals, Bandwidth of Transmission medium, Basic Elements of a Communication System (Block Diagram only)

SECTION B (20% weightage)

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2. Screw gauge - its use to determine thickness/ diameter of thin sheet/wire.
3. Simple Pendulum - dissipation of energy by plotting a graph between square of amplitude and time.
4. Metre Scale - mass of a given object by principle of moments
5. Young's modulus of elasticity of the material of a metallic wire
6. Surface tension of water by capillary rise and effect of detergents
7. Coefficient of Viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body.
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17. Refractive index of a glass slab using a travelling microscope
18. Characteristic curves of a p-n junction diode in forward and reverse bias.
19. Characteristic curves of a Zener diode and finding reverse break down voltage.
20. Characteristic curves of a transistor and finding current gain and voltage gain.
21. Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items.
22. Using multimeter to
    (i) Identify base of a transistor
    (ii) Distinguish between npn and pnp type transistor
    (iii) See the unidirectional flow of current in case of a diode and an LED.
    (iv) Check the correctness or otherwise of a given electronic component (diode, transistor or IC).

JEE ADVANCED

General

Units and dimensions, dimensional analysis, least count, significant figures, Methods of measurement and error analysis for physical quantities pertaining to the following experiments, Experiments based on using vernier calipers and screw gauge (micrometer), Determination of g using simple pendulum, Young's modulus by Searle's method, Specific heat of a liquid using calorimeter, focal length of a concave mirror and a convex lens using u-v method, Speed of sound using resonance column, Verification of Ohm's law using voltmeter and ammeter, and specific resistance of the material of a wire using meter bridge and post office box.

Mechanics

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Newton's Laws of Motion, Inertial and uniformly accelerated frames of reference, Static and dynamic friction, Kinetic and potential energy, Work and power, Conservation of linear momentum and mechanical energy.

Systems of Particles, Centre of mass and its motion, Impulse, Elastic and inelastic collisions.

Law of Gravitation, Gravitational potential and field, Acceleration due to gravity, Motion of planets and satellites in circular orbits, Escape velocity.

Rigid body, moment of inertia, parallel and perpendicular axes theorems, moment of inertia of uniform bodies with simple geometrical shapes, Angular momentum, Torque, Conservation of angular momentum, Dynamics of rigid bodies with fixed axis of rotation, Rolling without slipping of rings, cylinders and spheres, Equilibrium of rigid bodies, Collision of point masses with rigid bodies.

Linear and angular simple harmonic motions.

Hooke's law, Young's modulus.

Pressure in a fluid, Pascal's law, Buoyancy, Surface energy and surface tension, capillary rise, Viscosity (Poiseuille's equation excluded), Stoke's law, Terminal velocity, Streamline flow, Equation of continuity, Bernoulli's theorem and its applications.

Wave motion (plane waves only), longitudinal and transverse waves, Superposition of waves; progressive and stationary waves, Vibration of strings and air columns. Resonance, Beats, Speed of sound in gases, Doppler effect (in sound).

Thermal Physics

Thermal expansion of solids, liquids and gases, Calorimetry, latent heat, Heat conduction in one dimension, Elementary concepts of convection and radiation, Newton's law of cooling, Ideal gas laws, Specific heats (C_v and C_p for monatomic and diatomic gases), Isothermal and adiabatic processes, bulk modulus of gases, Equivalence of heat and work, First law of thermodynamics and its applications (only for ideal gases). Blackbody radiation, absorptive and emissive powers, Kirchhoff's law, Wien's displacement law, Stefan's law.

Electricity and Magnetism

Coulomb's law, Electric field and potential, Electrical Potential energy of a system of point charges and of electrical dipoles in a uniform electrostatic field, Electric field lines, Flux of electric field; Gauss's law and its application in simple cases, such as, to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Capacitance, Parallel plate capacitor with and without dielectrics, Capacitors in series and parallel, Energy stored in a capacitor.

Electric Current, Ohm's law, Series and parallel arrangements of resistances and cells, Kirchhoff's laws and simple applications, Heating effect of current.

Biot-Savart law and Ampere's law, magnetic field near a current-carrying straight wire, along the axis of a circular coil and inside a long straight solenoid, Force on a moving charge and on a current-carrying wire in a uniform magnetic field.

Magnetic Moment of a Current Loop, Effect of a uniform magnetic field on a current loop, Moving coil galvanometer, voltmeter, ammeter and their conversions.

Electromagnetic induction, Faraday's law, Lenz's law, Self and mutual inductance, RC, LR and LC circuits with DC and AC sources.

Optics

Rectilinear propagation of light, Reflection and refraction at plane and spherical surfaces, Total internal reflection, Deviation and dispersion of light by a prism, Thin lenses, Combinations of mirrors and thin lenses, Magnification.

Wave Nature of Light, Huygen's principle, interference limited to Young's double-slit experiment.

Modern Physics, Atomic nucleus, Alpha, beta and gamma radiations, Law of radioactive decay, Decay constant, Half-life and mean life, Binding energy and its calculation, Fission and fusion processes, Energy calculation in these processes.

Photoelectric Effect, Bohr's theory of hydrogen-like atoms, Characteristic and continuous X-rays, Moseley's law, de Broglie wavelength of matter waves.

DEDICATION

This book is dedicated to my
honourable grandfather

(LATE) SH. PITAMBER PANDEY

a Kumaoni poet; resident of village Dhaura
(Almora) Uttarakhand

1

General Physics

Topic 1 Units and Dimensions

Objective Questions I (Only one correct option)

1. Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)? **(2019 Main, 12 April II)**

(a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\frac{\mu_0}{\varepsilon_0}$ (c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{\varepsilon_0}{\mu_0}$

2. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? **(2019 Main, 10 April II)**

(a) $[M^{-1}L^{-2}T^4A^2]$ (b) $[M^{-2}L^0T^{-4}A^{-2}]$
(c) $[M^{-3}L^{-2}T^8A^4]$ (d) $[M^{-2}L^{-2}T^6A^3]$

3. If surface tension (S), moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be **(2019 Main, 8 April II)**

(a) $S^{1/2}I^{1/2}h^{-1}$ (b) $S^{3/2}I^{1/2}h^0$
(c) $S^{1/2}I^{1/2}h^0$ (d) $S^{1/2}I^{3/2}h^{-1}$

4. In SI units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is **(2019 Main, 11 April I)**

(a) $[A^{-1}TML^3]$ (b) $[AT^2M^{-1}L^{-1}]$
(c) $[AT^{-3}ML^{3/2}]$ (d) $[A^2T^3M^{-1}L^{-2}]$

5. Let l, r, c, and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{l}{rcv}$ in SI units will be **(2019 Main, 12 Jan I)**

(a) $[LT^2]$ (b) $[LTA]$ (c) $[A^{-1}]$ (d) $[LA^{-2}]$

6. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be **(2019 Main, 11 Jan II)**

(a) $[V^{-4}A^{-2}F]$ (b) $[V^{-2}A^2F^2]$
(c) $[V^{-2}A^2F^{-2}]$ (d) $[V^{-4}A^2F]$

7. The force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is **(2019 Main, 11 Jan I)**

(a) $[MLT^{-2}]$ (b) $[M^0L^2T^{-4}]$ (c) $[M^2LT^{-4}]$ (d) $[M^2L^2T^{-2}]$

8. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is **(2019 Main, 10 Jan I)**

(a) 40 (b) 16 (c) 640 (d) 410

9. In form of G (universal gravitational constant), h (Planck constant) and c (speed of light), the time period will be proportional to **(2019 Main, 11 Jan II)**

(a) $\sqrt{\frac{Gh}{c^5}}$ (b) $\sqrt{\frac{hc^5}{G}}$ (c) $\sqrt{\frac{c^3}{Gh}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

10. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = Time and A = electric current, then **(2013 Main)**

(a) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$ (b) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$
(c) $[\varepsilon_0] = [M^{-2}L^2T^{-1}A^{-2}]$ (d) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^2]$

11. Which of the following sets have different dimensions? **(2005, 2M)**

(a) Pressure, Young's modulus, Stress
(b) Emf, Potential difference, Electric potential
(c) Heat, Work done, Energy
(d) Dipole moment, Electric flux, Electric field

12. In the relation, $p = \frac{\alpha}{\beta} e^{-\frac{\alpha Z}{k\theta}}$

p is pressure, Z is distance, k is Boltzmann's constant and θ is the temperature. The dimensional formula of β will be **(2004, 2M)**

(a) $[M^0L^2T^0]$ (b) $[ML^2T]$
(c) $[ML^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

13. A quantity X is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of **(2001, 2M)**

(a) resistance (b) charge (c) voltage (d) current

14. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 : permittivity of free space; E : electric field) is **(2000, 2M)**

(a) $[MLT^{-1}]$ (b) $[ML^2 T^{-2}]$
(c) $[MLT^{-2}]$ (d) $[ML^2 T^{-1}]$

15. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction, respectively. What are the dimensions of Y in MKSQ system? **(1995, 2M)**

(a) $[M^{-3}L^{-1}T^3Q^4]$
(b) $[M^{-3}L^{-2}T^4Q^4]$
(c) $[M^{-2}L^{-2}T^4Q^4]$
(d) $[M^{-3}L^{-2}T^4Q]$

Match the Columns

16. Match Column I with Column II and select the correct answer using the codes given below the lists. **(2013 Main)**

	Column I		Column II
A.	Boltzmann's constant	p.	$[ML^2T^{-1}]$
B.	Coefficient of viscosity	q.	$[ML^{-1}T^{-1}]$
C.	Planck's constant	r.	$[MLT^{-3}K^{-1}]$
D.	Thermal conductivity	s.	$[ML^2T^{-2}K^{-1}]$

17. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II.** **(2007, 6M)**

	Column I		Column II
(A)	GM_eM_s G — universal gravitational constant, M_e — mass of the earth, M_s — mass of the sun.	(p)	(volt) (coulomb) (metre)
(B)	$\frac{3RT}{M}$ R — universal gas constant, T — absolute temperature, M — molar mass.	(q)	(kilogram) $(\text{metre})^3$ $(\text{second})^{-2}$
(C)	$\frac{F^2}{q^2B^2}$ F — force, q — charge, B — magnetic field.	(r)	$(\text{metre})^2$ $(\text{second})^{-2}$
(D)	$\frac{GM_e}{R_e}$ G — universal gravitational constant, M_e — mass of the earth, R_e — radius of the earth.	(s)	(farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

18. Match the physical quantities given in **Column I** with dimensions expressed in terms of mass (M), length (L), time (T), and charge (Q) given in **Column II** and write the correct answer against the matched quantity in a tabular form in your answer book. **(1993, 6 M)**

Column I	Column II
Angular momentum	$[ML^2T^{-2}]$
Latent heat	$[ML^2Q^{-2}]$
Torque	$[ML^2T^{-1}]$
Capacitance	$[ML^3T^{-1}Q^{-2}]$
Inductance	$[M^{-1}L^{-2}T^2Q^2]$
Resistivity	$[L^2T^{-2}]$

19. **Column I** gives three physical quantities. Select the appropriate units for the choices given in **Column II**. Some of the physical quantities may have more than one choice. **(1990, 3M)**

Column I	Column II
Capacitance	ohm-second
Inductance	Coulomb^2-joule^{-1}
Magnetic induction	Coulomb $(\text{volt})^{-1}$, Newton (ampere metre)$^{-1}$, volt-second (ampere)$^{-1}$

Objective Questions II (One or more correct option)

20. A length-scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann's constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression (s) for l is (are) dimensionally correct? **(2016 Adv.)**

(a) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (b) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(c) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (d) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

21. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then, the correct options is/are **(2015 Adv.)**

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$ (c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

22. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equations is/are **(2015 Adv.)**

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\varepsilon_0 I = \mu_0 V$
(c) $I = \varepsilon_0 cV$ (d) $\mu_0 cI = \varepsilon_0 V$

23. The SI unit of the inductance, the henry can by written as **(1998, 2M)**

(a) weber/ampere (b) volt-second/ampere
(c) joule/$(\text{ampere})^2$ (d) ohm-second

24. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current. **(1998, 2M)**

(a) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 I]$ (b) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 I^2]$

(c) $[\mu_0] = [MLT^{-2} I^{-2}]$ (d) $[\mu_0] = [ML^2 T^{-1} I]$

25. The pairs of physical quantities that have the same dimensions is (are) **(1995, 2M)**

(a) Reynolds number and coefficient of friction

(b) Curie and frequency of a light wave

(c) latent heat and gravitational potential

(d) Planck's constant and torque

26. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair (s). **(1986, 2M)**

(a) Torque and work

(b) Angular momentum and work

(c) Energy and Young's modulus

(d) Light year and wavelength

27. L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. The combinations which have the dimensions of frequency are **(1984, 2M)**

(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

Passage Based Questions

Paragraph

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, $[\mathbf{E}]$ and $[\mathbf{B}]$ stand for dimensions of electric and magnetic fields respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space, respectively. $[L]$ and $[T]$ are dimensions of length and time, respectively. All the quantities are given in SI units.

(*There are two questions based on PARAGRAPH "X", the question given below is one of them*) **(2018 Adv.)**

28. The relation between $[E]$ and $[B]$ is

(a) $[E] = [B]\ [L]\ [T]$ (b) $[E] = [B][L]^{-1}[T]$

(c) $[E] = [B][L][T]^{-1}$ (d) $[E] = [B][L]^{-1}[T]^{-1}$

29. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

(a) $[\mu_0] = [\varepsilon_0][L]^2[T]^{-2}$ (b) $[\mu_0] = [\varepsilon_0][L]^{-2}[T]^2$

(c) $[\mu_0] = [\varepsilon_0]^{-1}[L]^2[T]^{-2}$ (d) $[\mu_0] = [\varepsilon_0]^{-1}[L]^{-2}[T]^2$

Integer Answer Type Question

30. To find the distance d over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is **(2014 Adv.)**

Fill in the Blanks

31. The equation of state of a real gas is given by **(1997, 2M)**

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

where p, V and T are pressure, volume and temperature, respectively and R is the universal gas constant. The dimensions of the constant a in the above equation is …… .

32. The dimensions of electrical conductivity is ……… . **(1997, 1M)**

33. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction, respectively. The dimensions of Y in MKSQ system are ……… . **(1988, 2M)**

34. Planck's constant has dimensions ……… . **(1985, 2M)**

Analytical & Descriptive Questions

35. Write the dimensions of the following in terms of mass, time, length and charge. **(1982, 2M)**

(a) Magnetic flux (b) Rigidity modulus

36. A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a\ d^b\ E^c$, where p is the static pressure, d is the density of water and E is the total energy of the explosion. Find the values of a, b and c. **(1981, 3M)**

37. Give the MKS units for each of the following quantities. **(1980, 3M)**

(a) Young's modulus

(b) Magnetic induction

(c) Power of a lens

Topic 2 Significant Figures and Error Analysis

Objective Questions I (Only one correct option)

1. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is **(2019 Main, 9 April II)**

(a) 37.030 cm^2 (b) 37.0 cm^2

(c) 37.03 cm^2 (d) 37 cm^2

2. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is **(2019 Main, 9 April I)**

(a) 0.01 kg/m^3 (b) 0.10 kg/m^3

(c) 0.07 kg/m^3 (d) 0.31 kg/m^3

3. In a simple pendulum, experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained 55.0 cm. The percentage error in the determination of g is close to **(2019 Main, 8 April II)**

(a) 0.7% (b) 6.8% (c) 3.5% (d) 0.2%

4. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ? **(2019 Main, 10 Jan II)**

(a) $4300 \pm 80\,\text{cm}^3$ (b) $4260 \pm 80\,\text{cm}^3$

(c) $4264.4 \pm 81.0\,\text{cm}^3$ (d) $4264 \pm 81\,\text{cm}^3$

5. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1% , the maximum error in determining the density is **(2018 Main)**

(a) 6% (b) 2.5% (c) 3.5% (d) 4.5%

6. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ s and he measures the depth of the well to be $L = 20$m. Take the acceleration due to gravity $g = 10\,\text{ms}^{-2}$ and the velocity of sound is 300 ms^{-1}. Then the fractional error in the measurement, $\frac{\delta L}{L}$, is closest to **(2017 Adv.)**

(a) 1% (b) 5% (c) 3% (d) 0.2%

7. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is **(2015 Main)**

(a) 3% (b) 2% (c) 1% (d) 5%

8. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **(2014 Main)**

(a) 0.2 mA (b) 0.02 mA (c) 0.5 mA (d) 0.05 mA

9. A wire has a mass (0.3 ± 0.003) g, radius (0.5 ± 0.005) mm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of its density is **(2004, 2M)**

(a) 1 (b) 2 (c) 3 (d) 4

10. A cube has a side of length 1.2×10^{-2}m. Calculate its volume. **(2003, 2M)**

(a) $1.7 \times 10^{-6}\ \text{m}^3$ (b) $1.73 \times 10^{-6}\ \text{m}^3$

(c) $1.70 \times 10^{-6}\ \text{m}^3$ (d) $1.732 \times 10^{-6}\ \text{m}^3$

Integer Answer Type Question

11. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\,\text{s}^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is **(2015 Adv.)**

Topic 3 Experimental Physics

Objective Questions I (Only one correct option)

1. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is **(2019 Main, 12 Jan I)**

(a) 50 (b) 200 (c) 500 (d) 100

2. The pitch and the number of divisions, on the circular scale for a given screw gauge are 0.5 mm and 100, respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale for a thin sheet are 5.5 mm and 48 respectively, the thickness of this sheet is **(2019 Main, 9 Jan II)**

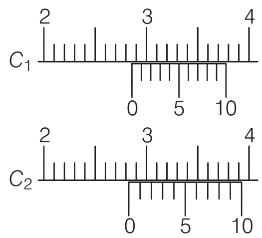
(a) 5.950 mm (b) 5.725 mm

(c) 5.755 mm (d) 5.740 mm

3. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet, if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? **(2016 Main)**

(a) 0.75 mm (b) 0.80 mm (c) 0.70 mm (d) 0.50 mm

4. There are two vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively, are **(2016 Adv.)**



(a) 2.87 and 2.87 (b) 2.87 and 2.83

(c) 2.85 and 2.82 (d) 2.87 and 2.86

5. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(a) A meter scale **(2014 Main)**

(b) A vernier caliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm

(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

(d) A screw gauge having 50 divisions in the circular scale

6. The diameter of a cylinder is measured using a vernier calipers with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm. The 24th division of the vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is **(2013 Adv.)**

(a) 5.112 cm (b) 5.124 cm (c) 5.136 cm (d) 5.148 cm

7. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement is **(2012)**

(a) due to the errors in the measurements of d and l are the same

(b) due to the error in the measurement of d is twice that due to the error in the measurement of l

(c) due to the error in the measurement of l is twice that due to the error in the measurement of d

(d) due to the error in the measurement of d is four times that due to the error in the measurement of l

8. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is **(2011)**

(a) 0.9% (b) 2.4% (c) 3.1% (d) 4.2%

9. A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier calipers, the least count is **(2010)**

(a) 0.02 mm (b) 0.05 mm (c) 0.1 mm (d) 0.2 mm

10. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum.

They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm, Least count for time = 0.1 s

Student	Length of the pendulum (cm)	Number of oscillations (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_{I}, E_{II} and E_{III} are the percentage errors in g, i.e. $\left(\frac{\Delta g}{g} \times 100\right)$ for students I, II and III, respectively **(2008, 3M)**

(a) $E_{\text{I}} = 0$ (b) E_{I} is minimum (c) $E_{\text{I}} = E_{\text{II}}$ (d) E_{II} is maximum

11. In the experiment to determine the speed of sound using a resonance column **(2007, 3M)**

(a) prongs of the tuning fork are kept in a vertical plane

(b) prongs of the tuning fork are kept in a horizontal plane

(c) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air

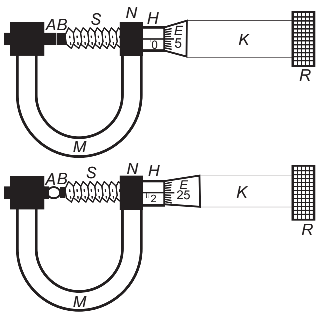
(d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

12. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8 \text{ m/s}^2$ (exact). The Young's modulus obtained from the reading is close to **(2007, 3M)**

(a) $(2.0 \pm 0.3) 10^{11}$ N/m^2 (b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2

(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2 (d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

13. The circular scale of a screw gauge has 50 divisions and pitch of 0.5 mm. Find the diameter of sphere. Main scale reading is 2. **(2006, 3M)**



(a) 1.2 mm (b) 1.25 mm

(c) 2.20 mm (d) 2.25 mm

Objective Questions II (One or more correct option)

14. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 , 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is (are) true? **(2016 Adv.)**

(a) The error in the measurement of r is 10%
(b) The error in the measurement of T is 3.57%
(c) The error in the measurement of T is 2%
(d) The error in the measurement of g is 11%

15. Consider a vernier caliper in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the vernier callipers, 5 divisions of the vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then **(2015 Adv.)**

(a) if the pitch of the screw gauge is twice the least count of the vernier caliper, the least count of the screw gauge is 0.01 mm
(b) if the pitch of the screw gauge is twice the least count of the Vernier caliper, the least count of the screw gauge is 0.05 mm
(c) if the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm
(d) if the least count of the linear scale of the screw gauge is twice the least count of the vernier caliper, the least count of the screw gauge is 0.005 mm.

16. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statement(s) is/are true? **(2010)**

(a) Error ΔT in measuring T, the time period, is 0.05 s
(b) Error ΔT in measuring T, the time period, is 1 s
(c) Percentage error in the determination of g is 5%
(d) Percentage error in the determination of g is 2.5%

Numerical Value Based Questions

17. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass m. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is .... (Take, $g = 10$ ms^{-2} and $\pi = 3.2$). **(2018 Adv.)**

Integer Answer Type Question

18. During Searle's experiment, zero of the vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is **(2014 Adv.)**

Analytical & Descriptive Questions

19. The edge of a cube is measured using a vernier caliper. (9 divisions of the main scale is equal to 10 divisions of vernier scale and 1 main scale division is 1 mm). The main scale division reading is 10 and 1 division of vernier scale was found to be coinciding with the main scale. The mass of the cube is 2.736 g. Calculate the density in g/cm^3 upto correct significant figures. **(2005, 2M)**

20. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in appropriate number of significant figures. **(2004, 2M)**

21. In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data. **(2004, 2M)**

22. N divisions on the main scale of a vernier calipers coincide with $(N + 1)$ divisions on the vernier scale. If each division on the main scale is of a units, determine the least count of instrument. **(2003, 2M)**

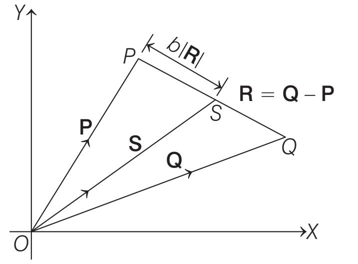
Topic 4 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{p}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point of the surface of the expanding sphere is proportional to **(2017 Adv.)**

(a) R
(b) $\frac{1}{R}$
(c) R^3
(d) $R^{\frac{2}{3}}$

2. Three vectors $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are shown in the figure. Let S be any point on the vector $\mathbf{R}$. The distance between the points P and S is $b\,[\mathbf{R}]$. The general relation among vectors $\mathbf{P}, \mathbf{Q}$ and $\mathbf{S}$ is **(2017 Adv.)**



(a) $\mathbf{S} = (1-b^2)\,\mathbf{P} + b\mathbf{Q}$ (b) $\mathbf{S} = (b-1)\,\mathbf{P} + b\mathbf{Q}$

(c) $\mathbf{S} = (1-b)\,\mathbf{P} + b\mathbf{Q}$ (d) $\mathbf{S} = (1-b)\,\mathbf{P} + b^2\mathbf{Q}$

Passage Based Questions

Paragraph

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So, the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$$

The above derivation makes the assumption that $\Delta x/x \ll 1, \Delta y/y \ll 1$. Therefore, the higher powers of these quantities are neglected.

(*There are two questions based on PARAGRAPH ''A'', the question given below is one of them*) **(2018 Adv.)**

3. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa $(\Delta a/a \ll 1)$, then what is the error Δr in determining r?

(a) $\frac{\Delta a}{(1+a)^2}$ (b) $\frac{-2\Delta a}{(1+a)^2}$ (c) $\frac{2\Delta a}{(1-a)^2}$ (d) $\frac{2a\Delta a}{(1-a^2)}$

4. In an experiment, the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0s. For $|x| \ll 1$, $\ln(1+x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ in s^{-1}, is

(a) 0.04 (b) 0.03

(c) 0.02 (d) 0.01

Passage

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ω_p, which is called the plasma frequency.

To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. **(2011)**

5. Taking the electronic charge as e and the permittivity as ε_0, use dimensional analysis to determine the correct expression for ω_p.

(a) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\frac{m\varepsilon_0}{Ne}}$

(c) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

6. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}\ m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

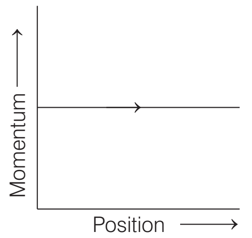
(a) 800 nm (b) 600 nm

(c) 300 nm (d) 200 nm

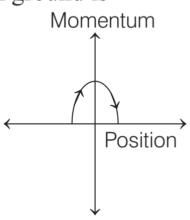
Passage

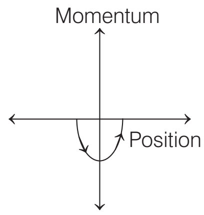
Phase space diagrams are useful tools in analysing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis.

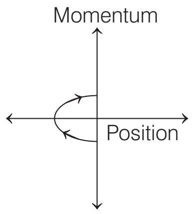
The phase space diagram is $x(t)$ vs $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative. **(2011)**



7. The phase space diagram for a ball thrown vertically up from ground is

(a)


(b)


(c)


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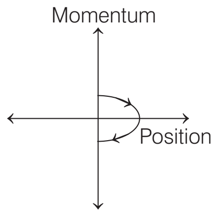
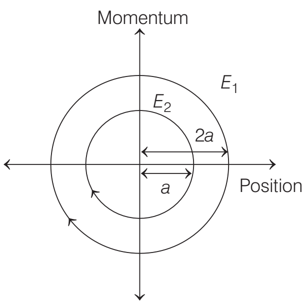
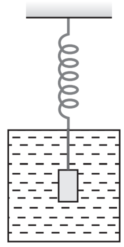


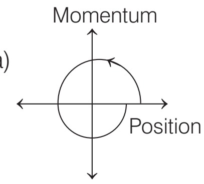
8. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then,

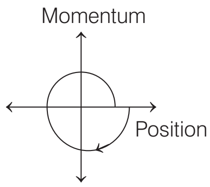


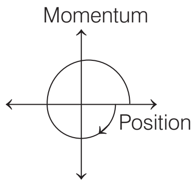
(a) $E_1 = \sqrt{2} E_2$ (b) $E_1 = 2E_2$
(c) $E_1 = 4E_2$ (d) $E_1 = 16E_2$

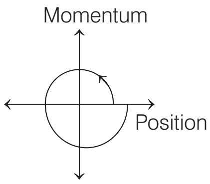
9. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(a)


(b)


(c)


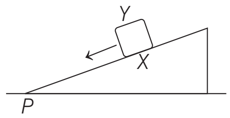
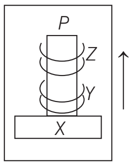
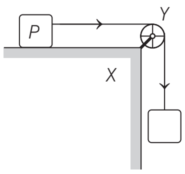
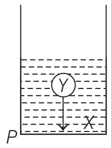
(d)


Match the Columns

10. **Column II** gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process (es) from **Column II**. **(2009)**

	Column I		Column II
(A)	The energy of the system is increased.	(p)	System : A capacitor, initially uncharged. Process : It is connected to a battery.
(B)	Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	(q)	System : A gas in an adiabatic container fitted with an adiabatic piston. Process : The gas is compressed by pushing the piston.
(C)	Internal energy of the system is converted into its mechanical energy.	(r)	System : A gas in a rigid container. Process : The gas gets cooled due to colder atmosphere surrounding it.
(D)	Mass of the system is decreased.	(s)	System : A heavy nucleus, initially at rest. Process : The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted.
		(t)	System : A resistive wire loop. Process : The loop is placed in a time varying magnetic field perpendicular to its plane.

11. **Column II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II** **(2009)**

Column I		Column II
(A) The force exerted by X on Y has a magnitude Mg.		(p) Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.
(B) The gravitational potential energy of X is continuously increasing.		(q) Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.
(C) Mechanical energy of the system $X + Y$ is continuously decreasing.		(r) A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.
(D) The torque of the weight of Y about point P is zero.		(s) A sphere Y of mass M is put in a non-viscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.
		(t) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

Answers

Topic 1

1. (a) **2.** (c) **3.** (c) **4.** (d)
5. (c) **6.** (d) **7.** (c) **8.** (a)
9. (a) **10.** (b) **11.** (d) **12.** (a)
13. (d) **14.** None of the four choices **15.** (b)
16. A–s, B–q, C–p, D–r **17.** A–p, q, B–r, s, C–r, s, D–r, s
18. See the solution
19. See the solution **20.** (b, d)
21. (a, c, d) **22.** (a, c)
23. (a, b, c, d) **24.** (b, c)
25. (a, b, c) **26.** (a, d)
27. (a, b,c) **28.** (c)
29. (d) **30.** (3)
31. $[ML^5T^{-2}]$ **32.** $[M^{-1}L^{-3}T^3A^2]$
33. $[M^{-3}L^{-3}T^4Q^4]$ **34.** $[ML^2T^{-1}]$
35. (a) $[ML^2T^{-1}Q^{-1}]$ (b) $[ML^{-1}T^{-2}]$
36. $a = \frac{-5}{6}, b = \frac{1}{2}, c = \frac{1}{3}$ **37.** (a) N/m^2 (b) Tesla (c) m^{-1}

Topic 2

1. (c) **2.** (*) **3.** (b) **4.** (b)
5. (d) **6.** (a) **7.** (a) **8.** (a)
9. (d) **10.** (a) **11.** $\pm$ 4%

Topic 3

1. (b) **2.** (c) **3.** (b) **4.** (b)
5. (b) **6.** (b)
7. (c) **8.** (c) **9.** (d) **10.** (b)
11. (a) **12.** (b) **13.** (a) **14.** (a, b, d)
15. (b, c) **16.** (a, c) **17.** (3) **18.** (4)
19. (2.66 g/cm^3)
20. (2.6 cm^2) **21.** $1.09 \times 10^{10} \text{ N/m}^2$ **22.** $\frac{a}{N+1}$

Topic 4

1. (a) **2.** (c) **3.** (b) **4.** (c)
5. (c) **6.** (b) **7.** (d) **8.** (c)
9. (b) **10.** A–p, q, s, t, B–q, C–s, D–s
11. A–p, t, B–q, s, t, C–p, r, s, t, D–q

Hints & Solutions

Topic 1 Units and Dimensions

1 **Key Idea** A formula is valid only, if the dimensions of LHS and RHS are same. So, we need to balance dimensions of given options with the dimension of electrical resistance.

Let dimensions of resistance R, permittivity ε_0 and permeability μ_0 are [R], $[\varepsilon_0]$ and $[\mu_0]$, respectively.

So, $$[R]=[\varepsilon_0]^{\alpha}[\mu_0]^{\beta} \quad \ldots(\text{i})$$

$$[R]=[\mathrm{M}^1\,\mathrm{L}^2\,\mathrm{T}^{-3}\,\mathrm{A}^{-2}],\ [\varepsilon_0]=[\mathrm{M}^{-1}\,\mathrm{L}^{-3}\,\mathrm{T}^4\mathrm{A}^2],$$

$$[\mu_0]=[\mathrm{M}^1\,\mathrm{L}^1\,\mathrm{T}^{-2}\,\mathrm{A}^{-2}]$$

Now, from Eq. (i), we get

$$[\mathrm{M}^1\,\mathrm{L}^2\,\mathrm{T}^{-3}\,\mathrm{A}^{-2}]=[\mathrm{M}^{-1}\,\mathrm{L}^{-3}\,\mathrm{T}^4\mathrm{A}^2]^{\alpha}\,[\mathrm{M}^1\,\mathrm{L}^1\,\mathrm{T}^{-2}\,\mathrm{A}^{-2}]^{\beta}$$

$$[\mathrm{M}^1\,\mathrm{L}^2\,\mathrm{T}^{-3}\,\mathrm{A}^{-2}]=\mathrm{M}^{-\alpha+\beta}\mathrm{L}^{-3\alpha+\beta}\,\mathrm{T}^{4\alpha-2\beta}\mathrm{A}^{2\alpha-2\beta}$$

On comparing both sides, we get

$$-\alpha+\beta=1 \quad \ldots(\text{ii})$$

$$-3\alpha+\beta=2 \quad \ldots(\text{iii})$$

$$4\alpha-2\beta=-3 \quad \ldots(\text{iv})$$

$$2\alpha-2\beta=-2 \quad \ldots(\text{v})$$

Value of α and β can be found using any two Eqs. from (ii) to (v),

On subtracting Eq. (iii) from Eq. (ii), we get

$$(-\alpha+\beta)-(-3\alpha+\beta)=1-2$$

$$\Rightarrow \quad 2\alpha=-1 \text{ or } \alpha=\frac{-1}{2}$$

Put the value of α in Eq. (ii), we get

$$\beta=+\frac{1}{2}$$

$$\therefore \quad [R]=[\varepsilon_0]^{-1/2}\,[\mu_0]^{1/2}=\left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$$

2. To find dimensions of capacitance in the given relation, we can use formula for energy.

Capacitors energy is

$$U=\frac{1}{2}CV^2$$

So, dimensionally,

$$\Rightarrow \quad [C]=\left[\frac{U}{V^2}\right]$$

As, V = potential = $\dfrac{\text{potential energy}}{\text{charge}}$

We have,

$$[C]=\frac{[U]}{\left[\dfrac{U^2}{q^2}\right]}=\frac{[q^2]}{[U]}=\left[\frac{\mathrm{A}^2\mathrm{T}^2}{\mathrm{ML}^2\mathrm{T}^{-2}}\right]$$

$$\therefore \quad X=[\mathrm{M}^{-1}\mathrm{L}^{-2}\mathrm{A}^2\mathrm{T}^4]$$

To get dimensions of magnetic field, we use force on a current carrying conductor in magnetic field,

$$F=BIl \Rightarrow [B]=\frac{[\mathrm{F}]}{[\mathrm{I}][\mathrm{l}]}=\left[\frac{\mathrm{MLT}^{-2}}{\mathrm{AL}}\right]$$

$$\therefore \quad Z=[\mathrm{M}^1\mathrm{L}^0\mathrm{T}^{-2}\mathrm{A}^{-1}]$$

Now, using given relation,

$$X=5YZ^2$$

$$[Y]=\frac{[X]}{[Z^2]}$$

$$=\left[\frac{\mathrm{M}^{-1}\mathrm{L}^{-2}\mathrm{A}^2\mathrm{T}^4}{(\mathrm{M}^1\mathrm{L}^0\mathrm{T}^{-2}\mathrm{A}^{-1})^2}\right]=\frac{\mathrm{M}^{-1}\mathrm{L}^{-2}\mathrm{A}^2\mathrm{T}^4}{\mathrm{M}^2\mathrm{T}^{-4}\mathrm{A}^{-2}}$$

$$\therefore \quad [Y]=[\mathrm{M}^{-3}\mathrm{L}^{-2}\mathrm{A}^4\mathrm{T}^8]$$

3 Suppose, linear momentum (p) depends upon the Planck's constant (h) raised to the power (a), surface tension(S) raised to the power (b) and moment of inertia (I) raised to the power (c).

Then,

$$p \propto (h)^a\,(S)^b\,(I)^c$$

or $$p=kh^aS^bI^c$$

where, k is a dimensionless proportionality constant.

Thus, $$[\mathrm{p}]=[\mathrm{h}]^a[\mathrm{S}]^b[\mathrm{I}]^c \quad \ldots(\text{i})$$

Then, the respective dimensions of the given physical quantities, i.e.

$$[\mathrm{p}]=[\text{mass}\times\text{velocity}]=[\mathrm{MLT}^{-1}]$$

$$[\mathrm{I}]=[\text{mass}\times\text{distance}^2]$$

$$=[\mathrm{ML}^2\mathrm{T}^0]$$

$$[\mathrm{S}]=[\text{force}\times\text{length}]=[\mathrm{ML}^0\mathrm{T}^{-2}]$$

$$[\mathrm{h}]=[\mathrm{ML}^2\mathrm{T}^{-1}]$$

Then, substituting these dimensions in Eq. (i), we get

$$[\mathrm{MLT}^{-1}]=[\mathrm{ML}^2\mathrm{T}^{-1}]^a\,[\mathrm{MT}^{-2}]^b\,[\mathrm{ML}^{+2}]^c$$

For dimensional balance, the dimensions on both sides should be same.

Thus, equating dimensions, we have

$$a+b+c=1$$

$$2(a+c)=1 \quad \text{or} \quad a+c=\frac{1}{2}$$

$$-a-2b=-1 \quad \text{or} \quad a+2b=1$$

Solving these three equations, we get

$$a=0,\ b=\frac{1}{2},\ c=\frac{1}{2}$$

$$\therefore \quad p=h^0S^{\frac{1}{2}}I^{\frac{1}{2}} \quad \text{or} \quad p=S^{\frac{1}{2}}I^{\frac{1}{2}}h^0$$

4 Dimensions of ε_0 (permittivity of free space) are

$$[\varepsilon_0]=\mathrm{M}^{-1}\mathrm{L}^{-3}\mathrm{T}^4\mathrm{A}^2$$

As, c = speed of light.

$\therefore$ Dimension of $[c]=[\mathrm{LT}^{-1}]$

So, dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ are

$$\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = \left[\sqrt{\frac{\varepsilon_0^2}{\varepsilon_0\mu_0}}\right] = [\varepsilon_0 c] \quad \left[\because c^2 = \frac{1}{\mu_0\varepsilon_0}\right]$$

$$= [M^{-1}L^{-3}T^4A^2][LT^{-1}] = [M^{-1}L^{-2}T^3A^2]$$

5 Dimensions of given quantities are

l = inductance = $[M^1L^2T^{-2}A^{-2}]$

r = resistance = $[M^1L^2T^{-3}A^{-2}]$

c = capacitance = $[M^{-1}L^{-2}T^4A^2]$

v = voltage = $[M^1L^2T^{-3}A^{-1}]$

So, dimensions of $\frac{l}{rcv}$ are

$$\left[\frac{l}{rcv}\right] = \frac{[ML^2T^{-2}A^{-2}]}{[M^1L^2T^{-2}A^{-1}]} = [A^{-1}]$$

6 Dimensions of speed are, $[V] = [LT^{-1}]$

Dimensions of acceleration are, $[A] = [LT^{-2}]$

Dimensions of force are, $[F] = [MLT^{-2}]$

Dimension of Young modulus is, $[Y] = [ML^{-1}T^{-2}]$

Let dimensions of Young's modulus is expressed in terms of speed, acceleration and force as;

$$[Y] = [V]^\alpha [A]^\beta [F]^\gamma \qquad \ldots(i)$$

Then substituting dimensions in terms of M, L and T we get,

$$[ML^{-1}T^{-2}] = [LT^{-1}]^\alpha [LT^{-2}]^\beta [MLT^{-2}]^\gamma$$
$$= [M^\gamma L^{\alpha+\beta+\gamma} T^{-\alpha-2\beta-2\gamma}]$$

Now comparing powers of basic quantities on both sides we get,

$$\gamma = 1$$
$$\alpha+\beta+\gamma = -1$$

and $$-\alpha-2\beta-2\gamma = -2$$

Solving these we get;

$$\alpha = -4,\ \beta = 2,\ \gamma = 1$$

Substituting $\alpha, \beta, \& \gamma$ in (i) we get;

$$[Y] = [V^{-4}A^2F^1]$$

7 Force of interaction between two atoms is given as

$$F = \alpha\beta \exp(-x^2/\alpha kT)$$

As we know, exponential terms are always dimensionless, so

dimensions of $\left(\frac{-x^2}{\alpha\, kT}\right) = [M^0L^0T^0]$

$\Rightarrow$ Dimensions of α = Dimension of (x^2/kT)

Now, substituting the dimensions of individual term in the given equation, we get

$$= \frac{[M^0L^2T^0]}{[M^1L^2T^{-2}]} \quad \{\because \text{ Dimensions of } kT \text{ equivalent to the dimensions of energy} = [M^1L^2T^{-2}]\}$$

$$= [M^{-1}L^0T^2] \qquad \ldots(i)$$

Now from given equation, we have dimensions of

F = dimensions of α × dimensions of β

$\Rightarrow$ Dimensions of β = Dimensions of $\left(\frac{F}{\alpha}\right)$

$$= \frac{[M^1L^1T^{-2}]}{[M^{-1}L^0T^2]} = [M^2L^1T^{-4}] \qquad [\because \text{using Eq. (i)}]$$

8 To convert a measured value from one system to another system, we use

$$N_1u_1 = N_2u_2$$

where, N is numeric value and u is unit.

We get

$$128 \cdot \frac{\text{kg}}{\text{m}^3} = N_2 \frac{50\,\text{g}}{(25\text{ cm})^3} \quad \left[\because \text{density} = \frac{\text{mass}}{\text{volume}}\right]$$

$$\Rightarrow \quad \frac{128\times1000\,\text{g}}{100\times100\times100\,\text{cm}^3} = \frac{N_2\times50\,\text{g}}{25\times25\times25\,\text{cm}^3}$$

$$\Rightarrow \quad N_2 = \frac{128\times1000\times25\times25\times25}{50\times100\times100\times100} = 40$$

9

> **Key Idea** According to dimensional analysis, if a physical quantity (let y) depends on another physical quantities (let A, B, C), then the dimension formula is given by
>
> $$y = A^aB^bC^c$$
>
> (where, a, b, c are the power of physical quantity and k = constant)

For the given question, the time (t) depends on the G (gravitational constant), h(Planck's constant) and c (velocity of light), then according to dimensional analysis

$$t = k(G)^a(h)^b(c)^c \qquad \ldots(i)$$

For calculating the values of a, b and c, compare the dimensional formula for both side.

LHS t = time = $[M^0L^0T^1]$

RHS $G = \frac{F\cdot r^2}{m_1m_2} = \frac{\text{kg ms}^{-2}\times\text{m}^2}{(\text{kg})^2} = [M^{-1}L^3T^{-2}]^a$

$h = [M^1L^2T^{-1}]^b$

$c = \frac{d}{t} = [M^0L^1T^{-1}]^c$

Compare both side for powers of M, L and T,

$$M \Rightarrow 0 = -a+b \qquad \ldots(i)$$
$$L \Rightarrow 0 = 3a+2b+c \qquad \ldots(ii)$$
$$T \Rightarrow 1 = -2a-b-c \qquad \ldots(iii)$$

Solving Eqs. (i), (ii), (iii), we get

$$a = \frac{1}{2}, \quad b = \frac{1}{2} \text{ and } c = \frac{-5}{2}$$

So, put these values in Eq. (i)

$$t = k\,G^{1/2}\,h^{1/2}\,c^{-5/2} \Rightarrow t = k\sqrt{\frac{Gh}{c^5}}$$

So, $$t \propto \sqrt{\frac{Gh}{c^5}}$$

10. From Coulomb's law, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2}$, $\varepsilon_0 = \frac{q_1q_2}{4\pi FR^2}$

Substituting the units, we have

$$\varepsilon_0 = \frac{C^2}{\text{N-m}^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$$

11. Dipole moment = (charge) × (distance)

Electric flux = (electric field) × (area)

Hence, the correct option is (d).

12. $$\left[\frac{\alpha Z}{k\theta}\right] = [M^0L^0T^0]$$

$$[\alpha] = \left[\frac{k\theta}{Z}\right]$$

Further $$[p] = \left[\frac{\alpha}{\beta}\right]$$

$\therefore$ $$[\beta] = \left[\frac{\alpha}{p}\right] = \left[\frac{k\theta}{Zp}\right]$$

Dimensions of $k\theta$ are that to energy. Hence,

$$[\beta] = \left[\frac{ML^2T^{-2}}{LML^{-1}T^{-2}}\right]$$

$$= [M^0L^2T^0]$$

13. $C = \dfrac{\Delta q}{\Delta V}$ or $\varepsilon_0 \dfrac{A}{L} = \dfrac{\Delta q}{\Delta V}$ or $\varepsilon_0 = \dfrac{(\Delta q)\,L}{A.(\Delta V)}$

$$X = \varepsilon_0 L \frac{\Delta V}{\Delta t} = \frac{(\Delta q)L}{A\,(\Delta V)} L \frac{\Delta V}{\Delta t}$$

but $[A] = [L^2]$

$\therefore$ $X = \dfrac{\Delta q}{\Delta t}$ = current

14. (None of the four choices) $\frac{1}{2}\varepsilon_0 E^2$ is the expression of energy density (Energy per unit volume)

$$\left[\frac{1}{2}\varepsilon_0 E^2\right] = \left[\frac{ML^2\ T^{-2}}{L^3}\right] = [ML^{-1}\ T^{-2}]$$

NOTE From this question, students can take a lesson that in IIT-JEE, also questions may be wrong or there may be no correct answer in the given choices. So, don't waste time if you are confident.

15. $$[Y] = \left[\frac{X}{Z^2}\right] = \left[\frac{\text{Capacitance}}{(\text{Magnetic induction})^2}\right]$$

$$= \left[\frac{M^{-1}L^{-2}Q^2T^2}{M^2Q^{-2}T^{-2}}\right] = [M^{-3}L^{-2}T^4Q^4]$$

16. (A) $$U = \frac{1}{2}kT$$

$\Rightarrow$ $[ML^2T^{-2}] = [k]\,K$

$\Rightarrow$ $[K] = [ML^2T^{-2}K^{-1}]$

(B) $$F = \eta A \frac{dv}{dx}$$

$\Rightarrow$ $$[\eta] = \frac{[MLT^{-2}]}{[L^2LT^{-1}L^{-1}]} = [ML^{-1}\ T^{-1}]$$

(C) $$E = h\nu$$

$\Rightarrow$ $[ML^2T^2] = [h][T^{-1}] \Rightarrow [h] = [ML^2T^{-1}]$

(D) $$\frac{dQ}{dt} = \frac{k\,A\Delta\theta}{l}$$

$\Rightarrow$ $$[k] = \frac{[ML^2T^{-3}L]}{[L^2K]} = [MLT^{-3}K^{-1}]$$

17. (A) $F = \dfrac{GM_eM_s}{r^2}$

= Gravitational force between sun and earth

$\Rightarrow$ $GM_eM_s = Fr^2$

$\therefore$ $[GM_eM_s] = [Fr^2] = [MLT^{-2}][L^2] = [ML^3T^{-2}]$

(B) $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ = rms speed of gas molecules

$\therefore$ $\dfrac{3RT}{M} = v_{rms}^2$

or $\left[\dfrac{3RT}{M}\right] = [v_{rms}^2] = [LT^{-1}]^2 = [L^2T^{-2}]$

(C) $F = Bqv$ = magnetic force on a charged particle

$\therefore$ $\dfrac{F}{Bq} = v$ or $\left[\dfrac{F^2}{B^2q^2}\right] = [v]^2 = [L^2T^{-2}]$

(D) $v_o = \sqrt{\dfrac{GM_e}{R_e}}$ = orbital velocity of earth's satellite

$\therefore$ $\dfrac{GM_e}{R_e} = v_o^2$ or $\left[\dfrac{GM_e}{R_e}\right] = [v_o^2] = [L^2T^{-2}]$

(p) $W = qV \Rightarrow$ (Coulomb) (Volt) = Joule

or [(Volt) (Coulomb) (Metre)] = [(Joule) (Metre)]

$= [ML^2T^{-2}][L] = [ML^3T^{-2}]$

(q) $[(\text{kilogram})\,(\text{metre})^3\,(\text{second})^{-2}] = [ML^3T^{-2}]$

(r) $[(\text{metre})^2\,(\text{second})^{-2}] = [L^2T^{-2}]$

(s) $U = \dfrac{1}{2}CV^2 \Rightarrow (\text{farad})\,(\text{volt})^2 = \text{Joule}$

or $[(\text{farad})\,(\text{volt})^2\,(\text{kg})^{-1}] = [(\text{Joule})(\text{kg})^{-1}]$

$= [ML^2T^{-2}][M^{-1}] = [L^2T^{-2}]$

18. Angular momentum, $L = I\omega$

$\therefore$ $[L] = [I\omega] = [ML^2][T^{-1}] = [ML^2T^{-1}]$

Latent heat, $L = \dfrac{Q}{m}$ (as $Q = mL$)

$\Rightarrow [L] = \left[\dfrac{Q}{m}\right] = \left[\dfrac{ML^2T^{-2}}{M}\right] = [L^2T^{-2}]$

Torque, $\tau = F \times r_\perp$

$\therefore$ $[\tau] = [F \times r_\perp] = [MLT^{-2}][L]$

$= [ML^2T^{-2}]$

Capacitance, $C = \dfrac{1}{2}\dfrac{q^2}{U}$ $\left(\text{as } U = \dfrac{1}{2}\dfrac{q^2}{C}\right)$

$\therefore$ $[C] = \left[\dfrac{q^2}{U}\right] = \left[\dfrac{Q^2}{ML^2T^{-2}}\right]$

$= [M^{-1}L^{-2}T^2Q^2]$

Inductance, $L = \frac{2U}{i^2}$ $\left(\text{as } U = \frac{1}{2}Li^2\right)$

$\therefore\ [L] = \left[\frac{U}{i^2}\right] = \left[\frac{Ut^2}{Q^2}\right]$ $\left(\text{as } i = \frac{Q}{t}\right)$

$= \left[\frac{ML^2T^{-2}T^2}{Q^2}\right] = [ML^2Q^{-2}]$

Resistivity, $\rho = \frac{RA}{l}$ $\left(\text{as } R = \rho\frac{l}{A}\right)$

$= \left[\frac{H}{i^2t}\right]\left[\frac{A}{l}\right]$ (as $H = i^2Rt$)

$= \left[\frac{Ht}{Q^2}\right]\left[\frac{A}{l}\right]$ $\left(\text{as } i = \frac{Q}{t}\right)$

$= \left[\frac{ML^2T^{-2}TL^2}{Q^2L}\right] = [ML^3T^{-1}Q^{-2}]$

The correct table is as under

I	II
Angular momentum	$[ML^2T^{-1}]$
Latent heat	$[L^2T^{-2}]$
Torque	$[ML^2T^{-2}]$
Capacitance	$[M^{-1}L^{-2}T^2Q^2]$
Inductance	$[ML^2Q^{-2}]$
Resistivity	$[ML^3T^{-1}Q^{-2}]$

19. $t \equiv \frac{L}{R}$

$\therefore\ L \equiv tR \equiv$ ohm-second

$U \equiv \frac{q^2}{2C}$

$\therefore\ C \equiv \frac{q^2}{U} \equiv \text{coulomb}^2/\text{joule}$

$q \equiv CV$

$\therefore\ C \equiv \frac{q}{V} \equiv$ coulomb/volt

$L \equiv \frac{-e}{di/dt}$

$\therefore\ L \equiv \frac{e\,(dt)}{(di)} \equiv$ volt-second/ampere

$F = ilB$

$\therefore\ B \equiv \frac{F}{il} \equiv$ Newton/ampere-metre

I	II
Capacitance	Coulomb/(volt)$^{-1}$coulomb^2/ joule
Inductance	Ohm-second, volt second/ampere^{-1}
Magnetic induction	Newton/(ampere - metre)

20. $[n] = [L^{-3}], [q] = [AT]$

$[\varepsilon] = [M^{-1}L^{-3}A^2T^4]$

$[T] = [L]$

$[l] = [L]$

$[k_B] = [M^1L^2T^{-2}K^{-1}]$

(a) RHS $= \sqrt{\frac{[L^{-3}A^2T^2]}{[M^{-1}L^{-3}T^4A^2][M^1L^2T^{-2}K^{-1}][K]}}$

$= \sqrt{\frac{[L^{-3}A^2T^2]}{[L^{-1}T^2A^2]}} = \sqrt{[L^{-2}]} = [L^{-1}]$ Wrong

(b) RHS $= \sqrt{\frac{[M^{-1}L^{-3}T^4A^2][M^1L^2T^{-2}K^{-1}][K]}{[L^{-3}][A^2T^2]}}$

$= \sqrt{\frac{[L^{-1}T^2A^2]}{[L^{-3}T^2A^2]}} = [L]$ Correct

(c) RHS $= \sqrt{\frac{[A^2T^2]}{[M^{-1}L^{-3}T^4A^2][L^{-2}][M^1L^2T^{-2}K^{-1}][K]}}$

$= \sqrt{[L^3]}$ Wrong

(d) RHS $= \sqrt{\frac{[A^2T^2]}{[M^{-1}L^{-3}T^4A^2][L^{-1}][M^1L^2T^{-2}K^{-1}]}}$

$= \sqrt{\frac{[A^2T^2]}{[L^{-2}T^2A^2]}} = [L]$ Correct

21. $M \propto h^a c^b G^c$

$M^1 \propto (ML^2T^{-1})^a (LT^{-1})^b (M^{-1}L^3T^{-2})^c$

$\propto M^{a-c}L^{2a+b+3c}T^{-a-b-2c}$

$a - c = 1$ ...(i)

$2a + b + 3c = 0$ ...(ii)

$a + b + 2c = 0$ ...(iii)

On solving (i), (ii), (iii),

$a = \frac{1}{2}, b = +\frac{1}{2}, c = -\frac{1}{2}$

$\therefore\ M \propto \sqrt{c}$ only $\rightarrow$ (a) is correct.

In the same way we can find that, $L \propto h^{1/2}c^{-3/2}G^{1/2}$

$L \propto \sqrt{h}, L \propto \sqrt{G} \rightarrow$ (c), (d) are also correct.

22. (a) Energy of inductor $\frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}I^2$

Energy of capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}\varepsilon_0\frac{A}{d}V^2$

$\mu_0\frac{A}{l}I^2$ and $\varepsilon_0\frac{A}{d}V^2$ have same dimension.

So, $\mu_0 I^2$ and $\varepsilon_0 V^2$ have same dimension.

(c) $Q = CV,\ \frac{Q}{t} = \frac{CV}{t}$

$I = \varepsilon_0 \frac{A}{l} \frac{V}{t} \frac{A}{lt}$ have unit of speed.

So, $\quad I = \varepsilon_0 cV$

23. (a) $L = \frac{\phi}{i}$ or henry $= \frac{\text{weber}}{\text{ampere}}$

(b) $e = -L\left(\frac{di}{dt}\right)$

$\therefore\ L = -\frac{e}{(di/dt)}$ or henry $= \frac{\text{volt-second}}{\text{ampere}}$

(c) $U = \frac{1}{2}Li^2 \quad \therefore \quad L = \frac{2U}{i^2}$ or henry $= \frac{\text{joule}}{(\text{ampere})^2}$

(d) $U = \frac{1}{2}Li^2 = i^2 Rt$

$\therefore \quad L = Rt$ or henry = ohm-second

24. $$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$[\varepsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[\text{IT}]^2}{[\text{MLT}^{-2}][\text{L}^2]} = [\text{M}^{-1}\ \text{L}^{-3}\ \text{T}^4\text{I}^2]$$

Speed of light, $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$$\therefore \quad [\mu_0] = \frac{1}{[\varepsilon_0][c]^2} = \frac{1}{[\text{M}^{-1}\ \text{L}^{-3}\ \text{T}^4\ \text{I}^2][\text{LT}^{-1}]^2} = [\text{MLT}^{-2}\text{I}^{-2}]$$

25. Reynold's number and coefficient of friction are dimensionless quantities.
Curie is the number of atoms decaying per unit time and frequency is the number of oscillations per unit time.

Latent heat and gravitational potential both have the same dimension corresponding to energy per unit mass.

26. (a) Torque and work both have the dimensions $[\text{ML}^2\text{T}^{-2}]$.

(d) Light year and wavelength both have the dimensions of length i.e, [L].

27. CR and $\frac{L}{R}$ both are time constants. Their unit is second.

$\therefore\ \frac{1}{CR}$ and $\frac{R}{L}$ have the SI unit $(\text{second})^{-1}$. Further,

resonance frequency, $\omega = \frac{1}{\sqrt{LC}}$

28. In terms of dimension, $F_e = F_m$

$\Rightarrow \quad qE = qvB$

or $\quad E = vB \qquad [E] = [B][\text{LT}^{-1}]$

29. $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\mu_0 = \varepsilon_0^{-1} \cdot c^{-2}$$

$$[\mu_0] = [\varepsilon_0]^{-1}[\text{L}^{-2}\ \text{T}^2]$$

30. Let $d = k(\rho)^a (S)^b (f)^c$

where, k is a dimensionless. Then,

$$[\text{L}] = \left[\frac{\text{M}}{\text{L}^3}\right]^a \left[\frac{\text{ML}^2\text{T}^{-2}}{\text{L}^2\text{T}}\right]^b \left[\frac{1}{\text{T}}\right]^c$$

Equating the powers of M and L, we have

$$0 = a + b \qquad \text{...(i)}$$

$$1 = -3a \qquad \text{...(ii)}$$

Solving these two equations, we get

$$b = \frac{1}{3} \quad \Rightarrow \quad \therefore \quad n = 3$$

31. $\left[\frac{a}{V^2}\right] = [p]$

$\therefore \quad [a] = [pV^2] = [\text{ML}^{-1}\text{T}^{-2}][\text{L}^6] = [\text{ML}^5\text{T}^{-2}]$

32. Electrical conductivity, $\sigma = \frac{J}{E} = \frac{i/A}{F/q}$

$$= \frac{qi}{FA} = \frac{(it)(i)}{FA} = \frac{i^2 t}{FA}$$

$$[\sigma] = \frac{[\text{A}^2][\text{T}]}{[\text{MLT}^{-2}][\text{L}^2]} = [\text{M}^{-1}\text{L}^{-3}\text{T}^3\text{A}^2]$$

33. $[X] = [C] = [\text{M}^{-1}\text{L}^{-2}\text{T}^2\text{Q}^2]$

$[Z] = [B] = [\text{M T}^{-1}\text{Q}^{-1}] \qquad \left(\because \text{Given } Y = \frac{X}{3Z^2}\right)$

$$\therefore \quad [Y] = \frac{[\text{M}^{-1}\text{L}^{-2}\text{T}^2\text{Q}^2]}{[\text{MT}^{-1}\text{Q}^{-1}]^2} = [\text{M}^{-3}\text{L}^{-2}\text{T}^4\text{Q}^4]$$

34. $E = h\nu$

$\therefore \quad h = \frac{E}{\nu}$

or $\quad [h] = \frac{[E]}{[\nu]} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{T}^{-1}]} = [\text{ML}^2\text{T}^{-1}]$

35. (a) Magnetic flux, $\phi = Bs = \left(\frac{F}{qv}\right)(s) \qquad [\because F = Bqv]$

$$\therefore \quad [\phi] = \left[\frac{\text{Fs}}{\text{qv}}\right] = \left[\frac{\text{MLT}^{-2}\text{L}^2}{\text{QLT}^{-1}}\right] = [\text{ML}^2\text{T}^{-1}\text{Q}^{-1}]$$

(b) [Rigidity Modulus] $= \left[\frac{\text{F}}{\text{A}}\right] = \left[\frac{\text{MLT}^{-2}}{\text{L}^2}\right] = [\text{ML}^{-1}\text{T}^{-2}]$

36. $[T] = [p^a d^b E^c] = [\text{ML}^{-1}\text{T}^{-2}]^a\ [\text{ML}^{-3}]^b[\text{ML}^2\text{T}^{-2}]^c$

Equating the powers on both sides, we have

$$a + b + c = 0 \qquad \text{... (i)}$$

$$-a - 3b + 2c = 0 \qquad \text{... (ii)}$$

$$-2a - 2c = 1 \qquad \text{... (iii)}$$

Solving these three equations, we have

$$a = -\frac{5}{6}, b = \frac{1}{2}$$

and $$c = \frac{1}{3}$$

37. (a) Young's modulus $= \dfrac{F/A}{\Delta l/l}$

Hence, the MKS units are N/m^2.

(c) Power of a lens (in dioptre) $= \dfrac{1}{f(\text{in metre})}$

Topic 2 Significant Figures and Error Analysis

1 Area of 1 square $= 5.29\ cm^2$

Area of seven such squares

$= 7$ times addition of area of 1 square

$= 5.29 + 5.29 + 5.29 \ldots 7$ times

$= 37.03\ cm^2$

As we know that, if in the measured values to be added/subtracted the least number of significant digits after the decimal is n.

Then, in the sum or difference also, the number of significant digits after the decimal should be n.

Here, number of digits after decimal in 5.29 is 2, so our answer also contains only two digits after decimal point.

$\therefore$ Area required $= 37.03\ cm^2$

2. Given, mass $= (10.00 \pm 0.10)$ kg

Edge length $= (0.10 \pm 0.01)$ m

Error in mass,

$$\frac{\Delta M}{M} = \frac{0.1}{10} \quad \ldots(i)$$

and error in length,

$$\frac{\Delta l}{l} = \frac{0.01}{0.1} \quad \ldots(ii)$$

Density of the cube is given by

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{l^3}$$

$\therefore$ Permissible error in density is

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} \pm 3\frac{\Delta l}{l} \quad \ldots(iii)$$

Substituting the value from Eqs. (i) and (ii) in Eq. (iii), we get

$$\frac{\Delta\rho}{\rho} = \frac{0.1}{10} + 3 \times \frac{0.01}{0.1} = \frac{1}{100} + \frac{3}{10} = \frac{31}{100}$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{31}{100} = 0.31$$

Since, $\dfrac{\Delta\rho}{\rho}$ should be unitless quantity.

But there is no option with unitless error. Hence, no option is correct.

3 Relation used for finding acceleration due to gravity by using a pendulum is

$$g = \frac{4\pi^2 l}{T^2}$$

So, fractional error in value of g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} \quad \ldots(i)$$

Given, $\Delta l = 0.1$ cm, $l = 55$ cm, $\Delta T = 1$ s and T for 20 oscillations $= 30$ s

Substituting above values in Eq. (i), we get

$$\frac{\Delta g}{g} = \frac{0.1}{55} + 2 \times \frac{1}{30}$$

Hence, percentage error in g is

$$= \frac{\Delta g}{g} \times 100 = \frac{10}{55} + \frac{20}{3} = 6.8\%$$

4 Volume of a cylinder of radius 'r' and height h is given by

$$V = \pi r^2 h$$

or $$V = \frac{1}{4}\pi D^2 h,$$

where D is the diameter of circular surface.

Here, $D = 12.6$ cm and $h = 34.2$ cm

$$\Rightarrow V = \frac{\pi}{4} \times (12.6)^2 \times (34.2)$$

$$V = 4262.22\ cm^3$$

$$V = 4260 \text{ (in three significant numbers)}$$

Now, error calculation can be done as

$$\frac{\Delta V}{V} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta h}{h}$$

$$= \frac{2 \times 0.1}{12.6} + \frac{0.1}{34.2}$$

$$\Rightarrow \frac{\Delta V}{V} = 0.0158 + 0.0029$$

$$\Rightarrow \Delta V = (0.01879) \times (4262.22)$$

$$\Rightarrow \Delta V = 79.7 \simeq 80\ cm^3$$

$\therefore$ For proper significant numbers, volume reading will be $V = 4260 \pm 80\ cm^3$

5. Given, $\dfrac{\Delta m}{m} \times 100 = 1.5\%$ and $\dfrac{\Delta l}{l} \times 100 = 1\%$

$$d = \frac{m}{l^3} \Rightarrow \frac{\Delta d}{d} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{3\Delta l}{l} \times 100$$

$$= 1.5 + 3 = 4.5\%$$

6. Distance covered by the stone

$$L = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times t^2$$

$$t_1 = \sqrt{\frac{L}{5}}$$

$\therefore$ Time Interval between dropping a stone and receiving the sound of impact.

$$dt = \frac{1}{\sqrt{5}}\frac{1}{2}L^{-1/2}dL + \left(\frac{1}{300}dL\right)$$

$$dt = \frac{1}{2\sqrt{5}}\frac{1}{\sqrt{20}}dL + \frac{dL}{300} = 0.01$$

$$dL\left(\frac{1}{20}+\frac{1}{300}\right) = 0.01$$

$$dL\left[\frac{15}{300}\right] = 0.01, \; dL = \frac{3}{16}$$

$$\frac{dL}{L}\times 100 = \frac{3}{16}\times\frac{1}{20}\times 100 = \frac{15}{16} \simeq 1\%$$

7. Time period is given by, $T = \frac{t}{n}$

Further, $T = 2\pi\sqrt{\frac{L}{g}}$

$$\Rightarrow \quad g = \frac{(4\pi^2)L}{T^2} = \frac{(4\pi^2)(L)}{\left(\frac{t}{n}\right)^2} = (4\pi^2 n^2)\frac{L}{t^2}$$

Percentage error in the value of 'g' will be

$$\frac{\Delta g}{g}\times 100 = \left(\frac{\Delta L}{L}\right)\times 100 + 2\left(\frac{\Delta t}{t}\right)\times 100$$

$$= \frac{0.1}{20}\times 100 + 2\times\left(\frac{1}{90}\right)\times 100 = 2.72\%$$

$\therefore$ The nearest answer is 3%

8. Given, $I = (e^{1000V/T} - 1)\,\text{mA},\; dV = \pm 0.01\,\text{V}$

$T = 300\,\text{K}$

So, $I = e^{1000V/T} - 1$

$$I + 1 = e^{1000V/T}$$

Taking log on both sides, we get

$$\log(I+1) = \frac{1000V}{T}$$

On differentiating, $\frac{dI}{I+1} = \frac{1000}{T}dV$

$$dI = \frac{1000}{T}\times(I+1)\;dV \Rightarrow \; dI = \frac{1000}{300}\times(5+1)\times 0.01$$

$$= 0.2 \quad \text{mA}$$

So, error in the value of current is 0.2 mA.

9. Density, $\rho = \frac{m}{\pi r^2 L}$

$$\therefore \quad \frac{\Delta\rho}{\rho}\times 100 = \left(\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}\right)\times 100$$

After substituting the values, we get the maximum percentage error in density = 4%

Hence, the correct option is (d).

10. $V = l^3 = (1.2\times 10^{-2}\text{ m})^3 = 1.728\times 10^{-6}\text{ m}^3$

$\because$ Length (l) has two significant figures, the volume (V) will also have two significant figures. Therefore, the `correct answer is $V = 1.7\times 10^{-6}\text{ m}^3$

11. $$E(t) = A^2 e^{-\alpha t} \quad \ldots(\text{i})$$

$$\alpha = 0.2\text{ s}^{-1}$$

$$\left(\frac{dA}{A}\right)\times 100 = 1.25\%$$

$$\left(\frac{dt}{t}\right)\times 100 = 1.50$$

$$\Rightarrow \quad (dt\times 100) = 1.5t$$

$$= 1.5\times 5 = 7.5$$

$$\therefore \quad \left(\frac{dE}{E}\right)\times 100 = \pm 2\left(\frac{dA}{A}\right)\times 100 \pm \alpha\,(dt\times 100)$$

Taking log on both sides of Eq. (i), we get

$$\log E = 2\log A - \alpha t$$

$$\frac{dE}{E} = \pm 2\frac{dA}{A} \pm \alpha dt$$

$$\therefore \quad \left(\frac{dE}{E}\right)\times 100 = \pm 2\left(\frac{dA}{A}\right)\times 100 \pm \alpha\,(dt\times 100)$$

$$= \pm 2\,(1.25) \pm 0.2\,(7.5)$$

$$= \pm 2.5 \pm 1.5 = \pm 4\%$$

Topic 3 Experimental Physics

1. In a screw gauge,

Least count

$$= \frac{\text{Measure of 1 main scale division (MSD)}}{\text{Number of division on circular scale}}$$

Here, minimum value to be measured/least count is $5\,\mu\text{m}$.

$$= 5\times 10^{-6}\text{ m}$$

$\therefore$ According to the given values,

$$5\times 10^{-6} = \frac{1\times 10^{-3}}{N}$$

or $$N = \frac{10^{-3}}{5\times 10^{-6}} = \frac{1000}{5} = 200 \text{ divisions}$$

2. For a measuring device, the least count is the smallest value that can be measured by measuring instrument.

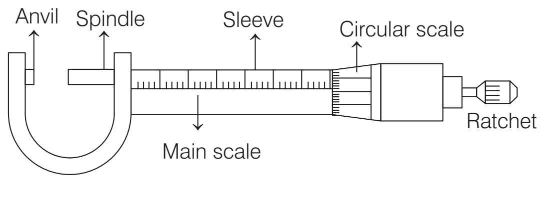
$$\text{Least count} = \frac{\text{Minimum reading on main-scale}}{\text{Total divisions on the scale}}$$

Here, screw gauge is used for measurement therefore,

$$\text{LC} = \frac{\text{Pitch}}{\text{number of division}}$$

$$\text{LC} = \frac{0.5}{100}\text{ mm}$$

$$\text{LC} = 5\times 10^{-3}\text{ mm} \quad \ldots(\text{i})$$



According to question, the zero line of its circular scale lies 3 division below the mean line and the readings of main scale = 5.5 mm

The reading of circular scale = 48

then the actual value is given by actual value of thickness (t) = (main scale reading) + (circular scale reading + number of division below mean line) × LC

$\Rightarrow \quad t = 5.5\text{ mm} + (48 + 3) \times 5 \times 10^{-3}\text{ mm}$

$\Rightarrow \quad t = 5.755\text{ mm}$

3. Least count

$$= \frac{\text{pitch}}{\text{number of divisions on circular scale}} = \frac{0.5\text{ mm}}{50}$$

$\therefore$ LC = 0.01

Negative zero error = $-5 \times \text{LC} = -0.005$ mm

Measured value = main scale reading + screw gauge reading − zero error

$$= 0.5\text{ mm} + \{25 \times 0.01 - (-0.05)\}\text{ mm}$$
$$= 0.8\text{ mm}$$

4. For vernier C_1,

$$10\text{ VSD} = 9\text{ MSD} = 9\text{ mm}$$

1 VSD = 0.9 mm

$\Rightarrow$ LC = 1 MSD − 1 VSD = 1 mm − 0.9 mm = 0.1 mm

Reading of C_1 = MSR + (VSR) (LC)

= 28 mm + (7)(0.1)

Reading of C_1 = 28.7 mm = 2.87 cm

For vernier C_2 : the vernier C_2 is abnormal

So, we have to find the reading form basics.

The point where both of the marks are matching :

distance measured from main scale = distance measured from vernier scale

28 mm + (1 mm) (8) = (28 mm + x) + (1.1 mm) (7)

Solving we get, $\quad x = 0.3$ mm

So, reading of $\quad C_2$ = 28 mm + 0.3 mm = 2.83 cm

5. If student measures 3.50 cm, it means that there is an uncertainly of order 0.01 cm.

For vernier scale with 1 MSD = $\frac{1}{10}$ cm

and $\qquad$ 9 MSD = 10 VSD

LC of vernier caliper = 1MSD − 1VSD

$$= \frac{1}{10}\left(1 - \frac{9}{10}\right) = \frac{1}{100}\text{ cm} = 0.01\text{ cm}$$

6. 1 MSD = 5.15 cm − 5.10 cm = 0.05 cm

$$1\text{ VSD} = \frac{2.45\text{ cm}}{50} = 0.049\text{ cm}$$

$\therefore \qquad$ LC = 1 MSD − 1 VSD = 0.01 cm

Hence, diameter of cyclinder = (Main scale reading) + (Vernier scale reading) (LC)

= 5.10 + (24) (0.001) = 5.124 cm

7. $\Delta d = \Delta l = \frac{0.5}{100}$ mm = 0.005 mm

$$Y = \frac{4MLg}{\pi l d^2} \Rightarrow \left(\frac{\Delta Y}{Y}\right)_{\max} = \left(\frac{\Delta l}{l}\right) + 2\left(\frac{\Delta d}{d}\right)$$

$$\left(\frac{\Delta l}{l}\right) = \frac{0.5/100}{0.25} = 0.02 \text{ and } \frac{2\Delta d}{d} = \frac{(2)(0.5/100)}{0.5} = 0.02$$

or $\qquad \frac{\Delta l}{l} = 2 \cdot \frac{\Delta d}{d}$

8. Least count of screw gauge = $\frac{0.5}{50}$ = 0.01 mm = Δr

Diameter, $r = 2.5\text{ mm} + 20 \times \frac{0.5}{50} = 2.70$ mm

$$\frac{\Delta r}{r} = \frac{0.01}{2.70} \text{ or } \frac{\Delta r}{r} \times 100 = \frac{1}{2.7}$$

Now, density, $d = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

Here, r is the diameter.

$$\therefore \quad \frac{\Delta d}{d} \times 100 = \left\{\frac{\Delta m}{m} + 3\left(\frac{\Delta r}{r}\right)\right\} \times 100$$
$$= \frac{\Delta m}{m} \times 100 + 3 \times \left(\frac{\Delta r}{r}\right) \times 100$$
$$= 2\% + 3 \times \frac{1}{2.7} = 3.11\%$$

9. Least count of vernier calipers

LC = 1MSD − 1 VSD

$$= \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

20 divisions of vernier scale = 16 divisions of main scale

$\therefore$ 1 VSD $= \frac{16}{20}$ mm = 0.8 mm

$\therefore \qquad$ LC = 1MSD − 1VSD = 1 mm − 0.8 mm = 0.2 mm

$\therefore$ Correct option is (d).

10. $T = 2\pi\sqrt{\frac{l}{g}}$ or $\frac{t}{n} = 2\pi\sqrt{\frac{l}{g}}$

$$\therefore \qquad g = \frac{(4\pi^2)(n^2)l}{t^2}$$

$$\%\text{ error in } g = \frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} + \frac{2\Delta t}{t}\right) \times 100$$

$$E_{\text{I}} = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{128}\right) \times 100 = 0.3125\%$$

$$E_{\text{II}} = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{64}\right) \times 100 = 0.46875\%$$

$$E_{\text{III}} = \left(\frac{0.1}{20} + \frac{2 \times 0.1}{36}\right) \times 100 = 1.055\%$$

Hence E_{I} is minimum.

$\therefore$ Correct option is (b).

11. Length of air column in resonance is odd integer multiple of $\lambda / 4$. The prongs of the tuning fork are kept in the vertical plane.

12. $$Y = \frac{FL}{Al} = \frac{4FL}{\pi d^2 l} = \frac{(4)(1.0 \times 9.8)(2)}{\pi(0.4 \times 10^{-3})^2 (0.8 \times 10^{-3})}$$

$$= 2.0 \times 10^{11} \text{ N/m}^2$$

Further $\frac{\Delta Y}{Y} = 2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)$

$\therefore \quad \Delta Y = \left\{2\left(\frac{\Delta d}{d}\right) + \left(\frac{\Delta l}{l}\right)\right\} Y$

$$= \left\{2 \times \frac{0.01}{0.4} + \frac{0.05}{0.8}\right\} \times 2.0 \times 10^{11}$$

$$= 0.225 \times 10^{11} \text{ N/m}^2$$

$$= 0.2 \times 10^{11} \text{ N/m}^2 \qquad \text{(By rounding off)}$$

or $(Y + \Delta Y) = (2 \pm 0.2) \times 10^{11} \text{ N/m}^2$

13. Least count (LC) $= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

$$= \frac{0.5}{50} = 0.01 \text{ mm}$$

Now, diameter of ball

$$= (2 \times 0.5 \text{ mm}) + (25 - 5)(0.01) = 1.2 \text{ mm}$$

14. Mean time period $= \frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5}$

$= 0.556 \cong 0.56$ sec as per significant figures

Error in reading $= |T_{\text{mean}} - T_1| = 0.04$

$|T_{\text{mean}} - T_2| = 0.00$

$|T_{\text{mean}} - T_3| = 0.01$

$|T_{\text{mean}} - T_4| = 0.02$

$|T_{\text{mean}} - T_5| = 0.03$

Mean error $= 0.1/5 = 0.02$

% error in $T = \frac{\Delta T}{T} \times 100 = \frac{0.02}{0.56} \times 100 = 3.57\%$

% error in $r = \frac{0.001 \times 100}{0.010} = 10\%$

% error in $R = \frac{0.001 \times 100}{0.60} = 1.67\%$

% error in $\frac{\Delta g}{g} \times 100 = \frac{\Delta(R - r)}{R - r} \times 100 + 2 \times \frac{\Delta T}{T}$

$$= \frac{0.002 \times 100}{0.05} + 2 \times 3.57 = 4\% + 7\% = 11\%$$

15. **For vernier calipers**

1 MSD $= \frac{1}{8}$ cm

5 VSD = 4 MSD

$\therefore$ 1 VSD $= \frac{4}{5}$ MSD $= \frac{4}{5} \times \frac{1}{8} = \frac{1}{10}$ cm

Least count of vernier calipers = 1 MSD − 1 VSD

$$= \frac{1}{8} \text{ cm} - \frac{1}{10} \text{ cm}$$

$$= 0.025 \text{ cm}$$

(a) and (b)

Pitch of screw gauge $= 2 \times 0.025 = 0.05$ cm

Least count of screw gauge $= \frac{0.05}{100}$ cm $= 0.005$ mm

(c) and (d)

Least count of linear scale of screw gauge = 0.05

Pitch $= 0.05 \times 2 = 0.1$ cm

Least count of screw gauge $= \frac{0.1}{100}$ cm

$$= 0.01 \text{ mm}$$

16. $T = \frac{40 \text{ s}}{20} = 2$ s

Further, $t = nT = 20T$ or $\Delta t = 20\,\Delta T$

$\therefore \quad \frac{\Delta t}{t} = \frac{\Delta T}{T}$ or $\Delta T = \frac{T}{t} \cdot \Delta t = \left(\frac{2}{40}\right)(1) = 0.05$ s

Further, $\quad T = 2\pi\sqrt{\frac{l}{g}}$ or $T \propto g^{-1/2}$

$\therefore \quad \frac{\Delta T}{T} \times 100 = -\frac{1}{2} \times \frac{\Delta g}{g} \times 100$

or % error in determination of g is

$$\frac{\Delta g}{g} \times 100 = -200 \times \frac{\Delta T}{T}$$

$$= -\frac{200 \times 0.05}{2}$$

$$= -5\%$$

$\therefore$ Correct options are (a) and (c).

17. Given, $\quad d = 0.5$ mm,

$Y = 2 \times 10^{11} \text{ Nm}^{-2}$,

$l = 1$ m

$$\Delta l = \frac{Fl}{AY} = \frac{mgl}{\frac{\pi d^2}{4} Y} = \frac{1.2 \times 10 \times 1}{\frac{\pi}{4} \times (5 \times 10^{-4})^2 \times 2 \times 10^{11}}$$

$$= 0.3 \text{ mm}$$

LC of vernier $= \left(1 - \frac{9}{10}\right)$ mm $= 0.1$ mm

So, 3rd division of vernier scale will coincide with main scale.

18. $Y = \frac{F/A}{\frac{\Delta l}{l}}$, $\Delta l = 25 \times 10^{-50}$ m

$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

$$\frac{\Delta Y}{Y} \times 100 = \frac{10^{-5}}{25 \times 10^{-5}} \times 100 = 4\%$$

19. 1 MSD = 1 mm

9 MSD = 10 VSD

$\therefore$ Least count,

$$LC = 1\text{ MSD} - 1\text{ VSD} = 1\text{ mm} - \frac{9}{10}\text{ mm} = \frac{1}{10}\text{ mm}$$

Measure reading of edge = MSR + VSR (LC)

$$= 10 + 1 \times \frac{1}{10} = 10.1\text{ mm}$$

Volume of cube, $V = (1.01)^3\text{ cm}^3 = 1.03\text{ cm}^3$

[After rounding off upto 3 significant digits, as edge length is measured upto 3 significant digits]

$\therefore$ Density of cube $= \frac{2.736}{1.03} = 2.6563\text{ g/cm}^3$

$= 2.66\text{ g/cm}^3$

(After rounding off to 3 significant digits)

20. Least count of screw gauge $= \frac{1\text{mm}}{100} = 0.01\text{ mm}$

Diameter of wire $= (1 + 47 \times 0.01)\text{mm} = 1.47\text{ mm}$

Curved surface area (in cm^2) $= (2\pi)\left(\frac{d}{2}\right)(L)$

or $S = \pi dL = (\pi)(1.47 \times 10^{-1})(5.6)\text{ cm}^2$

$= 2.5848\text{ cm}^2$

Rounding off to two significant digits

$$S = 2.6\text{ cm}^2$$

21. Young's modulus of elasticity is given by

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{l/L} = \frac{FL}{lA} = \frac{FL}{l\left(\frac{\pi d^2}{4}\right)}$$

Substituting the values, we get

$$Y = \frac{50 \times 1.1 \times 4}{(1.25 \times 10^{-3}) \times \pi \times (5.0 \times 10^{-4})^2}$$

$$= 2.24 \times 10^{11}\text{ N/m}^2$$

Now, $\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$

$$= \left(\frac{0.1}{110}\right) + \left(\frac{0.001}{0.125}\right) + 2\left(\frac{0.001}{0.05}\right)$$

$$= 0.0489$$

$$\Delta Y = (0.0489)\,Y = (0.0489) \times (2.24 \times 10^{11})\text{ N/m}^2$$

$$= 1.09 \times 10^{10}\text{ N/m}^2$$

22. $(N+1)$ divisions on the vernier scale

$= N$ divisions on main scale

$\therefore$ 1 division on vernier scale

$= \frac{N}{N+1}$ divisions on main scale

Each division on the main scale is of a units.

$\therefore$ 1 division on vernier scale $= \left(\frac{N}{N+1}\right)a$ units $= a'$(say)

Least count = 1 main scale division – 1 vernier scale division

$$= a - a' = a - \left(\frac{N}{N+1}\right)a = \frac{a}{N+1}$$

Topic 4 Miscellaneous Problems

1. $m = \frac{4\pi R^3}{3} \times \rho$

On taking log both sides, we have

$$\ln(m) = \ln\left(\frac{4\pi}{3}\right) + \ln(\rho) + 3\ln(R)$$

On differentiating with respect to time,

$$0 = 0 + \frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt}$$

$\Rightarrow$ $\left(\frac{dR}{dt}\right) = -R \times \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$

$\because$ $\frac{dR}{dt} = v$

$\therefore$ $v = -R \propto \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$

$\therefore$ $v \propto R$

2. $\mathbf{S} = \mathbf{P} + b\mathbf{R} = \mathbf{P} + b(\mathbf{Q} - \mathbf{P}) = \mathbf{P}(1-b) + b\mathbf{Q}$

3. $r = \frac{1-a}{1+a}$

$\ln r = \ln(1-a) - \ln(1+a)$

Differentiating, we get

$$\frac{dr}{r} = -\frac{da}{1-a} - \frac{da}{1+a}$$

or, we can write

$$\frac{\Delta r}{r} = -\left[\frac{\Delta a}{1-a} + \frac{\Delta a}{1+a}\right]$$

$$\frac{\Delta r}{r} = \frac{-2\Delta a}{1-a^2}$$

or $\Delta r = -\left(\frac{2\Delta a}{1-a^2}\right)(r) = \frac{-2\Delta a}{(1+a)^2}$

4. $N = N_0 e^{-\lambda t}$

$\text{In}N = \text{In}N_0 - \lambda t$

Differentiating w.r.t λ, we get

$$\frac{1}{N}\cdot\frac{dN}{d\lambda} = 0 - t$$

$\Rightarrow$ $|d\lambda| = \frac{dN}{Nt} = \frac{40}{2000 \times 1} = 0.02$

5. N = Number of electrons per unit volume

$\therefore$ $[\text{N}] = [\text{L}^{-3}]$, $[\text{e}] = [\text{q}] = [\text{It}] = [\text{AT}]$

$[\varepsilon_0] = [\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$

Substituting the dimensions, we can see that

$$\omega_p = \left[\sqrt{\frac{Ne^2}{m\varepsilon_0}}\right] = [T^{-1}]$$

Angular frequency has also the dimension $[T^{-1}]$.

6. $$\omega = 2\pi f = \frac{2\pi c}{\lambda}$$

$$\therefore \quad \lambda = \frac{2\pi c}{\omega} = \frac{2\pi c}{\sqrt{\frac{Ne^2}{m\varepsilon_0}}}$$

Substituting the values, we get $\lambda \simeq 600\,\text{nm}$.

7. Momentum is first positive but decreasing. Displacement (or say position) is initially zero. It will first increase. At highest point, momentum is zero and displacement is maximum. After that momentum is downwards (negative) and increasing but displacement is decreasing. Only (d) option satisfies these conditions.

8. $E = \frac{1}{2} m\omega^2 A^2$ or $E \propto A^2$,

$$\frac{E_2}{E_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{a}{2a}\right)^2 \text{ or } E_1 = 4E_2$$

9. In all the given four figures, at mean position, the position coordinate is zero. At the same time, mass is starting from the extreme position in all four cases. In Fig. (c) and (d), extreme position is more than the initial extreme position. But due to viscosity position should be the less.

Hence, the answer should be either option (a) or option (b).

Correct answer is (b), because mass starts from positive extreme position (from uppermost position). Then, it will move downwards or momentum should be negative.

10. (t) A time varying magnetic field will produce a non-conservative electric field. Due to this electric field, a current starts flowing in the resistive loop and heat is produced.

11. (p) Constant velocity means, net acceleration or net force = 0.

$\therefore$ Net force exerted by X and $Y = Mg$ in upward direction (opposite to its weight Mg). Since Y is moving with constant velocity, some friction is there between X and Y.

Therefore, some work is done against friction and mechanical energy of $(X + Y)$ is continuously decreasing.

(s) Potential energy of Y is decreasing but same volume of X rises up. Hence, potential energy of X is increasing. Some part of mechanical energy of $X + Y$ is lost in the form of heat in doing work against viscous forces. Net force on Y in this case is downwards before Y attains its terminal velocity.

(t) After attaining the terminal velocity, net force on Y becomes zero. Hence, force on Y from X is Mg, upwards. Rest of the logics are same as discussed in part (s).

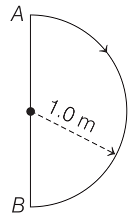
2

Kinematics

Topic 1 Basic Definitions

Objective Questions I (Only one correct option)

1. In 1.0 s, a particle goes from point A to point B, moving in a semicircle (see figure). The magnitude of the average velocity is **(1999, 2M)**



(a) 3.14 m/s (b) 2.0 m/s
(c) 1.0 m/s (d) zero

2. A particle is moving Eastwards with a velocity of 5 m/s. In 10 s, the velocity changes to 5 m/s Northwards. The average acceleration in this time is **(1982, 3M)**

(a) zero

(b) $\frac{1}{\sqrt{2}}$ m/s^2 towards North-Eeast

(c) $\frac{1}{\sqrt{2}}$ m/s^2 towards North-West

(d) $\frac{1}{2}$ m/s^2 towards North

Fill in the Blank

3. A particle moves in a circle of radius R. In half, the period of revolution its displacement is …… and distance covered is ………. . **(1983, 2M)**

Topic 2 One Dimensional Motion

Objective Questions I (Only one correct option)

1 A particle is moving with speed $v = b\sqrt{x}$ along positive X-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$). **(2019 Main, 12 April II)**

(a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$ (c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

2 A ball is thrown upward with an initial velocity v_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where, m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is **(2019 Main, 10 April I)**

(a) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}v_0\right)$ (b) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}v_0\right)$

(c) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}v_0\right)$ (d) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}v_0\right)$

3 The bob of a simple pendulum has mass 2g and a charge of $5.0\,\mu$C. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is (take $g = 10\,\text{m/s}^2$) **(2019 Main, 08 April I)**

(a) $\tan^{-1}(2.0)$ (b) $\tan^{-1}(0.2)$ (c) $\tan^{-1}(5.0)$ (d) $\tan^{-1}(0.5)$

4 A particle moves from the point $(2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ m at $t = 0$ with an initial velocity $(5.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})\,\text{ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})\,\text{ms}^{-2}$. What is the distance of the particle from the origin at time 2 s? **(2019 Main, 11 Jan II)**

(a) 5 m (b) $20\sqrt{2}$ m
(c) $10\sqrt{2}$ m (d) 15 m

5. In a car race on a straight path, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to **(2019 Main, 09 Jan II)**

(a) $\frac{2a_1a_2}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}\,t$

(c) $\sqrt{a_1a_2}\,t$ (d) $\frac{a_1+a_2}{2}t$

6. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is **(2014 Main)**

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2u^2$
(c) $2gH = nu^2(n-2)$ (d) $gH = (n-2)^2u^2$

7. A small block slides without friction down an inclined plane starting from rest. Let s_n be the distance travelled from $t = n - 1$ to $t = n$. Then, $\frac{s_n}{s_{n+1}}$ is **(2004, 2M)**

(a) $\frac{2n-1}{2n}$ (b) $\frac{2n+1}{2n-1}$

(c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n}{2n+1}$

Objective Questions II (One or more correct option)

8. The position vector **r** of particle of mass m is given by the following equation **(2016 Adv.)**

$\mathbf{r}(t) = \alpha t^3 \hat{\mathbf{i}} + \beta t^2 \hat{\mathbf{j}}$ where, $\alpha = \frac{10}{3}\,\text{ms}^{-3}$, $\beta = 5\,\text{ms}^{-2}$ and $m = 0.1$ kg. At $t = 1$s, which of the following statement(s) is (are) true about the particle?

(a) The velocity **v** is given by $\mathbf{v} = (10\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\,\text{ms}^{-1}$

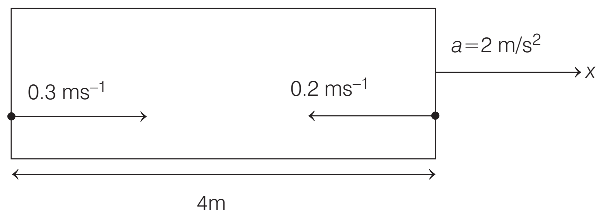
(b) The angular momentum **L** with respect to the origin is given by $\mathbf{L} = (5/3)\hat{\mathbf{k}}$ Nms

(c) The force **F** is given by $\mathbf{F} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$N

(d) The torque τ with respect to the origin is given by $\tau = -\frac{20}{3}\hat{\mathbf{k}}$ Nm

Integer Answer Type Questions

9. A rocket is moving in a gravity free space with a constant acceleration of $2\,\text{ms}^{-2}$ along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of $0.3\,\text{ms}^{-1}$ relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of $0.2\,\text{ms}^{-1}$ from its right end relative to the rocket. The time in seconds when the two balls hit each. **(2014 Adv.)**



Analytical & Descriptive Questions

10. A particle of mass 10^{-2} kg is moving along the positive X-axis under the influence of a force $F(x) = -k/2x^2$ where $k = 10^{-2}$ Nm^2. At time $t = 0$, it is at $x = 1.0$ m and its velocity $v = 0$. **(1998, 8M)**

(a) Find its velocity when it reaches $x = 0.5$ m. (b) Find the time at which it reaches $x = 0.25$ m.

True/False

11. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (Neglect air resistance). **(1983, 2M)**

Topic 3 Relative Motion

Objective Questions I (Only one correct option)

1. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ? **(2019 Main, 9 April I)**

(a) 60° (b) 120°

(c) 90° (d) 150°

2. Ship A is sailing towards north-east with velocity $\mathbf{v} = 30\hat{\mathbf{i}} + 50\hat{\mathbf{j}}$ km/h, where $\hat{\mathbf{i}}$ points east and $\hat{\mathbf{j}}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/h. A will be at minimum distance from B in **(2019 Main, 8 April I)**

(a) 4.2 h (b) 2.6 h

(c) 3.2 h (d) 2.2 h

3. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, then speed of the plane is **(2019 Main, 12 Jan I)**

(a) $\frac{\sqrt{3}}{2}v$ (b) v (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{2}$

4. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction and (ii) in the opposite direction is **(2019 Main, 12 Jan I)**

(a) $\frac{3}{2}$ (b) $\frac{25}{11}$

(c) $\frac{11}{5}$ (d) $\frac{5}{2}$

5. A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in km/h is **(1988, 2M)**

(a) 1 (b) 3

(c) 4 (d) $\sqrt{41}$

6. A river is flowing from West to East at a speed of 5 m/min. A man on the South bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river in the shortest time. He should swim in a direction **(1983, 1M)**

(a) due North (b) 30° East of North

(c) 30° West of North (d) 60° East of North

Assertion and Reason

Mark your answer as

(*a*) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*

(*b*) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*

(*c*) *If Statement I is true; Statement II is false*

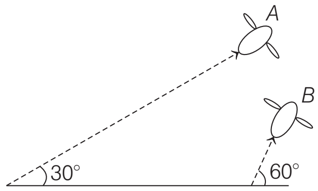
(*d*) *If Statement I is false; Statement II is true*

7. **Statement I** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement II If the observer and the object are moving at velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\mathbf{v}_2 - \mathbf{v}_1$. **(2008, 3M)**

Integer Answer Type Questions

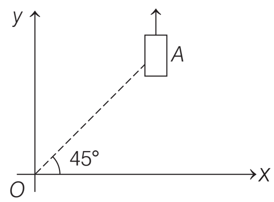
8. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}\ \text{ms}^{-1}$. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is **(2014 Adv.)**



Analytical & Descriptive Questions

9. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley. **(2002, 5M)**

(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

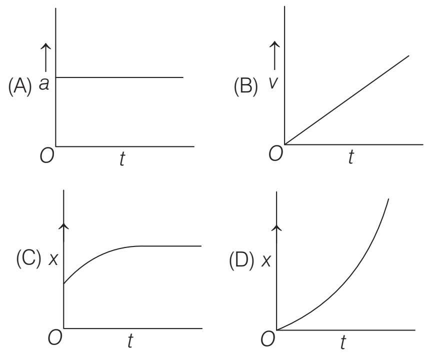


(b) Find the speed of the ball with respect to the surface, if $\phi = 4\theta/3$.

Topic 4 Graphs

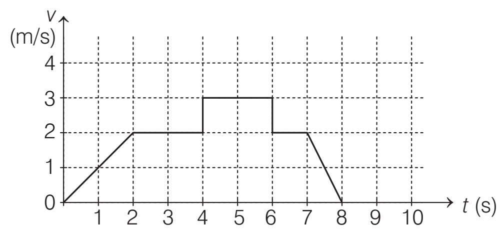
Objective Questions I (Only one correct option)

1. A particle starts from origin O from rest and moves with a uniform acceleration along the positive X-axis. Identify all figures that correctly represent the motion qualitatively.
(a = acceleration, v = velocity,
x = displacement, t = time) **(2019 Main, 8 April II)**



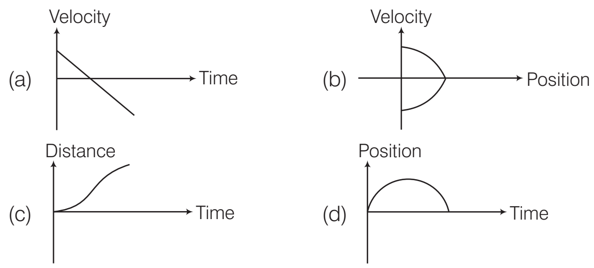
(a) (A) (b) (A), (B), (C)
(c) (B), (C) (d) (A), (B), (D)

2. A particle starts from the origin at time $t = 0$ and moves along the positive X-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5$ s? **(2019 Main, 10 Jan II)**

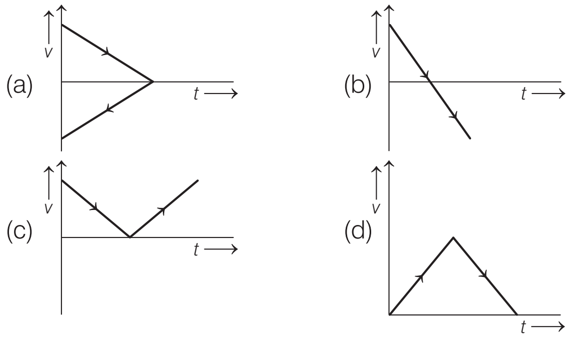


(a) 6 m
(b) 3 m
(c) 10 m
(d) 9 m

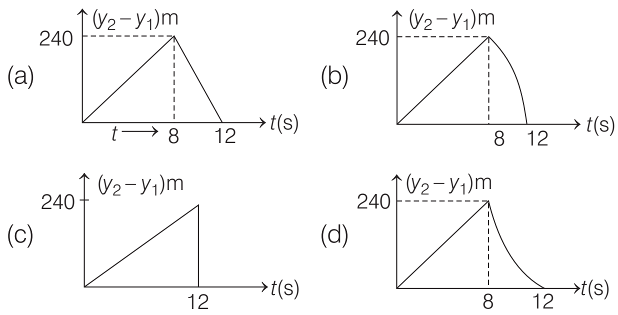
3. All the graphs below are intended to represent the same motion. One of them does it **incorrectly**. Pick it up. **(2018 Main)**



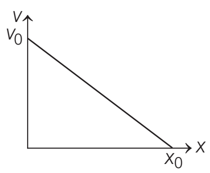
4. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity *vs* time? **(2017 Main)**

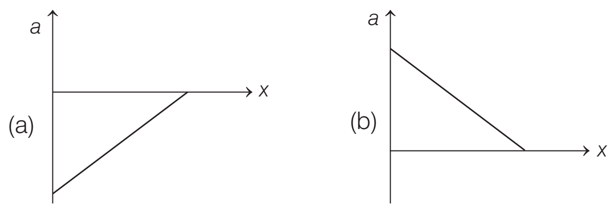


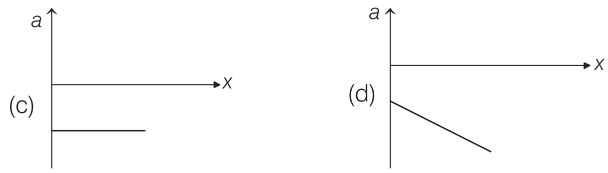
5. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s, respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ \text{m/s}^2$) **(2015 Main)**



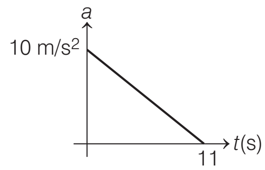
6. The given graph shows the variation of velocity with displacement. Which one of the graph given below correctly represents the variation of acceleration with displacement? **(2005, 2M)**





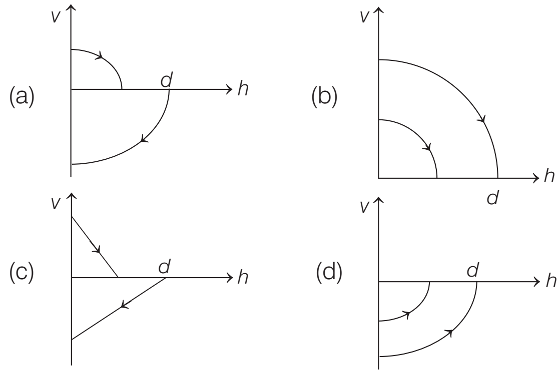


7. A particle starts from rest. Its acceleration (a) *versus* time (t) is as shown in the figure. The maximum speed of the particle will be **(2004, 2M)**



(a) 110 m/s (b) 55 m/s (c) 550 m/s (d) 660 m/s

8. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with height h above the ground as **(2000, 2M)**



Objective Question II (One or more correct option)

9. A particle of mass m moves on the x-axis as follows : it starts from rest at $t = 0$ from the point $x = 0$ and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate times ($0 < t < 1$). If α denotes the instantaneous acceleration of the particle, then **(1993, 2M)**

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$
(b) $|\alpha|$ cannot exceed 2 at any point in its path
(c) $|\alpha|$ must be ≥ 4 at some point or points in its path
(d) α must change sign during the motion, but no other assertion can be made with the information given **(1984, 2M)**

Topic 5 Projectile Motion

Objective Questions I (Only one correct option)

1. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights h_1 and h_2. Which of the following is correct? **(12 April 2019 Shift II)**

(a) $R^2 = 4h_1h_2$ (b) $R^2 = 16h_1h_2$
(c) $R^2 = 2h_1h_2$ (d) $R^2 = h_1h_2$

2. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is **(2019 Main, 12 April I)**

(a) $\frac{R}{4g}$ (b) $\frac{R}{g}$ (c) $\frac{R}{2g}$ (d) $\frac{2R}{g}$

3. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$.

If it were launched at an angle θ_0 with speed v_0, then (Take, $g = 10\,\text{ms}^{-2}$) **(2019 Main, 12 April I)**

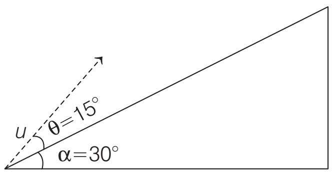
(a) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\text{ms}^{-1}$

(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\text{ms}^{-1}$

(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\text{ms}^{-1}$

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\text{ms}^{-1}$

4. A plane is inclined at an angle $\alpha = 30^\circ$ with respect to the horizontal. A particle is projected with a speed $u = 2\text{ ms}^{-1}$, from the base of the plane, making an angle $\theta = 15^\circ$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to [Take, $g = 10\text{ ms}^{-2}$] **(2019 Main, 10 April I)**



(a) 26 cm (b) 20 cm (c) 18 cm (d) 14 cm

5. The position vector of particle changes with time according to the relation $\mathbf{r}(t) = 15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}$. What is the magnitude of the acceleration (in ms^{-2}) at $t = 1$? **(2019 Main, 9 April II)**

(a) 50 (b) 100 (c) 25 (d) 40

6. The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be **(2019 Main, 09 April II)**

(a) $a + \frac{b^2}{2c}$ (b) $a + \frac{b^2}{4c}$
(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{c}$

7. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1$ s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\,\text{ms}^{-2}$, the value of R is **(2019 Main, 11 Jan I)**

(a) 10.3 m (b) 2.8 m (c) 5.1 m (d) 2.5 m

8. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s, respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets on the ground fired by the two guns is **(Main 2019, 10 Jan I)**

(a) 1 : 4 (b) 1 : 16 (c) 1 : 8 (d) 1 : 2

9. A projectile is given an initial velocity of $(\mathbf{i} + 2\mathbf{j})$ m/s, where, $\mathbf{i}$ is along the ground and $\mathbf{j}$ is along the vertical. If $g = 10\text{ m/s}^2$, then the equation of its trajectory is **(2013 Main)**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$
(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

Fill in the Blank

10. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a, b are constants, and x and y are respectively, the horizontal and vertical distances of the projectile from the point of projection. The maximum height attained is ……… and the angle of projection from the horizontal is ……… . **(1997C, 1M)**

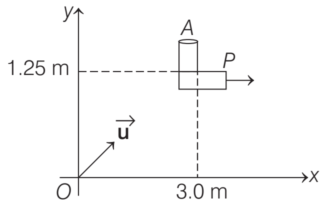
True/False

11. A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path. **(1984, 2M)**

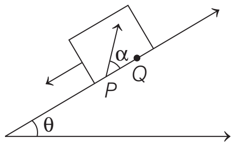
Integer Answer Type Questions

12. A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is. **(2011)**

13. An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$, the plank starts moving along the $+x$-direction with an acceleration 1.5 m/s^2. At the same instant, a stone is projected from the origin with a velocity $\mathbf{u}$ as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in x-y plane. Find $\mathbf{u}$ and the time after which the stone hits the object. (Take $g = 10 \text{m/s}^2$). **(2000, 10M)**



14. A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u and the direction of projection makes an angle α with the bottom as shown in the figure.



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected. **(1998, 8M)**

15. A cart is moving along x-direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart, the stone is thrown in y-z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L.

A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine $(g = 9.8 \text{ m/s}^2)$ **(1997, 5M)**

(a) the speed of the combined mass immediately after the collision with respect to an observer on the ground.

(b) the length L of the string such that tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

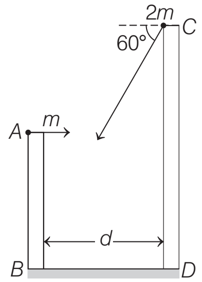
16. Two guns situated on the top of a hill of height 10 m fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at point P $(g = 10 \text{ m/s}^2)$. Find **(1996, 5M)**

(a) the time interval between the firings and

(b) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane.

17. Two towers AB and CD are situated a distance d apart as shown in figure. AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD.

Simultaneously, another object of mass $2m$ is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other. **(1994, 6M)**

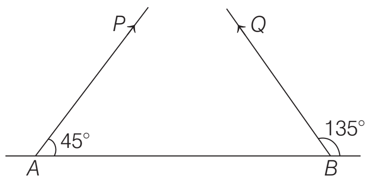


(a) Calculate the distance d between the towers.

(b) Find the position where the objects hit the ground.

18. A body falling freely from a given height H hits an inclined plane in its path at a height h. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h / H), the body will take maximum time to reach the ground? **(1986, 6M)**

19. Particles P and Q of mass 20 g and 40 g respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the figure. Each particle has an initial speed of 49 m/s. The separation AB is 245 m.



Both particles travel in the same vertical plane and undergo a collision. After the collision, P retraces its path. (a) Determine the position Q where it hits the ground. (b) How much time after the collision does the particle Q take to reach the ground? (Take $g = 9.8 \text{ m/s}^2$). **(1982, 8M**

Topic 6 Miscellaneous Problems

Objective Questions I (Only one correct option)

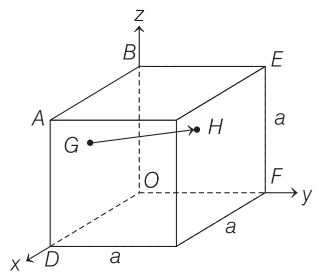
1. Let $|\mathbf{A}_1| = 3$, $|\mathbf{A}_2| = 5$ and $|\mathbf{A}_1 + \mathbf{A}_2| = 5$. The value of $(2\mathbf{A}_1 + 3\mathbf{A}_2) \cdot (3\mathbf{A}_1 - 2\mathbf{A}_2)$ is **(2019 Main, 08 April II)**

(a) -106.5 (b) -112.5
(c) -99.5 (d) -118.5

2. Two vectors $\mathbf{A}$ and $\mathbf{B}$ have equal magnitudes. The magnitude of $(\mathbf{A} + \mathbf{B})$ is 'n' times the magnitude of $(\mathbf{A} - \mathbf{B})$. The angle between $\mathbf{A}$ and $\mathbf{B}$ is **(2019 Main, 10 Jan II)**

(a) $\sin^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$ (b) $\sin^{-1}\left(\frac{n - 1}{n + 1}\right)$
(c) $\cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$ (d) $\cos^{-1}\left(\frac{n - 1}{n + 1}\right)$

3. In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be **(2019 Main, 10 Jan I)**



(a) $\frac{1}{2}a(\hat{\mathbf{i}} - \hat{\mathbf{k}})$ (b) $\frac{1}{2}a(\hat{\mathbf{j}} - \hat{\mathbf{i}})$
(c) $\frac{1}{2}a(\hat{\mathbf{j}} - \hat{\mathbf{k}})$ (d) $\frac{1}{2}a(\hat{\mathbf{k}} - \hat{\mathbf{i}})$

4. In three dimensional system, the position coordinates of a particle (in motion) are given below

$x = a\cos\omega t$
$y = a\sin\omega t$
$z = a\omega t$

The velocity of particle will be **(2019 Main, 09 Jan II)**

(a) $\sqrt{2}\, a\omega$ (b) $2\, a\omega$
(c) $a\omega$ (d) $\sqrt{3}\, a\omega$

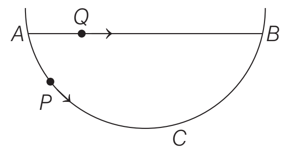
5. A particle is moving with a velocity $\mathbf{v} = k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where k is a constant.

The general equation for its path is **(2019 Main, 9 Jan I)**

(a) $y = x^2 +$ constant
(b) $y^2 = x +$ constant
(c) $xy =$ constant
(d) $y^2 = x^2 +$ constant

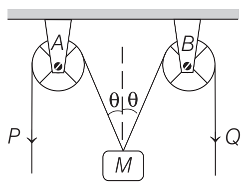
6. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at $t = 0$ along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. **(1993, 2M)**

Then



(a) $t_P < t_Q$
(b) $t_P = t_Q$
(c) $t_P > t_Q$
(d) $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of chord } AB}$

7. In the arrangement shown in the figure, the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed **(1982, 3M)**



(a) $2U\cos\theta$ (b) $\frac{U}{\cos\theta}$
(c) $\frac{2U}{\cos\theta}$ (d) $U\cos\theta$

Objective Questions II (One or more correct option)

8. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$ where $a, b\ (< a)$ and p are positive constants of appropriate dimensions. Then, **(1999, 3M)**

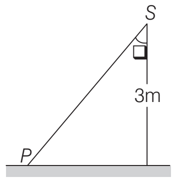
(a) the path of the particle is an ellipse
(b) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$
(c) the acceleration of the particle is always directed towards a focus
(d) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a

Numerical Value

9. Two vectors A and B are defined as $\mathbf{A} = a\hat{\mathbf{i}}$ and $\mathbf{B} = a(\cos\omega t\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}})$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\mathbf{A} + \mathbf{B}| = \sqrt{3}|\mathbf{A} - \mathbf{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is ....

10. A particle of mass 10^{-3} kg and charge 1.0 C is initially at rest. At time $t = 0$, the particle comes under the influence of an electric field $\mathbf{E}(t) = E_0 \sin\omega t\, \hat{\mathbf{i}}$, where $E_0 = 1.0\,\text{NC}^{-1}$ and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then, the maximum speed in m s^{-1}, attained by the particle at subsequent times is ....

Fill in the Blanks



11. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when $\theta = 45°$ (see fig.) is...... m/s. **(1987, 2M)**

12. Four persons K, L, M, N are initially at the four corners of a square of side d. Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N and N directly towards K. The four persons will meet at a time ...... **(1984, 2M)**

Answers

Topic 1

1. (b) **2.** (c) **3.** $2R$, πR

Topic 2

1. (b) **2.** (b) **3.** (d) **4.** (b)
5. (c)
6. (c) **7.** (c) **8.** (a, d) **9.** (2 or 8)
10. (a) $v = -1$ m/s, (b) $t = 1.48$ s **11.** T

Topic 3

1. (b) **2.** (b) **3.** (d) **4.** (c)
5. (b) **6.** (a) **7.** (b) **8.** (5)
9. (a) 45° (b) 2 m/s

Topic 4

1. (d) **2.** (d) **3.** (c) **4.** (b)
5. (b) **6.** (a) **7.** (b) **8.** (a)
9. (c)

Topic 5

1. (b) **2.** (d) **3.** (c) **4.** (b)
5. (a) **6.** (c) **7.** (b) **8.** (b)
9. (b) **10.** $a^2/4b$, $\tan^{-1}(a)$ **11.** T
12. 5 ms^{-2} **13.** $\mathbf{u} = (3.75\hat{\mathbf{i}} + 6.25\hat{\mathbf{j}})$ m/s, 1 s
14. (a) $\dfrac{u^2 \sin 2\alpha}{g\cos\theta}$ (b) $\dfrac{u\cos(\alpha + \theta)}{\cos\theta}$ (down the plane)
15. (a) 2.5 m/s (b) 0.32 m **16.** (a) 1 s (b) $(5\sqrt{3}\text{m}, 5\text{m})$
17. (a) Approximately 17.32 m (b) 11.55 m from B **18.** $\dfrac{1}{2}$
19. Just midway between A and B, 3.53 s

Topic 6

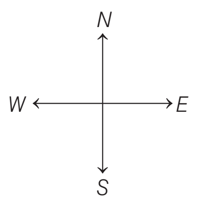
1. (d) **2.** (c) **3.** (b) **4.** (a)
5. (d) **6.** (a) **7.** (b) **8.** (a, b, c)
9. (2.0) **10.** (2) **11.** 0.6 **12.** d/v

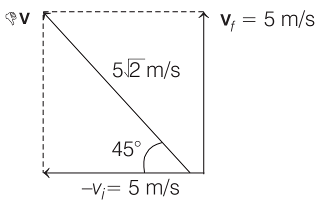
Hints & Solutions

Topic 1 Basic Definitions

1. $$|\text{Average velocity}| = \left|\frac{\text{displacement}}{\text{time}}\right| = \frac{AB}{\text{time}} = \frac{2}{1} = 2\text{ m/s}$$

2.



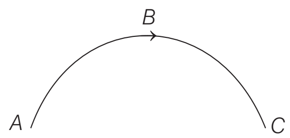


$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t}$$

$\Delta\mathbf{v} = 5\sqrt{2}$ m/s in North-West direction.

$$\mathbf{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\text{ m/s}^2 \text{ (in North-West direction)}$$

3. Displacement $= AC = 2R$



Distance $= ABC = \pi R$

Topic 2 One Dimensional Motion

1. Given, speed,

$$v = b\sqrt{x}$$

Now, differentiating it with respect to time, we get

$$\frac{dv}{dt} = \frac{d}{dt} b\sqrt{x}$$

Now, acceleration

$$\Rightarrow \quad a = \frac{b}{2\sqrt{x}} \cdot \frac{dx}{dt} \qquad \left[\because \frac{dv}{dt} = a\right]$$

$$\Rightarrow \quad a = \frac{b}{2\sqrt{x}} \cdot v = \frac{b}{2\sqrt{x}} \cdot b\sqrt{x} = \frac{b^2}{2}$$

As acceleration is constant, we use

$$v = u + at \qquad \ldots(\text{i})$$

Now, it is given that $x = 0$ at $t = 0$.

So, initial speed of particle is

$$u = b\sqrt{x}\Big|_{x=0} = b \times 0 = 0$$

Hence, when time $t = \tau$, speed of the particle using Eq. (i) is

$$v = u + at = 0 + \frac{b^2}{2} \cdot \tau = \frac{b^2}{2} \cdot \tau$$

2. Given, drag force, $F = m\gamma v^2$ … (i)

As we know, general equation of force

$$= ma \qquad \ldots (\text{ii})$$

Comparing Eqs. (i) and (ii), we get

$$a = \gamma v^2$$

$\therefore$ Net retardation of the ball when thrown vertically upward is

$$a_{\text{net}} = -(g + \gamma v^2) = \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{(g + \gamma v^2)} = -dt \qquad \ldots (\text{iii})$$

By integrating both sides of Eq. (iii) in known limits, i.e.

When the ball thrown upward with velocity v_0 and then reaches to its zenith, i.e. for maximum height at time $t = t$, $v = 0$

$$\Rightarrow \quad \int_{v_0}^{0} \frac{dv}{(\gamma v^2 + g)} = \int_0^t -dt$$

or

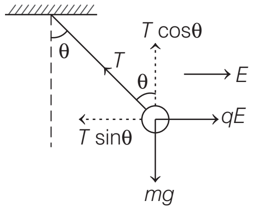
$$\frac{1}{\gamma} \int_{v_0}^{0} \frac{1}{\left[\left(\sqrt{\frac{g}{\gamma}}\right)^2 + v^2\right]} dv = -\int_0^t dt$$

$$\Rightarrow \quad \frac{1}{\gamma} \cdot \frac{1}{\sqrt{g/\gamma}} \cdot \left[\tan^{-1}\left(\frac{v}{\sqrt{g/\gamma}}\right)\right]_{v_0}^{0} = -t$$

$$\left[\because \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{\gamma g}} \cdot \tan^{-1}\left(\frac{\sqrt{\gamma}\, v_0}{\sqrt{g}}\right) = t$$

3. Forces on the bob are as shown



For equilibrium,

$$T\cos\theta = mg \qquad \ldots(\text{i})$$

and

$$T\sin\theta = qE \qquad \ldots(\text{ii})$$

Dividing Eq. (ii) by Eq. (i), we get

$$\tan\theta = \frac{qE}{mg}$$

Here, $q = 5\mu\text{C} = 5 \times 10^{-6}$ C, $E = 2000$ V / m,

$m = 2\text{g} = 2 \times 10^{-3}$ kg, $g = 10\ \text{ms}^{-2}$

$$\therefore \quad \tan\theta = \frac{5 \times 10^{-6} \times 2000}{2 \times 10^{-3} \times 10} = \frac{1}{2} = 0.5$$

So, the angle made by the string of the pendulum with the vertical is

$$\theta = \tan^{-1}(0.5)$$

4. Given, initial position of particle

$$\mathrm{r}_0 = (2\hat{\mathrm{i}} + 4\hat{\mathrm{j}})\mathrm{m},$$

Initial velocity of particle, $\mathrm{u} = (5\hat{\mathrm{i}} + 4\hat{\mathrm{j}})\mathrm{m/s}$

Aceleration of particle, $\mathrm{a} = (4\hat{\mathrm{i}} + 5\hat{\mathrm{j}})\mathrm{m/s^2}$

According to second equation of motion,

position of particle at time t is, $\mathrm{r} = \mathrm{r}_0 + \mathrm{u}t + \frac{1}{2}\mathrm{a}t^2$

At $t = 2$s, position of particle is,

$$\mathrm{r} = (2\hat{\mathrm{i}} + 4\hat{\mathrm{j}}) + (5\hat{\mathrm{i}} + 4\hat{\mathrm{j}}) \times 2 + \frac{1}{2}(4\hat{\mathrm{i}} + 4\hat{\mathrm{j}}) \times 4$$

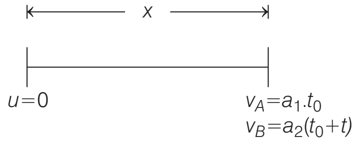
or $\quad \mathrm{r} = (2 + 10 + 8)\hat{\mathrm{i}} + (4 + 8 + 8)\hat{\mathrm{j}}$

$\Rightarrow \quad \mathrm{r} = 20\hat{\mathrm{i}} + 20\hat{\mathrm{j}}$

$\therefore$ Distance of particle from origin is,

$$|\mathrm{r}| = 20\sqrt{2}\ \mathrm{m}$$

5. Let car B takes time $(t_0 + t)$ and car A takes time t_0 to finish the race.



Then,

Given, $$v_A - v_B = v = (a_1 - a_2)t_0 - a_2 t \quad \text{...(i)}$$

$$s_B = s_A = \frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$$

or $$\sqrt{a_1}\ t_0 = \sqrt{a_2}(t_0 + t)$$

or $$\sqrt{a_1}t_0 = \sqrt{a_2}t_0 + \sqrt{a_2}t$$

or $$(\sqrt{a_1} - \sqrt{a_2})\ t_0 = \sqrt{a_2}\ t$$

or $$t_0 = \frac{\sqrt{a_2}\,.\,t}{(\sqrt{a_1} - \sqrt{a_2})} \quad \text{...(ii)}$$

Substituting the value of t_0 from Eq. (ii) into Eq. (i), we get

$$v = (a_1 - a_2)\frac{\sqrt{a_2}\ t}{\sqrt{a_1} - \sqrt{a_2}} - a_2 t$$

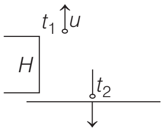
$$= (\sqrt{a_1} - \sqrt{a_2})(\sqrt{a_1} + \sqrt{a_2})\cdot\frac{\sqrt{a_2}t}{(\sqrt{a_1} - \sqrt{a_2})} - a_2 t$$

or $$v = (\sqrt{a_1} + \sqrt{a_2})\cdot\sqrt{a_2}\ t - a_2 t$$

$$= \sqrt{a_1 a_2}\cdot t + a_2 t - a_2 t$$

or $$v = \sqrt{a_1 \cdot a_2}\ t$$

6. Time taken to reach the maximum height, $t_1 = \dfrac{u}{g}$



If t_2 is the time taken to hit the ground, then

i.e. $$-H = ut_2 - \frac{1}{2}gt_2^2$$

But $$t_2 = nu_1 \quad \text{[Given]}$$

So, $$-H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g^2}$$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\frac{n^2u^2}{g}$$

$\Rightarrow$ $$2gH = nu^2(n-2)$$

7. Distance travelled in n^{th} second is, $s_n = u + an - \dfrac{1}{2}a$

Given, $u = 0$

$\therefore$ $$\frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a\,(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$$

$\therefore$ Correct option is (c).

8. $\mathbf{r} = \alpha t^3\hat{\mathbf{i}} + \beta t^2\hat{\mathbf{j}}$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3\alpha t^2\hat{\mathbf{i}} + 2\beta t\hat{\mathbf{j}}$$

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = 6\alpha t\,\hat{\mathbf{i}} + 2\beta\hat{\mathbf{j}}$$

At $t = 1\,\text{s}$,

(a) $\mathbf{v} = 3\times\dfrac{10}{3}\times 1\hat{\mathbf{i}} + 2\times 5\times 1\hat{\mathbf{j}} = (10\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\,\text{m/s}$

(b) $\mathbf{L} = \mathbf{r}\times\mathbf{p} = \left(\dfrac{10}{3}\times 1\hat{\mathbf{i}} + 5\times 1\hat{\mathbf{j}}\right)\times 0.1(10\hat{\mathbf{i}} + 10\hat{\mathbf{j}})$

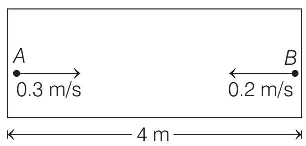
$$= \left(-\frac{5}{3}\hat{\mathbf{k}}\right)\text{N-ms}$$

(c) $\mathbf{F} = m\mathbf{a} = m\times\left(6\times\dfrac{10}{3}\times 1\hat{\mathbf{i}} + 2\times 5\hat{\mathbf{j}}\right) = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{N}$

(d) $\tau = r\times\mathbf{F} = \left(\dfrac{10}{3}\hat{\mathbf{i}} + 5\hat{\mathbf{j}}\right)\times(2\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$$= +\frac{10}{3}\hat{\mathbf{k}} + 10(-\hat{\mathbf{k}}) = \left(\frac{-20}{3}\hat{\mathbf{k}}\right)\text{N-m}$$

9.



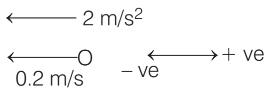
Motion of ball A relative to rocket.

Consider motion of two balls with respect to rocket.

Maximum distance of ball A from left wall

$$= \frac{u^2}{2a} = \frac{0.3\times 0.3}{2\times 2} = \frac{0.09}{4} \approx 0.02\text{ m} \qquad (\text{as } 0 = u^2 - 2aS)$$

So, collision of two balls will take place very near to left wall.



Motion of ball B relative to rocket

For B $\quad S = ut + \dfrac{1}{2}at^2 \;\Rightarrow\; -4 = -0.2\,t - \left(\dfrac{1}{2}\right)2t^2$

Solving this equation, we get,

$$t = 1.9\text{ s}$$

$\therefore$ Nearest Integer = 2 s

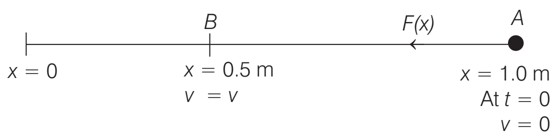
Alternate

$$S_1 = 0.2t + \frac{1}{2}\times 2\times t^2 \;\Rightarrow\; S_2 = 0.3t - \frac{1}{2}\times 2\times t^2$$

$$S_1 + S_2 = 4 \;\Rightarrow\; 0.5t = 4$$

$\therefore$ $$t = 8\text{ s}$$

10. (a) $F(x) = \dfrac{-k}{2x^2}$, k and x^2 both are positive. Hence, $F(x)$ is always negative.



Applying work-energy theorem between points A and B.

Change in kinetic energy between A and B = work done by the force between A and B

$$\therefore \ \frac{1}{2}mv^2 = \int_{x=1.0\text{ m}}^{x=0.5\text{ m}} F\,(dx) = \int_{1.0}^{0.5}\left(\frac{-k}{2x^2}\right)(dx)$$

$$= \frac{-k}{2}\int_{1.0}^{0.5}\frac{dx}{x^2} = \frac{k}{2}\left(\frac{1}{x}\right)_{1.0}^{0.5}$$

$$= \left(\frac{k}{2}\right)\left(\frac{1}{0.5}-\frac{1}{1.0}\right) = \frac{k}{2}$$

$$\therefore \qquad v = \pm\sqrt{\frac{k}{m}}$$

Substituting the values, $v = \pm\sqrt{\dfrac{10^{-2}\text{ N-m}^2}{10^{-2}\text{kg}}} = \pm 1\text{ m/s}$

Therefore, velocity of particle at $x = 1.0$ m is

$$v = -1.0\text{ m/s}$$

Negative sign indicates that velocity is in negative x-direction.

(b) Applying work-energy theorem between any intermediate value $x = x$, we get

$$\frac{1}{2}mv^2 = \int_{1.0}^{x}\frac{-k\,dx}{2x^2} = \frac{k}{2}\left(\frac{1}{x}\right)_{1.0}^{x} = \frac{k}{2}\left(\frac{1}{x}-1\right)$$

$$\therefore \ v^2 = \frac{k}{m}\left(\frac{1}{x}-1\right)$$

$$\therefore \quad v = \sqrt{\frac{1}{x}-1} = \sqrt{\frac{1-x}{x}}$$

$$\frac{k}{m} = \frac{10^{-2}\ \lambda\text{ Nm}^2}{10^{-2}\text{ kg}}$$

but $v = -\left(\dfrac{dx}{dt}\right) = \sqrt{\dfrac{1-x}{x}}$

$$\therefore \qquad \int\sqrt{\frac{x}{1-x}}\,dx = -\int dt$$

or $\displaystyle\int_1^{0.25}\sqrt{\frac{x}{1-x}}\,dx = -\int_0^t dt$

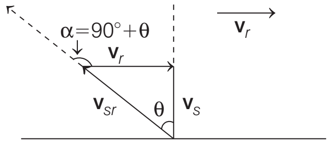
Solving this, we get $t = 1.48$ s.

11. Acceleration due to gravity is independent of the mass of the body.

Topic 3 Relative Motion

1. Let the velocity of the swimmer is $v_s = 4$ km/h and velocity of river is $v_r = 2$ km/h

Also, angle of swimmer with the flow of the river (down stream) is α as shown in the figure below



From diagram, angle θ is

$$\sin\theta = \frac{v_r}{v_{sr}} = \frac{2\text{ km/h}}{4\text{ km/h}} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

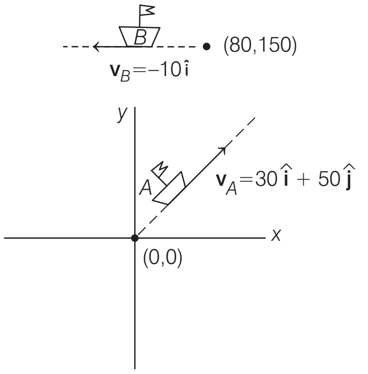
Clearly, $\qquad \alpha = 90^\circ + 30^\circ = 120^\circ$

2. Considering the initial position of ship A as origin, so the velocity and position of ship will be

$$\mathbf{v}_A = (30\hat{\mathbf{i}} + 50\hat{\mathbf{j}})\text{and } \mathbf{r}_A = (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}})$$

Now, as given in the question, velocity and position of ship B will be, $\mathbf{v}_B = -10\hat{\mathbf{i}}$ and $\mathbf{r}_B = (80\hat{\mathbf{i}} + 150\hat{\mathbf{j}})$

Hence, the given situation can be represented graphically as



After time t, coordinates of ships A and B are

$$(80-10t, 150)\text{ and }(30t, 50t).$$

So, distance between A and B after time t is

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d = \sqrt{(80-10t-30t)^2 + (150-50t)^2}$$

$$\Rightarrow \qquad d^2 = (80-40t)^2 + (150-50t)^2$$

Distance is minimum when $\dfrac{d}{dt}(d^2) = 0$

After differentiating, we get

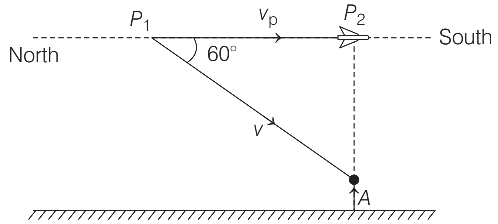
$$\Rightarrow \qquad \frac{d}{dt}[(80-40t)^2 + (150-50t)^2] = 0$$

$$\Rightarrow \qquad 2(80-40t)(-40) + 2(150-50t)(-50) = 0$$

$$\Rightarrow \qquad -3200 + 1600t - 7500 + 2500t = 0$$

$$\Rightarrow \qquad 4100t = 10700 \Rightarrow t = \frac{10700}{4100} = 2.6\text{ h}$$

3. Let P_1 be the position of plane at $t = 0$, when sound waves started towards person A and P_2 is the position of plane observed at time instant t as shown in the figure below.



In triangle P_1P_2A,

P_1P_2 = speed of plane × time = $v_P \times t$

P_1A = speed of sound × time = $v \times t$

Now, from ΔP_1P_2A

$$\cos\theta = \frac{\text{base}}{\text{hypotenues}}$$

$$\cos 60° = \frac{P_1P_2}{P_1A} = \frac{v_P \times t}{v \times t}$$

$$\frac{1}{2} = \frac{v_P}{v}$$

$$v_P = \frac{v}{2}$$

4. When trains are moving in same direction relative speed $= |v_1 - v_2|$ and in opposite direction relative speed $= |v_1 + v_2|$

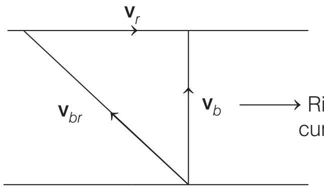
Hence, ratio of time when trains move in same direction with time when trains move in opposite direction is

$$\frac{t_1}{t_2} = \frac{\left(\dfrac{l_1 + l_2}{|v_1 - v_2|}\right)}{\left(\dfrac{l_1 + l_2}{|v_1 + v_2|}\right)} = \frac{|v_1 + v_2|}{|v_1 - v_2|}$$

where, $l_1 + l_2$ = sum of lengths of trains which is same as distance covered by trains to cross each other

So, $$\frac{t_1}{t_2} = \frac{80 + 30}{80 - 30} = \frac{110}{50} = \frac{11}{5}$$

5. Shortest possible path comes when absolute velocity of boatman comes perpendicular to river current as shown in figure.



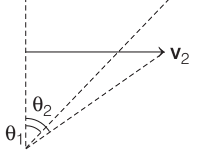
$$t = \frac{\omega}{v_b} = \frac{\omega}{\sqrt{v_{br}^2 - v_r^2}}$$

$$\therefore \quad \frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}}$$

Solving this equation, we get $v_r = 3$ km/ h

6. To cross the river in shortest time, one has to swim perpendicular to the river current.

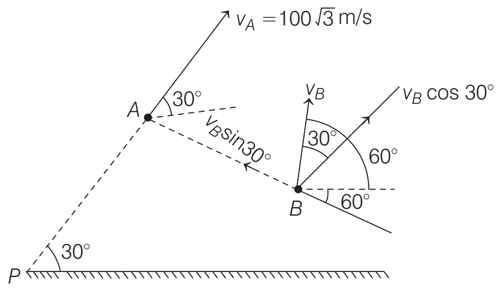
7.



$$\theta_2 > \theta_1 \quad \therefore \quad \omega_2 > \omega_1$$

Statement II, is the formula of relative velocity. But it does not explain Statement I correctly. The correct explanation of statement I is due to visual perception of motion. The object appears to be moving faster, when its angular velocity is greater w.r.t. observer.

8. Relative velocity of B with respect to A is perpendicular to line PA. Therefore, along line PA, velocity components of A and B should be same.



$$\Rightarrow \quad v_A = 100\sqrt{3} = v_B \cos 30°$$

$$\Rightarrow \quad v_B = 200 \text{ m/s}$$

$$t_0 = \frac{500}{v_B \sin 30°} = \frac{500}{200 \times 1/2} = 5 \text{ s}$$

9. (a) Let A stands for trolley and B for ball.

Relative velocity of B with respect to A ($\mathbf{v}_{BA}$) should be along OA for the ball to hit the trolley. Hence, $\mathbf{v}_{BA}$ will make an angle of 45° with positive X-axis.

$$\theta = 45°$$

(b) Let v = absolute velocity of ball.

$$\phi = \frac{4\theta}{3} = \frac{4}{3}(45°) = 60° \rightarrow \text{with } X\text{-axis}$$

$$\therefore \quad \mathbf{v}_B = (v\cos\theta)\,\hat{\mathbf{i}} + (v\sin\theta)\,\hat{\mathbf{j}}$$

$$= \frac{v}{2}\,\hat{\mathbf{i}} + \frac{\sqrt{3}\,v}{2}\,\hat{\mathbf{j}}$$

$$\mathbf{v}_A = (\sqrt{3} - 1)\,\hat{\mathbf{j}}$$

$$\therefore \quad \mathbf{v}_{BA} = \frac{v}{2}\,\hat{\mathbf{i}} + \left(\frac{\sqrt{3}v}{2} - \sqrt{3} + 1\right)\hat{\mathbf{j}}$$

Since, $\mathbf{v}_{BA}$ is at 45°

$$\therefore \quad \frac{v}{2} = \frac{\sqrt{3}v}{2} - \sqrt{3} + 1$$

or $\quad v = 2$ m/s

Topic 4 Graphs

1. Since, the particle starts from rest, this means, initial velocity, $u = 0$

Also, it moves with uniform acceleration along positive X-axis. This means, its acceleration (a) is constant.

$\therefore$ Given, a - t graph in (A) is correct.

As we know, for velocity-time graph, slope = acceleration.

Since, the given v-t graph in (B) represents that its slope is constant and non-zero.

$\therefore$ Graph in (B) is also correct.

Also, the displacement of such a particle w.r.t. time is given by

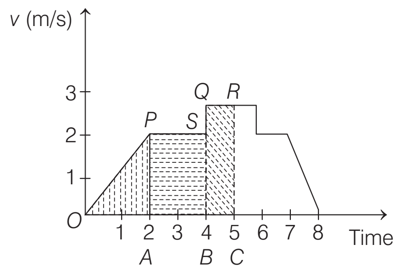
$$x = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2}at^2 \Rightarrow x \propto t^2$$

So, x *versus* t graph would be a parabola with starting from origin.

This is correctly represented in displacement-time graph given in (D).

2. **Key Idea** Area under the velocity-time curve represents displacement.

To get exact position at $t = 5$ s, we need to calculate area of the shaded part in the curve as shown below



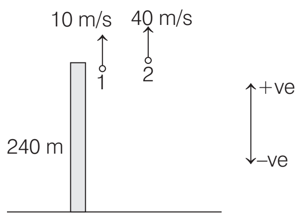
$\therefore$ Displacement of particle = Area of triangle OPA + Area of square $PABSP$ + Area of rectangle $QBCRQ$

$$= \left(\frac{1}{2} \times 2 \times 2\right) + (2 \times 2) + (3 \times 1)$$
$$= 2 + 4 + 3 = 9 \text{ m}$$

3. In graph (c) initial slope is zero which is not possible, since initial velocity is not zero. In all other three graphs: initial velocity is not zero.

4. Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

5.



Let us first find, time of collision of two particles with ground in putting proper values in the equation

$$S = ut + \frac{1}{2}at^2$$
$$-240 = 10t_1 - \frac{1}{2} \times 10 \times t_1^2$$

Solving, we get the position value of $t_1 = 8$ sec

Therefore, the first particle will strike the ground at 8 sec.

Similarly,

$$-240 = 40t_2 - \frac{1}{2} \times 10 \times t_2^2$$

Solving this equation, we get positive value of $t_2 = 12$ sec

Therefore, second particle strikes the ground at 12 sec.

If y is measured from ground. Then,

from 0 to 8 sec

$$y_1 = 240 + S_1$$
$$= 240 + u_1 t + \frac{1}{2}a_1 t^2$$

or $$y_1 = 240 + 10t - \frac{1}{2} \times 10 \times t^2$$

Similarly, $$y_2 = 240 + 40t - \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow \quad t_2 - y_1 = 30t$$

$\therefore$ $(y_2 - y_1)$ versus t graph is a straight line passing through origin

At $t = 8$ sec, $y_2 - y_1 = 240$ m

From 8 sec to 12 sec

$$y_1 = 0$$
$$\Rightarrow \quad y_2 = 240 + 40t - \frac{1}{2} \times 10 \times t^2$$
$$= 240 + 40t - 5t$$
$$\therefore \quad (y_2 - y_1) = 240 + 40t - 5t^2$$

Therefore, $(y_2 - y_1)$ *versus* t graph is parabolic, substituting the values we can check that at $t = 8$ sec, $y_2 - y_1$ is 240 m and at $t = 12$ sec, $y_2 - y_1$ is zero.

6. The v-x equation from the given graph can be written as

$$v = \left(-\frac{v_0}{x_0}\right)x + v_0 \quad \text{...(i)}$$

$$a = \frac{dv}{dt} = \left(-\frac{v_0}{x_0}\right)\frac{dx}{dt} = \left(-\frac{v_0}{x_0}\right)v$$

Substituting v from Eq. (i) we get,

$$a = \left(-\frac{v_0}{x_0}\right)\left[\left(-\frac{v_0}{x_0}\right)x + v_0\right] \Rightarrow a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus, a-x graph is a straight line with positive slope and negative intercept.

7. Area under acceleration-time graph gives the change in velocity.

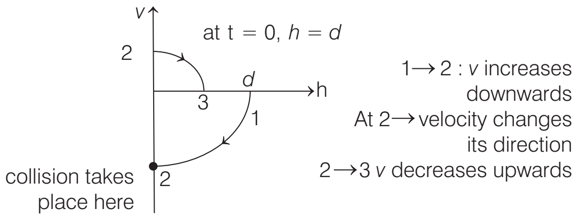
Hence, $v_{max} = 1/2 \times 10 \times 11 = 55$ m/s

8. (i) For uniformly accelerated/decelerated motion

$$v^2 = u^2 \pm 2gh$$

i.e. v - h graph will be a parabola (because equation is quadratic).

(ii) Initially velocity is downwards (−ve) and then after collision, it reverses its direction with lesser magnitude, i.e. velocity is upwards (+ve). Graph (a) satisfies both these conditions.



NOTE At time $t = 0$ corresponds to the point on the graph where $h = d$.

9. Since, the body is at rest at $x = 0$ and $x = 1$. Hence, α cannot be positive for all time in the interval $0 \leq t \leq 1$.

Therefore, first the particle is accelerated and then retarded.

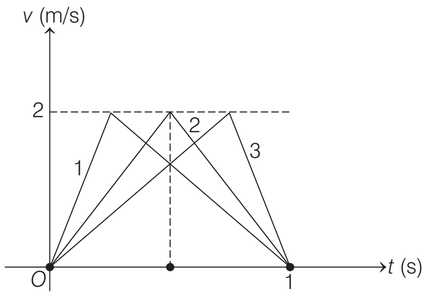
Now, total time $t = 1\,\text{s}$ (given)

Total displacement, $s = 1\,\text{m}$ (given)

$$s = \text{Area under } v\text{-}t \text{ graph}$$

$\therefore$ Height or $v_{max} = \dfrac{2s}{t} = 2\,\text{m/s}$ is also fixed.

$$\left[\text{Area or } s = \frac{1}{2} \times t \times v_{max}\right]$$



If height and base are fixed, area is also fixed .

In case 2 : Acceleration = Retardation = $4\ \text{m/s}^2$

In case 1 : Acceleration > $4\ \text{m/s}^2$ while

Retardation < $4\,\text{m/s}^2$

While in case 3 : Acceleration < $4\,\text{m/s}^2$ and

Retardation > $4\,\text{m/s}^2$

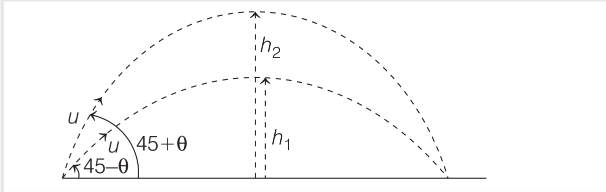
Hence, $|\alpha| \geq 4$ at some point or points in its path.

Topic 5 Projectile Motion

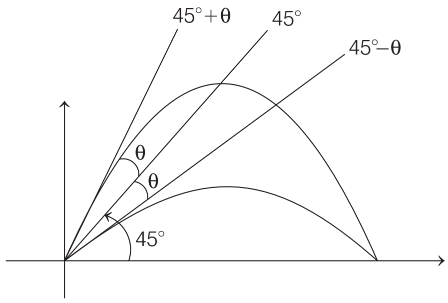
1 **Key Idea** Same range for two particles thrown with some initial speed may occurs when they are projected at complementary angle.

$\therefore$ $$\theta_1 + \theta_2 = 90°$$

where, θ_1 and θ_2 are angles of projection.



As maximum range occurs at $\theta = 45°$ for a given initial projection speed, we take angles of projection of two particles as



$$\theta_1 = 45° + \theta, \theta_2 = 45° - \theta$$

where, θ is angle of projectiles with 45° line. So, range of projectiles will be

$$R = R_1 = R_2 = \frac{u^2 \sin 2(\theta_1)}{g}$$

$$\Rightarrow R = \frac{u^2 \sin 2(45° + \theta)}{g} \Rightarrow R = \frac{u^2 \sin(90° + 2\theta)}{g}$$

$$\Rightarrow R = \frac{u^2 \cos 2\theta}{g} \qquad \Rightarrow R^2 = \frac{u^4 \cos^2 2\theta}{g^2} \qquad \ldots\text{(i)}$$

Maximum heights achieved in two cases are

$$h_1 = \frac{u^2 \sin^2(45° + \theta)}{2g}$$

and $$h_2 = \frac{u^2 \sin^2(45° - \theta)}{2g}$$

So, $$h_1 h_2 = \frac{u^4 \sin^2(45° + \theta) \sin^2(45° - \theta)}{4g^2}$$

Using $2\sin A \cdot \sin B = \cos(A - B) - \cos(A + B)$, we have

$$\sin(45° + \theta)\sin(45° - \theta) = \frac{1}{2}(\cos 2\theta - \cos 90°)$$

$$\Rightarrow \sin(45° + \theta)\sin(45° - \theta) = \frac{\cos 2\theta}{2} \qquad [\because \cos 90° = 0]$$

So, we have

$$h_1 h_2 = \frac{u^4 \left(\dfrac{\cos 2\theta}{2}\right)^2}{4g^2} \Rightarrow h_1 h_2 = \frac{u^4 \cos^2 2\theta}{16g^2} \qquad \ldots\text{(ii)}$$

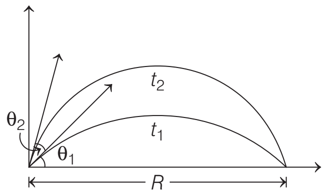
From Eqs. (i) and (ii), we get

$$\Rightarrow \qquad h_1 h_2 = \frac{R^2}{16} \Rightarrow R^2 = 16\, h_1 h_2$$

2. **Key Idea** Range of a projectile motion is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Time of flight is given by $t = \dfrac{2u\sin\theta}{g}$



Given, range of the fired shell,

$$R = R$$

and time of flights are t_1 and t_2.

Let θ_1 and θ_2 are the two angles at which shell is fired. As, range in both cases is same, i.e.

$$R_1 = R_2 = R$$

Here, $$R_1 = \frac{u^2 \sin 2\theta_1}{g}$$

and $$R_2 = \frac{u^2 \sin 2\theta_2}{g}$$

$$\Rightarrow \quad R = \frac{u^2 \sin 2\theta_1}{g} = \frac{u^2 \sin 2\theta_2}{g} \quad \ldots\text{(i)}$$

$$\Rightarrow \quad \sin 2\theta_1 = \sin 2\theta_2$$

$$\Rightarrow \quad \sin 2\theta_1 = \sin(180 - 2\theta_2)\ [\because \sin(180-\theta) = \sin\theta]$$

$$\Rightarrow \quad 2(\theta_1 + \theta_2) = 180 \text{ or } \theta_1 + \theta_2 = 90$$

$$\Rightarrow \quad \theta_2 = 90 - \theta_1 \quad \ldots\text{(ii)}$$

So, time of flight in first case,

$$t_1 = \frac{2u\sin\theta_1}{g} \quad \ldots\text{(iii)}$$

and time of flight in second case,

$$t_2 = \frac{2u\sin\theta_2}{g} = \frac{2u\sin(90-\theta_1)}{g} = \frac{2u\cos\theta_1}{g} \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$t_1 t_2 = \frac{2u\sin\theta_1}{g} \times \frac{2u\cos\theta_1}{g}$$

$$\Rightarrow \quad t_1 t_2 = \frac{4u^2 \sin\theta_1 \cos\theta_1}{g^2}$$

$$\Rightarrow \quad t_1 t_2 = \frac{2u^2 \sin 2\theta_1}{g^2} \quad (\because \sin 2\theta = 2\sin\theta\cos\theta) \ldots\text{(v)}$$

From Eq. (i), we get

$$R = \frac{u^2 \sin 2\theta_1}{g}$$

$$\therefore \quad t_1 t_2 = \frac{2R}{g}$$

3. Given, $g = 10\,\text{m/s}^2$

Equation of trajectory of the projectile,

$$y = 2x - 9x^2 \quad \ldots\text{(i)}$$

In projectile motion, equation of trajectory is given by

$$y = x\tan\theta_0 - \frac{g x^2}{2v_0^2 \cos^2\theta_0} \quad \ldots\text{(ii)}$$

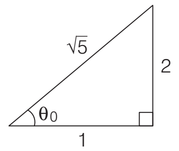
By comparison of Eqs. (i) and (ii), we get

$$\tan\theta_0 = 2 \quad \ldots\text{(iii)}$$

and $$\frac{g}{2v_0^2\cos^2\theta_0} = 9 \text{ or } v_0^2 = \frac{g}{9 \times 2\cos^2\theta_0} \quad \ldots\text{(iv)}$$

From Eq. (iii), we can get value of $\cos\theta$ and $\sin\theta$

$$\cos\theta_0 = \frac{1}{\sqrt{5}} \text{ and } \sin\theta_0 = \frac{2}{\sqrt{5}} \quad \ldots\text{(v)}$$



Using value of $\cos\theta_0$ from Eq. (v) to Eq. (iv), we get

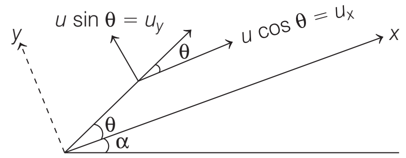
$$v_0^2 = \frac{10 \times (\sqrt{5})^2}{2 \times (1)^2 \times 9} = \frac{10 \times 5}{2 \times 9}$$

$$\Rightarrow \quad v_0^2 = \frac{25}{9} \text{ or } v_0 = \frac{5}{3}\,\text{m/s} \quad \ldots\text{(vi)}$$

From Eq. (v), we get

$$\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

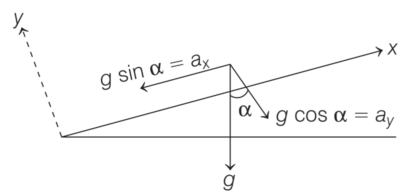
4. When a projectile is projected at an angle θ with an inclined plane making angle α with the horizontal, then



Components of u along and perpendicular to plane are

$$u_x = u\cos\theta \quad \text{and} \quad u_y = u\sin\theta$$

We can also resolve acceleration due to gravity into its components along and perpendicular to plane as shown below



So, we can now apply formula for range, i.e. net horizontal displacement of the particle as

$$R = u_x T + \frac{1}{2} a_x T^2 \quad \ldots\text{(i)}$$

where, T = time of flight.

Using formula for time of flight, we have

$$T = \frac{2u_y}{a_y} = \frac{2u\sin\theta}{g\cos\alpha} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

Range up the inclined plane is

$$R = u_x T + \frac{1}{2}a_x T^2$$

$$= u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

Here, $u = 2\ \text{ms}^{-1}$, $g = 10\ \text{ms}^{-2}$, $\theta = 15°$, $\alpha = 30°$

So, $\quad T = \dfrac{2u\sin\theta}{g\cos\alpha} = \dfrac{2\times 2\sin 15°}{10\times\cos 30°}$

$$= \frac{2\times 2\times 0.258\times 2}{10\times 1.732} = 0.1191$$

Now,

$$R = 2\times\cos 15°\times 0.1191 - \frac{1}{2}\times 10\sin 30°\,(0.1191)^2$$

$$= 2\times 0.965\times 0.1191 - \frac{5}{2}(0.1191)^2$$

$$= 0.229 - 0.0354 = 0.1936\ \text{m} \approx 0.20\ \text{m} = 20\ \text{cm}$$

5. Position vector of particle is given as

$$\mathbf{r} = 15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}$$

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}[15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}]$$

$$= 30t\,\hat{\mathbf{i}} - 40\,t\,\hat{\mathbf{j}}$$

Acceleration of particle is

$$\mathbf{a} = \frac{d}{dt}(\mathbf{v}) = \frac{d}{dt}(30t\hat{\mathbf{i}} - 40t\hat{\mathbf{j}}) = 30\hat{\mathbf{i}} - 40\hat{\mathbf{j}}$$

So, magnitude of acceleration at t = 1s is

$$|\mathbf{a}|_{t=1\,\text{s}} = \sqrt{a_x^2 + a_y^2} = \sqrt{30^2 + 40^2}$$

$$= 50\ \text{ms}^{-2}$$

6. Position of particle is, $x(t) = at + bt^2 - ct^3$

So, its velocity is, $v = \dfrac{dx}{dt} = a + 2bt - 3ct^2$

and acceleration is, $a = \dfrac{dv}{dt} = 2b - 6ct$

Acceleration is zero, then $2b - 6ct = 0$

$\Rightarrow \quad t = \dfrac{2b}{6c} = \dfrac{b}{3c}$

Substituting this 't' in expression of velocity, we get

$$v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = a + \frac{2b^2}{3c} - \frac{b^2}{3c} = a + \frac{b^2}{3c}$$

7. Components of velocity at an instant of time t of a body projected at an angle θ is

$$v_x = u\cos\theta + g_x t \text{ and } \quad v_y = u\sin\theta + g_y t$$

Here, components of velocity at $t = 1$ s, is

$$v_x = u\cos 60° + 0 \qquad [\text{as } g_x = 0]$$

$$= 10\times\frac{1}{2} = 5\ \text{m/s}$$

and $\quad v_y = u\sin 60° + (-10)\times(1)$

$$= 10\times\frac{\sqrt{3}}{2} + (-10)\times(1) = 5\sqrt{3} - 10$$

$\Rightarrow \quad |v_y| = |10 - 5\sqrt{3}|$ m/s

Now, angle made by the velocity vector at time of $t = 1$ s

$$|\tan\alpha| = \left|\frac{v_y}{v_x}\right| = \frac{|10 - 5\sqrt{3}|}{5}$$

$\Rightarrow \qquad \tan\alpha = |2 - \sqrt{3}|$ or $\alpha = 15°$

$\therefore$ Radius of curvature of the trajectory of the projected body

$$R = v^2/g\cos\alpha = \frac{(5)^2 + (10 - 5\sqrt{3})^2}{10\times 0.97}$$

$$[\because v^2 = v_x^2 + v_y^2 \text{ and } \cos 15° = 0.97]$$

$\Rightarrow \qquad R = 2.77\ \text{m} \approx 2.8\ \text{m}$

8. Bullets from guns can reach upto a distance of maximum range which occurs when projection is made at angle of 45°.

Maximum range for gun

$$A = R_1 = \frac{u_1^2}{g}$$

Maximum range for gun

$$B = R_2 = \frac{u_2^2}{g}$$

So, ratio of covered areas

$$= \frac{\pi(R_1)^2}{\pi(R_2)^2} = \frac{\pi\cdot u_1^4/g^2}{\pi\cdot u_2^4/g^2} = \frac{u_1^4}{u_2^4}$$

Here, $u_1 = 1$ km/s and $u_2 = 2$ km/s

So, ratio of areas $= \dfrac{1^4}{2^4} = \dfrac{1}{16} = 1:16$

9. Initial velocity $= (\mathbf{i} + 2\mathbf{j})$ m/s

Magnitude of initial velocity, $u = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$ m/s

Equation of trajectory of projectile is

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta) \qquad \left[\tan\theta = \frac{y}{x} = \frac{2}{1} = 2\right]$$

$$\therefore \quad y = x\times 2 - \frac{10(x)^2}{2(\sqrt{5})^2}[1 + (2)^2]$$

$$= 2x - \frac{10(x^2)}{2\times 5}(1 + 4)$$

$$= 2x - 5x^2$$

10. $y = ax - bx^2$

For height (or y) to be maximum.

$$\frac{dy}{dx} = 0 = a - 2bx \qquad \ldots\text{(i)}$$

$\therefore \qquad x = \dfrac{a}{2b}$

(i) $\therefore \quad y_{max}$ = maximum height

$$H = a\,(a/2b) - b\,(a/2b)^2$$

$$H = a^2/4b$$

(ii) $\left(\dfrac{dy}{dx}\right)_{(x=0)} = a = \tan\theta_0$

where, θ_0 is the angle of projection.

$\therefore \qquad \theta_0 = \tan^{-1}(a)$

11. From conservation of mechanical energy at highest point, potential energy is maximum. Therefore, kinetic energy and hence speed should be minimum.

12. $t = T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 10\times \sin 60°}{10} = \sqrt{3}$ s

Displacement of train in time $t = \dfrac{1}{2}at^2$

Displacement of boy with respect to train = 1.15 m

$\therefore$ Displacement of boy with respect to ground

$$= \left(1.15 + \frac{1}{2}at^2\right)$$

Displacement of ball with respect to ground = $(u\cos 60°)\,t$

To catch the ball back at initial height,

$$1.15 + \frac{1}{2}at^2 = (u\cos 60°)\,t$$

$$\therefore \qquad 1.15 + \frac{1}{2}a\,(\sqrt{3})^2 = 10\times\frac{1}{2}\times\sqrt{3}$$

Solving this equation, we get

$$a = 5\ \text{m/s}^2$$

13. Let t be the time after which the stone hits the object and θ be the angle which the velocity vector **u** makes with horizontal. According to question, we have following three conditions.

Vertical displacement of stone is 1.25 m.

$$1.25 = (u\sin\theta)\,t - \frac{1}{2}gt^2$$

where, $\qquad g = 10\ \text{m/s}^2$

or $\qquad (u\sin\theta)\,t = 1.25 + 5t^2 \qquad$ ..(i)

Horizontal displacement of stone

= 3 + displacement of object A

$\therefore \qquad (u\cos\theta)\,t = 3 + \dfrac{1}{2}at^2,$

where, $a = 1.5\ \text{m/s}^2$

or $\qquad (u\cos\theta)\,t = 3 + 0.75\,t^2 \qquad$ ...(ii)

Horizontal component of velocity (of stone) = vertical component (because velocity vector is inclined at 45° with horizontal)

$\therefore \qquad (u\cos\theta) = gt - (u\sin\theta) \qquad$ …(iii)

(The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion. Therefore, $gt > u\sin\theta$. In upward motion $u\sin\theta > gt$).

Multiplying Eq. (iii) with t, we can write

or $\qquad (u\cos\theta)\,t + (u\sin\theta)\,t = 10t^2 \qquad$ ....(iv)

Now Eqs. (iv), (ii) and (i) gives

$$4.25t^2 - 4.25 = 0$$

or $\qquad t = 1$s

Substituting $t = 1$s in Eqs. (i) and (ii), we get

$$u\sin\theta = 6.25\ \text{m/s}$$

or $\qquad u_y = 6.25$ m/s

and $\qquad u\cos\theta = 3.75$ m/s

or $\qquad u_x = 3.75$ m/s

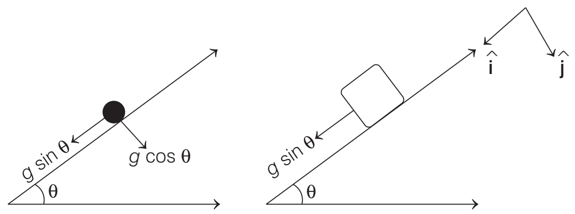
Therefore, $\qquad \mathbf{u} = u_x\hat{\mathbf{i}} + u_y\hat{\mathbf{j}}$ m/s

or $\qquad \mathbf{u} = (3.75\hat{\mathbf{i}} + 6.25\,\hat{\mathbf{j}})$ m/s

NOTE Most of the problems of projectile motion are easily solved by breaking the motion of the particle in two suitable mutually perpendicular directions, say x and y. Find u_x, u_y, a_x and a_y and then apply

$$v_x = u_x + a_x t;\ s_y = u_y t + \frac{1}{2}a_y t^2 \text{ etc.}$$

14. (a) Accelerations of particle and block are shown in figure.

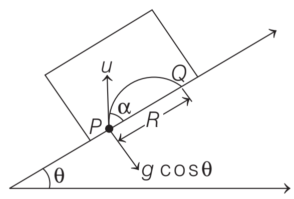


Acceleration of particle with respect to block

= acceleration of particle − acceleration of block

$= (g\sin\theta\,\hat{\mathbf{i}} + g\cos\theta\,\hat{\mathbf{j}}) - (g\sin\theta)\,\hat{\mathbf{i}} = g\cos\theta\,\hat{\mathbf{j}}$

Now, motion of particle with respect to block will be a projectile as shown.

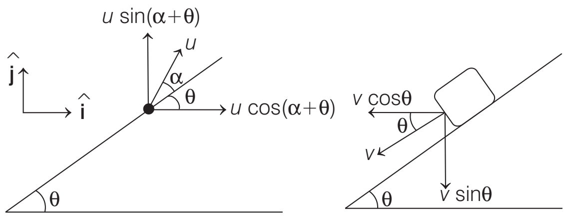


The only difference is, g will be replaced by $g\cos\theta$

$\therefore \qquad PQ = \text{Range}(R) = \dfrac{u^2\sin 2\alpha}{g\cos\theta}$

$\Rightarrow \qquad PQ = \dfrac{u^2\sin 2\alpha}{g\cos\theta}$

(b) Horizontal displacement of particle with respect to ground is zero. This implies that initial velocity with respect to ground is only vertical or there is no horizontal component of the absolute velocity of the particle.



Let v be the velocity of the block down the plane.

Velocity of particle

$$= u\cos(\alpha+\theta)\,\hat{\mathbf{i}} + u\sin(\alpha+\theta)\,\hat{\mathbf{j}}$$

Velocity of block $= -v\cos\theta\,\hat{\mathbf{i}} - v\sin\theta\,\hat{\mathbf{j}}$

$\therefore$ Velocity of particle with respect to ground

$$= \{u\cos(\alpha+\theta) - v\cos\theta\}\,\hat{\mathbf{i}}$$

$$+ \{u\sin(\alpha+\theta) - v\sin\theta\}\,\hat{\mathbf{j}}$$

Now, as we said earlier that horizontal component of absolute velocity should be zero.

Therefore, $u\cos(\alpha+\theta) - v\cos\theta = 0$

or $\quad v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ (down the plane)

15. (a) Let $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ be the unit vectors along x, y and z-directions respectively.

Given, $\quad \mathbf{v}_{\text{cart}} = 4\hat{\mathbf{i}}$ m/s

$$\therefore \quad \mathbf{v}_{\text{stone, cart}} = (6\sin 30^\circ)\hat{\mathbf{j}} + (6\cos 30^\circ)\hat{\mathbf{k}}$$

$$= (3\,\hat{\mathbf{j}} + 3\sqrt{3}\hat{\mathbf{k}})\ \text{m/s}$$

$$\therefore \quad \mathbf{v}_{\text{stone}} = \mathbf{v}_{\text{stone, cart}} + \mathbf{v}_{\text{cart}}$$

$$= (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\sqrt{3}\,\hat{\mathbf{k}})\ \text{m/s}$$

This is the absolute velocity of stone (with respect to ground). At highest point of its trajectory, the vertical component (z) of its velocity will become zero, whereas the x and y-components will remain unchanged. Therefore, velocity of stone at highest point will be,

$$\mathbf{v} = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\ \text{m/s}$$

or speed at highest point,

$$v = |\mathbf{v}| = (\sqrt{(4)^2 + (3)^2}\ \text{m/s} = 5\ \text{m/s}$$

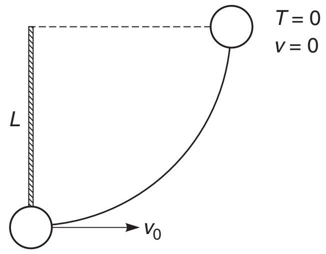
Now, applying law of conservation of linear momentum, let v_0 be the velocity of combined mass after collision.

Then, $\quad mv = (2m)\,v_0$

$$\therefore \quad v_0 = \frac{v}{2} = \frac{5}{2}\ \text{m/s} = 2.5\ \text{m/s}$$

$\therefore$ Speed of combined mass just after collision is 2.5 m/s.

(b) Tension in the string becomes zero at horizontal position. It implies that velocity of combined mass also becomes zero in horizontal position. Applying conservation of energy, we have



$$0 = v_0^2 - 2\,gL$$

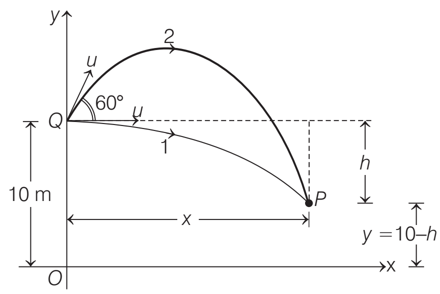
$$\therefore \quad L = \frac{v_0^2}{2g} = \frac{(2.5)^2}{2\,(9.8)} = 0.32\ \text{m}$$

Hence, length of the string is 0.32 m.

16. $u = 5\sqrt{3}$ m/s

$$\therefore \quad u\cos 60^\circ = (5\sqrt{3})\left(\frac{1}{2}\right)\text{m/s} = 2.5\sqrt{3}\text{m/s}$$

and $\quad u\sin 60^\circ = (5\sqrt{3})\left(\dfrac{\sqrt{3}}{2}\right)\text{m/s} = 7.5\ \text{m/s}$



Since, the horizontal displacement of both the shots are equal. The second should be fired early because its horizontal component of velocity $u\cos 60^\circ$ or $2.5\sqrt{3}$ m/s is less than the other which is u or $5\sqrt{3}$ m/s.

Now, let first shot takes t_1 time to reach the point P and the second t_2. Then,

$$x = (u\cos 60^\circ)\,t_2 = ut_1$$

or $\quad x = 2.5\sqrt{3}\,t_2 = 5\sqrt{3}\,t_1 \quad$ ...(i)

or $\quad t_2 = 2t_1 \quad$ ...(ii)

and $\quad h = |(u\sin 60^\circ)t_2 - \dfrac{1}{2}gt_2^2| = \dfrac{1}{2}gt_1^2$

or $\quad h = \dfrac{1}{2}gt_2^2 - (u\sin 60^\circ)\,t_2 = \dfrac{1}{2}gt_1^2$

Taking $\quad g = 10\ \text{m/s}^2$

$$h = 5t_2^2 - 7.5\,t_2 = 5t_1^2 \quad \text{...(iii)}$$

Substituting $t_2 = 2t_1$ in Eq. (iii), we get

$$5(2t_1)^2 - 7.5\,(2t_1) = 5t_1^2 \text{ or } 15t_1^2 = 15t_1$$

$\Rightarrow \quad t_1 = 1\text{s}$

and $\quad t_2 = 2t_1 = 2\text{s}$

$$x = 5\sqrt{3}\,t_1 = 5\sqrt{3}\ \text{m} \quad \text{[From Eq. (i)]}$$

and $\quad h = 5t_1^2 = 5\,(1)^2 = 5\text{ m} \quad$ [From Eq. (iii)]

$\therefore \quad y = 10 - h = (10 - 5) = 5\text{ m}$

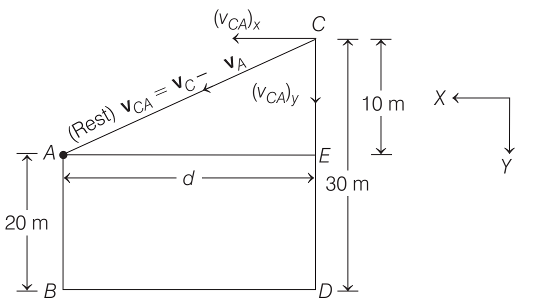
Hence,

(a) Time interval between the firings $= t_2 - t_1 = (2 - 1)\text{ s}$

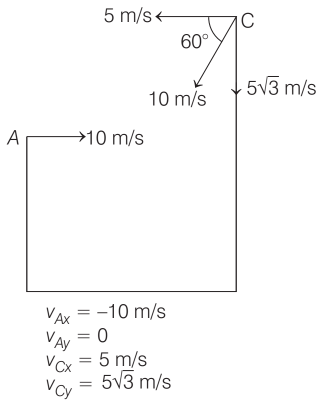
$$\Delta t = 1\text{ s}$$

(b) Coordinates of point $P = (x, y) = (5\sqrt{3}\text{ m}, 5\text{ m})$

17. (a) Acceleration of A and C both is 9.8 m/s^2 downwards. Therefore, relative acceleration between them is zero i.e. the relative motion between them will be uniform.



Now, assuming A to be at rest, the condition of collision will be that $\mathbf{v}_{CA} = \mathbf{v}_C - \mathbf{v}_A =$ relative velocity of C w.r.t. A should be along CA.



$$\therefore \quad \frac{(v_{CA})_y}{(v_{CA})_x} = \frac{CE}{AE} = \frac{10}{d}$$

$$\text{or} \quad \frac{v_{Cy} - v_{Ay}}{v_{Cx} - v_{Ax}} = \frac{10}{d} \quad \text{or} \quad \frac{5\sqrt{3} - 0}{5 - (-10)} = \frac{10}{d}$$

$$\therefore \quad d = 10\sqrt{3}\text{ m} \approx 17.32\text{ m}$$

(b) Time of collision, $t = \dfrac{AC}{|\mathbf{v}_{CA}|}$

$$|\mathbf{v}_{CA}| = \sqrt{(v_{CAx}^2) + (v_{CAy})^2}$$

$$= \sqrt{\{5 - (-10)\}^2 + \{5\sqrt{3} - 0\}^2} = 10\sqrt{3}\text{m/s}$$

$$CA = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20\text{ m}$$

$$\therefore \quad t = \frac{20}{10\sqrt{3}} = \frac{2}{\sqrt{3}}\text{ s}$$

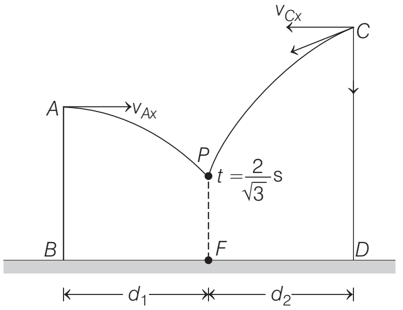
Horizontal (or x) component of momentum of A, i.e. $p_{Ax} = mv_{Ax} = -10\text{m}$.

Similarly, x component of momentum of C, i.e.

$$p_{Cx} = (2m)v_{Cx} = (2m)(5) = +10\text{ m}$$

Since, $\quad p_{Ax} + p_{Cx} = 0$

i.e. x-component of momentum of combined mass after collision will also be zero, i.e. the combined mass will have momentum or velocity in vertical or y-direction only.



Hence, the combined mass will fall at point F just below the point of collision P.

Here $\quad d_1 = |(v_{Ax})|\,t = (10)\dfrac{2}{\sqrt{3}} = 11.55\text{ m}$

$\therefore \quad d_2 = (d - d_1) = (17.32 - 11.55) = 5.77\text{m}$

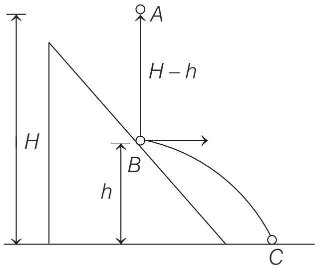
d_2 should also be equal to

$$|v_{Cx}|\,t = (5)\left(\frac{2}{\sqrt{3}}\right) = 5.77\text{ m}$$

Therefore, position from B is d_1 i.e. 11.55 m and from D is d_2 or 5.77 m.

18. Let t_1 be the time from A to B and t_2 the time from B to C.

Then, $t_1 = \sqrt{\dfrac{2(H-h)}{g}}$ and $t_2 = \sqrt{\dfrac{2h}{g}}$



Then, the total time

$$t = t_1 + t_2 = \sqrt{\frac{2}{g}}\,[\sqrt{H-h} + \sqrt{h}]$$

For t to be maximum $\dfrac{dt}{dh} = 0$

$$\text{or} \quad \sqrt{\frac{2}{g}}\left[\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right] = 0$$

or $$\frac{1}{\sqrt{h}} = \frac{1}{\sqrt{H-h}}$$

or $$2h = H \quad \Rightarrow \quad \therefore \quad \frac{h}{H} = \frac{1}{2}$$

19. (a) Range of both the particles is

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(49)^2 \sin 90^\circ}{9.8}$$

By symmetry we can say that they will collide at highest point.

Just before collision		Just after collision	
$u \cos 45^\circ$ →	$u \cos 45^\circ$ ←	$u \cos 45^\circ$ ←	v →
• 20g	• 40g	• 20g	• 40g

Let v be the velocity of Q just after collision. Then, from conservation of linear momentum, we have

$$20\,(u \cos 45^\circ) - 40\,(u \cos 45^\circ) = 40\,(v) - 20\,(u \cos 45^\circ)$$

$$\therefore \qquad v = 0$$

i.e. particle Q comes to rest. So, particle Q will fall vertically downwards and will strike just midway between A and B.

(b) Maximum height,

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(49)^2 \sin^2 45^\circ}{2 \times 9.8} = 61.25\,\text{m}$$

Therefore, time taken by Q to reach the ground,

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 61.25}{9.8}} = 3.53 \text{ s}$$

Topic 6 Miscellaneous Problems

1. For vector $\mathbf{A}_1 + \mathbf{A}_2$, we have

$$|\mathbf{A}_1 + \mathbf{A}_2|^2 = (\mathbf{A}_1 + \mathbf{A}_2) \cdot (\mathbf{A}_1 + \mathbf{A}_2) \qquad [\because \mathbf{x} \cdot \mathbf{x} = |\mathbf{x}|^2]$$

$$\Rightarrow |\mathbf{A}_1 + \mathbf{A}_2|^2 = |\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2\mathbf{A}_1 \cdot \mathbf{A}_2$$

Given, $|\mathbf{A}_1| = 3$, $|\mathbf{A}_2| = 5$ and $|\mathbf{A} + \mathbf{A}_2| = 5$

So, we have

$$(5)^2 = 9 + 25 + 2\mathbf{A}_1 \cdot \mathbf{A}_2 \Rightarrow \mathbf{A}_1 \cdot \mathbf{A}_2 = -\frac{9}{2}$$

Now, $(2\mathbf{A}_1 + 3\mathbf{A}_2) \cdot (3\mathbf{A}_1 - 2\mathbf{A}_2)$

$$= 6|\mathbf{A}_1|^2 - 4\mathbf{A}_1 \cdot \mathbf{A}_2 + 9\mathbf{A}_1 \cdot \mathbf{A}_2 - 6|\mathbf{A}_2|^2$$

$$= 6|\mathbf{A}_1|^2 - 6|\mathbf{A}_2|^2 + 5\mathbf{A}_1 \cdot \mathbf{A}_2$$

Substituting values, we have

$$(2\mathbf{A}_1 + 3\mathbf{A}_2) \cdot (3\mathbf{A}_1 - 2\mathbf{A}_2)$$
$$= 6(9) - 6(25) + 5\left(-\frac{9}{2}\right) = -118.5$$

Alternate Solution

As we know, $|\mathbf{A}_1 + \mathbf{A}_2|$ can also be written as

$$|\mathbf{A} + \mathbf{A}_2| = \sqrt{|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2|\mathbf{A}_1||\mathbf{A}_2|\cos\theta}$$

Substituting the given values, we get

$$5 = \sqrt{(3)^2 + (5)^2 + 2 \times 3 \times 5 \cos\theta}$$

or $$\cos\theta = -\frac{9}{2 \times 3 \times 5} = -\frac{3}{10}$$

So, $(2\mathbf{A}_1 + 3\mathbf{A}_2) \cdot (3\mathbf{A}_1 - 2\mathbf{A}_2)$

$$= 6|\mathbf{A}_1|^2 - 6|\mathbf{A}_2|^2 + 5\mathbf{A}_1 \cdot \mathbf{A}_2$$

$$= 6|\mathbf{A}_1|^2 - 6|\mathbf{A}_2|^2 + 5|\mathbf{A}_1||\mathbf{A}_2|\cos\theta$$

$$= 6 \times 9 - 6 \times 25 + 5 \times 2 \times 3 \times \left(\frac{-3}{10}\right) = -118.5$$

2. Given, $|\mathbf{A}| = |\mathbf{B}|$

or $$A = B \qquad \ldots\text{(i)}$$

Let magnitude of $(\mathbf{A} + \mathbf{B})$ is R and for $(\mathbf{A} - \mathbf{B})$ is R'.

Now, $$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

and $$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$R^2 = 2A^2 + 2A^2\cos\theta \qquad \ldots\text{(ii)}$$

[$\because$ using Eq. (i)]

Again, $$\mathbf{R}' = \mathbf{A} - \mathbf{B}$$

$$\Rightarrow \quad R'^2 = A^2 + B^2 - 2AB\cos\theta$$

$$R'^2 = 2A^2 - 2A^2\cos\theta \qquad \ldots\text{(iii)}$$

[$\because$ using Eq. (i)]

Given, $$R = nR' \text{ or } \left(\frac{R}{R'}\right)^2 = n^2$$

Dividing Eq. (ii) with Eq. (iii), we get

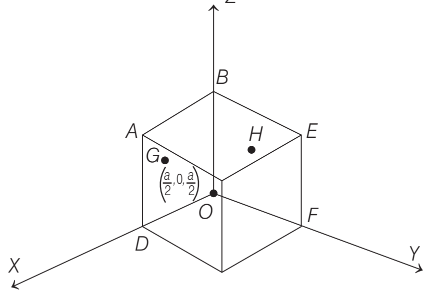
$$\frac{n^2}{1} = \frac{1 + \cos\theta}{1 - \cos\theta}$$

or $$\frac{n^2 - 1}{n^2 + 1} = \frac{(1 + \cos\theta) - (1 - \cos\theta)}{(1 + \cos\theta) + (1 - \cos\theta)}$$

$$\Rightarrow \quad \frac{n^2 - 1}{n^2 + 1} = \frac{2\cos\theta}{2} = \cos\theta \text{ or } \theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$$

3. In the given cube, coordinates of point G(centre of face $ABOD$) are $x_1 = \frac{a}{2}, y_1 = 0, z_1 = \frac{a}{2}$

where, a = side of cube



and coordinates of point H are

$$x_2 = 0,\ y_2 = \frac{a}{2},\ z_2 = \frac{a}{2}$$

So, vector GH is

$$\mathbf{GH} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}$$

$$= -\frac{a}{2}\hat{\mathbf{i}} + \frac{a}{2}\hat{\mathbf{j}} = \frac{a}{2}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

4. Given that the position coordinates of a particle

$$\left.\begin{aligned} x &= a\cos\omega t \\ y &= a\sin\omega t \\ z &= a\omega t \end{aligned}\right\} \quad \ldots\text{(i)}$$

So, the position vector of the particle is

$$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

$$\Rightarrow \quad \hat{\mathbf{r}} = a\cos\omega t\,\hat{\mathbf{i}} + a\sin\omega t\,\hat{\mathbf{j}} + a\,\omega t\,\hat{\mathbf{k}}$$

$$\hat{\mathbf{r}} = a[\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}} + \omega t\,\hat{\mathbf{k}}]$$

therefore, the velocity of the particle is

$$\because \quad \hat{\mathbf{v}} = \frac{d\mathbf{r}}{dt} = \frac{d[a][\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}} + \omega t\,\hat{\mathbf{k}}]}{dt}$$

$$\Rightarrow \quad \hat{\mathbf{v}} = -a\omega\sin\omega t\,\hat{\mathbf{i}} + a\omega\cos\omega t\,\hat{\mathbf{j}} + a\omega\hat{\mathbf{k}})$$

The magnitude of velocity is

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\text{or} \quad |\mathbf{v}| = \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + (a\omega)^2}$$

$$= \omega a\sqrt{(-\sin\omega t)^2 + (\cos\omega t)^2 + (1)^2} = \sqrt{2}\omega a$$

5. Given, velocity of a particle is

$$\mathbf{v} = k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}}) \quad \text{...(i)}$$

Suppose, it's position is given as

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\therefore \quad \mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{dx}{dt} = y \quad \text{...(iii)}$$

$$\text{and} \quad \frac{dy}{dt} = x \quad \text{...(iv)}$$

Dividing Eq. (iii) and Eq. (iv), we get

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{y}{x} \Rightarrow x\frac{dx}{dt} = y\frac{dy}{dt} \quad \text{or } xdx = ydy$$

Integrating both sides, we get

$$\int xdx = \int ydy \text{ or } \frac{x^2}{2} + \frac{c_1}{2} = \frac{y^2}{2} + \frac{c_2}{2}$$

where, c_1 and c_2 are the constants of integration.

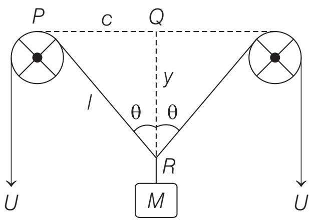
$$\Rightarrow \quad x^2 + c = y^2 \qquad [\text{here, } c \text{ (constant)} = c_1 - c_2]$$

$$\text{or} \quad y^2 = x^2 + \text{constant}$$

6. For particle P, motion between AC will be an accelerated one while between CB a retarded one. But in any case horizontal component of its velocity will be greater than or equal to v. On the other hand, in case of particle Q, it is always equal to v. Horizontal displacement for both the particles are equal. Therefore, $t_P < t_Q$.

7. In the right angle ΔPQR

$$l^2 = c^2 + y^2$$



c=constant, l and y are variable

Differentiating this equation with respect to time, we get

$$2l\frac{dl}{dt} = 0 + 2y\frac{dy}{dt} \quad \text{or} \quad \left(-\frac{dy}{dt}\right) = \frac{l}{y}\left(-\frac{dl}{dt}\right)$$

$$\text{Here,} \quad -\frac{dy}{dt} = v_M$$

$$\Rightarrow \quad \frac{l}{y} = \frac{1}{\cos\theta} \quad \text{and} \quad -dl/dt = U$$

$$\text{Hence,} \quad v_M = \frac{U}{\cos\theta}$$

8. $x = a\cos(pt) \Rightarrow \quad \cos(pt) = \dfrac{x}{a}$ …(i)

$$y = b\sin(pt)$$

$$\Rightarrow \quad \sin(pt) = y/b \quad \ldots\text{(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

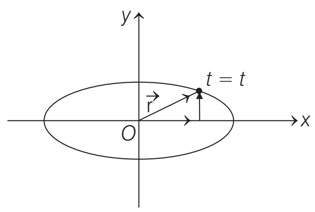
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Therefore, path of the particle is an ellipse. Hence, option (a) is correct.

From the given equations, we can find

$$\frac{dx}{dt} = v_x = -ap\sin$$

$$\Rightarrow \frac{d^2x}{dt^2} = a_x = -ap^2\cos pt$$



$$\frac{dy}{dt} = v_y = bp\cos pt$$

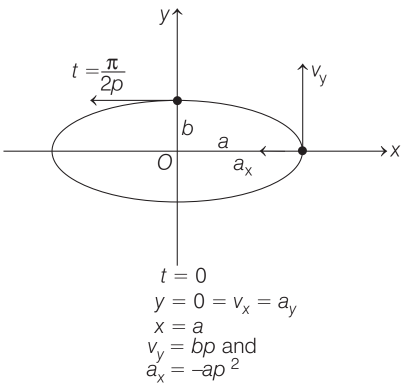
$$\text{and} \quad \frac{d^2y}{dt^2} = a_y = -bp^2\sin pt$$

At time $t = \pi/2p$ or $pt = \pi/2$

a_x and v_y become zero (because $\cos\pi/2 = 0$),

only v_x and a_y are left.

or we can say that velocity is along negative x-axis and acceleration along y-axis.



Hence, at $t = \pi/2p$ velocity and acceleration of the particle are normal to each other. So, option (b) is also correct.

At $t = t$, position of the particle

$$\mathbf{r}(t) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = a\cos pt\,\hat{\mathbf{i}} + b\sin pt\hat{\mathbf{j}}$$

and acceleration of the particle is

$$\mathbf{a}(t) = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}} = -p^2[a\cos\ pt\ \hat{\mathbf{i}} + b\sin\ pt\ \hat{\mathbf{j}}]$$

$$= -p^2\,[x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}] = -p^2\,\mathbf{r}\,(t)$$

Therefore, acceleration of the particle is always directed towards origin.

Hence, option (c) is also correct.

At $t = 0$, particle is at $(a, 0)$ and at $t = \pi/2p$ particle is at $(0, b)$. Therefore, the distance covered is one-fourth of the elliptical path not a.

Hence, option (d) is wrong.

9. $\mathrm{A} = a\hat{\mathrm{i}}$ and $\mathrm{B} = a\cos\omega\hat{\mathrm{i}} + a\sin\omega t\hat{\mathrm{j}}$

$$\mathrm{A} + \mathrm{B} = (a + a\cos\omega t)\hat{\mathrm{i}} + a\sin\omega t\hat{\mathrm{j}}$$

$$\mathrm{A} - \mathrm{B} = (a - a\cos\omega t)\hat{\mathrm{i}} + a\sin\omega t\hat{\mathrm{j}}$$

$$|\mathrm{A} + \mathrm{B}| = \sqrt{3}|\mathrm{A} - \mathrm{B}|$$

$$\sqrt{(a + a\cos\omega t)^2 + (a\sin\omega t)^2} = \sqrt{3}\sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$$

$$\Rightarrow \quad 2\cos\frac{\omega t}{2} = \pm\sqrt{3}\times 2\sin\frac{\omega t}{2} \Rightarrow \tan\frac{\omega t}{2} = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{\omega t}{2} = n\pi \pm \frac{\pi}{6} \qquad \Rightarrow \quad \frac{\pi}{12}t = n\pi \pm \frac{\pi}{6}$$

$$\Rightarrow \quad t = (12n \pm 2)\text{s} = 2\text{s}, 10\text{s}, 14\text{s} \text{ and so on.}$$

10. $a = \frac{F}{m} = \frac{qE}{m} = 10^3\sin(10^3 t)$

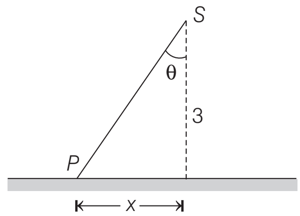
$$\frac{dv}{dt} = 10^3\sin(10^3 t) \Rightarrow \int_0^v dv = \int_0^t 10^3\sin(10^3 t)\,dt$$

$$\therefore \quad v = \frac{10^3}{10^3}[1 - \cos(10^3 t)]$$

Velocity will be maximum when $\cos(10^3 t) = -1$

$$v_{\max} = 2 \text{ m/s}$$

11. $\frac{x}{3} = \tan\theta$



$\therefore\ x = 3\tan\theta$

$$-\left(\frac{dx}{dt}\right) = 3\sec^2\theta\left(-\frac{d\theta}{dt}\right)$$

$$-\frac{dx}{dt} = v_P \text{ and}$$

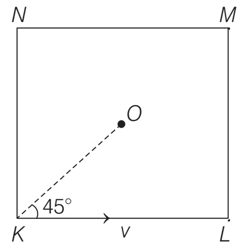
$$-\frac{d\theta}{dt} = \omega = 0.1 \text{ rad/s}, \theta = 45°$$

Substituting the values, we get

$$v_P = 3(\sec^2 45°)(0.1) = 0.6 \text{ m/s}$$

12. By symmetry, we can say that all will meet at the centre of square.

Component of velocity along KO (or LO etc) at any instant will be $v\cos 45°$ or $\frac{v}{\sqrt{2}}$.



Further distance KO

$$= \sqrt{2}\left(\frac{d}{2}\right) = \frac{d}{\sqrt{2}}$$

$$\therefore \quad \text{Time} = \frac{d/\sqrt{2}}{v/\sqrt{2}} = \frac{d}{v}$$

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http://tinyurl.com/yyd9keb5 *or*

3

Laws of Motion

Topic 1 Newton's Laws

Objective Questions I (Only one correct option)

1. A bullet of mass 20 g has an initial speed of 1ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2}N, the speed of the bullet after emerging from the other side of the wall is close to **(Main 2009, 10 April II)**

(a) 0.3 ms^{-1} (b) 0.4 ms^{-1} (c) 0.1 ms^{-1} (d) 0.7 ms^{-1}

Assertion and Reason

Mark your answer as

(*a*) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*

(*b*) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*

(*c*) *If Statement I is true; Statement II is false*

(*d*) *If Statement I is false; Statement II is true*

2. **Statement I** It is easier to pull a heavy object than to push it on a level ground.

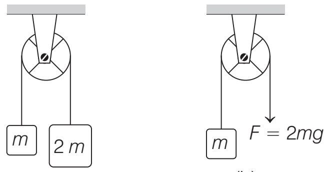
Statement II The magnitude of frictional force depends on the nature of the two surfaces in contact. **(2008, 3M)**

3. **Statement I** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

Statement II For every action there is an equal and opposite reaction. **(2007, 3M)**

True/ False

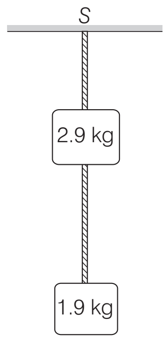
4. The pulley arrangements of figures (a) and (b) are identical. The mass of the rope is negligible. In Fig. (a), the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In Fig. (b), m is lifted up by pulling the other end of the rope with a constant downward force $F = 2mg$. The acceleration of m is the same in both cases. **(1984, 2M)**



(a) (b)

Analytical & Descriptive Questions

5. Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support S by two inextensible wires each of length 1 m (see figure). The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg/m. The whole system of blocks, wires and support have an upward acceleration of 0.2 m/s^2. The acceleration due to gravity is 9.8 m/s^2. **(1989, 6M)**



(a) Find the tension at the mid-point of the lower wire.

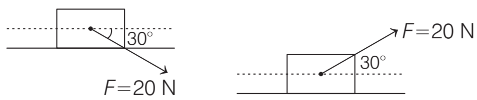
(b) Find the tension at the mid-point of the upper wire.

6. A uniform rope of length L and mass M lying on a smooth table is pulled by a constant force F. What is the tension in the rope at a distance l from the end where the force is applied? **(1978)**

Topic 2 Equilibrium of Forces

Objective Questions I (Only one correct option)

1. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20$N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block, the floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be (Take, $g = 10 \text{ ms}^{-2}$)



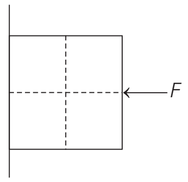
(A) (B)

(a) 0.4 ms^{-2} (b) 3.2 ms^{-2} (c) 0.8 ms^{-2} (d) 0 ms^{-2}

2. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the mass, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (Take, $g = 10 \text{ ms}^{-2}$) **(2019 Main, 9 Jan II)**

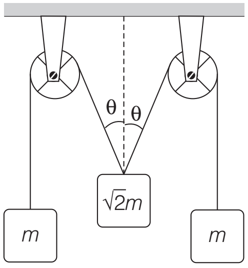
(a) 70 N (b) 200 N (c) 100 N (d) 140 N

3. A block of mass m is at rest under the action of force F against a wall as shown in figure. Which of the following statement is incorrect? **(2005, 2M)**



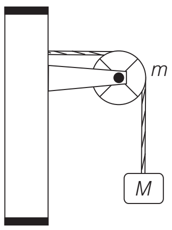
(a) $f = mg$ (where f is the frictional force)
(b) $F = N$ (where N is the normal force)
(c) F will not produce torque
(d) N will not produce torque

4. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be **(2001, 2M)**



(a) 0° (b) 30° (c) 45° (d) 60°

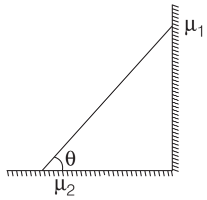
5. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by **(2001, 2M)**



(a) $\sqrt{2}\,Mg$ (b) $\sqrt{2}\,mg$
(c) $g\sqrt{(M+m)^2+m^2}$ (d) $g\sqrt{(M+m)^2+M^2}$

Objective Question II (One or more correct option)

6. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then **(2014 Adv.)**



(a) $\mu_1 = 0, \mu_2 \neq 0$ and $N_2 \tan\theta = \dfrac{mg}{2}$

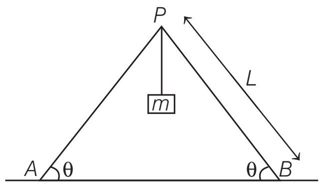
(b) $\mu_1 \neq 0, \mu_2 = 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

(c) $\mu_1 \neq 0, \mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(d) $\mu_1 = 0, \mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

Analytical & Descriptive Questions

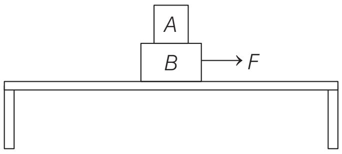
7. Two identical ladders are arranged as shown in the figure. Mass of each ladder is M and length L. The system is in equilibrium. Find direction and magnitude of frictional force acting at A or B. **(2005)**



Topic 3 Friction

Objective Questions I (Only one correct option)

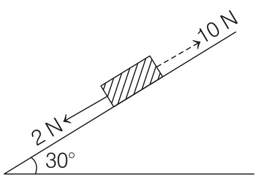
1. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is



[Take, $g = 10\ \text{m/s}^2$] **(2019 Main, 10 April II)**

(a) 12 N (b) 16 N (c) 8 N (d) 40 N

2. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is **(2019 Main, 12 Jan I)**

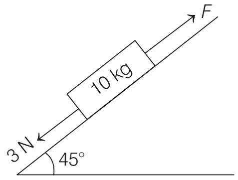


(Take, $g = 10\ \text{m/s}^2$)

(a) $\dfrac{2}{3}$ (b) $\dfrac{\sqrt{3}}{2}$

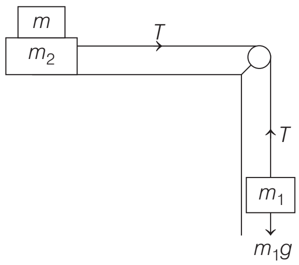
(c) $\dfrac{\sqrt{3}}{4}$ (d) $\dfrac{1}{2}$

3. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force F, such that the block does not move downward ? (Take, $g = 10\text{ms}^{-2}$) **(2019 Main, 9 Jan I)**



(a) 32 N (b) 25 N
(c) 23 N (d) 18 N

4. Two masses $m_1 = 5\,\text{kg}$ and $m_2 = 10$ kg connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is **(2018 Main)**



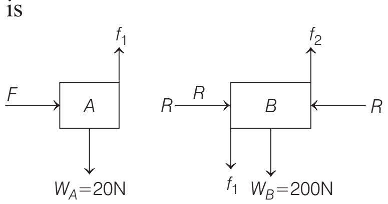
(a) 23.33 kg (b) 18.3 kg (c) 27.3 kg (d) 43.3 kg

5. A uniform wooden stick of mass 1.6 kg of length l rests in an inclined manner on a smooth, vertical wall of height $h(<l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio $\frac{h}{l}$ and the frictional force f at the bottom of the stick are $(g = 10\text{ms}^{-2})$ **(2016 Adv.)**

(a) $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$ (b) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$

(c) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}\text{N}$ (d) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}\text{N}$

6. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall in block B is **(2015 Main)**



(a) 100 N (b) 120 N (c) 80 N (d) 150 N

7. A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is **(2014 Main)**

(a) $\frac{1}{6}$ m (b) $\frac{2}{3}$ m
(c) $\frac{1}{3}$ m (d) $\frac{1}{2}$ m

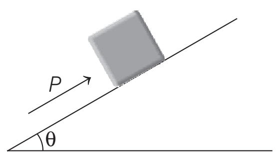
8. A block of mass $m_1 = 1\,\text{kg}$ another mass $m_2 = 2\,\text{kg}$ are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. **(2014 Adv.)**

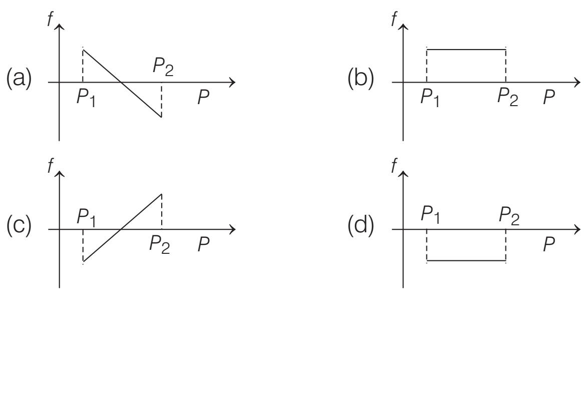
In List II expressions for the friction on the block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.
[**Useful information tan** $(5.5°) \approx 0.1$;
tan $(11.5°) \approx 0.2$; **tan** $(16.5°) \approx 0.3$]

	List I		List II
P.	$\theta = 5°$	1.	$m_2 g \sin\theta$
Q.	$\theta = 10°$	2.	$(m_1 + m_2)\, g \sin\theta$
R.	$\theta = 15°$	3.	$\mu\, m_2 g \cos\theta$
S.	$\theta = 20°$	4.	$\mu\,(m_1 + m_2)\, g \cos\theta$

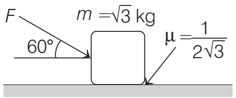
Codes
(a) P-1, Q-1, R-1, S-3 (b) P-2, Q-2, R-2, S-3
(c) P-2, Q-2, R-2, S-4 (d) P-2, Q-2, R-3, S-3

9. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f *versus* P graph will look like **(2010)**



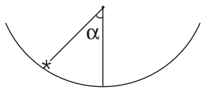


10. What is the maximum value of the force F such that the block shown in the arrangement, does not move? **(2003, 2M)**



(a) 20 N (b) 10 N (c) 12 N (d) 15 N

11. An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given **(2001, 2M)**



(a) $\cot \alpha = 3$ (b) $\tan \alpha = 3$
(c) $\sec \alpha = 3$ (d) $\operatorname{cosec} \alpha = 3$

12. A block of mass 0.1 kg is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is **(1994, 1M)**

(a) 2.5 N (b) 0.98 N (c) 4.9 N (d) 0.49 N

13. During paddling of a bicycle, the force of friction exerted by the ground on the two wheels is such that it acts **(1990, 2M)**

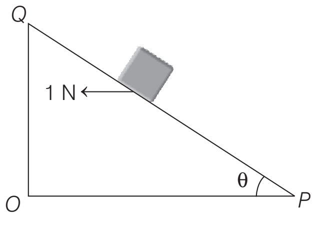
(a) in the backward direction on the front wheel and in the forward direction on the rear wheel
(b) in the forward direction on the front wheel and in the backward direction on the rear wheel
(c) in the backward direction on both the front and the rear wheels
(d) in the forward direction on both the front and the rear wheels

14. A block of mass 2 kg rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is **(1980, 2M)**

(a) 9.8 N (b) $0.7 \times 9.8 \times \sqrt{3}$ N
(c) $9.8 \times \sqrt{3}$ N (d) 0.7×9.8 N

Objective Questions II (One or more correct option)

15. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10 \text{ m/s}^2$) **(2012)**



(a) $\theta = 45°$.
(b) $\theta > 45°$ and a frictional force acts on the block towards P
(c) $\theta > 45°$ and a frictional force acts on the block towards Q
(d) $\theta < 45°$ and a frictional force acts on the block towards Q

Fill in the Blank

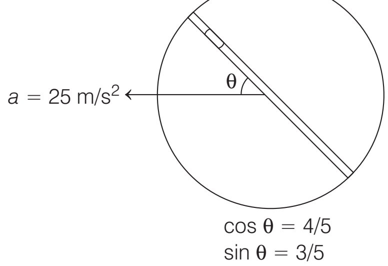
16. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is ……… N. **(1984, 2M)**

True/False

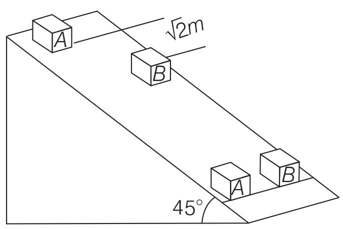
17. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. **(1981, 2M)**

Analytical & Descriptive Questions

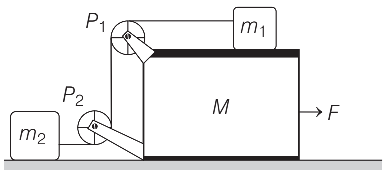
18. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The coefficient of friction between the block and all surfaces of groove in contact is $\mu = 2/5$. The disc has an acceleration of 25 m/s^2. Find the acceleration of the block with respect to disc. **(2006, 6M)**



19. Two blocks A and B of equal masses are released from an inclined plane of inclination 45° at $t = 0$. Both the blocks are initially at rest. The coefficient of kinetic friction between the block A and the inclined plane is 0.2 while it is 0.3 for block B. Initially the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line?
(Take $g = 10 \text{ m/s}^2$) **(2004, 5M)**



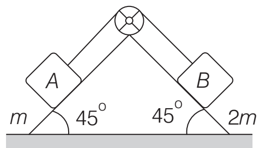
20. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the strings are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2.
The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to the mass M.
(Take $g = 10 \text{ m/s}^2$.) **(2000, 10M)**



(a) Draw a free body diagram of mass M, clearly showing all the forces.

(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular force F it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write equations of motion of all the masses. Find F, tension in the string and accelerations of the masses.

21. Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless, in extensible string and a frictionless pulley as shown in figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the blocks A and B are released from rest, find **(1997C, 5M)**



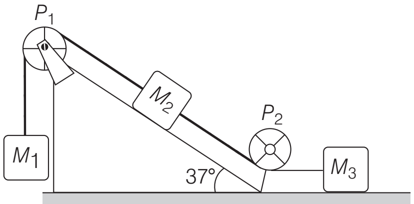
(a) the acceleration of A,

(b) tension in the string and

(c) the magnitude and direction of the force of friction acting on A.

22. A block of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied. **(1987, 7M)**

23. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which passes over massless and frictionless pulleys P_1 and P_2 as shown in figure.



The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal.

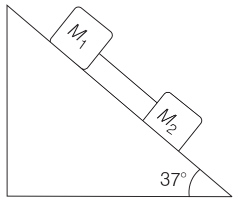
If the mass M_1 moves downwards with a uniform velocity, find **(1981, 6M)**

(a) the mass of M_1,

(b) the tension in the horizontal portion of the string.

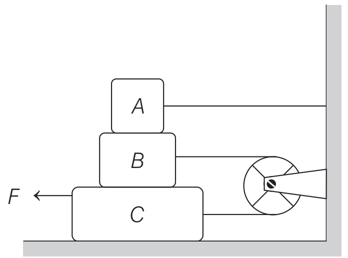
(Take $g = 9.8 \text{ m/s}^2$, $\sin 37° \cong 3/5$)

24. Two blocks connected by a massless string slides down an inclined plane having an angle of inclination of 37°. The masses of the two blocks are $M_1 = 4$ kg and $M_2 = 2$ kg respectively and the coefficients of friction of M_1 and M_2 with the inclined plane are 0.75 and 0.25 respectively. Assuming the string to be taut, find (a) the common acceleration of two masses and (b) the tension in the string. ($\sin 37° = 0.6, \cos 37° = 0.8$). (Take $g = 9.8 \text{ m/s}^2$) **(1979)**



25. A block of mass 2 kg slides on an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\sqrt{3}/\sqrt{2}$. What force along the plane should be applied to the block so that it moves (a) down and (b) up without any acceleration? (Take $g = 10 \text{ m/s}^2$.) **(1978)**

26. In the figure, the blocks A, B and C have masses 3 kg, 4 kg and 8 kg respectively. The coefficient of sliding friction between any two surfaces is 0.25. A is held at rest by a massless rigid rod fixed to the wall, while B and C are connected by a light flexible cord passing around a fixed frictionless pulley. Find the force F necessary to drag C along the horizontal surface to the left at a constant speed. Assume that the arrangement shown in the figure. i.e. B on C and A on B, is maintained throughout.(Take $g = 10 \text{ m/s}^2$). **(1978)**



Integer Answer Type Question

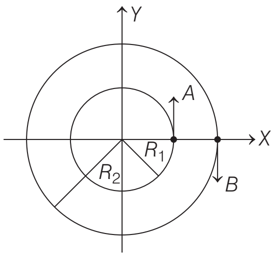
27. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ.

The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down.If we define $N = 10\mu$, then N is **(2011)**

Topic 4 Dynamics of Circular Motion

Objective Questions I (Only one correct option)

1. Two particles A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure. The relative velocity $\mathbf{v}_A - \mathbf{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by **(2019 Main, 12 Jan II)**

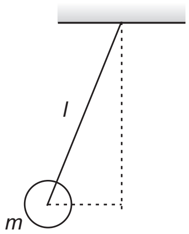


(a) $\omega(R_1 + R_2)\hat{\mathbf{i}}$ (b) $-\omega(R_1 + R_2)\hat{\mathbf{i}}$
(c) $\omega(R_1 - R_2)\hat{\mathbf{i}}$ (d) $\omega(R_2 - R_1)\hat{\mathbf{i}}$

2. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **(2019 Main, 11 Jan I)**

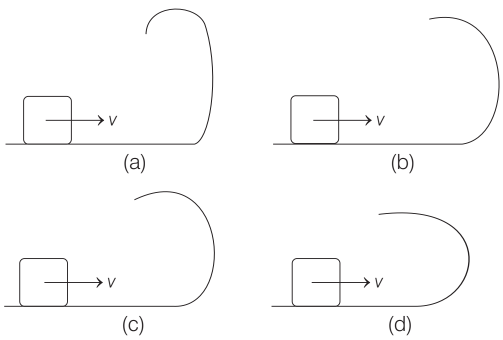
(a) $10\sqrt{2}$ m/s (b) 10 m/s
(c) $10\sqrt{3}$ m/s (d) Zero

3. A ball of mass (m) 0.5 kg is attached to the end of a string having length (l) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) is **(2011)**



(a) 9 (b) 18
(c) 27 (d) 36

4. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in **(2001, 2M)**



5. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is **(2000, 2M)**

(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$
(c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

6. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod. The angle made by the rod with the vertical is (Take $g = 10\ m/s^2$) **(1992, 2M)**

(a) zero (b) 30° (c) 45° (d) 60°

Objective Question II (One or more correct option)

7. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement θ ($|\theta| < \phi$), the tension in the string and the velocity of the bob are T and v respectively. The following relations hold good under the above conditions. **(1986, 2M)**

(a) $T\cos\theta = Mg$
(b) $T - Mg\cos\theta = \frac{Mv^2}{L}$
(c) The magnitude of the tangential acceleration of the bob $|a_T| = g\sin\theta$
(d) $T = Mg\cos\theta$

True/False

8. A simple pendulum with a bob of mass m swings with an angular amplitude of 40°. When its angular displacement is 20°, the tension in the string is greater than $mg\cos 20°$. **(1984, 2M)**

Analytical & Descriptive Questions

9. Two blocks of mass $m_1 = 10$ kg and $m_2 = 5$ kg connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/s about a vertical axis passing through its centre O. The masses are placed along the diameter of table on either side of the centre O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table. **(1997, 5M)**

(a) Calculate the frictional force on m_1.
(b) What should be the minimum angular speed of the turn table, so that the masses will slip from this position?

(c) How should the masses be placed with the string remaining taut so that there is no frictional force acting on the mass m_1?

10. A hemispherical bowl of radius $R = 0.1\text{m}$ is rotating about its own axis (which is vertical) with an angular velocity ω. A particle of mass 10^{-2} kg on the frictionless inner surface of the bowl is also rotating with the same ω. The particle is at a height h from the bottom of the bowl. **(1993, 3 + 2M)**

(a) Obtain the relation between h and ω. What is the minimum value of ω needed, in order to have a non-zero value of h?

(b) It is desired to measure g (acceleration due to gravity) using the set-up by measuring h accurately. Assuming that R and ω are known precisely and that the least count in the measurement of h is 10^{-4} m, what is minimum possible error Δg in the measured value of g?

Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. Two forces P and Q of magnitude $2F$ and $3F$, respectively are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is **(2019 Main, 10 Jan II)**

(a) $60°$ (b) $120°$ (c) $30°$ (d) $90°$

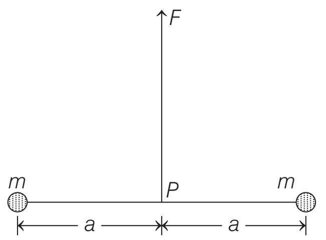
2. A particle is moving in a circular path of radius a under the action of an attractive potential energy $U = -\dfrac{k}{2r^2}$. Its total energy is **(2018 Main)**

(a) $-\dfrac{3}{2}\cdot\dfrac{k}{a^2}$ (b) $-\dfrac{k}{4a^2}$ (c) $\dfrac{k}{2a^2}$ (d) zero

3. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is **(2009)**

(a) $\dfrac{a}{gk}$ (b) $\dfrac{a}{2gk}$ (c) $\dfrac{2a}{gk}$ (d) $\dfrac{a}{4gk}$

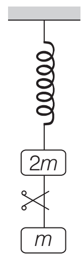
4. Two particles of mass m each are tied at the ends of a light string of length $2a$. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance a from the centre P (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes $2x$, is **(2007, 3M)**



(a) $\dfrac{F}{2m}\dfrac{a}{\sqrt{a^2 - x^2}}$ (b) $\dfrac{F}{2m}\dfrac{x}{\sqrt{a^2 - x^2}}$

(c) $\dfrac{F}{2m}\dfrac{x}{a}$ (d) $\dfrac{F}{2m}\dfrac{\sqrt{a^2 - x^2}}{x}$

5. System shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of mass $2m$ and m just after the string is cut will be **(2006, 3M)**



(a) $g/2$ upwards, g downwards
(b) g upwards, $g/2$ downwards
(c) g upwards, $2g$ downwards
(d) $2g$ upwards, g downwards

6. A ship of mass 3×10^7 kg initially at rest, is pulled by a force of 5×10^4 N through a distance of 3 m. Assuming that the resistance due to water is negligible, the speed of the ship is **(1980, 2M)**

(a) 1.5 m/s (b) 60 m/s (c) 0.1 m/s (d) 5 m/s

Objective Questions II (One or more correct option)

7. A reference frame attached to the earth **(1986, 2M)**

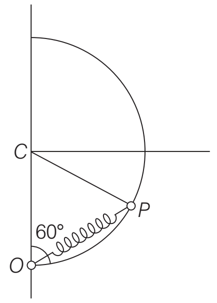
(a) is an inertial frame by definition
(b) cannot be an inertial frame because the earth is revolving round the sun
(c) is an inertial frame because Newton's laws are applicable in this frame
(d) cannot be an inertial frame because the earth is rotating about its own axis

True/ False

8. Two identical trains are moving on rails along the equator on the earth in opposite directions with the same speed. They will exert the same pressure on the rails. **(1985, 3M)**

Analytical & Descriptive Question

9. A smooth semicircular wire track of radius R is fixed in a vertical plane (figure). One end of a massless spring of natural length $3R/4$ is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle $60°$ with the vertical. The spring constant $k = mg/R$. Consider the instant when the ring is making an angle $60°$ with the vertical. The spring is released, (a) Draw the free body diagram of the ring. (b) Determine the tangential acceleration of the ring and the normal reaction. **(1996, 5M)**



Answers

Topic 1

1. (d) **2.** (b) **3.** (b) **4.** F

5. (a) 20 N, (b) 50 N **6.** $T = F\left(1 - \frac{l}{L}\right)$

Topic 2

1. (c) **2.** (c) **3.** (d) **4.** (c)
5. (d)

6. (c, d) **7.** $f = \left(\frac{M+m}{2}\right) g \cot \theta$

Topic 3

1. (b) **2.** (b) **3.** (a)
4. (a)
5. (d) **6.** (b) **7.** (a) **8.** (d)
9. (a) **10.** (a) **11.** (a) **12.** (b)
13. (a) **14.** (a) **15.** (a, c) **16.** (5)
17. F **18.** 10 m/s^2 **19.** $s_A = 8\sqrt{2}$ m , 2s

20. (b) $f_1 = 30$ N, $f_2 = 15$ N, $F = 60$ N, $T = 18$ N, $a = \frac{3}{5} \text{ m/s}^2$

21. (a) acceleration = 0 (b) $\frac{2\sqrt{2}}{3} mg$ (c) $\frac{mg}{3\sqrt{2}}$ (down the plane)

22. $mg \sin \theta, \theta = \tan^{-1}(\mu)$ from horizontal

23. (a) 4.2 kg, (b) 9.8 N **24.** (a) $a = 1.3 \text{ m/s}^2$ (b) $T = 5.2$ N

25. (a) 11.21 N (b) 31.21 **26.** 80 N

27. 5

Topic 4

1. (d) **2.** (b)
3. (d) **4.** (a) **5.** (a)
6. (c) **7.** (b, c) **8.** T

9. (a) 36 N (towards centre) (b) 11.67 rad/s
(c) m_1 should be placed at 0.1 m from the centre O

10. (a) $h = R - \frac{g}{\omega^2}, \omega_{\min} = \sqrt{\frac{g}{R}} = 9.89$ rad/s
(b) 9.8×10^{-3} m / s^2

Topic 5

1. (b) **2.** (d)
3. (b) **4.** (b) **5.** (a) **6.** (c)
7. (b, d) **8.** F **9.** (b) $\frac{5\sqrt{3}}{8} g, \frac{3mg}{8}$

Hints & Solutions

Topic 1 Newton's Laws

1. Given, resistance offered by the wall

$$= F = -2.5 \times 10^{-2} \text{ N}$$

So, deacceleration of bullet,

$$a = \frac{F}{m} = \frac{-2.5 \times 10^{-2}}{20 \times 10^{-3}} = -\frac{5}{4} \text{ms}^{-2}$$

$$(\because m = 20 \text{ g} = 20 \times 10^{-3} \text{ kg})$$

Now, using the equation of motion,

$$v^2 - u^2 = 2as$$

We have,

$$v^2 = 1 + 2\left(-\frac{5}{4}\right)(20 \times 10^{-2})$$

$$(\because u = 1 \text{ ms}^{-1} \text{ and } s = 20 \text{ cm} = 20 \times 10^{-2} \text{ m})$$

$$\Rightarrow \quad v^2 = \frac{1}{2}$$

$$\therefore \quad v = \frac{1}{\sqrt{2}} \approx 0.7 \text{ ms}^{-1}$$

2. Both statements are correct. But statement II, does not explain correctly, statement I.

Explanation There is an increase in normal reaction when the object is pushed and there is a decrease in normal reaction when the object is pulled (but strictly not horizontally).

3. The cloth can be pulled out without dislodging the dishes from the table due to law of inertia, which is Newton's first law. While, the statement II is true, but it is Newton's third law.

4. In these cases, $a = \dfrac{\text{Net pulling force}}{\text{Total mass}}$

Net pulling force in both the cases is

$$2mg - mg = mg$$

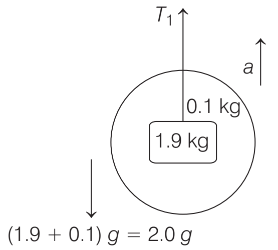
But mass (to be pulled) in case (a) is $3m$ and in case (b) is m.

Therefore, $a_1 = \dfrac{mg}{3m} = \dfrac{g}{3}$ [in case (a)]

and $a_2 = \dfrac{mg}{m} = g$ [in case (b)]

or $a_1 < a_2$

5. (a) To find tension at mid-point of the lower wire, we cut the string at this point. Draw the free body diagram of lower portion.

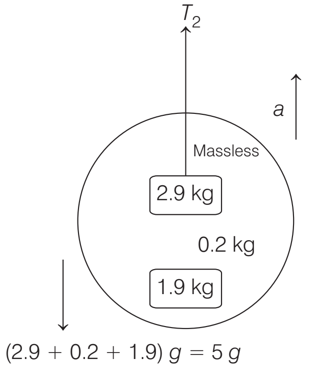


The equation of motion gives

$$T_1 - 2.0g = (2.0)\,a$$

or $$T_1 = (2.0)\,(g + a)$$
$$= (2.0)\,(9.8 + 0.2) = 20\,\text{N}$$

(b) To find tension at mid-point of upper wire



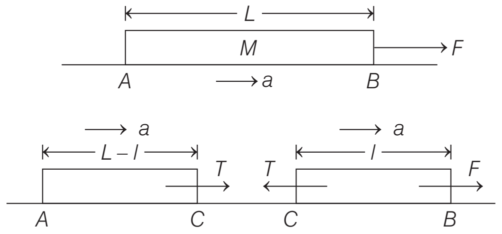
The equation of motion gives

$$T_2 - 5g = 5a$$

$\therefore$ $$T_2 = 5\,(g + a)$$
$$= 5\,(9.8 + 0.2) = 50\,\text{N}$$

6. Acceleration of rope, $a = F/M$

Now, to find tension at point C, a distance l from point B, we can write equation of motion of any one part (AC or CB), both moving with acceleration a.

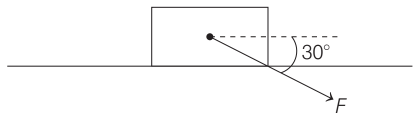


Equation of motion of part AC is

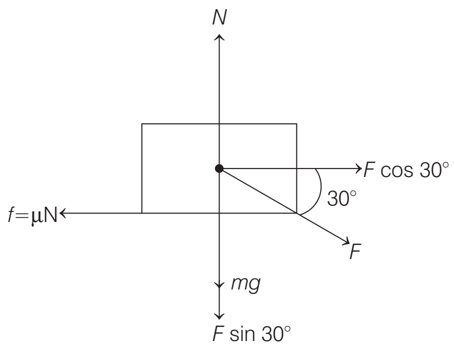
$$T = (\text{mass of } AC) \times (\text{acceleration})$$
$$= \frac{M}{L}\,(L - l)\left(\frac{F}{M}\right) = F\left(1 - \frac{l}{L}\right)$$

Topic 2 Equilibrium of Forces

1. Case I Block is pushed over surface



Free body diagram of block is



In this case, normal reaction,

$$N = mg + F \sin 30° = 5 \times 10 + 20 \times \frac{1}{2} = 60\,\text{N}$$

[Given, $m = 5\,\text{kg}$, $F = 20\,\text{N}$]

Force of function, $f = \mu N$
$= 0.2 \times 60$ [$\because \mu = 0.2$]
$= 12\,\text{N}$

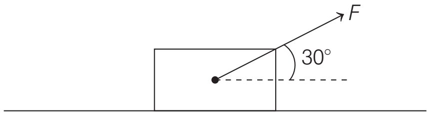
So, net force causing acceleration (a_1) is

$$F_{\text{net}} = ma_1 = F \cos 30° - f$$

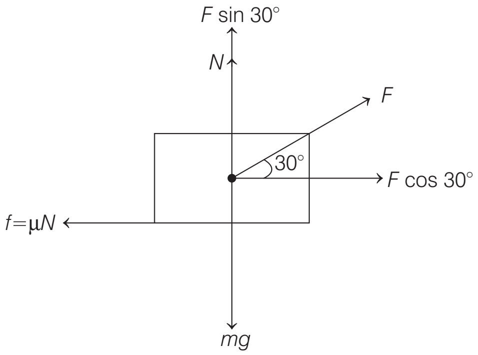
$\Rightarrow$ $$ma_1 = 20 \times \frac{\sqrt{3}}{2} - 12$$

$\therefore$ $$a_1 = \frac{10\sqrt{3} - 12}{5} \approx 1\,\text{ms}^{-2}$$

Case II Block is pulled over the surface



Free body diagram of block is,



Net force causing acceleration is

$$F_{\text{net}} = F\cos 30° - f = F\cos 30° - \mu N$$

$$\Rightarrow \quad F_{\text{net}} = F\cos 30° - \mu(mg - F\sin 30°)$$

If acceleration is now a_2, then

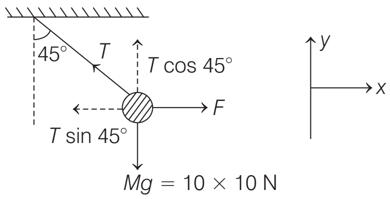
$$a_2 = \frac{F_{\text{net}}}{m} = \frac{F\cos 30° - \mu(mg - F\sin 30°)}{m}$$

$$= \frac{20 \times \frac{\sqrt{3}}{2} - 0.2\left(5 \times 10 - 20 \times \frac{1}{2}\right)}{5} = \frac{10\sqrt{3} - 8}{5}$$

$$\Rightarrow \text{a}_2 \approx 1.8\,\text{ms}^{-2}$$

So, difference $= a_2 - a_1 = 1.8 - 1 = 0.8\,\text{ms}^{-2}$

2. FBD of the given system is follow



Let T = tension in the rope.

For equilibrium condition of the mass,

$$\Sigma F_x = 0 \qquad \text{(force in } x\text{-direction)}$$

$$\Sigma F_y = 0 \qquad \text{(force in } y\text{-direction)}$$

When $\Sigma F_x = 0$, then

$$\because \qquad F = T\sin 45° \qquad \text{...(i)}$$

When $\Sigma F_y = 0$, then

$$Mg = T\cos 45° \qquad \text{...(ii)}$$

Using Eqs. (i) and (ii),

$$\Rightarrow \quad \frac{F}{Mg} = \frac{T\sin 45°}{T\cos 45°} \quad \Rightarrow \frac{F}{Mg} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\Rightarrow \quad F = Mg = 10 \times 10 = 100\,\text{N}$$

3. This is the equilibrium of coplanar forces. Hence,

$$\Sigma F_x = 0$$

$$\therefore \qquad F = N$$

$$\therefore \qquad \Sigma F_y = 0,\; f = mg$$

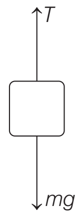
$$\Sigma \tau_c = 0$$

$$\therefore \qquad \tau_N + \tau_f = 0$$

Since, $\tau_f \neq 0$

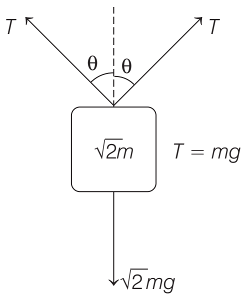
$$\therefore \qquad \tau_N \neq 0$$

4. Free body diagram of m is



$$T = mg \qquad \text{...(i)}$$

Free body diagram of mass $\sqrt{2}\,m$ is

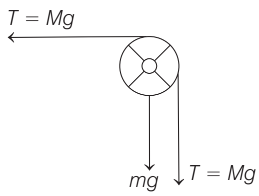


$$2T\cos\theta = \sqrt{2}\,mg \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\cos\theta = \frac{1}{\sqrt{2}} \text{ or } \theta = 45°$$

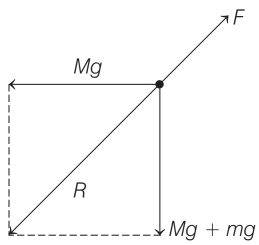
5. Free body diagram of pulley is shown in figure. Pulley is in equilibrium under four forces. Three forces as shown in figure and the fourth, which is equal and opposite to the resultant of these three forces, is the force applied by the clamp on the pulley (say F).



Resultant R of these three forces is

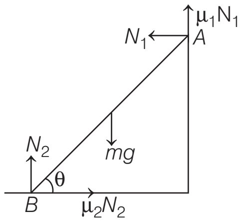
$$R = \left(\sqrt{(M+m)^2 + M^2}\right) g$$

Therefore, the force F is equal and opposite to R as shown in figure.



$$\therefore \quad F = \left(\sqrt{(M+m)^2 + M^2}\right) g$$

6. μ_2 can never be zero for equilibrium.



When $\mu_1 = 0$, we have

$$N_1 = \mu_2 N_2 \quad \ldots\text{(i)}$$

$$N_2 = mg \quad \ldots\text{(ii)}$$

$$\tau_B = 0$$

$$\Rightarrow \quad mg\frac{L}{2}\cos\theta = N_1 L\sin\theta$$

$$\Rightarrow \quad N_1 = \frac{mg\cot\theta}{2} \quad \Rightarrow \quad N_1\tan\theta = \frac{mg}{2}$$

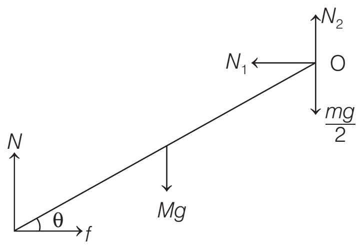
When, $\mu_1 \neq 0$ we have

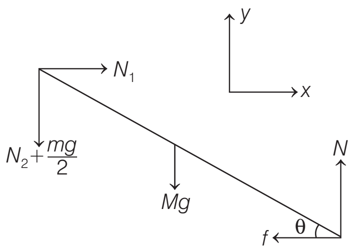
$$\mu_1 N_1 + N_2 = mg \quad \ldots\text{(iii)}$$

$$\mu_2 N_2 = N_1 \quad \ldots\text{(iv)}$$

$$\Rightarrow \quad N_2 = \frac{mg}{1+\mu_1\mu_2}$$

7. Drawing force diagrams of the rod, we have





This is the equilibrium of coplanar forces, hence using the equations $\Sigma F_x = 0$, $\Sigma_y = 0$ and net moment about point $O = 0$ we have the equations,

$$N + N_2 = Mg + \frac{mg}{2} \quad \ldots\text{(i)}$$

$$N_1 = f \quad \ldots\text{(ii)}$$

$$N = N_2 + Mg + \frac{mg}{2} \quad \ldots\text{(iii)}$$

and $$Mg\frac{L}{2}\cos\theta + fL\sin\theta = NL\cos\theta \quad \ldots\text{(iv)}$$

Solving these four equations, we have $f = \left(\frac{M+m}{2}\right)g\cot\theta$

NOTE Force of friction f cannot be obtained by considering the equilibrium of whole system as a whole. Think why ?

Topic 3 Friction

1. Acceleration a of system of blocks A and B is

$$a = \frac{\text{Net force}}{\text{Total mass}} = \frac{F - f_1}{m_A + m_B}$$

where, f_1 = friction between B and the surface

$$= \mu(m_A + m_B)g$$

So,

$$a = \frac{F - \mu(m_A + m_B)g}{(m_A + m_B)} \quad \ldots\text{(i)}$$

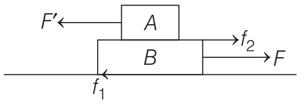
Here, $\mu = 0.2$, $m_A = 1\,\text{kg}$, $m_B = 3\,\text{kg}$, $g = 10\,\text{ms}^{-2}$

Substituting the above values in Eq. (i), we have

$$a = \frac{F - 0.2(1+3)\times 10}{1+3}$$

$$a = \frac{F-8}{4} \quad \ldots\text{(ii)}$$

Due to acceleration of block B, a pseudo force F' acts on A.



This force F' is given by $F' = m_A a$

where, a is acceleration of A and B caused by net force acting on B.

For A to slide over B; pseudo force on A, i.e. F' must be greater than friction between A and B.

$$\Rightarrow \quad m_A a \geq f_2$$

We consider limiting case,

$$m_A a = f_2$$

$$\Rightarrow \quad m_A a = \mu(m_A)g$$

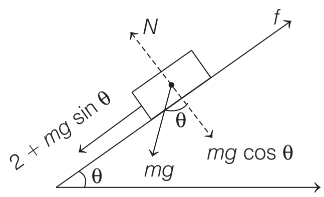
$$\Rightarrow \quad a = \mu g = 0.2\times 10 = 2\,\text{ms}^{-2} \quad \ldots\text{(iii)}$$

Putting the value of a from Eq. (iii) into Eq. (ii), we get

$$\frac{F-8}{4} = 2$$

$$\therefore \quad F = 16\,\text{N}$$

2. Block does not move upto a maximum applied force of 2N down the inclined plane.

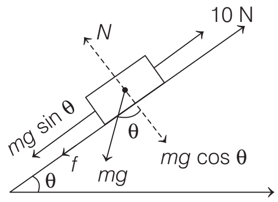


So, equating forces, we have;

$$2 + mg\sin\theta = f$$

or $$2 + mg\sin\theta = \mu\, mg\cos\theta \quad \ldots\text{(i)}$$

Similarly, block also does not move upto a maximum applied force of 10 N up the plane.



Now, equating forces, we have

$$mg\sin\theta + f = 10\,\text{N}$$

or $mg\sin\theta + \mu mg\cos\theta = 10$ ...(ii)

Now, solving Eqs. (i) and (ii), we get

$$mg\sin\theta = 4 \quad \text{...(iii)}$$

and $\mu\, mg\cos\theta = 6$ ...(iv)

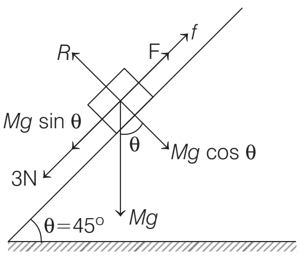
Dividing, Eqs. (iii) and (iv), we get

$$\mu\cot\theta = \frac{3}{2}$$

$$\Rightarrow \quad \mu = \frac{3\tan\theta}{2} = \frac{3\tan 30^\circ}{2}$$

$$\Rightarrow \quad \mu = \frac{\sqrt{3}}{2}$$

3. Free body diagram, for the given figure is as follows,



For the block to be in equilibrium i.e., so that it does not move downward, then

$$\Sigma f_x = 0$$

$\therefore \quad 3 + Mg\sin\theta - F - f = 0$

or $\quad 3 + Mg\sin\theta = F + f$

As, frictional force, $\quad f = \mu R$

$\therefore \quad 3 + Mg\sin\theta = F + \mu R$ ...(i)

Similarly, $\quad \Sigma f_y = 0$

$$-Mg\cos\theta + R = 0$$

or $\quad Mg\cos\theta = R$ ...(ii)

Substituting the value of 'R' from Eq. (ii) to Eq. (i), we get

$$3 + Mg\sin\theta = \text{F} + \mu(Mg\cos\theta) \quad \text{...(iii)}$$

Here, $\quad M = 10\,\text{kg}, \theta = 45^\circ, g = 10\,\text{m/s}^2$ and $\mu = 0.6$

Substituting these values is Eq. (iii), we get

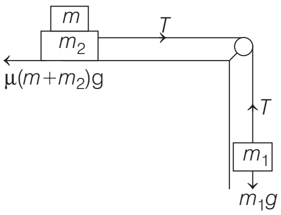
$$3 + (10\times 10\sin 45^\circ) - (0.6\times 10\times 10\cos 45^\circ) = F$$

$$\Rightarrow \quad F = 3 + \frac{100}{\sqrt{2}} - \frac{60}{\sqrt{2}} = 3 + \frac{40}{\sqrt{2}}$$

$$= 3 + 20\sqrt{2} = 31.8\,\text{N}$$

or $\quad F \simeq 32\,\text{N}$

4. None of the four options are correct.

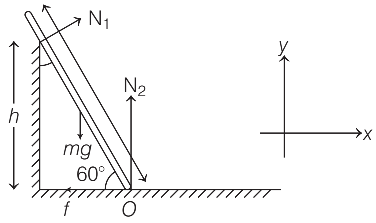


Substituting, $\quad m_1 = 5$ kg and $m_2 = 10$ kg

We get, $\mu(10 + m)g \geq 5g \quad \Rightarrow \quad 10 + m \geq \dfrac{5}{0.15}$

$$m \geq 23.33\text{ kg}$$

5.



$$\Sigma F_x = 0,$$

$$N_1\cos 30^\circ - f = 0 \quad \text{...(i)}$$

$$N_1\sin 30^\circ + N_2 - mg = 0 \quad \text{...(ii)}$$

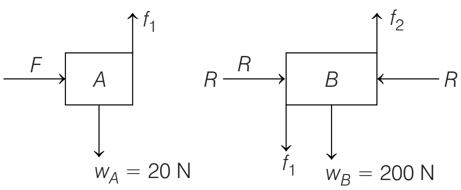
$$mg\frac{l}{2}\cos 60^\circ - N_1\frac{h}{\cos 30^\circ} = 0 \quad \text{...(iii)}$$

Also, given $\quad N_1 = N_2$ ...(iv)

Solving Eqs. (i), (ii), (iii) and (iv) we have

$$\frac{h}{l} = \frac{3\sqrt{3}}{16} \text{ and } f = \frac{16\sqrt{3}}{3}$$

6. **NOTE** It is not given in the question, best assuming that both blocks are in equilibrium. The free body diagram of two blocks is as shown below,



Reaction force, R = applied force F

For vertical equilibrium of A;

f_1 = friction between two blocks = W_A = 20 N

For vertical equilibrium of B;

f_2 = friction between block B and wall

$$= W_B + f_1 = 100 + 20 = 120\,\text{N}$$

7. A block of mass m is placed on a surface with a vertical cross-section, then

$$\tan\theta = \frac{dy}{dx} = \frac{d\left(\frac{x^3}{6}\right)}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, we get

$$\mu = \tan\theta$$

$$0.5 = \frac{x^2}{2}$$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = \pm 1$

Now, putting the value of x in $y = \frac{x^3}{6}$, we get

When $x = 1$	When $x = -1$
$y = \frac{(1)^3}{6} = \frac{1}{6}$	$y = \frac{(-1)^3}{6} = \frac{-1}{6}$

So, the maximum height above the ground at which the block can be placed without slipping is 1/6 m.

8. Block will not slip if

$$(m_1 + m_2)\, g \sin\theta \le \mu m_2 g \cos\theta$$

$\Rightarrow \quad 3\sin\theta \le \left(\frac{3}{10}\right)(2)\cos\theta$

$$\tan\theta \le 1/5$$

$\Rightarrow \quad \theta \le 11.5°$

(P) $\theta = 5°$ friction is static

$f = (m_1 + m_2)\, g \sin\theta$

(Q) $\theta = 10°$ friction is static

$f = (m_1 + m_2)\, g \sin\theta$

(R) $\theta = 15°$ friction is kinetic

$f = \mu m_2 g \cos\theta$

(S) $\theta = 20°$ friction is kinetic

$\Rightarrow f = \mu m_2 g \cos\theta$

9. When

$P = mg\,(\sin\theta - \mu\cos\theta)$

$f = \mu\, mg\cos\theta$ (upwards)

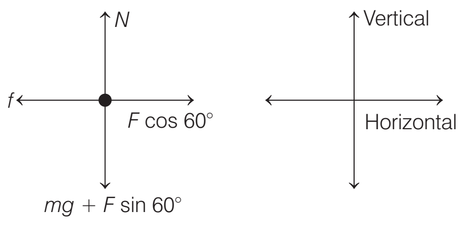
when $P = mg\sin\theta;\ f = 0$

and when $P = mg\,(\sin\theta + \mu\cos\theta)$

$f = \mu mg\cos\theta$ (downwards)

Hence, friction is first positive, then zero and then negative.

10. Free body diagram (FBD) of the block (shown by a dot) is shown in figure.



For vertical equilibrium of the block

$$N = mg + F\sin 60° = \sqrt{3}g + \sqrt{3}\frac{F}{2} \quad \ldots\text{(i)}$$

For no motion, force of friction

$$f \ge F\cos 60° \quad \text{or} \quad \mu N \ge F\cos 60°$$

or $\frac{1}{2\sqrt{3}}\left(\sqrt{3}g + \frac{\sqrt{3}F}{2}\right) \ge \frac{F}{2}$ or $g \ge \frac{F}{2}$

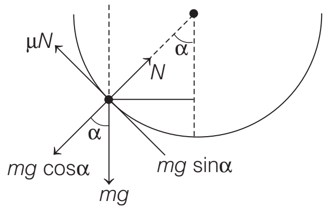
or $F \le 2g$ or 20 N

Therefore, maximum value of F is 20 N.

11. Equilibrium of insect gives

$$N = mg\cos\alpha \quad \ldots\text{(i)}$$

$$\mu N = mg\sin\alpha \quad \ldots\text{(ii)}$$

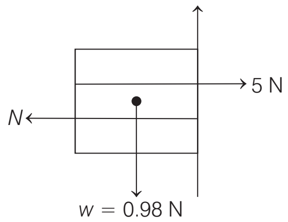


From Eqs. (i) and (ii), we get

$$\cot\alpha = \frac{1}{\mu} = 3$$

12. $N = 5$ N

$(f)_{max} = \mu N = (0.5)(5) = 2.5$ N



For vertical equilibrium of the block,

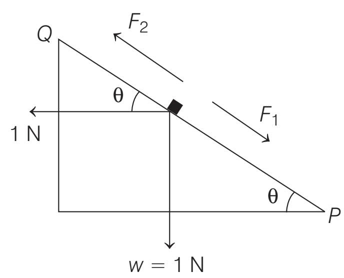
$f = mg = 0.98\text{ N} < (f)_{max}$

14. Since, $\mu mg\cos\theta > mg\sin\theta$

$\therefore$ Force of friction is $f = mg\sin\theta$

$$2 \times 9.8 \times \frac{1}{2} = 9.8\text{ N}$$

15. $w = mg = 0.1 \times 10 = 1$ N



F_1 = component of weight $= 1 \cdot \sin\theta = \sin\theta$

F_2 = component of applied force = $1 \cdot \cos\theta = \cos\theta$

Now, at $\theta = 45°$: $F_1 = F_2$ and block remains stationary without the help of friction.

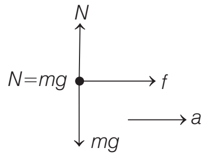
For $\theta > 45°$, $F_1 > F_2$, so friction will act towards Q.

For $\theta < 45°$, $F_2 > F_1$ and friction will act towards P.

16. The block will move due to friction.

The frictional force (f) is therefore

$$f = ma = (1)(5) = 5\,\text{N}$$



17. The frictional force exerted by the surface on the person is in the direction of his motion, since the legs pushes the surface backward direction.

18. Normal reaction in vertical direction $N_1 = mg$

Normal reaction from side to the groove $N_2 = ma \sin 37°$

Therefore, acceleration of block with respect to disc

$$a_r = \frac{ma\cos 37° - \mu N_1 - \mu N_2}{m}$$

Substituting the values we get, $a_r = 10\,\text{m/s}^2$

19. Acceleration of A down the plane,

$$a_A = g\sin 45° - \mu_A\, g\cos 45°$$

$$= (10)\left(\frac{1}{\sqrt{2}}\right) - (0.2)(10)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\,\text{m/s}^2$$

Similarly acceleration of B down the plane,

$$a_B = g\sin 45° - \mu_B\, g\cos 45°$$

$$= (10)\left(\frac{1}{\sqrt{2}}\right) - (0.3)(10)\left(\frac{1}{\sqrt{2}}\right) = 3.5\sqrt{2}\,\text{m/s}^2$$

The front face of A and B will come in a line when,

$$s_A = s_B + \sqrt{2}$$

or
$$\frac{1}{2}a_A t^2 = \frac{1}{2}a_B t^2 + \sqrt{2}$$

$$\frac{1}{2} \times 4\sqrt{2} \times t^2 = \frac{1}{2} \times 3.5\sqrt{2} \times t^2 + \sqrt{2}$$

Solving this equation, we get $t = 2\,\text{s}$

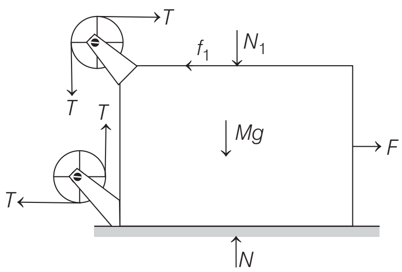
Further, $$s_A = \frac{1}{2}a_A t^2 = \frac{1}{2} \times 4\sqrt{2} \times (2)^2 = 8\sqrt{2}\,\text{m}$$

Hence, both the blocks will come in a line after A has travelled a distance $8\sqrt{2}$ m down the plane.

20. Given, $m_1 = 20\text{kg}$, $m_2 = 5\text{kg}$, $M = 50\text{kg}$,

$\mu = 0.3$ and $g = 10\,\text{m/s}^2$.

(a) Free body diagram of mass M is given as



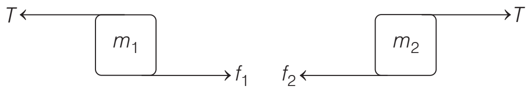
(b) The maximum value of f_1 is

$$(f_1)_{\max} = (0.3)(20)(10) = 60\text{N}$$

The maximum value of f_2 is

$$(f_2)_{\max} = (0.3)(5)(10) = 15\text{N}$$

Forces on m_1 and m_2 in horizontal direction are as follows



Now, there are only two possibilities :

(1) Either both m_1 and m_2 will remain stationary (w.r.t. ground) or

(2) Both m_1 and m_2 will move (w.r.t. ground)

First case is possible when

$$T \le (f_1)_{\max} \quad \text{or} \quad T \le 60\text{ N}$$

and $$T \le (f_2)_{\max} \quad \text{or} \quad T \le 15\text{ N}$$

These conditions will be satisfied when $T \le 15$ N

say $T = 14$ N, then $f_1 = f_2 = 14$N

Therefore, the condition $f_1 = 2f_2$ will not be satisfied. Thus, m_1 and m_2 both cannot remain stationary.

In the second case, when m_1 and m_2 both move

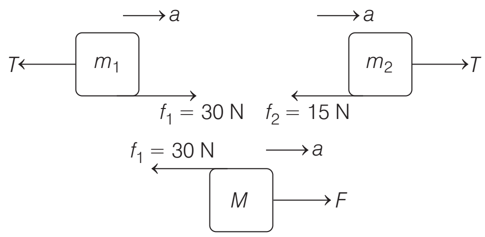
$$f_2 = (f_2)_{\max} = 15\text{N}$$

$$\therefore \quad f_1 = 2f_2 = 30\text{ N}$$

Now, since $f_1 < (f_1)_{\max}$, there is no relative motion between m_1 and M i.e. all the masses move with same acceleration, say a

$$\therefore \quad f_2 = 15\text{N} \quad \text{and} \quad f_1 = 30\text{N}$$

Free body diagrams and equations of motion are as follows



For m_1, $\quad 30 - T = 20a \quad$ ...(i)

For m_2, $\quad T - 15 = 5a \quad$ ...(ii)

For M, $\quad F - 30 = 50a \quad$ ...(iii)

Solving these three equations, we get

$$F = 60\text{ N}$$

$$T = 18\text{N} \quad \text{and} \quad a = \frac{3}{5}\,\text{m/s}^2$$

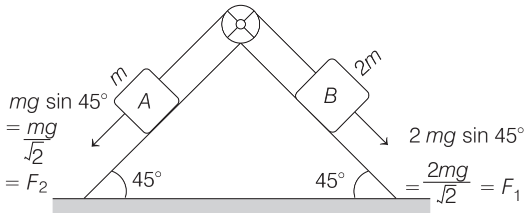
NOTE

- Friction always opposes the relative motion between two surfaces in contact.
- Whenever there is relative motion between two surfaces in contact, always maximum friction (kinetic) acts, but if there is no relative motion then friction force (f) may be less than its limiting value also. So, don't apply maximum force.

21. **(a) Acceleration of block *A***

Maximum friction force that can be obtained at A is

$$(f_{\max})_A = \mu_A (mg \cos 45°)$$
$$= \frac{2}{3}(mg/\sqrt{2}) = \frac{\sqrt{2}mg}{3}$$



Similarly,

$$(f_{\max})_B = \mu_B (2mg \cos 45°)$$
$$= \frac{1}{3}(2mg/\sqrt{2}) = \frac{\sqrt{2}mg}{3}$$

Therefore, maximum value of friction that can be obtained on the system is

$$(f_{\max}) = (f_{\max})_A + (f_{\max})_B = \frac{2\sqrt{2}\,mg}{3} \quad \text{...(i)}$$

Net pulling force on the system is

$$F = F_1 - F_2 = \frac{2mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = \frac{mg}{\sqrt{2}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we can see that

Net pulling force $< f_{\max}$. Therefore, the system will not move or the acceleration of block A will be zero.

(b) and (c) Tension in the string and friction at *A*

Net pulling force on the system (block A and B)

$$F = F_1 - F_2 = mg/\sqrt{2}$$

Therefore, total friction force on the blocks should also be equal to $\frac{mg}{\sqrt{2}}$

or $$f_A + f_B = F = mg/\sqrt{2}$$

Now, since the blocks will start moving from block B first (if they move), therefore, f_B will reach its limiting value first and if still some force is needed, it will be provided by f_A.

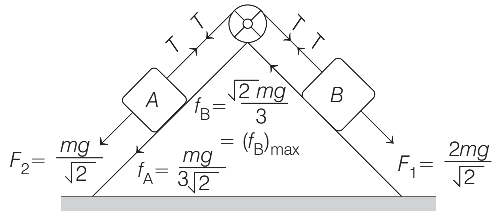
Here, $(f_{\max})_B < F$

Therefore, f_B will be in its limiting value and rest will be provided by f_A.

Hence, $$f_B = (f_{\max})_B = \frac{\sqrt{2}mg}{3}$$

and $$f_A = F - f_B$$
$$= \frac{mg}{\sqrt{2}} - \frac{\sqrt{2}mg}{3} = \frac{mg}{3\sqrt{2}}$$

The FBD of the whole system will be as shown in the figure.



Therefore, friction on A is

$$f_A = mg/3\sqrt{2} \quad \text{(down the plane)}$$

Now, for tension T in the string, we may consider either equilibrium of A or B.

Equilibrium of A gives

$$T = F_2 + f_A = \frac{mg}{\sqrt{2}} + \frac{mg}{3\sqrt{2}} = \frac{4mg}{3\sqrt{2}}$$

or $$T = \frac{2\sqrt{2}mg}{3}$$

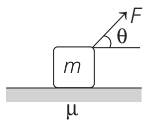
Similarly, equilibrium of B gives $T + f_B = F_1$

or $$T = F_1 - f_B$$
$$= \frac{2mg}{\sqrt{2}} - \frac{\sqrt{2}mg}{3} = \frac{4mg}{3\sqrt{2}}$$

or $$= \frac{2\sqrt{2}\,mg}{3}$$

Therefore, tension in the string is $\frac{2\sqrt{2}\,mg}{3}$.

22. Let F be applied at angle θ as shown in figure.



Normal reaction in this case will be,

$$N = mg - F \sin\theta$$

The limiting friction is therefore

$$f_L = \mu N = \mu(mg - F \sin\theta)$$

For the block to move,

$$F \cos\theta = f_L = \mu\,(mg - F \sin\theta)$$

or $$F = \frac{\mu mg}{\cos\theta + \mu \sin\theta} \quad \text{...(i)}$$

For F to be minimum, denominator should be maximum.

or $$\frac{d}{d\theta}(\cos\theta + \mu \sin\theta) = 0$$

or $$-\sin\theta + \mu \cos\theta = 0$$

or $$\tan\theta = \mu \quad \text{or} \quad \theta = \tan^{-1}(\mu)$$

Substituting this value of θ in Eq. (i), we get

$$F_{\min} = mg \sin\theta$$

23. Constant velocity means net acceleration of the system is zero. Or net pulling force on the system is zero. While calculating the pulling force, tension forces are not taken into consideration. Therefore,

(a) $$M_1 g = M_2 g \sin 37° + \mu\, M_2 g \cos 37° + \mu\, M_3 g$$

or $$M_1 = M_2 \sin 37° + \mu M_2 \cos 37° + \mu M_3$$

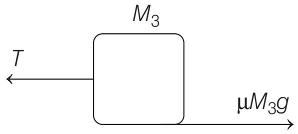
Substituting the values

$$M_1 = (4)\left(\frac{3}{5}\right) + (0.25)(4)\left(\frac{4}{5}\right) + (0.25)(4)$$

$$= 4.2\,\text{kg}$$

(b) Since, M_3 is moving with uniform velocity

$$T = \mu\ M_3 g = (0.25)(4)(9.8) = 9.8\,\text{N}$$



24. Maximum force of friction between M_1 and inclined plane

$$f_1 = \mu_1 M_1 g \cos\theta = (0.75)(4)(9.8)(0.8) = 23.52\,\text{N}$$

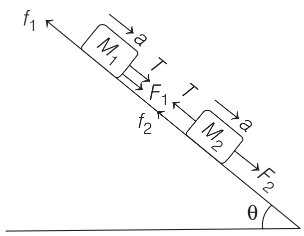
$$M_1 g \sin\theta = (4)(9.8)(0.6) = 23.52\,\text{N} = F_1 \qquad \text{(say)}$$

Maximum force of friction between M_2 and inclined plane

$$f_2 = \mu_2 M_2 g \cos\theta$$

$$= (0.25)(2)(9.8)(0.8) = 3.92\,\text{N}$$

$$M_2 g \sin\theta = (2)(9.8)(0.6) = 11.76\,\text{N} = F_2 \qquad \text{(say)}$$



Both the blocks will be moving downwards with same acceleration a. Different forces acting on two blocks are as shown in above figure.

Equation of motion of M_1

$$T + F_1 - f_1 = M_1 a$$

or $$T = 4a \qquad \ldots\text{(i)}$$

Equation of motion of M_2

$$F_2 - T - f_2 = M_2 a$$

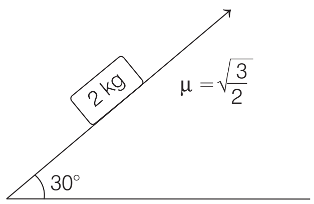
or $$7.84 - T = 2a \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$a = 1.3\,\text{m/s}^2$$

and $$T = 5.2\,\text{N}$$

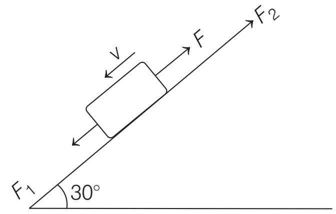
25. $mg\sin\theta = (2)(10)\left(\frac{1}{2}\right) = 10\,\text{N} = F_1$ (say)



$$\mu\ mg\cos\theta = \left(\sqrt{\frac{3}{2}}\right)(2)(10)\left(\frac{\sqrt{3}}{2}\right)$$

$$= 21.21\,\text{N} = F_2 \text{ (say)}$$

(a) Force required to move the block down the plane with constant velocity.



F_1 will be acting downwards, while F_2 upwards.

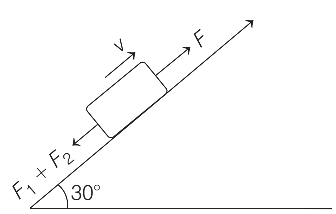
Since $F_2 > F_1$, force required

$$F = F_2 - F_1 = 11.21\,\text{N}$$

(b) Force required to move the block up the plane with constant velocity.

F_1 and F_2 both will be acting downwards.

$\therefore \qquad F = F_1 + F_2 = 31.21\,\text{N}$



26. Maximum friction between A and $B = \mu\ m_A g$

or $$f_1 = 0.25\,(3)(10) = 7.5\,\text{N}$$

Maximum friction between B and $C = \mu\ (m_A + m_B)\,g$

or $$f_2 = 0.25\,(3+4)(10)$$

$$= 17.5\,\text{N}$$

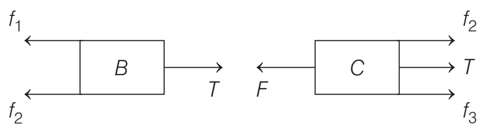
Maximum friction between C and ground

$$f_3 = \mu\ (m_A + m_B + m_C)\,g$$

$$= 0.25\,(3+4+8)(10)$$

$$= 37.5\,\text{N}$$

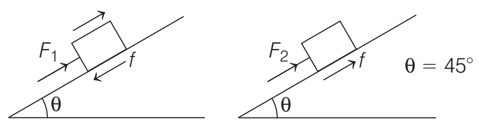
Block C and hence block B are moving in opposite directions with constant velocities and block A is at rest. Hence, net force on all three blocks should be zero. Free body diagrams have been shown below (Only horizontal forces are shown)



For equilibrium of B, $T = f_1 + f_2 = 25\,\text{N}$

For equilibrium of C, $F = T + f_2 + f_3 = 80\,\text{N}$

27.



Moving upwards   Just remains stationary

$$F_1 = mg\sin\theta + \mu mg\cos\theta$$

$$F_2 = mg \sin\theta - \mu mg \cos\theta$$

Given that $F_1 = 3F_2$ or $(\sin 45° + \mu \cos 45°)$

$= 3(\sin 45° - \mu \cos 45°)$

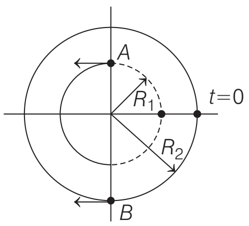
On solving, we get $\mu = 0.5 \therefore \; N = 10\mu = 5$

Topic 4 Dynamics of Circular Motion

1. Angle covered by each particle in time duration 0 to $\frac{\pi}{2\omega}$ is

$$\theta = \omega \times t = \omega \times \frac{\pi}{2\omega} = \frac{\pi}{2} \text{ rad}$$

So, positions of particles at $t = \frac{\pi}{2\omega}$ is as shown below;



Velocities of particles at $t = \frac{\pi}{2\omega}$ are

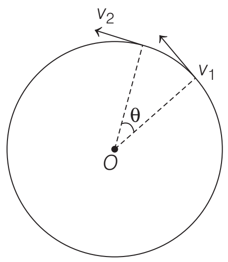
$$\mathbf{v}_A = -\omega R_1 \hat{\mathbf{i}} \quad \text{and} \quad \mathbf{v}_B = -\omega R_2 \hat{\mathbf{i}}$$

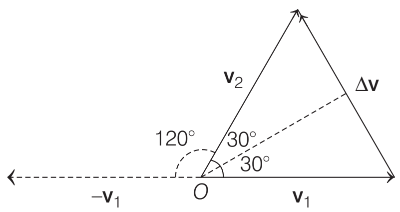
The relative velocity of particles is

$$\mathbf{v}_A - \mathbf{v}_B = -\omega R_1 \hat{\mathbf{i}} - (-\omega R_2 \hat{\mathbf{i}})$$

$$= -\omega(R_1 - R_2)\hat{\mathbf{i}} = \omega(R_2 - R_1)\hat{\mathbf{i}}$$

2. Let v_1 be the velocity of the particle moving along the circular path initially, v_1 and v_2 be the velocity when it moves through an angle of 60° as shown below.





From the figure,

$$\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$

$\Rightarrow \quad |\Delta \mathbf{v}| = 2v \sin\frac{\theta}{2} = 2\,\mathbf{v} \sin 30° \qquad [\because |\mathbf{v}_1| = |\mathbf{v}_2|]$

$= 2v \times \frac{1}{2} = v \qquad$ (Given, $v = 10$ m / s)

$\Rightarrow \quad |\Delta\mathbf{v}| = 10$ m/s

Alternate method

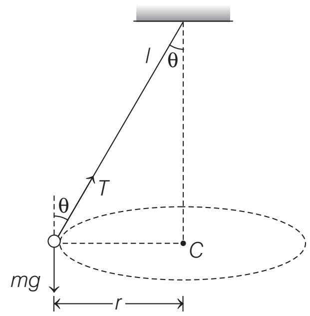
$\because \quad \Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = \mathbf{v}_2 + (-\mathbf{v}_1)$

$\therefore |\Delta\mathbf{v}|^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos 120°$

$= v^2 + v^2 + 2v \times v \times \left(-\frac{1}{2}\right)$

$\Rightarrow |\Delta\,\mathbf{v}| = v = 10$ m/s.

3.



$$r = l \sin\theta$$

$T\cos\theta$ component will cancel mg.

$T\sin\theta$ component will provide necessary centripetal force to the ball towards centre C.

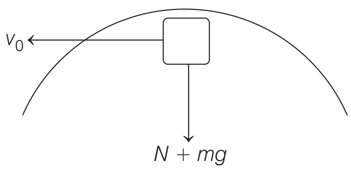
$\therefore \quad T\sin\theta = mr\omega^2 = m(l\sin\theta)\omega^2$ or $T = ml\omega^2$

$\therefore \quad \omega = \sqrt{\frac{T}{ml}}$

or $\quad \omega_{\max} = \sqrt{\frac{T_{\max}}{ml}} = \sqrt{\frac{324}{0.5 \times 0.5}} = 36$ rad/s

4. Since, the block rises to the same heights in all the four cases, from conservation of energy, speed of the block at highest point will be same in all four cases. Say it is v_0.

Equation of motion will be



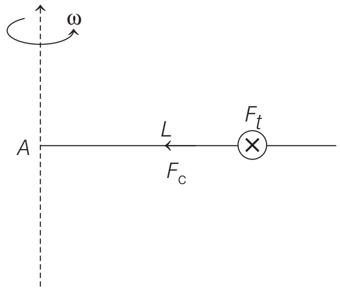
$$N + mg = \frac{mv_0^2}{R}$$

or $$N = \frac{mv_0^2}{R} - mg$$

R (the radius of curvature) in first case is minimum. Therefore, normal reaction N will be maximum in first case.

NOTE In the question, it should be mentioned that all the four tracks are frictionless. Otherwise, v_0 will be different in different tracks.

5. Tangential force (F_t) of the bead will be given by the normal reaction (N), while centripetal force (F_c) is provided by friction (f_r). The bead starts sliding when the centripetal force is just equal to the limiting friction.



F_t is inwards

Therefore,

$$F_t = ma = m\alpha\, L = N$$

$\therefore$ Limiting value of friction

$$(f_r)_{max} = \mu N = \mu m\alpha L \qquad \ldots(i)$$

Angular velocity at time t is $\omega = \alpha\, t$

$\therefore$ Centripetal force at time t will be

$$F_c = mL\,\omega^2 = mL\,\alpha^2\, t^2 \qquad \ldots(ii)$$

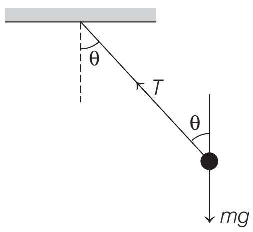
Equating Eqs. (i) and (ii), we get

$$t = \sqrt{\frac{\mu}{\alpha}}$$

For $t > \sqrt{\frac{\mu}{\alpha}}$, $F_c > (f_r)_{max}$ i.e. the bead starts sliding.

In the figure, F_t is perpendicular to the paper inwards.

6. FBD of bob is $T \sin\theta = \frac{mv^2}{R}$



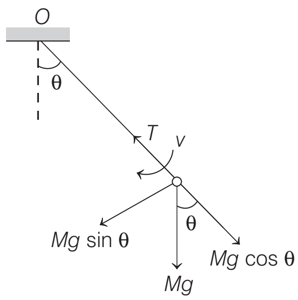
and $T \cos\theta = mg$

$\therefore \quad \tan\theta = \frac{v^2}{Rg} = \frac{(10)^2}{(10)(10)}$

$\tan\theta = 1$

or $\quad \theta = 45°$

7. Motion of pendulum is the part of a circular motion. In circular motion, it is better to resolve the forces in two perpendicular directions. First along radius (towards centre) and second along tangential. Along radius, net force should be equal to $\frac{mv^2}{R}$ and along tangent it should be equal to ma_T, where a_T is the tangential acceleration in the figure.



$$T - Mg\cos\theta = \frac{Mv^2}{L}$$

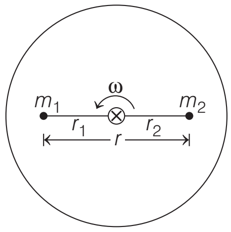
and $\quad Mg\sin\theta = Ma_T$ or $a_T = g\sin\theta$

8. $T - mg\cos 20° = \frac{mv^2}{R}$ or $T = mg\cos 20° + \frac{mv^2}{R}$

9. Given, $m_1 = 10\,\text{kg}$,

$$m_2 = 5\,\text{kg}, \omega = 10\,\text{rad/s}$$
$$r = 0.3\,\text{m}, r_1 = 0.124\,\text{m}$$
$$\therefore \quad r_2 = r - r_1 = 0.176\,\text{m}$$

(a) Masses m_1 and m_2 are at rest with respect to rotating table. Let f be the friction between mass m_1 and table.



Free body diagram of m_1 and m_2 with respect to ground

$$T = m_2 r_2 \omega^2 \qquad \ldots(i)$$

Since, $\quad m_2 r_2 \omega^2 < m_1 r_1 \omega^2$

Therefore, $\quad m_1 r_1 \omega^2 > T$

and friction on m_1 will be inward (toward centre)

$$f + T = m_1 r_1 \omega^2 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$f = m_1 r_1 \omega^2 - m_2 r_2 \omega^2 \qquad \ldots(iii)$$
$$= (m_1 r_1 - m_2 r_2)\,\omega^2$$
$$= (10 \times 0.124 - 5 \times 0.176)\,(10)^2\ \text{N} = 36\ \text{N}$$

Therefore, frictional force on m_1 is 36 N (inwards)

(b) From Eq. (iii)

$$f = (m_1 r_1 - m_2 r_2)\,\omega^2$$

Masses will start slipping when this force is greater than f_{max} or

$$(m_1 r_1 - m_2 r_2)\,\omega^2 > f_{max} > \mu m_1 g$$

$\therefore$ Minimum values of ω is

$$\omega_{min} = \sqrt{\frac{\mu m_1 g}{m_1 r_1 - m_2 r_2}} = \sqrt{\frac{0.5 \times 10 \times 9.8}{10 \times 0.124 - 5 \times 0.176}}$$

$$\omega_{min} = 11.67 \quad \text{rad/s}$$

(c) From Eq. (iii), frictional force $f = 0$

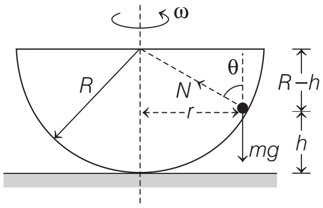
where, $\quad m_1 r_1 = m_2 r_2$

or $\quad \frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{5}{10} = \frac{1}{2}$ and $r = r_1 + r_2 = 0.3\,\text{m}$

$\therefore \quad r_1 = 0.1\,\text{m}$ and $r_2 = 0.2\,\text{m}$

i.e. mass m_2 should placed at 0.2 m and m_1 at 0.1m from the centre O.

10.



Given, $R = 0.1\text{m}$, $m = 10^{-2}$ kg

(a) FBD of particle in ground frame of reference is shown in figure. Hence,

$$\tan\theta = \frac{r}{R-h}$$

$$N\cos\theta = mg \qquad \ldots\text{(i)}$$

and $$N\sin\theta = mr\omega^2 \qquad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we obtain

$$\tan\theta = \frac{r\omega^2}{g} \text{ or } \frac{r}{R-h} = \frac{r\omega^2}{g}$$

or $$\omega^2 = \frac{g}{R-h} \qquad \ldots\text{(iii)}$$

This is the desired relation between ω and h.
From Eq. (iii),

$$h = R - \frac{g}{\omega^2}$$

For non-zero value of h,

$$R > \frac{g}{\omega^2} \text{ or } \omega > \sqrt{g/R}$$

Therefore, minimum value of ω should be

$$\omega_{\min} = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{0.1}} \text{ rad/s}$$

or $$\omega_{\min} = 9.89 \text{ rad/s}$$

(b) Eq. (iii) can be written as $h = R - \dfrac{g}{\omega^2}$

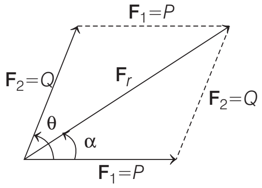
If R and ω are known precisely, then $\Delta h = -\dfrac{\Delta g}{\omega^2}$

or $\Delta g = \omega^2\,\Delta h$ (neglecting the negative sign)

$$(\Delta g)_{\min} = (\omega_{\min})^2\,\Delta h,\ (\Delta g)_{\min} = 9.8\times 10^{-3}\ \text{m/s}^2$$

Topic 5 Miscellaneous Problems

1. Resultant force $\mathbf{F}_r$ of any two forces $\mathbf{F}_1$ (i.e. P) and $\mathbf{F}_2$ (i.e. Q) with an angle θ between them can be given by vector addition as



In first case $F_1 = 2F$ and $F_2 = 3F$

$$\Rightarrow \quad F_r^2 = 4F^2 + 9F^2 + 2\times 2\times 3F^2\cos\theta$$

$$\Rightarrow \quad F_r^2 = 13F^2 + 12F^2\cos\theta \qquad \ldots\text{(ii)}$$

In second case $F_1 = 2F$ and $F_2 = 6F$

(∵ Force Q gets doubled)

and $$F_r' = 2F_r \qquad \text{(Given)}$$

By putting these values in Eq. (i), we get

$$(2F_r)^2 = (2F)^2 + (6F)^2 + 2\times 2\times 6F^2\cos\theta$$

$$\Rightarrow \quad 4F_r^2 = 40F^2 + 24F^2\cos\theta \qquad \ldots\text{(iii)}$$

From Eq. (ii) and Eq. (iii), we get;

$$52F^2 + 48F^2\cos\theta = 40F^2 + 24F^2\cos\theta$$

$\Rightarrow$ $12 + 24\cos\theta = 0$ or $\cos\theta = -1/2$

or $$\theta = 120^\circ \qquad (\because \cos 120^\circ = -1/2)$$

2. Given, $U = -\dfrac{k}{2r^2} \Rightarrow F_r = -\dfrac{dU}{dr} = -\dfrac{k}{r^3}$

Since, the particle moves in a circular path of radius 'a', the required centripetal force is provided by the above force.

Hence, $\dfrac{mv^2}{a} = \dfrac{k}{a^3} \Rightarrow mv^2 = \dfrac{k}{a^2}$

Kinetic energy, $K = \dfrac{1}{2}mv^2 = \dfrac{k}{2a^2}$

Total energy $= K + U = -\dfrac{k}{2a^2} + \dfrac{k}{2a^2} = 0$

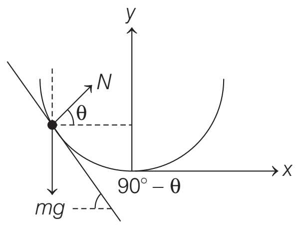
3. $N\sin\theta = mg$

$$\Rightarrow \quad N\cos\theta = ma$$

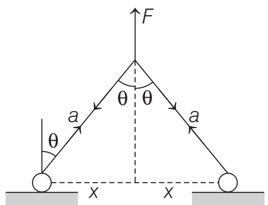
$$\tan\theta = \frac{g}{a}$$

$$\Rightarrow \quad \cot\theta = \frac{a}{g} = \tan(90^\circ - \theta) = \frac{dy}{dx} = 2kx$$

$$\therefore \quad x = \frac{a}{2kg}$$



4. As, from FBD (Free Body Diagram), $2T\cos\theta = F$



So, $$T = \frac{F}{2}\sec\theta$$

Acceleration of particle

$$= \frac{T\sin\theta}{m} = \frac{F\tan\theta}{2m}$$

$$= \frac{F}{2m}\frac{x}{\sqrt{a^2 - x^2}}$$

5. Initially under equilibrium of mass m

$$T = mg$$

Now, the string is cut. Therefore, $T = mg$ force is decreased on mass m upwards and downwards on mass $2m$.

$\therefore$ $$a_m = \frac{mg}{m} = g \quad \text{(downwards)}$$

and $$a_{2m} = \frac{mg}{2m} = \frac{g}{2} \quad \text{(upwards)}$$

6. As, $a = \frac{F}{m} = \frac{5\times 10^4}{3\times 10^7} = \frac{5}{3}\times 10^{-3}\ \text{m/s}^2$

So, $v = \sqrt{2as} = \sqrt{2\times\frac{5}{3}\times 10^{-3}\times 3} = 0.1\ \text{m/s}$

7. A rotating/revolving frame is accelerating and hence non-inertial.

8. Angular speed and hence the required centripetal force for both the trains will be different. So, normal reaction (or pressure) will also be different.

9. $CP = CO$

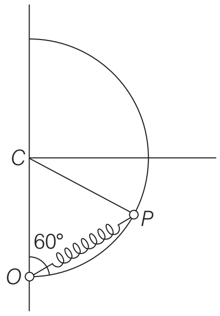
Radius of circle (R)

$\therefore$ $\angle CPO = \angle POC = 60°$

$\therefore$ $\angle OCP$ is also $60°$.

Therefore, $\Delta\, OCP$ is an equilateral triangle.

Hence, $OP = R$



Natural length of spring is $3R/4$.

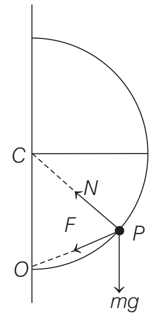
$\therefore$ Extension in the spring

$$x = R - \frac{3R}{4} = \frac{R}{4}$$

$\Rightarrow$ Spring force,

$$F = kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$$

The free body diagram of the ring will be as shown below.



Here, $F = kx = \frac{mg}{4}$ and N = Normal reaction.

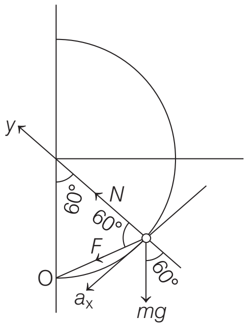
(b) **Tangential acceleration, a_T** The ring will move towards the x-axis just after the release. So, net force along x-axis

$$F_x = F\sin 60° + mg\sin 60°$$

$$= \left(\frac{mg}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$$

$$F_x = \frac{5\sqrt{3}}{8}mg$$

The free body diagram of the ring is shown below.



Therefore, tangential acceleration of the ring,

$$a_T = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8}g \Rightarrow a_T = \frac{5\sqrt{3}}{8}g$$

Normal reaction, N Net force along y-axis on the ring just after the release will be zero.

$$F_y = 0$$

$\therefore$ $$N + F\cos 60° = mg\cos 60°$$

$\therefore$ $$N = mg\cos 60° - F\cos 60°$$

$$= \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right) = \frac{mg}{2} - \frac{mg}{8}$$

$$N = \frac{3mg}{8}$$

4

Work, Power and Energy

Topic 1 Work Done and Power

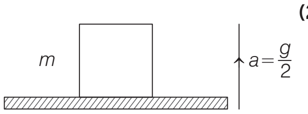
Objective Questions I (Only one correct option)

1. A uniform cable of mass M and length L is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)$th part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be **(2019 Main, 9 April I)**

(a) $\frac{2MgL}{n^2}$ (b) $nMgL$

(c) $\frac{MgL}{n^2}$ (d) $\frac{MgL}{2n^2}$

2. A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upwards as shown in figure. Work done by normal reaction on block in time t is **(2019 Main, 10 Jan I)**



(a) $\frac{mg^2t^2}{8}$ (b) $\frac{3mg^2t^2}{8}$

(c) 0 (d) $-\frac{mg^2t^2}{8}$

3. A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? **(2019 Main, 9 Jan II)**

(a) 850 J (b) 900 J (c) 950 J (d) 875 J

4. When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is **(2014 Main)**

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$

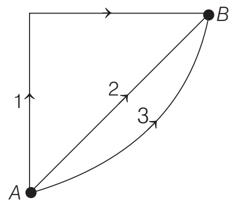
(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

5. The work done on a particle of mass m by a force,

$K\left[\frac{x}{(x^2 + y^2)^{3/2}}\hat{\mathbf{i}} + \frac{y}{(x^2 + y^2)^{3/2}}\hat{\mathbf{j}}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x-y plane is **(2013 Adv.)**

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$ (c) $\frac{K\pi}{2a}$ (d) 0

6. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m. Find the correct relation between W_1, W_2 and W_3 **(2003, 2M)**



(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

7. A force $\mathbf{F} = -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$ (where, k is a positive constant) acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive X-axis to the point $(a, 0)$ and then parallel to the Y-axis to the point (a, a). The total work done by the force F on the particle is **(1998, 2M)**

(a) $-2ka^2$ (b) $2ka^2$ (c) $-ka^2$ (d) ka^2

8. A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is **(1985, 2M)**

(a) MgL (b) $MgL/3$ (c) $MgL/9$ (d) $MgL/18$

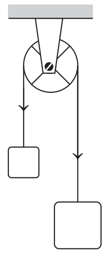
9. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to **(1984, 2M)**
(a) $t^{1/2}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2

Integer Answer Type Questions

10. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is **(2013 Adv.)**

Analytical & Descriptive Questions

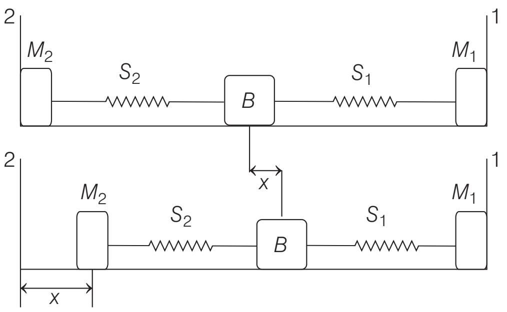
11. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\,ms^{-2}$, find the work done (in Joule) by string on the block of mass 0.36 kg during the first second after the system is released from rest. **(2009)**



Topic 2 Conservation of Mechanical Energy

Objective Questions I (Only one correct option)

1. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively. The other ends are attached to two supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere.



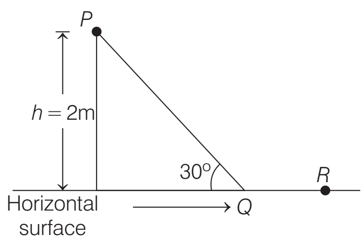
The block B is displaced towards wall 1 by a small distance x and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is **(2008, 3M)**
(a) 4 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

2. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is **(2002, 2M)**
(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$
(c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

Topic 3 Problems with Friction

Objective Questions II (Only one correct option)

1. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released from rest, from the point P and it comes to rest at a point R. The energies lost by the ball, over the parts PQ and QR of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x (= QR), are respectively close to **(2016 Main)**



(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

Assertion and Reason

Mark your answer as
(*a*) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*
(*b*) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*
(*c*) *If Statement I is true; Statement II is false*
(*d*) *If Statement I is false; Statement II is true*

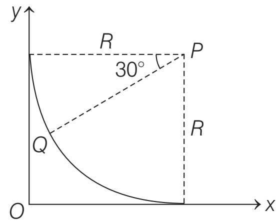
2. **Statement I** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Statement II The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. **(2007, 3M)**

Passage Based Questions

Passage 1

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure, is 150 J. (Take the acceleration due to gravity, $g = 10\ \text{ms}^{-2}$) **(2013 Adv.)**



3. The speed of the block when it reaches the point Q is

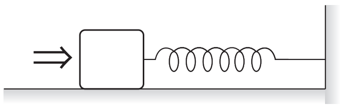
(a) $5\ \text{ms}^{-1}$ (b) $10\ \text{ms}^{-1}$ (c) $10\sqrt{3}\ \text{ms}^{-1}$ (d) $20\ \text{ms}^{-1}$

4. The magnitude of the normal reaction that acts on the block at the point Q is

(a) 7.5 N (b) 8.6 N (c) 11.5 N (d) 22.5 N

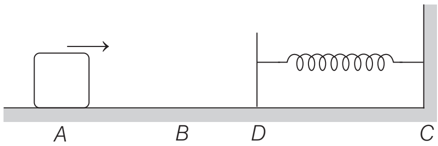
Integer Answer Type Questions

5. A block of mass 0.18 kg is attached to a spring of force constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = \frac{N}{10}$. Then N is **(2011)**

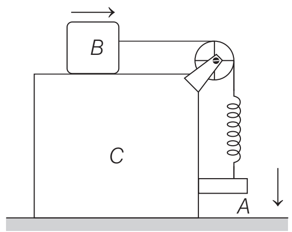


Analytical & Descriptive Questions

6. A 0.5 kg block slides from the point A (see fig.) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 m and force constant 2 N/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0.2 respectively. If the distances AB and BD are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely. (Take $g = 10\ \text{m/s}^2$). **(1983, 7M)**



7. Two blocks A and B are connected to each other by a string and a spring; the string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of blocks is 0.2. Force constant of the spring is 1960 N/m. If mass of block A is 2 kg. Calculate the mass of block B and the energy stored in the spring. **(1982, 5M)**

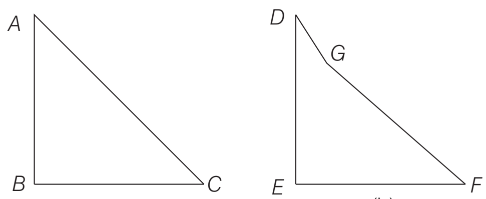


8. A lead bullet just melts when stopped by an obstacle. Assuming that 25 per cent of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C.

(Melting point of lead = 327° C, specific heat of lead = 0.03 cal/g-°C, latent heat of fusion of lead = 6 cal/g, $J = 4.2$ J/cal). **(1981, 3M)**

9. A body of mass 2 kg is being dragged with a uniform velocity of 2 m/s on a rough horizontal plane. The coefficient of friction between the body and the surface is 0.20, $J = 4.2$ J/cal and $g = 9.8\ \text{m/s}^2$. Calculate the amount of heat generated in 5 s. **(1980, 5M)**

10. In the figures (a) and (b) AC, DG and GF are fixed inclined planes, $BC = EF = x$ and $AB = DE = y$. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed v_C. The same block is released from rest from the point D. It slides down DGF and reaches the point F with speed v_F. The coefficients of kinetic frictions between the block and both the surfaces AC and DGF are μ. Calculate v_C and v_F. **(1980, 6M)**

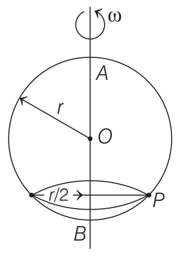


(a) (b)

Topic 4 Problems of Circular Motion

Objective Questions I (Only one correct option)

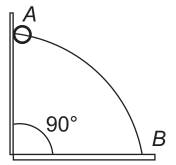
1 A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then, the value of ω^2 is equal to **(Main 2019, April 12 II)**



(a) $\dfrac{\sqrt{3}g}{2r}$ (b) $2g/(r\sqrt{3})$

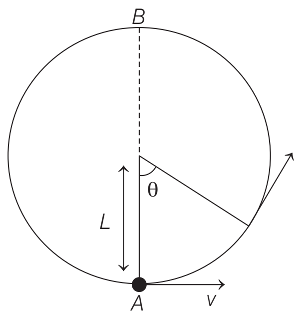
(c) $(g\sqrt{3})/r$ (d) $2g/r$

2. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is **(2014 Adv.)**



(a) always radially outwards
(b) always radially inwards
(c) radially outwards initially and radially inwards later
(d) radially inwards initially and radially outwards later

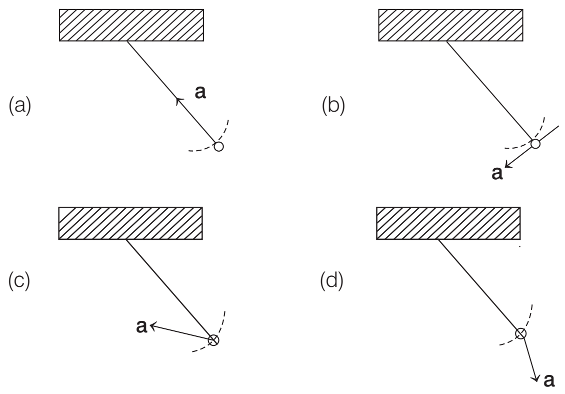
3. A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, statisfies **(2008, 3M)**



(a) $\theta = \dfrac{\pi}{4}$ (b) $\dfrac{\pi}{4} < \theta < \dfrac{\pi}{2}$

(c) $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{4}$ (d) $\dfrac{3\pi}{4} < \theta < \pi$

4. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector **a** is correctly shown in **(2002, 2M)**



5. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$ **(1998, 2M)**

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

6. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is

(a) $2\pi mk^2 r^2$ (b) $mk^2 r^2 t$ **(1994, 1M)**

(c) $\dfrac{(mk^4 r^2 t^5)}{3}$ (d) zero

Objective Question II (One or more correct option)

7. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that **(1987, 2M)**

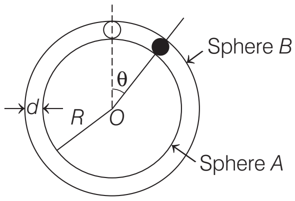
(a) its velocity is constant (b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a circular path

Integer Answer Type Question

8. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are massless and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio l_1 / l_2 is **(2013 Adv.)**

Analytical & Descriptive Questions

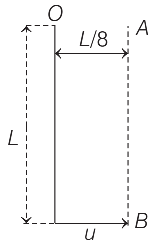
9. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see fig.). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ. **(2002, 5M)**



(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as function of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axis.

10. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $L/8$ from O as shown in figure. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u. **(1999, 10M)**



11. A bullet of mass M is fired with a velocity 50 m/s at an angle θ with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass $3M$ suspended by a massless string of length 10/3 m and gets embedded in the bob. After the collision, the string moves through an angle of 120°. Find **(1988, 6M)**

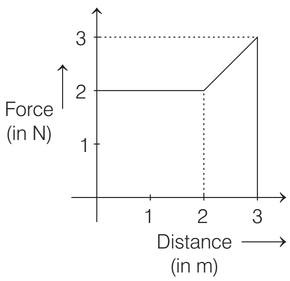
(a) the angle θ,

(b) the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. (Take $g = 10\ \text{m/s}^2$)

Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

1 A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is **(2019 Main, 08 April I)**



(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

2 A particle which is experiencing a force, is given by $\mathbf{F} = 3\hat{\mathbf{i}} - 12\hat{\mathbf{j}}$, undergoes a displacement of $\mathbf{d} = 4\hat{\mathbf{i}}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ? **(2019 Main, 10 Jan II)**

(a) 9 J (b) 15J (c) 12 J (d) 10 J

3. A body of mass $m = 10^{-2}$kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10\,\mathbf{ms}^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be **(2017 Main)**

(a) $10^{-3}\,\text{kgs}^{-1}$ (b) $10^{-4}\,\text{kgm}^{-1}$
(c) $10^{-1}\,\text{kgm}^{-1}\text{s}^{-1}$ (d) $10^{-3}\,\text{kgm}^{-1}$

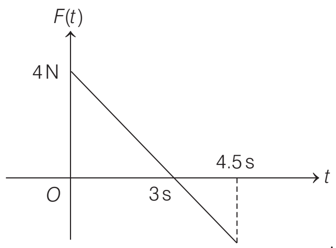
4. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be **(2017Main)**

(a) 22 J (b) 9 J (c) 18 J (d) 4.5 J

5. A person trying to loose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. (Take, $g = 9.8\ \text{ms}^{-2}$) **(2016 Main)**

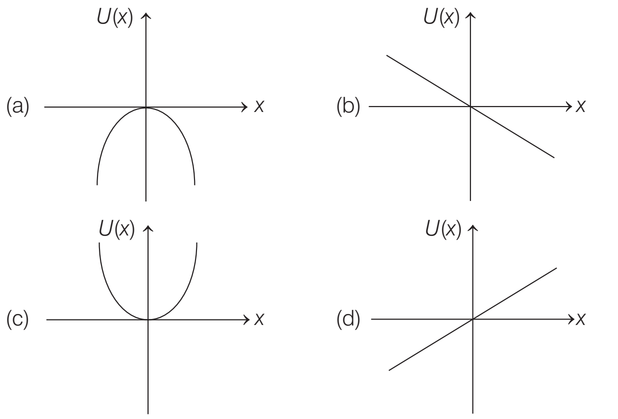
(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg
(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

6. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 s is **(2010)**

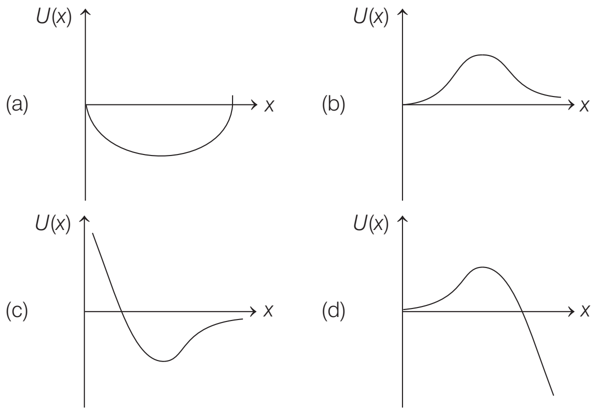


(a) 4.50 J (b) 7.50 J (c) 5.06 J (d) 14.06 J

7. A particle is placed at the origin and a force $F = kx$ is acting on it (where, k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ *versus* x will be (where, U is the potential energy function) **(2004, 2M)**



8. A particle, which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here, k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is **(2002, 2M)**



9. A wind-powered generator converts wind energy into electric energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to **(2000, 2M)**

(a) v (b) v^2
(c) v^3 (d) v^4

10. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then, the long piece will have a force constant of **(1999, 2M)**

(a) $(2/3)k$ (b) $(3/2)k$
(c) $3k$ (d) $6k$

11. Two masses of 1 g and 4 g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is **(1980, 2M)**

(a) $4:1$ (b) $\sqrt{2}:1$
(c) $1:2$ (d) $1:16$

12. If a machine is lubricated with oil, **(1980, 2M)**

(a) the mechanical advantage of the machine increases
(b) the mechanical efficiency of the machine increases
(c) both its mechanical advantage and efficiency increases
(d) its efficiency increases, but its mechanical advantage decreases

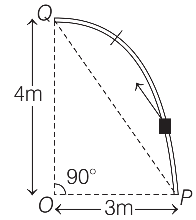
Objective Questions I (One or more than one)

13. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the X-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? **(2018 Adv.)**

(a) The force applied on the particle is constant
(b) The speed of the particle is proportional to time
(c) The distance of the particle from the origin increases linearly with time
(d) The force is conservative

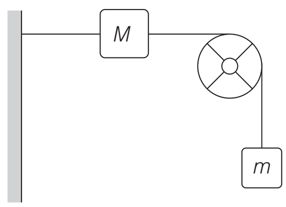
Integer Answer Type Questions

14. Consider an elliptically shaped rail PQ in the vertical plane with $OP = 3$ m and $OQ = 4$ m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ J. The value of n is (take acceleration due to gravity = $10\ \text{ms}^{-2}$) **(2014 Adv.)**



Analytical & Descriptive Questions

15. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2 m from the wall, has a point mass $M = 2$ kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? (Take $g = 9.8\ \text{m/s}^2$) **(1985, 6M)**



16. The displacement x of a particle moving in one dimension, under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$ where x is in metre and t in second. Find **(1980)**

(a) the displacement of the particle when its velocity is zero, and
(b) the work done by the force in the first 6 s.

Match the Column

17. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in Column I (a and U_0 are constants). Match the potential energies in Column I to the corresponding statements in Column II **(2015 Adv.)**

Column I	Column II
A. $U_1(x)=\frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	P. The force acting on the particle is zero at $x=a$
B. $U_2(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2$	Q. The force acting on the particle is zero at $x=0$
C. $U_3(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2\exp\left[-\left(\frac{x}{a}\right)^2\right]$	R. The force acting on the particle is zero at $x=-a$
D. $U_4(x)=\frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	S. The particle experiences an attractive force towards $x=0$ in the region $\lvert x\rvert<a$
	T. The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x=-a$.

Answers

Topic 1

1. (d) **2.** (b) **3.** (b)
4. (c) **5.** (d) **6.** (b) **7.** (c)
8. (d) **9.** (c) **10.** 5 **11.** 8

Topic 2

1. (c) **2.** (b)

Topic 3

1. (c) **2.** (c) **3.** (b)
4. (a) **5.** (4) **6.** 4.24 m
7. 10 kg, 0.098 J **8.** 409.8 m/s
9. 9.33 cal **10.** $v_C=v_F=\sqrt{2(gy-\mu gx)}$

Topic 4

1. (b) **2.** (c) **3.** (d) **4.** (c)
5. (d) **6.** (b) **7.** (c, d) **8.** 5

9. (a) $N=mg(3\cos\theta-2)$

(b) For $\theta\le\cos^{-1}\left(\frac{2}{3}\right)$, $N_B=0, N_A=mg(3\cos\theta-2)$ and for

$\theta\ge\cos^{-1}\left(\frac{2}{3}\right)$; $N_A=0,\ N_B=mg\,(2-3\cos\theta)$

10. $u=\sqrt{gL\left(2+\frac{3\sqrt{3}}{2}\right)}$

11. (a) $\theta=30^\circ$ (b) The desired coordinates are (108.25 m, 31.25 m)

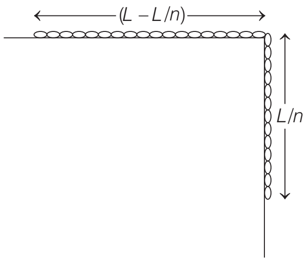
Topic 5

1. (c) **2.** (b) **3.** (b) **4.** (d)
5. (d) **6.** (c) **7.** (a) **8.** (d)
9. (c) **10.** (b) **11.** (c) **12.** (b)
13. (a, b) **14.** 50 J **15.** 3.29 m/s
16. (a) zero (b) zero
17. A–P, Q, R, T B–Q, S C–P, Q, R, S D–P, R, T

Hints & Solutions

Topic 1 Work Done and Power

1.



Given, mass of the cable is M.

So, mass of $\frac{1}{n}$th part of the cable, i.e. hanged part of the cable is $= M / n$ ...(i)

Now, centre of mass of the hanged part will be its middle point. So, its distance from the top of the table will be $L / 2n$.

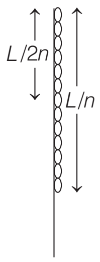
$\therefore$ Initial potential energy of the hanged part of cable,

$$U_i = \left(\frac{M}{n}\right)(-g)\left(\frac{L}{2n}\right)$$

$$\Rightarrow \quad U_i = -\frac{MgL}{2n^2} \quad \text{...(ii)}$$

When whole cable is on the table,
its potential energy will be zero.

$$\therefore \quad U_f = 0 \quad \text{...(iii)}$$



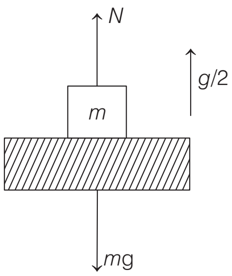
Now, using work-energy theorem,

$$W_{\text{net}} = \Delta U = U_f - U_i$$

$$\Rightarrow \quad W_{\text{net}} = 0 - \left(-\frac{MgL}{2n^2}\right) \quad \text{[using Eqs. (ii) and (iii)]}$$

$$\Rightarrow \quad W_{\text{net}} = \frac{MgL}{2n^2}$$

2 Normal reaction force on the block is



$$N = ma_{\text{net}}$$

where, a_{net} = net acceleration of block.

$$= g + a = g + \frac{g}{2} = \frac{3g}{2}$$

$$\Rightarrow \quad N = m\left(g + \frac{g}{2}\right) = \frac{3mg}{2}$$

Now, in time 't' block moves by a displacement s given by

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{g}{2}\right)t^2 \quad (\because u = 0)$$

Here, $\quad a = \frac{g}{2}$ (given)

$\therefore$ Work done = Force $\times$ Displacement

$$\Rightarrow \quad W = \frac{3mg}{2} \times \frac{gt^2}{4} = \frac{3mg^2t^2}{8}.$$

3. Here, the displacement of an object is given by

$$x = (3t^2 + 5)\,\text{m}$$

Therefore, velocity $(v) = \frac{dx}{dt} = \frac{d(3t^2 + 5)}{dt}$

or $\quad v = 6t$ m/s ...(i)

The work done in moving the object from $t = 0$ to $t = 5$ s

$$W = \int_{x_0}^{x_5} F \cdot dx \quad \text{...(ii)}$$

The force acting on this object is given by

$$F = ma = m \times \frac{dv}{dt}$$

$$= m \times \frac{d(6t)}{dt} \quad [\because \text{using (i)}]$$

$$F = m \times 6 = 6\,\text{m} = 12\,\text{N}$$

Also, $\quad x_0 = 3t^2 + 5 = 3 \times (0)^2 + 5 = 5\,\text{m}$

and at $t = 5$ s,

$$x_5 = 3 \times (5)^2 + 5 = 80\,\text{m}$$

Put the values in Eq. (ii),

$$W = 12 \times \int_{x_0}^{x_5} dx = 12[80 - 5]$$

$$W = 12 \times 75 = 900\,\text{J}$$

Alternative Method

To using work – kinetic energy theorem is,

$$W = \Delta\text{K}\cdot\text{E} = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$= \frac{1}{2}m \times (30^2 - 0^2) = \frac{1}{2} \times 2 \times 900 = 900\,\text{J}$$

4. Thinking Process We know that change in potential energy of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^f \mathbf{F} \cdot d\,\mathbf{r}$$

Given, $F = ax + bx^2$

We know that work done in stretching the rubber band by L is $|dW| = |Fdx|$

$$|W| = \int_0^L (ax + bx^2)\, dx$$

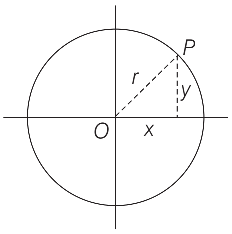
$$= \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$$

$$= \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2}\right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3}\right]$$

$$= |W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

5. $\mathbf{r} = \mathbf{OP} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$$\mathbf{F} = \frac{k}{(x^2 + y^2)^{3/2}} \; (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) = \frac{k}{r^3}(\mathbf{r})$$



Since, $\mathbf{F}$ is along $\mathbf{r}$ or in radial direction.
Therefore, work done is zero.

6. Gravitational field is a conservative force field. In a conservative force field work done is path independent

$\therefore \quad W_1 = W_2 = W_3$

7. $dW = \mathbf{F} \cdot \mathbf{ds}$, where $\mathbf{ds} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}}$

and $\quad \mathbf{F} = -k\,(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$

$\therefore \quad dW = -k\,(ydx + xdy)$

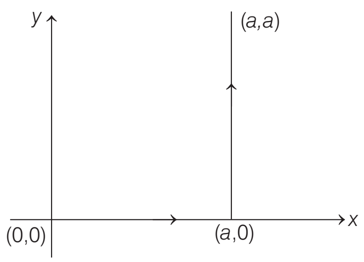
$\quad = -kd(xy)$

$\therefore \quad W = \int_{0,0}^{a,a} dW = -k\int_{0,0}^{a,a} d\,(xy) = -k\,[xy]_{0,\,0}^{a,\,a}$

$W = -ka^2$

Alternate Answer

While moving from (0, 0) to (a, 0) along positive X-axis,



$y = 0 \therefore \mathbf{F} = -kx\hat{\mathbf{j}}$ i.e. force is in negative y-direction while the displacement is in positive x-direction. Therefore, $W_1 = 0$ (Force $\perp$ displacement).

Then, it moves from (a, 0) to (a, a) along a line parallel to Y-axis ($x = +a$). During this $\mathbf{F} = -k\,(y\hat{\mathbf{i}} + a\hat{\mathbf{j}})$

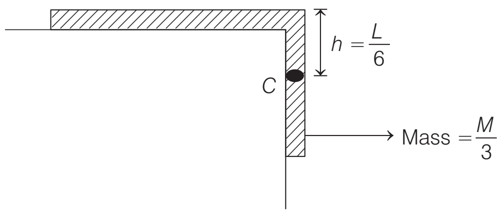
The first component of force, $-k\,y\hat{\mathbf{i}}$ will not contribute any work, because this component is along negative x-direction $(-\hat{\mathbf{i}})$ while displacement is in positive y-direction $(a, 0)$ to (a, a).

The second component of force i.e. $-k\,a\,\hat{\mathbf{j}}$ will perform negative work $\begin{bmatrix} \mathbf{F}_y = -ka\hat{\mathbf{j}} \\ \mathbf{s} = a\hat{\mathbf{j}} \end{bmatrix}$, $W_2 = (-ka)(a) = -ka^2$

$\therefore \quad W = W_1 + W_2 = -ka^2$

NOTE In the given force, work done is path independent. It depends only on initial and final positions. Therefore, first method is brief and correct.

8. Mass of hanging portion is $M/3$ (one-third) and centre of mass c, is at a distance $h = \frac{L}{6}$ below the table top. Therefore, the required work done is,



$$W = mgh = \left(\frac{M}{3}\right)(g)\left(\frac{L}{6}\right) = \frac{MgL}{18}$$

9. P = constant

$\therefore$ Work done in time t.

$$W = Pt$$

From work-energy theorem, net work done is change in kinetic energy. Therefore,

$$\frac{1}{2}mv^2 = Pt \quad \text{or} \quad v \propto t^{1/2}$$

Integrating, we get $\quad s \propto t^{3/2}$

10. As, $\quad W = \frac{1}{2}mv^2$

So, $\quad Pt = \frac{1}{2}mv^2$

$\therefore \quad v = \sqrt{\frac{2Pt}{m}}$

$\quad = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5$ m/s

11. $a = \dfrac{\text{Net pulling force}}{\text{Total mass}}$

$$= \frac{0.72g - 0.36g}{0.72 + 0.36} = \frac{g}{3}$$

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{g}{3}\right)(1)^2 = \frac{g}{6}$$

$$T - 0.36g = 0.36a = 0.36\frac{g}{3}$$

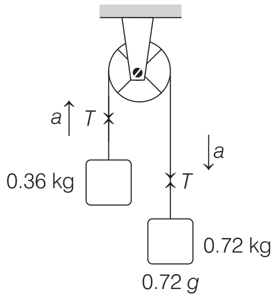
$\therefore \quad T = 0.48g$

Now, $\quad W_T = TS\cos 0°$

(on 0.36 kg mass)

$$= (0.48g)\left(\frac{g}{6}\right)(1)$$

$$= 0.08(g^2) = 0.08(10)^2 = 8\text{ J}$$



Topic 2 Conservation of Mechanical Energy

1. From energy conservation,

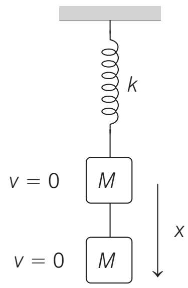
$$\frac{1}{2}kx^2 = \frac{1}{2}(4k)y^2$$

$$\therefore \quad \frac{y}{x} = \frac{1}{2}$$

2. Let x be the maximum extension of the spring. From conservation of mechanical energy

decrease in gravitational potential energy = increase in elastic potential energy

$$\therefore \quad Mg\,x = \frac{1}{2}kx^2 \quad \text{or} \quad x = \frac{2Mg}{k}$$



Topic 3 Problems with Friction

1. As energy loss is same, thus

$$\mu\, mg\cos\theta \cdot (PQ) = \mu mg \cdot (QR)$$

$$\therefore \quad QR = (PQ)\cos\theta$$

$$\Rightarrow \quad QR = 4 \times \frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3} \approx 3.5\text{ m}$$

Further, decrease in potential energy = loss due to friction

$$\therefore \quad mgh = (\mu mg\cos\theta)d_1 + (\mu mg)d_2$$

$$m \times 10 \times 2 = \mu \times m \times 10 \times \frac{\sqrt{3}}{2} \times 4 + \mu \times m \times 10 \times 2\sqrt{3}$$

$$\Rightarrow \quad 4\sqrt{3}\,\mu = 2$$

$$\Rightarrow \quad \mu = \frac{1}{2\sqrt{3}} = 0.288 = 0.29$$

2. **In statement I** Decrease in mechanical energy in case I will be $\quad \Delta U_1 = \frac{1}{2}mv^2$

But decrease in mechanical energy in case II will be

$$\Delta U_2 = \frac{1}{2}mv^2 - mgh$$

$$\Rightarrow \quad \Delta U_2 < \Delta U_1$$

or statement I is correct.

In statement II Coefficient of friction will not change or this statement is wrong.

3. Height fallen up to point Q

$$h = R\sin 30° = 40 \times \frac{1}{2} = 20\text{ m}$$

Work done against friction = Initial mechanical energy − Final mechanical energy

$$= mgh - \frac{1}{2}mv^2$$

Putting the values, we get

$$150 = 1 \times 10 \times 20 - \frac{1}{2} \times 1 \times v^2 \Rightarrow v = 10\text{ m/s}$$

4. At point Q, component of weight along PQ (radially outwards) is $mg\cos 60°$ or $\frac{mg}{2}$.

Normal reaction is radially inwards

$$N - \frac{mg}{2} = \frac{mv^2}{R}$$

or $$N = \frac{mg}{2} + \frac{mv^2}{R}$$

$$= \frac{1 \times 10}{2} + \frac{1 \times (10)^2}{40} = 7.5\text{ N}$$

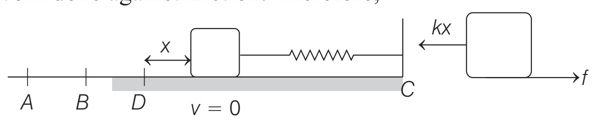
5. Decrease in mechanical energy = Work done against friction

$$\therefore \quad \frac{1}{2}mv^2 - \frac{1}{2}kx^2 = \mu\, mgx \quad \text{or} \quad v = \sqrt{\frac{2\mu\, mgx + kx^2}{m}}$$

Substituting the values, we get

$$v = 0.4\text{ m/s} = \left(\frac{4}{10}\right)\text{ m/s} \Rightarrow N = 4$$

6. From A to B, there will be no loss of energy. Now, let block compresses the spring by an amount x and comes momentarily to rest. Then, loss of energy will be equal to the work done against friction. Therefore,



$$\mu_k mg\,(BD + x) = \frac{1}{2}mv^2 - \frac{1}{2}kx^2$$

Substituting the values

$$(0.2)(0.5)(10)(2.14 + x) = \frac{1}{2}(0.5)(3)^2 - \frac{1}{2}(2)(x)^2$$

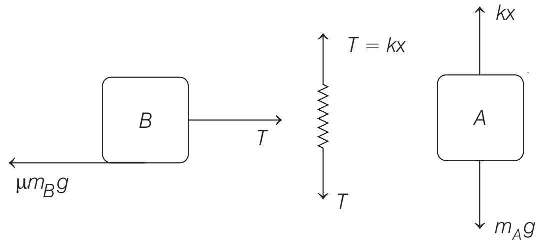
Solving this equation, we get $x = 0.1\text{m}$

Now, spring exerts a force $kx = 0.2\text{N}$ on the block. But to stop the block from moving limiting static friction is $\mu_s mg = (0.22)(0.5)(10) = 1.1\text{N}$. Since, $1.1\text{ N} > 0.2\text{ N}$, block will not move further and it will permanently stop there.

Therefore, total distance covered before it comes to rest permanently is

$$d = AB + BD + x = 2 + 2.14 + 0.1 = 4.24\text{ m}$$

7. Normal reaction between blocks A and C will be zero. Therefore, there will be no friction between them.
Both A and B are moving with uniform speed. Therefore, net force on them should be zero.



For equilibrium of A

$$m_A g = kx$$

$$\therefore \quad x = \frac{m_A g}{k} = \frac{(2)(9.8)}{1960}$$

$$= 0.01\text{m}$$

For equilibrium of B

$$\mu m_B g = T = kx = m_A g$$

$$\therefore \quad m_B = \frac{m_A}{\mu} = \frac{2}{0.2}$$

$$= 10 \text{ kg}$$

Energy stored in spring

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(1960)(0.01)^2$$

$$= 0.098\text{J}$$

8. Heat energy required to just melt the bullet.

$$Q = Q_1 + Q_2$$

Here, $Q_1 = ms\Delta\theta$

$$= (m \times 10^3)(0.03 \times 4.2)(327 - 27)$$

$$= (3.78 \times 10^4 \text{ m})$$

$$Q_2 = mL = (m \times 10^3)(6 \times 4.2)$$

$$= (2.52 \times 10^4\, m)$$

$$\therefore \quad Q = (6.3 \times 10^4\, m)$$

If v be the speed of bullet, then 75% of $1/2mv^2$ should be equal to Q. Thus,

$$0.75 \times \frac{1}{2} \times m \times v^2 = 6.3 \times 10^4\, m$$

$$\Rightarrow \quad v = 409.8 \text{ m/s}$$

9. $s = vt = 2 \times 5 = 10$ m

Q = work done against friction

$= \mu mgs = 0.2 \times 2 \times 9.8 \times 10$

$= 39.2 \text{ J} = 9.33$ cal

10. In both the cases, work done by friction will be $-\mu Mgx$.

$$\therefore \quad \frac{1}{2}Mv_C^2 = \frac{1}{2}Mv_F^2$$

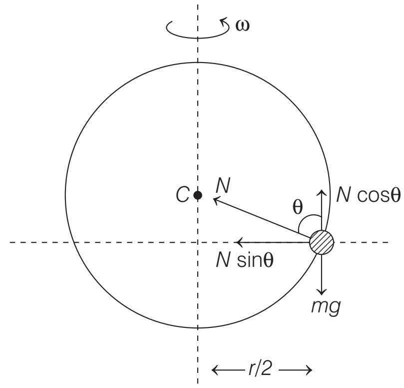
$$= Mgy - \mu Mgx$$

$$\therefore \quad v_C = v_F = \sqrt{2gy - 2\mu gx}$$

Topic 4 Problems of Circular Motion

1. *(b)* **Key Idea** For revolution in a circular path, there should be a force which balances the necessary centripetal force.

Let N = normal reaction of wire loop acting towards centre.



Then, component $N\cos\theta$ balances weight of bead,

$$\Rightarrow \quad N\cos\theta = mg \quad \text{...(i)}$$

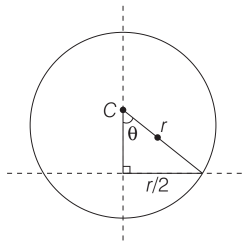
and component $N\sin\theta$ provides necessary centripetal pull on the bead,

$$\Rightarrow \quad N\sin\theta = m\left(\frac{r}{2}\right)\omega^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\tan\theta = \frac{r\omega^2}{2g} \quad \text{...(iii)}$$

Now, from geometry of figure,



$$\tan\theta = \frac{\frac{r}{2}}{\sqrt{r^2 - \left(\frac{r}{2}\right)^2}} = \frac{r}{2\left(\frac{\sqrt{3}}{2}\right)r} = \frac{1}{\sqrt{3}} \quad \text{...(iv)}$$

Put this value in Eq. (iii), we get

$$\omega^2 = \frac{2g}{\sqrt{3}r}$$

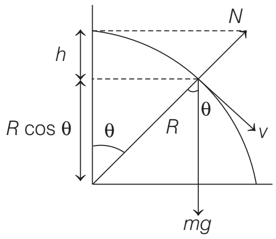
2. $h = R - R\cos\theta$

Using conservation of energy

$$mgR(1 - \cos\theta) = \frac{1}{2}mv^2$$

Radial force equation is

$$mg\cos\theta - N = \frac{mv^2}{R}$$



Here, N = normal force on bead by wire

$$N = mg\cos\theta - \frac{mv^2}{R}$$

$$= mg(3\cos\theta - 2)$$

$$N = 0 \text{ at } \cos\theta = \frac{2}{3}$$

So, normal force act radially outward on bead, if $\cos\theta > \frac{2}{3}$ and normal force act radially inward on bead, if $\cos\theta < 2/3$

$\therefore$ Force on ring is opposite to normal force on bead.

3. As, $v = \sqrt{5gL}$ ... (i)

So, $\left(\frac{v}{2}\right)^2 = v^2 - 2gh$ ... (ii)

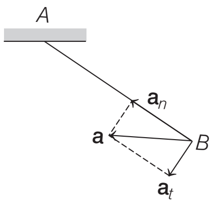
$h = L(1 - \cos\theta)$ ... (iii)

Solving Eqs. (i), (ii) and (iii), we get

$$\cos\theta = -\frac{7}{8}$$

or $\theta = \cos^{-1}\left(-\frac{7}{8}\right) = 151°$

4. Net acceleration **a** of the bob in position B has two components.

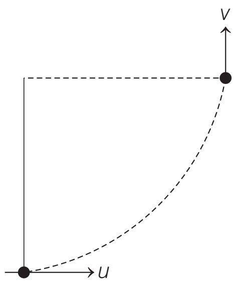


(a) $\mathbf{a}_n =$ radial acceleration (towards BA)

(b) $\mathbf{a}_t =$ tangential acceleration (perpendicular to BA)

Therefore, direction of **a** is correctly shown in option (c).

5. From energy conservation, $v^2 = u^2 - 2gL$ …(i)



Now, since the two velocity vectors shown in figure are mutually perpendicular, hence the magnitude of change of velocity will be given by $|\Delta \mathbf{v}| = \sqrt{u^2 + v^2}$

Substituting value of v^2 from Eq.(i), we get

$$|\Delta \mathbf{v}| = \sqrt{u^2 + u^2 - 2gL} = \sqrt{2(u^2 - gL)}$$

6. $a_c = k^2rt^2$

or $\frac{v^2}{r} = k^2rt^2$

or $v = krt$

Therefore, tangential acceleration, $a_t = \frac{dv}{dt} = kr$

or Tangential force, $F_t = ma_t = mkr$

Only tangential force does work.

$$\text{Power} = F_t\, v = (mkr)(krt)$$

or $\text{Power} = mk^2r^2\, t$

7. The given case is of uniform circular motion, in which speed of kinetic energy remains constant. Direction of velocity and acceleration keep on changing although its magnitude remains constant.

8. Velocity of first bob at highest point.

$$v_1 = \sqrt{gR} = \sqrt{gl_1}$$

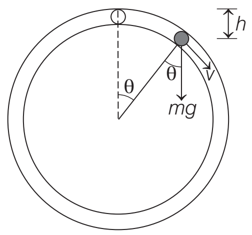
(to just complete the vertical circle)

= velocity of second bob just after elastic collision.

= velocity of second bob at the bottommost point = $\sqrt{5gl_2}$

$\therefore$ $\frac{l_1}{l_2} = 5$

9. (a) $h = \left(R + \frac{d}{2}\right)(1 - \cos\theta)$



Velocity of ball at angle θ is

$$v^2 = 2gh = 2\left(R + \frac{d}{2}\right)(1 - \cos\theta)\, g \quad \text{…(i)}$$

Let N be the normal reaction (away from centre) at angle θ.

Then, $mg\cos\theta - N = \frac{mv^2}{\left(R + \frac{d}{2}\right)}$

Substituting value of v^2 from Eq. (i), we get

$$mg\cos\theta - N = 2\,mg\,(1 - \cos\theta)$$

$\therefore$ $N = mg\,(3\cos\theta - 2)$

(b) The ball will lose contact with the inner sphere when $N = 0$

or $3\cos\theta - 2 = 0$ or $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

After this it makes contact with outer sphere and normal reaction starts acting towards the centre.

Thus for $\theta \le \cos^{-1}\left(\frac{2}{3}\right)$

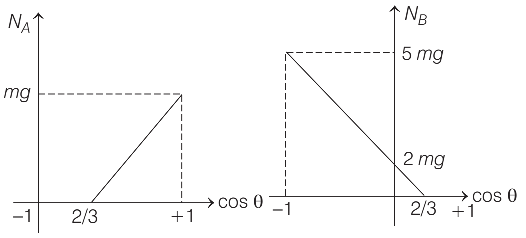
$$N_B = 0$$

and $N_A = mg\,(3\cos\theta - 2)$

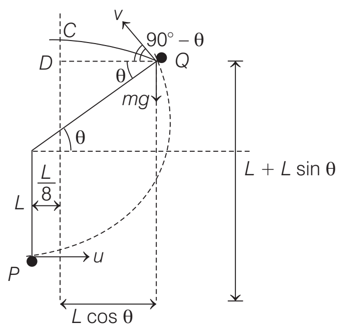
and for $\theta \geq \cos^{-1}\left(\frac{2}{3}\right)$

$N_A = 0$ and $N_B = mg\,(2 - 3\cos\theta)$

The corresponding graphs are as follows.



10. Let the string slacks at point Q as shown in figure. From P to Q path is circular and beyond Q path is parabolic. At point C, velocity of particle becomes horizontal, therefore, QD = half the range of the projectile.



Now, we have following equations

(1) $T_Q = 0$. Therefore, $mg\sin\theta = \dfrac{mv^2}{L}$ ...(i)

(2) $v^2 = u^2 - 2gh = u^2 - 2gL\,(1 + \sin\theta)$ ...(ii)

(3) $QD = \dfrac{1}{2}$ (Range)

$$\Rightarrow \left(L\cos\theta - \frac{L}{8}\right) = \frac{v^2 \sin 2\,(90^\circ - \theta)}{2g} = \frac{v^2\ \sin 2\theta}{2g} \quad \text{...(iii)}$$

Eq. (iii) can be written as $\left(\cos\theta - \dfrac{1}{8}\right) = \left(\dfrac{v^2}{gL}\right)\sin\theta\cos\theta$

Substituting value of $\left(\dfrac{v^2}{gL}\right) = \sin\theta$ from Eq. (i), we get

$$\left(\cos\theta - \frac{1}{8}\right) = \sin^2\theta \cdot \cos\theta = (1 - \cos^2\theta)\cos\theta$$

or $\cos\theta - 1/8 = \cos\theta - \cos^3\theta$

$\therefore$ $\cos^3\theta = 1/8$ or $\cos\theta = 1/2$ or $\theta = 60^\circ$

From Eq. (i), $v^2 = gL\sin\theta = gL\sin 60^\circ$

or $v^2 = \dfrac{\sqrt{3}}{2}\,gL$

$\therefore$ Substituting this value of v^2 in Eq. (ii)

$u^2 = v^2 + 2gL\,(1 + \sin\theta)$

$$= \frac{\sqrt{3}}{2}\,g\,L + 2gL\left(1 + \frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}\,gL + 2gL$$

$$= gL\left(2 + \frac{3\sqrt{3}}{2}\right)$$

$$u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$$

11. (a) At the highest point, velocity of bullet is $50\cos\theta$. So, by conservation of linear momentum

$$M\,(50\cos\theta) = 4Mv$$

$\therefore$ $v = \left(\dfrac{50}{4}\right)\cos\theta$ ...(i)

At point B, $T = 0$ but $v \neq 0$

Hence, $4\,Mg\cos 60^\circ = \dfrac{(4\,M\,)\,v^2}{l}$

or $v^2 = \dfrac{g}{2}\,l = \dfrac{50}{3}$ ...(ii)

$\left(\text{as } l = \dfrac{10}{3}\text{ m and } g = 10\text{m/s}^2\right)$

Also, $v^2 = u^2 - 2gh = u^2 - 2g\left(\dfrac{3}{2}l\right)$

$$= u^2 - 3\,(10)\left(\frac{10}{3}\right)$$

or $v^2 = u^2 - 100$

Solving Eqs. (i), (ii) and (iii), we get

$\cos\theta = 0.86$ or $\theta \approx 30^\circ$

(b) $x = \dfrac{\text{Range}}{2} = \dfrac{1}{2}\left(\dfrac{u^2\sin 2\theta}{g}\right)$

$$= \frac{50 \times 50 \times \sqrt{3}}{2 \times 10 \times 2} = 108.25\text{ m}$$

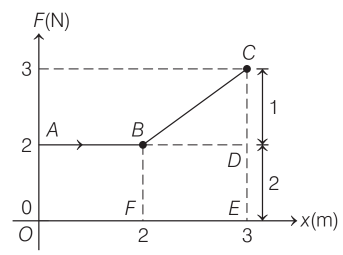
$$y = H = \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{50 \times 50 \times 1}{2 \times 10 \times 4} = 31.25\text{m}$$

Hence, the desired coordinates are (108.25 m, 31.25 m).

Topic 5 Miscellaneous Problems

1. **Key Idea** Area under force-displacement graph gives the value of work done.



$\therefore$ Work done on the particle = Area under the curve ABC

W = Area of square $ABFO$ + Area of ΔBCD + Area of rectangle $BDEF$

$$= 2\times 2 + \frac{1}{2}\times 1\times 1 + 2\times 1 = 6.5\text{ J}$$

Now, from work-energy theorem,

$$\Delta W = K_f - K_i$$

$$\Rightarrow \quad K_f = \Delta W = 6.5\text{ J} \qquad [\because K_i = 0]$$

2. We know that, work done in displacing a particle at displacement $\mathbf{d}$ under force $\mathbf{F}$ is given by

$$\Delta W = \mathbf{F}\cdot\mathbf{d}$$

By substituting given values, we get

$$\Rightarrow \quad \Delta W = (3\hat{\mathbf{i}} - 12\hat{\mathbf{j}})\cdot(4\hat{\mathbf{i}})$$

$$\Rightarrow \quad \Delta W = 12\text{ J} \qquad \ldots\text{(i)}$$

Now, using work-energy theorem, we get

work done (ΔW) = change in kinetic energy (ΔK)

$$\text{or} \quad \Delta W = K_2 - K_1 \qquad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$K_2 - K_1 = 12\text{ J}$$

$$\text{or} \quad K_2 = K_1 + 12\text{ J}$$

Given, initial kinetic energy, $K_1 = 3$ J

$\therefore$ Final kinetic energy, $K_2 = 3\text{ J} + 12\text{ J} = 15\text{ J}$

3. Given, force, $F = -kv^2$

$\therefore$ Acceleration, $a = \frac{-k}{m}v^2$

$$\text{or} \quad \frac{dv}{dt} = \frac{-k}{m}v^2$$

$$\Rightarrow \quad \frac{dv}{v^2} = -\frac{k}{m}\,.\,dt$$

Now, with limits, we have

$$\int_{10}^{v}\frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

$$\Rightarrow \quad \left(-\frac{1}{v}\right)_{10}^{v} = -\frac{k}{m}t$$

$$\Rightarrow \quad \frac{1}{v} = 0.1 + \frac{kt}{m}$$

$$\Rightarrow \quad v = \frac{1}{0.1 + \frac{kt}{m}} = \frac{1}{0.1 + 1000k}$$

$$\Rightarrow \quad \frac{1}{2}\times m\times v^2 = \frac{1}{8}\times v_0^2$$

$$\Rightarrow \quad v = \frac{v_0}{2} = 5$$

$$\Rightarrow \quad \frac{1}{0.1 + 1000\,k} = 5$$

$$\Rightarrow \quad 1 = 0.5 + 5000k$$

$$\Rightarrow \quad k = \frac{0.5}{5000} \quad \Rightarrow \quad k = 10^{-4}\text{kg/m}$$

4. From Newton's second law, $\frac{\Delta p}{\Delta t} = F$

$$\Rightarrow \quad \Delta p = F\Delta t$$

$$\therefore \quad p = \int dp = \int_0^1 F\,dt$$

$$\Rightarrow \quad p = \int_0^1 6t\,dt = 3\,\mathbf{kg}\left(\frac{\mathbf{m}}{\mathbf{s}}\right)$$

Also, $\Delta k = \frac{\Delta p^2}{2m} = \frac{3^2}{2\times 1} = 4.5$

So, work done $= \Delta k = 4.5$ J

5. Work done in lifting mass $= (10\times 9.8\times 1)\times 1000$

If m is mass of fat burnt, then energy

$$= m\times 3.8\times 10^7\times\frac{20}{100}$$

Equating the two, we get

$$\therefore \quad m = \frac{49}{3.8} \approx 12.89\times 10^{-3}\text{ kg}$$

6. Area under F-t graph = momentum = $p = \sqrt{2\,km}$

$$\therefore \quad k = \frac{A^2}{2m} \qquad (A = \text{net area of } F\text{ - }t \text{ graph})$$

$$= \frac{\left\{\left(\frac{4\times 3}{2}\right) - \left(\frac{1.5\times 2}{2}\right)\right\}^2}{2\times 2}$$

$$= 5.0625\text{ J}$$

7. From $F = -\frac{dU}{dx}$

$$\int_0^{U(x)} dU = -\int_0^x F dx = -\int_0^x (kx)\,dx$$

$$\therefore \quad U(x) = -\frac{kx^2}{2} \quad \text{as } U(0) = 0$$

8.

$$F = -\frac{dU}{dx}$$

$$\therefore \quad dU = -F\cdot dx$$

$$\text{or} \quad U(x) = -\int_0^x (-kx + ax^3)\,dx$$

$$U(x) = \frac{kx^2}{2} - \frac{ax^4}{4}$$

$$U(x) = 0 \text{ at } x = 0 \text{ and } x = \sqrt{\frac{2k}{a}}$$

$$U(x) = \text{negative for } x > \sqrt{\frac{2k}{a}}$$

From the given function, we can see that $F = 0$ at $x = 0$ i.e. slope of U - x graph is zero at $x = 0$.

9. Power $= \mathbf{F}\bullet\mathbf{v} = Fv$

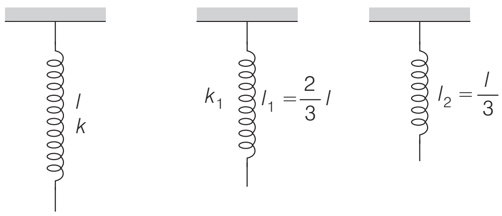
$$F = v\left(\frac{dm}{dt}\right) = v\left\{\frac{d\,(\rho\times\text{volume})}{dt}\right\}$$

$$= \rho v \left\{ \frac{d\ (\text{volume})}{dt} \right\} = \rho v\ (Av) = \rho A v^2$$

$\therefore$ Power, $P = \rho A v^3$ or $P \propto v^3$

10. $l_1 = 2l_2 \Rightarrow l_1 = \frac{2}{3} l$

Force constant $k \propto \frac{1}{\text{length of spring}}$



$\therefore$ $k_1 = \frac{3}{2} k$

11. $p = \sqrt{2Km}$ or $p \propto \sqrt{m}$,

$\frac{m_1}{m_2} = \frac{1}{4} \Rightarrow \therefore \frac{p_1}{p_2} = \frac{1}{2}$

13. (a, b) $K = \frac{1}{2} mv^2$

$\Rightarrow \frac{dK}{dt} = mv \frac{dv}{dt}$

Given, $\frac{dK}{dt} = \gamma t$

$\Rightarrow mv \frac{dv}{dt} = \gamma t$

$\Rightarrow \int_0^v v\,dv = \int_0^t \frac{\gamma}{m} t\,dt$

$\Rightarrow \frac{v^2}{2} = \frac{\gamma}{m} \frac{t^2}{2}$

$\Rightarrow v = \sqrt{\frac{\gamma}{m}}\, t$

$\Rightarrow a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}}$

$\therefore F = ma = \sqrt{\gamma m} = \text{constant}$

$\therefore V = \frac{ds}{dt} = \sqrt{\frac{\gamma}{m}}\, t \Rightarrow s = \sqrt{\frac{\gamma}{m}} \frac{t^2}{2}$

NOTE Force is constant. In the website of IIT, option (d) is given correct. In the opinion of author all constant forces are not necessarily conservative. For example : viscous force at terminal velocity is a constant force but it is not conservative.

14. From work-energy theorem

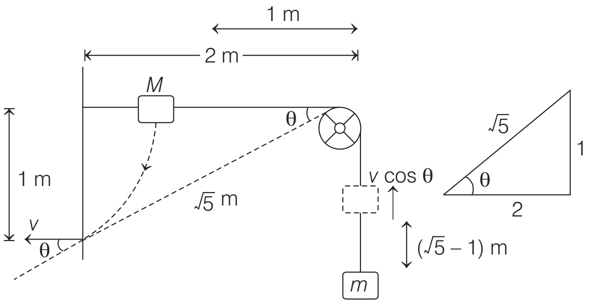
Work done by all forces = change in kinetic energy

or $W_F + W_{mg} = K_f - K_i$

$18 \times 5 + (1 \times 10)(-4) = K_f$

$90 - 40 = K_f$ or $K_f = 50\ \text{J} = 5 \times 10\ \text{J}$

15. Let M strikes with speed v. Then, velocity of m at this instant will be $v \cos\theta$ or $\frac{2}{\sqrt{5}} v$. Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5} - 1)$ m. From energy conservation, decrease in potential energy of M = increase in potential energy of m + increase in kinetic energy of both the blocks.



or $(2)(9.8)(1) = (0.5)(9.8)(\sqrt{5} - 1) + \frac{1}{2} \times 2 \times v^2$

$$+ \frac{1}{2} \times 0.5 \times \left(\frac{2v}{\sqrt{5}} \right)^2$$

Solving this equation, we get, $v = 3.29$ m/s.

16. Given, $t = \sqrt{x} + 3$ or $\sqrt{x} = (t - 3)$ ...(i)

$\therefore$ $x = (t-3)^2 = t^2 - 6t + 9$ ...(ii)

Differentiating this equation with respect to time, we get

velocity $v = \frac{dx}{dt} = 2t - 6$ ...(iii)

(a) $v = 0$ when $2t - 6 = 0$ or $t = 3$ s

Substituting in Eq. (i), we get

$$\sqrt{x} = 0$$

or $x = 0$

i.e. displacement of particle when velocity is zero is also zero.

(b) From Eq. (iii), speed of particle at $t = 0$ is

$$v_i = |v| = 6\ \text{m/s}$$

and at $t = 6$ s

$$v_f = |v| = 6\ \text{m/s}$$

From work energy theorem,

Work done = change in kinetic energy

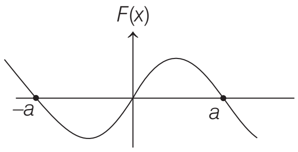
$$= \frac{1}{2} m [v_f^2 - v_i^2] = \frac{1}{2} m [(6)^2 - (6)^2] = 0$$

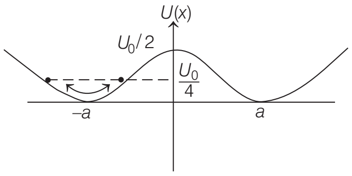
17. (A) $F_x = \frac{-dU}{dx} = -\frac{2U_0}{a^3} [x - a][x][x + a]$

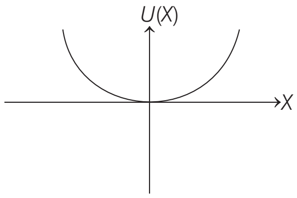
$F = 0$ at $x = 0,\ x = a, x = -a$

and $U = 0$ at $x = -a$

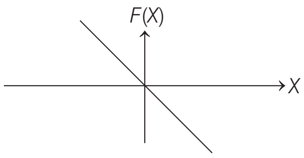
and $x = a$



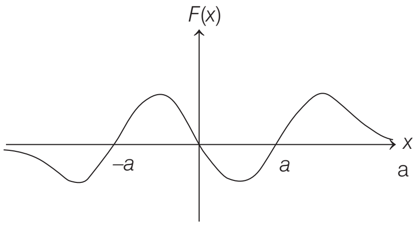


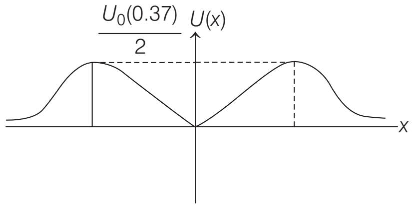


(B) $F_x = -\dfrac{dU}{dx} - U_0\left(\dfrac{x}{a}\right)$

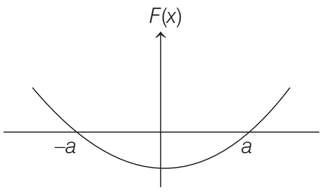


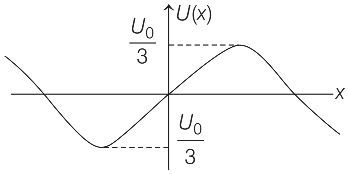
(C) $F_x = -\dfrac{dU}{dx} = U_0\,\dfrac{e^{-x^2/x^2}}{a^3}\,[x]\,[x-a]\,[x+a]$





(D) $F_x = -\dfrac{dU}{dx} = -\dfrac{U_0}{2a^3}\,[(x-a)\,(x+a)]$





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http://tinyurl.com/yyxyyt8y *or*

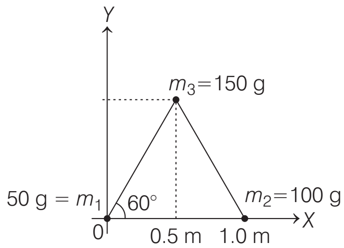
5

Centre of Mass

Topic 1 Centre of Mass

Objective Questions I (Only one correct option)

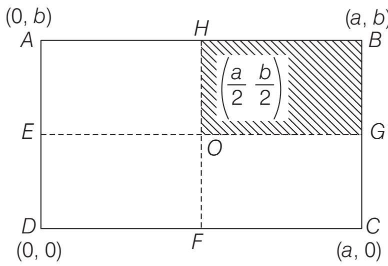
1. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be **(2019 Main, 12 April II)**



(a) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$ (b) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$

(c) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$ (d) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$

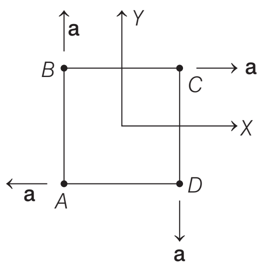
2. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be **(2019 Main, 8 April II)**



(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

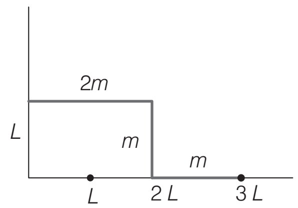
(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

3. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles (in ms^{-2}) is **(2019 Main, 8 April I)**



(a) $\frac{a}{5}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ (b) $a(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (c) zero (d) $\frac{a}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

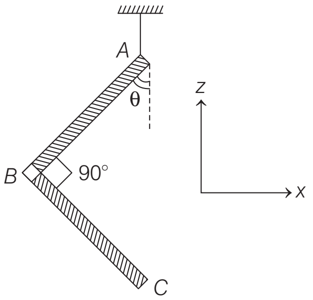
4. The position vector of the centre of mass $\mathbf{r}_{\text{cm}}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is **(2019 Main, 12 Jan I)**



(a) $\mathbf{r} = \frac{13}{8} L\,\hat{\mathbf{x}} + \frac{5}{8} L\,\hat{\mathbf{y}}$ (b) $\mathbf{r} = \frac{11}{8} L\,\hat{\mathbf{x}} + \frac{3}{8} L\,\hat{\mathbf{y}}$

(c) $\mathbf{r} = \frac{3}{8} L\,\hat{\mathbf{x}} + \frac{11}{8} L\,\hat{\mathbf{y}}$ (d) $\mathbf{r} = \frac{5}{8} L\,\hat{\mathbf{x}} + \frac{13}{8} L\,\hat{\mathbf{y}}$

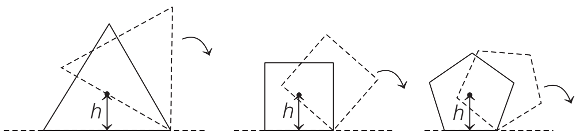
5. An L-shaped object made of thin rods of uniform mass density is suspended with a string as shown in figure. If $AB = BC$ and the angle is made by AB with downward vertical is θ, then **(2019 Main, 9 Jan)**



(a) $\tan\theta = \dfrac{2}{\sqrt{3}}$ (b) $\tan\theta = \dfrac{1}{2\sqrt{3}}$

(c) $\tan\theta = \dfrac{1}{2}$ (d) $\tan\theta = \dfrac{1}{3}$

6. Consider regular polygons with number of sides $n = 3, 4, 5 \ldots\ldots$ as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the centre of mass for each each polygon is Δ. Then, Δ depends on n and h as **(2017 Adv.)**



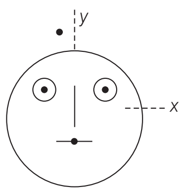
(a) $\Delta = h\sin^2\left(\dfrac{\pi}{n}\right)$ (b) $\Delta = h\sin\left(\dfrac{2\pi}{n}\right)$

(c) $\Delta = h\tan^2\left(\dfrac{\pi}{2n}\right)$ (d) $\Delta = h\left(\dfrac{1}{\cos\left(\dfrac{\pi}{n}\right)} - 1\right)$

7. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h, then z_0 is equal to **(2015 Main)**

(a) $\dfrac{3h}{4}$ (b) $\dfrac{h^2}{4R}$ (c) $\dfrac{5h}{8}$ (d) $\dfrac{3h^2}{8R}$

8. Look at the drawing given in the figure, which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6 m.



The coordinates of the centres of the different parts are : outer circle (0, 0), left inner circle $(-a, a)$, right inner circle (a, a), vertical line $(0, 0)$ and horizontal line $(0, -a)$. The y-coordinate of the centre of mass of the ink in this drawing is **(2009)**

(a) $\dfrac{a}{10}$ (b) $\dfrac{a}{8}$ (c) $\dfrac{a}{12}$ (d) $\dfrac{a}{3}$

9. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is **(2002, S)**

(a) 30 m/s (b) 20 m/s

(c) 10 m/s (d) 5 m/s

10. Two particles A and B initially at rest, move towards each other by mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$, the speed of the centre of mass of the system is **(1982, 3M)**

(a) 3 v (b) v

(c) 1.5 v (d) zero

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I

(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(c) If Statement I is true; Statement II is false

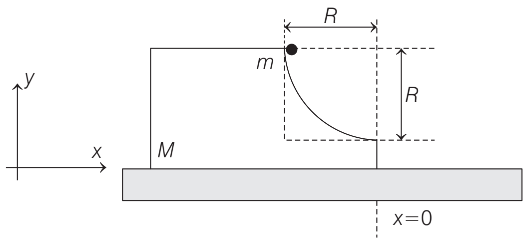
(d) If Statement I is false; Statement II is true

11. **Statement I** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.

Statement II The linear momentum of an isolated system remains constant. **(2007, 3M)**

Objective Questions II (One or more correct option)

12. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surfaced of a fixed table. Initially the right edge of the block is at $x = 0$, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following option is/are correct? **(2017 Adv.)**



(a) The velocity of the point mass m is $v = \sqrt{\dfrac{2gR}{1 + \dfrac{m}{M}}}$

(b) The x component of displacement of the centre of mass of the block M is $-\dfrac{mR}{M + m}$

(c) The position of the point mass is $x = -\sqrt{2}\,\dfrac{mR}{M+m}$

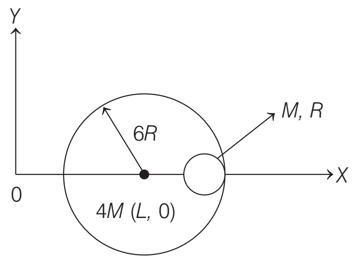
(d) The velocity of the block M is $V = -\dfrac{m}{M}\sqrt{2gR}$

True/False

13. Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is 0.75 m/s. **(1989, 2M)**

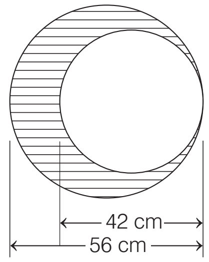
Analytical & Descriptive Questions

14. A small sphere of radius R is held against the inner surface of a larger sphere of radius $6R$. The masses of large and small spheres are $4M$ and M respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position. **(1996, 3 M)**



15. A ball of mass 100 g is projected vertically upwards from the ground with a velocity of 49 m/s. At the same time, another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time, the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. **(1985, 8M)**

16. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in figure.
Find the position of the centre of mass of the remaining portion. **(1980)**



Topic 2 Linear Momentum, Mechanical Energy and Their Conservation

Objective Question I (Only one correct option)

1. A person of mass M is sitting on a swing to length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $l(l << L)$, is close to **(2019 Main, 12 April I)**

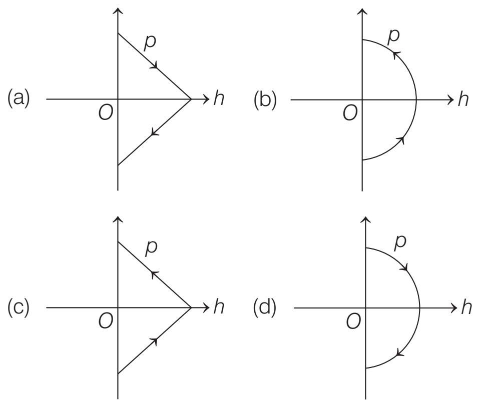
(a) $Mgl\,(1-\theta_0^2)$ (b) $Mgl\,(1+\theta_0^2)$

(c) Mgl (d) $Mgl\left(1+\dfrac{\theta_0^2}{2}\right)$

2. A wedge of mass $M = 4$ m lies on a frictionless plane. A particle of mass m aproaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by **(2019 Main, 9 April II)**

(a) $\dfrac{2v^2}{7g}$ (b) $\dfrac{v^2}{g}$ (c) $\dfrac{2v^2}{5g}$ (d) $\dfrac{v^2}{2g}$

3. A ball is thrown vertically up (taken as + Z-axis) from the ground. The correct momentum-height (p-h) diagram is **(2019 Main, 9 April I)**



4. A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T, so that its momentum changes from p to $3p$. Here, k is a constant. The value of T is **(JEE Main 2019, 11 Jan Shift II)**

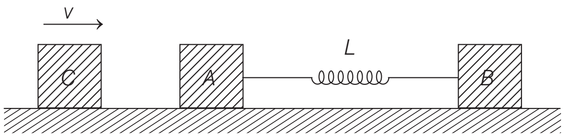
(a) $\sqrt{\dfrac{2p}{k}}$ (b) $2\sqrt{\dfrac{p}{k}}$ (c) $\sqrt{\dfrac{2k}{p}}$ (d) $2\sqrt{\dfrac{k}{p}}$

5. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **(2009)**
(a) linear momentum of the system does not change in time
(b) kinetic energy of the system does not change in time
(c) angular momentum of the system does not change in time
(d) potential energy of the system does not change in time

6. A particle moves in the x-y plane under the influence of a force such that its linear momentum is $\mathbf{p}(t) = A[\hat{\mathbf{i}}\cos(kt) - \hat{\mathbf{j}}\sin(kt)]$, where, A and k are constants. The angle between the force and the momentum is **(2007, 3M)**
(a) 0° (b) 30° (c) 45° (d) 90°

Objective Question II (One or more correct option)

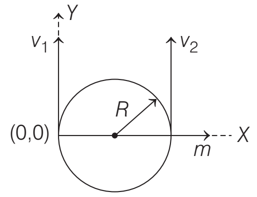
7. Two blocks A and B each of mass m, are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in figure. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A and B, and collides elastically with A. Then **(1993, 2M)**



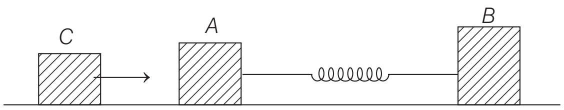
(a) the kinetic energy of the A-B system, at maximum compression of the spring, is zero
(b) the kinetic energy of the A-B system, at maximum compression of the spring, is $mv^2/4$
(c) the maximum compression of the spring is $v\sqrt{(m/k)}$
(d) the maximum compression of the spring is $v\sqrt{\frac{m}{2k}}$

Analytical & Descriptive Questions

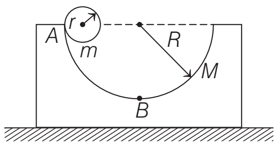
8. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point $(2R, 0)$ at time $t = 0$ and a man starts moving with a velocity v_1 along the positive Y-axis from origin at time $t = 0$. Calculate the linear momentum of the particle w.r.t. man as a function of time. **(2003, 2M)**



9. Two bodies A and B of masses m and $2m$ respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A as shown in figure. At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further at this instant the compression of the spring is found to be x_0. Determine (a) the common velocity of A and B at time t_0 and (b) the spring constant. **(1984, 6M)**



10. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see fig.). The cylinder slips on the semicircular frictionless track.
(a) How far has the block moved when the cylinder reaches the bottom (point B) of the track ?
(b) How fast is the block moving when the cylinder reaches the bottom of the track? **(1983, 7M)**



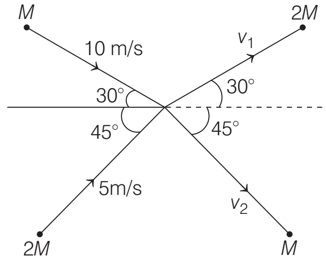
11. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? **(1979)**

Topic 3 Impulse, Explosions and Collisions

Objective Questions I (Only one correct option)

1. A man (mass = 50kg) and his son (mass = 20kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is **(2019 Main, 12 April I)**
(a) 0.28 ms^{-1} (b) 0.20 ms^{-1} (c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

2. Two particles of masses M and $2M$, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speed v_1 and v_2 are nearly **(2019 Main, 10 April I)**



(a) 6.5 m/s and 3.2 m/s (b) 3.2 m/s and 6.3 m/s
(c) 3.2 m/s and 12.6 m/s (d) 6.5 m/s and 6.3 m/s

3. A particle of mass m is moving with speed $2v$ and collides with a mass $2m$ moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45° with respect to the original direction.

The speed of each of the moving particle will be **(2019 Main, 9 April II)**

(a) $\sqrt{2}\,v$ (b) $\frac{v}{\sqrt{2}}$ (c) $\frac{v}{(2\sqrt{2})}$ (d) $2\sqrt{2}\,v$

4. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one-fourth of its original speed. What is the mass of the second body ? **(2019 Main, 9 April I)**

(a) 1.5 kg (b) 1.2 kg (c) 1.8 kg (d) 1.0 kg

5. A body of mass m_1 moving with an unknown velocity of $v_1\hat{\mathbf{i}}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{\mathbf{i}}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{\mathbf{i}}$ and $v_4\hat{\mathbf{i}}$, respectively.

If $m_2 = 0.5m_1$ and $v_3 = 0.5\,v_1$, then v_1 is **(2019 Main, 8 April II)**

(a) $v_4 + v_2$ (b) $v_4 - \frac{v_2}{4}$

(c) $v_4 - \frac{v_2}{2}$ (d) $v_4 - v_2$

6. An α-particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing 64% of its initial kinetic energy. The mass of the nucleus is **(2019 Main, 12 Jan II)**

(a) 1.5 m (b) 4 m (c) 3.5 m (d) 2 m

7. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same just before the collision. The subsequent motion of the combined body will be **(2019 Main, 12 Jan I**

(a) in the same circular orbit of radius R
(b) in an elliptical orbit
(c) such that it escapes to infinity
(d) in a circular orbit of a different radius

8. A simple pendulum is made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then, M is given by **(2019 Main, 12 Jan I)**

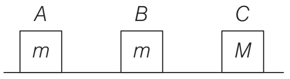
(a) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (b) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

(c) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$ (d) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$

9. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward with a velocity 100 ms^{-1} from the ground. The bullet gets embedded in the wood. Then, the maximum height to which the combined system reaches above the top of the building before falling below is (Take, $g = 10\,\text{ms}^{-2}$) **(2019 Main, 10 Jan I)**

(a) 20 m (b) 30 m (c) 10 m (d) 40 m

10. Three blocks A, B and C are lying on a smooth horizontal surface as shown in the figure. A and B have equal masses m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$? **(2019 Main, 9 Jan I)**



(a) 4 (b) 2 (c) 3 (d) 5

11. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is **(2018 Main)**

(a) $\frac{v_0}{\sqrt{2}}$ (b) $\frac{v_0}{4}$ (c) $\sqrt{2}\,v_0$ (d) $\frac{v_0}{2}$

12. It is found that, if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is P_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively **(2018 Main)**

(a) (0, 1) (b) (.89, .28) (c) (.28, .89) (d) (0, 0)

13. A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **(2015 Main)**

(a) 50 % (b) 56 % (c) 62 % (d) 44 %

14. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **(2013 Main)**

Statement I A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{m}{M + m}\right)$.

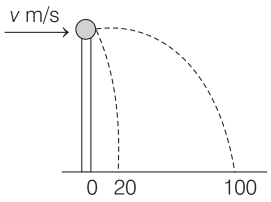
Statement II Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement I is true, Statement II is true, and Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

15. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **(2013 Adv.)**

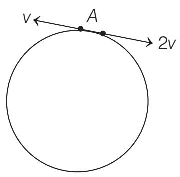
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+\alpha$ (c) $\frac{\pi}{4}-\alpha$ (d) $\frac{\pi}{2}$

16. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity v of the bullet is **(2011)**



(a) 250 m/s (b) $250\sqrt{2}$ m/s (c) 400 m/s (d) 500 m/s

17. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$ respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **(2009)**



(a) 4 (b) 3 (c) 2 (d) 1

18. Two particles of masses m_1 and m_2 in projectile motion have velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively at time $t = 0$. They collide at time t_0. Their velocities become $\mathbf{v}'_1$ and $\mathbf{v}'_2$ at time $2t_0$ while still moving in air. The value of $|(m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2) - (m_1\mathbf{v}_1 + m_2\mathbf{v}_2)|$ is **(2001, S)**

(a) zero (b) $(m_1 + m_2)g\,t_0$

(c) $2\,(m_1 + m_2)g\,t_0$ (d) $\frac{1}{2}(m_1 + m_2)\,gt_0$

19. An isolated particle of mass m is moving in horizontal plane (x-y), along the X-axis, at a certain height above the ground. It suddenly explodes into two fragment of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at **(1997 C, 1M)**

(a) $y = -5$ cm (b) $y = +20$ cm
(c) $y = +5$ cm (d) $y = -20$ cm

20. A shell is fired from a cannon with a velocity v (m/s) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (m/s) of the other piece immediately after the explosion is

(a) $3\,v\cos\theta$ (b) $2\,v\cos\theta$ **(1986, 2M)**

(c) $\frac{3}{2}\,v\cos\theta$ (d) $\sqrt{\frac{3}{2}}\,v\cos\theta$

21. A ball hits the floor and rebounds after an inelastic collision. In this case, **(1986, 2M)**

(a) the momentum of the ball just after the collision is the same as that just before the collision
(b) the mechanical energy of the ball remains the same in the collision
(c) the total momentum of the ball and the earth is conserved
(d) the total mechanical energy of the ball and the earth is conserved

Numerical Value

22. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is .... **(2018 Adv.)**

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) If Statement I is true; Statement II is false
(d) If Statement I is false; Statement II is true

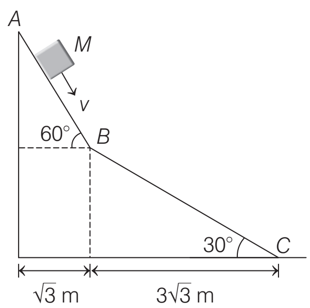
23. Statement I In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.

Statement II In an elastic collision, the linear momentum of the system is conserved. **(2007, 3M)**

Passage Based Questions

Passage 1

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic($g = 10\,\text{m/s}^2$).



24. The speed of the block at point B immediately after it strikes the second incline is **(2008, 4M)**

(a) $\sqrt{60}$ m/s (b) $\sqrt{45}$ m/s (c) $\sqrt{30}$ m/s (d) $\sqrt{15}$ m/s

25. The speed of the block at point C, immediately before it leaves the second incline is **(2008, 4M)**
(a) $\sqrt{120}$ m/s (b) $\sqrt{105}$ m/s (c) $\sqrt{90}$ m/s (d) $\sqrt{75}$ m/s

26. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is **(2008, 4M)**
(a) $\sqrt{30}$ m/s (b) $\sqrt{15}$ m/s (c) zero (d) $-\sqrt{15}$ m/s

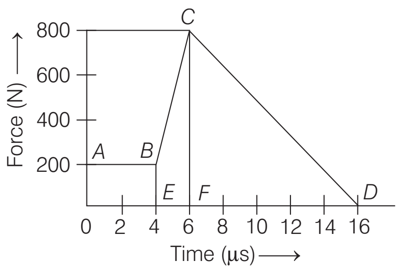
Objective Questions II (One or more correct option)

27. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of $2\ \text{ms}^{-1}$. Which of the following statement(s) is/are correct for the system of these two masses? **(2010)**
(a) Total momentum of the system is $3\ \text{kg-ms}^{-1}$
(b) Momentum of 5 kg mass after collision is $4\ \text{kg-ms}^{-1}$
(c) Kinetic energy of the centre of mass is 0.75 J
(d) Total kinetic energy of the system is 4 J

28. Two balls, having linear momenta $\mathbf{p}_1 = p\,\hat{\mathbf{i}}$ and $\mathbf{p}_2 = -\,p\,\hat{\mathbf{i}}$, undergo a collision in free space. There is no external force acting on the balls. Let $\mathbf{p}'_1$ and $\mathbf{p}'_2$ be their final momenta. The following option figure (s) is (are) not allowed for any non-zero value of $p, a_1, a_2, b_1, b_2, c_1$ and c_2. **(2008, 4M)**
(a) $\mathbf{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}},\ \mathbf{p}'_2 = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}}$
(b) $\mathbf{p}'_1 = c_1\hat{\mathbf{k}},\ \mathbf{p}'_2 = c_2\hat{\mathbf{k}}$
(c) $\mathbf{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}},\ \mathbf{p}'_2 = a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} - c_1\hat{\mathbf{k}}$
(d) $\mathbf{p}'_1 = a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}},\ \mathbf{p}'_2 = a_2\hat{\mathbf{i}} + b_1\hat{\mathbf{j}}$

Fill in the Blanks

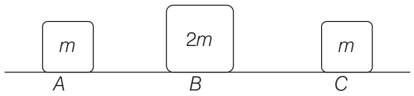
29. The magnitude of the force (in Newtons) acting on a body varies with time t (in microseconds) as shown in the figure. AB, BC and CD are straight line segments. The magnitude of the total impulse of the force on the body from $t = 4\ \mu\text{s}$ to $t = 16\mu\text{s}$ is ……… N-s. **(1994, 2M)**



30. A particle of mass $4m$ which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed v each in mutually perpendicular directions. The total energy released in the process of explosion is ……… . **(1987, 2M)**

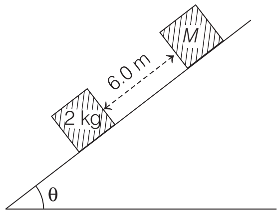
Integer Answer Type Questions

31. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed $9\ \text{ms}^{-1}$ and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in ms^{-1}) of the object C. **(2009)**

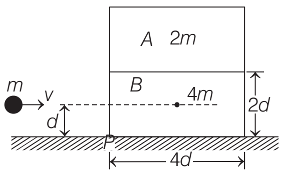


Analytical & Descriptive Questions

32. Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown. The coefficient of friction between each block and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M.
[Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\ \text{m/s}^2$] **(1999, 10M)**



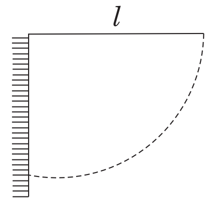
33. A block A of mass $2m$ is placed on another block B of mass $4m$ which in turn is placed on a fixed table. The two blocks have a same length $4d$ and they are placed as shown in figure. The coefficient of friction (both static and kinetic) between the block B and table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass (CM) of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table. **(1991, 4 + 4M)**
(a) What is the minimum value of v (call it v_0) required to make the block A to topple?



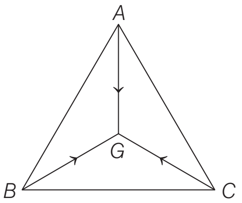
(b) If $v = 2\,v_0$, find the distance (from the point P in the figure) at which the mass m falls on the table after collision. (Ignore the role of friction during the collision.)

34. An object of mass 5 kg is projected with a velocity of 20 m/s at an angle of 60° to the horizontal. At the highest point of its path, the projectile explodes and breaks up into two fragments of masses 1 kg and 4 kg. The fragments separate horizontally after the explosion. The explosion releases internal energy such that the kinetic energy of the system at the highest point is doubled. Calculate the separation between the two fragments when they reach the ground. **(1990, 8M)**

35. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see fig.) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degrees ? **(1987, 7M)**

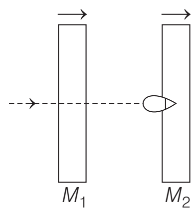


36. Three particles A, B and C of equal mass move with equal speed v along the medians of an equilateral triangle as shown in figure. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed v. What is the velocity of C? **(1982, 2M)**



37. A body of mass 1 kg initially at rest, explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal mass fly-off perpendicular to each other with a speed of 30 m/s each. What is the velocity of the heavier fragment ? **(1981, 3M)**

38. A 20 g bullet pierces through a plate of mass $M_1 = 1$ kg and then comes to rest inside a second plate of mass $M_2 =$ 2.98 kg as shown in the figure. It is found that the two plates initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is between M_1 and M_2. Neglect any loss of material of the plates due to the action of bullet. Both plates are lying on smooth table. **(1979)**



39. A body of mass m moving with a velocity v in the x-direction collides with another body of mass M moving in the y-direction with a velocity V. They coalesce into one body during collision. Find

(a) the direction and magnitude of the momentum of the composite body.

(b) the fraction of the initial kinetic energy transformed into heat during the collision. **(1978)**

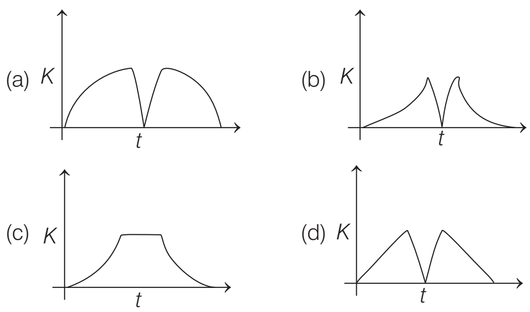
Topic 4 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\,\text{ms}^{-2}$, the value of x will be close to **(JEE Main 2019, 11 Jan Shift I)**

(a) 8 cm (b) 4 cm
(c) 40 cm (d) 80 cm

2. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. **(2014 Adv.)**

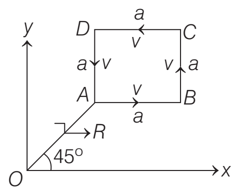


3. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mV and the speed of light is $3 \times 10^8\ \text{ms}^{-1}$. The final momentum of the object is **(2013 Adv.)**

(a) 0.3×10^{-17} kg-ms^{-1} (b) 1.0×10^{-17} kg-ms^{-1}
(c) 3.0×10^{-17} kg-ms^{-1} (d) 9.0×10^{-17} kg-ms^{-1}

Objective Questions II (One or more correct option)

4. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the xy-plane as shown in the figure. **(2016 Main)**



Which of the following statements is **false** for the angular momentum **L** about the origin?

(a) $\mathbf{L} = \dfrac{-mv}{\sqrt{2}} R\hat{\mathbf{k}}$ when the particle is moving from A to B

(b) $\mathbf{L} = mv\left(\dfrac{R}{\sqrt{2}} - a\right)\hat{\mathbf{k}}$ when the particle is moving from C to D

(c) $\mathbf{L} = mv\left(\dfrac{R}{\sqrt{2}} + a\right)\hat{\mathbf{k}}$ when the particle is moving from B to C

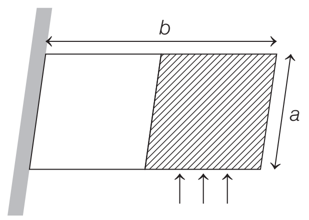
(d) $\mathbf{L} = \dfrac{mv}{\sqrt{2}} R\hat{\mathbf{k}}$ when the particle is moving from D to A

Numerical Value

5. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4$ kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time $t = 0$, so that it starts moving along the X-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is .... (Take, $e^{-1} = 0.37$). **(2018 Adv.)**

Analytical & Descriptive Questions

6. There is a rectangular plate of mass M kg of dimensions $(a \times b)$. The plate is held in horizontal position by striking n small balls uniformly each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v?

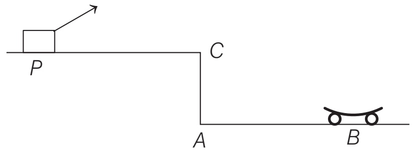


It is given $n = 100$, $M = 3$ kg, $m = 0 \cdot 01$ kg; $b = 2$ m; $a = 1$ m; $g = 10\ \text{m/s}^2$. **(2006, 6M)**

7. Two point masses m_1 and m_2 are connected by a spring of natural length l_0. The spring is compressed such that the two point masses touch each other and then they are fastened by a string. Then the system is moved with a velocity v_0 along positive x-axis. When the system reaches the origin the string breaks
($t = 0$). The position of the point mass m_1 is given by
$x_1 = v_0 t - A(1 - \cos \omega t)$ where A and ω are constants.
Find the position of the second block as a function of time. Also, find the relation between A and l_0. **(2003, 4M)**

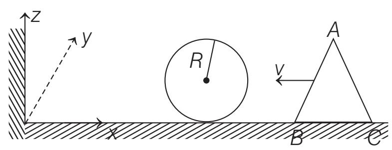
8. A car P is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30° with the horizontal. The first cannon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout.

If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact? **(2011, 10M)**



9. A cylindrical solid of mass 10^{-2} kg and cross-sectional area $10^{-4}\ \text{m}^2$ is moving parallel to its axis (the x-axis) with a uniform speed of 10^3 m/s in the positive direction. At $t = 0$, its front face passes the plane $x = 0$. The region to the right of this plane is filled with stationary dust particles of uniform density $10^{-3}\ \text{kg/m}^3$. When a dust particle collides with the face of the cylinder, it sticks to its surface. Assuming that the dimensions of the cylinder remain practically unchanged and that the dust sticks only to the front face of the cylinder find the x-coordinate of the front of the cylinder at $t = 150$ s. **(1998, 5M)**

10. A wedge of mass m and triangular cross-section ($AB = BC = CA = 2R$) is moving with a constant velocity $(-v\hat{\mathbf{i}})$ towards a sphere of radius R fixed on a smooth horizontal table as shown in the figure.



The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time Δt during which the sphere exerts a constant force $\mathbf{F}$ on the wedge. **(1998, 8M)**

(a) Find the force $\mathbf{F}$ and also the normal force $\mathbf{N}$ exerted by the table on the wedge during the time Δt.

(b) Let h denote the perpendicular distance between the centre of mass of the wedge and the line of action of F. Find the magnitude of the torque due to the normal force $\mathbf{N}$ about the centre of the wedge during the interval Δt.

11. A uniform thin rod of mass M and length L is standing vertically along the Y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive X-axis, and the rod starts falling. **(1993, 1+5M)**

(a) What is the path followed by the centre of mass of the rod during its fall?

(b) Find the equation of the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point ?

Answers

Topic 1

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)
5. (d)	**6.** (d)	**7.** (a)	**8.** (a)
9. (c)	**10.** (d)	**11.** (d)	**12.** (a)
13. F	**14.** $(L + 2R, 0)$	**15.** 6.53 s	

16. 9 cm from centre of bigger circle (leftwards)

Topic 2

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)
5. (a)	**6.** (d)	**7.** (b, d)	

8. $-mv_2 \sin \frac{v_2}{R} t\,\hat{\mathbf{i}} + m\left(v_2 \cos \frac{v_2}{R} t - v_1\right)\hat{\mathbf{j}}$

9. (a) $\frac{v_0}{3}$ (b) $\frac{2mv_0^2}{3x_0^2}$ **10.** $\frac{m(R-r)}{M+m}, m\sqrt{\frac{2g(R-r)}{M\,(M+m)}}$

11. No

Topic 3

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)	
5. (d)	**6.** (b)	**7.** (b)	**8.** (a)	
9. (d)	**10.** (a)	**11.** (c)	**12.** (b)	
13. (b)	**14.** (d)	**15.** (a)	**16.** (d)	
17. (c)	**18.** (c)	**19.** (a)	**20.** (a)	
21. (c)	**22.** (30)	**23.** (d)	**24.** (b)	**25.** (b)

26. (c) **27.** (a, c) **28.** (a, d) **29.** 5×10^{-3}

30. $\frac{3}{2} mv^2$ **31.** 4 **32.** $e = 0.84, M = 15.12$ kg

33. (a) $\frac{5}{2}\sqrt{6\mu g d}$ (b) $6d\sqrt{3\mu}$ **34.** 44.25 m **35.** 4

36. Velocity of C is v in a direction opposite to Velocity of B.

37. $10\sqrt{2}$ m/s at 45° **38.** 25%

39. (a) At an angle $\tan^{-1}\left(\frac{MV}{mv}\right)$ with positive x-axis. Magnitude is $\sqrt{(mv)^2 + (MV)^2}$ (b) Fraction $= \frac{Mm(v^2 + V^2)}{(M+m)(mv^2 + MV^2)}$

Topic 4

1. (*)	**2.** (b)	**3.** (b)	**4.** (b, d)
5. (6.30)	**6.** 10 m/s		

7. $x_2 = v_0 t + \frac{m_1}{m_2} A(1 - \cos \omega t),\ l_0 = \left(\frac{m_1}{m_2} + 1\right)A$

8. 12s, 15.75 m/s **9.** 10^5 m

10. (a) $\frac{2mv}{\sqrt{3}\Delta t}(\sqrt{3}\hat{\mathbf{i}} - \hat{\mathbf{k}}), \left(\frac{2mv}{\sqrt{3}\Delta t} + mg\right)\hat{\mathbf{k}}$ (b) $\frac{4mv}{\sqrt{3}\Delta t} h$

11. (a) Straight line (b) $\frac{x^2}{\left(\frac{L}{2} - r\right)^2} + \frac{y^2}{r^2} = 1$, ellipse

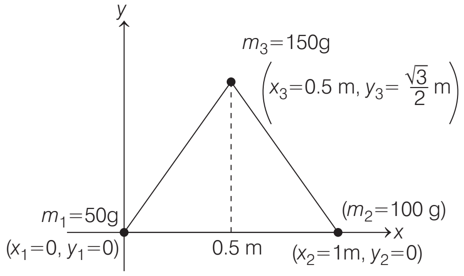
Hints & Solutions

Topic 1 Centre of Mass

1. The height of equilateral Δ is

$$h = y_3 = \sqrt{(1)^2 - (0.5)^2} = \sqrt{3}/2\,\text{m}$$

Thus, coordinates of three masses are (0, 0), (1, 0) and $\left(0.5, \frac{\sqrt{3}}{2}\right)$



Using, $X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$,

we have

$$X_{CM} = \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150}$$

$$= \frac{175}{300} = \frac{7}{12}\text{m}$$

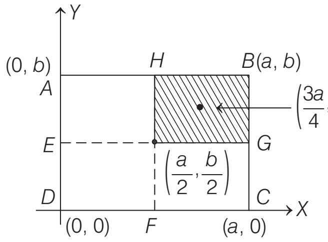
Similarly,

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150}$$

$$= \frac{\sqrt{3}}{4}\text{ m}$$

2. The given rectangular thin sheet $ABCD$ can be drawn as shown in the figure below,



Here,

Area of complete lamina, $A_1 = ab$

Area of shaded part of lamina $= \dfrac{a}{2} \times \dfrac{b}{2} = \dfrac{ab}{4}$

(x_1, y_1) = coordinates of centre of mass of complete lamina

$$= \left(\frac{a}{2}, \frac{a}{2}\right)$$

(x_2, y_2) = coordinates of centre of mass of shaded part of lamina $= \left(\dfrac{3a}{4}, \dfrac{3a}{4}\right)$

$\therefore$ Using formula for centre of mass, we have

$$X_{\text{CM}} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$= \frac{ab\left(\frac{a}{2}\right) - \frac{ab}{4}\left(\frac{3a}{4}\right)}{ab - \frac{ab}{4}} = \frac{\frac{8a^2b - 3a^2b}{16}}{\frac{3ab}{4}} = \frac{5a}{12}$$

Similarly, $Y_{\text{CM}} = \dfrac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$

$$= \frac{ab\left(\frac{b}{2}\right) - \frac{ab}{4}\left(\frac{3b}{4}\right)}{ab - \frac{ab}{4}} = \frac{5b}{12}$$

The coordinate of the centre of mass is $\left(\dfrac{5a}{12}, \dfrac{5b}{12}\right)$.

Alternate Solution

Let m be the mass of entire rectangular lamina.

So, the mass of the shaded portion of lamina $= \dfrac{m}{4}$

Using the relation,

$$X_{\text{CM}} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}, \text{ we get}$$

$$X_{\text{CM}} = \frac{m\left(\frac{a}{2}\right) - \frac{m}{4}\left(\frac{3a}{4}\right)}{m - \frac{m}{4}} = \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

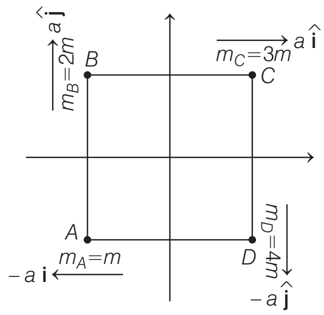
Similarly, $Y_{\text{CM}} = \dfrac{m_1 y_1 - m_2 y_2}{m_1 - m_2}$, we get

$$Y_{\text{CM}} = \frac{m\left(\frac{a}{2}\right) - \frac{m}{4}\left(\frac{3b}{4}\right)}{m - \frac{m}{4}} = \frac{\frac{b}{2} - \frac{3b}{16}}{\frac{3}{4}} = \frac{5b}{12}$$

$\therefore$ The coordinates of the centre of mass of remaining portion will be $\left(\dfrac{5a}{12}, \dfrac{5b}{12}\right)$.

3. For a system of discrete masses, acceleration of centre of mass (CM) is given by

$$\mathbf{a}_{\text{CM}} = \frac{m_A \mathbf{a}_A + m_B \mathbf{a}_B + m_C \mathbf{a}_C + m_D \mathbf{a}_D}{m_A + m_B + m_C + m_D}$$



where, $m_A = m, m_B = 2m$, $m_C = 3m$ and $m_D = 4m$, $|\mathbf{a}_A| = |\mathbf{a}_B| = |\mathbf{a}_C| = |\mathbf{a}_D| = a$ (according to the question)

$$a_{\text{CM}} = \frac{-ma\hat{\mathbf{i}} + 2ma\hat{\mathbf{j}} + 3ma\hat{\mathbf{i}} - 4ma\hat{\mathbf{j}}}{m + 2m + 3m + 4m}$$

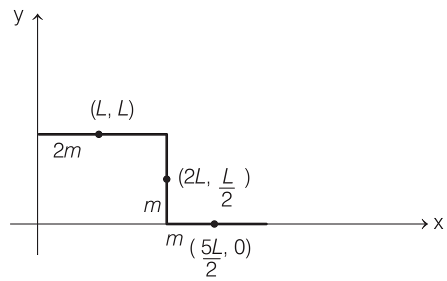
$$= \frac{2a\hat{\mathbf{i}} - 2a\hat{\mathbf{j}}}{10} = \frac{a}{5} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}})\,\text{ms}^{-2}$$

4. Coordinates of centre of mass (COM) are given by

$$X_{\text{COM}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

and $$Y_{\text{COM}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

For given system of rods, masses and coordinates of centre of rods are as shown.



So, $X_{\text{COM}} = \left(\dfrac{2mL + m2L + m\frac{5L}{2}}{4\,m}\right) = \dfrac{13}{8}L$

and $Y_{\text{COM}} = \dfrac{2mL + m \times \frac{L}{2} + m \times 0}{4\,m} = \dfrac{5L}{8}$

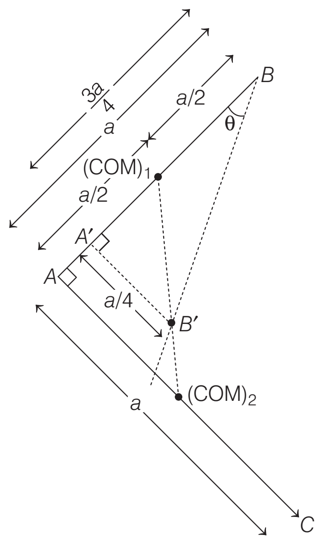
So, position vector of COM is

$$\mathbf{r}_{\text{com}} = X_{\text{COM}}\hat{\mathbf{x}} + Y_{\text{COM}}\hat{\mathbf{y}}$$

$$= \frac{13}{8}L\hat{\mathbf{x}} + \frac{5}{8}L\hat{\mathbf{y}}$$

5. **Key Idea** The centre of mass of a thin rod of uniform density lies at its centre.

The given system of rods can be drawn using geometry as,

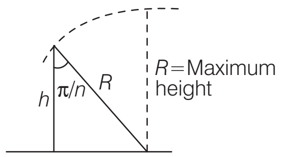


where, $(\text{COM})_1$ and $(\text{COM})_2$ are the centre of mass of both rods AB and AC, respectively.

So, in $\Delta A'BB'$,

$$\tan\theta = \frac{A'B'}{A'B} = \frac{\frac{a}{4}}{\frac{3a}{4}} = \frac{1}{3} \text{ or } \tan\theta = \frac{1}{3}$$

6.



$$\cos\left(\frac{\pi}{n}\right) = \frac{h}{R}$$

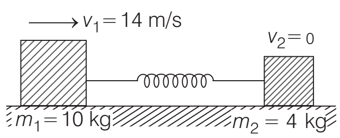
$$\Delta = R - h = \frac{h}{\cos\left(\frac{\pi}{n}\right)} - h = h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right) - 1}\right]$$

7. Centre of mass of uniform solid cone of height h is at a height of $\frac{h}{4}$ from base. Therefore from vertex it's $\frac{3h}{4}$.

8. $$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4 + m_5y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$= \frac{(6m)(0) + (m)(a) + m(a) + m(0) + m(-a)}{6m + m + m + m + m} = \frac{a}{10}$$

9.



$$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{10 \times 14 + 4 \times 0}{10 + 4} = \frac{140}{14} = 10\,\text{m/s}$$

10. Net force on centre of mass is zero. Therefore, centre of mass always remains at rest.

11. If a force is applied at centre of mass of a rigid body, its torque about centre of mass will be zero, but acceleration will be non-zero. Hence, velocity will change.

12. Δx_{cm} of the block and point mass system = 0

$\therefore \quad m(x + R) + Mx = 0$

where, x is displacement of the block.

Solving this equation, we get

$$x = -\frac{mR}{M + m}$$

From conservation of momentum and mechanical energy of the combined system

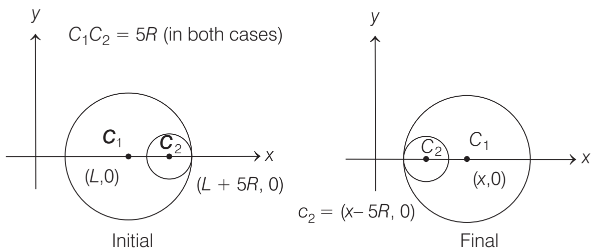
$$0 = mv - MV \Rightarrow mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Solving these two equations, we get

$$\therefore \quad v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$$

13. Since, net force on the system is zero. Velocity of centre of mass will remain constant.

14. Since, all the surfaces are smooth, no external force is acting on the system in horizontal direction. Therefore, the centre of mass of the system in horizontal direction remains stationary.



x-coordinate of CM initially will be given by

$$x_i = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{(4M)(L) + M(L + 5R)}{4M + M} = (L + R) \qquad \ldots\text{(i)}$$

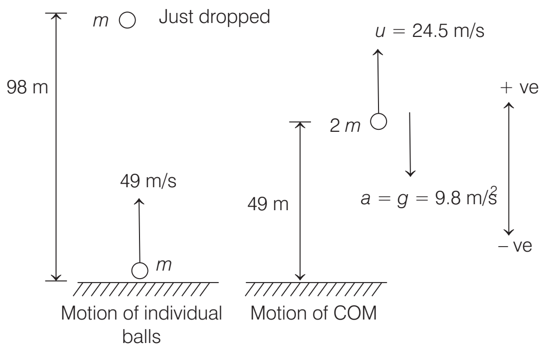
Let $(x, 0)$ be the coordinates of the centre of large sphere in final position. Then, x-coordinate of CM finally will be

$$x_f = \frac{(4M)(x) + M(x - 5R)}{4M + M} = (x - R) \qquad \ldots\text{(ii)}$$

Equating Eqs. (i) and (ii), we have $x = L + 2R$

Therefore, coordinates of large sphere, when the smaller sphere reaches the other extreme position, are $(L + 2R, 0)$.

15. Both the balls are of equal masses. Therefore, their centre of mass is at height $98/2 = 49\,\text{m}$ from ground. Acceleration of both the balls is g downwards. Therefore, acceleration of their centre of mass is also g downwards. Further, initially one ball has a velocity 49 m/s.



While the other is at rest. Therefore, initial velocity of their centre of mass is 49/2 = 24.5 m/s upwards. So, looking the motion of their centre of mass. Let it strikes after time t with the ground. Putting the proper values with sign in equation.

$$s = ut + \frac{1}{2}at^2$$

$$-49 = 24.5\,t - 4.9\,t^2$$

or $$t^2 - 5\,t - 10 = 0$$

or $$t = \frac{5 \pm \sqrt{25 + 40}}{2}$$

The positive value of t from this equation comes out to be, 6.53 s. Therefore, time of flight of the balls is 6.53 s.

16. Suppose r_1 be the distance of centre of mass of the remaining portion from centre of the bigger circle, then

$$A_1 r_1 = A_2 r_2$$

$$r_1 = \left(\frac{A_2}{A_1}\right) r_2$$

$$= \frac{\pi(42)^2}{\pi[(56)^2 - (42)^2]} \times 7 = 9\,\text{cm}$$

Topic 2 Linear Momentum, Mechanical Energy and Their Conservation

1. Initially, centre of mass is at distance L from the top end of the swing. It shifts to $(L - l)$ distance when the person stands up on the swing.

$\therefore$ Using angular momentum conservation law, if v_0 and v_1 are the velocities before standing and after standing of the person, then

$$Mv_0 L = Mv_1 (L - l)$$

$$\Rightarrow \quad v_1 = \left(\frac{L}{L - l}\right) v_0 \qquad \ldots\text{(i)}$$

Now, total work done by (person + gravitation) system will be equal to the change in kinetic energy of the person, i.e.

$$W_g + W_p = \text{KE}_1 - \text{KE}_0$$

$$\Rightarrow \quad -Mgl + W_p = \frac{1}{2}Mv_1^2 - \frac{1}{2}Mv_0^2$$

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}M(v_1^2 - v_0^2)$$

$$= Mgl + \frac{1}{2}M\left[\left(\frac{L}{L - l}\right)^2 v_0^2 - v_0^2\right] \qquad \text{[from Eq. (i)]}$$

$$= Mgl + \frac{1}{2}M\, v_0^2\left[\left(\frac{L - l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1 - \frac{l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1 + \frac{2l}{L}\right) - 1\right]$$

[using $(1 + x)^n = 1 + nx$ as higher terms can be neglected, if $n << 1$]

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}Mv_0^2 \times \frac{2l}{L}$$

or $$W_p = Mgl + Mv_0^2\,\frac{l}{L} \qquad \ldots\text{(ii)}$$

Here, $$v_0 = WA = \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$$

$$\Rightarrow \quad v_0 = \theta_0\sqrt{gL}$$

$\therefore$ Using this value of v_0 in Eq.(ii), we get

$$W_p = Mgl + M\theta_0^2\, gL \cdot \frac{l}{L}$$

$$\Rightarrow \quad W_p = Mgl\,[1 + \theta_0^2]$$

2. **Key Idea** Since, the ground is frictionless, so whe the particle will collide and climb over the wedge, then the wedge will also move. Thus, by using conservation laws for momentum and energy, maximum height climbed by the particle can be calculated.

Initial condition can be shown in the figure below

As mass m collides with wedge, let both wedge and mass move with speed v'. Then,

By applying linear momentum conservation, we have

Initial momentum of the system =

Final momentum of the system

$$mv + 0 = (m + 4m)\,v'$$

$$\Rightarrow \quad v' = \frac{v}{5} \qquad \ldots\text{(i)}$$

Now, if m rises upto height h over wedge, then by applying conservation of mechanical energy, we have

Initial energy of the system = Final energy of the system

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv'^2 + mgh + \frac{1}{2}(4m)v'^2$$

$$mv^2 = (m + 4m)v'^2 + 2mgh$$

$$\Rightarrow \quad v^2 = 5v'^2 + 2gh$$

$$\Rightarrow \quad v^2 = \frac{1}{5}v^2 + 2gh \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \frac{4}{5}v^2 = 2gh \Rightarrow h = \frac{2v^2}{5g}$$

3. When a ball is thrown vertically upward, then the acceleration of the ball,
a = acceleration due to gravity (g) (acting in the downward direction).
Now, using the equation of motion,

$$v^2 = u^2 - 2gh$$

or $$h = \frac{-v^2 + u^2}{2g} \qquad \ldots\text{(i)}$$

As we know, momentum, $p = mv$ or $v = p/m$
So, substituting the value of v in Eq. (i), we get

$$h = \frac{u^2 - (p/m)^2}{2g}$$

As we know that, at the maximum height, velocity of the ball thrown would be zero.

So, for the flight when the ball is thrown till is reaches the maximum height (h).
$v \rightarrow$ changes from u to 0
$\Rightarrow p \rightarrow$ changes from mu to 0
Similarly, when it reacher it's initial point, then
$h \rightarrow$ changes from h_{max} to 0
Also, $p \rightarrow$ changes from 0 to some values.

Thus, these conditions are only satisfied in the plot given in option (d).

4. Here,

$$F = kt$$

When $t = 0$, linear momentum $= p$
When $t = T$, linear momentum $= 3p$
According to Newton's second law of motion,
applied force, $F = \dfrac{dp}{dt}$
or $dp = F \cdot dt$
or $dp = kt \cdot dt$
Now, integrate both side with proper limit

$$\int_p^{3p} dp = k\int_0^T t\,dt \text{ or } [p]_p^{3p} = k\left[\frac{t^2}{2}\right]_0^T$$

or $$(3p - p) = \frac{1}{2}k[T^2 - 0]$$

or $$T^2 = \frac{4p}{k} \text{ or } T = 2\sqrt{\frac{p}{k}}$$

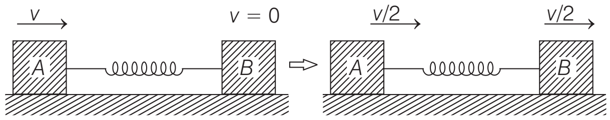
5. On a system of particles if, $\Sigma \mathbf{F}_{ext.} = 0$
then, $\mathbf{p}_{system}$ = constant
No other conclusions can be drawn.

6. $$\mathbf{F} = \frac{d\mathbf{p}}{dt} = -kA\sin(kt)\,\hat{\mathbf{i}} - kA\cos(kt)\,\hat{\mathbf{j}}$$

$$\mathbf{p} = A\cos(kt)\,\hat{\mathbf{i}} - A\sin(kt)\,\hat{\mathbf{j}}$$

Since, $\mathbf{F} \cdot \mathbf{p} = 0$
$\therefore$ Angle between $\mathbf{F}$ and $\mathbf{p}$ should be 90°.

7. After collision between C and A, C stops while A moves with speed of C i.e. v [in head on elastic collision, two equal masses exchange their velocities]. At maximum compression, A and B will move with same speed $v/2$ (From conservation of linear momentum).



Let x be the maximum compression in this position.
$\therefore$ KE of A-B system at maximum compression

$$= \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 \text{ or } K_{max} = mv^2/4$$

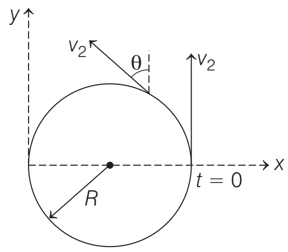
From conservation of mechanical energy in two positions shown in above figure

or $$\frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}kx^2 = \frac{1}{4}mv^2$$

$\Rightarrow$ $$x = v\sqrt{\frac{m}{2k}} \text{ (Maximum compression)}$$

8. Angular speed of particle about centre of the circle



$$\omega = \frac{v_2}{R}, \theta = \omega t = \frac{v_2}{R}t$$

$$\mathbf{v}_p = (-v_2\sin\theta\,\hat{\mathbf{i}} + v_2\cos\theta\,\hat{\mathbf{j}})$$

or $$\mathbf{v}_p = \left(-v_2\sin\frac{v_2}{R}t\,\hat{\mathbf{i}} + v_2\cos\frac{v_2}{R}t\,\hat{\mathbf{j}}\right)$$

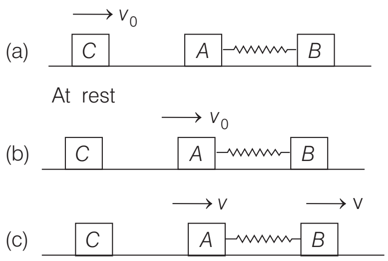
and $$\mathbf{v}_m = v_1\hat{\mathbf{j}}$$

$\therefore$ Linear momentum of particle w.r.t. man as a function of time is

$$\mathbf{L}_{pm} = m(\mathbf{v}_p - \mathbf{v}_m)$$

$$= m\left[\left(-v_2\sin\frac{v_2}{R}t\right)\hat{\mathbf{i}} + \left(v_2\cos\frac{v_2}{R}t - v_1\right)\hat{\mathbf{j}}\right]$$

9. (a) Collision between A and C is elastic and mass of both the blocks is same. Therefore, they will exchange their velocities i.e. C will come to rest and A will be moving will velocity v_0. Let v be the common velocity of A and B, then from conservation of linear momentum, we have



$$m_A v_0 = (m_A + m_B) v \quad \text{or} \quad m v_0 = (m + 2m) v \quad \text{or} \quad v = \frac{v_0}{3}$$

(b) From conservation of energy, we have

$$\frac{1}{2} m_A v_0^2 = \frac{1}{2}(m_A + m_B) v^2 + \frac{1}{2} k x_0^2$$

or
$$\frac{1}{2} m v_0^2 = \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2 + \frac{1}{2} k x_0^2$$

or
$$\frac{1}{2} k x_0^2 = \frac{1}{3} m v_0^2 \quad \text{or} \quad k = \frac{2 m v_0^2}{3 x_0^2}$$

10. (a) The centre of mass of $M + m$ in this case will not move in horizontal direction. Let M moves towards left by a distance x then m will move towards right by a distance $R - r - x$ (with respect to ground). For centre of mass not to move along horizontal we should have

$$Mx = m(R - r - x), \quad x = \frac{m(R - r)}{M + m}$$

(b) Let v_1 be the speed of m towards right and v_2 the speed of M towards left. From conservation of linear momentum.

$$m v_1 = M v_2 \quad \text{...(i)}$$

From conservation of mechanical energy

$$mg(R - r) = \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2 \quad \text{...(ii)}$$

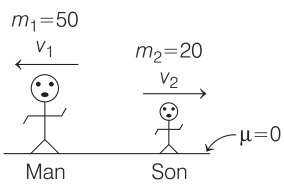
Solving these two equations, we get

$$v_2 = m\sqrt{\frac{2g(R - r)}{M(M + m)}}$$

11. No, the situation given does not violate conservation of linear momentum. Because linear momentum of ball-earth system remains constant.

Topic 3 Impulse, Explosions and Collisions

1. The given situation can be shown as below



After collision

Using momentum conservation law,

$$(\text{Total momentum})_{\text{before collision}}$$
$$= (\text{Total momentum})_{\text{after collision}}$$
$$(m_1 \times 0) + (m_2 \times 0) = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$
$$0 = m_1(-v_1)\hat{\mathbf{i}} + m_2 v_2 \hat{\mathbf{i}}$$

$$\Rightarrow \quad m_1 v_1 = m_2 v_2$$
$$\Rightarrow \quad 50 v_1 = 20 v_2$$
$$\Rightarrow \quad v_2 = 2.5 v_1 \quad \text{...(i)}$$

Again, relative velocity = 0.70 m/s

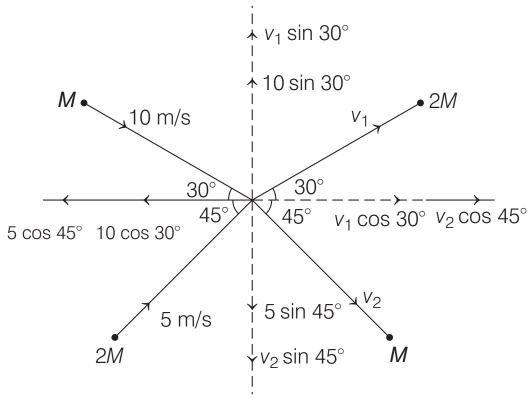
But from figure, relative velocity = $v_1 + v_2$

$$\therefore \quad v_1 + v_2 = 0.7 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$v_1 + 2.5 v_1 = 0.7$$
$$\Rightarrow \quad v_1(3.5) = 0.7$$
$$v_1 = \frac{0.7}{3.5} = 0.20 \text{ m/s}$$

2. The given condition can be drawn as shown below



Applying linear momentum conservation law in x-direction, we get

Initial momentum = Final momentum

$$(M \times 10\cos 30°) + (2M \times 5\cos 45°)$$
$$= (M \times v_2 \cos 45°) + (2M \times v_1 \cos 30°)$$
$$\Rightarrow \quad \left(M \times 10 \times \frac{\sqrt{3}}{2}\right) + \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$
$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) + \left(2M \times v_1 \times \frac{\sqrt{3}}{2}\right)$$
$$\Rightarrow \quad 5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + v_1\sqrt{3} \quad \text{... (i)}$$

Similarly, applying linear momentum conservation law in y-direction, we get

$$(M \times 10 \sin 30°) - (2M \times 5 \sin 45°)$$
$$= (M \times v_2 \sin 45°) - (2M \times v_1 \sin 30°)$$
$$\Rightarrow \left(M \times 10 \times \frac{1}{2}\right) - \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$
$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) - \left(2M \times v_1 \times \frac{1}{2}\right)$$

$$\Rightarrow \quad 5-5\sqrt{2}=\frac{v_2}{\sqrt{2}}-v_1 \quad \ldots \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

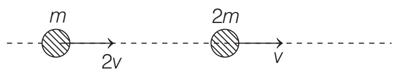
$$(5\sqrt{3}+5\sqrt{2})-(5-5\sqrt{2})$$
$$=\left(\frac{v_2}{\sqrt{2}}+v_1\sqrt{3}\right)-\left(\frac{v_2}{\sqrt{2}}-v_1\right)$$
$$\Rightarrow \quad 5\sqrt{3}+10\sqrt{2}-5=v_1\sqrt{3}+v_1$$
$$\Rightarrow \quad v_1=\left(\frac{5\sqrt{3}+10\sqrt{2}-5}{1+\sqrt{3}}\right)=\frac{8.66+14.142-5}{1+1.732}$$
$$=\frac{17.802}{2.732}\Rightarrow v_1=6.516\text{ m/s}\approx 6.5\text{ m/s} \quad \ldots \text{(iii)}$$

Substituting the value from Eq. (iii) in Eq. (i), we get

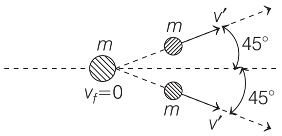
$$5\sqrt{3}+5\sqrt{2}=\frac{v_2}{\sqrt{2}}+6.51\times\sqrt{3}$$
$$\Rightarrow \quad v_2=(5\sqrt{3}+5\sqrt{2}-6.51\times\sqrt{3})\sqrt{2}$$
$$v_2=(8.66+7.071-11.215)\,1.414$$
$$\Rightarrow \quad v_2=4.456\times 1.414$$
$$\Rightarrow \quad v_2\approx 6.3\text{ m/s}$$

3. According to the questions,

Initial condition,



Final condition,



As we know that, in collision, linear momentum is conserved in both x and y directions separately.

So, $(p_x)_{\text{initial}}=(p_x)_{\text{final}}$

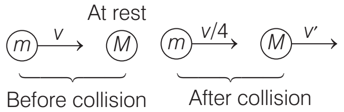
$$m(2v)+2m(v)=0+mv'\cos 45^\circ+mv'\cos 45^\circ$$
$$\Rightarrow \quad 4mv=\frac{2m}{\sqrt{2}}v' \Rightarrow v'=2\sqrt{2}v$$

So, each particle will move with a speed of $2\sqrt{2}\,v$.

4. **Key Idea** For an elastic collision, coefficient of restitution (e), i.e. the ratio of relative velocity of separation after collision to the relative velocity of approach before collision is 1.

Given, mass of small body, $m=2$ kg

Given situation is as shown



Using momentum conservation law for the given system,

$$(\text{Total momentum})_{\text{before collision}}=(\text{Total momentum})_{\text{after collision}}$$

$$\Rightarrow \quad m(v)+M(0)=m\left(\frac{v}{4}\right)+M(v') \ldots \text{(i)}$$

$\because$ $e=1$ and we know that,

$$e=-\frac{v_2-v_1}{u_2-u_1}$$
$$\Rightarrow \quad 1=-\frac{v'-v/4}{0-v}$$
$$\Rightarrow \quad v=v'-v/4$$

or $\quad v'=5v/4 \quad \ldots$(ii)

Using value from Eq. (ii) into Eq. (i), we get

$$mv=\frac{mv}{4}+M\left(\frac{5v}{4}\right)$$
$$\Rightarrow \quad m\left(v-\frac{v}{4}\right)=M\left(\frac{5v}{4}\right)$$
$$\Rightarrow \quad \frac{3}{4}mv=\frac{5}{4}Mv$$
$$M=\frac{3}{5}m=\frac{3}{5}\times 2=1.2\text{ kg}$$

5. **Key Idea** Total linear momentum is conserved in all collisions, i.e. the initial momentum of the system is equal to final momentum of the system.

Given,

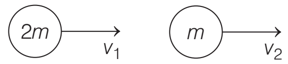
$$m_2=0.5m_1 \quad \Rightarrow \quad m_1=2m_2$$

Let $m_2=m$, then, $m_1=2m$

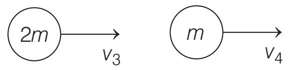
Also, $v_3=0.5v_1$

Given situation of collinear collision is as shown below

Before collision,



After collision,

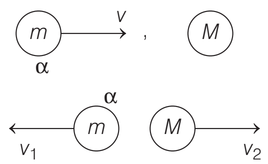


$\therefore$ According to the conservation of linear momentum,

Initial momentum = Final momentum

$$m_1v_1\hat{\mathbf{i}}+m_2v_2\hat{\mathbf{i}}=m_1v_3\hat{\mathbf{i}}+m_2v_4\hat{\mathbf{i}}$$
$$\Rightarrow 2mv_1\hat{\mathbf{i}}+mv_2\hat{\mathbf{i}}=2m(0.5v_1)\hat{\mathbf{i}}+mv_4\hat{\mathbf{i}}$$
$$\Rightarrow v_4=v_1+v_2 \quad \Rightarrow \quad v_1=v_4-v_2$$

6. We have following collision, where mass of α particle $=m$ and mass of nucleus $=M$



Let α particle rebounds with velocity v_1, then

Given; final energy of α = 36% of initial energy

$$\Rightarrow \qquad \frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv^2$$

$$\Rightarrow \qquad v_1 = 0.6\,v \qquad \text{...(i)}$$

As unknown nucleus gained 64% of energy of α, we have

$$\frac{1}{2}Mv_2^2 = 0.64 \times \frac{1}{2}\,mv^2$$

$$\Rightarrow \qquad v_2 = \sqrt{\frac{m}{M}} \times 0.8\,v \qquad \text{...(ii)}$$

From momentum conservation, we have

$$mv = Mv_2 - mv_1$$

Substituting values of v_1 and v_2 from Eqs. (i) and (ii), we have

$$mv = M\sqrt{\frac{m}{M}} \times 0.8v - m \times 0.6v$$

$$\Rightarrow \qquad 1.6mv = \sqrt{mM} \times 0.8v$$

$$\Rightarrow \qquad 2m = \sqrt{mM}$$

$$\Rightarrow \qquad 4m^2 = mM \quad \Rightarrow \quad M = 4m$$

7 According to the given condition in the question, after collision the mass of combined system is doubled. Also, this system would be displaced from its circular orbit.

So, the combined system revolves around centre of mass of 'system + earth' under action of a central force.

Hence, orbit must be elliptical.

8 Pendulum's velocity at lowest point just before striking mass m is found by equating it's initial potential energy (PE) with final kinetic energy (KE).

Initially, when pendulum is released from angle θ_0 as shown in the figure below,

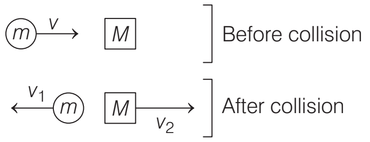
We have,

$$mgh = \frac{1}{2}mv^2$$

Here, $\qquad h = l - l\cos\theta_0$

So, $\qquad v = \sqrt{2gl\,(1-\cos\theta_0)} \qquad \text{…(i)}$

With velocity v, bob of pendulum collides with block. After collision, let v_1 and v_2 are final velocities of masses m and M respectively as shown



Then if pendulum is deflected back upto angle θ_1, then

$$v_1 = \sqrt{2gl\,(1-\cos\theta_1)} \quad \text{…(ii)}$$

Using definition of coefficient of restitution to get

$$e = \frac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$$

$$1 = \frac{v_2 - (-v_1)}{v - 0} \Rightarrow v = v_2 + v_1 \qquad \text{…(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \sqrt{2gl(1-\cos\theta_0)} = v_2 + \sqrt{2gl(1-\cos\theta_1)}$$

$$\Rightarrow \quad v_2 = \sqrt{2gl}\,(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}) \qquad \text{…(iv)}$$

According to the momentum conservation, initial momentum of the system = final momentum of the system

$$\Rightarrow \quad mv = Mv_2 - mv_1$$

$$\Rightarrow Mv_2 = m(v + v_1)$$

$$Mv_2 = m\sqrt{2gl}\,(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1})$$

Dividing Eq. (v) and Eq. (iv), we get

$$\Rightarrow \frac{M}{m} = \frac{\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}}$$

$$= \frac{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} + \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} - \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}$$

$$\frac{M}{m} = \frac{\sin\left(\frac{\theta_0}{2}\right) + \sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right) - \sin\left(\frac{\theta_1}{2}\right)}$$

For small θ_0, we have

$$\frac{M}{m} = \frac{\frac{\theta_0}{2} + \frac{\theta_1}{2}}{\frac{\theta_0}{2} - \frac{\theta_1}{2}}$$

$$\text{or} \qquad M = m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$$

9. **Key Idea** As bullet gets embedded in the block of wood so, it represents a collision which is perfectly inelastic and hence only momentum of the system is conserved.

Velocity of bullet is very high compared to velocity of wooden block so, in order to calculate time for collision, we take relative velocity nearly equal to velocity of bullet.

So, time taken for particles to collide is

$$t = \frac{d}{v_{\text{rel}}} = \frac{100}{100} = 1\text{s}$$

Speed of block just before collision is;

$$v_1 = gt = 10 \times 1 = 10\ \text{ms}^{-1}$$

Speed of bullet just before collision is

$$v_2 = u - gt$$

$$= 100 - 10 \times 1 = 90\ \text{ms}^{-1}$$

Let $v =$ velocity of bullet + block system, then by conservation of linear momentum, we get

$-(0.03 \times 10) + (0.02 \times 90) = (0.05)\, v$

$\Rightarrow \qquad v = 30\ \text{ms}^{-1}$

Now, maximum height reached by bullet and block is

$$h = \frac{v^2}{2g} \Rightarrow h = \frac{30 \times 30}{2 \times 10}$$

$\Rightarrow \qquad h = 45$ m

$\therefore$ Height covered by the system from point of collision = 45 m

Now, distance covered by bullet before collision in 1 sec.

$$= 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95\ \text{m}$$

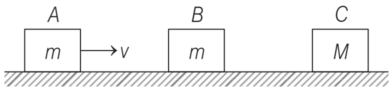
Distance of point of collision from the top of the building

$= 100 - 95 = 5\ \text{m}$

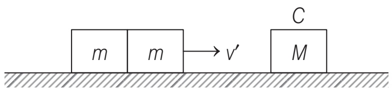
$\therefore$ Maximum height to which the combined system reaches above the top of the building before falling below $= 45 - 5 = 40\ \text{m}$

10. **Key Idea** For a perfectly inelastic collision, the momentum of the system remains conserved but there is some of loss of kinetic energy. Also, after collision the objects of the system are stuck to each other and move as a combined system.

Initially, block A is moving with velocity v as shown in the figure below,

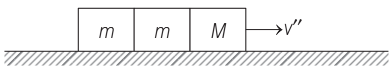


Now, A collides with B such that they collide inelastically. Thus, the combined mass (say) move with the velocity 'v'' as shown below,



Then, if this combined system is collided inelastically again with the block C.

So, now the velocity of system be v'' as shown below.



Thus, according to the principle of conservation of momentum,

initial momentum of the system

= final momentum of the system

$\Rightarrow \qquad mv = (2m + M)v''$

or $\qquad v'' = \left(\dfrac{mv}{2m+M}\right) \qquad \ldots$ (i)

Initial kinetic energy of the system,

$$(\text{KE})_i = \frac{1}{2}mv^2 \qquad \ldots\text{(ii)}$$

Final kinetic energy of the system, $(\text{KE})_f$

$$= \frac{1}{2}(2m + M)(v'')^2$$

$$= \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2 \qquad [\because \text{using Eq. (i)}]$$

$$= \frac{1}{2} \cdot \frac{v^2 m^2}{(2m+M)} \qquad \ldots\text{(iii)}$$

Dividing Eq. (iii) and Eq. (ii), we get

$$\frac{(\text{KE})_f}{(\text{KE})_i} = \frac{\dfrac{\frac{1}{2}m^2 v^2}{(2m+M)}}{\frac{1}{2}mv^2} = \frac{m}{2m + M} \qquad \ldots\text{(iv)}$$

It is given that $\frac{5}{6}$th of $(\text{KE})_i$ is lost in this process.

$\Rightarrow \qquad (\text{KE})_f = \frac{1}{6}(\text{KE})_i$

$\Rightarrow \qquad \dfrac{(\text{KE})_f}{(\text{KE})_i} = \dfrac{1}{6} \qquad \ldots$ (v)

Comparing Eq. (iv) and Eq. (v), we get

$$\frac{m}{2m + M} = \frac{1}{6} \Rightarrow 6m = 2m + M$$

$$4m = M \Rightarrow \frac{M}{m} = 4$$

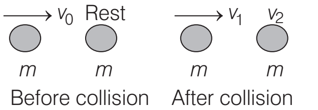
11. From conservation of linear momentum,

$$mv_0 + 0 = mv_1 + mv_2$$

$$v_0 = v_1 + v_2 \qquad \ldots\text{(i)}$$

Further, $\qquad K_f = \frac{3}{2}K,$

$\therefore \qquad \dfrac{3}{2}\left[\dfrac{1}{2}mv_0^2\right] = \dfrac{1}{2}mv_1^2 + \dfrac{1}{2}mv_2^2$



$$\frac{3}{2}v_0^2 = v_1^2 + v_2^2 \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii),

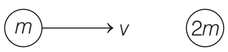
$$v_1 = \frac{v_0}{2}(1+\sqrt{2}\,)$$

$$v_2 = \frac{v_0}{2}(1-\sqrt{2})$$

$$v_{\text{rel}} = v_1 - v_2$$

$$\frac{v_0}{2}[1+\sqrt{2}-1+\sqrt{2}] = \frac{v_0}{2} \times 2\sqrt{2} = \sqrt{2}v_0$$

12. **Case I** Just before Collision,



Just after collision

From momentum conservation,

$$2v_2 - v_1 = v$$

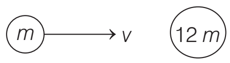
From the definition of e (= 1 for elastic collision)

$$v_2 + v_1 = v \quad \Rightarrow \quad 3v_2 = 2v$$

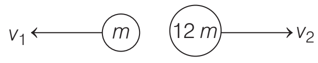
$$v_2 = \frac{2v}{3} \quad \Rightarrow \quad v_1 = \frac{v}{3}$$

$$p_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{1 - \frac{1}{9}}{1} = \frac{8}{9} = 0.89$$

Case II Just before Collision



Just after Collision,



From momentum conservation,

$$12v_2 - v_1 = v$$

From the definition of e(= 1 for elastic collision),

$$v_2 + v_1 = v$$

$$13v_2 = 2v$$

$$v_2 = \frac{2v}{13} \quad \Rightarrow \quad v_1 = v - \frac{2v}{13} = \frac{11v}{13}$$

$\Rightarrow$

$$p_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{1 - \frac{121}{169}}{1} = \frac{48}{169} = 0.28$$

13. In all type of collisions, momentum of the system always remains constant. In perfectly inelastic collision, particles stick together and move with a common velocity.

Let this velocity is $\mathbf{v}_c$. Then,
initial momentum of system = final momentum of system

or $\quad m(2v)\hat{\mathbf{i}} + 2m(v)\hat{\mathbf{j}} = (m + 2m)\mathbf{v}_c$

$\therefore \quad \mathbf{v}_c = \frac{2}{3}(v\hat{\mathbf{i}} + v\hat{\mathbf{j}})$

$$|\mathbf{v}_c| \text{ or } v_c \text{ or speed} = \sqrt{\left(\frac{2}{3}v\right)^2 + \left(\frac{2}{3}v\right)^2} = \frac{2\sqrt{2}}{3}v$$

Initial kinetic energy

$$K_i = \frac{1}{2}(m)(2v)^2 + \frac{1}{2}(2m)(v)^2 = 3mv^2$$

Final kinetic energy

$$K_f = \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}v\right)^2 = \frac{4}{3}mv^2$$

$$\text{Fractional loss} = \left(\frac{K_i - K_f}{K_i}\right) \times 100$$

$$= \left[\frac{(3mv^2) - \left(\frac{4}{3}mv^2\right)}{(3mv^2)}\right] \times 100 = 56\%$$

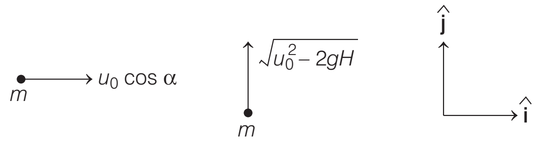
14. Maximum energy loss

$$= \frac{p^2}{2m} - \frac{p^2}{2(m+M)} \qquad \left(\because \text{KE} = \frac{p^2}{2m}\right)$$

Before collision the mass m and after collision the mass is $m + M$

$$= \frac{p^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\} \qquad \left(f = \frac{M}{m+M}\right)$$

15. From momentum conservation equation, we have,



$$\mathbf{p}_i = \mathbf{p}_f$$

$$\therefore \quad m(u_0 \cos\alpha)\hat{\mathbf{i}} + m(\sqrt{u_0^2 - 2gH})\hat{\mathbf{j}} = (2m)\mathbf{v} \quad \ldots\text{(i)}$$

$$H = \frac{u_0^2 \sin^2\alpha}{2g} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii)

$$\mathbf{v} = \frac{u_0 \cos\alpha}{2}\hat{\mathbf{i}} + \frac{u_0 \cos\alpha}{2}\hat{\mathbf{j}}$$

Since both components of $\mathbf{v}$ are equal. Therefore, it is making 45° with horizontal.

16. $\quad R = u\sqrt{\frac{2h}{g}} \Rightarrow 20 = v_1\sqrt{\frac{2 \times 5}{10}}$

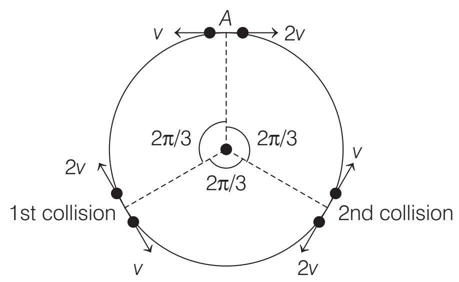
and $\quad 100 = v_2\sqrt{\frac{2 \times 5}{10}}$

$\Rightarrow \quad v_1 = 20\text{ m/s}, v_2 = 100\text{ m/s}.$

Applying momentum conservation just before and just after the collision

$(0.01)(v) = (0.2)(20) + (0.01)(100)\ v = 500\text{ m/s}$

17. At first collision one particle having speed $2v$ will rotate 240° $\left(\text{or } \frac{4\pi}{3}\right)$ while other particle having speed v will rotate 120° $\left(\text{or } \frac{2\pi}{3}\right)$. At first collision, they will exchange their velocities. Now, as shown in figure, after two collisions they will again reach at point A.



18. $|(m_1 \mathbf{v}_1' + m_2 \mathbf{v}_2') - (m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2)|$

$= |$change in momentum of the two particles$|$

$= |$External force on the system$| \times$ time interval

$$= (m_1 + m_2)\, g\, (2t_0) = 2\,(m_1 + m_2)\, g t_0$$

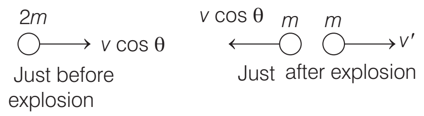
19. Before explosion, particle was moving along x-axis *i.e.,* it has no y-component of velocity. Therefore, the centre of mass will not move in y-direction or we can say $y_{CM} = 0$.

Now, $y_{CM} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

Therefore, $0 = \dfrac{(m/4)(+15) + (3m/4)(y)}{(m/4 + 3m/4)}$

or $y = -5$ cm

20. Let v' be the velocity of second fragment. From conservation of linear momentum,



$$2m\,(v\cos\theta) = mv' - m\,(v\cos\theta)$$

$\therefore$ $v' = 3v\cos\theta$

21. In an inelastic collision, only momentum of the system may remain conserved. Some energy can be lost in the form of heat, sound etc.

22. $\because\ H = \dfrac{u^2 \sin^2 45^\circ}{2g} = 120 \text{ m} \Rightarrow \dfrac{u^2}{4g} = 120 \text{ m}$

If speed is v after the first collision, then speed should remain $\dfrac{1}{\sqrt{2}}$ times, as kinetic energy has reduced to half.

$\Rightarrow$ $v = \dfrac{u}{\sqrt{2}}$

$\therefore$ $h_{\max} = \dfrac{v^2 \sin^2 30^\circ}{2g} = \dfrac{(u/\sqrt{2})^2 \sin^2 30^\circ}{2g}$

$$= \left(\frac{u^2/4g}{4}\right) = \frac{120}{4} = 30$$

23. In case of elastic collision, coefficient of restitution $e = 1$.

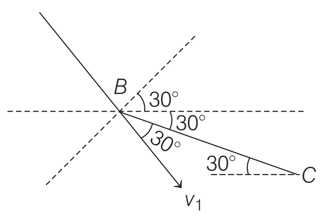
Magnitude of relative velocity of approach

= Magnitude of relative velocity of separation.

But relative speed of approach

$\neq$ Relative speed of separation

24. Between A and B, height fallen by block



$$h_1 = \sqrt{3}\tan 60^\circ = 3 \text{ m.}$$

$\therefore$ Speed of block just before striking the second incline,

$$v_1 = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 3} = \sqrt{60} \text{ ms}^{-1}$$

In perfectly inelastic collision, component of v_1 perpendicular to BC will become zero, while component of v_1 parallel to BC will remain unchanged.

$\therefore$ Speed of block B immediately after it strikes the second incline is,

v_2 = component of v_1 along BC

$$= v_1 \cos 30^\circ = (\sqrt{60})\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{45} \text{ ms}^{-1}$$

25. Height fallen by the block from B to C

$$h_2 = 3\sqrt{3}\tan 30^\circ = 3 \text{ m}$$

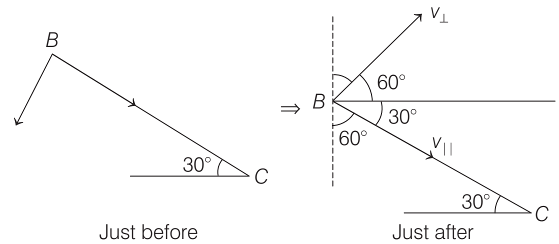
Let v_3 be the speed of block, at point C, just before it leaves the second incline, then :

$$v_3 = \sqrt{v_2^2 + 2gh_2}$$

$$= \sqrt{45 + 2 \times 10 \times 3}$$

$$= \sqrt{105} \text{ ms}^{-1}$$

26. In elastic collision, compound of v_1 parallel to BC will remain unchanged, while component perpendicular to BC will remain unchanged in magnitude but its direction will be reversed.



$$v_{11} = v_1 \cos 30^\circ$$

$$= (\sqrt{60})\left(\frac{\sqrt{3}}{2}\right) = \sqrt{45} \text{ ms}^{-1}$$

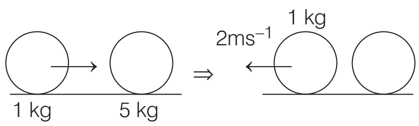
$$v_\perp = v_1 \sin 30^\circ = (\sqrt{60})\left(\frac{1}{2}\right) = \sqrt{15} \text{ ms}^{-1}$$

Now vertical component of velocity of block

$$v = v_\perp \cos 30^\circ - v_{||} \cos 60^\circ$$

$$= (\sqrt{15})\left(\frac{\sqrt{3}}{2}\right) - (\sqrt{45})\left(\frac{1}{2}\right) = 0$$

27.



$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2$$

$$-2 = \left(\frac{1-5}{1+5}\right) v_1 + 0 \qquad (\text{as } v_2 = 0)$$

$$\therefore \qquad v_1 = 3 \text{ ms}^{-1}$$

$$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2 + \left(\frac{2m_1}{m_1 + m_2}\right) v_1 = 1 \text{ ms}^{-1}$$

$$P_{CM} = P_i = (1)(3) = 3 \text{ kg - m/s}$$

$$P_5' = (5)(1) = 5 \text{ kg - m/s}$$

$$K_{CM} = \frac{p_{CM}^2}{2M_{CM}} = \frac{9}{2 \times 6} = 0.75 \text{ J}$$

$$K_{total} = \frac{1}{2} \times 1 \times (3)^2 = 4.5 \text{J}$$

28. Initial momentum of the system $\mathbf{p}_1 + \mathbf{p}_2 = 0$

$\therefore$ Final momentum $\mathbf{p}_1' + \mathbf{p}_2'$ should also be zero.

Option (b) is allowed because if we put $c_1 = -c_2 \neq 0$, $\mathbf{p}_1' + \mathbf{p}_2'$ will be zero. Similary, we can check other options.

29. Impulse $= \int F dt$ = area under F-t graph

$\therefore$ Total impulse from $t = 4\,\mu$s to $t = 16\,\mu$s

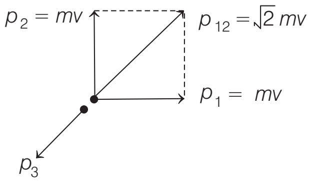
= Area $EBCD$

= Area of trapezium $EBCF$ + Area of triangle FCD

$$= \frac{1}{2}(200 + 800)\, 2 \times 10^{-6} + \frac{1}{2} \times 800 \times 10 \times 10^{-6}$$

$$= 5 \times 10^{-3} \text{ N-s}$$

30. From conservation of linear momentum p_3 should be $\sqrt{2}\, mv$ in a direction opposite to $\mathbf{p}_{12}$ (resultant of $\mathbf{p}_1$ and $\mathbf{p}_2$). Let v' be the speed of third fragment, then



$$(2m)\, v' = \sqrt{2} mv$$

$$\therefore \qquad v' = \frac{v}{\sqrt{2}}$$

$\therefore$ Total energy released is,

$$E = 2\left(\frac{1}{2} mv^2\right) + \frac{1}{2}(2m) v'^2$$

$$= mv^2 + m\left(\frac{v}{\sqrt{2}}\right)^2 = \frac{3}{2} mv^2$$

31. After elastic collision,

$$v_A' = \left(\frac{m - 2m}{m + 2m}\right)(9) + \frac{2(2m)}{m + 2m}(0) = -3 \text{ms}^{-1}$$

Now, from conservation of linear momentum after all collisions are complete,

$$m(+9 \text{ ms}^{-1}) = m(-3 \text{ ms}^{-1}) + 3m(v_C) \text{ or } v_C = 4 \text{ ms}^{-1}$$

32. Let v_1 = velocity of block 2 kg just before collision,

v_2 = velocity of block 2 kg just after collision,

and v_3 = velocity of block M just after collision.

Applying work-energy theorem

(change in kinetic energy = work done by all the forces) at different stages as shown in figure.

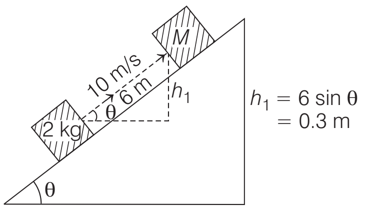


Figure 1.

$$\Delta KE = W_{friction} + W_{gravity}$$

$$\left[\frac{1}{2} m \{v_1^2 - (10)^2\}\right] = -6\mu\, mg \cos\theta - mgh_1$$

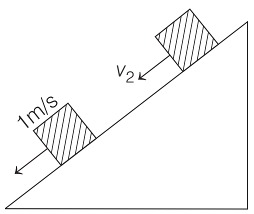
or $$v_1^2 - 100 = -2[6\mu g \cos\theta + gh_1]$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - (0.05)^2} \approx 0.99$$

$$\therefore v_1^2 = 100 - 2[(6)(0.25)(10)(0.99) + (10)(0.3)]$$

$$\Rightarrow \qquad v_1 \approx 8 \text{m/s}$$

Figure 2. $\Delta KE = W_{friction} + W_{gravity}$



$$\frac{1}{2} m[(1)^2 - (v_2^2)] = -6\mu\, mg \cos\theta + mgh_1$$

or $$1 - v_2^2 = 2[-6\mu\, g\cos\theta + gh_1]$$

$$= 2[-(6)(0.25)(10)(0.99) + (10)(0.3)]$$

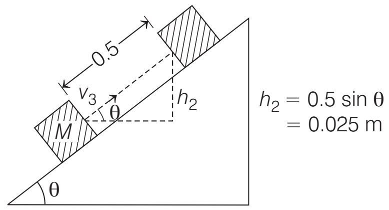
$$= -23.7$$

$$\therefore \qquad v_2^2 = 24.7$$

or $$v_2 \approx 5 \text{ m/s}$$

Figure 3. $\Delta KE = W_{friction} + W_{gravity}$

$$\frac{1}{2} M[0 - v_3^2] = -(0.5)(\mu)(M) g \cos\theta - Mgh_2$$



or $\quad -v_3^2 = -\mu\, g\cos\theta - 2gh_2$

or $\quad v_3^2 = (0.25)(10)(0.99) + 2(10)(0.025)$

or $\quad v_3^2 = 2.975 \quad \Rightarrow \quad \therefore \quad v_3 \approx 1.72\,\text{m/s}$

Now,

(a) Coefficient of restitution

$$= \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$= \frac{v_2 + v_3}{v_1} = \frac{5 + 1.72}{8} = \frac{6.72}{8} \quad \text{or} \quad e \approx 0.84$$

(b) Applying conservation of linear momentum before and after collision

$$2v_1 = Mv_3 - 2v_2$$

$$\therefore \quad M = \frac{2(v_1 + v_2)}{v_3}$$

$$= \frac{2(8+5)}{1.72} = \frac{26}{1.72}$$

$$M \approx 15.12\,\text{kg}$$

33. If v_1 and v_2 are the velocities of object of mass m and block of mass $4m$, just after collision then by conservation of momentum,

$$mv = mv_1 + 4mv_2, \text{ i.e. } v = v_1 + 4v_2 \qquad \ldots\text{(i)}$$

Further, as collision is elastic

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}4mv_2^2, \text{ i.e. } v^2 = v_1^2 + 4v_2^2 \qquad \ldots\text{(ii)}$$

Solving, these two equations we get either

$$v_2 = 0 \quad \text{or} \quad v_2 = \frac{2}{5}v$$

Therefore, $\quad v_2 = \frac{2}{5}v$

Substituting in Eq. (i) $v_1 = \frac{3}{5}v$

when $v_2 = 0$, $v_1 = v_2$, but it is physically unacceptable.

(a) Now, after collision the block B will start moving with velocity v_2 to the right. Since, there is no friction between blocks A and B, the upper block A will stay at its position and will topple if B moves a distance s such that

$$s > 2d \qquad \ldots\text{(iii)}$$

However, the motion of B is retarded by frictional force $f = \mu\,(4m + 2m)g$ between table and its lower surface. So, the distance moved by B till it stops

$$0 = v_2^2 - 2\left(\frac{6\mu mg}{4m}\right)s, \text{ i.e. } s = \frac{v_2^2}{3\mu g}$$

Substituting this value of s in Eq. (iii), we find that for toppling of A

or $\quad v_2^2 > 6\,\mu g d$

$$\frac{2}{5}v > \sqrt{6\mu g d} \quad [\text{as } v_2 = 2v/5]$$

i.e. $\quad v > \frac{5}{2}\sqrt{6\,\mu g d}$

or $\quad v_{\min} = v_0 = \frac{5}{2}\sqrt{6\,\mu g d}$

(b) If $v = 2v_0 = 5\sqrt{6\mu g d}$, the object will rebound with speed

$v_1 = \frac{3}{5}v = 3\sqrt{6\,\mu g d}$

and as time taken by it to fall down

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2d}{g}} \qquad [\text{as } h = d]$$

The horizontal distance moved by it to the left of P in this time $x = v_1 t = 6d\sqrt{3\mu}$

NOTE

- Toppling will take place if line of action of weight does not pass through the base area in contact.
- v_1 and v_2 can be obtained by using the equations of head on elastic collision

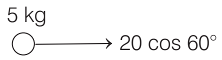
$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_2 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

and $$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

34. Let v_1 and v_2 be the velocities after explosion in the directions shown in figure. From conservation of linear momentum, we have

$$5\,(20\cos 60^\circ) = 4v_1 - 1 \times v_2$$

or $\quad 4v_1 - v_2 = 50 \qquad \ldots\text{(i)}$



Just before explosion

Just after explosion

Further, it is given that, kinetic energy after explosion becomes two times.

Therefore,

$$\frac{1}{2} \times 4 \times v_1^2 + \frac{1}{2} \times 1 \times v_2^2 = 2\left[\frac{1}{2} \times 5 \times (20\cos 60^\circ)^2\right]$$

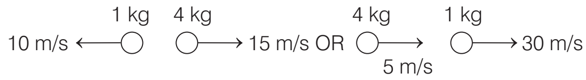
or $\quad 4v_1^2 + v_2^2 = 1000 \qquad \ldots\text{(ii)}$

Solving Eqs. (i) and (ii), we have

$$v_1 = 15\,\text{m/s}, \quad v_2 = 10\,\text{m/s}$$

or $\quad v_1 = 5\,\text{m/s} \quad \text{and} \quad v_2 = -30\,\text{m/s}.$

In both the cases relative velocity of separation in horizontal direction is 25 m/s.



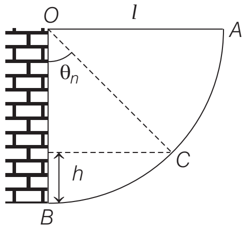
$\therefore$ $x = 25t$ = distance between them when they strike the ground.

Here, $t = \dfrac{T}{2}$ (T = time of flight of projectile)

$$= \frac{u \sin\theta}{g} = \frac{20 \sin 60°}{9.8} = 1.77 \text{s}$$

$\therefore$ $x = 25 \times 1.77 \text{ m} = 44.25 \text{m}$

35. As shown in figure initially when the bob is at A, its potential energy is mgl. When the bob is released and it strikes the wall at B, its potential energy mgl is converted into its kinetic energy. If v be the velocity with which the bob strikes the wall, then



$$mgl = \frac{1}{2} mv^2 \text{ or } v = \sqrt{(2gl)} \quad \ldots\text{(i)}$$

Speed of the bob after rebounding (first time)

$$v_1 = e\sqrt{(2gl)} \quad \ldots\text{(ii)}$$

The speed after second rebound is $v_2 = e^2 \sqrt{(2gl)}$

In general after n rebounds, the speed of the bob is

$$v_n = e^n \sqrt{(2gl)} \quad \ldots\text{(iii)}$$

Let the bob rises to a height h after n rebounds. Applying the law of conservation of energy, we have

$$\frac{1}{2} mv_n^2 = mgh$$

$$\therefore \quad h = \frac{v_n^2}{2g} = \frac{e^{2n} \cdot 2gl}{2g} = e^{2n} \cdot l = \left(\frac{2}{\sqrt{5}}\right)^{2n} \cdot l = \left(\frac{4}{5}\right)^n l \quad \ldots\text{(iv)}$$

If θ_n be the angle after n collisions, then

$$h = l - l \cos\theta_n = l\,(1 - \cos\theta_n)$$

...(v)

From Eqs. (iv) and (v), we have

$$\left(\frac{4}{5}\right)^n l = l\,(1 - \cos\theta_n) \quad \text{or} \quad \left(\frac{4}{5}\right)^n = (1 - \cos\theta_n)$$

For θ_n to be less than 60°, i.e. $\cos\theta_n$ is greater than 1/2, i.e. $(1 - \cos\theta_n)$ is less than 1/2, we have

$$\left(\frac{4}{5}\right)^n < \left(\frac{1}{2}\right)$$

This condition is satisfied for $n = 4$.

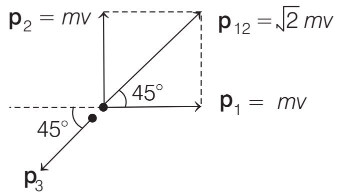
$\therefore$ Required number of collisions = 4.

36. Before collision, net momentum of the system was zero. No external force is acting on the system. Hence, momentum after collision should also be zero. A has come to rest. Therefore, B and C should have equal and opposite momenta or velocity of C should be v in opposite direction of velocity of B.

37. From conservation of linear momentum $\mathbf{p}_3$ should be equal and opposite to $\mathbf{p}_{12}$ (resultant of $\mathbf{p}_1$ and $\mathbf{p}_2$). So, let v' be the velocity of third fragment, then

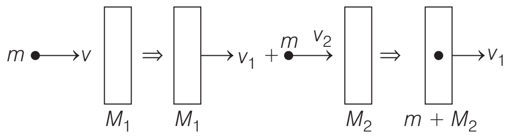
$$(3m)\, v' = \sqrt{2} mv \quad \Rightarrow \quad \therefore \quad v' = \frac{\sqrt{2}}{3} v$$

Here, $v = 30 \text{ m/s}$ (given) $\Rightarrow v' = \dfrac{\sqrt{2}}{3} \times 30 = 10\sqrt{2} \text{ m/s}$



This velocity is at 45° as shown in figure.

38. Applying conservation of linear momentum twice. We have



$$mv = M_1 v_1 + mv_2 \quad \ldots\text{(i)}$$

$$mv_2 = (M_2 + m)\, v_1 \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\frac{v_2}{v} = \frac{M_2 + m}{M_1 + M_2 + m}$$

Substituting the values of $m : M_1$ and M_2 we get percentage of velocity retained by bullet

$$\frac{v_2}{v} \times 100 = \left(\frac{2.98 + 0.02}{1 + 2.98 + 0.02}\right) \times 100 = 75\%$$

$\therefore$ % loss = 25%

39. (a) From conservation of linear momentum, momentum of composite body

$$\mathbf{p} = (\mathbf{p}_i)_1 + (\mathbf{p}_i)_2 = (mv)\,\hat{\mathbf{i}} + (MV)\,\hat{\mathbf{j}}$$

$$\therefore \quad |\mathbf{p}| = \sqrt{(mv)^2 + (MV)^2}$$

Let it makes an angle α with positive X-axis, then

$$\alpha = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}\left(\frac{MV}{mv}\right)$$

(b) Fraction of initial kinetic energy transformed into heat during collision

$$= \frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1 = \frac{p^2/2(M+m)}{\frac{1}{2} mv^2 + \frac{1}{2} MV^2} - 1$$

$$= \frac{(mv)^2 + (MV)^2}{(M+m)(mv^2 + MV^2)} - 1$$

$$= \frac{Mm\,(v^2 + V^2)}{(M+m)(mv^2 + MV^2)}$$

Topic 4 Miscellaneous Problems

1 Initial compression of the spring,

$$mg = k\left(\frac{x_0}{100}\right) \qquad (x_0 \text{ in cm})$$

$$\Rightarrow \qquad x_0 = \frac{3 \times 10 \times 100}{1.25 \times 10^6} = \frac{3}{1250}$$

Which is very small and can be neglected.

Applying conservation of momentum before and after the collision i.e., momentum before collision = momentum after collision.

$m \times \sqrt{2gh} = (m + M)\, v$ ($\because$ velocity of the block just before the collision is

$$v^2 - 0^2 = 2gh$$

or $$v = \sqrt{2gh})$$

After, substituting the given values, we get

$$1 \times \sqrt{2 \times 10 \times 100} = 4v \text{ or } 4v = 20\sqrt{5}$$

so $$v = 5\sqrt{5} \text{ m / s}$$

Let this be the maximum velocity, then for the given system, using

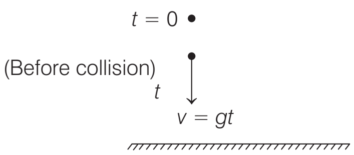
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\therefore \qquad \frac{1}{2} \times 4 \times 125 = \frac{1}{2} \times 1.25 \times 10^6 \times \left(\frac{x}{100}\right)^2$$

$$\Rightarrow \qquad 4 = 10^4 \times \frac{x^2}{10^4} \text{ or } x = 2 \text{ cm}$$

$\therefore$ No option given is correct.

2.



$$K = \frac{1}{2}mg^2t^2$$

$K \propto t^2$ Therefore, K-t graph is parabola.

During collision, retarding force is just like the spring force ($F \propto x$), therefore kinetic energy first decreases to elastic potential energy and then increases.

3. Final momentum of object $= \dfrac{\text{Power} \times \text{time}}{\text{Speed of light}}$

$$= \frac{30 \times 10^{-3} \times 100 \times 10^{-9}}{3 \times 10^8}$$

$$= 1.0 \times 10^{-17} \text{ kg-m/s}$$

4. We can apply $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$ for different parts.

For example :

In part (a), coordinates of A are $\left(\dfrac{R}{\sqrt{2}}, \dfrac{R}{\sqrt{2}}\right)$

Therefore, $\mathbf{r} = \dfrac{R}{\sqrt{2}}\hat{\mathbf{i}} + \dfrac{R}{\sqrt{2}}\hat{\mathbf{j}}$ and $\mathbf{v} = v\hat{\mathbf{i}}$

So, substituting in $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$ we get,

$$\mathbf{L} = -\frac{mvR}{\sqrt{2}}\hat{\mathbf{k}}$$

Hence, option (a) is correct. Similarly, we can check other options also.

5. Linear impulse, $J = mv_0$

$$\therefore \qquad v_0 = \frac{J}{m} = 2.5 \text{ m/s}$$

$$\therefore \qquad v = v_0 e^{-t/\tau}$$

$$\frac{dx}{dt} = v_0 e^{-t/\tau} \qquad \int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$$

$$x = v_0 \left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}}\right]_0^\tau$$

$$x = 2.5\,(-4)\,(e^{-1} - e^0)$$

$$= 2.5(-4)\,(0.37 - 1)$$

$$x = 6.30 \text{ m}$$

6. $F = \dfrac{\Delta p}{\Delta t} = n \times \left(a \times \dfrac{b}{2}\right) \times (2mv)$

Equating the torque about hinge side, we have

$$n \times \left(a \times \frac{b}{2}\right) \times (2mv) \times \frac{3b}{4} = Mg \times \frac{b}{2}$$

Substituting the given values, we get

$$v = 10 \text{ m/s}$$

7. (a) $$x_1 = v_0\, t - A\,(1 - \cos \omega t)$$

$$x_{CM} = \frac{m_1\, x_1 + m_2\, x_2}{m_1 + m_2} = v_0\, t$$

$$\therefore \qquad x_2 = v_0\, t + \frac{m_1}{m_2} A\,(1 - \cos \omega t)$$

(b) $$a_1 = \frac{d^2 x_1}{dt^2} = -\omega^2 A \cos \omega t$$

The separation $x_2 - x_1$ between the two blocks will be equal to l_0 when $a_1 = 0$

or $\cos \omega t = 0$

$$x_2 - x_1 = \frac{m_1}{m_2} A\,(1 - \cos \omega t) + A\,(1 - \cos \omega t)$$

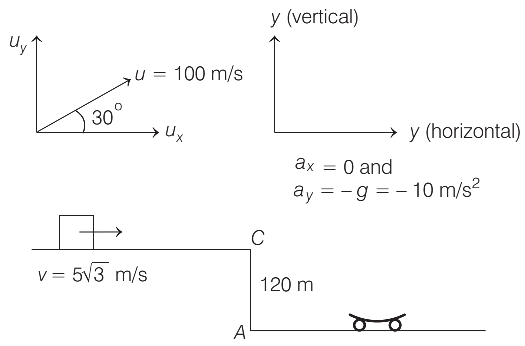
or $$l_0 = \left(\frac{m_1}{m_2} + 1\right) A \quad (\because \cos \omega t = 0)$$

Thus, the relation between l_0 and A is,

$$l_0 = \left(\frac{m_1}{m_2} + 1\right) A$$

8. (a) 100 m/s velocity of the ball is relative to ground.

[Unless and until it is mentioned in the question, the velocity is always relative to ground]



Horizontal component of velocity of cannon ball,

$$u_x = u\cos 30°$$

or $$u_x = (100)\times\frac{\sqrt{3}}{2} = 50\sqrt{3}\text{ m/s}$$

and vertical component of its velocity,

$$u_y = u\sin 30°$$

$$u_y = 100\times\frac{1}{2} = 50\text{m/s}$$

Vertical displacement of the ball when it strikes the carriage is – 120 m or

$$s_y = u_y\, t + \frac{1}{2}a_y\, t^2 \Rightarrow -120 = (50\,t) + \left(\frac{1}{2}\right)(-10)t^2$$

$$\Rightarrow t^2 - 10t - 24 = 0 \Rightarrow \quad t = 12\text{s} \quad \text{or} \quad -2\text{s}$$

Ignoring the negative time, we have

$$t_0 = 12\text{s}$$

(b) When it strikes the carriage, its horizontal component of velocity is still $50\sqrt{3}$ m/s. It strikes to the carriage. Let v_2 be the velocity of (carriage + ball) system after collision. Then, applying conservation of linear momentum in horizontal direction

(mass of ball) (horizontal component of its velocity before collision) = (mass of ball + carriage) (v_2)

$$\therefore \quad (1\text{kg})(50\sqrt{3}\text{ m/s}) = (10\text{kg})(v_2)$$

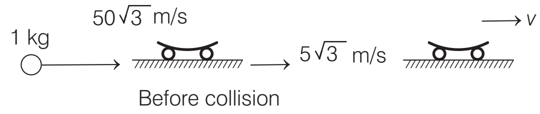
$$\therefore \quad v_2 = 5\sqrt{3}\text{ m/s}$$

The second ball is fired when the first ball strikes the carriage i.e. after 12 s. In these 12 s, the car will move forward a distance of $12v_1$ or $60\sqrt{3}$ m.

The second ball also takes 12 s to travel a vertical displacement of – 120 m. This ball will strike the carriage only when the carriage also covers the same distance of $60\sqrt{3}$ m in these 12 s. This is possible only when resistive forces are zero because velocity of car (v_1) = velocity of carriage after first collision. (v_2) = $5\sqrt{3}$ m/s.

Hence, at the time of second collision

Horizontal component of velocity of ball = $50\sqrt{3}$ m/s and horizontal velocity of carriage + first ball = $5\sqrt{3}$ m/s. Let v be the desired velocity of carriage after second collision. Then, conservation of linear momentum in horizontal direction gives



$$11v = (1)(50\sqrt{3}) + (10)(5\sqrt{3})$$

$$= 100\sqrt{3}$$

$$\therefore \quad v = \frac{100\sqrt{3}}{11}\text{ m/s}$$

or $$v = 15.75\text{ m/s}$$

In this particular problem, values are so adjusted that even if we take the velocity of ball with respect to car, we get the same results of both the parts, although the method will be wrong.

9. Given, $m_0 = 10^{-2}$ kg, $A = 10^{-4}$ m^2, $v_0 = 10^3$ m/s

and $\rho_{dust} = \rho = 10^{-3}$ kg/m^3.

$$m = m_0 + \text{mass of dust collected so far}$$

$$= m_0 + Ax\rho_{dust}$$

or $$m = m_0 + Ax\rho$$

The linear momentum at $t = 0$ is

$$p_0 = m_0 v_0$$

and momentum at $t = t$ is

$$p_t = mv = (m_0 + Ax\rho)\, v$$

From law of conservation of momentum

$$p_0 = p_t$$

$$\therefore \quad m_0 v_0 = (m_0 + Ax\rho)\, v$$

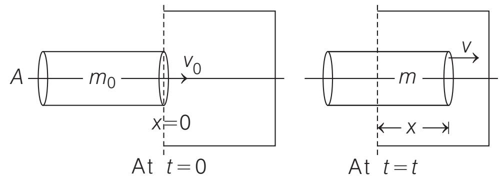
$$\therefore \quad m_0 v_0 = (m_0 + Ax\rho)\frac{dx}{dt}$$

or $$(m_0 + A\,\rho\, x)\, dx = m_0 v_0\, dt$$

or $$\int_0^x (m_0 + A\rho x)\, dx = \int_0^{150} m_0 v_0\, dt$$

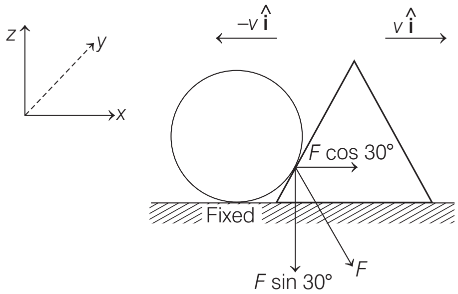
$$\Rightarrow \quad \left(m_0 x + A\rho\frac{x^2}{2}\right)_0^x = (m_0 v_0 t)_0^{150}$$

Hence, $$m_0 x + A\rho\frac{x^2}{2} = 150\, m_0 v_0$$



Solving this quadratic equation and substituting the values of m_0 A, ρ and v_0, we get positive value of x as 10^5m. Therefore, $x = 10^5$m.

10. (a) (i) Since, the collision is elastic, the wedge will return with velocity $v\,\hat{\mathbf{i}}$.



Now, linear impulse in x-direction
= change in momentum in x-direction.

$\therefore\ (F\cos 30°)\,\Delta t = mv - (-mv) = 2mv$

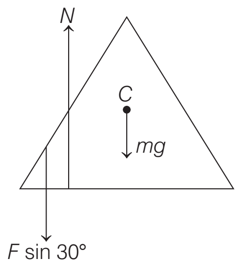
$$\therefore \qquad F = \frac{2mv}{\Delta t\cos 30°} = \frac{4mv}{\sqrt{3}\,\Delta t} \Rightarrow F = \frac{4mv}{\sqrt{3}\,\Delta t}$$

$$\therefore \qquad \mathbf{F} = (F\cos 30°)\,\hat{\mathbf{i}} - (F\sin 30°)\,\hat{\mathbf{k}}$$

$$\text{or} \qquad \mathbf{F} = \left(\frac{2\,mv}{\Delta t}\right)\hat{\mathbf{i}} - \left(\frac{2\,mv}{\sqrt{3}\Delta t}\right)\hat{\mathbf{k}}$$

(ii) Taking the equilibrium of wedge in vertical z-direction during collision.

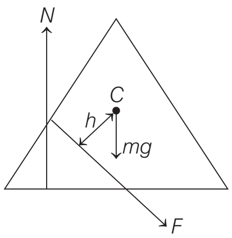
$$N = mg + F\sin 30° \Rightarrow N = mg + \frac{2\,mv}{\sqrt{3}\,\Delta t}$$



or in vector form

$$\mathbf{N} = \left(mg + \frac{2mv}{\sqrt{3}\Delta t}\right)\hat{\mathbf{k}}$$

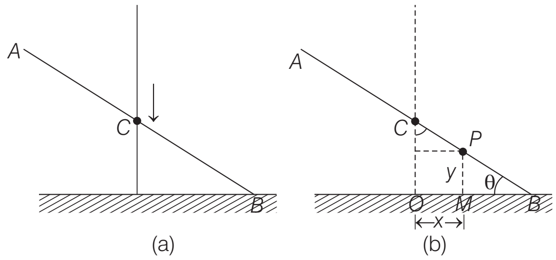
(b) For rotational equilibrium of wedge [about (CM)] anticlockwise torque of F = clockwise torque due to N



$\therefore$ Magnitude of torque of N about CM = magnitude of torque of F about CM

$$= F\,.h\,|\tau_N| = \left(\frac{4\,mv}{\sqrt{3}\,\Delta t}\right)h$$

11. (a) Since, only two forces are acting on the rod, its weight Mg (vertically downwards) and a normal reaction N at point of contact B (vertically upwards).



No horizontal force is acting on the rod (surface is smooth).

Therefore, CM will fall vertically downwards towards negative Y-axis i.e. the path of CM is a straight line.

(b) Refer figure (b). We have to find the trajectory of a point $P\ (x, y)$ at a distance r from end B.

$$CB = L/2$$

$$\therefore \qquad OB = (L/2)\cos\theta;$$

$$MB = r\cos\theta$$

$$\therefore \qquad x = OB - MB$$

$$= \cos\theta\,\{(L/2 - r)\}$$

$$\text{or} \qquad \cos\theta = \frac{x}{\{(L/2) - r\}} \qquad \ldots\text{(i)}$$

Similarly, $\quad y = r\sin\theta$

$$\text{or} \qquad \sin\theta = \frac{y}{r} \qquad \ldots\text{(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$\sin^2\theta + \cos^2\theta = \frac{x^2}{\{(L/2) - r\}^2} + \frac{y^2}{r^2}$$

$$\frac{x^2}{\{(L/2) - r\}^2} + \frac{y^2}{r^2} = 1 \qquad \ldots\text{(iii)}$$

This is an equation of an ellipse. Hence, path of point P is an ellipse whose equation is given by (iii).

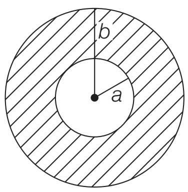
6

Rotation

Topic 1 Moment of Inertia

Objective Questions I (Only one correct option)

1. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **(2019 Main, 12 April I)**



(a) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (b) $\frac{a+b}{2}$

(c) $\sqrt{\frac{a^2+b^2+ab}{3}}$ (d) $\frac{a+b}{3}$

2. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis.

The ratio I_1 / I_2 is given by **(2019 Main, 10 Apri II)**

(a) 285 (b) 185 (c) 65 (d) 140

3. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$, where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is **(2019 Main, 10 April I)**

(a) $\frac{MR^2}{2}$ (b) $\frac{MR^2}{6}$

(c) $\frac{MR^2}{3}$ (d) $\frac{2MR^2}{3}$

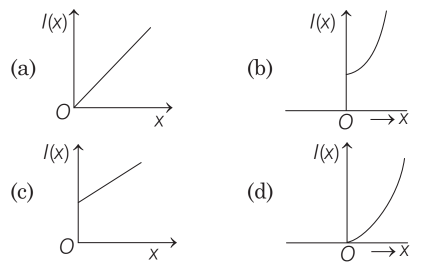
4. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I, then the angular acceleration of the disc is **(2019 Main, 9 April I)**

(a) $\frac{k}{2I}\theta$ (b) $\frac{k}{I}\theta$ (c) $\frac{k}{4I}\theta$ (d) $\frac{2k}{I}\theta$

5. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is **(2019 Main, 8 April I)**

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{8}{5}$ (d) $\frac{3}{2}$

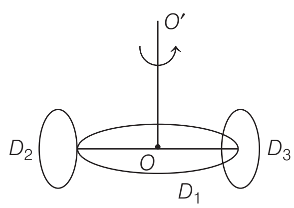
6. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with x correctly? **(2019 Main, 12 Jan II)**



7. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm) about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is **(2019 Main, 12 Jan I)**

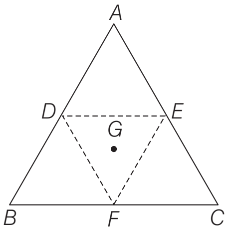
(a) 16 cm (b) 14 cm (c) 12 cm (d) 18 cm

8. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1, as shown in the figure will be **(2019 Main, 11 Jan II)**



(a) $\frac{2}{3}MR^2$ (b) $\frac{4}{5}MR^2$ (c) $3MR^2$ (d) MR^2

9. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I.

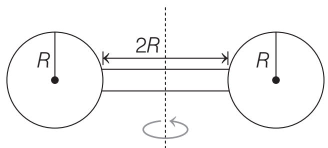


Then **(2019 Main, 11 Jan I)**

(a) $I' = \frac{3}{4}I_0$ (b) $I' = \frac{15}{16}I_0$ (c) $I' = \frac{I_0}{4}$ (d) $I' = \frac{9}{16}I_0$

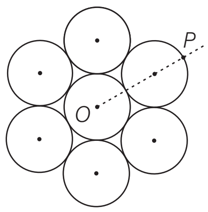
10. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure).

The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is **(2019 Main, 10 Jan II)**



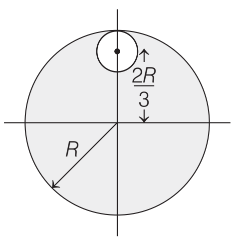
(a) $\frac{137}{15}MR^2$ (b) $\frac{209}{15}MR^2$ (c) $\frac{17}{15}MR^2$ (d) $\frac{152}{15}MR^2$

11. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about an axis normal to the plane and passing through the point P is **(2018 Main)**



(a) $\frac{181}{2}MR^2$ (b) $\frac{19}{2}MR^2$ (c) $\frac{55}{2}MR^2$ (d) $\frac{73}{2}MR^2$

12. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **(2018 Main)**



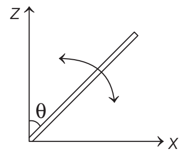
(a) $\frac{37}{9}MR^2$ (b) $4MR^2$

(c) $\frac{40}{9}MR^2$ (d) $10MR^2$

13. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum? **(2017 Main)**

(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{2}}$

14. A cylinder uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see the figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical, is **(2017 Main)**

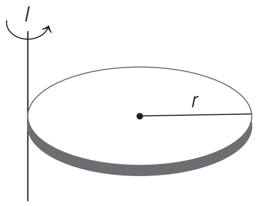


(a) $\frac{2g}{3l}\sin\theta$ (b) $\frac{3g}{2l}\cos\theta$ (c) $\frac{2g}{3l}\cos\theta$ (d) $\frac{3g}{2l}\sin\theta$

15. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is **(2015 Main)**

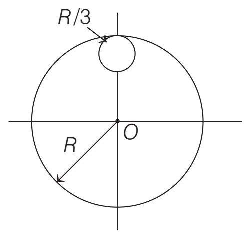
(a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{4MR^2}{9\sqrt{3}\pi}$ (c) $\frac{MR^2}{16\sqrt{2}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

16. A solid sphere of radius R has moment of inertia I about its geometrical axis. It is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to **(2006, 3M)**



(a) $\frac{2}{\sqrt{15}}R$ (b) $\frac{2}{\sqrt{5}}R$ (c) $\frac{3}{\sqrt{15}}R$ (d) $\frac{\sqrt{3}}{\sqrt{15}}R$

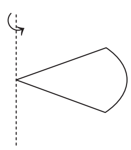
17. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is **(2005)**



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$

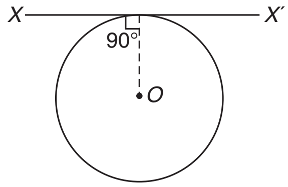
(c) $10MR^2$ (d) $\frac{37}{9}MR^2$

18. One quarter section is cut from a uniform circular disc of radius R. This section has a mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is **(2001)**



(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$ (c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}MR^2$

19. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is **(2000)**



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$ (c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

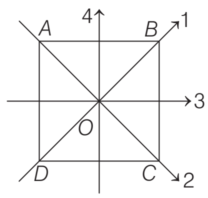
20. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to **(1998, 2M)**

(a) I (b) $I \sin^2 \theta$

(c) $I \cos^2 \theta$ (d) $I \cos^2 (\theta/2)$

Objective Question II (One or more correct option)

21. The moment of inertia of a thin square plate $ABCD$, of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is **(1992, 2M)**



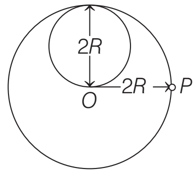
(a) $I_1 + I_2$ (b) $I_3 + I_4$

(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$

where, I_1, I_2, I_3 and I_4 are respectively moments of inertia about axes 1, 2, 3 and 4 which are in the plane of the plate.

Integer Answer Type Questions

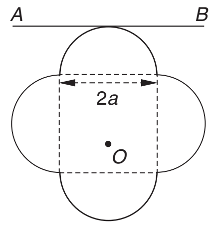
22. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is **(2012)**



23. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is **(2011)**

Fill in the Blank

24. A symmetric lamina of mass M consists of a square shape with a semi-circular section over of the edge of the square as shown in figure. The side of the square is $2a$. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is $1.6\ Ma^2$. The moment of inertia of the lamina about the tangent AB in the plane of the lamina is ……… **(1997, 2M)**



Topic 2 Angular Momentum and its Conservation

Objective Questions I (Only one correct option)

1. The time dependence of the position of a particle of mass $m = 2$ is given by $\mathbf{r}(t) = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$. Its angular momentum, with respect to the origin, at time $t = 2$ is

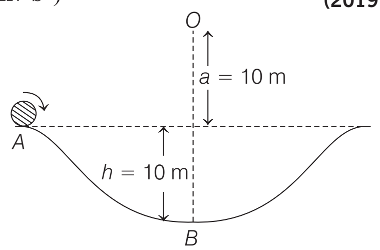
(a) $36\,\hat{\mathbf{k}}$ (b) $-34\,(\hat{\mathbf{k}} - \hat{\mathbf{i}})$

(c) $-48\,\hat{\mathbf{k}}$ (d) $48\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

2. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its centre. Two beads of mass m and negligible size are at the centre of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be **(2019 Main, 09 April II)**

(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+m}$ (c) $\frac{M\omega_0}{M+2m}$ (d) $\frac{M\omega_0}{M+6m}$

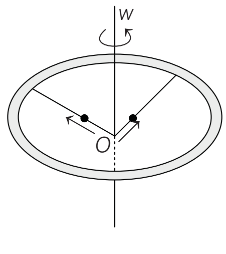
3. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take, $g = 10\ \text{m/s}^2$) **(2019 Main, 12 Jan II)**



(a) $8\ \text{kg-m}^2/\text{s}$ (b) $3\ \text{kg-m}^2/\text{s}$

(c) $2\ \text{kg-m}^2/\text{s}$ (d) $6\ \text{kg-m}^2/\text{s}$

4. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure.



At some instant, the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant, the distance of the other mass from O is **(2015 Adv.)**

(a) $\frac{2}{3}R$ (b) $\frac{1}{3}R$

(c) $\frac{3}{5}R$ (d) $\frac{4}{5}R$

5. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical support. About the point of suspension **(2014 Main)**

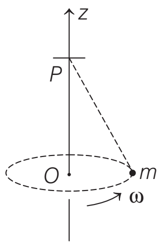
(a) angular momentum is conserved

(b) angular momentum changes in magnitude but not in direction

(c) angular momentum changes in direction but not in magnitude

(d) angular momentum changes both in direction and magnitude

6. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\mathbf{L}_O$ and $\mathbf{L}_P$ respectively, then



(2012)

(a) $\mathbf{L}_O$ and $\mathbf{L}_P$ do not vary with time

(b) $\mathbf{L}_O$ varies with time while $\mathbf{L}_P$ remains constant

(c) $\mathbf{L}_O$ remains constant while $\mathbf{L}_P$ varies with time

(d) $\mathbf{L}_O$ and $\mathbf{L}_P$ both vary with time

7. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is **(2004)**

(a) $2K$ (b) $K/2$ (c) $K/4$ (d) $4K$

8. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved ? **(2003)**

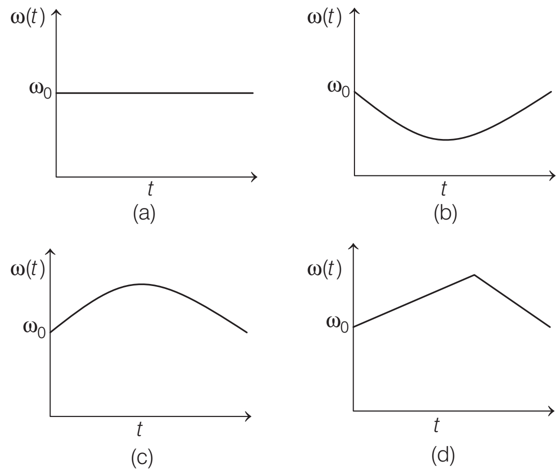
(a) Centre of circle

(b) On the circumference of the circle

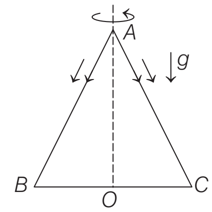
(c) Inside the circle

(d) Outside the circle

9. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as **(2002)**



10. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and other along AC as shown. Neglecting frictional effects, the quantities that are conserved as beads slides down are **(2000)**



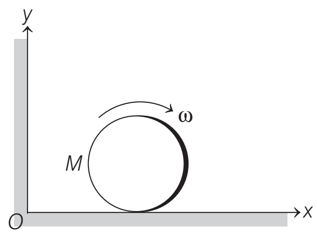
(a) angular velocity and total energy (kinetic and potential)

(b) total angular momentum and total energy

(c) angular velocity and moment of inertia about the axis of rotation

(d) total angular momentum and moment of inertia about the axis of rotation

11. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is **(1999, 2M)**



(a) $\left(\frac{1}{2}\right)MR^2\omega$ (b) $MR^2\omega$

(c) $\left(\frac{3}{2}\right)MR^2\omega$ (d) $2MR^2\omega$

12. A mass m is moving with a constant velocity along a line parallel to the X-axis, away from the origin. Its angular momentum with respect to the origin **(1997C, 1M)**

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing

13. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **(1990, 2M)**

(a) zero (b) $mv^3/(4\sqrt{2}\,g)$
(c) $mv^3/(\sqrt{2}\,g)$ (d) $m\sqrt{2gh^3}$

14. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity **(1983, 1M)**

(a) $\omega M/(M+m)$ (b) $\omega (M-2m)/(M+2m)$
(c) $\omega M/(M+2m)$ (d) $\omega (M+2m)/M$

Objective Question II (One or more correct option)

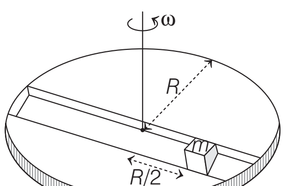
15. The torque τ on a body about a given point is found to be equal to $\mathbf{A}\times\mathbf{L}$, where $\mathbf{A}$ is a constant vector and $\mathbf{L}$ is the angular momentum of the body about that point. From this it follows that **(1998, 2M)**

(a) $\dfrac{d\mathbf{L}}{dt}$ is perpendicular to $\mathbf{L}$ at all instants of time
(b) the component of $\mathbf{L}$ in the direction of $\mathbf{A}$ does not change with time
(c) the magnitude of $\mathbf{L}$ does not change with time
(d) $\mathbf{L}$ does not change with time

Passage Based Questions

Passage 1

A frame of the reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\mathbf{F}_{\text{rot}}$ experienced by a particle of mass m moving on the rotating disc and the force $\mathbf{F}_{\text{in}}$ experienced by the particle in an inertial frame of reference is, $\mathbf{F}_{\text{rot}} = \mathbf{F}_{\text{in}} + 2m(\mathbf{v}_{\text{rot}}\times\vec{\omega}) + m(\vec{\omega}\times\mathbf{r})\times\vec{\omega}$,



where, $\mathbf{v}_{\text{rot}}$ is the velocity of the particle in the rotating frame of reference and $\mathbf{r}$ is the position vector of the particle with respect to the centre of the disc. Now, consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its centre. We assign a coordinate system with the origin at the centre of the disc, the X-axis along the slot, the Y-axis perpendicular to the slot and the z-axis along th rotation axis ($\omega = \omega\mathbf{k}$). A small block of mass m is gently placed in the slot at $\mathbf{r} = (R/2)\hat{\mathbf{i}}$ at $t = 0$ and is constrained to move only along the slot. **(2016 Adv.)**

16. The distance r of the block at time t is

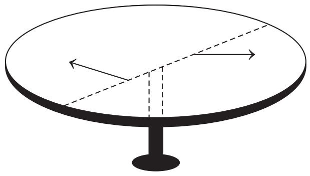
(a) $\dfrac{R}{2}\cos 2\omega t$ (b) $\dfrac{R}{2}\cos\omega t$
(c) $\dfrac{R}{4}(e^{\omega t} + e^{-\omega t})$ (d) $\dfrac{R}{4}(e^{2\omega t} + e^{-2\omega t})$

17. The net reaction of the disc on the block is

(a) $m\omega^2 R\sin\omega t\hat{\mathbf{j}} - mg\hat{\mathbf{k}}$
(b) $\dfrac{1}{2}m\omega^2 R\,(e^{\omega t} - e^{-\omega t})\hat{\mathbf{j}} + mg\hat{\mathbf{k}}$
(c) $\dfrac{1}{2}m\omega^2 R\,(e^{2\omega t} - e^{-2\omega t})\hat{\mathbf{j}} + mg\hat{\mathbf{k}}$
(d) $-m\omega^2 R\cos\omega t\hat{\mathbf{j}} - mg\hat{\mathbf{k}}$

Integer Answer Type Questions

18. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of $9\,\text{ms}^{-1}$ with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **(2014 Adv.)**



19. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad/s about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **(2013 Adv.)**

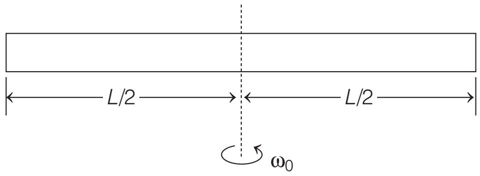
20. A binary star consists of two stars A (mass $2.2M_S$) and B (mass $11M_S$), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is **(2010)**

Fill in the Blanks

21. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle. (Neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle. Then $n =$ …… . **(1993, 1M)**

22. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially, the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpendicular to the rod and passing through the mid-point of the rod (see figure).

There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is…… . **(1988, 2M)**



True/False

23. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{5}$. **(1986, 3M)**

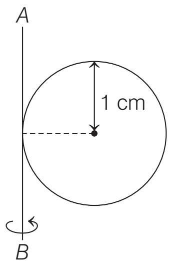
Analytical & Descriptive Questions

24. A particle is projected at time $t = 0$ from a point P on the ground with a speed v_0, at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t = \dfrac{v_0}{g}$. **(1984, 6M)**

Topic 3 Pure Rolling or Rolling without Slipping

Objective Questions I (Only one correct option)

1 A metal coin of mass 5g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to **(2019 Main, 10 April II)**



(a) 4.0×10^{-6} N-m (b) 2.0×10^{-5} N-m

(c) 1.6×10^{-5} N-m (d) 7.9×10^{-6} N-m

2 Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of **(2019 Main, 9 April II)**

(a) 5 s (b) 2 s (c) 3 s (d) 2.5 s

3 The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $R/2$ and (iii) a solid sphere of radius $R/4$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum height they climb is **(2019 Main, 9 April I)**

(a) 10 : 15 : 7 (b) 4 : 3 : 2

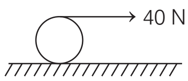
(c) 14 : 15 : 20 (d) 2 : 3 : 4

4 A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\dfrac{h_{sph}}{h_{cyl}}$ is given by

(2019 Main, 8 Ap[ril II)

(a) $\dfrac{2}{\sqrt{5}}$ (b) $\dfrac{14}{15}$ (c) 1 (d) $\dfrac{4}{5}$

5 A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)



(2019 Main, 11 Jan II)

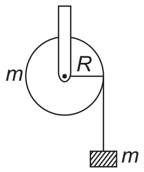
(a) 10 rad/ s^2 (b) 16 rad/ s^2

(c) 20 rad/ s^2 (d) 12 rad/ s^2

6 A homogeneous solid cylindrical roller of radius R and mass m is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is **(2019 Main, 10 Jan I)**

(a) $\frac{F}{2\,m\,R}$ (b) $\frac{2F}{3\,m\,R}$ (c) $\frac{3F}{2\,m\,R}$ (d) $\frac{F}{3\,m\,R}$

7. A mass m supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **(2014 Main)**

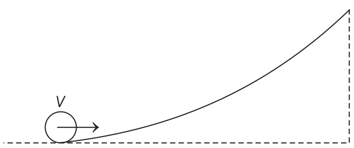


(a) $2g/3$ (b) $g/2$
(c) $5g/6$ (d) g

8. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? **(2012)**

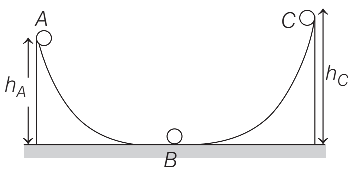
(a) Both cylinders P and Q reach the ground at the same time
(b) Cylinder P has larger linear acceleration than cylinder Q
(c) Both cylinders reach the ground with same translational kinetic energy
(d) Cylinder Q reaches the ground with larger angular speed

9. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is **(2007, 3M)**



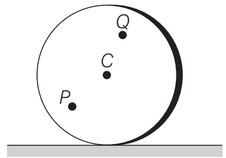
(a) ring (b) solid sphere
(c) hollow sphere (d) disc

10. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. If surface BC is frictionless and K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively, then **(2006, 5M)**



(a) $h_A > h_C$; $K_B > K_C$ (b) $h_A > h_C$; $K_C > K_A$
(c) $h_A = h_C$; $K_B = K_C$ (d) $h_A < h_C$; $K_B > K_C$

11. A disc is rolling (without slipping) on a horizontal surface. C is its centre and Q and P are two points equidistant from C. Let v_P, v_Q and v_C be the magnitude of velocities of points P, Q and C respectively, then **(2004)**



(a) $v_Q > v_C > v_P$ (b) $v_Q < v_C < v_P$
(c) $v_Q = v_P$, $v_C = \frac{1}{2}v_P$ (d) $v_Q < v_C > v_P$

12. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are **(2002)**

(a) up the incline while ascending and down the incline while descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending

Assertion and Reason

Mark your answer as

(*a*) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*
(*b*) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*
(*c*) *If Statement I is true; Statement II is false*
(*d*) *If Statement I is false; Statement II is true*

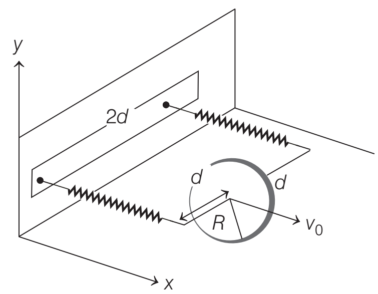
13. **Statement I** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Statement II By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline. **(2008, 3M)**

Passage Based Questions

Passage 1

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk diammetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L.



The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\mathbf{v}_0 = v_0\hat{\mathbf{i}}$. The coefficient of friction is μ.

14. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is **(2008, 4M)**

(a) $-kx$ (b) $-2kx$ (c) $-\frac{2kx}{3}$ (d) $-\frac{4kx}{3}$

15. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to **(2008 , 4M)**

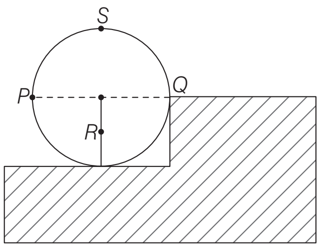
(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$ (c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$

16. The maximum value of v_0 for which the disk will roll without slipping is **(2008, 4M)**

(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$ (c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$

Objective Questions II (One or more correct option)

17. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct? **(2017 Adv.)**



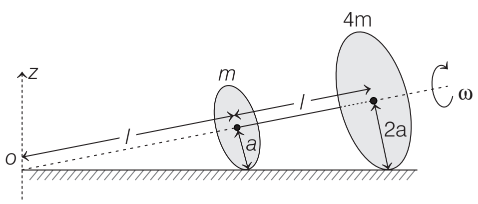
(a) If the force is applied normal to the circumference at point P, then τ is zero

(b) If the force is applied tangentially at point S, then $\tau \neq 0$ but the wheel never climbs the step

(c) If the force is applied at point P tangentially, then τ decreases continuously as the wheel climbs

(d) If the force is applied normal to the circumference at point X, then τ is constant

18. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}\,a$ through their centers. This assembly is laid on a firm and flat surface and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\mathbf{L}$ (see the figure). Which of the following statement(s) is (are) true? **(2016 Adv.)**



(a) The magnitude of the z-component of $\mathbf{L}$ is $55\,ma^2\omega$

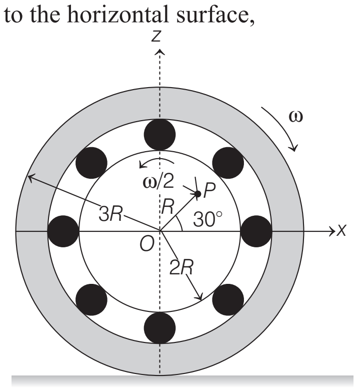
(b) The magnitude of angular momentum of centre of mass of the assembly about the point O is $81\,ma^2\omega$

(c) The centre of mass of the assembly rotates about the Z-axis with an angular speed of $\frac{\omega}{5}$

(d) The magnitude of angular momentum of the assembly about its centre of mass is $17\,ma^2\,\frac{\omega}{2}$

19. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed $\omega/2$.

The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, **(2012)**



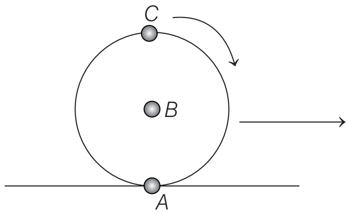
(a) the point O has a linear velocity $3R\omega\,\hat{\mathbf{i}}$

(b) the point P has a linear velocity $\frac{11}{4}R\omega\,\hat{\mathbf{i}} + \frac{\sqrt{3}}{4}R\omega\,\hat{\mathbf{k}}$

(c) the point P has a linear velocity $\frac{13}{4}R\omega\,\hat{\mathbf{i}} - \frac{\sqrt{3}}{4}R\omega\,\hat{\mathbf{k}}$

(d) the point P has a linear velocity
$\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\,\hat{\mathbf{i}} + \frac{1}{4}R\omega\,\hat{\mathbf{k}}$

20. A sphere is rolling without slipping on a fixed horizontal plane surface.

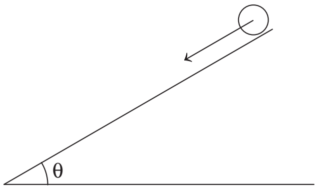


In the figure, A is the point of contact. B is the centre of the sphere and C is its topmost point. Then, **(2009)**

(a) $\mathbf{v}_C - \mathbf{v}_A = 2(\mathbf{v}_B - \mathbf{v}_C)$ (b) $\mathbf{v}_C - \mathbf{v}_B = \mathbf{v}_B - \mathbf{v}_A$

(c) $|\mathbf{v}_C - \mathbf{v}_A| = 2|\mathbf{v}_B - \mathbf{v}_C|$ (d) $|\mathbf{v}_C - \mathbf{v}_A| = 4|\mathbf{v}_B|$

21. A solid sphere is in pure rolling motion on an inclined surface having inclination θ **(2006, 5M)**



(a) frictional force acting on sphere is $f = \mu mg \cos\theta$
(b) f is dissipative force
(c) friction will increase its angular velocity and decrease its linear velocity
(d) If θ decreases, friction will decrease

Numerical Value Based Question

22. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is .... (Take, $g = 10\,\text{ms}^{-2}$) **(2018 Main)**

Fill in the Blanks

23. A uniform disc of mass m and radius R is rolling up a rough inclined plane which makes an angle of 30° with the horizontal. If the coefficients of static and kinetic friction are each equal to μ and the only forces acting are gravitational and frictional, then the magnitude of the frictional force acting on the disc is ……… and its direction is ……… (write up or down) the inclined plane. **(1997C, 1M)**

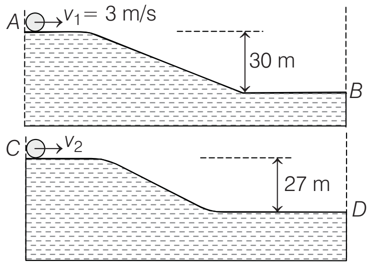
24. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x-y plane) with its axis fixed along the Y-axis and free to rotate about its axis. The platform is given a motion in the x-direction given by $x = A\cos(\omega t)$. There is no slipping between the cylinder and platform. The maximum torque acting on the cylinder during its motion is ……… . **(1988, 2M)**

True / False

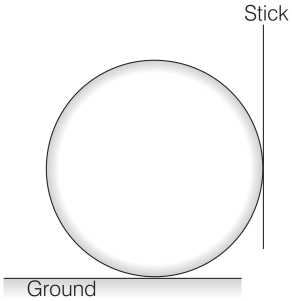
25. A ring of mass 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius are given the same kinetic energy and released simultaneously on a flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from the ring and the cylinder. The rolling friction in both cases is negligible. The cylinder will reach the wall first. **(1989, 2M)**

Integer Answer Type Question

26. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10\,\text{m/s}^2$) **(2015 Adv.)**



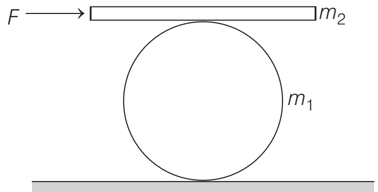
27. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $\left(\frac{P}{10}\right)$. The value of P is **(2011)**



Analytical & Descriptive Questions

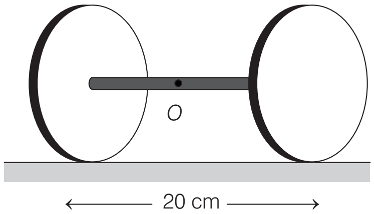
28. A solid cylinder rolls without slipping on an inclined plane inclined at an angle θ. Find the linear acceleration of the cylinder. Mass of the cylinder is M. **(2005, 4M)**

29. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find **(1999, 10M)**



(a) the accelerations of the plank and the centre of mass of the cylinder and
(b) the magnitudes and directions of frictional forces at contact points.

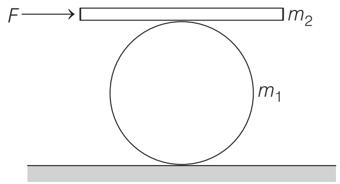
30. Two thin circular discs of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disc through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck.



Its friction with the floor of the truck is large enough, so that the object can roll on the truck without slipping. Take X-axis as the direction of motion of the truck and Z-axis as the vertically upwards direction. If the truck has an acceleration 9 m/s^2, calculate **(1997, 5M)**

(a) the force of friction on each disc and
(b) the magnitude and direction of the frictional torque acting on each disc about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ in x, y and z-directions.

31. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0 m above the ground level and the top of the track is 2.6 m above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in figure) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass ? Explain. **(1987, 7M)**



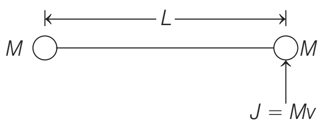
Topic 4 Collision in Rotational Motion

Objective Questions I (Only one correct option)

1 Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$ are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is **(2019 Main, 10 April I)**

(a) $-\frac{I_1\omega_1^2}{24}$ (b) $-\frac{I_1\omega_1^2}{12}$
(c) $\frac{3}{8} I_1\omega_1^2$ (d) $\frac{I_1\omega_1^2}{6}$

2. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its end, what would be its angular velocity ? **(2003)**

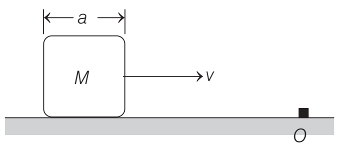


(a) v/L (b) $2v/L$ (c) $v/3L$ (d) $v/4L$

3. A smooth sphere A is moving on a frictionless horizontal plane with angular velocity ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision their angular speeds are ω_A and ω_B respectively. Then, **(1999, 2M)**

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$ (c) $\omega_A = \omega$ (d) $\omega_B = \omega$

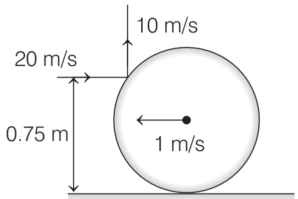
4. A cubical block of side a moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is **(1999, 2M)**



(a) $3v/4a$ (b) $3v/2a$
(c) $\sqrt{3}/\sqrt{2}\,a$ (d) zero

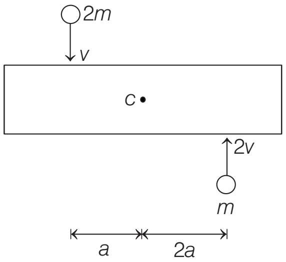
Objective Questions II (One or more correct option)

5. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision, **(2011)**



(a) the ring has pure rotation about its stationary CM
(b) the ring comes to a complete stop
(c) friction between the ring and the ground is to the left
(d) there is no friction between the ring and the ground

6. A uniform bar of length 6 a and mass 8 m lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speed $2v$ and v respectively, strike the bar [as shown in the figure] and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_c respectively, we have after collision **(1991)**



(a) $v_c = 0$ (b) $\omega = \dfrac{3v}{5a}$ (c) $\omega = \dfrac{v}{5a}$ (d) $E = \dfrac{3}{5}mv^2$

Analytical Answer Type Questions

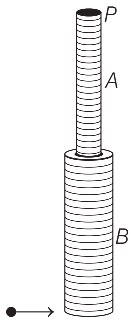
7. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end A of the rod with a velocity v_0 in a direction perpendicular to AB. The collision is elastic. After the collision, the particle comes to rest. **(2000)**

(a) Find the ratio m/M.

(b) A point P on the rod is at rest immediately after collision. Find the distance AP.

(c) Find the linear speed of the point P a time $\pi L/3v_0$ after the collision.

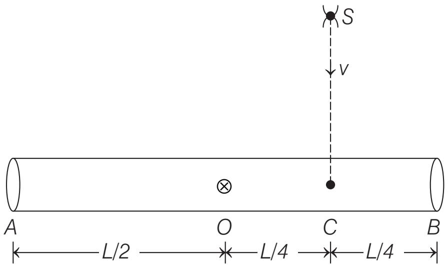
8. Two uniform rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg, respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in figure. Such that it can freely rotate about point P in a vertical plane.



A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it. What should be the velocity of the object, so that the system could just be raised to the horizontal position? **(1994, 6M)**

9. A homogeneous rod AB of length $L = 1.8$ m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (figure).

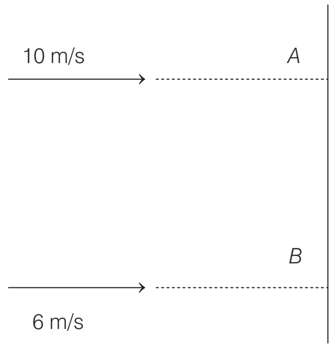
The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed v on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω. **(1992, 8M)**



(a) Determine the angular velocity ω in terms of v and L.

(b) If the insect reaches the end B when the rod has turned through an angle of $90°$, determine v.

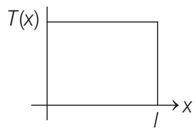
10. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length is $\sqrt{3}$ m. Two particles, each of mass 0.08 kg are moving on the same surface and towards the bar in a direction perpendicular to the bar one with a velocity of 10 m/s, and the other with 6 m/s, as shown in figure. The first particle strikes the bar at points A and the other at point B. Points A and B are at a distance of 0.5 m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of kinetic energy of the system in the above collision process. **(1991)**

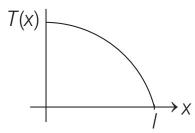


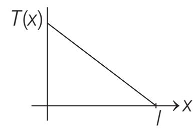
Topic 5 Miscellaneous Problems

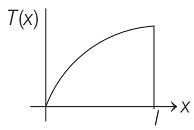
Objective Questions I (Only one correct option)

1 A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a distance x from the axis, then which of the following graphs depicts it most closely? **(2019 Main, 12 April I)**

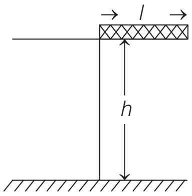
(a)


(b)


(c)


(d)


2 A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to **(2019 Main, 8 April II)**



(a) 0.02 (b) 0.3 (c) 0.5 (d) 0.28

3 A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass m at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by **(2019 Main, 12 Jan I)**

(a) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$

(b) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$

(c) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

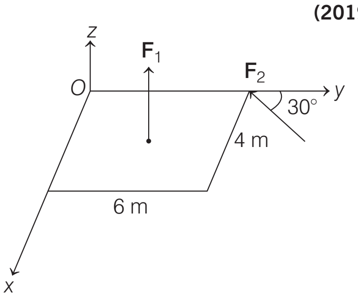
(d) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

4 The magnitude of torque on a particle of mass 1 kg is 2.5 N-m about the origin. If the force acting on it is 1 N and the distance of the particle from the origin is 5 m, then the angle between the force and the position vector is (in radian) **(Main 2019, 11 Jan II)**

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

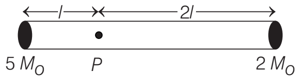
5 A slab is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ of same magnitude F as shown in the figure. Force $\mathbf{F}_2$ is in xy-plane while force $\mathbf{F}_1$ acts along Z-axis at the point $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$. The moment of these forces about point O will be **(2019 Main, 11 Jan I)**



(a) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (b) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

(c) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (d) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

6 A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be **(2019 Main, 10 Jan II)**

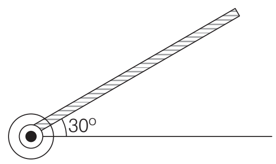


(a) $\frac{g}{13l}$ (b) $\frac{g}{2l}$ (c) $\frac{7g}{3l}$ (d) $\frac{g}{3l}$

7 To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque applied by the machine on the mop is **(2019 Main, 10 Jan I)**

(a) $\frac{2}{3}\mu FR$ (b) $\frac{\mu FR}{6}$ (c) $\frac{\mu FR}{3}$ (d) $\frac{\mu FR}{2}$

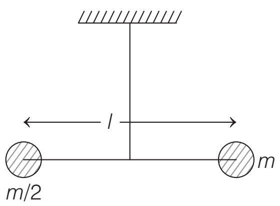
8 A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be
(Take, $g = 10\ ms^{-2}$) **(2019 Main, 09 Jan II)**



(a) $\frac{\sqrt{30}}{2}$ (b) $\sqrt{30}$

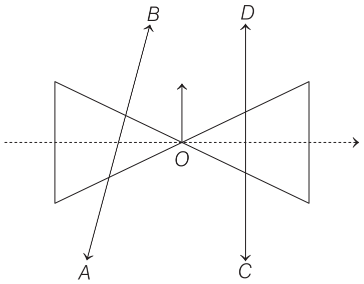
(c) $\frac{\sqrt{20}}{3}$ (d) $\sqrt{\frac{30}{2}}$

9 Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be **(2019 Main, 9 Jan I)**



(a) $\frac{2k\theta_0^2}{l}$ (b) $\frac{k\theta_0^2}{l}$ (c) $\frac{3k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

10. A roller is made by joining together two corners at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see the figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see the figure). It is given a light push, so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to **(2016 Main)**

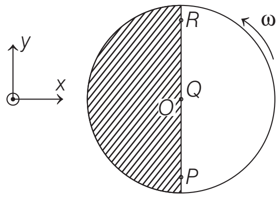


(a) turn left (b) turn right
(c) go straight (d) turn left and right alternately

11. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **(2013 Main)**

(a) $r\omega_0/4$ (b) $r\omega_0/3$ (c) $r\omega_0/2$ (d) $r\omega_0$

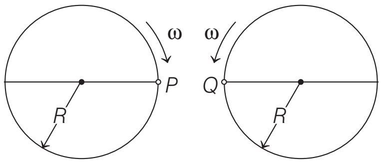
12. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O.



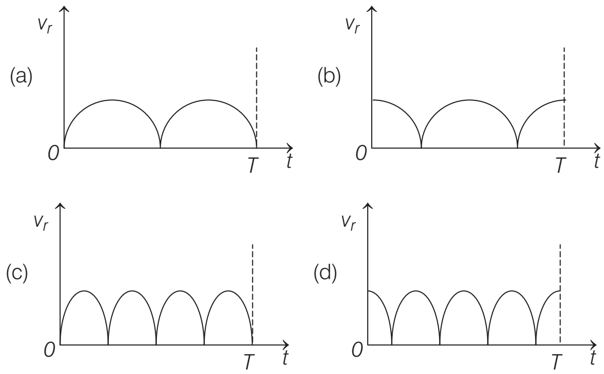
The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc.
Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **(2012)**

(a) P lands in the shaded region and Q in the unshaded region
(b) P lands in the unshaded region and Q in the shaded region
(c) both P and Q land in the unshaded region
(d) both P and Q land in the shaded region

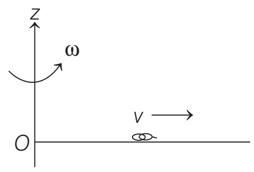
13. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane.



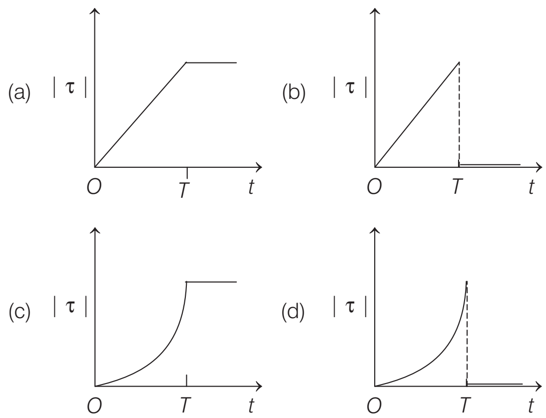
At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **(2012)**



14. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. **(2012)**



The magnitude of the torque $|\tau|$ on the system about O, as a function of time is best represented by which plot?

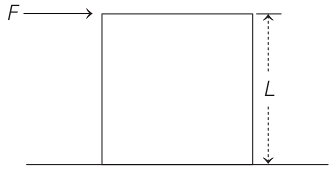


15. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then, **(2009)**

(a) at $\theta = 30°$, the block will start sliding down the plane
(b) the block will remain at rest on the plane up to certain θ and then it will topple
(c) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles
(d) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

16. A particle moves in a circular path with decreasing speed. Choose the correct statement. **(2005)**
(a) Angular momentum remains constant
(b) Acceleration ($\mathbf{a}$) is towards the centre
(c) Particle moves in a spiral path with decreasing radius
(d) The direction of angular momentum remains constant

17. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high, so that the block does not slide before toppling, the minimum force required to topple the block is **(2000)**



(a) infinitesimal (b) $mg/4$
(c) $mg/2$ (d) $mg\,(1-\mu)$

18. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of
(a) 0.42 m from mass of 0.3 kg **(1995, S)**
(b) 0.70 m from mass of 0.7 kg
(c) 0.98 m from mass of 0.3 kg
(d) 0.98 m from mass of 0.7 kg

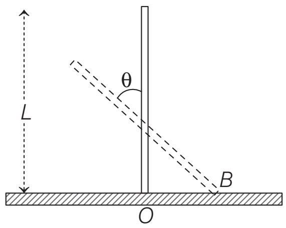
19. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is **(1992, 2M)**
(a) $\dfrac{M\omega^2 L}{2}$ (b) $M\omega^2 L$ (c) $\dfrac{M\omega^2 L}{4}$ (d) $\dfrac{M\omega^2 L^2}{2}$

Objective Question II (One or more correct option)

20. The potential energy of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ? **(2018 Adv.)**
(a) $v = \sqrt{\dfrac{k}{2m}}\,R$ (b) $v = \sqrt{\dfrac{k}{m}}\,R$
(c) $L = \sqrt{mk}\,R^2$ (d) $L = \sqrt{\dfrac{mk}{2}}\,R^2$

21. Consider a body of mass 1.0 kg at rest at the origin at time $t = 0$. A force $\mathrm{F} = (\alpha t\,\hat{\mathrm{i}} + \beta\,\hat{\mathrm{j}})$ is applied on the body, where $\alpha = 1.0\,\mathrm{Ns}^{-1}$ and $\beta = 1.0\mathrm{N}$. The torque acting on the body about the origin at time $t = 1.0\mathrm{s}$ is τ. Which of the following statements is (are) true ? **(2108 Adv.)**
(a) $|\tau| = \dfrac{1}{3}\,\mathrm{N\text{-}m}$
(b) The torque τ is in the direction of the unit vector $+\hat{k}$
(c) The velocity of the body at $t = 1\,\mathrm{s}$ is $\mathrm{v} = \dfrac{1}{2}(\hat{\mathrm{i}} + 2\hat{\mathrm{j}})\ \mathrm{ms}^{-1}$
(d) The magnitude of displacement of the body at $t = 1\,\mathrm{s}$ is $\dfrac{1}{6}\mathrm{m}$

22. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct? **(2017 Adv.)**



(a) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$
(b) The trajectory of the point A is parabola
(c) The mid-point of the bar will fall vertically downward
(d) When the bar makes an angle θ with the vertical, the displacement of its mid-point from the initial position is proportional to $(1 - \cos\theta)$

Matching Type Questions

23. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: p is the linear momentum, L is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are conserved for that path. **(2018 Adv.)**

	List-I		List-II
P.	$\mathrm{r}(t) = \alpha\,t\,\mathrm{i} + \beta\,t\,\mathrm{j}$	1.	p
Q.	$\mathrm{r}(t) = \alpha\cos\omega t\,\mathrm{i} + \beta\sin\omega t\,\mathrm{j}$	2.	L
R.	$\mathrm{r}(t) = \alpha\,(\cos\omega t\,\mathrm{i} + \sin\omega t\,\mathrm{j})$	3.	K
S.	$\mathrm{r}(t) = a\,t\,\mathrm{i} + \dfrac{\beta}{2}t^2\,\mathrm{j}$	4.	U
		5.	E

(a) P → 1, 2, 3, 4, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 5
(b) P → 1, 2, 3, 4, 5; Q → 3, 5; R → 2, 3, 4, 5; S → 2, 5
(c) P → 2, 3, 4; Q → 5; R → 1, 2, 4; S → 2, 5
(d) P → 1, 2, 3, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 2, 5

Passage Based Questions

Passage 1

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

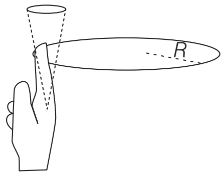


Figure 1

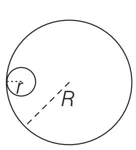


Figure 2

24. The total kinetic energy of the ring is **(2017 Adv.)**

(a) $M\omega_0^2(R-r)^2$ (b) $\frac{1}{2}M\omega_0^2(R-r)^2$

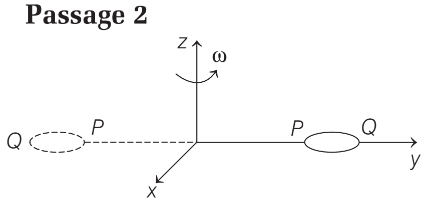
(c) $M\omega_0^2R^2$ (d) $\frac{3}{2}M\omega_0^2(R-r)^2$

25. The minimum value of ω_0 below which the ring will drop down is **(2017 Adv.)**

(a) $\sqrt{\dfrac{g}{2\mu(R-r)}}$ (b) $\sqrt{\dfrac{3g}{2\mu(R-r)}}$

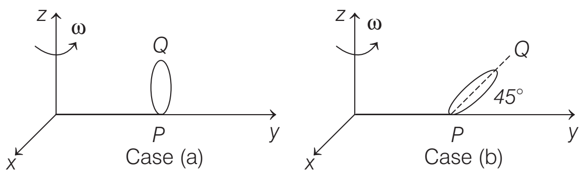
(c) $\sqrt{\dfrac{g}{\mu(R-r)}}$ (d) $\sqrt{\dfrac{2g}{\mu(R-r)}}$

Passage 2



The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the Z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.

Now, consider two similar systems as shown in the figure : Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to X-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the Z-axis.



26. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct? **(2012)**

(a) It is $\sqrt{2}\omega$ for both the cases

(b) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b)

(c) It is ω for case (a); and $\sqrt{2}\omega$ for case (b)

(d) It is ω for both the cases

27. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct? **(2012)**

(a) It is vertical for both the cases (a) and (b)

(b) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b)

(c) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

(d) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

Passage 3

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

28. The ratio $\frac{x_1}{x_2}$ is **(2007, 4M)**

(a) 2 (b) 1/2 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

29. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is **(2007, 4M)**

(a) $\frac{2I\omega}{3t}$ (b) $\frac{9I\omega}{2t}$

(c) $\frac{9I\omega}{4t}$ (d) $\frac{3I\omega}{2t}$

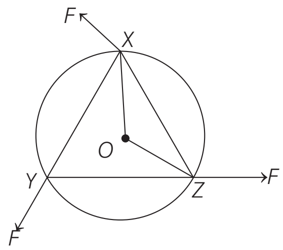
30. The loss of kinetic energy during the above process is **(2007, 4M)**

(a) $\frac{I\omega^2}{2}$ (b) $\frac{I\omega^2}{3}$

(c) $\frac{I\omega^2}{4}$ (d) $\frac{I\omega^2}{6}$

Integer Answer Type Question

31. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is **(2012)**



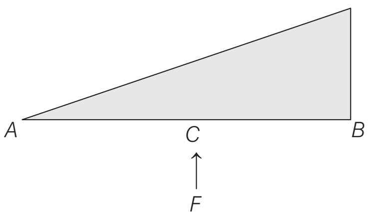
Fill in the Blanks

32. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is .... and on B is .... **(1997, 2M)**

33. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $3a/4$ above the base. The minimum value of F for which the cube begins to tip about the edge is .... (Assume that the cube does not slide). **(1984, 2M)**

True / False

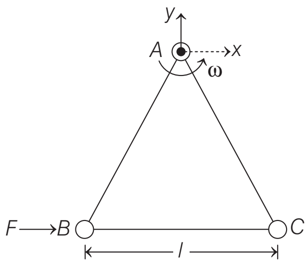
34. A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and (a) passing through A, (b) passing through B, by the application of the same force, F, at C (mid-point of AB) as shown in the figure. The angular acceleration in both the cases will be the same. **(1985, 3M)**



Analytical & Descriptive Questions

35. A rod of length L and mass M is hinged at point O. A small bullet of mass m hits the rod as shown in the figure. The bullet gets embedded in the rod. Find angular velocity of the system just after impact. **(2005, 2M)**

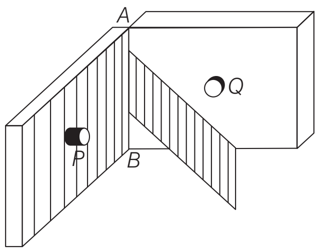
36. Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω. **(2002, 5M)**



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the X-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T.

37. Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen, so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s. **(2001, 10M)**

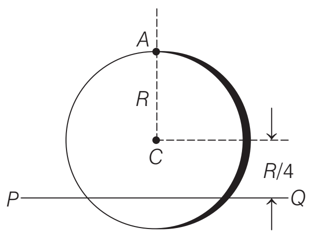


(a) Find the location of the centre of mass of the laminar sheet from AB.

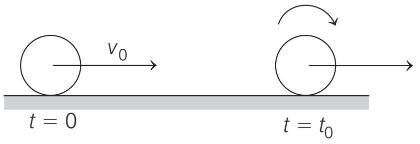
(b) At what angular velocity does the laminar sheet come back after the first impact ?

(c) After how many impacts, does the laminar sheet come to rest ?

38. A uniform circular disc has radius R and mass m. A particle, also of mass m, is fixed at a point A on the edge of the disc as shown in the figure. The disc can rotate freely about a horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall, so that it starts rotation about PQ. Find the linear speed of the particle as it reaches its lowest position. **(1998, 8M)**



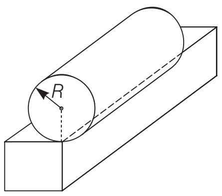
39. A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor, so that it starts off with a purely sliding motion at $t = 0$. After t_0 seconds, it acquires a purely rolling motion as shown in figure. **(1997 C, 5M)**



(a) Calculate the velocity of the centre of mass of the disc at t_0.

(b) Assuming the coefficient of friction to be μ, calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0.

40. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in figure. There is sufficient friction present at the edge, so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine **(1995, 10M)**

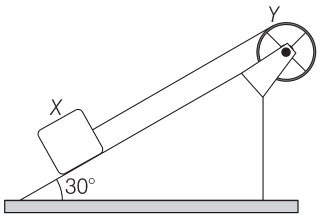


(a) the angle θ_c through which the cylinder rotates before it leaves contact with the edge,

(b) the speed of the centre of mass of the cylinder before leaving contact with the edge and

(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

41. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in figure.



The drum is given an initial angular velocity such that the block X starts moving up the plane. **(1994, 6M)**

(a) Find the tension in the string during the motion.

(b) At a certain instant of time, the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest.

42. A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $R/2$. **(1990, 8M)**

Answers

Topic 1

1. (c) 2. (d) 3. (d) 4. (d)
5. (*) 6. (b) 7. (a) 8. (c)
9. (b) 10. (a) 11. (a) 12. (b)
13. (d) 14. (d) 15. (b) 16. (a)
17. (a) 18. (a) 19. (d) 20. (a)
21. (a, b, c) 22. (3) 23. (9) 24. 4.8 Ma^2

Topic 2

1. (c) 2. (d) 3. (d)
4. (d) 5. (c) 6. (c) 7. (b)
8. (a) 9. (c) 10. (b) 11. (c)
12. (b) 13. (b) 14. (c) 15. (a, b, c)
16. (c) 17. (b) 18. 4 19. (8)
20. (6) 21. −3 22. $\frac{M\omega_0}{M+6m}$
23. F 24. $\frac{mv_0^3}{2\sqrt{2}g}$ in a direction perpendicular to paper inwards

Topic 3

1. (b) 2. (b) 3. (c) 4. (b)
5. (b) 6. (b)
7. (b) 8. (d) 9. (d) 10. (d)
11. (a) 12. (b) 13. (d) 14. (d)
15. (d) 16. (c) 17. (a, c) 18. (c, d)
19. (a, b) 20. (b, c) 21. (c, d) 22. (0.75)
23. $\frac{mg}{6}$, up
24. $\frac{1}{3}MRA\omega^2$ 25. F 26. 7 27. 3.6
28. $\frac{2}{3}g\sin\theta$ 29. $a_{CM} = \frac{4F}{3m_1+8m_2}$, $a_{plank} = \frac{8F}{3m_1+8m_2} = 2a_{CM}$
(b) $\frac{3Fm_1}{3m_1+8m_2}, \frac{Fm_1}{3m_1+8m_2}$
30. (a) $6\hat{\mathbf{i}}$ (b) $0.6(\hat{\mathbf{k}}-\hat{\mathbf{j}})$, $0.6(-\hat{\mathbf{j}}-\hat{\mathbf{k}})$, 0.85 N-m
31. 2.13 m, yes

Topic 4

1. (a)
2. (a) 3. (c) 4. (a) 5. (a, c)
6. (a, c, d) 7. (a) $\frac{1}{4}$ (b) $\frac{2}{3}L$ (c) $\frac{v_0}{2\sqrt{2}}$ 8. 6.3 m/s
9. (a) $\frac{12v}{7L}$ (b) 3.5 ms^{-1} 10. 2.72 J

Topic 5

1. (b) 2. (c) 3. (c) 4. (d)
5. (b) 6. (a) 7. (a) 8. (b)
9. (b) 10. (a)
11. (c) 12. (*) 13. (a) 14. (b)
15. (b) 16. (d) 17. (c) 18. (c)
19. (a) 20. (b,c) 21. (a,c) 22. (a,c,d)
23. (a) 24. Not clear 25. (c)
26. (d) 27. (a) 28. (c) 29. (a)
30. (b) 31. (2) 32. $\left(\frac{d-x}{d}\right)W, \frac{xW}{d}$
33. $\frac{2}{3}mg$ 34. F 35. $\frac{3mv}{L(3m+M)}$
36. (a) $\sqrt{3}\,ml\omega^2$ (b) $(F_{net})_x = \frac{-F}{4}$, $(F_{net})_y = \sqrt{3}\,ml\omega^2$
37. (a) 0.1 m (b) 1 rad/s (c) sheet will never come to rest
38. $\sqrt{5gR}$
39. (a) $\frac{2}{3}v_0$ (b) $\frac{v_0}{3\mu g}$, For $t \le t_0, W_f = \frac{m\mu gt}{2}[3\mu gt - 2v_0], \frac{-mv_0^2}{6}$
40. (a) $\theta = \cos^{-1}\frac{4}{7}$ (b) $\sqrt{\frac{4gR}{7}}$ (c) 6
41. (a) 1.63 N (b) 1.22 m 42. $v = \sqrt{\frac{14Rg}{3}}$

Hints & Solutions

Topic 1 Moment of Inertia

1. **Key Idea** Radius of gyration K of any structure is given by

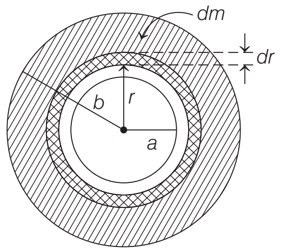
$$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}$$

To find K, we need to find both moment of inertia I and mass M of the given structure.

Given, variation in mass per unit area (surface mass density),

$$\sigma = \frac{\sigma_0}{r} \qquad \ldots \text{(i)}$$

Calculation of Mass of Disc



Let us divide whole disc in small area elements, one of them shown at r distance from the centre of the disc with its width as dr.

Mass of this element is

$$dm = \sigma \cdot dA$$

$$\Rightarrow \quad dm = \frac{\sigma_0}{r} \times 2\pi r dr \quad \text{[from Eq. (i)]} \ldots \text{(ii)}$$

Mass of the disc can be calculated by integrating it over the given limits of r,

$$\int_0^M dm = \int_a^b \sigma_0 \times 2\pi \times dr$$

$$\Rightarrow \quad M = \sigma_0 2\pi (b - a) \qquad \ldots \text{(iii)}$$

Calculation of Moment of Inertia

$$I = \int_0^M r^2 dm = \int_a^b r^2 \cdot \frac{\sigma_0}{r} \times 2\pi r dr = \sigma_0 2\pi \int_a^b r^2 dr = \sigma_0 2\pi \left[\frac{r^3}{3}\right]_a^b$$

$$\Rightarrow \quad I = \frac{1}{3} \sigma_0 2\pi [b^3 - a^3] \qquad \ldots \text{(iv)}$$

Now, radius of gyration,

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{2\pi\sigma_0}{3}(b^3 - a^3)}{2\pi\sigma_0 (b - a)}}$$

$$\Rightarrow \quad K = \sqrt{\frac{1}{3} \frac{(b^3 - a^3)}{b - a}}$$

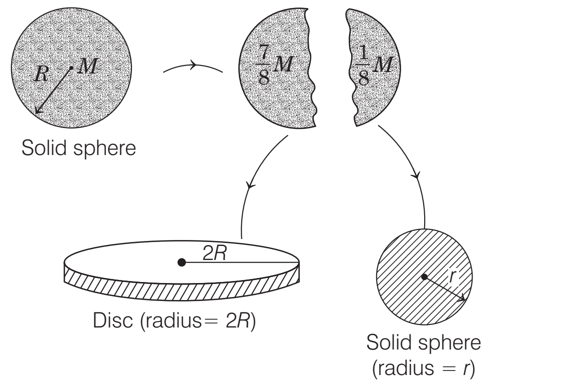
As we know, $b^3 - a^3 = (b - a)(b^2 + a^2 + ab)$

$$\therefore \quad K = \sqrt{\frac{1}{3}(b^2 + a^2 + ab)}$$

or

$$K = \sqrt{\frac{(a^2 + b^2 + ab)}{3}}$$

2. The given situation is shown in the figure given below



Density of given sphere of radius R is

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4}{3}\pi R^3}$$

Let radius of sphere formed from second part is r, then mass of second part = volume × density

$$\frac{1}{8} M = \frac{4}{3}\pi r^3 \times \frac{M}{\frac{4}{3}\pi R^3}$$

$$\therefore \quad r^3 = \frac{R^3}{8} \Rightarrow r = \frac{R}{2}$$

Now, I_1 = moment of inertia of disc (radius $2R$ and mass $\frac{7}{8}M$) about its axis

$$= \frac{\text{Mass} \times (\text{Radius})^2}{2} = \frac{\frac{7}{8}M \times (2R)^2}{2} = \frac{7}{4}MR^2$$

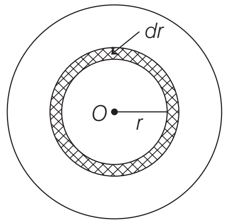
and I_2 = moment of inertia of sphere (radius $\frac{R}{2}$ and mass $\frac{1}{8}M$) about its axis

$$= \frac{2}{5} \times \text{Mass} \times (\text{Radius})^2 = \frac{2}{5} \times \frac{1}{8} M \times \left(\frac{R}{2}\right)^2 = \frac{MR^2}{80}$$

$$\therefore \text{ Ratio } \frac{I_1}{I_2} = \frac{\frac{7}{4}MR^2}{\frac{1}{80}MR^2} = 140$$

3. Given, Surface mass density, $\sigma = kr^2$

So, mass of the disc can be calculated by considering small element of area $2\pi r dr$ on it and then integrating it for complete disc, i.e.



$$dm = \sigma\, dA = \sigma \times 2\pi r dr$$

$$\int dm = M = \int_0^R (kr^2)\, 2\pi r dr$$

$$\Rightarrow \qquad M = 2\pi k \frac{R^4}{4} = \frac{1}{2}\pi k R^4 \qquad \ldots \text{(i)}$$

Moment of inertia about the axis of the disc,

$$I = \int dI = \int dm r^2 = \int \sigma dA r^2$$

$$= \int_0^R kr^2\, (2\pi r dr)\, r^2$$

$$\Rightarrow \qquad I = 2\pi k \int_0^R r^5\, dr = \frac{2\pi k R^6}{6} = \frac{\pi k R^6}{3} \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$I = \frac{2}{3} MR^2$$

4. Given, kinetic energy $= k\theta^2$

We know that, kinetic energy of a rotating body about its axis $= \frac{1}{2} I\omega^2$

where, I is moment of inertia and ω is angular velocity.

$$\text{So,} \qquad \frac{1}{2} I\omega^2 = k\theta^2 \;\text{ or }\; \omega^2 = \frac{2k\theta^2}{I}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{2k}{I}}\,\theta \qquad \ldots \text{(i)}$$

Differentiating the above equation w.r.t. time on both sides, we get

$$\frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \frac{d\theta}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega \qquad \left[\because \omega = \frac{d\theta}{dt}\right]$$

$\therefore$ Angular acceleration,

$$\alpha = \frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\,\theta \quad \text{[using Eq. (i)]}$$

$$\text{or} \qquad \alpha = \frac{2k}{I}\,\theta$$

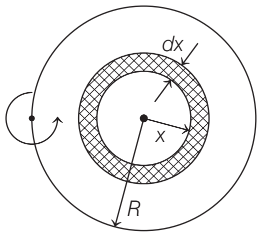
Alternate Solution

$$\text{As,} \qquad \omega^2 = \frac{2k\theta^2}{I}$$

$$\Rightarrow \qquad 2\omega \frac{d\omega}{dt} = \frac{2k}{I} \cdot 2\theta \frac{d\theta}{dt} \;\text{ or }\; \omega\, d\omega = \frac{2k}{I}\theta\, d\theta$$

$$\omega \frac{d\omega}{d\theta} (= \alpha) = \frac{2k}{I} \cdot \theta \;\text{ or }\; \alpha = \frac{2k}{I}\,\theta$$

5. Consider an elementary ring of thickness dx and radius x.



Moment of inertia of this ring about a perpendicular axes through centre is

$$dI_c = dm \cdot x^2 = \rho_0 x (2\pi x) dx \cdot x^2 = 2\pi\rho_0 x^4 dx$$

Moment of inertia of this elementary ring about a perpendicular axes at a point through edge, (by parallel axes theorem)

$$dI = dm x^2 + dm R^2$$

$$= 2\pi\rho_0 x^4 dx + 2\pi\rho_0 R^2 x^2 dx$$

Moment of inertia of complete disc is

$$I = \int_0^R dI = \int_0^R 2\pi\rho_0 x^4 dx + \int_0^R 2\pi\rho_0 R^2 x^2 dx$$

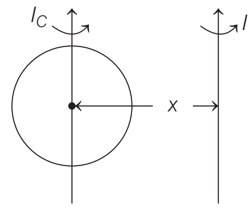
$$= \frac{2\pi\rho_0 R^5}{5} + \frac{2\pi\rho_0 R^5}{3} = \frac{16\pi\rho_0 R^5}{15}$$

$\therefore \quad a = \frac{16}{15}$ (No option matches)

6. Moment of inertia of a solid sphere about an axis through its centre of mass is

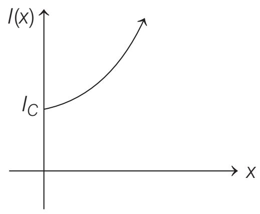
$$I_C = \frac{2}{5} MR^2$$

Moment of inertia about a parallel axis at a distance x from axis through its COM is



$I = I_C + Mx^2$ (by parallel axis theorem)

So graph of I *versus* x is parabolic are shown



7. Moment of inertia of hollow cylinder about its axis is

$$I_1 = \frac{M}{2}(R_1^2 + R_2^2)$$

where, R_1 = inner radius and
R_2 = outer radius.

Moment of inertia of thin hollow cylinder of radius R about its axis is.

$$I_2 = MR^2$$

Given, $I_1 = I_2$ and both cylinders have same mass (M).

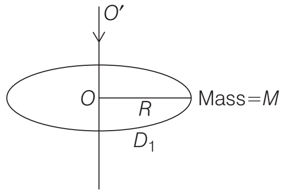
So, we have

$$\frac{M}{2}(R_1^2 + R_2^2) = MR^2$$

$$(10^2 + 20^2)/2 = R^2$$
$$R^2 = 250 = 15.8$$
$$R \approx 16\,\text{cm}$$

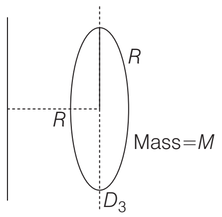
8 For disc D_1, moment of inertia across axis OO' will be

$$I_1 = \frac{1}{2}MR^2 \qquad \ldots\text{(i)}$$



For discs D_2 and D_3, OO' is an axis parallel to the diameter of disc. Using parallel axis theorem,

$$I_2 = I_3 = I_{\text{diameter}} + Md^2 \qquad \ldots\text{(ii)}$$



Here, $\quad I_{\text{diameter}} = \frac{1}{4}MR^2$

and $\quad d = R$

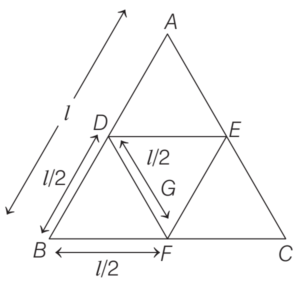
$\therefore \quad I_2 = I_3 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$

Now, total MI of the system

$$I = I_1 + I_2 + I_3 = \frac{1}{2}MR^2 + 2 \times \frac{5}{4}MR^2 = 3MR^2$$

9 *(b)* Suppose the mass of the ΔABC be 'M' and length of the side be 'l'.

When the ΔDEF is being removed from it, then the mass of the removed Δ will be '$M/4$' and length of its side will be '$l/2$' as shown below



Since we know, moment of inertia of the triangle about the axis, passing through its centre of gravity is,

$I = kml^2$, where k is a constant.

Then for ΔDEF, moment of inertia of the triangle about the axis,

$$I = k\left(\frac{M}{4}\right)\left(\frac{l}{2}\right)^2 = \frac{kMl^2}{16} \qquad \ldots\text{(i)}$$

Moment of inertia of ΔABC is

$$I_0 = kMl^2 \qquad \ldots\text{(ii)}$$

The moment of inertia of the remaining part will be

$$I' = I_0 - I = kMl^2 - \frac{kMl^2}{16} \quad [\because \text{using Eqs. (i) and (ii)}]$$
$$= \frac{15kMl^2}{16} \text{ or } I' = \frac{15}{16}I_0$$

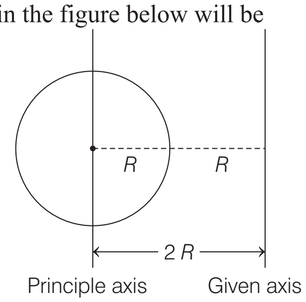
10

> **Key Idea** This problem will be solved by applying parallel axis theorem, which states that moment of inertia of a rigid body about any axis is equals to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the axis.

We know that moment of inertia (MI) about the principle axis of the sphere is given by

$$I_{\text{sphere}} = \frac{2}{5}MR^2 \qquad \ldots\text{(i)}$$

Using parallel axis theorem, moment of inertia about the given axis in the figure below will be



$$I_1 = \frac{2}{5}MR^2 + M(2R)^2$$
$$I_1 = \frac{22}{5}MR^2 \qquad \ldots\text{(ii)}$$

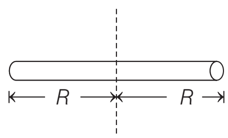
Considering both spheres at equal distance from the axis, moment of inertia due to both spheres about this axis will be

$$2I_1 = 2 \times 22/5\, MR^2$$

Now, moment of inertia of rod about its perpendicular bisector axis is given by

$$I_2 = \frac{1}{12}ML^2$$

Here, given that $\quad L = 2R$



$$\therefore \quad I_2 = \frac{1}{12} M(2R)^2 = \frac{1}{3} MR^2 \quad \ldots \text{(iii)}$$

So, total moment of inertia of the system is

$$I = 2I_1 + I_2 = 2 \times \frac{22}{5} MR^2 + \frac{1}{3} MR^2$$

$$\Rightarrow \quad I = \left(\frac{44}{5} + \frac{1}{3}\right) MR^2 = \frac{137}{15} MR^2$$

11. From theorem of parallel axis,

$$\mathrm{I} = \mathrm{I}_{cm} + 7M(3R)^2$$

$$= \left[\frac{MR^2}{2} + 6 \times \left\{\frac{MR^2}{2} + M(2R)^2\right\}\right] + 7M(3R)^2 = \frac{181MR^2}{2}$$

12. $I_{\text{Remaining}} = I_{\text{Total}} - I_{\text{Cavity}}$

$$\Rightarrow \quad I = \frac{9MR^2}{2} - \left[\frac{M}{2}\left(\frac{R}{3}\right)^3 + M\left(\frac{2R}{3}\right)^2\right] = 4MR^2$$

13. MI of a solid cylinder about its perpendicular bisector of length is

$$I = M\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$$

$$\Rightarrow I = \frac{mR^2}{4} + \frac{ml^2}{12} = \frac{m^2}{4\pi\rho l} + \frac{ml^2}{12} \quad [\because \rho\pi r^2 l = m]$$

For I to be maximum,

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho}\left(\frac{1}{l^2}\right) + \frac{ml}{6} = 0$$

$$\Rightarrow \quad \frac{m^2}{4\mu\pi\rho} = \frac{ml^3}{6} \Rightarrow l^3 = \frac{3m}{2\pi\rho}$$

$$\Rightarrow \quad l = \left(\frac{3}{2}\right)^{1/3}\left(\frac{m}{\pi\rho}\right)^{1/3}$$

$$\rho = \frac{m}{\pi R^2 l} \Rightarrow R^2 = \frac{m}{\pi\rho l}$$

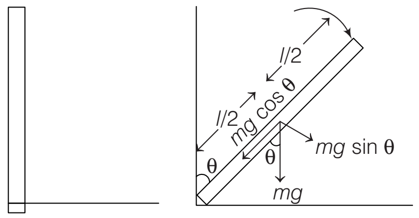
$$\Rightarrow \quad R^2 = \frac{m}{\pi\rho}\left(\frac{2}{3}\right)^{1/3}\left(\frac{\pi\rho}{m}\right)^{1/3} = \left(\frac{m}{\pi\rho}\right)^{2/3}\left(\frac{2}{3}\right)^{1/3}$$

$$\Rightarrow \quad R = \left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/6}$$

$$\frac{l}{R} = \frac{\left(\frac{3}{2}\right)^{1/3}\left(\frac{m}{\pi\rho}\right)^{1/3}}{\left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/6}} = \left(\frac{3}{2}\right)^{1/3} + \left(\frac{3}{2}\right)^{1/6}$$

$$\therefore \quad \frac{l}{R} = \sqrt{\frac{3}{2}}$$

14. As the rod rotates in vertical plane so a torque is acting on it, which is due to the vertical component of weight of rod.



Initial condition At any time t

Now, Torque τ = force × perpendicular distance of line of action of force from axis of rotation

$$= mg \sin\theta \times \frac{l}{2}$$

Again, Torque, $\tau = I\alpha$

Where, I = moment of inertia $= \dfrac{ml^2}{3}$

[Force and Torque frequency along axis of rotation passing through in end]

α = angular acceleration

$$\therefore \quad mg\sin\theta \times \frac{l}{2} = \frac{ml^2}{3}\alpha$$

$$\therefore \quad \alpha = \frac{3g\sin\theta}{2l}$$

15. Maximum possible volume of cube will occur when

$$\sqrt{3}a = 2R \quad (a = \text{side of cube})$$

$$\therefore \quad a = \frac{2}{\sqrt{3}}R$$

Now, density of sphere, $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

Mass of cube, m = (volume of cube)(ρ) = $(a^3)(\rho)$

$$= \left[\frac{2}{\sqrt{3}}R\right]^3\left[\frac{m}{\frac{4}{3}\pi R^3}\right] = \left(\frac{2}{\sqrt{3}\pi}\right)M$$

Now, moment of inertia of the cube about the said axis is

$$I = \frac{ma^2}{6} = \frac{\left(\frac{2}{\sqrt{3}\pi}\right)M\left(\frac{2}{\sqrt{3}}R\right)^2}{\sigma}$$

$$= \frac{4MR^2}{9\sqrt{3}\pi}$$

16. $$\frac{2}{5}MR^2 = \frac{1}{2}Mr^2 + Mr^2$$

or $$\frac{2}{5}MR^2 = \frac{3}{2}Mr^2$$

$$\therefore \quad r = \frac{2}{\sqrt{15}}R$$

17. $I_{\text{remaining}} = I_{\text{whole}} - I_{\text{removed}}$

or $\quad I = \dfrac{1}{2}(9M)(R)^2 - \left[\dfrac{1}{2}m\left(\dfrac{R}{3}\right)^2 + \dfrac{1}{2}m\left(\dfrac{2R}{3}\right)^2\right] \quad$ ...(i)

Here, $m = \dfrac{9M}{\pi R^2} \times \pi\left(\dfrac{R}{3}\right)^2 = M$

Substituting in Eq. (i), we have

$$I = 4MR^2$$

18. Mass of the whole disc $= 4M$

Moment of inertia of the disc about the given axis

$$= \frac{1}{2}(4M)R^2 = 2MR^2$$

$\therefore$ Moment of inertia of quarter section of the disc

$$= \frac{1}{4}(2MR^2) = \frac{1}{2}MR^2$$

19. Mass of the ring $M = \rho L$. Let R be the radius of the ring, then

$$L = 2\pi R$$

or $\quad R = \dfrac{L}{2\pi}$

Moment of inertia about an axis passing through O and parallel to XX' will be

$$I_0 = \frac{1}{2}MR^2$$

Therefore, moment of inertia about XX' (from parallel axis theorem) will be given by

$$I_{XX'} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

Substituting values of M and R

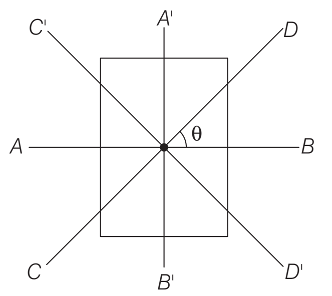
$$I_{XX'} = \frac{3}{2}(\rho L)\left(\frac{L^2}{4\pi^2}\right) = \frac{3\rho L^3}{8\pi^2}$$

20. $A'B' \perp AB$ and $C'D' \perp CD$

From symmetry $\quad I_{AB} = I_{A'B'}$

and $\quad I_{CD} = I_{C'D'}$

From theorem of perpendicular axes,



$$I_{ZZ} = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$$
$$= 2I_{AB} = 2I_{CD}$$
$$I_{AB} = I_{CD}$$

Alternate

The relation between I_{AB} and I_{CD} should be true for all values of θ

At $\quad \theta = 0, I_{CD} = I_{AB}$

Similarly, at $\theta = \pi/2, I_{CD} = I_{AB}$ (by symmetry)

Keeping these things in mind, only option (a) is correct.

21. Since, it is a square lamina

$$I_3 = I_4$$

and $\quad I_1 = I_2 \quad$ (by symmetry)

From perpendicular axes theorem,

Moment of inertia about an axis perpendicular to square plate and passing from O is

$$I_o = I_1 + I_2 = I_3 + I_4$$

or $\quad I_o = 2I_2 = 2I_3$

Hence, $\quad I_2 = I_3$

Rather we can say $I_1 = I_2 = I_3 = I_4$

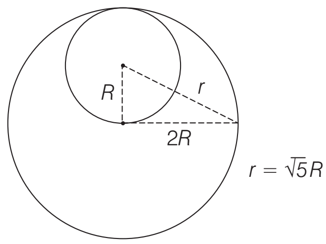
Therefore, I_o can be obtained by adding any two i.e.

$$I_o = I_1 + I_2 = I_1 + I_3$$
$$= I_1 + I_4 = I_2 + I_3$$
$$= I_2 + I_4 = I_3 + I_4$$

22. T = Total portion

R = Remaining portion and

C = Cavity and let σ = mass per unit area.



Then, $\quad m_T = \pi(2R)^2\sigma = 4\pi R^2\sigma$

$\quad m_C = \pi(R)^2\sigma = \pi R^2\sigma$

For I_P

$$I_R = I_T - I_C$$
$$= \frac{3}{2}m_T(2R)^2 - \left[\frac{1}{2}m_C R^2 + m_C r^2\right]$$
$$= \frac{3}{2}(4\pi R^2\sigma)(4R^2) - \left[\frac{1}{2}(\pi R^2\sigma) + (\pi R^2\sigma)(5R^2)\right]$$
$$= (18.5\,\pi R^4\sigma)$$

For I_O $\quad I_R = I_T - I_C$

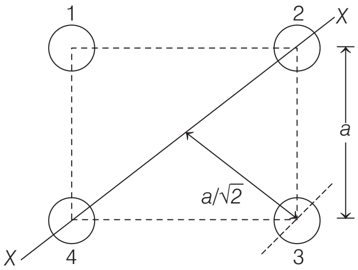
$$= \frac{1}{2}m_T(2R)^2 - \frac{3}{2}m_C R^2$$
$$= \frac{1}{2}(4\pi R^2\sigma)(4R^2) - \frac{3}{2}(\pi R^2\sigma)(R^2) = 6.5\,\pi R^4\sigma$$

$$\therefore \quad \frac{I_P}{I_O} = \frac{18.5\pi R^4\sigma}{6.5\pi R^4\sigma} = 2.846$$

Therefore, the nearest integer is 3.

23. $r = \dfrac{d}{2} = \dfrac{\sqrt{5}}{2}$ cm $= \dfrac{\sqrt{5}}{2} \times 10^{-2}$ m $\Rightarrow m = 0.5$ kg

$$a = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$$



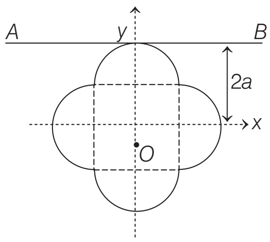
$$I_{XX} = I_1 + I_2 + I_3 + I_4$$

$$= \left[\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right] + \frac{2}{5}mr^2 + \left[\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right] + \frac{2}{5}mr^2$$

Substituting the values, we get

$$I_{XX} = 9 \times 10^{-4} \text{ kg-m}^2 \quad \therefore \quad N = 9$$

24. Assuming the lamina to be in x-y plane.



From, the perpendicular axis theorem,

$$I_x + I_y = I_z$$

but $\quad I_x = I_y \quad$ (by symmetry)

and $\quad I_z = 1.6\, Ma^2 \quad$ (given)

$\therefore \quad I_x = \dfrac{I_z}{2} = 0.8\, Ma^2$

Now, from the parallel axis theorem,

$$I_{AB} = I_x + M\,(2a)^2 = 0.8\, Ma^2 + 4\, Ma^2 = 4.8\, Ma^2$$

Topic 2 Angular Momentum and its Conservation

1. Position of particle is, $\mathbf{r} = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$

where, t is instantaneous time.

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}}$$

Now, angular momentum of particle with respect to origin is given by

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

$$= m\{(2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}})\}$$

$$= m(-12t^2(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) - 6t^2(\hat{\mathbf{j}} \times \hat{\mathbf{i}}))$$

As, $\quad \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0$

$\Rightarrow \quad \mathbf{L} = m(-12t^2\hat{\mathbf{k}} + 6t^2\hat{\mathbf{k}})$

As, $\quad \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$

$\Rightarrow \quad \mathbf{L} = -m(6t^2)\hat{\mathbf{k}}$

So, angular momentum of particle of mass 2 kg at time $t = 2$ s is

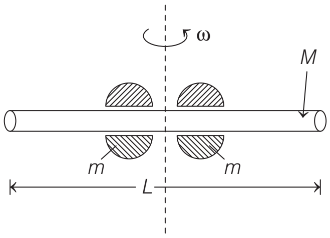
$$\mathbf{L} = (-2 \times 6 \times 2^2)\hat{\mathbf{k}} = -48\,\hat{\mathbf{k}}$$

2. As there is no external torque on system.

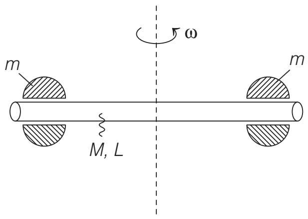
$\therefore$ Angular momentum of system is conserved.

$\Rightarrow \quad I_i\omega_i = I_f\omega_f$

Initially,



Finally,

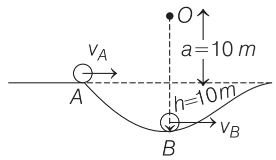


$$\Rightarrow \quad \frac{ML^2}{12}\cdot\omega_0 + 0 = \left(\frac{ML^2}{12} + 2(m)\left(\frac{L}{2}\right)^2\right)\omega$$

So, final angular speed of system is

$$\Rightarrow \quad \omega = \frac{\frac{ML^2}{12}\cdot\omega_0}{\left(\frac{ML^2 + 6mL^2}{12}\right)} = \frac{M\omega_0}{M + 6m}$$

3. The given figure is shown below as



As friction is absent, energy at A = energy at B

$$\Rightarrow \quad \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad v_A^2 + 2gh = v_B^2$$

or $\quad v_B^2 = (5)^2 + 2 \times 10 \times 10 = 225$

$\Rightarrow \quad v_B = 15 \text{ ms}^{-1}$

Angular momentum about point 'O',

$$= mv_B r_B$$
$$= 20 \times 10^{-3} \times 15 \times 20 = 6 \text{ kg-m}^2 \text{ s}^{-1}$$

4. Let the other mass at this instant is at a distance of x from the centre O. Applying law of conservation of angular momentum, we have $\quad I_1\omega_1 = I_2\omega_2$

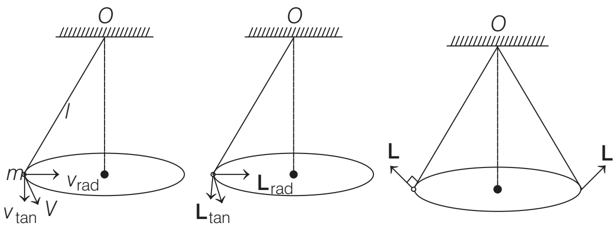
$\therefore$

$$(MR^2)(\omega) = \left[MR^2 + \frac{M}{8}\left(\frac{3}{5}R\right)^2 + \frac{M}{8}x^2\right]\left(\frac{8}{9}\omega\right)$$

Solving this equation, we get $x = \dfrac{4}{5}R$.

NOTE If we take identical situations with both point masses, then answer will be (c). But in that case, angular momentum is not conserved.

5. Angular momentum of the pendulum about the suspension point O is



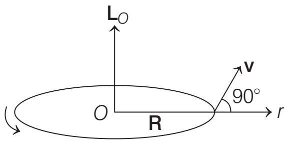
Then, v can be resolved into two components, radial component r_{rad} and tangential component r_{tan}. Due to v_{rad}, L will be tangential and due to v_{tan}, L will be radially outwards as shown. So, net angular momentum will be as shown in figure whose magnitude will be constant ($|L| = mvl$). But its direction will change as shown in the figure.

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

where, r = radius of circle.

6. Angular momentum of a particle about a point is given by $\mathbf{L} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$

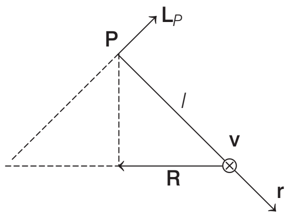
For $\mathbf{L}_O$



$|\mathbf{L}| = (mvr \sin\theta) = m(R\omega)(R)\sin 90° = \text{constant}$

Direction of $\mathbf{L}_O$ is always upwards. Therefore, complete $\mathbf{L}_O$ is constant, both in magnitude as well as direction.

For $\mathbf{L}_P$



$|\mathbf{L}_P| = (mvr \sin\theta) = (m)(R\omega)(l)\sin 90° = (mRl\omega)$

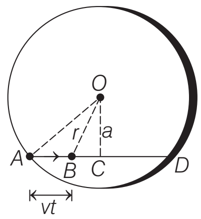
Magnitude of $\mathbf{L}_P$ will remain constant but direction of $\mathbf{L}_P$ keeps on changing.

7. From conservation of angular momentum ($I\omega = \text{constant}$), angular velocity will remain half. As, $K = \dfrac{1}{2}I\omega^2$

The rotational kinetic energy will become half.

8. In uniform circular motion, the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remain conserved.

9. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore, ω will initially increase and then decrease.



Let R be the radius of platform, m the mass of disc and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B

$$I_2 = mr^2 + \frac{MR^2}{2}$$

Here, $\quad r^2 = a^2 + [\sqrt{R^2 - a^2} - vt]^2$

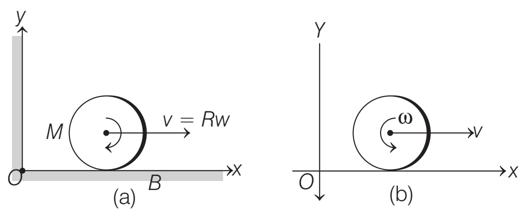
From conservation of angular momentum

$$\omega_0 I_1 = \omega(t) I_2$$

Substituting the values, we can see that variation of $\omega(t)$ is non-linear.

10. Net external torque on the system is zero. Therefore, angular momentum is conserved. Force acting on the system are only conservative. Therefore, total mechanical energy of the system is also conserved.

11. From the theorem



$$\mathbf{L}_0 = \mathbf{L}_{CM} + M(\mathbf{r} \times \mathbf{v}) \qquad \ldots(i)$$

We may write

Angular momentum about O = Angular momentum about CM + Angular momentum of CM about origin

$$\therefore \quad L_0 = I\omega + MRv$$

$$= \frac{1}{2}MR^2\omega + MR(R\omega) = \frac{3}{2}MR^2\omega$$

NOTE That in this case [Figure (a)] both the terms in Eq. (i), i.e. L_{CM} and M (r × v) have the same direction Ä. That is why, we have used $L_0 = I\omega + MRv$. We will use $L_0 = I\omega \sim MRv$ if they are in opposite direction as shown in figure (b).

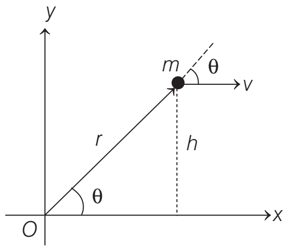
12. $|\mathbf{v}| = v =$ constant and $|\mathbf{r}| = r$ (say)

Angular momentum of the particle about origin O will be given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$$

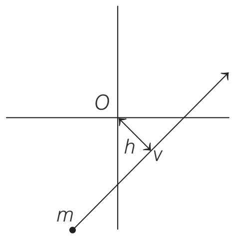
or $\quad |\mathbf{L}| = L = mrv\sin\theta = mv(r\sin\theta) = mvh$

Now, m, v and h all are constants.



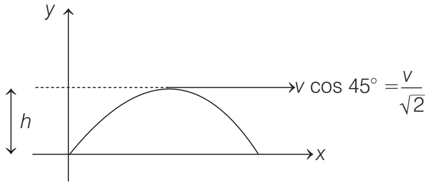
Therefore, angular momentum of particle about origin will remain constant. The direction of $\mathbf{r} \times \mathbf{v}$ also remains the same (negative z).

NOTE Angular momentum of a particle moving with constant velocity about any point is always constant. e.g. Angular momentum of the particle shown in figure about origin O will be



$L = mvh =$ constant

13. $L = m\dfrac{v}{\sqrt{2}}r_\perp$



Here, $r_\perp = h = \dfrac{v^2 \sin^2 45°}{2g} = \dfrac{v^2}{4g}$

$$\therefore \quad L = m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{v^2}{4g}\right) = \frac{mv^3}{4\sqrt{2}g}$$

14. $I_1\omega_1 = I_2\omega_2$

$$\therefore \quad \omega_2 = \frac{I_1}{I_2}\omega = \left(\frac{Mr^2}{Mr^2 + 2mr^2}\right)\omega$$

$$= \left(\frac{M}{M + 2m}\right)\omega$$

15. (a) $\tau = \mathbf{A} \times \mathbf{L}$

i.e. $\quad \dfrac{d\mathbf{L}}{dt} = \mathbf{A} \times \mathbf{L}$

This relation implies that $\dfrac{d\mathbf{L}}{dt}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{L}$.

(c) Here , $\quad \mathbf{L} \cdot \mathbf{L} = L^2$

Differentiating w.r.t. time, we get

$$\mathbf{L} \cdot \frac{d\mathbf{L}}{dt} + \frac{d\mathbf{L}}{dt} \cdot \mathbf{L} = 2L\frac{dL}{dt}$$

$$\Rightarrow \quad 2\,\mathbf{L} \cdot \frac{d\mathbf{L}}{dt} = 2L\frac{dL}{dt}$$

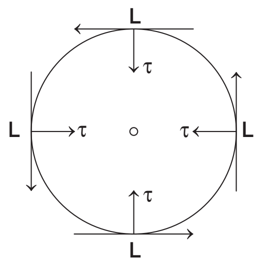
But since, $\quad \mathbf{L} \perp \dfrac{d\mathbf{L}}{dt}$

$$\therefore \quad \mathbf{L} \cdot \frac{d\mathbf{L}}{dt} = 0$$

Therefore, from Eq. (i) $\quad \dfrac{dL}{dt} = 0$

or magnitude of $\mathbf{L}$ i.e. L does not change with time.

(b) So far we are confirm about two points



(i) τ or $\dfrac{d\mathbf{L}}{dt} \perp \mathbf{L}$ and

(ii) $|\mathbf{L}| = L$ is not changing with time, therefore, it is a case when direction of $\mathbf{L}$ is changing but its magnitude is constant and τ is perpendicular to $\mathbf{L}$ at all points.

This can be written as

If $\quad \mathbf{L} = (a\cos\theta)\,\hat{\mathbf{i}} + (a\sin\theta)\,\hat{\mathbf{j}}$

Here, a = positive constant

Then $\quad \tau = (a\sin\theta)\hat{\mathbf{i}} - (a\cos\theta)\hat{\mathbf{j}}$

So, that $\mathbf{L} \cdot \tau = 0$ and $\mathbf{L} \perp \tau$

Now, $\mathbf{A}$ is constant vector and it is always perpendicular to τ. Thus, $\mathbf{A}$ can be written as $\mathbf{A} = A\hat{\mathbf{k}}$

we can see that $\mathbf{L} \cdot \mathbf{A} = 0$ i.e. $\mathbf{L} \perp \mathbf{A}$ also.

Thus, we can say that component of **L** along **A** is zero or component of **L** along **A** is always constant.

Finally, we conclude that τ, **A** and **L** are always mutually perpendicular.

16. Force on block along slot $= m\omega^2 r = ma = m\left(\dfrac{vdv}{dr}\right)$

$$\int_0^v vdv = \int_{R/2}^r \omega^2 rdr \quad \Rightarrow \quad \frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right)$$

$$\Rightarrow \qquad v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt} \Rightarrow \int_{R/4}^r \frac{dr}{\sqrt{r^2 - \dfrac{R^2}{4}}} = \int_0^t \omega\, dt$$

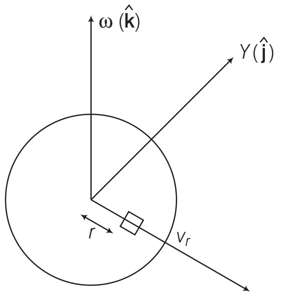
$$\ln\left(\frac{r + \sqrt{r^2 - \dfrac{R^2}{4}}}{\dfrac{R}{2}}\right) - \ln\left(\frac{R/2 + \sqrt{\dfrac{R^2}{4} - \dfrac{R^2}{4}}}{\dfrac{R}{4}}\right) = \omega t$$

$$\Rightarrow \quad r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow \quad r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

$$\Rightarrow \qquad r = \frac{\dfrac{R^2}{4}e^{2\omega t} + \dfrac{R^2}{4}}{Re^{\omega t}} = \frac{R}{4}\,(e^{\omega t} + e^{-\omega t})$$

17.



$$\mathbf{F}_{\text{rot}} = \mathbf{F}_{\text{in}} + 2m(v_{\text{rot}}\,\hat{\mathbf{i}}) \times \omega\hat{\mathbf{k}} + m(\omega\hat{\mathbf{k}} \times r\hat{\mathbf{i}}) \times \omega\hat{\mathbf{k}}$$

$$mr\omega^2\hat{\mathbf{i}} = \mathbf{F}_{\text{in}} + 2mv_{\text{rot}}\omega(-\hat{\mathbf{j}}) + m\omega^2 r\hat{\mathbf{i}}$$

$$\mathbf{F}_{\text{in}} = 2mv_r\omega\hat{\mathbf{j}}$$

$$r = \frac{R}{4}[e^{\omega t} + e^{-\omega t}]$$

$$\frac{dr}{dt} = v_r = \frac{R}{4}[\omega e^{\omega t} - \omega e^{-\omega t}]$$

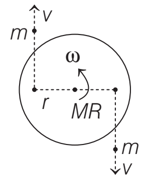
$$\mathbf{F}_{\text{in}} = 2m\frac{R\omega}{4}[e^{\omega t} - e^{-\omega t}]\omega\hat{\mathbf{j}}$$

$$\mathbf{F}_{\text{in}} = \frac{mR\omega^2}{2}[e^{\omega t} - e^{-\omega t}]\hat{\mathbf{j}}$$

Also, reaction is due to disc surface then

$$\mathbf{F}_{\text{reaction}} = \frac{mR\omega^2}{2}[e^{\omega t} - e^{-\omega t}]\hat{\mathbf{j}} + mg\hat{\mathbf{k}}$$

18. Applying conservation of angular momentum



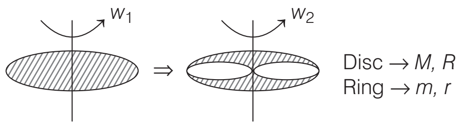
$$2mvr - \frac{MR^2}{2}\omega = 0$$

$$\omega = \frac{4mvr}{MR^2}$$

Substituting the values, we get

$$\omega = \frac{(4)(5 \times 10^{-2})(9)\left(\dfrac{1}{4}\right)}{45 \times 10^{-2} \times \dfrac{1}{4}} \Rightarrow \omega = 4 \text{ rad/s}$$

19. $I_1\omega_1 = I_2\omega_2$



$$\therefore\ \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left[\frac{\dfrac{1}{2}MR^2}{\dfrac{1}{2}MR^2 + 2(mr^2)}\right]\omega_1$$

$$= \left[\frac{50(0.4)^2}{50(0.4)^2 + 8 \times (6.25) \times (0.2)^2}\right](10) = 8 \text{ rad/s}$$

20. $\dfrac{L_{\text{Total}}}{L_B} = \dfrac{(I_A + I_B)\,\omega}{I_B \cdot \omega}$ (as ω will be same in both cases)

$$= \frac{I_A}{I_B} + 1 = \frac{m_A r_A^2}{m_B r_B^2} + 1 = \frac{r_A}{r_B} + 1 \qquad (\text{as } m_A r_A = m_B r_B)$$

$$= \frac{11}{2.2} + 1 = 6 \qquad \left(\text{as } r \propto \frac{1}{m}\right)$$

21. $mvr = k$ (a constant) $\Rightarrow v = \dfrac{k}{mr}$

$$T = \frac{mv^2}{r} = \left(\frac{m}{r}\right)\left(\frac{k}{mr}\right)^2 = \frac{k^2}{m}\cdot\frac{1}{r^3}$$

$$= Ar^{-3} \qquad \left(\text{where, } A = \frac{k^2}{m}\right)$$

Hence, $n = -3$

22. $I_1\omega_1 = I_2\omega_2$

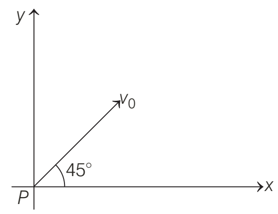
$$\therefore \quad \omega_2 = \frac{I_1}{I_2}\cdot\omega_1 = \frac{(ML^2/12)}{[(ML^2/12) + 2m(L/2)^2]}\omega_0$$

$$= \left(\frac{ML^2}{ML^2 + 6mL^2}\right)\omega_0 = \left(\frac{M}{M + 6m}\right)\omega_0$$

23. $I_1\omega_1 = I_2\omega_2$

$$\therefore\ \omega_2 = \frac{I_1}{I_2}\cdot\omega_1 = \left[\frac{\dfrac{MR^2}{2}}{\dfrac{MR^2}{2} + \dfrac{M}{4}\cdot\dfrac{R^2}{2}}\right]\omega = \frac{4}{5}\omega$$

24. In terms of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$



$$\mathbf{u} = \frac{v_0}{\sqrt{2}}\hat{\mathbf{i}} + \frac{v_0}{\sqrt{2}}\hat{\mathbf{j}}$$

$$\Rightarrow \quad \mathbf{a} = -g\hat{\mathbf{j}}$$

$$t = \frac{v_0}{g}$$

$$\Rightarrow \mathbf{v} = \mathbf{u} + \mathbf{a}t = \frac{v_0}{\sqrt{2}}\hat{\mathbf{i}} - \left(v_0 - \frac{v_0}{\sqrt{2}}\right)\hat{\mathbf{j}}$$

$$\mathbf{r} = \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$= \frac{v_0^2}{\sqrt{2}g}\hat{\mathbf{i}} + \frac{v_0^2}{\sqrt{2}g}\hat{\mathbf{j}} - \frac{v_0^2}{2g}\hat{\mathbf{j}}$$

Now, angular momentum about point P at given time

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

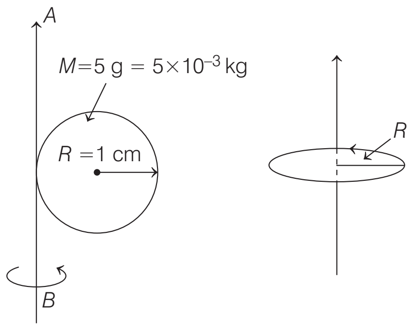
$$= m\left[-\frac{v_0^3}{\sqrt{2}g} + \frac{v_0^3}{2g} - \frac{v_0^3}{2g} + \frac{v_0^3}{2\sqrt{2}g}\right]\hat{\mathbf{k}}$$

$$= -\frac{mv_0^3}{2\sqrt{2}g}\hat{\mathbf{k}}$$

Thus, magnitude of angular momentum is $\frac{mv_0^3}{2\sqrt{2}g}$ in $-\hat{\mathbf{k}}$ direction i.e. in a direction perpendicular to paper inwards

Topic 3 Pure Rolling or Rolling without Slipping

1. Moment of inertia (MI) of a disc about a tangential axis in the plane of disc can be obtained as below.



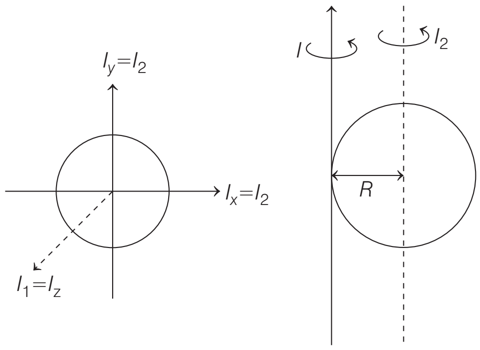
Moment of inertia of disc about it's axis,

$$I_1 = \frac{MR^2}{2}$$

From perpendicular axes theorem, moment of inertia of disc about an axis along it's diameter is

$$I_x + I_y = I_z \Rightarrow 2I_2 = I_1$$

$$\Rightarrow \quad I_2 = \frac{I_1}{2} = \frac{MR^2}{4}$$



So, moment of inertia about a tangential axis from parallel axes theorem is

$$I = I_2 + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$

Now, using torque, $\tau = I\alpha$, we have

$$\tau = I\alpha = \frac{5}{4}MR^2\left(\frac{\omega_f - \omega_i}{\Delta t}\right)$$

Here,

$M = 5 \times 10^{-3}$ kg, $R = 1 \times 10^{-2}$ m

$$\omega_f = 25\,\text{rps} = 25 \times 2\pi\frac{\text{rad}}{\text{s}} = 50\pi\frac{\text{rad}}{\text{s}},$$

$\omega_i = 0,\ \Delta t = 5$ s

$$\text{So, } \tau = \frac{\frac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 50\pi}{5}$$

$$\approx 2 \times 10^{-5}\ \text{N-m}$$

2. Rotation kinetic energy of a body is given by

$$\text{KE}_{\text{rotational}} = \frac{1}{2}I\omega^2$$

where, $\quad \omega = \omega_0 + \alpha t$

$$\text{So,} \quad \text{KE}_{\text{rotational}} = \frac{1}{2}I(\omega_0 + \alpha t)^2 \qquad \ldots\text{(i)}$$

Here, $I = 1.5\ \text{kgm}^2$,

$\text{KE} = 1200$ J and

$\alpha = 20\ \text{rad/s}^2$ and $\omega_0 = 0$

Substituting these values in Eq. (i), we get

$$1200 = \frac{1}{2}(1.5)(20 \times t)^2$$

$$\Rightarrow \quad t^2 = \frac{2 \times 1200}{1.5 \times 400} = 4$$

$$\therefore \quad t = 2\ \text{s}$$

3. From question,

let height attained by ring $= h_1$

Height attained by cylinder $= h_2$

Height attained by sphere $= h_3$

As we know that for a body which is rolling up an inclined plane (without slipping), follows the law of conservation of energy.

$\therefore$ For ring, using energy conservation law at its height h_1.

$$(KE)_{linear} + (KE)_{rotational} = (PE)$$

$$\Rightarrow \quad \frac{1}{2}m_1 v_0^2 + \frac{1}{2}I_1\omega^2 = m_1 g h_1$$

$$\Rightarrow \quad \frac{1}{2}m_1 v_0^2 + \frac{1}{2}m_1 R^2\omega^2 = m_1 g h_1$$

$$(\because I = mR^2 \text{ for ring})$$

$$\Rightarrow \quad gh_1 = \frac{v_0^2}{2} + \frac{v_0^2}{2} \qquad (\because v_0 = \omega R)$$

$$\Rightarrow \quad h_1 = v_0^2 / g \qquad \ldots\text{(i)}$$

Similarly, for solid cylinder, applying the law of conservation of energy,

$$\frac{1}{2}m_2 v_0^2 + \frac{1}{2}I_2\omega^2 = m_2 g h_2$$

$$\Rightarrow \quad \frac{1}{2}m_2 v_0^2 + \frac{1}{2}\left(\frac{1}{2} \times m_2 \times \left(\frac{R}{2}\right)^2\right)\omega^2 = m_2 g h_2$$

$$\left[\begin{aligned} &\because I = \frac{1}{2}mR^2 \text{ for cylinder} \\ &\text{and } \; R = \frac{R}{2} \end{aligned}\right]$$

$$\Rightarrow \quad \frac{1}{2}m_2 v_0^2 + \frac{1}{2} \times \frac{1}{8} m_2 R^2 \times \frac{v_0^2}{(R/2)^2} = m_2 g h_2$$

$$\Rightarrow \quad \frac{1}{2}v_0^2 + \frac{1}{4}v_0^2 = gh_2$$

$$\Rightarrow \quad gh_2 = \frac{3}{4}v_0^2$$

$$\Rightarrow \quad h_2 = \frac{3}{4}\left(\frac{v_0^2}{g}\right) \qquad \ldots\text{(ii)}$$

Similarly, for solid sphere,

$$\frac{1}{2}m_3 v_0^2 + \frac{1}{2}I_3\omega^2 = m_3 g h_3$$

$$\Rightarrow \frac{1}{2}m_3 v_0^2 + \frac{1}{2}\left[\frac{2}{5}m_3\left(\frac{R}{4}\right)^2\right]\omega^2 = m_3 g h_3$$

$$\left[\begin{aligned} &\because I = \frac{2}{5}mR^2 \text{ for solid sphere} \\ &\text{and } \; R = \frac{R}{4} \end{aligned}\right]$$

$$\Rightarrow \quad \frac{1}{2}m_3 v_0^2 + \frac{1}{2} \times \frac{2}{5} \times m_3 \frac{R^2}{16} \times \frac{v_0^2}{(R/4)^2} = m_3 g h_3$$

$$\Rightarrow \quad \frac{1}{2}v_0^2 + \frac{1}{5}v_0^2 = gh_3$$

$$\Rightarrow \quad gh_3 = \frac{7}{10}v_0^2$$

$$\text{or} \quad h_3 = \frac{7}{10}\left(\frac{v_0^2}{g}\right) \qquad \ldots\text{(iii)}$$

$\therefore$ Taking the ratio of h_1, h_2 and h_3 by using Eqs. (i), (ii) and (iii), we get

$$h_1 : h_2 : h_3 = \frac{v_0^2}{g} : \frac{3}{4}\frac{v_0^2}{g} : \frac{7}{10}\frac{v_0^2}{g}$$

$$= 1 : \frac{3}{4} : \frac{7}{10}$$

$\Rightarrow$ $h_1 : h_2 : h_3 = 40 : 30 : 28 = 20 : 15 : 14$

$\therefore$ The most appropriate option is (c).

Although, it is still not in the correct sequence.

Alternate Solution

Total kinetic energy of a rolling body is also given as

$$E_{total} = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$$

where, K is the radius of gyration.

Using conservation law of energy,

$$\frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right] = mgh$$

$$\text{or} \quad h = \frac{v^2}{2g}\left[1 + \frac{K^2}{R^2}\right]$$

For ring, $\dfrac{K^2}{R^2} = 1$

$$\Rightarrow h_1 = \frac{v^2}{2g}[1+1] = \frac{2v^2}{2g} = \frac{v^2}{g}$$

For solid cylinder, $\dfrac{K^2}{R^2} = \dfrac{(R/2\sqrt{2})^2}{(R/2)^2} = \dfrac{R^2}{8} \times \dfrac{4}{R^2} = \dfrac{1}{2}$

$$\Rightarrow \quad h_2 = \frac{v^2}{2g}\left[1 + \frac{1}{2}\right] = \frac{3v^2}{4g}$$

For solid sphere, $\dfrac{K^2}{R^2} = \dfrac{2}{5}$

$$\Rightarrow \quad h_3 = \frac{v^2}{2g}\left[1 + \frac{2}{5}\right] = \frac{7v^2}{10g}$$

So,the ratio of h_1, h_2 and h_3 is

$$h_1 : h_2 : h_3 = \frac{v^2}{g} : \frac{3v^2}{4g} : \frac{7}{10}\frac{v^2}{g}$$

$$= 1 : \frac{3}{4} : \frac{7}{10} = 20 : 15 : 14$$

4 When a spherical/circular body of radius r rolls without slipping, its total kinetic energy is

$$K_{total} = K_{translation} + K_{rotation}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}\left[\because \omega = \frac{v}{r}\right]$$

Let v be the linear velocity and R be the radius for both solid sphere and solid cylinder.

$\therefore$ Kinetic energy of the given solid sphere will be

$$K_{\text{sph}} = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{sph}}\frac{v^2}{R^2}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mR^2\times\frac{v^2}{R^2} = \frac{7}{10}mv^2 \quad \ldots\text{(i)}$$

Similarly, kinetic energy of the given solid cylinder will be

$$K_{\text{cyl}} = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{cyl}}\frac{v^2}{R}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{mR^2}{2}\times\frac{v^2}{R^2} = \frac{3}{4}mv^2 \quad \ldots\text{(ii)}$$

Now, from the conservation of mechanical energy,

$$mgh = K_{\text{total}}$$

$\therefore$ For solid sphere,

$$mgh_{\text{sph}} = \frac{7}{10}mv^2 \quad \ldots\text{(iii) [using Eq. (i)]}$$

Similarly, for solid cylinder,

$$mgh_{\text{cyl}} = \frac{3}{4}mv^2 \quad \ldots\text{(iv) [using Eq. (ii)]}$$

Taking the ratio of Eqs. (iii) and (iv), we get

$$\frac{mgh_{\text{sph}}}{mgh_{\text{cyl}}} = \frac{\frac{7}{10}mv^2}{\frac{3}{4}mv^2} \Rightarrow \frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{7}{10}\times\frac{4}{3} = \frac{14}{15}$$

Alternate Solution

Total kinetic energy for a rolling body without slipping can also be given as

$$K_{\text{total}} = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$$

where, K is the radius of gyration.

$\therefore$ From law of conservation,

$$mgh = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$$

or
$$h \propto \left(1 + \frac{K^2}{R^2}\right)$$

As we know that, for solid sphere,

$$K = \sqrt{\frac{2}{5}}R \Rightarrow \frac{K^2}{R^2} = \frac{2}{5}$$

Similarly, for solid cylinder,

$$K = \frac{R}{\sqrt{2}} \Rightarrow \frac{K^2}{R^2} = \frac{1}{2}$$

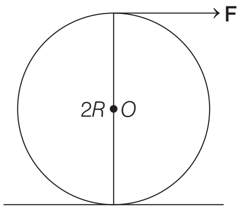
So,
$$\frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{1+\frac{2}{5}}{1+\frac{1}{2}} = \frac{\frac{7}{5}}{\frac{3}{2}} = \frac{7}{5}\times\frac{2}{3} = \frac{14}{15}$$

5 Given, $m = 5\,\text{kg}$, $R = 0.5\,\text{m}$.

Horizontal force, $F = 40\,\text{N}$

As, Cylinder is rolling without slipping.

Hence, torque is producing rotation about centre O.



So, $\tau = \text{r}\times\text{F}$ (Here, $r = R$)

$\theta = 90°$

So, $\tau = \mathbf{r}\times\mathbf{F} = RF$

or $\tau = 0.5\times 40 = 20\,\text{N-m}$ ...(i)

If α is acceleration of centre of mass 'O' then torque is, $\tau = I\alpha$

where, $I = MR^2$

$\therefore \quad \tau = MR^2\alpha$ ...(ii)

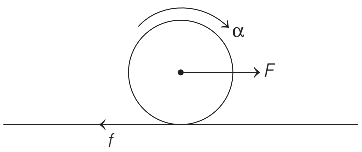
Comparing Eq. (i) with Eq. (ii),

$$MR^2\alpha = 20$$

$$\Rightarrow \quad \alpha = \frac{20}{5\times(0.5)^2}$$

or $\alpha = 16\,\text{rad/s}^2$

6 When force F is applied at the centre of roller of mass m as shown in the figure below



Its acceleration is given by

$$\frac{(F - f)}{m} = a \quad \ldots\text{(i)}$$

where, f = force of friction and

m = mass of roller.

Torque on roller is provided by friction f and it is

$$\tau = fR = I\alpha \quad \ldots\text{(ii)}$$

where, I = moment of inertia of solid cylindrical roller.

$= mR^2/2$

and α = angular acceleration of cylinder $= a/R$.

Hence,
$$\tau = \frac{mR^2}{2}\cdot\frac{a}{R} = \frac{maR}{2}$$

From Eq. (ii), ($\tau = fR$)

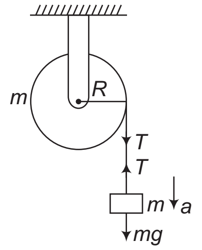
$$f = \frac{ma}{2} \quad \ldots\text{(iii)}$$

From Eqs. (i) and (iii), we get

$$F = \frac{3}{2}ma \Rightarrow a = \frac{2F}{3m}$$

So,
$$\alpha = \frac{2F}{3mR} \quad \left[\because \alpha = \frac{a}{R}\right]$$

7. For the mass m, $mg - T = ma$



As we know, $a = R\alpha$

So, $mg - T = mR\alpha$ ...(i)

Torque about centre of pully

$$T \times R = mR^2\alpha \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get, $a = g/2$

Hence, the acceleration with the mass of a body fall is $g/2$.

8. $I_P > I_Q$

In case of pure rolling, $a = \dfrac{g \sin\theta}{1 + I/mR^2}$

$a_Q > a_P$ as its moment of inertia is less. Therefore, Q reaches first with more linear speed and more translational kinetic energy.

Further, $\omega = \dfrac{v}{R}$ or $\omega \propto v$

$\therefore \quad \omega_Q > \omega_P$ as $v_P > v_Q$

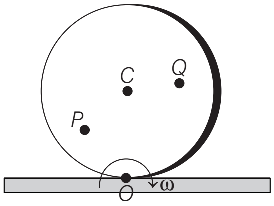
9. $$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$$

$$\therefore \quad I = \frac{1}{2}mR^2$$

$\therefore$ Body is disc.

10. On smooth part BC, due to zero torque, angular velocity and hence the rotational kinetic energy remains constant. While moving from B to C translational kinetic energy converts into gravitational potential energy. So, $h_A < h_c$ and $k_B > k_C$.

11. In case of pure rolling bottom most point is the instantaneous centre of zero velocity.



Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.

$$r_Q > r_C > r_P$$

$$\therefore \quad v_Q > v_C > v_P$$

12. $mg \sin\theta$ component is always down the plane whether it is rolling up or rolling down. Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.

13. In case of pure rolling on inclined plane,

$$a = \frac{g\sin\theta}{1 + I/mR^2}$$

$$I_{\text{solid}} < I_{\text{hollow}}$$

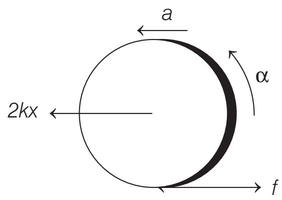
$$\therefore \quad a_{\text{solid}} > a_{\text{hollow}}$$

$\therefore$ Solid cylinder will reach the bottom first. Further, in case of pure rolling on stationary ground, work done by friction is zero. Therefore, mechanical energy of both the cylinders will remain constant.

$\therefore$ $(KE)_{\text{Hollow}} = (KE)_{\text{Solid}}$ = decrease in PE = mgh

$\therefore$ Correct option is (d).

14. $a = R\alpha$



$$\therefore \quad \frac{2kx - f}{M} = R\left[\frac{fR}{\frac{1}{2}MR^2}\right]$$

Solving this equation, we get

$$f = \frac{2kx}{3}$$

$$\therefore \quad |F_{\text{net}}| = 2kx - f = 2kx - \frac{2kx}{3} = \frac{4kx}{3}$$

This is opposite to displacement.

$$\therefore \quad F_{\text{net}} = -\frac{4kx}{3}$$

15. $$F_{\text{net}} = -\left(\frac{4kx}{3}\right)x$$

$$\therefore \quad a = \frac{F_{\text{net}}}{M} = -\left(\frac{4k}{3M}\right)x = -\omega^2 x$$

$$\therefore \quad \omega = \sqrt{\frac{4k}{3M}}$$

16. In case of pure rolling, mechanical energy will remain conserved.

$$\therefore \quad \frac{1}{2}Mv_0^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_0}{R}\right)^2 = 2\left[\frac{1}{2}kx^2_{\max}\right]$$

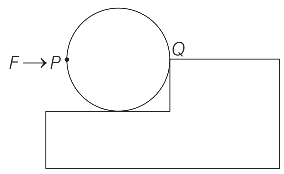
$$\therefore \quad x_{\max} = \sqrt{\frac{3M}{4k}}\,v_0$$

As, $$f = \frac{2kx}{3}$$

$$\therefore \quad F_{\max} = \mu Mg = \frac{2kx_{\max}}{3} = \frac{2k}{3}\sqrt{\frac{3M}{4k}}\,v_0$$

$$\therefore \quad v_0 = \mu g\sqrt{\frac{3M}{k}}$$

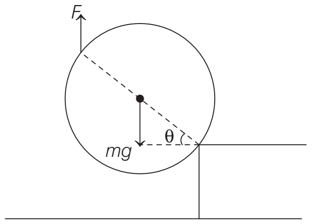
17.



(a) If force is applied normal to surface at P, then line of action of force will pass from Q and thus, $\tau = 0$.

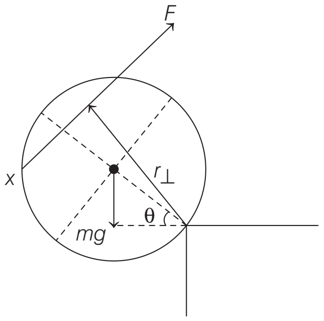
(b) Wheel can climb.

(c) $\tau = F(2R\cos\theta) - mgR\cos\theta$, $\tau \propto \cos\theta$



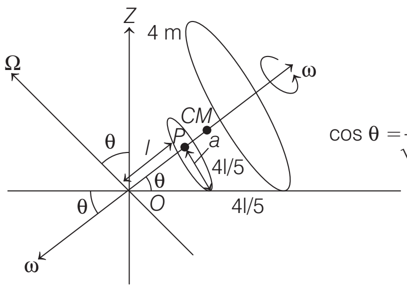
Hence, as θ increases
τ decreases. So its correct.

(d)



$\tau = Fr_\perp - mg\cos\theta$; τ increases with θ.

18.



$$\cos\theta = \frac{l}{\sqrt{l^2+a^2}} = \frac{\sqrt{24}}{5}$$

(a) $$L_z = L_{CM-O}\cos\theta - L_{D-CM}\sin\theta$$

$$= \frac{81\sqrt{24}}{5}a^2 m\omega \times \frac{\sqrt{24}}{5} - \frac{17\,ma^2\omega}{2} \times \frac{1}{\sqrt{24}}$$

$$= \frac{81\times 24 ma^2\omega}{25} - \frac{17ma^2\omega}{2\sqrt{24}}$$

(b) $$L_{CM-O} = (5m)\left[\frac{9l}{5}\omega\right]\frac{9l}{5} = \frac{81ml^2\omega}{5}$$

$$= \frac{81ml^2}{5} \times \frac{a\omega}{l}$$

$$L_{CM-O} = \frac{81mla\omega}{5} = \frac{81\sqrt{24}\,a^2 m\omega}{5}$$

(c) Velocity of point P : $a\omega = 1\omega$ then

$\omega = \dfrac{a\omega}{1}$ = Angular velocity of C.M. w.r.t. point O.

Angular velocity of CM w.r.t. Z-axis $= \omega_0\cos\theta$

$$\omega_{CM-z} = \frac{a\omega}{1}\,\frac{\sqrt{24}}{5} = \frac{a\omega}{\sqrt{24}\,a}\,\frac{\sqrt{24}}{5}$$

$$\omega_{CM-z} = \frac{\omega}{5}$$

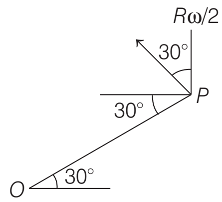
(d) $$L_{D-CM} = \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2\omega}{2}$$

19. Velocity of point O is

$v_O = (3R\omega)\,\hat{\mathbf{i}}$

$\mathbf{v}_{PO}$ is $\dfrac{R\cdot\omega}{2}$ in the direction shown in figure.

In vector form



$$\mathbf{v}_{PO} = -\frac{R\omega}{2}\sin 30°\,\hat{\mathbf{i}} + \frac{R\omega}{2}\cos 30°\,\hat{\mathbf{k}}$$

$$= -\frac{R\omega}{4}\,\hat{\mathbf{i}} + \frac{\sqrt{3}R\omega}{4}\,\hat{\mathbf{k}}$$

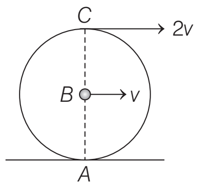
But $\mathbf{v}_{PO} = \mathbf{v}_P - \mathbf{v}_O$

∴ $\mathbf{v}_P = \mathbf{v}_{PO} + \mathbf{v}_O$

$$= \left(-\frac{R\omega}{4}\,\hat{\mathbf{i}} + \frac{\sqrt{3}R\omega}{4}\,\hat{\mathbf{k}}\right) + 3R\omega\,\hat{\mathbf{i}}$$

$$= \frac{11}{4}R\omega\,\hat{\mathbf{i}} + \frac{\sqrt{3}}{4}R\omega\,\hat{\mathbf{k}}$$

20. $v_A = 0, v_B = v$ and $v_C = 2v$



21. In case of pure rolling,

$$f = \frac{mg\sin\theta}{1+\dfrac{mR^2}{I}} \qquad \text{(upwards)}$$

∴ $f \propto \sin\theta$

Therefore, as θ decreases force of friction will also decrease.

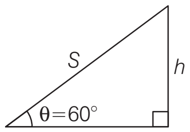
22. $$a = \frac{g\sin\theta}{1+\dfrac{I}{MR^2}}$$

$$a_{\text{ring}} = \frac{g\sin\theta}{2} \qquad (I = MR^2)$$

$$a_{\text{disc}} = \frac{2g\sin\theta}{3} \qquad \left(I = \frac{MR^2}{2}\right)$$

$$s = \frac{h}{\sin\theta} = \frac{1}{2}at^2$$

$$= \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2$$



$$\Rightarrow \quad t_1 = \sqrt{\frac{4h}{g\sin^2\theta}} = \sqrt{\frac{16h}{3g}}$$

$$s = \frac{h}{\sin\theta} = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2$$

$$\Rightarrow \quad t_2 = \sqrt{\frac{3h}{g\sin^2\theta}} = \sqrt{\frac{4h}{g}}$$

$$t_2 - t_1 = \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\sqrt{h}\left[\frac{4}{\sqrt{3}} - 2\right] = 2 - \sqrt{3}$$

Soving this equation we get, $h = 0.75$ m.

23. The equations of motion are

$$a = \frac{mg\sin\theta - f}{m}$$

$$= \frac{mg\sin 30° - f}{m} = \frac{g}{2} - \frac{f}{m} \quad \text{...(i)}$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{I} = \frac{fR}{mR^2/2} = \frac{2f}{mR} \quad \text{...(ii)}$$

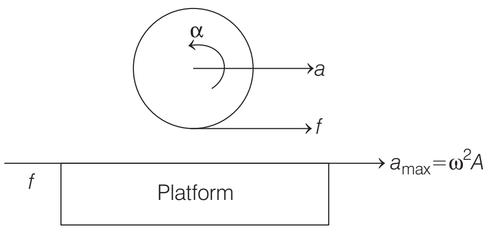
For rolling (no slipping)

$$a = R\alpha$$

or $\quad g/2 - f/m = 2f/m$

$\therefore \quad \frac{3f}{m} = g/2 \quad$ or $\quad f = mg/6$

24. Considering the motion of cylinder.



$$a = \frac{f}{M} \Rightarrow \alpha = \frac{fR}{\frac{MR^2}{2}}$$

$$= \frac{2f}{MR}$$

or $\quad R\alpha = \frac{2f}{M}$

Now, $\quad a + R\alpha = a_{max} = \omega^2 A$

or $\quad \frac{3f}{m} = \omega^2 A$

$\therefore \quad f = \frac{M\omega^2 A}{3}$

$\therefore \quad \tau_{max} = fR = \frac{M\omega^2 AR}{3}$

25. In case of ring, $\frac{K_R}{K_T} = 1$ (pure rolling)

or $\quad K_R = K_T = \frac{K}{2}$

$\therefore \quad \frac{1}{2}(0.3)v_1^2 = \frac{K}{2} \quad$ (i)

In case of disc, $\frac{K_R}{K_T} = \frac{1}{2}$ or $K_T = \frac{2}{3}K$

$\therefore \quad \frac{1}{2}(0.4)v_2^2 = \frac{2}{3}K \quad$ ...(ii)

From Eqs. (i) and (ii), $\frac{v_1}{v_2} = 1$ i.e. $v_1 = v_2$

or both will reach simultaneously.

NOTE In the question,
K = kinetic energy given to ring and cylinder,
K_R = rotational kinetic energy and
K_T = translational kinetic energy.

26. In case of pure rolling, mechanical energy remains constant (as work-done by friction is zero). Further in case of a disc,

$$\frac{\text{translational kinetic energy}}{\text{rotational kinetic energy}} = \frac{K_T}{K_R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2}$$

$$= \frac{mv^2}{\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2} = \frac{2}{1}$$

or, $\quad K_T = \frac{2}{3}$ (Total kinetic energy)

or, $\quad$ Total kinetic energy

$$K = \frac{3}{2}K_T = \frac{3}{2}\left(\frac{1}{2}mv^2\right) = \frac{3}{4}mv^2$$

Decrease in potential energy = increase in kinetic energy

or, $\quad mgh = \frac{3}{4}m(v_f^2 - v_i^2)$ or $v_f = \sqrt{\frac{4}{3}gh + v_i^2}$

As final velocity in both cases is same.

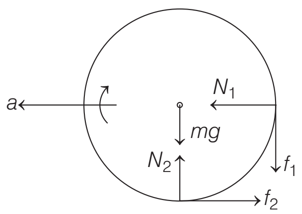
So, value of $\sqrt{\frac{4}{3}gh + v_i^2}$ should be same in both cases.

$\therefore \quad \sqrt{\frac{4}{3} \times 10 \times 30 + (3)^2} = \sqrt{\frac{4}{3} \times 10 \times 27 + (v_2)^2}$

Solving this equation, we get

$$v_2 = 7 \text{ m/s}$$

27.



There is no slipping between ring and ground. Hence f_2 is not maximum. But there is slipping between ring and stick. Therefore, f_1 is maximum. Now let us write the equations.

$$I = mR^2 = (2)(0.5)^2$$

$$= \frac{1}{2} \text{ kg-m}^2$$

$$N_1 - f_2 = ma$$

or $\quad N_1 - f_2 = (2)(0.3) = 0.6 \text{ N} \quad \ldots\text{(i)}$

$$a = R\alpha = \frac{R\tau}{I} = \frac{R(f_2 - f_1)R}{I} = \frac{R^2(f_2 - f_1)}{I}$$

$\therefore \quad 0.3 = \dfrac{(0.5)^2 (f_2 - f_1)}{(1/2)}$

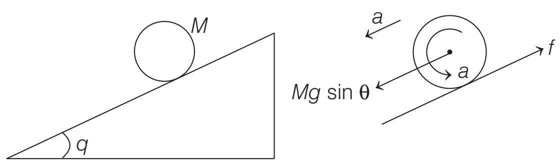
or $\quad f_2 - f_1 = 0.6 \text{ N} \quad \ldots\text{(ii)}$

$$N_1^2 + f_1^2 = (2)^2 = 4 \quad \ldots\text{(iii)}$$

Further $\quad f_1 = \mu N_1 = \left(\frac{P}{10}\right) N_1 \quad \ldots\text{(iv)}$

Solving above four equations we get, $P \simeq 3.6$

28. For rolling without slipping, we have



$$a = R\alpha$$

or $\quad \dfrac{Mg\sin\theta - f}{M} = R\left(\dfrac{fR}{\frac{1}{2}MR^2}\right)$

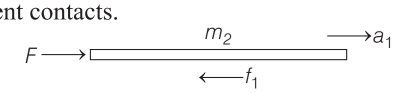
or $\quad \dfrac{Mg\sin\theta - f}{M} = \dfrac{2f}{M}$

$\therefore \quad f = \dfrac{Mg\sin\theta}{3}$

Therefore, linear acceleration of cylinder,

$$a = \frac{Mg\sin\theta - f}{M} = \frac{2}{3} g\sin\theta$$

29. We can choose any arbitrary directions of frictional forces at different contacts.



In the final answer the negative values will show the opposite directions.

Let f_1 = friction between plank and cylinder

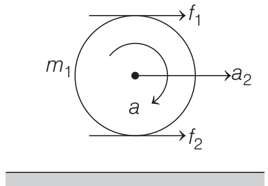
f_2 = friction between cylinder and ground

a_1 = acceleration of plank

a_2 = acceleration of centre of mass of cylinder

and α = angular acceleration of cylinder about its CM.

Since, there is no slipping anywhere



$\therefore \quad a_1 = 2a_2 \quad \ldots\text{(i)}$

$$a_1 = \frac{F - f_1}{m_2} \quad \ldots\text{(ii)}$$

$$a_2 = \frac{f_1 + f_2}{m_1} \quad \ldots\text{(iii)}$$

$$\alpha = \frac{(f_1 - f_2)R}{I} = \frac{(f_1 - f_2)R}{\frac{1}{2}m_1 R^2}$$

$$\alpha = \frac{2(f_1 - f_2)}{m_1 R} \quad \ldots\text{(iv)}$$

$$a_2 = R\alpha = \frac{2(f_1 - f_2)}{m_1} \quad \ldots\text{(v)}$$

(a) Solving Eqs. (i) to (v), we get

$$a_1 = \frac{8F}{3m_1 + 8m_2} \text{ and } a_2 = \frac{4F}{3m_1 + 8m_2}$$

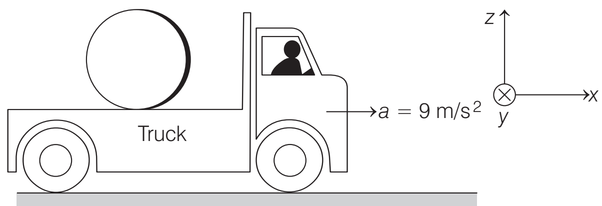
(b) $\quad f_1 = \dfrac{3m_1 F}{3m_1 + 8m_2};$

$$f_2 = \frac{m_1 F}{3m_1 + 8m_2}$$

Since, all quantities are positive, they are correctly shown in figures.

30. Given, mass of disc $m = 2$ kg and radius $R = 0.1$ m

(a) FBD of any one disc is



Frictional force on the disc should be in forward direction.

Let a_0 be the linear acceleration of CM of disc and α the angular acceleration about its CM. Then,

$$a_0 = \frac{f}{m} = \frac{f}{2} \qquad \ldots\text{(i)}$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{\frac{1}{2}mR^2} = \frac{2f}{mR} = \frac{2f}{2\times 0.1} = 10f \qquad \ldots\text{(ii)}$$

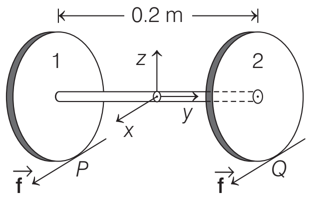
Since, there is no slipping between disc and truck. Therefore,

$$\therefore \alpha_0 + R\,\alpha = a \;\text{ or }\; \left(\frac{f}{2}\right) + (0.1)\,(10f) = a$$

or $\quad \dfrac{3}{2}f = a \Rightarrow \; f = \dfrac{2a}{3} = \dfrac{2\times 9.0}{3}\text{N}$

$\therefore \quad f = 6\text{N}$

Since, this force is acting in positive x-direction.



Therefore, in vector form $\mathbf{f} = (6\hat{\mathbf{i}})$ N

(b) $\tau = \mathbf{r} \times \mathbf{f}$

Here, $\mathbf{f} = (6\hat{\mathbf{i}})$ N (for both the discs)

$$\mathbf{r}_P = \mathbf{r}_1 = -\,0.1\,\hat{\mathbf{j}} - 0.1\hat{\mathbf{k}}$$

and $\quad \mathbf{r}_Q = \mathbf{r}_2 = 0.1\,\hat{\mathbf{j}} - 0.1\,\hat{\mathbf{k}}$

Therefore, frictional torque on disk 1 about point O (centre of mass).

$$\tau_1 = \mathbf{r}_1 \times \mathbf{f} = (-\,0.1\,\hat{\mathbf{j}} - 0.1\,\hat{\mathbf{k}}) \times (6\hat{\mathbf{i}})\text{ N-m}$$

$$= (0.6\,\hat{\mathbf{k}} - 0.6\,\hat{\mathbf{j}})$$

or $\quad \tau_1 = 0.6\,(\hat{\mathbf{k}} - \hat{\mathbf{j}})$ N-m

and $\quad |\,\tau_1\,| = \sqrt{(0.6)^2 + (0.6)^2}$

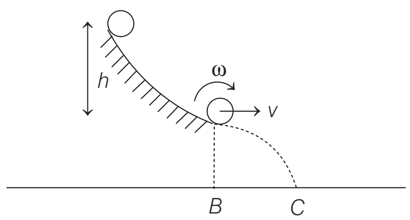
$$= 0.85 \text{ N-m}$$

Similarly, $\quad \tau_2 = \mathbf{r}_2 \times \mathbf{f} = 0.6(-\hat{\mathbf{j}} - \hat{\mathbf{k}})$

and $\quad |\,\tau_1\,| = |\,\tau_2\,| = 0.85$ N- m

31. $h = 2.6 - 1.0 = 1.6$ m

During pure rolling mechanical energy remains conserved



So, at bottom of track total kinetic energy of sphere will be mgh.

The ratio of $\quad \dfrac{K_R}{K_T} = \dfrac{2}{5}$

or $\quad K_T = \dfrac{5}{7}mgh = \dfrac{1}{2}mv^2$

$$\therefore \quad v = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}\times 9.8\times 1.6}$$

$$= 4.73 \text{ m/s}$$

In projectile motion

Time to fall to ground $= \sqrt{\dfrac{2\times 1}{9.8}} = 0.45$ s

$\therefore$ The desired distance $BC = vt = 2.13$ m

In air, during its flight as a projectile only mg is acting on the sphere which passes through its centre of mass. Therefore, net torque about centre of mass is zero or angular velocity will remain constant.

Topic 4 Collision in Rotational Motion

1 Initial kinetic energy of the given system,

$$\text{KE}_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right)^2$$

$$= \left(\frac{1}{2} + \frac{1}{16}\right)I_1\omega_1^2 = \frac{9}{16}I_1\omega_1^2 \qquad \ldots\text{(i)}$$

Now, using angular momentum conservation law (assuming angular velocity after contact is ω)

Initial angular momentum = Final angular momentum

$$I_1\omega_1 + \left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right) = I_1\omega' + \frac{I_1}{2}\omega'$$

$$\Rightarrow \quad \frac{5}{4}\omega_1 = \frac{3}{2}\omega' \text{ or } \omega' = \frac{5}{6}\omega_1 \qquad \ldots\text{(ii)}$$

Now, final kinetic energy (after contact) is

$$\text{KE}_f = \frac{1}{2}I_1\omega'^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\omega'^2$$

$$= \frac{1}{2}I_1\left(\frac{5}{6}\omega_1\right)^2 + \frac{1}{4}I_1\left(\frac{5}{6}\omega_1\right)^2 \text{ [using Eq. (ii)]}$$

$$= \left(\frac{25}{72} + \frac{25}{144}\right)I_1\omega_1^2$$

$$= \frac{25}{48}I_1\omega_1^2 \qquad \ldots\text{(iii)}$$

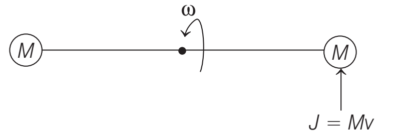
Hence, change in KE,

$$\Delta\text{KE} = \text{KE}_f - \text{KE}_i$$

$$= \frac{25}{48}I_1\omega_1^2 - \frac{9}{16}I_1\omega_1^2 \qquad \text{[using Eqs. (i)]}$$

$$\Delta\text{KE} = -\frac{1}{24}I_1\omega_1^2$$

2. Let ω be the angular velocity of the rod. Applying, angular impulse = change in angular momentum about centre of mass of the system

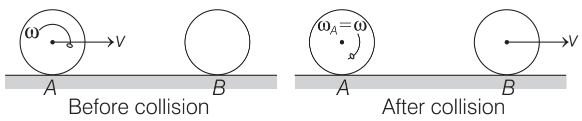


$$J.\frac{L}{2} = I_c\ \omega$$

$$\therefore \qquad (Mv)\left(\frac{L}{2}\right) = (2)\left(\frac{ML^2}{4}\right)\omega$$

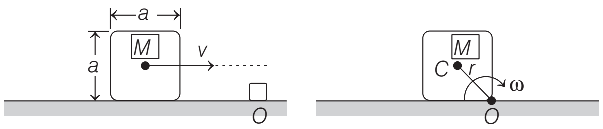
$$\therefore \qquad \omega = \frac{v}{L}$$

3. Since, it is head on elastic collision between two identical spheres, they will exchange their linear velocities, i.e., A comes to rest and B starts moving with linear velocity v. As there is no friction anywhere, torque on both the spheres about their centre of mass is zero and their angular velocities remain unchanged. Therefore,



$$\omega_A = \omega \quad \text{and} \quad \omega_B = 0.$$

4. $r = \sqrt{2}\,\dfrac{a}{2}$ or $r^2 = \dfrac{a^2}{2}$



Net torque about O is zero.

Therefore, angular momentum (L) about O will be conserved, or $L_i = L_f$

$$Mv\left(\frac{a}{2}\right) = I_o\,\omega = (I_{CM} + Mr^2)\,\omega$$

$$= \left\{\left(\frac{Ma^2}{6}\right) + M\left(\frac{a^2}{2}\right)\right\}\omega$$

$$= \frac{2}{3}Ma^2\omega$$

$$\omega = \frac{3v}{4a}$$

5. The data is incomplete. Let us assume that friction from ground on ring is not impulsive during impact.

From linear momentum conservation in horizontal direction, we have

$$(-2 \times 1) + (0.1 \times 20)$$
$$= (0.1 \times 0) + (2 \times v) \qquad \overset{-\text{ve} \quad +\text{ve}}{\longleftrightarrow}$$

Here, v is the velocity of CM of ring after impact.

Solving the above equation, we have $v = 0$

Thus, CM becomes stationary.

∴ Correct option is (a).

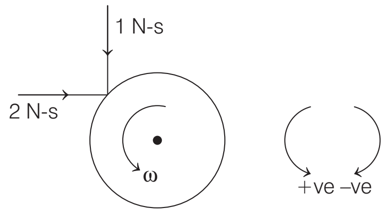
Linear impulse during impact

(i) In horizontal direction

$$J_1 = \Delta p = 0.1 \times 20 = 2\ \text{N-s}$$

(ii) In vertical direction $J_2 = \Delta p = 0.1 \times 10 = 1\ \text{N-s}$

Writing the equation (about CM)



Angular impulse = Change in angular momentum

We have,

$$1 \times \left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right) - 2 \times 0.5 \times \frac{1}{2} = 2 \times (0.5)^2\left[\omega - \frac{1}{0.5}\right]$$

Solving this equation ω comes out to be positive or ω anti-clockwise. So just after collision rightwards slipping is taking place.

Hence, friction is leftwards.

Therefore, option (c) is also correct.

6. $P_i = 0$

$\therefore \quad P_f = 0$ or $v_c = 0$

$$L_i = L_f \text{ or } (2mv)\,a + (2mv)\,(2a) = I\omega \qquad \ldots\text{(i)}$$

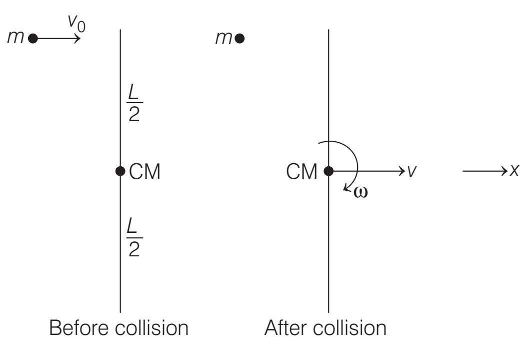
Here, $I = \dfrac{(8m)\,(6a)^2}{12} + m\,(2a)^2 + (2m)\,(a^2) = 30\,ma^2$

Substituting in Eq. (i), we get

$$\omega = \frac{v}{5a}$$

Further, $E = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2} \times (30ma^2)\left(\dfrac{v}{5a}\right)^2 = \dfrac{3mv^2}{5}$

7. (a) Let just after collision, velocity of CM of rod is v and angular velocity about CM is ω. Applying following three laws.



(1) External force on the system (rod + mass) in horizontal plane along x-axis is zero.

$\therefore$ Applying conservation of linear momentum in x-direction.

$$mv_0 = mv \qquad \ldots(\text{i})$$

(2) Net torque on the system about CM of rod is zero.

$\therefore$ Applying conservation of angular momentum about CM of rod, we get $mv_0\left(\frac{L}{2}\right) = I\,\omega$

or $$mv_0 \frac{L}{2} = \frac{ML^2}{12}\omega$$

or $$mv_0 = \frac{ML\omega}{6} \qquad \ldots(\text{ii})$$

(3) Since, the collision is elastic, kinetic energy is also conserved.

$$\therefore \quad \frac{1}{2}mv_0^2 = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

or $$mv_0^2 = Mv^2 + \frac{ML^2}{12}\omega^2 \qquad \ldots(\text{iii})$$

From Eqs. (i), (ii) and (iii), we get the following results

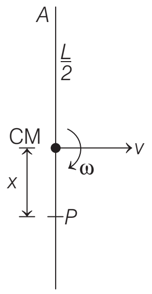
$$\frac{m}{M} = \frac{1}{4}$$

$$v = \frac{mv_0}{M} \text{ and } \omega = \frac{6\,mv_0}{ML}$$

(b) Point P will be at rest if $x\,\omega = v$

or $$x = \frac{v}{\omega} = \frac{mv_0/M}{6mv_0/ML}$$

or $$x = L/6$$



$$\therefore \quad AP = \frac{L}{2} + \frac{L}{6}$$

or $$AP = \frac{2}{3}L$$

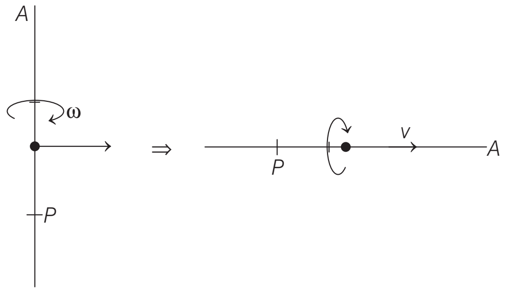
(c) After time $$t = \frac{\pi L}{3v_0}$$

angle rotated by rod, $$\theta = \omega t = \frac{6mv_0}{ML}\cdot\frac{\pi L}{3v_0}$$

$$= 2\pi\left(\frac{m}{M}\right)$$

$$= 2\pi\left(\frac{1}{4}\right) \therefore \quad \theta = \frac{\pi}{2}$$

Therefore, situation will be as shown below

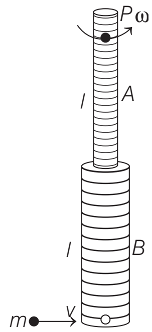


$\therefore$ Resultant velocity of point P will be

$$|\mathbf{v}_P| = \sqrt{2}v = \sqrt{2}\left(\frac{m}{M}\right)v_0$$

$$= \frac{\sqrt{2}}{4}v_0 = \frac{v_0}{2\sqrt{2}} \quad \text{or} \quad |\mathbf{v}_P| = \frac{v_0}{2\sqrt{2}}$$

8. System is free to rotate but not free to translate. During collision, net torque on the system (rod A + rod B + mass m) about point P is zero.



Therefore, angular momentum of system before collision
= angular momentum of system just after collision (about P).

Let ω be the angular velocity of system just after collision, then

$$L_i = L_f$$

$$\Rightarrow \quad mv(2l) = I\omega \qquad \ldots(\text{i})$$

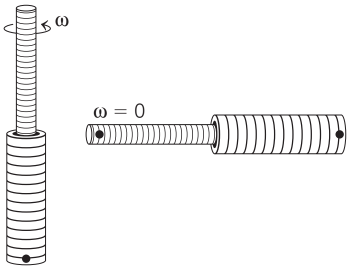
Here, I = moment of inertia of system about P

$$= m(2l)^2 + m_A(l^2/3) + m_B\left[\frac{l^2}{12} + \left(\frac{l}{2} + l\right)^2\right]$$

Given, $l = 0.6$ m, $m = 0.05$ kg, $m_A = 0.01$ kg and $m_B = 0.02$ kg.

Substituting the values, we get

$$I = 0.09 \text{ kg-m}^2$$



Therefore, from Eq. (i)

$$\omega = \frac{2mvl}{I} = \frac{(2)(0.05)(v)(0.6)}{0.09}$$

$$\omega = 0.67v \qquad \ldots(\text{ii})$$

Now, after collision, mechanical energy will be conserved.

Therefore, decrease in rotational KE
= increase in gravitational PE

or $\frac{1}{2} I\omega^2 = mg\ (2l) + m_A\ g\left(\frac{l}{2}\right) + m_B g\ (l + \frac{l}{2})$

or $\omega^2 = \frac{gl\ (4m + m_A + 3\ m_B)}{I}$

$= \frac{(9.8)\ (0.6)\ (4 \times 0.05 + 0.01 + 3 \times 0.02)}{0.09}$

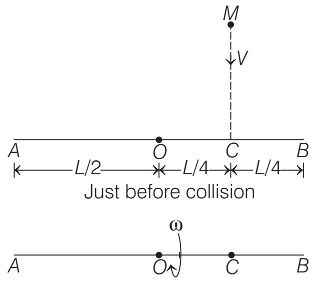
$= 17.64\ (\text{rad/s})^2$

$\therefore \quad \omega = 4.2\ \text{rad/s}$ ...(iii)

Equating Eqs. (ii) and (iii), we get

$$v = \frac{4.2}{0.67}\ \text{m/s} \quad \text{or} \quad v = 6.3\ \text{m/s}$$

9. In this problem we will write K for the angular momentum because L has been used for length of the rod.



Just before collision

Just after collision

(a) Angular momentum of the system (rod + insect) about the centre of the rod O will remain conserved just before collision and after collision, i.e. $K_i = K_f$.

or $\quad Mv\frac{L}{4} = I\ \omega = \left[\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2\right]\omega$

or $\quad Mv\frac{L}{4} = \frac{7}{48} ML^2\ \omega$

i.e. $\quad \omega = \frac{12}{7}\frac{v}{L}$ ...(i)

(b) Due to the torque of weight of insect about O, angular momentum of the system will not remain conserved (although angular velocity ω is constant). As the insect moves towards B, moment of inertia of the system increases, hence, the angular momentum of the system will increase.

Let at time t_1 the insect be at a distance x from O and by then the rod has rotated through an angle θ. Then, angular momentum at that moment,

$$K = \left[\frac{ML^2}{12} + Mx^2\right]\omega$$

Hence, $\frac{dK}{dt} = 2M\ \omega x\frac{dx}{dt}$ (ω = constant)

$\Rightarrow \quad \tau = 2M\omega\ x\frac{dx}{dt} \Rightarrow Mgx\cos\theta = 2\ M\ \omega\ x\frac{dx}{dt}$

$\Rightarrow \quad dx = \left(\frac{g}{2\omega}\right)\cos\omega t\ dt$ $(\because \theta = \omega t)$

At time $t = 0$, $x = L/4$ and at time $t = T/4$ or $\pi/2\omega$, $x = L/2$.

Substituting these limits, we get

$$\int_{L/4}^{L/2} dx = \frac{g}{2\omega}\int_0^{\pi/2\omega} (\cos\omega t)\ dt$$

$$[x]_{L/4}^{L/2} = \frac{g}{2\omega^2}[\sin\omega t]_0^{\pi/2\omega}$$

$$\Rightarrow \quad \left(\frac{L}{2} - \frac{L}{4}\right) = \frac{g}{2\omega^2}\left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$\frac{L}{4} = \frac{g}{2\omega^2} \quad \text{or} \quad \omega = \sqrt{\frac{2g}{L}}$$

Substituting in Eq. (i), we get

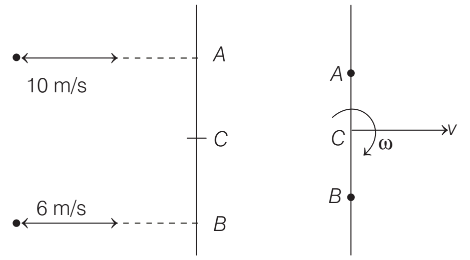
$$\sqrt{\frac{2g}{L}} = \frac{12}{7}.\frac{v}{L}$$

or $\quad v = \frac{7}{12}\sqrt{2gL}$

$= \frac{7}{12}\sqrt{2 \times 10 \times 1.8}$

$\therefore \quad v = 3.5\text{m/s}$

10. Let v be the velocity of centre of mass (also at C) of rod and two particles and ω the angular velocity of the system.



From conservation of linear momentum

$$(0.08)\ (10 + 6) = [0.08 + 0.08 + 0.16]\ v$$

$\therefore \quad v = 4\ \text{m/s}$

$AC = CB = 0.5\text{m}$

Similarly, conservation of angular momentum about point C.

$(0.08)\ (10)\ (0.5) - (0.08)\ (6)\ (0.5) = I\omega$ ...(i)

Here, $\quad I = I_{\text{rod}} + I_{\text{two particles}}$

$= \frac{(1.6)\ (\sqrt{3})^2}{12} + 2\ (0.08)\ (0.5)^2$

$= 0.08\ \text{kg-m}^2$

Substituting in Eq. (i), we get

$$\omega = 2\ \text{rad/s}$$

Loss of kinetic energy

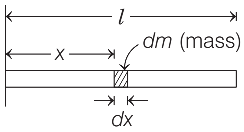
$= \frac{1}{2}(0.08)\ (10)^2 + \frac{1}{2}(0.08)\ (6)^2$

$- \frac{1}{2}(0.08 + 0.08 + 0.16)\ (4)^2 - \frac{1}{2}(0.08)\ (2)^2$

$= 4 + 1.44 - 2.56 - 0.16 = 2.72\ \text{J}$

Topic 5 Miscellaneous Problems

1. **Key Idea** In a uniform rod, mass per unit length remains constant. If it is denoted by λ, then $\lambda = \frac{m}{l}$ = constant for all segments of rod.



To find tension at x distance from fixed end, let us assume an element of dx length and dm mass. Tension on this part due to rotation is

$$dT = Kx \quad \ldots(i)$$

As, $K = m\omega^2$

For this element, $K = (dm)\omega^2 \quad \ldots(ii)$

$\therefore$ $dT = (dm)\omega^2 x \quad \ldots(iii)$

To find complete tension in the rod, we need to integrate Eq. (iii),

$$\int_0^T dT = \int_0^m (dm)\,\omega^2 x \quad \ldots(iv)$$

Using linear mass density,

$$\lambda = \frac{m}{l} = \frac{dm}{dx}$$

$\Rightarrow$ $dm = \frac{m}{l} \cdot dx \quad \ldots(v)$

Putting the value of Eq. (v) in Eq. (iv), we get

$$T = \int_x^l \frac{m}{l} \cdot \omega^2 x \cdot dx = \frac{m}{l} \cdot \omega^2 \left[\frac{x^2}{2}\right]_x^l$$

$\Rightarrow$ $T = \frac{m\omega^2}{2l}[l^2 - x^2]$ or $T \propto -x^2$

2. **Key Idea** The rectangular box rotates due to torque of weight about its centre of mass.

Now, angular impulse of weight = Change in angular momentum.

$\therefore$ $mg\frac{l}{2} \times \tau = \frac{ml^2}{3}\omega$

$\Rightarrow$ $\omega = \frac{3g \times \tau}{2 \times l}$

Substituting the given values, we get

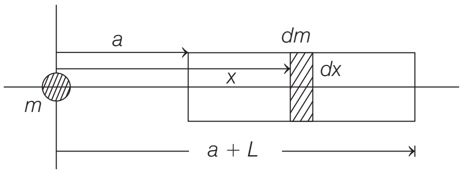
$$= \frac{3 \times 10 \times 0.01}{2 \times 0.3} = 0.5 \text{ rad s}^{-1}$$

Time of fall of box,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} \approx 1 \text{ s}$$

So, angle turned by box in reaching ground is
$\theta = \omega t = 0.5 \times 1 = 0.5$ rad

3. Given, situation is,



Force of attraction between mass 'm' and an elemental mass 'dm' of rod is

$$dF = \frac{Gmdm}{x^2} = \frac{Gm(A + Bx^2)\,dx}{x^2}$$

Total attraction force is sum of all such differential forces produced by elemental parts of rod from $x = a$ to $x = a + L$.

$$\therefore\ F = \int dF = \int_{x=a}^{x=a+L} \frac{Gm(A + Bx^2)}{x^2}\,dx$$

$$= Gm\int_{x=a}^{x=a+L}\left(\frac{A}{x^2} + B\right)dx$$

$$= Gm\left[-\frac{A}{x} + Bx\right]_{x=a}^{x=a+L}$$

$$= Gm\left(\frac{-A}{a+L} + B(a+L) + \frac{A}{a} - Ba\right)$$

$$= Gm\left(\frac{A}{a} - \frac{A}{a+L} + BL\right) = Gm\left\{A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right\}$$

4. Given, $m = 1$ kg

$|\tau| = 2.5$ N-m, $F = 1$ N and $r = 5$ m

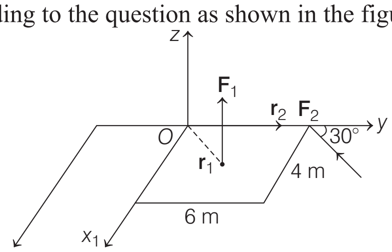
We know that, torque $|\tau| = rF\sin\theta$

$\Rightarrow$ $2.5 = 5 \times 1 \times \sin\theta$

$\Rightarrow$ $\sin\theta = \frac{1}{2}$

or $\theta = \frac{\pi}{6}$ rad

5. According to the question as shown in the figure below,



$$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} \text{ and } \mathbf{r}_2 = 6\hat{\mathbf{j}}$$

$$\mathbf{F}_1 = F\,\hat{\mathbf{k}}$$

and $\mathbf{F}_2 = (-\sin 30^\circ\,\hat{\mathbf{i}} - \cos 30^\circ\,\hat{\mathbf{j}})\,F$

Moment of force is given as, $\tau = \mathbf{r} \times \mathbf{F}$

where, $\mathbf{r}$ is the perpendicular distance and $\mathbf{F}$ is the force.

$\therefore$ Moment due to $\mathbf{F}_1$

$$\tau_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (F\,\hat{\mathbf{k}})$$

$$= -2F\,\hat{\mathbf{j}} + 3F\,\hat{\mathbf{i}} \quad \ldots(i)$$

Moment due to $\mathbf{F}_2$

$$\tau_2 = (6\hat{\mathbf{j}}) \times (-\sin 30^\circ \hat{\mathbf{i}} - \cos 30\hat{\mathbf{j}})\, F$$

$$= 6\sin 30^\circ F\ \hat{\mathbf{k}} = 3F\ \hat{\mathbf{k}} \qquad \ldots\text{(ii)}$$

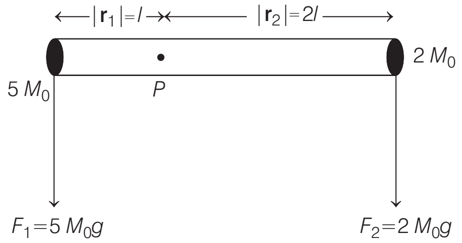
$\therefore$ Resultant torque,

$$\tau = \tau_1 + \tau_2 = 3F\ \hat{\mathbf{i}} - 2F\ \hat{\mathbf{j}} + 3F\ \hat{\mathbf{k}}$$

$$= (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\,F.$$

6. **Key Idea** When a rod is pivoted at any point, its angular acceleration is given by $\tau_{net} = I\alpha$.

The given condition can be drawn in the figure below



Torque (τ) about $\quad P = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 \qquad \ldots$ (i)

$\Rightarrow \quad \tau = l \times 5M_0 g$ (outwards) $- 2l \times 2M_0 g$ inwards)

$\Rightarrow \quad \tau = 5M_0 gl - 4M_0 gl$ (outwards)

$\Rightarrow \quad \tau = M_0 gl$ (outwards)

or $\quad \tau = M_0 gl \qquad \ldots$(ii)

Now we know that, torque is also given by

$$\tau = I\alpha \qquad \ldots\text{(iii)}$$

Here, I = moment of inertia (w.r.t. point P) of rod and α = angular acceleration.

For point P, $I = (5M_0) \times l^2 + (2M_0)(2l)^2 \qquad [\because I = MR^2]$

$$\Rightarrow \quad I = 13M_0 l^2 \qquad \ldots\text{(iv)}$$

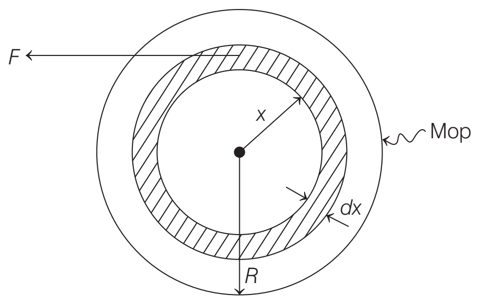
Putting value of I from Eq. (iv) in Eq. (iii), we get

$$\tau = (13M_0 l^2)\,\alpha \qquad \ldots\text{(v)}$$

From Eqs. (ii) and (v), we get

$$M_0 gl = 13M_0 l^2 \alpha \Rightarrow \alpha = \frac{g}{13l}$$

7 Let a small strip of mop has width dx and radius x, as shown below,



Torque applied to move this strip is

$d\tau$ = Force on strip

$\times$ Perpendicular distance from the axis

$\Rightarrow d\tau$ = Force per unit area $\times$ Area of strip

$\times$ Perpendicular distance from the axis.

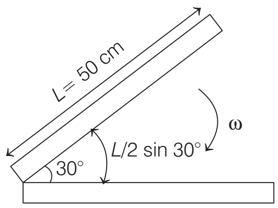
$$= \frac{\mu F}{\pi R^2} \cdot 2\pi x dx \cdot x \;\Rightarrow\; d\tau = \frac{2\mu F x^2}{R^2} \cdot dx$$

So, total torque to be applied on the mop is

$$\tau = \int_{x=0}^{x=R} d\tau = \int_0^R \frac{2\mu F x^2}{R^2} \cdot dx$$

$$= \frac{2\mu F}{R^2} \times \frac{R^3}{3} = \frac{2}{3}\mu FR \text{ (N-m)}$$

8 Lose of potential energy of rod = Gain of kinetic energy



$$\therefore \quad \Delta PE = \frac{1}{2} I\omega^2$$

(where, I = MOI of rod and ω = angular frequency of rod)

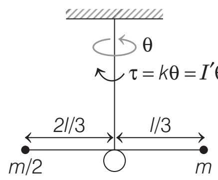
$$\Rightarrow \quad Mg \times \frac{L}{2}\sin 30^\circ = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow \quad Mg\frac{L}{2} \times \frac{1}{2} = \frac{1}{2} \times I \times \omega^2 \;\Rightarrow\; \frac{Mg\ L \times 2}{4 \times I} = \omega^2$$

$$\Rightarrow \quad \sqrt{\frac{MgL}{4 \times \frac{ML^2}{3}} \times \frac{2}{1}} = \omega \qquad \left[\because I = \frac{ML^2}{3} \text{ for rod}\right]$$

$$\omega = \sqrt{30} \text{ rad/s}$$

9 Since in the given question, rotational torque, $\tau \propto$ angular displacement.

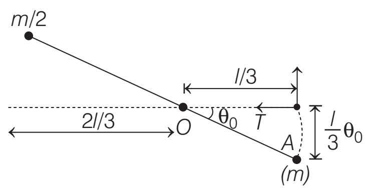


Thus, when it will be released, the system will execute SHM with a time period, $T = 2\pi\sqrt{\frac{I}{k}}$

(Where I is moment of inertia and k is torsional constant)

and the angular frequency is given as, $\omega = \sqrt{\frac{k}{I}}$.

If we know look at the top view of the above figure, we have



At some angular displacement 'θ_0', at point 'A' the maximum velocity will be

$$v_{\max} = \frac{l}{3}\theta_0\omega = \frac{l}{3}\theta_0\sqrt{\frac{k}{I}} \qquad \text{...(i)}$$

Then, tension in the rod when it passes through mean position will be

$$T = \frac{m \times v_{\max}^2}{\frac{l}{3}} = \frac{ml^2\theta_0^2 k \times 3}{9 \times l \times I} \qquad \text{[using Eq. (i)]}$$

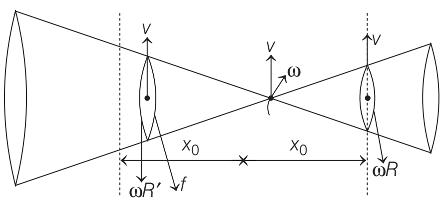
$$= \frac{ml\,\theta_0^2 k}{3I}$$

The moment of inertia I at point O,

$$= \frac{m}{2}\left(\frac{2l}{3}\right)^2 + m\left(\frac{l}{3}\right)^2 = \frac{2l^2 m}{9} + \frac{ml^2}{9} = \frac{3ml^2}{9} = \frac{ml^2}{3}$$

$$\Rightarrow \quad T = \frac{ml\,\theta_0^2\;k \times 3}{3 \times ml^2} = \frac{\theta_0^2 k}{l} = \frac{k\theta_0^2}{l}$$

10.



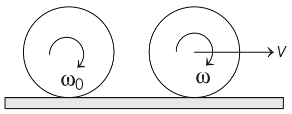
At distance x_0 from O, $v = \omega R$

Distance less than x_0, $v > \omega R$

Initially, there is pure rolling at both the contacts. As the cone moves forward, slipping at AB will start in forward direction, as radius at left contact decreases.

Thus, the cone will start turning towards left. As it moves, further slipping at CD will start in backward direction which will also turn the cone towards left.

11.



$$\omega = v/r$$

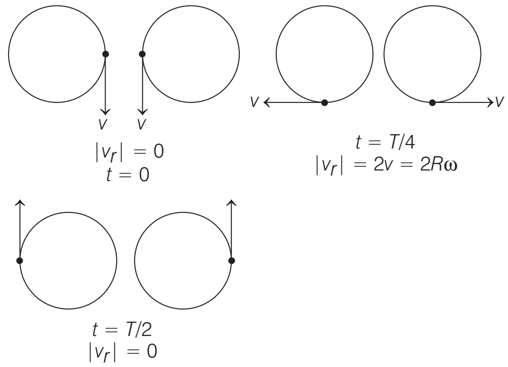
From conservation of angular momentum about bottom most point

$$mr^2\omega_0 = mvr + mr^2 \times v/r$$

$$\Rightarrow \qquad v = \frac{\omega_0 r}{2}$$

12. Language of question is not very clear. For example, disc is rotating. Its different points have different velocities. Relative velocity of pebble with respect to which point, it is not clear. Further, actual initial positions of P and Q are also not given.

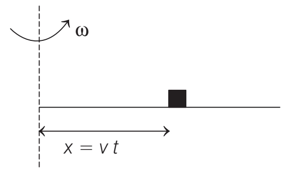
13. Language of question is wrong because relative speed is not the correct word. Relative speed between two is always zero. The correct word is magnitude of relative velocity.



Corresponding to above values, the correct graph is (a).

14. $|\mathbf{L}|$ or $L = I\omega$ (about axis of rod)

$$I = I_{\text{rod}} + mx^2 = I_{\text{rod}} + mv^2t^2$$



Here, m = mass of insect

$$\therefore \qquad L = (I_{\text{rod}} + mv^2t^2)\,\omega$$

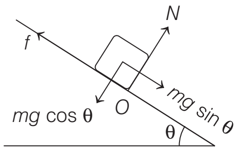
Now $|\tau| = \frac{dL}{dt} = (2mv^2t\omega)$ or $|\tau| \propto t$

i.e. the graph is straight line passing through origin.

After time T, L = constant

$$\therefore \qquad |\tau| \text{ or } \frac{dL}{dt} = 0$$

15. Condition of sliding is



$$mg\sin\theta > \mu mg\cos\theta$$

or $\qquad \tan\theta > \mu$ or $\tan\theta > \sqrt{3}$ $\qquad$ ...(i)

Condition of toppling is

Torque of $mg\sin\theta$ about 0 > torque of $mg\cos\theta$ about

$$\therefore \qquad (mg\sin\theta)\left(\frac{15}{2}\right) > (mg\cos\theta)\left(\frac{10}{2}\right)$$

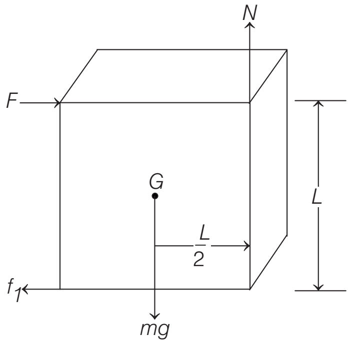
or $\qquad \tan\theta > \frac{2}{3}$ $\qquad$ ...(ii)

With increase in value of θ, condition of sliding is satisfied first.

16. $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

Direction of $(\mathbf{r} \times \mathbf{v})$, hence the direction of angular momentum remains the same.

17. At the critical condition, normal reaction N will pass through point P. In this condition, $\tau_N = 0 = \tau_{fr}$ (about P)
the block will topple when



$$\tau_F > \tau_{mg} \quad \text{or} \quad FL > (mg)\frac{L}{2}$$

$$\therefore \qquad F > \frac{mg}{2}$$

Therefore, the minimum force required to topple the block is

$$F = \frac{mg}{2}$$

18. Work done, $W = \frac{1}{2}I\omega^2$

If x is the distance of mass 0.3 kg from the centre of mass, we will have,

$$I = (0.3)x^2 + (0.7)(1.4 - x)^2$$

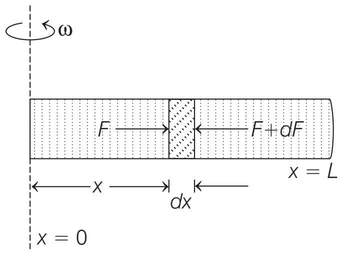
For work to be minimum, the moment of inertia (I) should be minimum, or $\frac{dI}{dx} = 0$

or $\quad 2(0.3x) - 2(0.7)(1.4 - x) = 0$

or $\quad (0.3)x = (0.7)(1.4 - x)$

$$\Rightarrow \qquad x = \frac{(0.7)(1.4)}{0.3 + 0.7} = 0.98\,\text{m}$$

19. Mass of the element dx is $m = \frac{M}{L}dx$.



This element needs centripetal force for rotation.

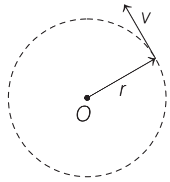
$$\therefore \qquad dF = mx\omega^2 = \left(\frac{M}{L}x\omega^2\,dx\right)$$

$$\therefore \qquad F = \int_0^L dF = \frac{M}{L}.\omega^2\int_0^L x\,dx = \frac{M\omega^2 L}{2}$$

This is the force exerted by the liquid at the other end.

20. $V = \frac{Kr^2}{2}$

$$F = -\frac{dV}{dr} = -Kr\,(\text{towards centre})\left[F = -\frac{dV}{dr}\right]$$



$$kR = \frac{mv^2}{R}\ \text{(Centripetal force)}$$

$$v = \sqrt{\frac{kR^2}{m}} = \sqrt{\frac{k}{m}}R$$

$$L = mvR = \sqrt{mk}R^2$$

21. $F = (\alpha t)\hat{i} + \beta\hat{j}$ $\qquad$ [at $t = 0, v = 0, r = 0$]

$\alpha = 1, \beta = 1 \Rightarrow F = t\hat{i} + \hat{j}$

$$m\frac{dv}{dt} = t\hat{i} + \hat{j}$$

On integrating,

$$mv = \frac{t^2}{2}\hat{i} + t\hat{j} \qquad [m = 1\,\text{kg}]$$

$$\frac{dr}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j} = v \Rightarrow v = \frac{1}{2}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \text{ at } t = 1\,\text{s}$$

Again, on integrating,

$$r = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j} \qquad [r = 0 \text{ at } t = 0]$$

At $t = 1\,\text{s}$, $\tau = (r \times F) = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j}) = -\frac{1}{3}\hat{\mathbf{k}}$

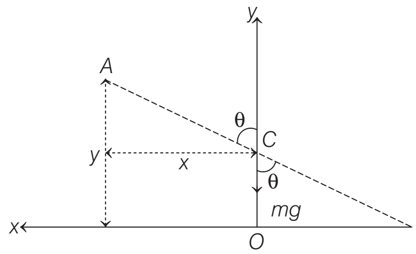
22. When the bar makes an angle θ, the height of its COM (mid-point) is $\frac{L}{2}\cos\theta$.

$\therefore$ Displacement $= L - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$

Since, force on COM is only along the vertical direction, hence COM is falling vertically downward. Instantaneous torque about point of contact is

$$\tau = mg \times \frac{L}{2}\sin\theta$$

or $\quad \tau \propto \sin\theta$



Now, $x = \frac{L}{2}\sin\theta$

$y = L\cos\theta$

$$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1$$

Path of A is an ellipse.

23. When force $F = 0 \Rightarrow$ potential energy U = constant

$F \neq 0 \Rightarrow$ force is conservative $\Rightarrow$ Total energy E = constant

List-I

(P) $\mathbf{r}(t) = \alpha t\hat{\mathbf{i}} + \beta t\hat{\mathbf{j}}$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} = \text{constant} \Rightarrow \mathbf{p} = \text{constant}$$

$$|\mathbf{v}| = \sqrt{\alpha^2+\beta^2} = \text{constant} \Rightarrow K = \text{constant}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{a} = 0 \Rightarrow F = 0 \Rightarrow U = \text{constant}$$

$$E = U + K = \text{constant}$$

$$\mathbf{L} = m(\mathbf{r}\times\mathbf{v}) = 0$$

$$\mathbf{L} = \text{constant}$$

$P \to 1, 2, 3, 4, 5$

(Q) $\mathbf{r}(t) = \alpha\cos\omega t\hat{\mathbf{i}} + \beta\sin\omega t\hat{\mathbf{j}}$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\omega\sin\omega t(-\hat{\mathbf{i}}) + \beta\omega\cos\omega t\hat{\mathbf{j}} \neq \text{constant}$$

$\Rightarrow$ $\mathbf{p} \neq$ constant

$$|\mathbf{v}| = \omega\sqrt{(\alpha\sin\omega t)^2 + (\beta\cos\omega t)^2} \neq \text{constant}$$

$\Rightarrow$ $K \neq$ constant

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -\omega^2\mathbf{r} \neq 0$$

$\Rightarrow$ E = constant = $K + U$

But $K \neq$ constant $\Rightarrow U \neq$ constant

$$\mathbf{L} = m(\mathbf{r}\times\mathbf{v}) = m\omega\alpha\beta\,(\hat{\mathbf{k}}) = \text{constant}$$

$Q \to 2, 5$

(R) $\mathbf{r}(t) = \alpha\,(\cos\omega t\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}})$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\omega\,[\sin\omega t(-\hat{\mathbf{i}}) + \cos\omega t\hat{\mathbf{j}}] \neq \text{constant}$$

$\Rightarrow \mathbf{p} \neq$ constant

$$|\mathbf{v}| = \alpha\omega = \text{constant} \Rightarrow K = \text{constant}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -\omega^2\mathbf{r} \neq 0 \Rightarrow E = \text{constant}, U = \text{constant}$$

$$\mathbf{L} = m(\mathbf{r}\times\mathbf{v}) = m\omega\alpha^2\hat{\mathbf{k}} = \text{constant}$$

$R \to 2, 3, 4, 5$

(S) $\mathbf{r}(t) = \alpha t\hat{\mathbf{i}} + \frac{\beta}{2}t^2\hat{\mathbf{j}}$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \alpha\hat{\mathbf{i}} + \beta t\hat{\mathbf{j}} \neq \text{constant} \Rightarrow \mathbf{p} \neq \text{constant}$$

$$|\mathbf{v}| = \sqrt{\alpha^2 + (\beta t)^2} \neq \text{constant} \Rightarrow K \neq \text{constant}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \beta\hat{\mathbf{j}} \neq 0 \Rightarrow E = \text{constant} = K + U$$

But $K \neq$ constant

$\therefore$ $U \neq$ constant

$$\mathbf{L} = m(\mathbf{r}\times\mathbf{v}) = \frac{1}{2}\alpha\beta t^2\hat{\mathbf{k}} \neq \text{constant}$$

$S \to 5$

24. Question is not very clear.

25. If height of the cone $h >> r$

Then, $\mu N = mg$

$$\mu m(R-r)\omega_0^2 = mg$$

$$\omega_0 = \sqrt{\frac{g}{\mu(R-r)}}$$

26-27. (i) Every particle of the disc is rotating in a horizontal circle.

(ii) Actual velocity of any particle is horizontal.

(iii) Magnitude of velocity of any particle is

$$v = r\omega$$

where, r is the perpendicular distance of that particle from actual axis of rotation (Z-axis).

(iv) When it is broken into two parts then actual velocity of any particle is resultant of two velocities

$$v_1 = r_1\omega_1 \text{ and } v_2 = r_2\omega_2$$

Here,

r_1 = perpendicular distance of centre of mass from Z-axis.

ω_1 = angular speed of rotation of centre of mass from Z-axis.

r_2 = distance of particle from centre of mass and

ω_2 = angular speed of rotation of the disc about the axis passing through centre of mass.

(v) Net v will be horizontal, if v_1 and v_2 both are horizontal. Further, v_1 is already horizontal, because centre of mass is rotating about a vertical Z-axis. To make v_2 also horizontal, second axis should also be vertical.

28. $$\frac{1}{2}I(2\omega)^2 = \frac{1}{2}kx_1^2 \qquad \text{...(i)}$$

$$\frac{1}{2}(2I)(\omega)^2 = \frac{1}{2}kx_2^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{x_1}{x_2} = \sqrt{2}$$

29. Let ω' be the common velocity. Then from conservation of angular momentum, we have

$$(I + 2I)\omega' = I(2\omega) + 2I(\omega)$$

$$\omega' = \frac{4}{3}\omega$$

From the equation,

angular impulse = change in angular momentum, for any of the disc, we have

$$\tau\cdot t = I(2\omega) - I\left(\frac{4}{3}\omega\right) = \frac{2I\omega}{3}$$

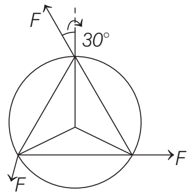
$$\therefore \qquad \tau = \frac{2I\omega}{3t}$$

30. Loss of kinetic energy $= K_i - K_f$

$$= \left\{\frac{1}{2} I (2\omega)^2 + \frac{1}{2} (2I)(\omega)^2\right\} - \frac{1}{2}(3I)\left(\frac{4}{3}\omega\right)^2 = \frac{1}{3} I\omega^2$$

31. Angular impulse = change in angular momentum

$\therefore \int \tau \, dt = I\omega$



$$\Rightarrow \quad \omega = \frac{\int \tau \, dt}{I} = \frac{\int_0^t 3F \sin 30° \, R \, dt}{I}$$

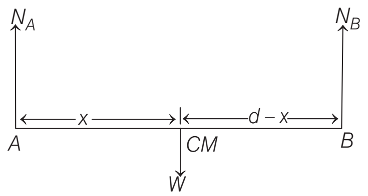
Substituting the values, we have

$$\omega = \frac{3\,(0.5)\,(0.5)\,(0.5)\,(1)}{\dfrac{1.5\,(0.5)^2}{2}} = 2 \text{ rad/s}$$

32. Net torque of all the forces about B should be zero.

$$\therefore \quad W\,(d - x) = N_A \cdot d$$

or $$N_A = \left(\frac{d - x}{d}\right) W$$



For vertical equilibrium of rod

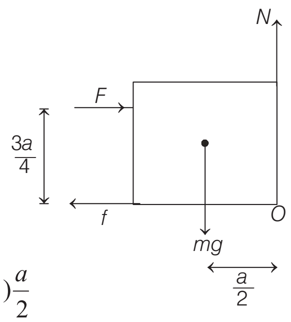
$$N_A + N_B = W$$

$$\therefore \quad N_B = \frac{x}{d} W = W - N_A$$

$$= W - \left(\frac{d - x}{d}\right) W = \frac{x}{d} W$$

33. Taking moments about point O

Moments of N (normal reaction) and f (force of friction) are zero. In critical case normal reaction will pass through O. To tip about the edge, moment of F should be greater than moment of mg. Or,



$$F\left(\frac{3a}{4}\right) > (mg)\frac{a}{2}$$

or $$F > \frac{2}{3} mg$$

34. $\alpha = \dfrac{\tau}{I}$

$\tau = F \times r_\perp$: Torque is same in both the cases but moment of inertia depends on distribution of mass from the axis. Distribution of mass in both the cases is different. Therefore, moment of inertia will be different or the angular acceleration α will be different.

35. Angular momentum of the system about an axis perpendicular to plane of paper and passing through O will remain conserved.

$$L_i = L_f$$

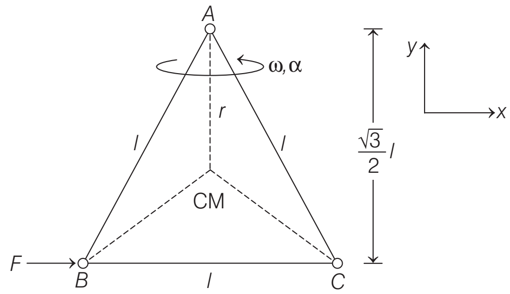
$$\therefore \quad mvL = I\omega = \left(mL^2 + \frac{ML^2}{3}\right)\omega$$

$$\therefore \quad \omega = \frac{3mv}{L\,(3m + M)}$$

36. (a) The distance of centre of mass (CM) of the system about point A will be $r = \dfrac{l}{\sqrt{3}}$

Therefore, the magnitude of horizontal force exerted by the hinge on the body is

F = centripetal force or $F = (3m)\, r\omega^2$



or $$F = (3m)\left(\frac{l}{\sqrt{3}}\right)\omega^2$$

or $$F = \sqrt{3}\, ml\omega^2$$

(b) Angular acceleration of system about point A is

$$\alpha = \frac{\tau_A}{I_A} = \frac{(F)\left(\dfrac{\sqrt{3}}{2} l\right)}{2ml^2} = \frac{\sqrt{3}\,F}{4ml}$$

Now, acceleration of CM along x-axis is

$$a_x = r\alpha = \left(\frac{l}{\sqrt{3}}\right)\left(\frac{\sqrt{3}\,F}{4ml}\right) \quad \text{or} \quad a_x = \frac{F}{4m}$$

Let F_x be the force applied by the hinge along X-axis.

Then, $$F_x + F = (3m) a_x$$

or $$F_x + F = (3m)\left(\frac{F}{4m}\right)$$

or $$F_x + F = \frac{3}{4} F$$

or $$F_x = -\frac{F}{4}$$

Further if F_y be the force applied by the hinge along Y-axis. Then,

$$F_y = \text{ centripetal force}$$

or $$F_y = \sqrt{3}\, ml\omega^2$$

37. Let r be the perpendicular distance of CM from the line AB and ω the angular velocity of the sheet just after colliding with rubber obstacle for the first time.

Obviously the linear velocity of CM before and after collision will be $v_i = (r)(1 \text{ rad/s}) = r$ and $v_f = r\omega$.

$\mathbf{v}_i$ and $\mathbf{v}_f$ will be in opposite directions.

Now, linear impulse on CM

= change in linear momentum of CM

or $\quad 6 = m(v_f + v_i) = 30(r + r\omega)$

or $\quad r(1+\omega) = \dfrac{1}{5}$ ...(i)

Similarly, angular impulse about AB = change in angular momentum about AB

Angular impulse = Linear impulse

× perpendicular distance of impulse from AB

Hence, $\quad 6(0.5 \text{ m}) = I_{AB}(\omega + 1)$

(Initial angular velocity = 1 rad/s)

or $\quad 3 = [I_{CM} + Mr^2](1+\omega)$

or $\quad 3 = [1.2 + 30r^2](1+\omega)$ ...(ii)

Solving Eqs. (i) and (ii) for r, we get

$r = 0.4$ m and $r = 0.1$ m

But at $r = 0.4$ m, ω comes out to be negative (-0.5 rad/s) which is not acceptable. Therefore,

(a) r = distance of CM from $AB = 0.1$ m

(b) Substituting $r = 0.1$ m in Eq. (i), we get $\omega = 1$ rad/s *ie,* the angular velocity with which sheet comes back after the first impact is 1 rad/s.

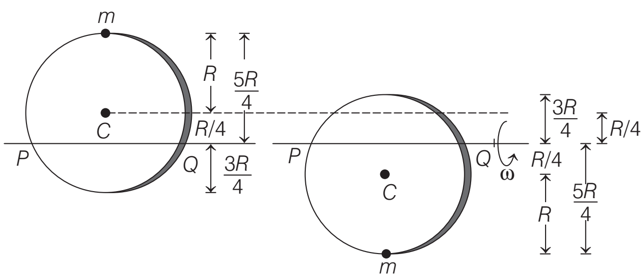
(c) Since, the sheet returns with same angular velocity of 1 rad/s, the sheet will never come to rest.

38. Initial and final positions are shown below.

Decrease in potential energy of mass

$$= mg\left\{2 \times \frac{5R}{4}\right\} = \frac{5\,mgR}{2}$$

Decrease in potential energy of disc $= mg\left\{2 \times \dfrac{R}{4}\right\} = \dfrac{mgR}{2}$



Therefore, total decrease in potential energy of system

$$= \frac{5mgR}{2} + \frac{mgR}{2} = 3mgR$$

Gain in kinetic energy of system $= \dfrac{1}{2} I\omega^2$

where, I = moment of inertia of system (disc + mass) about axis PQ

= moment of inertia of disc

+ moment of inertia of mass

$$= \left\{\frac{mR^2}{4} + m\left(\frac{R}{4}\right)^2\right\} + m\left(\frac{5R}{4}\right)^2$$

$$I = \frac{15\,mR^2}{8}$$

From conservation of mechanical energy,

Decrease in potential energy = Gain in kinetic energy

$$\therefore \quad 3\,mgR = \frac{1}{2}\left(\frac{15\,mR^2}{8}\right)\omega^2$$

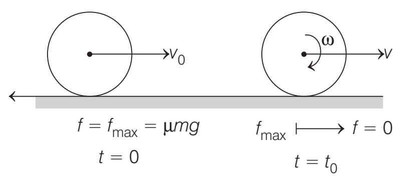
$$\Rightarrow \quad \omega = \sqrt{\frac{16g}{5R}}$$

Therefore, linear speed of particle at its lowest point

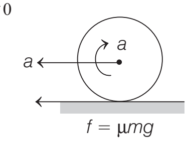
$$v = \left(\frac{5R}{4}\right)\omega = \frac{5R}{4}\sqrt{\frac{16g}{5R}}$$

$$v = \sqrt{5gR}$$

39. (a) Between the time $t = 0$ to $t = t_0$. There is forward sliding, so friction f is leftwards and maximum i.e. μmg. For time $t > t_0$, friction f will become zero, because now pure rolling has started i.e. there is no sliding (no relative motion) between the points of contact.



So, for time $t < t_0$



Linear retardation, $a = \dfrac{f}{m} = \mu g$

and angular acceleration, $\alpha = \dfrac{\tau}{I} = \dfrac{fR}{\frac{1}{2}mR^2} = \dfrac{2\mu g}{R}$

Now, let v be the linear velocity and ω, the angular velocity of the disc at time $t = t_0$, then

$$v = v_0 - at_0 = v_0 - \mu g t_0 \quad \text{...(i)}$$

and $\quad \omega = \alpha\, t_0 = \dfrac{2\mu g t_0}{R}$ ...(ii)

For pure rolling to take place

$$v = R\omega$$

i.e. $\quad v_0 - \mu g t_0 = 2\mu g t_0 \quad \Rightarrow t_0 = \dfrac{v_0}{3\mu\, g}$

Substituting in Eq. (i), we have

$$v = v_0 - \mu g\left(\frac{v_0}{3\mu\, g}\right) \Rightarrow v = \frac{2}{3}v_0$$

(b) **Work done by friction**

For $t \le t_0$, linear velocity of disc at any time t is $v = v_0 - \mu g t$ and angular velocity is $\omega = \alpha t = \dfrac{2\mu g t}{R}$. From Work-energy theorem, work done by friction upto time t = Kinetic energy of the disc at time t – Kinetic energy of the disc at time $t = 0$

$$\therefore\ W = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}m[v_0 - \mu g t]^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{2\mu g t}{R}\right)^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}[mv_0^2 + m\mu^2 g^2 t^2 - 2mv_0\mu g t + 2m\mu^2 g^2 t^2 - mv_0^2]$$

or $W = \dfrac{m\mu g t}{2}[3\mu g t - 2v_0]$

For $t > t_0$, friction force is zero i.e. work done in friction is zero. Hence, the energy will be conserved.

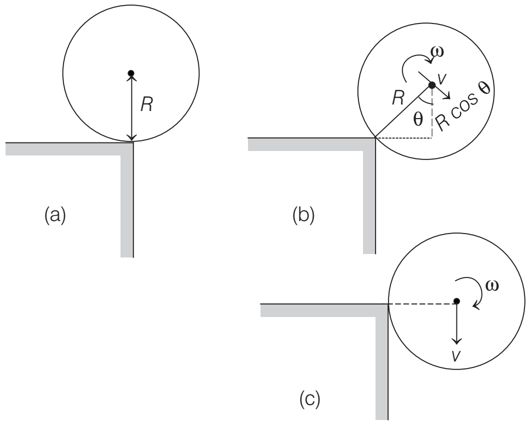
Therefore, total work done by friction over a time t much longer then t_0 is total work done upto time t_0 (because beyond this work done by friction is zero) which is equal to

$$W = \frac{m\mu g t_0}{2}[3\mu g t_0 - 2v_0]$$

Substituting $t_0 = v_0/3\mu g$, we get

$$W = \frac{mv_0}{6}[v_0 - 2v_0] \Rightarrow W = -\frac{mv_0^2}{6}$$

40. (a) The cylinder rotates about the point of contact. Hence, the mechanical energy of the cylinder will be conserved i.e.



$$\therefore \quad (PE + KE)_1 = (PE + KE)_2$$

$$\therefore \quad mgR + 0 = mgR\cos\theta + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

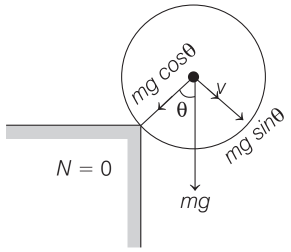
but $\omega = v/R$ (no slipping at point of contact)

and $\quad I = \dfrac{1}{2}mR^2$

Therefore,

$$mgR = mgR\cos\theta + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$$

or $\dfrac{3}{4}v^2 = gR(1 - \cos\theta)$



or $\quad v^2 = \dfrac{4}{3}gR(1 - \cos\theta)$

or $\quad \dfrac{v^2}{R} = \dfrac{4}{3}g(1 - \cos\theta) \qquad$ ...(i)

At the time of leaving contact, normal reaction $N = 0$ and $\theta = \theta_c$, hence,

$$mg\cos\theta = \frac{mv^2}{R} \text{ or } \frac{v^2}{R} = g\cos\theta \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{4}{3}g(1 - \cos\theta_c) = g\cos\theta_c$$

or $\dfrac{7}{4}\cos\theta_c = 1$ or $\cos\theta_c = 4/7$ or $\theta_c = \cos^{-1}(4/7)$

(b) $v = \sqrt{\dfrac{4}{3}gR(1 - \cos\theta)}$

At the time of losing contact

$\cos\theta = \cos\theta_c = 4/7$

$\therefore \quad v = \sqrt{\dfrac{4}{3}gR\left(1 - \dfrac{4}{7}\right)} \quad$ or $\quad v = \sqrt{\dfrac{4}{7}gR}$

Therefore, speed of CM of cylinder just before losing contact is $\sqrt{\dfrac{4}{7}gR}$

(c) At the moment, when cylinder loses contact

$$v = \sqrt{\frac{4}{7}gR}$$

Therefore, rotational kinetic energy, $K_R = \dfrac{1}{2}I\omega^2$

or $\quad K_R = \dfrac{1}{2}\left(\dfrac{1}{2}mR^2\right)\dfrac{v^2}{R^2} = \dfrac{1}{4}mv^2 = \dfrac{1}{4}m\left(\dfrac{4}{7}gR\right)$

or $\quad K_R = \dfrac{mgR}{7}$

Now, once the cylinder loses its contact, $N = 0$, i.e the frictional force, which is responsible for its rotation, also vanishes. Hence, its rotational kinetic energy now becomes constant, while its translational kinetic energy increases. Applying conservation of energy at (a) and (c).

Decrease in gravitational PE

= Gain in rotational KE + translational KE

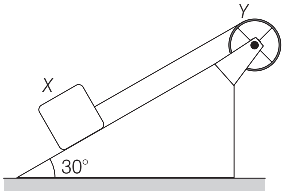
$\therefore$ Translational KE (K_T)

= Decrease in gravitational PE − K_R

or $$K_T = (mgR) - \frac{mgR}{7} = \frac{6}{7}mgR$$

$$\therefore \quad \frac{K_T}{K_R} = \frac{\frac{6}{7}mgR}{\frac{mgR}{7}} \text{ or } \frac{K_T}{K_R} = 6$$

41. Given, mass of block X, $m = 0.5$kg



Mass of drum Y, $M = 2$kg

Radius of drum, $R = 0.2$ m

Angle of inclined plane, $\theta = 30°$

(a) Let a be the linear retardation of block X and α be the angular retardation of drum Y. Then, $a = R\alpha$

$$mg \sin 30° - T = ma \quad \ldots\text{(i)}$$

or $$\frac{mg}{2} - T = ma \quad \ldots\text{(ii)}$$

$$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$$

or $$\alpha = \frac{2T}{MR} \quad \ldots\text{(iii)}$$

Solving Eqs. (i), (ii) and (iii) for T, we get,

$$T = \frac{1}{2}\frac{M\,mg}{M + 2m}$$

Substituting the value, we get

$$T = \left(\frac{1}{2}\right)\left\{\frac{(2)(0.5)(9.8)}{2 + (0.5)(2)}\right\} = 1.63 \text{ N}$$

(b) From Eq. (iii), angular retardation of drum

$$\alpha = \frac{2T}{MR} = \frac{(2)(1.63)}{(2)(0.2)} = 8.15 \text{ rad/s}^2$$

or linear retardation of block

$$a = R\alpha = (0.2)(8.15) = 1.63 \text{ m/s}^2$$

At the moment when angular velocity of drum is

$$\omega_0 = 10 \text{ rad/s}$$

The linear velocity of block will be

$$v_0 = \omega_0 R = (10)(0.2) = 2 \text{m/s}$$

Now, the distance (s) travelled by the block until it comes to rest will be given by

$$s = \frac{v_0^2}{2a} \qquad (\text{Using } v^2 = v_0^2 - 2as \text{ with } v = 0)$$

$$= \frac{(2)^2}{2(1.63)} \text{ or } s = 1.22 \text{ m}$$

42. Let M' be the mass of unwound carpet. Then,

$$M' = \left(\frac{M}{\pi R^2}\right)\pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

From conservation of mechanical energy :

$$MgR - M'g\frac{R}{2} = \frac{1}{2}\left(\frac{M}{4}\right)v^2 + \frac{1}{2}I\omega^2$$

or $$MgR - \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right) = \frac{Mv^2}{8} + \frac{1}{2}\left(\frac{1}{2}\times\frac{M}{4}\times\frac{R^2}{4}\right)\left(\frac{v}{R/2}\right)^2$$

or $$\frac{7}{8}MgR = \frac{3Mv^2}{16}$$

$$\therefore \quad v = \sqrt{\frac{14Rg}{3}}$$

Download Chapter Test
http://tinyurl.com/y2g5csax *or*

7

Gravitation

Topic 1 Gravitational Force and Acceleration due to Gravity

Objective Questions I (Only one correct option)

1 The ratio of the weights of a body on the earth's surface, so that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the earth. If R is the radius of the earth, what is the radius of the planet? (Take, the planets to have the same mass density) **(2019 Main, 12 April II)**

(a) $\frac{R}{3}$ (b) $\frac{R}{4}$

(c) $\frac{R}{9}$ (d) $\frac{R}{2}$

2 The time dependence of the position of a particle of mass $m = 2$ is given by $\mathbf{r}(t) = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$. Its angular momentum, with respect to the origin, at time $t = 2$ is **(2019 Main, 10 April II)**

(a) $36\,\hat{\mathbf{k}}$ (b) $-34\,(\hat{\mathbf{k}} - \hat{\mathbf{i}})$

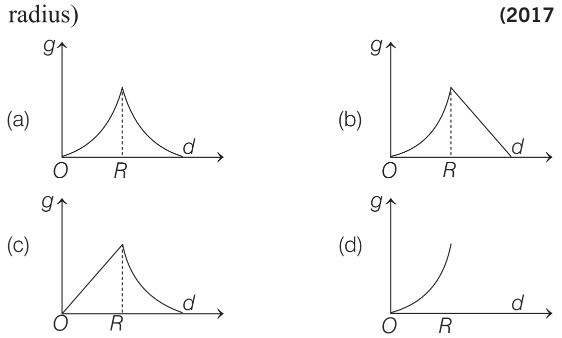
(c) $-48\,\hat{\mathbf{k}}$ (d) $48\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

3 The value of acceleration due to gravity at earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to (Take, radius of earth $= 6.4 \times 10^6$ m) **(2019 Main, 10 April I)**

(a) 9.0×10^6 m (b) 2.6×10^6 m

(c) 6.4×10^6 m (d) 1.6×10^6 m

4. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius) **(2017 Main)**



5. A planet of radius $R = \frac{1}{10} \times$ (radius of earth) has the same mass density as earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kg m^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of earth $= 6 \times 10^6$ m and the acceleration due to gravity of earth is $10\,\text{ms}^{-2}$) **(2014 Adv.)**

(a) 96 N (b) 108 N (c) 120 N (d) 150 N

6. If the radius of the earth were to shrink by one per cent, its mass remaining the same, the acceleration due to gravity on the earth's surface would **(1981, 2M)**

(a) decrease (b) remain unchanged

(c) increase (d) be zero

Objective Question II (One or more correct option)

7. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2, respectively. Then **(1994, 2M)**

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1^3}{r_2^3}$ if $r_1 < R$ and $r_2 < R$

(d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

Fill in the Blank

8. The numerical value of the angular velocity of rotation of the earth should be ……… rad/s in order to make the effective acceleration due to gravity at equator equal to zero. **(1984, 2M)**

Topic 2 Field Strength, Potential, Potential Energy and Escape Velocity

Objective Questions I (Only one correct option)

1 A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet? **(2019 Main, 10 April II)**

[Take, mass of planet $= 8 \times 10^{22}$ kg,

radius of planet $= 2 \times 10^6$ m,

gravitational constant $G = 6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$]

(a) 11 (b) 17
(c) 13 (d) 9

2 A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and $2M$. The gravitational field at distance $3a$ from the centre will be **(2019 Main, 9 April I)**

(a) $\dfrac{GM}{9a^2}$ (b) $\dfrac{2GM}{9a^2}$
(c) $\dfrac{GM}{3a^2}$ (d) $\dfrac{2GM}{3a^2}$

3 A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have, if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon. **(Main 2019, 8 April II)**

(a) $\dfrac{E}{64}$ (b) $\dfrac{E}{16}$ (c) $\dfrac{E}{32}$ (d) $\dfrac{E}{4}$

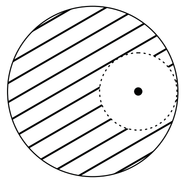
4. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2\ \text{km s}^{-1}$. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet) **(2017 Adv.)**

(a) $v_s = 72\ \text{km s}^{-1}$ (b) $v_s = 22\ \text{km s}^{-1}$
(c) $v_s = 42\ \text{km s}^{-1}$ (d) $v_s = 62\ \text{km s}^{-1}$

5. A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of earth $R, h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to (Neglect the effect of atmosphere) **(2016 Main)**

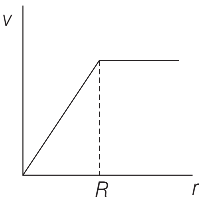
(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$
(c) $\sqrt{gR/2}$ (d) $\sqrt{gR}\ (\sqrt{2} - 1)$

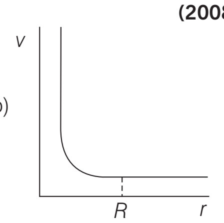
6. From a solid sphere of mass M and radius R, a spherical portion of radius $\left(\dfrac{R}{2}\right)$ is removed as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant) **(2015 Main)**

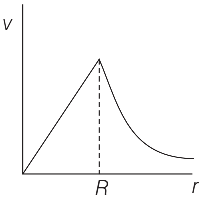


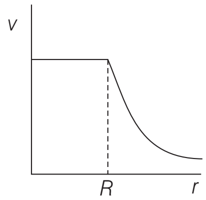
(a) $\dfrac{-GM}{R}$ (b) $\dfrac{-GM}{2R}$
(c) $\dfrac{-2GM}{3R}$ (d) $\dfrac{-2GM}{R}$

7. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 \text{ for } r \le R \\ 0 \text{ for } r > R \end{cases}$, where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r from the centre of the system is represented by **(2008, 3M)**

(a)


(b)


(c)


(d)


8. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the Earth, is **(1983, 1M)**

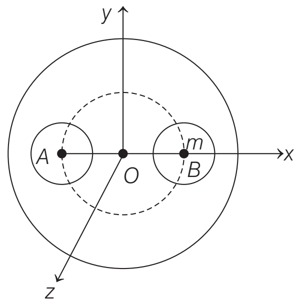
(a) $\dfrac{1}{2}\,mgR$ (b) $2\,mgR$
(c) mgR (d) $\dfrac{1}{4}\,mgR$

Objective Questions II (One or more correct option)

9. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and $4A$, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are v_P, v_Q and v_R, respectively. Then **(2012)**

(a) $v_Q > v_R > v_P$ (b) $v_R > v_Q > v_P$
(c) $v_R / v_P = 3$ (d) $v_P / v_Q = 1/2$

10. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in figure. **(1993, 2M)**



Then,

(a) the gravitational field due to this object at the origin is zero

(b) the gravitational field at the point B (2, 0, 0) is zero

(c) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$

(d) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$

Fill in the Blanks

11. A particle is projected vertically upwards from the surface of Earth (radius R) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of Earth is ……… . **(1997, 2M)**

12. The masses and radii of the Earth and the Moon are M_1, R_1 and M_2, R_2 respectively. Their centres are a distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is ……… . **(1988, 2M)**

Integer Answer Type Questions

13. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{4}$ th of its value at the surface of the planet. If the escape velocity from the planet is $v_{\text{sec}} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **(2015 Adv.)**

14. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 1/3 times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **(2010)**

Analytical & Descriptive Questions

15. There is a crater of depth $\frac{R}{100}$ on the surface of the moon (radius R). A projectile is fired vertically upward from the crater with velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile. **(2003, 4M)**

Topic 3 Motion of Satellites

Objective Questions I (Only one correct option)

1. A test particle is moving in a circular orbit in the gravitational field produced by mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T **(Main 2019, 9 April II)**

(a) $\frac{T^2}{R^3}$ is a constant (b) $\frac{T}{R^2}$ is a constant

(c) TR is a constant (d) $\frac{T}{R}$ is a constant

2. Two satellites A and B have masses m and $2m$ respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, T_A / T_B is **(2019 Main, 12 Jan II)**

(a) $\frac{1}{2}$ (b) 2 (c) $\sqrt{\frac{1}{2}}$ (d) 1

3. A satellite is revolving in a circular orbit at a height h from the earth surface such that $h << R$, where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is **(2019 Main, 11 Jan II)**

(a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{gR}$ (c) $\sqrt{2gR}$ (d) $\sqrt{gR}\,(\sqrt{2} - 1)$

4. A satellite is moving with a constant speed v in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **(2019 Main, 10 Jan I)**

(a) $\frac{1}{2}mv^2$ (b) mv^2

(c) $\frac{3}{2}mv^2$ (d) $2mv^2$

5. The energy required to take a satellite to a height 'h' above earth surface (where, radius of earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal is **(2019 Main, 9 Jan II)**

(a) 3.2×10^3 km (b) 1.28×10^4 km

(c) 6.4×10^3 km (d) 1.6×10^3 km

6. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? **(2013 Main)**
(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$ (c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{3R}$

7. A geostationary satellite orbits around the earth in a circular orbit of radius 36,000 km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_e = 6400km) will approximately be **(2002)**
(a) 1/2 h (b) 1 h (c) 2 h (d) 4 h

8. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth **(1998, 2M)**
(a) the acceleration of S is always directed towards the centre of the earth
(b) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(c) the total mechanical energy of S varies periodically with time
(d) the linear momentum of S remains constant in magnitude

Assertion and Reason

Mark your answer as
(*a*) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*
(*b*) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*
(*c*) *If Statement I is true; Statement II is false*
(*d*) *If Statement I is false; Statement II is true*

9. **Statement I** An astronaut in an orbiting space station above the earth experiences weightlessness.

Statement II An object moving around the earth under the influence of earth's gravitational force is in a state of 'free-fall'. **(2008, 3M)**

Fill in the Blank

10. A geostationary satellite is orbiting the earth at a height of $6R$ above the surface of the earth where R is the radius of earth. The time period of another satellite at a height of 3.5 R from the surface of the earth is ……… hours. **(1987, 2M)**

True / False

11. It is possible to put an artificial satellite into orbit in such a way that it will always remain directly over New Delhi. **(1984, 2M)**

Analytical & Descriptive Questions

12. An artificial satellite is moving in a circular orbit around the Earth with a speed equal to half the magnitude of escape velocity from the Earth. **(1990, 8M)**
(a) Determine the height of the satellite above the earth's surface.
(b) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the Earth, find the speed with which it hits the surface of the Earth.

13. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h, respectively. The radius of the orbit of S_1 is 10^4 km when S_2 is closest to S_1. Find **(1986, 6M)**
(a) the speed of S_2 relative to S_1,
(b) the angular speed of S_2 as actually observed by an astronaut in S_1.

Topic 4 Kepler's Laws and Motion of Planets

Objective Questions I (Only one correct option)

1 If the angular momentum of a planet of mass m, moving around sun in a circular orbit is L about the centre of the sun, its areal velocity is **(Main 2019, 9 Jan I)**
(a) $\frac{4L}{m}$ (b) $\frac{2L}{m}$ (c) $\frac{L}{2m}$ (d) $\frac{L}{m}$

2. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then **(2018 Main)**
(a) $T \propto R^{n/2}$ (b) $T \propto R^{3/2}$ for any value of n
(c) $T \propto R^{\frac{n}{2}+1}$ (d) $T \propto R^{\frac{n+1}{2}}$

3. A double star system consists of two stars A and B which have time periods T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option. **(2006, 3M)**
(a) If $T_A > T_B$ then $R_A > R_B$ (b) If $T_A > T_B$ then $M_A > M_B$
(c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (d) $T_A = T_B$

4. If the distance between the earth and the sun were half its present value, the number of days in a year would have been **(1996, 2M)**
(a) 64.5 (b) 129 (c) 182.5 (d) 730

5. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then **(1989, 2M)**
(a) T^2 is proportional to R^2
(b) T^2 is proportional to $R^{7/2}$
(c) T^2 is proportional to $R^{3/2}$
(d) T^2 is proportional to $R^{3.75}$

Fill in the Blanks

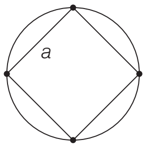
6. The ratio of earth's orbital angular momentum (about the sun) to its mass is $4.4 \times 10^{15}\ m^2/s$. The area enclosed by earth's orbit is approximately .... m^2. **(1997C, 1M)**

7. According to Kepler's second law, the radius vector to a planet from the sun sweeps out equal areas in equal intervals of time. This law is a consequence of the conservation of .... **(1985, 2M)**

Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. Four identical particles of mass M are located at the corners of a square of side a. What should be their speed, if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square ? **(Main 2019, 8 April I)**



(a) $1.35\sqrt{\frac{GM}{a}}$ (b) $1.16\sqrt{\frac{GM}{a}}$

(c) $1.21\sqrt{\frac{GM}{a}}$ (d) $1.41\sqrt{\frac{GM}{a}}$

2. The mass and the diameter of a planet are three times the respective values for the earth. The period of oscillation of a simple pendulum on the earth is 2 s. The period of oscillation of the same pendulum on the planet would be **(Main 2019, 11 Jan II)**

(a) $\frac{2}{\sqrt{3}}$ s (b) $\frac{3}{2}$ s (c) $2\sqrt{3}$ s (d) $\frac{\sqrt{3}}{2}$ s

3. Two stars of masses 3×10^{31} kg each and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is (Take, gravitational constant, $G = 6.67 \times 10^{-11}\ \text{N-m}^2\text{kg}^{-2}$) **(Main 2019, 10 Jan II)**

(a) 2.8×10^5 m/s (b) 3.8×10^4 m/s

(c) 2.4×10^4 m/s (d) 1.4×10^5 m/s

4. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is **(2014 Main)**

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

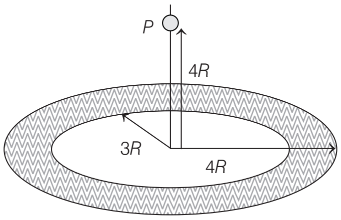
(c) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (d) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

5. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is **(2001)**

(a) 1 (b) $\sqrt{2}$

(c) 4 (d) 2

6. A thin uniform annular disc (*see figure*) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is **(2010)**



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

Objective Questions II (One or more correct option)

7. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct options is/are **(2015 Adv.)**

(a) $P\ (r=0)=0$ (b) $\frac{P\left(r=\frac{3R}{4}\right)}{P\left(r=\frac{2R}{3}\right)} = \frac{63}{80}$

(c) $\frac{P\left(r=\frac{3R}{5}\right)}{P\left(r=\frac{2R}{5}\right)} = \frac{16}{21}$ (d) $\frac{P\left(r=\frac{R}{2}\right)}{P\left(r=\frac{R}{3}\right)} = \frac{20}{27}$

8. Two bodies, each of mass M, are kept fixed with a separation $2L$. A particle of mass m is projected from the mid-point of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) **(2013 Adv.)**

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(d) The energy of the mass m remains constant

Analytical & Descriptive Questions

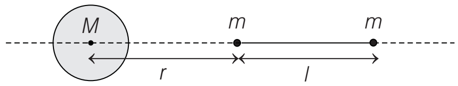
9. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the surface of the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a. **(1996, 5M)**

10. Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual separation a. Find the initial velocity that should be given to each particle and also the time period of the circular motion. **(1988, 5M)**

Integer Answer Type Question

11. A large spherical mass M is fixed at one position and two identical masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them.

All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is **(2015 Adv.)**



Match the following

12. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2, respectively.

Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given, $m_1 / m_2 = 2$ and $R_1 / R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **(2018 Adv.)**

	List-I		List-II
P.	v_1 / v_2	1.	1/8
Q.	L_1 / L_2	2.	1
R.	K_1 / K_2	3.	2
S.	T_1 / T_2	4.	8

(a) P → 4; Q → 2; R → 1; S → 3
(b) P → 3; Q → 2; R → 4; S → 1
(c) P → 2; Q → 3; R → 1; S → 4
(d) P → 2; Q → 3; R → 4; S → 1

Answers

Topic 1

1. (d) **2.** (c) **3.** (b)
4. (c) **5.** (b) **6.** (c) **7.** (a, b)
8. 1.24×10^{-3}

Topic 2

1. (a) **2.** (c) **3.** (b)
4. (c) **5.** (d) **6.** (a) **7.** (c)
8. (a) **9.** (b, d) **10.** (a, c, d) **11.** $h = R$
12. $v = 2\sqrt{\frac{G}{d}(M_1 + M_2)}$ **13.** 2 **14.** 3
15. $99.5\,R$

Topic 3

1. (d) **2.** (d) **3.** (d) **4.** (b)
5. (a)
6. (a) **7.** (c) **8.** (a)
9. (a) **10.** 8.48 **11.** F
12. (a) 6400 km (b) 7.9 km/s
13. (a) $-\pi \times 10^4$ km/h (b) 3×10^{-4} rad/s

Topic 4

1. (c) **2.** (d)
3. (d) **4.** (b) **5.** (b) **6.** 6.94×10^{22}
7. angular momentum

Topic 5

1. (b) **2.** (c) **3.** (a)
4. (d) **5.** (d) **6.** (a) **7.** (b, c)
8. (b, d) **9.** $\frac{3\sqrt{5}}{2}\sqrt{\frac{GM}{a}}$ **10.** $v = \sqrt{\frac{Gm}{a}},\ T = 2\pi\sqrt{\frac{a^3}{3Gm}}$
11. (7) **12.** (b)

Hints & Solutions

Topic 1 Gravitational Force and Acceleration due to Gravity

1. Let mass of given body is m. Then, it's weight on earth's surface $= mg_e$

where, g_e = acceleration due to gravity on earth's surface and weight on the surface of planet $= mg_p$

g_p = acceleration due to gravity on planet's surface.

Given,

$$\frac{mg_e}{mg_p} = \frac{9}{4} \Rightarrow \frac{g_e}{g_p} = \frac{9}{4}$$

But $g = \frac{GM}{R^2}$, so we have

$$\frac{\left(\frac{GM}{R^2}\right)}{\left(\frac{GM_p}{R_p^2}\right)} = \frac{9}{4}$$

where, M = mass of earth,

R = radius of earth,

M_p = mass of plane $= \frac{M}{9}$ (given)

and R_p = radius of planet.

$$\Rightarrow \frac{M}{M_p} \cdot \frac{R_p^2}{R^2} = \frac{9}{4} \Rightarrow 9 \cdot \left(\frac{R_p}{R}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \frac{R_p}{R} = \frac{1}{2} \Rightarrow R_p = \frac{R}{2}$$

2. Position of particle is, $\mathbf{r} = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$

where, t is instantaneous time.

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}}$$

Now, angular momentum of particle with respect to origin is given by

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$
$$= m\{(2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}})\}$$
$$= m(-12t^2(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) - 6t^2(\hat{\mathbf{j}} \times \hat{\mathbf{i}}))$$

As, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0$

$$\Rightarrow \mathbf{L} = m(-12t^2\hat{\mathbf{k}} + 6t^2\hat{\mathbf{k}})$$

As, $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$

$$\Rightarrow \mathbf{L} = -m(6t^2)\hat{\mathbf{k}}$$

So, angular momentum of particle of mass 2 kg at time $t = 2$ s is

$$\mathbf{L} = (-2 \times 6 \times 2^2)\hat{\mathbf{k}} = -48\,\hat{\mathbf{k}}$$

3. **Key Idea** The relation of gravitational acceleration at an altitude h above the earth's surface is given as

$$g_h = g\left(1 + \frac{h}{R_e}\right)^{-2}$$

where, g is the acceleration due to gravity at earth's surface and R_e is the radius of the earth.

Given that at some height h, acceleration due to gravity,

$$g_h = 4.9\ \text{m/s}^2 \approx \frac{g}{2} \qquad \ldots \text{(i)}$$

$\therefore$ The ratio of acceleration due to gravity at earth's surface and at some altitude h is

$$\left(1 + \frac{h}{R_e}\right) = \sqrt{\frac{g}{g_h}} = \sqrt{2} \qquad \text{[From Eq. (i)]}$$

$$\therefore \quad \frac{h}{R_e} = \sqrt{2} - 1$$

or $h = 0.414 \times R_e$

$h = 0.414 \times 6400$ km

($\because$ given, radius of earth, $R_e = 6400\ \text{km} = 6.4 \times 10^6$ m)

or $h = 2649.6\ \text{km} = 2.6 \times 10^6$ m

Thus, at 2.6×10^6 m above the earth's surface, the acceleration due to gravity decreases to 4.9 m/s^2.

4. Inside the earth surface

$$g = \frac{GM}{R^3}\,r$$

i.e. $g \propto r$

Out the earth surface $g = \frac{Gm}{r^2}$

i.e. $g \propto \frac{1}{r^2}$

So, till earth surface g increases linearly with distance r, shown only in graph (c).

5. Given, $R_{\text{planet}} = \frac{R_{\text{earth}}}{10}$ and density, $\rho = \frac{M_{\text{earth}}}{\frac{4}{3}\pi R_{\text{earth}}^3}$

$$= \frac{M_{\text{planet}}}{\frac{4}{3}\pi R^3_{\text{planet}}} \Rightarrow M_{\text{planet}} = \frac{M_{\text{earth}}}{10^3}$$

$$g_{\text{surface of planet}} = \frac{GM_{\text{planet}}}{R^2_{\text{planet}}} = \frac{GM_e \cdot 10^2}{10^3 \cdot R_e^2} = \frac{GM_e}{10R_e^2}$$

$$= \frac{g_{\text{surface of earth}}}{10}$$

$$g_{\text{depth of planet}} = g_{\text{surface of planet}}\left(\frac{x}{R}\right)$$

where, x = distance from centre of planet.

$\therefore$ Total force on wire

$$F = \int_{4R/5}^{R} \lambda \, dx \, g \left(\frac{x}{R}\right) = \frac{\lambda g}{R}\left[\frac{x^2}{2}\right]_{4R/5}^{R}$$

Here, $g = g_{\text{surface of planet}}, R = R_{\text{planet}}$

Substituting the given values, we get $F = 108$ N

6. $g = \dfrac{GM}{R^2}$ or $g \propto \dfrac{1}{R^2}$

g will increase if R decreases.

7. For $r \le R, F = \dfrac{GM}{R^3}.r$

or $F \propto r$

$\dfrac{F_1}{F_2} = \dfrac{r_1}{r_2}$ for $r_1 < R$

and $r_2 < R$

and for $r \ge R, F = \dfrac{GM}{r^2}$

or $F \propto \dfrac{1}{r^2}$

i.e. $\dfrac{F_1}{F_2} = \dfrac{r_2^2}{r_1^2}$ for $r_1 > R$

and $r_2 > R$

8. $g' = g - R\omega^2 \cos^2 \phi$

At equator $\phi = 0$

$\therefore$ $g' = g - R\omega^2$

$\Rightarrow$ $0 = g - R\omega^2$

$\therefore$ $\omega = \sqrt{\dfrac{g}{R}} = \sqrt{\dfrac{9.8}{6400 \times 10^3}}$

$= 1.24 \times 10^{-3}$ rad/s

Topic 2 Field Strength, Potential, Potential Energy and Escape Velocity

1. A satellite or spaceship in a circular orbit at a distance $(R + h)$ from centre of a planet experiences a gravitational force given by

$$F_g = \frac{GmM}{(R+h)^2}$$

where, M = mass of planet,
m = mass of spaceship,
R = radius of planet
and h = height of spaceship above surface.

This gravitational pull provides necessary centripetal pull for orbital motion of spaceship.

So, $F_g = F_{\text{centripetal}}$

$\Rightarrow$ $\dfrac{GmM}{(R+h)^2} = \dfrac{mv^2}{(R+h)}$

where, v = orbital speed of spaceship.
Orbital speed of spaceship is

$$v = \sqrt{\frac{GM}{(R+h)}} \quad \text{...(i)}$$

Here, $G = 6.67 \times 10^{-11}$ N-m^2/kg^2,
$M = 8 \times 10^{22}$ kg
$R + h = (2 \times 10^6 + 20 \times 10^3)$ m
$= 2.02 \times 10^6$ m

So, substituting these values in Eq. (i), we get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 8 \times 10^{22}}{2.02 \times 10^6}}$$

$= 1.6 \times 10^3$ ms^{-1}

Time period of rotation of spaceship will be

$$T = \frac{2\pi(R+h)}{v}$$

$\Rightarrow$ $T = \dfrac{2\pi \times 2.02 \times 10^6}{1.6 \times 10^3}$

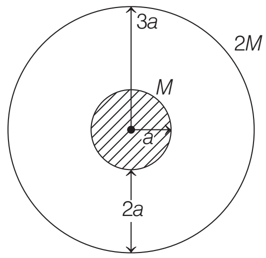
$\approx 8 \times 10^3$ s $= \dfrac{8 \times 10^3}{60 \times 60}$ (h)

$= 2.2$ h

So, number of revolutions made by spaceship in 24 h,

$$n = \frac{24}{T} = \frac{24}{2.2} \approx 11 \text{ rev}$$

2. According to question, diagram will be as follows



Gravitational field due to solid sphere of radius a at a distance, $r = 3a$, i.e. $(r > R)$ is

$$E_1 = \frac{GM}{r^2} = \frac{GM}{(3a)^2} = \frac{GM}{9a^2}$$

Similarly, gravitational field due to spherical shell at a distance, $r = 3a$,

i.e. $E_2 = \dfrac{GM}{R^2}$

$= \dfrac{G(2M)}{(3a)^2} = \dfrac{2GM}{9a^2}$

Both fields are attractive in nature, so direction will be same.

Net field, $E_{\text{net}} = \dfrac{GM}{9a^2} + \dfrac{2GM}{9a^2}$

$$E_{\text{net}} = \frac{GM}{3a^2}$$

3. Given, volume of earth (V_e) is 64 times of volume of moon (V_m), i.e.

$$\frac{V_e}{V_m} = 64 = \frac{\frac{4}{3}\pi R_e^3}{\frac{4}{3}\pi R_m^3}$$

where, R_e and R_m are the radius of earth and moon, respectively.

Then, $$\frac{R_e}{R_m} = 4 \quad \text{...(i)}$$

Also, since the density of moon and earth are equal, i.e.

$$\rho_m = \rho_e$$

$\Rightarrow \frac{M_e}{V_e} = \frac{M_m}{V_m}$, where M_e and M_m are the mass of the earth and moon, respectively.

$$\Rightarrow \frac{M_e}{M_m} = \frac{V_e}{V_m} = 64 \quad \text{...(ii)}$$

The minimum energy or escape energy delivered by the rocket launcher, so that the rocket never returns to earth is

$$E_e = \frac{GM_e m}{R_e} = E$$

where, m is the mass of the rocket.

Similarly, minimum energy that a launcher should have to escape or to never return, if rocket is launched from surface of the moon is

$$E_m = \frac{GM_m m}{R_m}$$

$\therefore$ Ratio of escape energies E_e and E_m is

$$\frac{E_e}{E_m} = \frac{\left(\frac{GM_e m}{R_e}\right)}{\left(\frac{GM_m m}{R_m}\right)} = \frac{M_e}{M_m} \cdot \frac{R_m}{R_e}$$

$$= 64 \times \frac{1}{4} = 16 \quad \text{[using Eqs. (i) and (ii)]}$$

$$\therefore \quad E_m = \frac{E_e}{16} = \frac{E}{16}$$

4. Given, $v_e = 11.2 \text{ km/s} = \sqrt{\frac{2GM_e}{R_e}}$

From energy conservation,

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 + 0$$

Here, r = distane of rocket from sun

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Given, $M_s = 3 \times 10^5 M_e$ and $r = 2.5 \times 10^4 R_e$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G\, 3 \times 10^5 M_e}{2.5 \times 10^4 R_e}}$$

$$= \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3 \times 10^5}{2.5 \times 10^4}\right)}$$

$$= \sqrt{\frac{2GM_e}{R_e} \times 13}$$

$$\Rightarrow \quad v_s \simeq 42 \text{ km/s}$$

5. $v_{\text{orbital}} = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$v_{\text{escape}} = \sqrt{2gR}$

$\therefore$ Extra velocity required $= v_{\text{escape}} - v_{\text{orbital}} = \sqrt{gR}\,(\sqrt{2} - 1)$

6. $V_R = V_T - V_C$

V_R = Potential due to remaining portion

V_T = Potential due to total sphere

V_C = Potential due to cavity

Radius of cavity is $\frac{R}{2}$. Hence, volume and mass is $\frac{M}{8}$.

$$\therefore \; V_R = -\frac{GM}{R^3}\left[1.5R^2 - 0.5\left(\frac{R}{2}\right)^2\right] + \frac{G\left(\frac{M}{8}\right)}{\left(\frac{R}{2}\right)}\left(\frac{3}{2}\right) = -\frac{GM}{R}$$

7. For $\quad r \le R, \; \frac{mv^2}{r} = \frac{GmM}{r^2} \quad \text{...(i)}$

Here, $$M = \left(\frac{4}{3}\pi r^3\right)\rho_0$$

Substituting in Eq. (i), we get $v \propto r$

i.e. v-r graph is a straight line passing through origin.

For $r > R$,

$$\frac{mv^2}{r} = \frac{Gm\left(\frac{4}{3}\pi R^3\right)\rho_0}{r^2} \text{ or } v \propto \frac{1}{\sqrt{r}}$$

The corresponding v-r graph will be as shown in option (c).

8. $\Delta U = \frac{mgh}{1 + h/R}$

Given, $h = R$

$$\Delta U = \frac{mgR}{1 + R/R} = \frac{1}{2}mgR$$

9. Surface area of Q is four times. Therefore, radius of Q is two times. Volume is eight times. Therefore, mass of Q is also eight times.

So, let $\quad M_P = M$ and $R_P = r$

Then, $\quad M_Q = 8\text{M}$ and $R_Q = 2r$

Now, mass of R is $(M_P + M_Q)$ or 9 M. Therefore, radius of R is $(9)^{1/3} r$. Now, escape velocity from the surface of a planet is given by

$$v = \sqrt{\frac{2GM}{r}} \quad (r = \text{radius of that planet})$$

$$\therefore \quad v_P = \sqrt{\frac{2GM}{r}}$$

$$v_Q = \sqrt{\frac{2G(8M)}{(2r)}} \Rightarrow v_R = \sqrt{\frac{2G(9M)}{(9)^{1/3} r}}$$

From here we can see that,

$$\frac{v_P}{v_Q} = \frac{1}{2} \text{ and } v_R > v_Q > v_P$$

10. The gravitational field is zero at the centre of a solid sphere. The small spheres can be considered as negative mass m located at A and B. The gravitational field due to these masses at O is equal and opposite. Hence, the resultant field at O is zero, options c and d are correct because plane of these circles is y-z, i.e. perpendicular to X-axis i.e. potential at any point on these two circles will be equal due to the positive mass M and negative masses $-m$ and $-m$.

11. Kinetic energy needed to escape $= \dfrac{GM_e m}{R}$

Therefore, energy given to the particle $= \dfrac{GM_e m}{2R}$

Now, from conservation of mechanical energy. Kinetic energy at the surface of earth = Difference in potential energy at a height h and on the surface of earth

$$\therefore \quad \frac{GM_e m}{2R} = \frac{-GM_e m}{(R+h)} - \left(\frac{-GM_e m}{R}\right)$$

$$\frac{1}{2R} = \frac{1}{R} - \frac{1}{R+h}$$

$$\frac{1}{2R} = \frac{1}{R+h}$$

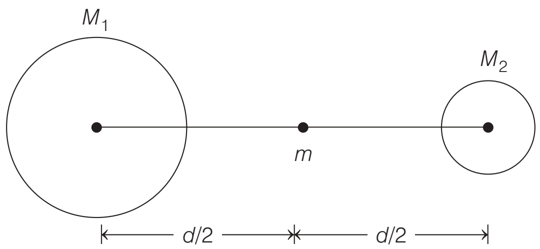
$$h + R = 2R$$

$$\Rightarrow \quad h = R$$

12. Total mechanical energy of mass m at a point midway between two centres is

$$E = -\frac{GM_1 m}{d/2} - \frac{GM_2 m}{d/2} = -\frac{2\,Gm}{d}(M_1 + M_2)$$

Binding energy $= \dfrac{2\,Gm}{d}(M_1 + M_2)$

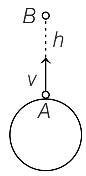


Kinetic energy required to escape the mass to infinity is,

$$\frac{1}{2} m v_e^2 = \frac{2Gm}{d}(M_1 + M_2)$$

$$\therefore \quad v_e = 2\sqrt{\frac{G(M_1 + M_2)}{d}}$$

13. At height h



$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} \quad \ldots\text{(i)}$$

Given, $g' = \dfrac{g}{4}$

Substituting in equation (i) we get,

$$h = R$$

Now, from A to B,

decrease in kinetic energy = increase in potential energy

$$\Rightarrow \quad \frac{1}{2} m v^2 = \frac{mgh}{1 + \frac{h}{R}} \Rightarrow \frac{v^2}{2} = \frac{gh}{1 + \frac{h}{R}} = \frac{1}{2} gR \quad (h = R)$$

$$\Rightarrow \quad v^2 = gR \quad \text{or} \quad v = \sqrt{gR}$$

Now, $\quad v_{\text{esc}} = \sqrt{2gR} = v\sqrt{2}$

$$\Rightarrow \quad N = 2$$

14. $g = \dfrac{GM}{R^2} = \dfrac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$ or $g \propto \rho R$ or $R \propto \dfrac{g}{\rho}$

Now escape velocity, $v_e = \sqrt{2gR}$

or $\quad v_e \propto \sqrt{gR}$ or $\quad v_e \propto \sqrt{g \times \dfrac{g}{\rho}} \propto \sqrt{\dfrac{g^2}{\rho}}$

$$\therefore \quad (v_e)_{\text{planet}} = (11\,\text{km s}^{-1})\sqrt{\frac{6}{121} \times \frac{3}{2}} = 3\,\text{km s}^{-1}$$

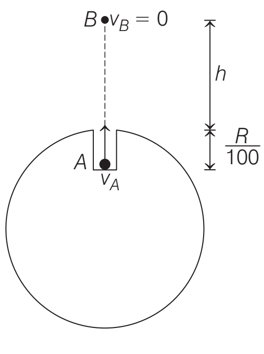
$\therefore$ The correct answer is 3.

15. Speed of particle at A, v_A

= escape velocity on the surface of earth $= \sqrt{\dfrac{2GM}{R}}$

At highest point $B, v_B = 0$

Applying conservation of mechanical energy, decrease in kinetic energy = increase in gravitational potential energy



or $\frac{1}{2}mv_A^2 = U_B - U_A = m(v_B - v_A)$

or $\frac{v_A^2}{2} = v_B - v_A$

$$\therefore \quad \frac{GM}{R} = -\frac{GM}{R+h} - \left[-\frac{GM}{R^3}\left(1.5R^2 - 0.5\left(R - \frac{R}{100}\right)^2\right]\right.$$

or $$\frac{1}{R} = -\frac{1}{R+h} + \frac{3}{2R} - \left(\frac{1}{2}\right)\left(\frac{99}{100}\right)^2 \cdot \frac{1}{R}$$

Solving this equation, we get

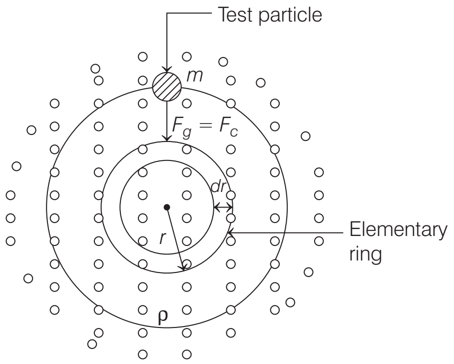
$$h = 99.5R$$

Topic 3 Motion of Satellites

1. Let the mass of the test particle be m and its orbital linear speed be v. Force of gravity of the mass-density would provide the necessary centripetal pull on test particle.

$\Rightarrow$ Gravitational force (F_g) = Centripetal force (F_c)

Let us now assume an elementary ring at a distance r from the centre of the mass density, such that mass of this elementary ring



$$\Rightarrow \quad \frac{GMm}{R^2} = \frac{mv^2}{R}$$

where, $$M = \int_0^R \rho \cdot 4\pi r^2 \cdot dr$$

$$\Rightarrow \quad \frac{G\left(\int_0^R \frac{K}{r^2} \cdot 4\pi r^2 \cdot dr\right)}{R^2} = \frac{v^2}{R}$$

$$\left[\because \text{given, } \rho(r) = \frac{K}{r^2}\right]$$

$$\Rightarrow \quad G \cdot 4\pi K = v^2$$

$\therefore$ Orbital speed of mass, $m = v = \sqrt{G4\pi K}$ ... (i)

Time period of rotation of the test particle is

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{G \cdot 4\pi K}} \quad [\because \text{using Eq. (i)}]$$

Hence, $$\frac{T}{R} = \sqrt{\frac{\pi}{GK}} = \text{a constant}$$

Alternate Solution

Since, $$F_g = F_c$$

This relation can also be written directly as,

$$\frac{G\int_0^R \left(\frac{K}{r^2} \cdot 4\pi r^2 \cdot dr\right)}{R^2} = m\omega^2 R = m\left(2\frac{\pi}{T}\right)^2 R$$

$$\Rightarrow \quad \frac{G \cdot 4\pi KR}{R^2} = \frac{4\pi^2}{T^2} R$$

or $$\frac{T^2}{R^2} = \frac{\pi}{GK}$$

$$\Rightarrow \quad \frac{T}{R} = \sqrt{\frac{\pi}{GK}} = \text{a constant.}$$

2. Orbital speed of a satellite in a circular orbit is

$$v_0 = \sqrt{\left(\frac{GM}{r_0}\right)}$$

where r_0 is the radius of the circular orbit.

So, kinetic energies of satellites A and B are

$$T_A = \frac{1}{2}m_A v_{OA}^2 = \frac{GMm}{2R}$$

$$T_B = \frac{1}{2}m_B v_{OB}^2 = \frac{GM(2m)}{2(2R)} = \frac{GMm}{2R}$$

So, ratio of their kinetic energies is

$$\frac{T_A}{T_B} = 1$$

3. Orbital velocity of the satellite is given as,

$$v_O = \sqrt{\frac{GM}{R+h}}$$

Since, $R >> h$

$$\therefore \quad v_O = \sqrt{\frac{GM}{R}} = \sqrt{gR} \quad \left[\because g = \frac{GM}{R^2}\right]$$

Escape velocity of the satellite,

$$v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Since, we know that in order to escape the earth's gravitational field a satellite must get escape velocity.

$\therefore$ Change in velocity,

$$\Delta v = v_e - v_O$$
$$= \sqrt{gR}\,(\sqrt{2} - 1)$$

4. In circular orbit of a satellite, potential energy

$$= -2 \times (\text{kinetic energy}) = -2 \times \frac{1}{2}mv^2 = -mv^2$$

Just to escape from the gravitational pull, its total mechanical energy should be zero. Therefore, its kinetic energy should be $+mv^2$.

5. The energy required for taking a satellite upto a height h from earth's surface is the difference between the energy at h height and energy at surface, then

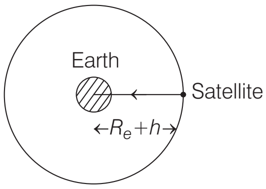
$$\Rightarrow \quad E_1 = U_f - U_i$$

$$E_1 = -\frac{GM_e m}{R_e + h} + \frac{GM_e m}{R_e} \quad \ldots\text{(i)}$$

(where, U = potential energy)

$\therefore$ Orbital velocity of satellite,

$$v_o = \sqrt{\frac{GM_e}{(R_e + h)}} \quad \text{(where, } M_e = \text{mass of earth)}$$



So energy required to perform circular motion

$$\Rightarrow \quad E_2 = \frac{1}{2}mv_o^2 = \frac{GM_e m}{2(R_e + h)}$$

$$E_2 = \frac{GM_e m}{2(R_e + h)} \quad \ldots\text{(ii)}$$

According to the question,

$$E_1 = E_2$$

$$\therefore \quad \frac{-GM_e m}{R_e + h} + \frac{GM_e m}{R_e} = \frac{GM_e m}{2(R_e + h)}$$

$$\Rightarrow \quad 3R_e = 2R_e + 2h$$

$$h = \frac{R_e}{2}$$

As radius of earth, $R_e \approx 6.4 \times 10^3$ km

Hence, $\quad h = \dfrac{6.4 \times 10^3}{2}$ km or $= 3.2 \times 10^3$ km

6. E = Energy of satellite − energy of mass on the surface of planet

$$= -\frac{GMm}{2r} - \left(-\frac{GMm}{R}\right)$$

Here, $r = R + 2R = 3R$

Substituting in about equation we get, $E = \dfrac{5GMm}{6R}$

7. Time period of a satellite very close to earth's surface is 84.6 min. Time period increases as the distance of the satellite from the surface of earth increases. So, time period of spy satellite orbiting a few 100 km above the earth's surface should be slightly greater than 84.6 min. Therefore, the most appropriate option is (c) or 2 h.

8. Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since, the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

9. Force acting on astronaut is utilised in providing necessary centripetal force, thus he feels weightlessness, as he is in a state of free fall.

10. $$T \propto r^{3/2}$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2}$$

$$\text{or} \quad T_2 = \left(\frac{r_2}{r_1}\right)^{3/2}$$

$$T_1 = \left(\frac{3.5R}{7R}\right)^{3/2}(24)\,\text{h} = 8.48\ \text{h}$$

(T_1 = 24 h for geostationary satellite)

11. New Delhi is not on the equatorial plane.

12. (a) Orbital speed of a satellite at distance r from centre of earth,

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \quad \ldots\text{(i)}$$

Given, $$v_o = \frac{v_e}{2} = \frac{\sqrt{2GM/R}}{2} = \sqrt{\frac{GM}{2R}} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$h = R = 6400\,\text{km}$$

(b) Decrease in potential energy

= increase in kinetic energy

or $\dfrac{1}{2}mv^2 = \Delta U$

$$\therefore \quad v = \sqrt{\frac{2(\Delta U)}{m}}$$

$$= \sqrt{\frac{2\left(\dfrac{mgh}{1+h/R}\right)}{m}} = \sqrt{gR} \quad (h = R)$$

$$= \sqrt{9.8 \times 6400 \times 10^3} = 7919\,\text{m/s} = 7.9\,\text{km/s}$$

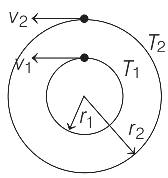
13. $T \propto r^{3/2}$ or $r \propto T^{2/3}$ $\Rightarrow$ $\dfrac{r_2}{r_1} = \left(\dfrac{T_2}{T_1}\right)^{2/3}$

$$r_2 = \left(\frac{T_2}{T_1}\right)^{2/3} r_1 = \left(\frac{8}{1}\right)^{2/3}(10^4)$$

$$= 4 \times 10^4\ \text{km}$$

Now, $$v_1 = \frac{2\pi r_1}{T_1} = \frac{(2\pi)(10^4)}{1} = 2\pi \times 10^4\,\text{km/h}$$

$$v_2 = \frac{2\pi r_2}{T_2} = \frac{(2\pi)(4 \times 10^4)}{8} = (\pi \times 10^4)\,\text{km/h}$$



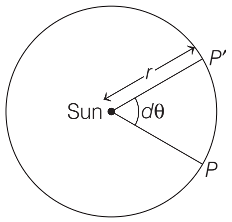
(a) Speed of S_2 relative to $S_1 = v_2 - v_1 = -\pi \times 10^4$ km/h

(b) Angular speed of S_2 as observed by S_1

$$\omega_r = \frac{|v_2 - v_1|}{|r_2 - r_1|} = \frac{\left(\pi \times 10^4 \times \frac{5}{18} \text{ m/s}\right)}{(3 \times 10^7 \text{ m})}$$
$$= 0.3 \times 10^{-3} \text{ rad/s}$$
$$= 3.0 \times 10^{-4} \text{ rad/s}$$

Topic 4 Kepler's Laws and Motion of Planets

1. According to Kepler's second law, "the line joining the planet to sun sweeps out equal areas in equal interval of time". This means the rate of change of area with time is constant.



The area covered from P to P' is dA, which is given by

$$dA = \frac{d\theta}{2\pi} \times \pi r^2$$

$$\Rightarrow \quad dA = \frac{1}{2} r^2 d\theta \text{ or } \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

where, $\frac{dA}{dt}$ = a real velocity.

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega \text{ or } \frac{dA}{dt} = \frac{1}{2} r^2 \cdot \frac{L}{mr^2}$$

(because angular momentum, $L = mr^2\omega$)

$$\frac{dA}{dt} = \frac{L}{2m}$$

2. $$F \propto \frac{1}{R^n}$$

$$\frac{mv^2}{R} \propto \frac{1}{R^n} \quad \text{or} \quad v \propto \frac{I}{R^{\frac{n-1}{2}}}$$

$$T = \frac{2\pi R}{v} \quad \Rightarrow \quad T \propto \frac{R}{v}$$

$$\Rightarrow \quad T \propto R^{1 + \frac{n-1}{2}} \quad \text{or} \quad T \propto R^{\frac{n+1}{2}}$$

3. In case of binary star system, angular velocity and hence, the time period of both the stars are equal.

4. From Kepler's third law $T^2 \propto r^3$ or $T \propto (r)^{3/2}$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \quad \text{or} \quad T_2 = T_1 \left(\frac{r_2}{r_1}\right)^{3/2} = (365)\left(\frac{1}{2}\right)^{3/2}$$

$$\Rightarrow \quad T_2 \approx 129 \text{ days}$$

5. $$\frac{mv^2}{R} \propto R^{-5/2}$$

$$\therefore \quad v \propto R^{-3/4}$$

Now, $$T = \frac{2\pi R}{v} \quad \text{or } T^2 \propto \left(\frac{R}{v}\right)^2$$

or $$T^2 \propto \left(\frac{R}{R^{-3/4}}\right)^2 \quad \text{or } T^2 \propto R^{7/2}$$

6. Areal velocity of a planet round the sun is constant and is given by

$$\frac{dA}{dt} = \frac{L}{2m}$$

where, L = angular momentum of planet (earth) about sun
and m = mass of planet (earth).

Given, $$\frac{L}{m} = 4.4 \times 10^{15} \text{ m}^2/\text{s}$$

$\therefore$ Area enclosed by earth in time T (365 days) will be

$$\text{Area} = \frac{dA}{dt} \cdot T = \frac{L}{2m} \cdot T$$
$$= \frac{1}{2} \times 4.4 \times 10^{15} \times 365 \times 24 \times 3600 \text{ m}^2$$

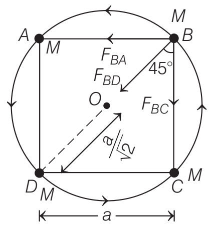
$$\text{Area} \approx 6.94 \times 10^{22} \text{ m}^2$$

7. $\frac{dA}{dt} = \frac{L}{2m}$ = constant, because angular momentum of planet (L) about the centre of sun is constant.

Thus, this law comes from law of conservation of angular momentum.

Topic 5 Miscellaneous Problems

1. In given configuration of masses, net gravitational force provides the necessary centripetal force for rotation.



Net force on mass M at position B towards centre of circle is

$$F_{BO \text{ net}} = F_{BD} + F_{BA} \sin 45° + F_{BC} \cos 45°$$
$$= \frac{GM^2}{(\sqrt{2}a)^2} + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right) + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right)$$

[where, diagonal length BD is $\sqrt{2}a$]

$$= \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\left(\frac{2}{\sqrt{2}}\right) = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force.

Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

Here, $F_{\text{centripetal}} = \dfrac{Mv^2}{r} = \dfrac{Mv^2}{\dfrac{a}{\sqrt{2}}} = \dfrac{\sqrt{2}Mv^2}{a} \quad \left(\because r = \dfrac{a}{\sqrt{2}}\right)$

So, for rotation about the centre,

$$F_{\text{centripetal}} = F_{BO\,(\text{net})}$$

$$\Rightarrow \quad \sqrt{2}\frac{Mv^2}{a} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$\Rightarrow \quad v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right) = \frac{GM}{a}(1.35) \Rightarrow v = 1.16\sqrt{\frac{GM}{a}}$$

2. Period of motion of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \ldots\text{(i)}$$

On the surface of earth, let period of motion is T_e and acceleration due to gravity is g_e

$$\therefore \qquad T_e = 2\pi\sqrt{\frac{l}{g_e}} \qquad \ldots\text{(ii)}$$

On the another planet, let period of motion is T_p and gravitational acceleration is g_p

$$\therefore \qquad T_p = 2\pi\sqrt{\frac{l}{g_p}} \qquad \ldots\text{(iii)}$$

($\because$ Pendulum is same, so l will be same)

From Eqs. (ii) and (iii),

$$\frac{T_e}{T_p} = \frac{2\pi\sqrt{\dfrac{l}{g_e}}}{2\pi\sqrt{\dfrac{l}{g_p}}} = \sqrt{\frac{g_p}{g_e}} \qquad \ldots\text{(iv)}$$

Now, $\quad g_e = \dfrac{GM_e}{R_e^2}$ and $g_p = \dfrac{GM_p}{R_p^2}$

Given, $\quad M_p = 3M_e$ and $R_p = 3R_e$

$$\therefore \qquad g_p = \frac{G \times 3M_e}{9R_e^2} = \frac{1}{3}\cdot\frac{GM_e}{R_e^2} = \frac{1}{3}g_e$$

$$\Rightarrow \qquad \frac{g_p}{g_e} = \frac{1}{3} \text{ or } \sqrt{\frac{g_p}{g_e}} = \frac{1}{\sqrt{3}} \qquad \ldots\text{(v)}$$

From Eqs. (iv) and (v), $T_p = \sqrt{3}\,T_e$

or $\qquad T_p = 2\sqrt{3}$ s $\qquad (\because T_e = 2\text{ s})$

3. Let us assume that stars are moving in $x\,y$-plane with origin as their centre of mass as shown in the figure below

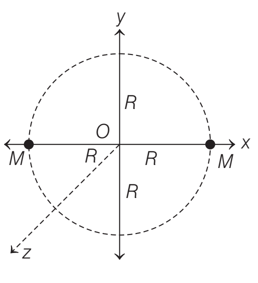
According to question,

mass of each star, $M = 3 \times 10^{31}$ kg

and diameter of circle,

$$2R = 2 \times 10^{11}\text{ m}$$

$$\Rightarrow \quad R = 10^{11}\text{ m}$$



Potential energy of meteorite at O, origin $\hat{\mathbf{j}}$ is, $\quad U_{\text{total}} = -\dfrac{2\,GMm}{r}$

If v is the velocity of meteorite at O then

Kinetic energy K of the meteorite is

$$K = \frac{1}{2}mv^2$$

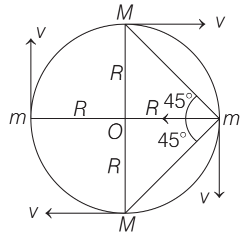
To escape from this dual star system, total mechanical energy of the meteorite at infinite distance from stars must be at least zero. By conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{2\,GMm}{R} = 0$$

$$\Rightarrow v^2 = \frac{4\,GM}{R} = \frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}$$

$$\Rightarrow \quad v = 2.8 \times 10^5\text{ m/s}$$

4. Net force acting on any one particle M,



$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45° + \frac{GM^2}{(R\sqrt{2})^2}\cos 45°$$

$$= \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$$

This force will equal to centripetal force.

So, $\quad \dfrac{Mv^2}{R} = \dfrac{GM^2}{R^2}\left(\dfrac{1}{4} + \dfrac{1}{\sqrt{2}}\right)$

$$v = \sqrt{\frac{GM}{4R}(1 + 2\sqrt{2})} = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

Hence, speed of each particle in a circular motion is

$$\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}.$$

5. $\qquad T \propto \dfrac{1}{\sqrt{g}}$, i.e. $\dfrac{T_2}{T_1} = \sqrt{\dfrac{g_1}{g_2}}$

where, $g_1 =$ acceleration due to gravity on earth's surface $= g$

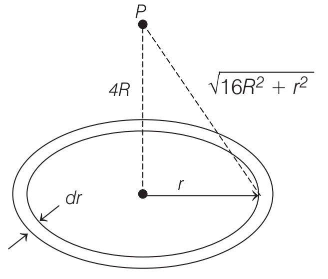
$g_2 =$ acceleration due to gravity at a height $h = R$ from earth's surface $= g/4$

$$\left[\text{Using } g(h) = \frac{g}{\left(1 + \dfrac{h}{R}\right)^2}\right]$$

$$\Rightarrow \qquad \frac{T_2}{T_1} = \sqrt{\frac{g}{g/4}} = 2$$

6. $W = \Delta U = U_f - U_i = U_\infty - U_P = -U_P = -mV_P$

$$= -V_P \qquad (\text{as } m = 1)$$



Potential at point P will be obtained by integration as given below.

Let dM be the mass of small ring as shown

$$dM = \frac{M}{\pi(4R)^2 - \pi\,(3R)^2}\,(2\pi r)\,dr = \frac{2Mr\,dr}{7R^2}$$

$$dV_P = -\frac{G\,dM}{\sqrt{16R^2 + r^2}}$$

$$= -\frac{2GM}{7R^2}\int_{3R}^{4R}\frac{r}{\sqrt{16R^2 + r^2}}\,dr$$

$$= -\frac{2GM}{7R}\,(4\sqrt{2} - 5)$$

$$\therefore \quad W = +\frac{2GM}{7R}\,(4\sqrt{2} - 5)$$

7. Gravitational field at a distance r due to mass 'm'

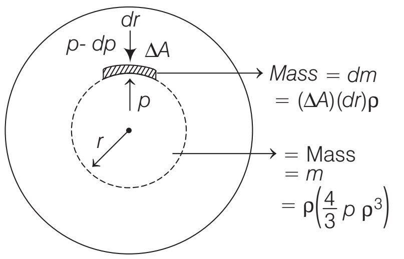
$$E = \frac{G\rho\frac{4}{3}\pi r^3}{r^2} = \frac{4G\rho\pi r}{3}$$

Consider a small element of width dr and area ΔA at a distance r.

Pressure force on this element outwards = gravitational force on 'dm' from 'm' inwards

$$\Rightarrow \quad (dp)\Delta A = E(dm)$$

$$\Rightarrow \quad -\,dp\cdot\Delta A = \left(\frac{4}{3}G\pi\rho r\right)(\Delta A\;dr\cdot\rho)$$



$$-\int_O^P dp = \int_R^r \left(\frac{4G\rho^2\pi}{3}\right) r\,dr$$

$$-p = \frac{4G\rho^2\pi}{3\times 2}\,[r^2 - R^2] \quad \Rightarrow \quad p = c(R^2 - r^2)$$

$$r = \frac{3R}{4},\; p_1 = c\left(R^2 - \frac{9R^2}{16}\right) = c\left(\frac{7R^2}{16}\right)$$

$$r = \frac{2R}{3},\; p_2 = c\left(R^2 - \frac{4R^2}{9}\right) = c\left(\frac{5R^2}{9}\right)$$

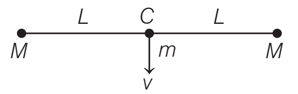
$$\frac{p_1}{p_2} = \frac{63}{80}$$

$$r = \frac{3R}{5},\; p_3 = c\left(R^2 - \frac{9}{25}R^2\right) = c\left(\frac{16R^2}{25}\right)$$

$$r = \frac{2R}{5},\; p_4 = c\left(R^2 - \frac{4R^2}{25}\right) = c\left(\frac{21R^2}{25}\right)$$

$$\frac{p_3}{p_4} = \frac{16}{21}$$

8.



Let v is the minimum velocity. From energy conservation,

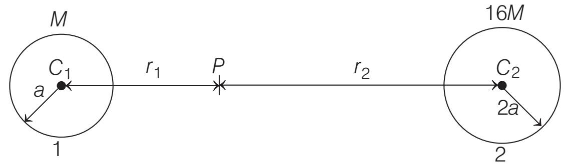
$$U_c + K_c = U_\infty + K_\infty$$

$$\therefore \quad mV_c + \frac{1}{2}mv^2 = 0 + 0$$

$$\therefore \quad v = \sqrt{-2V_c} = \sqrt{(-2)\left(\frac{-2GM}{L}\right)}$$

$$= 2\sqrt{\frac{GM}{L}}$$

9. Let there are two stars 1 and 2 as shown below.



Let P is a point between C_1 and C_2, where gravitational field strength is zero or at P field strength due to star 1 is equal and opposite to the field strength due to star 2. Hence,

$$\frac{GM}{r_1^2} = \frac{G\,(16\,M)}{r_2^2} \quad \text{or} \quad \frac{r_2}{r_1} = 4 \quad \text{also} \quad r_1 + r_2 = 10\,a$$

$$\therefore \quad r_2 = \left(\frac{4}{4+1}\right)(10\,a) = 8a \quad \text{and} \quad r_1 = 2a$$

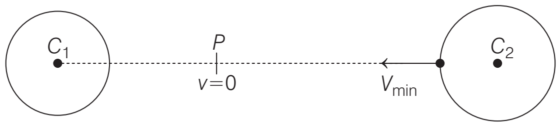
Now, the body of mass m is projected from the surface of larger star towards the smaller one. Between C_2 and P it is attracted towards 2 and between C_1 and P it will be attracted towards 1. Therefore, the body should be projected to just cross point P because beyond that the particle is attracted towards the smaller star itself.

From conservation of mechanical energy $\frac{1}{2}mv^2_{\text{min}}$

= Potential energy of the body at P

– Potential energy at the surface of larger star.

$$\therefore \quad \frac{1}{2}mv_{\min}^2 = \left[-\frac{GMm}{r_1} - \frac{16GMm}{r_2}\right] - \left[-\frac{GMm}{10a-2a} - \frac{16GMm}{2a}\right]$$



$$= \left[-\frac{GMm}{2a} - \frac{16GMm}{8a}\right] - \left[-\frac{GMm}{8a} - \frac{8GMm}{a}\right]$$

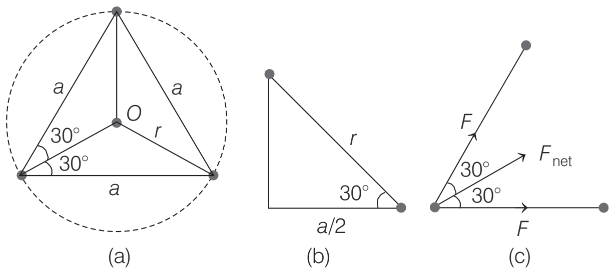
or $$\frac{1}{2}mv_{\min}^2 = \left(\frac{45}{8}\right)\frac{GMm}{a}$$

$$\therefore \quad v_{\min} = \frac{3\sqrt{5}}{2}\left(\sqrt{\frac{GM}{a}}\right)$$

10. Centre should be at O and radius r.
We can calculate r from figure (b).

$$\frac{a/2}{r} = \cos 30° = \frac{\sqrt{3}}{2} \Rightarrow \therefore \quad r = \frac{a}{\sqrt{3}}$$

Further net force on any particle towards centre



$$F_{\text{net}} = 2F\cos 30° = 2\left(\frac{Gm^2}{a^2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}\,Gm^2}{a^2}$$

This net force should be equal to $\frac{mv^2}{r}$

$$\therefore \quad \frac{\sqrt{3}\,Gm^2}{a^2} = \frac{mv^2}{a/\sqrt{3}}$$

$$\Rightarrow \quad v = \sqrt{\frac{Gm}{a}}$$

Time period of circular motion

$$T = \frac{2\pi r}{v} = \frac{2\pi\,(a/\sqrt{3})}{\sqrt{Gm/a}} = 2\pi\sqrt{\frac{a^3}{3\,Gm}}$$

11. For point mass at distance $r = 3l$

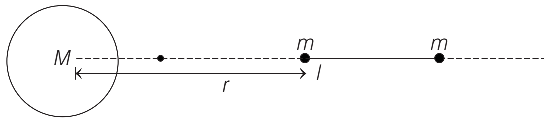
$$\frac{GMm}{(3l)^2} - \frac{Gm^2}{l^2} = ma \qquad \text{....(i)}$$

For point mass at distance $r = 4l$

$$\frac{GMm}{(4l)^2} + \frac{Gm^2}{l^2} = ma \qquad \text{(ii)}$$

Equating the two equations, we have

$$\frac{GMm}{9l^2} - \frac{Gm^2}{l^2} = \frac{GMm}{16l^2} + \frac{Gm^2}{l^2}$$



$$\frac{7GMm}{144} = \frac{2\,Gm^2}{l^2}$$

$$m = \frac{7M}{288}$$

12. As, $v = \sqrt{\frac{GM}{R}}$

Let $R_1 = R$, then $R_2 = 4R$

If $m_2 = m$, then $m_1 = 2m$

List-I

(P) $\frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{4R}{R}} = 2:1$

(Q) $L = mvR$

$$\frac{L_1}{L_2} = \frac{R(2m)v_1}{4R(m)v_2} = \frac{1}{2}(2) = 1:1$$

(R) $\frac{K_1}{K_2} = \frac{\frac{1}{2}(2m)v_1^2}{\frac{1}{2}(m)v_2^2} = 2(4) = 8:1$

(S) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} = 1:8$

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http://tinyurl.com/y4j53yoh *or*

8

Simple Harmonic Motion

Topic 1 Energy in Simple Harmonic Motion

Objective Questions I (Only one correct option)

1. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then **(2019 Main, 11 Jan III)**

(a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$

(c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

2. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210\,\text{s}$ will be **(2019 Main, 11 Jan I)**

(a) 2

(b) 1

(c) $\frac{1}{9}$

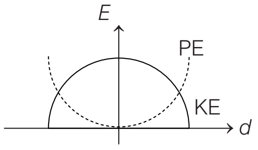
(d) 3

3. A particle is executing simple harmonic motion (SHM) of amplitude A, along the X-axis, about $x = 0$. when its potential energy (PE) equals kinetic energy (KE), the position of the particle will be **(2019 Main, 9 Jan II)**

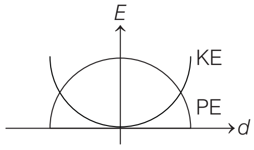
(a) A (b) $\frac{A}{2}$

(c) $\frac{A}{2\sqrt{2}}$ (d) $\frac{A}{\sqrt{2}}$

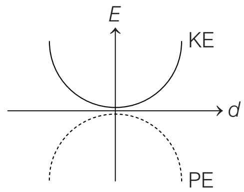
4. For a simple pendulum, a graph is plotted between its Kinetic Energy (KE) and Potential Energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) **(2015 Main)**



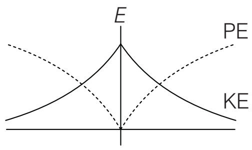
(a)



(b)



(c)



(d)

5. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is **(1987, 2M)**

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

Objective Question II (One or more correct option)

6. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01m has a total mechanical energy of 160 J. Its **(1989, 2M)**

(a) maximum potential energy is 100 J

(b) maximum kinetic energy is 100 J

(c) maximum potential energy is 160 J

(d) maximum potential energy is zero

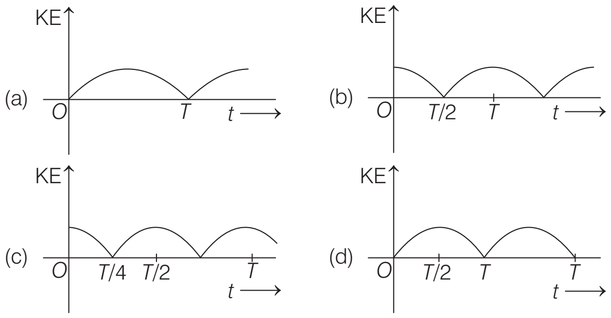
Fill in the Blank

7. An object of mass 0.2 kg executes simple harmonic oscillations along the X-axis with a frequency of $(25/\pi)$Hz. At the position $x = 0.04$, the object has kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is ……m. **(1994, 2M)**

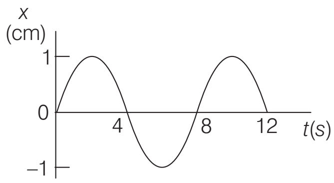
Topic 2 Graphs in Simple Harmonic Motion

Objective Questions I (Only one correct option)

1. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look, like **(2017 Main)**

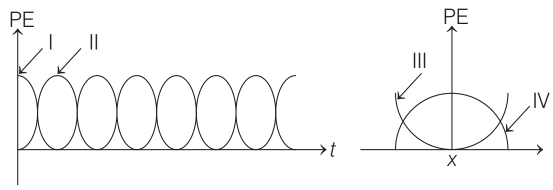


2. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **(2009)**



(a) $\frac{\sqrt{3}}{32}\pi^2$ cms^{-2}
(b) $\frac{-\pi^2}{32}$ cms^{-2}
(c) $\frac{\pi^2}{32}$ cms^{-2}
(d) $-\frac{\sqrt{3}}{32}\pi^2$cms^{-2}

3. For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x. **(2003, 2M)**



(a) I, III
(b) II, IV
(c) II, III
(d) I, IV

Topic 3 Time Period of Simple Harmonic Motion

Objective Questions I (Only one correct option)

1. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10\pi t + \phi)$.
Here, t is in seconds.
The time taken for its amplitude of vibration to drop to half of its initial value is close to **(2019 Main, 10 April I)**
(a) 27 s
(b) 13 s
(c) 4 s
(d) 7 s

2. A simple harmonic motion is represented by $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm. The amplitude and time period of the motion are **(2019 Main, 12 Jan II)**
(a) 10 cm, $\frac{3}{2}$ s
(b) 5 cm, $\frac{2}{3}$ s
(c) 5 cm, $\frac{3}{2}$ s
(d) 10 cm, $\frac{2}{3}$ s

3. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time (in seconds) is **(2019 Main, 10 Jan II)**
(a) $\frac{4\pi}{3}$
(b) $\frac{8\pi}{3}$
(c) $\frac{7}{3}\pi$
(d) $\frac{3}{8}\pi$

4. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 92 s and 95 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be **(2016 Main)**
(a) $(92 \pm 2\text{ s})$
(b) $(92 \pm 5\text{s})$
(c) $(92 \pm 1.8\text{s})$
(d) $(92 \pm 3\text{s})$

5. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2$, $(k = 1\text{m/s}^2)$, where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is
(Take, $g = 10\text{ m/s}^2$) **(2005, 2M)**
(a) 6/5
(b) 5/6
(c) 1
(d) 4/5

6. The period of oscillation of simple pendulum of length L suspended from the roof of the vehicle which moves without friction, down an inclined plane of inclination α, is given by **(2000, 2M)**
(a) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$
(b) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$
(c) $2\pi\sqrt{\frac{L}{g}}$
(d) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

7. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to **(1993, 2M)**

(a) $2\pi\,(m/k)^{1/2}$ (b) $2\pi\sqrt{\dfrac{m\,(YA+kL)}{YAk}}$

(c) $2\pi\,[(mYA/kL)]^{1/2}$ (d) $2\pi\,(mL/YA)^{1/2}$

8. A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by **(1992, 2M)**

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\dfrac{M\eta}{L}}$

(c) $2\pi\sqrt{\dfrac{ML}{\eta}}$ (d) $2\pi\sqrt{\dfrac{M}{\eta L}}$

9. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with a small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is **(1990, 2M)**

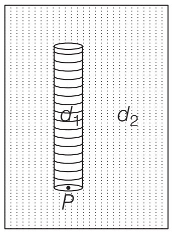
(a) $\dfrac{1}{2\pi}\left(\dfrac{k-A\rho g}{M}\right)^{1/2}$ (b) $\dfrac{1}{2\pi}\left(\dfrac{k+A\rho g}{M}\right)^{1/2}$

(c) $\dfrac{1}{2\pi}\left(\dfrac{k+\rho gL^2}{M}\right)^{1/2}$ (d) $\dfrac{1}{2\pi}\left(\dfrac{k+A\rho g}{A\rho g}\right)^{1/2}$

Analytical & Descriptive Questions

10. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **(2004, 4M)**

11. A thin rod of length L and uniform cross-section is pivoted at its lowest point P inside a stationary homogeneous and non-viscous liquid. The rod is free to rotate in a vertical plane about a horizontal axis passing through P.



The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **(1996, 5M)**

12. A thin fixed ring of radius 1 m has a positive charge 1×10^{-5} C uniformly distributed over it. A particle of mass 0.9 g and having a negative charge of 1×10^{-6} C is placed on the axis at a distance of 1 cm from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillations. **(1982, 5M)**

13. An ideal gas is enclosed in a vertical cylindrical container and supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. Atmospheric pressure is p_0 and when the piston is in equilibrium, the volume of the gas is V_0. The piston is now displaced slightly from its equilibrium position. Assuming that the system is completely isolated from its surroundings, show that the piston executes simple harmonic motion and find the frequency of oscillation. **(1981, 6M)**

14. A point mass m is suspended at the end of massless wire of length L and cross-sectional area A. If Y is the Young's modulus of elasticity of the material of the wire, obtain the expression for the frequency of the simple harmonic motion along the vertical line. **(1978)**

Integer Answer Type Question

15. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n\times10^9$ Nm^{-2}, the value of n is. **(2010)**

Topic 4 Spring Based Problems

Objective Questions I (Only one correct option)

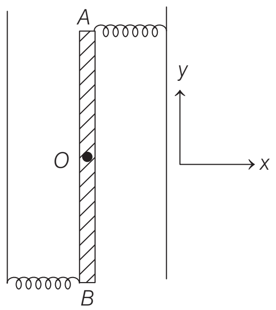
1. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1 / k_2 of the corresponding force constants k_1 and k_2 will be **(2019 Main, 12 April II)**

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

2. A massless spring ($k = 800\,\text{N/m}$), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released, so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K) **(2019 Main, 9 April II)**

(a) 10^{-4} K (b) 10^{-3} K (c) 10^{-1} K (d) 10^{-5} K

3. Two light identical springs of spring constant k are attached horizontally at the two ends of an uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure.

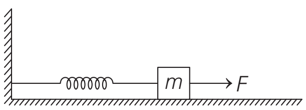


The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is **(2019 Main, 12 Jan I)**

(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

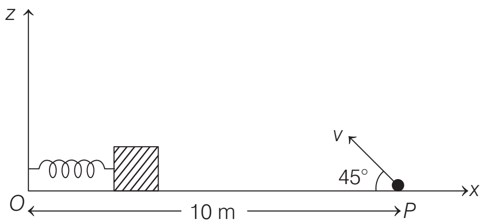
(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

4. A block of mass m lying on a smooth horizontal surface is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is **(2019 Main, 09 Jan I)**



(a) $\frac{\pi F}{\sqrt{mk}}$ (b) $\frac{F}{\sqrt{mk}}$ (c) $\frac{2F}{\sqrt{mk}}$ (d) $\frac{F}{\pi\sqrt{mk}}$

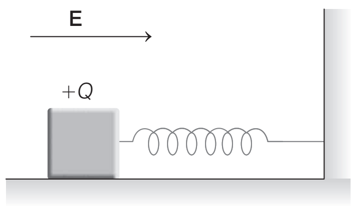
5. A small block is connected to one end of a massless spring of unstretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure.



Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take, $g = 10\,\text{m/s}^2$) **(2012)**

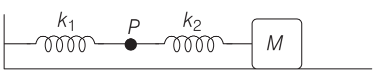
(a) $\sqrt{50}$ m/s (b) $\sqrt{51}$ m/s (c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

6. A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field **E** is switched-on as shown, then the SHM of the block will be **(2011)**



(a) of the same frequency and with shifted mean position
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position
(d) of changed frequency and with the same mean position

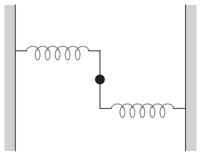
7. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **(2009)**



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$ (c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

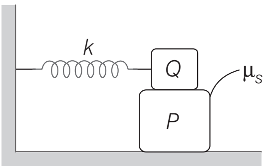
8. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and rod is free to oscillate in the horizontal plane.

The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is **(2009)**



(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

9. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by a distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks? **(2004, 2M)**



(a) $kA/2$ (b) kA (c) $\mu_s mg$ (d) Zero

10. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then, the long piece will have a force constant of **(1999, 2M)**

(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$ (c) $3k$ (d) $6k$

11. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the one amplitude of vibration of M to that of N is **(1988, 1M)**

(a) k_1/k_2 (b) $\sqrt{k_2/k_1}$ (c) k_2/k_1 (d) $\sqrt{k_1/k_2}$

Objective Question II (One or more correct option)

12. A particle of mass m is attached to one end of a massless spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5\,u_0$, it collides elastically with a rigid wall. After this collision **(2013 Adv.)**

(a) the speed of the particle when it returns to its equilibrium position is u_0

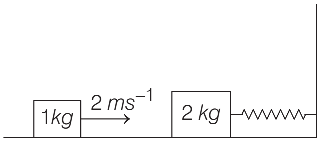
(b) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(c) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(d) the time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

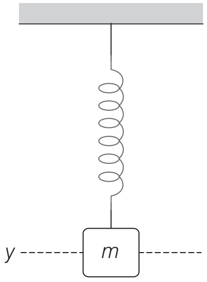
Numerical Value

13. A spring block system is resting on a frictionless floor as shown in the figure. The spring constant is $2.0\ \text{Nm}^{-1}$ and the mass of the block is $2.0\,\text{kg}$. Ignore the mass of the spring. Initially, the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of $2.0\ \text{ms}^{-1}$ collides elastically with the first block. The collision is such that the $2.0\,\text{kg}$ block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is .... **(2018 Adv.)**



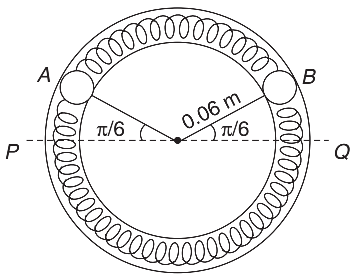
Analytical & Descriptive Questions

14. A mass m is undergoing SHM in the vertical direction about the mean position y_0 with amplitude A and angular frequency ω. At a distance y from the mean position, the mass detaches from the spring. Assume that the spring contracts and does not obstruct the motion of m. Find the distance y (measured from the mean position) such that the height h attained by the block is maximum. ($A\omega^2 > g$). **(2005)**



15. Two identical balls A and B, each of mass 0.1 kg, are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π m and spring constant 0.1 N/m.

Initially, both the balls are displaced by an angle $\theta = \pi/6$ rad with respect to the diameter PQ of the circle (as shown in figure) and released from rest. **(1993, 6M)**



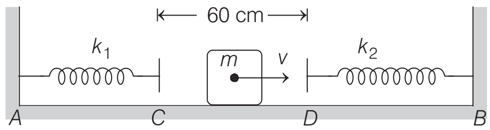
(a) Calculate the frequency of oscillation of ball B.

(b) Find the speed of ball A when A and B are at the two ends of the diameter PQ

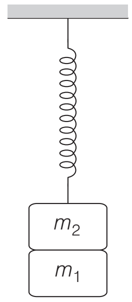
(c) What is the total energy of the system?

16. Two light springs of force constants k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure.

The distance CD between the free ends of the spring is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation of the block.
(Take, $k_1 = 1.8$ N/m, $k_2 = 3.2$ N/m, $m = 200$ g) **(1985, 6M)**



17. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k (Fig.). When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2. **(1981, 3M)**



18. A mass M attached to a spring oscillates with a period of 2 s. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hooke's law is obeyed. **(1979)**

Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is **(Main 2019, 9 April I)**

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$
(c) $4T\sqrt{\frac{1}{14}}$ (d) $4T\sqrt{\frac{1}{15}}$

2. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to **(2019 Main, 8 April II)**

(a) 20 s (b) 50 s
(c) 100 s (d) 10 s

3. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by **(2019 Main, 11 Jan II)**

(a) 1 rad/s (b) 10^{-5} rad/s
(c) 10^{-3} rad/s (d) 10^{-1} rad/s

4. A cylindrical plastic bottle of negligible mass is filled with 310 mL of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm, then ω is close to (Take, density of water $= 10^3$ kg/m^3) **(2019 Main, 10 Jan II)**

(a) 2.50 rad s^{-1} (b) 5.00 rad s^{-1}
(c) 1.25 rad s^{-1} (d) 3.75 rad s^{-1}

5. A rod of mass 'M' and length '$2L$' is suspended at its middle by a wire. It exhibits torsional oscillations. If two masses each of 'm' are attached at distance '$L/2$' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to **(2019 Main, 9 Jan II)**

(a) 0.57 (b) 0.37
(c) 0.77 (d) 0.17

6. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} per second. What is the force constant of the bonds connecting one atom with the other? (Take, molecular weight of silver = 108 and Avogadro number $= 6.02 \times 10^{23}$ g mol^{-1}) **(2018 Main)**

(a) 5.5 N/m (b) 6.4 N/m (c) 7.1 N/m (d) 2.2 N/m

7. A magnetic needle of magnetic moment 6.7×10^{-2} **Am**^{2} and moment of inertia 7.5×10^{-6} kg **m**^{2} is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is **(2017 Main)**

(a) 8.89 s (b) 6.98 s (c) 8.76 s (d) 6.65s

8. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2}{3}A$ from equilibrium position. The new amplitude of the motion is **(2016 Main)**

(a) $\frac{A}{3}\sqrt{41}$ (b) $3A$ (c) $A\sqrt{3}$ (d) $\frac{7}{3}A$

9. A particle moves with simple harmonic motion in a straight line. In first τ sec, after starting from rest it travels a distance a and in next τ sec, it travels $2a$, in same direction, then **(2014 Main)**

(a) amplitude of motion is $3a$
(b) time period of oscillations is 8π
(c) amplitude of motion is $4a$
(d) time period of oscillations is 6π

10. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are **(2011)**

(a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$

(c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

11. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from O to $A/2$ is T_1 and to go from $A/2$ to A is T_2, then **(2001, 2M)**

(a) $T_1 < T_2$ (b) $T_1 > T_2$

(c) $T_1 = T_2$ (d) $T_1 = 2T_2$

12. A particle free to move along the X-axis has potential energy given by $U(x) = k[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then, **(1999, 2M)**

(a) at points away from the origin, the particle is in unstable equilibrium
(b) for any finite non-zero value of x, there is a force directed away from the origin
(c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin
(d) for small displacements from $x = 0$, the motion is simple harmonic

13. A particle of mass m is executing oscillation about the origin on the X-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is **(1998, 2M)**

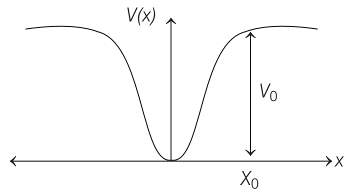
(a) proportional to $1/\sqrt{a}$ (b) independent of a

(c) proportional to $\sqrt{a}$ (d) proportional to $a^{3/2}$

Passage Based Questions

Passage

When a particle of mass m moves on the X-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{m/k}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the X-axis. Its potential energy is $V(x) = \alpha x^4 (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \ge X_0$ (see figure). **(2010)**



14. If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$ (b) $E > 0$ (c) $V_0 > E > 0$ (d) $E > V_0$

15. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

16. The acceleration of this particle for $|x| > X_0$ is

(a) proportional to V_0 (b) proportional to $\frac{V_0}{mX_0}$

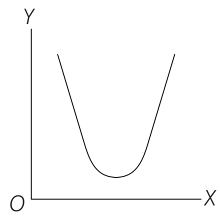
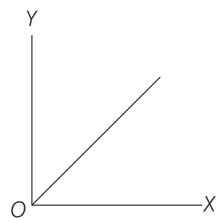
(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (d) zero

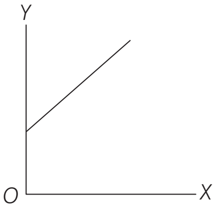
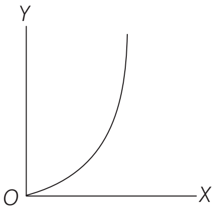
Fill in the Blank

17. Two simple harmonic motions are represented by the equations $y_1 = 10 \sin(3\pi t + \pi/4)$ and $y_2 = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$. Their amplitudes are in the ratio of …… . **(1986, 2M)**

Match the Columns

18. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. **(2008, 7M)**

	Column I		Column II
(A)	Potential energy of a simple pendulum (Y-axis) as a function of displacement (X-axis).	(p)	
(B)	Displacement (Y-axis) as a function of time (X-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	(q)	

	Column I		Column II
(C)	Range of a projectile (Y-axis) as a function of its velocity (X-axis) when projected at a fixed angle.	(r)	
(D)	The square of the time period (Y-axis) of a simple pendulum as a function of its length (X-axis).	(s)	

19. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situations in Column I with the characteristics in Column II. **(2007, 6M)**

	Column I		Column II
(A)	The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p)	The object executes a simple harmonic motion.
(B)	The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q)	The object does not change its direction.
(C)	The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r)	The kinetic energy of the object keeps on decreasing.
(D)	The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s)	The object can change its direction only once.

Objective Questions II (One or more correct option)

20. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0 and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass $m\ (< M)$ is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M ? **(2016 Adv.)**

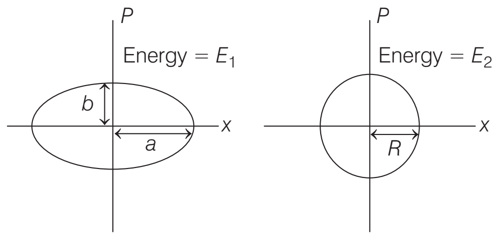
(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(b) The final time period of oscillation in both the cases is same

(c) The total energy decreases in both the cases

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases

21. Two independent harmonic oscillators of equal masses are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equations is/are **(2015 Adv.)**



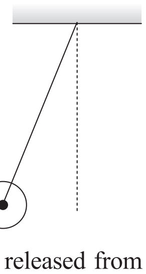
(a) $E_1\omega_1 = E_2\omega_2$

(b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$

(d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

22. A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius $R\ (< L)$ is attached at its centre to the free end of the rod. Consider two ways the disc is attached. **case** ***A***—the disc is not free to rotate about its centre and **case** ***B***—the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true? **(2011)**



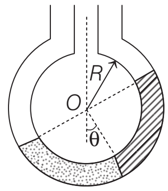
(a) Restoring torque in case A = Restoring torque in case B
(b) Restoring torque in case A < Restoring torque in case B
(c) Angular frequency for case A > Angular frequency for case$B$
(d) Angular frequency for case A < Angular frequency for case B

23. Function $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM **(2006, 5M)]**
(a) For any value of A, B and C (except $C = 0$)
(b) If $A = -B, C = 2B$, amplitude = $|B\sqrt{2}|$
(c) If $A = B$; $C = 0$
(d) If $A = B$; $C = 2B$, amplitude = $|B|$

24. Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by 45°, then **(1999, 3M)**
(a) the resultant amplitude is $(1+\sqrt{2})a$
(b) the phase of the resultant motion relative to the first is 90°
(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion
(d) the resulting motion is not simple harmonic

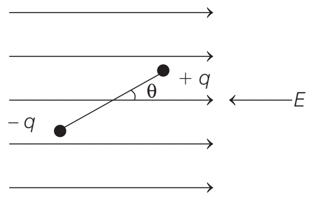
Analytical & Descriptive Questions

25. Two non-viscous, incompressible and immiscible liquids of densities ρ and $1.5\,\rho$ are poured into the two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in figure. Each liquid occupies one-fourth the circumference of the tube. **(1991, 4 + 4 M)**



(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.
(b) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

26. A point particle of mass M attached to one end of a massless rigid non- conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say of about 5 degrees) with the field direction. Find an expression for the minimum time needed for the rod to become parallel to the field after it is set free. **(1989, 8M)**



Answers

Topic 1

1. (b) **2.** (*) **3.** (d)
4. (a) **5.** (c) **6.** (b, c) **7.** 0.06

Topic 2

1. (c) **2.** (d) **3.** (a)

Topic 3

1. (d) **2.** (d) **3.** (b)
4. (a) **5.** (a) **6.** (a) **7.** (b)
8. (d) **9.** (b) **10.** $\frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$
11. $\omega = \sqrt{\frac{3g(d_2 - d_1)}{2d_1 L}}$ **12.** 0.628 s
13. $\frac{1}{2\pi}\sqrt{\frac{\gamma\,(p_0A^2 + MgA)}{V_0M}}$ **14.** $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ **15.** 4

Topic 4

1. (c) **2.** (d) **3.** (c) **4.** (b)
5. (a) **6.** (a) **7.** (d)
8. (c) **9.** (a) **10.** (b) **11.** (b)
12. (a, d) **13.** (2.09) **14.** $\frac{g}{\omega^2}$
15. (a) $\frac{1}{\pi}$ Hz (b) 0.0628 m/s (c) 3.9×10^{-4} J
16. 2.82 s **17.** $\omega = \sqrt{\frac{k}{m_2}}$, $A = \frac{m_1 g}{k}$ **18.** $M = 1.6$ kg

Topic 5

1. (d) **2.** (a) **3.** (c) **4.** (*)
5. (b) **6.** (c) **7.** (d) **8.** (d)
9. (d) **10.** (b) **11.** (a) **12.** (d)
13. (a) **14.** (c) **15.** (b) **16.** (d)
17. 1 : 1
18. A $\rightarrow$ p or p, s B $\rightarrow$ q, s or q, r, s C $\rightarrow$ s D $\rightarrow$ q
19. A $\rightarrow$ p B $\rightarrow$ q,r C $\rightarrow$ p D $\rightarrow$ q, r
20. (a, b, d) **21.** (b, d) **22.** (a, d) **23.** (b, d)
24. (a, c) **25.** (a) $\theta = \tan^{-1}\left(\frac{1}{5}\right)$ (b) $2\pi\sqrt{\frac{R}{6.11}}$
26. $\frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

Hints & Solutions

Topic 1 Energy in Simple Harmonic Motion

1. Kinetic energy of a pendulum is maximum at its mean position. Also, maximum kinetic energy of pendulum

$$K_{\max} = \frac{1}{2} m\omega^2 a^2$$

where,angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}}$$

or $\omega = \sqrt{\frac{g}{l}}$ or $\omega^2 = \frac{g}{l}$

and a = amplitude.

As amplitude is same in both cases so;

$$K_{\max} \propto \omega^2$$

or $K_{\max} \propto \frac{1}{l}$ [$\because g$ is constant]

According to given data, $K_1 \propto \frac{1}{l}$

and $K_2 \propto \frac{1}{2l}$

$\therefore$ $\frac{K_1}{K_2} = \left(\frac{1/l}{1/2l}\right) = 2$

or $K_1 = 2K_2 \Rightarrow K_2 = \frac{K_1}{2}$

2. Here given, displacement, $x(t) = A \sin \frac{\pi t}{90}$

where A is amplitude of S.H.M., t is time taken by particle to reach a point where its potential energy $U = \frac{1}{2}kx^2$ and kinetic energy $= \frac{1}{2}k(A^2 - x^2)$ here k is force constant and x is position of the particle.

Potential energy (U) at $t = 210$ s is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \sin^2\left(\frac{210}{90}\pi\right)$$

$$= \frac{1}{2}kA^2 \sin^2\left(2\pi + \frac{3}{9}\pi\right) = \frac{1}{2}kA^2 \sin^2\left(\frac{\pi}{3}\right)$$

Kinetic energy at $t = 210$ s, is

$$K = \frac{1}{2}k(A^2 - x^2)$$

$$= \frac{1}{2}kA^2\left[1 - \sin^2\left(\frac{210\pi}{90}\right)\right]$$

$$= \frac{1}{2}kA^2 \cos^2(210\,\pi/90)$$

$$\Rightarrow \quad K = \frac{1}{2}kA^2 \cos^2(\pi/3)$$

So, ratio of kinetic energy to potential energy is

$$\frac{K}{U} = \frac{\frac{1}{2}kA^2\cos^2(\pi/3)}{\frac{1}{2}kA^2\sin^2(\pi/3)} = \cot^2(\pi/3) = \frac{1}{3}$$

$\therefore$ No option given is correct.

3. Here, A = amplitude of particle in SHM

We know that in SHM potential energy (U) of a particle is given by the relation at distance x from the mean position is

$$U = \frac{1}{2}kx^2$$

and at the same point kinetic energy (K)

$$= \text{Total energy} - \text{Potential energy}$$

$$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

According to the question,

potential energy = kinetic energy

$\therefore$ $\frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

or $kx^2 = \frac{kA^2}{2}$ or $x = \pm\frac{A}{\sqrt{2}}$

4. Taking minimum potential energy at mean position to be zero, the expression of KE and PE are

$$\text{KE} = \frac{1}{2}m\omega^2(A^2 - d^2) \text{ and PE} = \frac{1}{2}m\omega^2 d^2$$

Both graphs are parabola. At $d = 0$, the mean position,

$$\text{PE} = 0 \text{ and KE} = \frac{1}{2}m\omega^2 A^2 = \text{maximum}$$

At $d = \pm A$, the extreme positions,

$$\text{KE} = 0 \text{ and PE} = \frac{1}{2}m\omega^2 A^2 = \text{maximum}$$

Therefore, the correct graph is (a).

5. In SHM, frequency with which kinetic energy oscillates is two times the frequency of oscillation of displacement.

$\therefore$ Correct answer is (c).

6. $\frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 10^6 \times (10^{-2})^2 = 100$ J

This is basically the energy of oscillation of the particle.

K, U and E at mean position $(x = 0)$ and extreme position $(x = \pm A)$ are shown in figure.

$x = 0$	$x = A$
K = 100 J = Maximum	K = 0 J
U = 60 J = Minimum	U = 160 J = Maximum
E = 160 J = Constant	E = 160 J = Constant

$\therefore$ Correct options are (b) and (c).

7. Since, $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Rightarrow \quad \frac{25}{\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{0.2}}$

or $\quad k = 50 \times 50 \times 0.2 = 500\,\text{N/m}$

If A is the amplitude of oscillation,

Total energy = KE + PE

$$\frac{1}{2}kA^2 = 0.5 + 0.4$$

$$A = \sqrt{\left(\frac{2 \times 0.9}{500}\right)}$$

$$= 0.06\,\text{m}$$

Topic 2 Graphs in Simple Harmonic Motion

1. KE is maximum at mean position and minimum at extreme position $\left(\textbf{at } t = \frac{T}{4}\right)$.

2. $T = 8\text{s}, \ \omega = \frac{2\pi}{T} = \left(\frac{\pi}{4}\right)\text{rads}^{-1}$

$\Rightarrow \quad x = A\sin\omega t$

$\therefore \quad a = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$

Substituting $\quad t = \frac{4}{3}$ s, we get

$$a = -\left(\frac{\sqrt{3}}{32}\pi^2\right)\text{cm-s}^{-2}$$

3. Potential energy is minimum (in this case zero) at mean position $(x = 0)$ and maximum at extreme positions $(x = \pm A)$.

At time $t = 0, x = A$. Hence, PE should be maximum. Therefore, graph I is correct. Further in graph III, PE is minimum at $x = 0$. Hence, this is also correct .

Topic 3 Time Period of Simple Harmonic Motion

1. Given, displacement is

$$x(t) = e^{-0.1t}\cos(10\pi t + \phi)$$

Here, amplitude of the oscillator is

$$A = e^{-0.1t} \qquad \ldots \text{(i)}$$

Let it takes t seconds for amplitude to be dropped by half.

At $\quad t = 0 \Rightarrow A = 1 \quad$ [from Eq. (i)]

At $\quad t = t \Rightarrow A' = \frac{A}{2} = \frac{1}{2}$

So, Eq. (i) can be written as

$$e^{-0.1t} = \frac{1}{2}$$

or $\quad e^{0.1t} = 2$

or $\quad 0.1t = \ln(2)$

or $\quad t = \frac{1}{0.1}\ln(2) = 10\ln(2)$

Now, $\ln(2) = 0.693$

$\therefore \quad t = 10 \times 0.693 = 6.93$ s

or $\quad t \approx 7$ s

2. Equation for SHM is given as

$$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

$$= 5 \times 2\left(\frac{1}{2} \times \sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$= 5 \times 2\left(\cos\frac{\pi}{3} . \sin 3\pi t + \sin\frac{\pi}{3} . 3\pi t\right)$$

$$= 5 \times 2\sin\left(3\pi t + \frac{\pi}{3}\right)$$

[using, $\sin(a + b) = \sin a\cos b + \cos a\sin b$]

or $\quad y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

Comparing this equation with the general equation of SHM, i.e.

$$y = A\sin\left(\frac{2\pi t}{T} + \phi\right),$$

We get, amplitude, $A = 10$ cm

and $\quad 3\pi = \frac{2\pi}{T}$

or Time period, $\quad T = \frac{2}{3}$ s

3. In simple harmonic motion, position (x), velocity (v) and acceleration (a) of the particle are given by

$$x = A\sin\omega t$$

$$v = \omega\sqrt{A^2 - x^2} \text{ or } v = A\omega\cos\omega t$$

and $\quad a = -\omega^2 x \quad$ or $a = -\omega^2 A\sin\omega t$

Given, amplitude $A = 5$ cm and displacement $x = 4$ cm.

At this time (when $x = 4$ cm), velocity and acceleration have same magnitude.

$\Rightarrow \quad |v_{x=4}| = |a_{x=4}|$ or $|\omega\sqrt{5^2 - 4^2}| = |-4\omega^2|$

$\Rightarrow \quad 3\omega = +4\omega^2 \Rightarrow \omega = (3/4)$ rad/s

So, time period, $T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{3} \times 4 = \frac{8\pi}{3}$ s

4. True value $= \frac{90 + 91 + 95 + 92}{4} = 92$

Mean absolute error

$$= \frac{|92 - 90| + |92 - 91| + |92 - 95| + |92 - 92|}{4}$$

$$= \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Value $= (92 \pm 1.5)$

Since, least count is 1 sec

$\therefore$ Value = $(92 \pm 2\text{s})$

5. $y = kt^2 \Rightarrow \dfrac{d^2 y}{dt^2} = 2k$

or $a_y = 2\text{m/s}^2$ (as $k = 1\text{m/s}^2$)

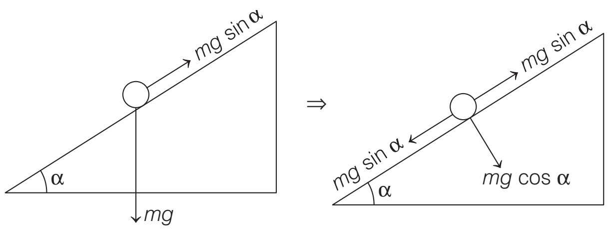
$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

and $$T_2 = 2\pi\sqrt{\frac{l}{g + a_y}}$$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{g + a_y}{g} = \frac{10 + 2}{10} = \frac{6}{5}$$

$\therefore$ Correct answer is (a).

6. Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as follows



$\therefore$ Net force on the bob is $F_{\text{net}} = mg\cos\alpha$

or net acceleration of the bob is $g_{\text{eff}} = g\cos\alpha$

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

or $$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

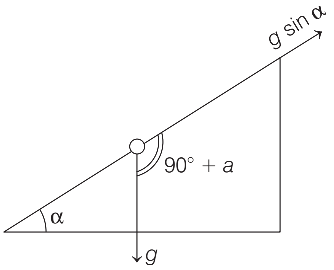
NOTE

- Whenever point of suspension is accelerating

Take, $T = 2\pi\sqrt{\dfrac{L}{g_{\text{eff}}}}$

where, $g_{\text{eff}} = g - a$

a = acceleration of point of suspension.



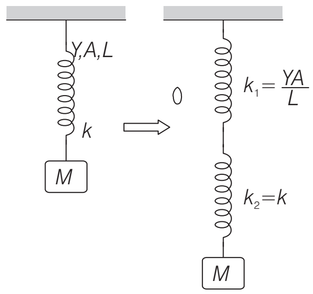
In this question, $a = g\sin\alpha$ (down the plane)

$\therefore$ $|g - a| = g_{\text{eff}}$

$$= \sqrt{g^2 + (g\sin\alpha)^2 + 2(g)(g\sin\alpha)\cos(90° + \alpha)}$$

$$= g\cos\alpha$$

7. $$k_{\text{eq}} = \frac{k_1 k_2}{k_1 + k_2} = \frac{\dfrac{YA}{L}}{\dfrac{YA}{L} + k} = \frac{YAk}{YA + Lk}$$



$$\therefore \quad T = 2\pi\sqrt{\frac{m}{k_{\text{eq}}}}$$

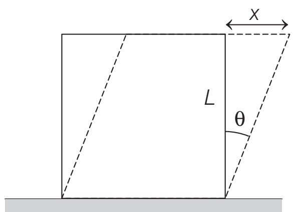
$$= 2\pi\sqrt{\frac{m(YA + Lk)}{YAk}}$$

NOTE Equivalent force constant for a wire is given by $k = \dfrac{YA}{L}$. Because in case of a wire, $F = \left(\dfrac{YA}{L}\right)\Delta L$ and in case of spring, $F = k.\Delta x$. Comparing these two, we find k of wire $= \dfrac{YA}{L}$.

8. Modulus of rigidity, $\eta = F/A\,\theta$

Here, $A = L^2$

and $\theta = \dfrac{x}{L}$



Therefore, restoring force is

$$F = -\eta A\theta = -\eta Lx$$

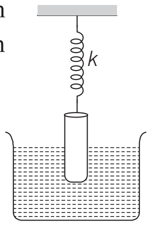
or acceleration, $a = \dfrac{F}{M} = -\dfrac{\eta L}{M}x$

Since, $a \propto -x$, oscillations are simple harmonic in nature, time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|}$$

$$= 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{M}{\eta L}}$$

9. When cylinder is displaced by an amount x from its mean position, spring force and upthrust both will increase. Hence,



Net restoring force = extra spring force + extra upthrust

or $\quad F = -(kx + Ax\rho g)$

or $\quad a = -\left(\dfrac{k + \rho A g}{M}\right)x$

Now, $\quad f = \dfrac{1}{2\pi}\sqrt{\left|\dfrac{a}{x}\right|} = \dfrac{1}{2\pi}\sqrt{\dfrac{k + \rho A g}{M}}$

$\therefore$ Correct option is (b).

10. Half of the volume of sphere is submerged.

For equilibrium of sphere, weight = upthrust

$\therefore \quad V\rho_s g = \dfrac{V}{2}(\rho_L)(g)$

$$\rho_s = \frac{\rho_L}{2} \qquad \ldots(\text{i})$$

When slightly pushed downwards by x, weight will remain as it is while upthrust will increase. The increased upthrust will become the net restoring force (upwards).

$F = -$ (extra upthrust)

$= -$ (extra volume immersed) $(\rho_L)(g)$

or $\quad ma = -(\pi R^2)x\rho_L g \quad (a = \text{acceleration})$

$\therefore \quad \dfrac{4}{3}\pi R^3\left(\dfrac{\rho}{2}\right)a = -(\pi R^2 \rho g)x$

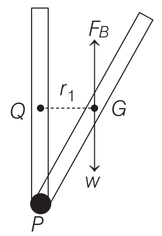
$\therefore \quad a = -\left(\dfrac{3g}{2R}\right)x$

as $a \propto -x$, motion is simple harmonic.

Frequency of oscillation, $f = \dfrac{1}{2\pi}\sqrt{\left|\dfrac{a}{x}\right|}$

$= \dfrac{1}{2\pi}\sqrt{\dfrac{3g}{2R}}$

11. Let S be the area of cross-section of the rod.



In the displaced position, as shown in figure, weight (w) and upthrust (F_B) both pass through its centre of gravity G.

Here, w = (volume) (density of rod)

$g = (SL)(d_1)g$

F_B = (Volume) (density of liquid) g

$= (SL)(d_2)g$

Given that, $d_1 < d_2$. Therefore, $w < F_B$

Therefore, net force acting at G will be

$F = F_B - w = (SLg)(d_2 - d_1)$ upwards. Restoring torque of this force about point P is

$\tau = F \times r_\perp = (SLg)(d_2 - d_1)(QG)$

or $\quad \tau = -(SLg)(d_2 - d_1)\left(\dfrac{L}{2}\sin\theta\right)$

Here, negative sign shows the restoring nature of torque.

or $$\tau = -\left\{\frac{SL^2 g\,(d_2 - d_1)}{2}\right\}\theta \qquad \ldots(\text{i})$$

$\sin\theta \approx \theta$ for small values of θ

From Eq. (i), we see that $\tau \propto -\theta$

Hence, motion of the rod will be simple harmonic.

Rewriting Eq. (i) as

$$I\frac{d^2\theta}{dt^2} = -\left\{\frac{SL^2 g\,(d_2 - d_1)}{2}\right\}\theta \qquad \ldots(\text{ii})$$

Here, I = moment of inertia of rod about an axis passing through P.

$$I = \frac{ML^2}{3} = \frac{(SLd_1)L^2}{3}$$

Substituting this value of I in Eq. (ii), we have

$$\frac{d^2\theta}{dt^2} = -\left\{\frac{3}{2}\,\frac{g\,(d_2 - d_1)}{d_1 L}\right\}\theta$$

Comparing this equation with standard differential equation of SHM, i.e. $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$

The angular frequency of oscillation is

$$\omega = \sqrt{\frac{3g\,(d_2 - d_1)}{2\,d_1 L}}$$

12. Given, $Q = 10^{-5}$ C

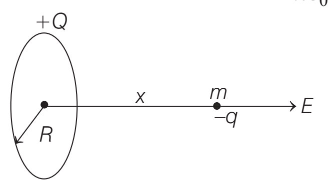
$q = 10^{-6}$ C, $\quad R = 1$ m

and $\quad m = 9 \times 10^{-4}$ kg

Electric field at a distance x from the centre on the axis of a ring is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{3/2}}$$

If $x << R$, $\quad R^2 + x^2 \approx R^2$ and $E \approx \dfrac{Qx}{4\pi\varepsilon_0 R^3}$



Net force on negatively charged particle would be qE and towards the centre of ring. Hence, we can write

$$F = -\frac{Q\,q\,x}{(4\pi\varepsilon_0)R^3}$$

or $\quad$ acceleration $a = \frac{F}{m} = -\frac{Qqx}{(4\pi\varepsilon_0)\,mR^3}$

as $a \propto -x$, motion of the particle is simple harmonic in nature. Time period of which will be given by

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} \text{ or } T = 2\pi\sqrt{\frac{(4\pi\varepsilon_0)mR^3}{Qq}}$$

Substituting the values, we get

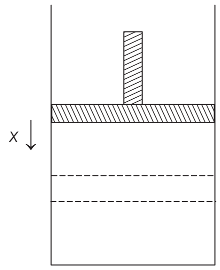
$$T = 2\pi\sqrt{\frac{(9\times10^{-4})(1)^3}{(9\times10^9)(10^{-5})(10^{-6})}}$$

$$= 0.628\text{ s}$$

13. In equilibrium pressure inside the cylinder

= pressure just outside it

or $\quad p = p_0 + \frac{Mg}{A}$ $\quad$ ...(i)



When piston is displaced slightly by an amount x, change in volume, $\quad dV = -Ax$

Since, the cylinder is isolated from the surroundings, process is adiabatic in nature. In adiabatic process,

$$\frac{dp}{dV} = -\gamma\frac{p}{V}$$

or increase in pressure inside the cylinder,

$$dp = -\frac{(\gamma p)}{V}(dV) = \gamma\left(\frac{p_0 + \frac{Mg}{A}}{V_0}\right)(Ax)$$

This increase in pressure when multiplied with area of cross-section A will give a net upward force (or the restoring force). Hence,

$$F = -(dp)\,A = -\gamma\left(\frac{p_0A^2 + MgA}{V_0}\right)x$$

or $\quad a = \frac{F}{M} = -\gamma\left(\frac{p_0A^2 + MgA}{V_0M}\right)x$

Since, $a \propto -x$, motion of the piston is simple harmonic in nature. Frequency of this oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{\gamma\,(p_0A^2 + MgA)}{V_0M}}$$

14. Force constant of a wire $k = \frac{YA}{L}$

Frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

or $\quad f = \frac{1}{2\pi}\sqrt{\frac{(YA/L)}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

15. $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{YA}{lm}} = \sqrt{\frac{(n\times10^9)(4.9\times10^{-7})}{1\times0.1}}$

Putting, $\omega = 140\text{ rad s}^{-1}$ in above equation we get, $n = 4$

$\therefore$ Answer is 4.

Topic 4 Spring Based Problems

1. If parameters like material, number of loops per unit length, area of cross-section, etc., are kept same, then force constant of spring is inversely proportional to its length.

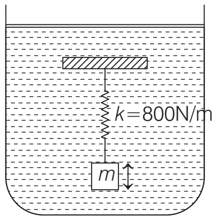
In given case, all other parameters are same for both parts of spring.

So, $\quad k_1 \propto \frac{1}{l_1}$ and $k_2 \propto \frac{1}{l_2}$

$\therefore \quad \frac{k_1}{k_2} = \frac{l_2}{l_1}$

$= \frac{l_2}{nl_2} = \frac{1}{n}$ $\quad [\because l_1 = nl_2]$

2. The given situation is shown in the figure given below



When vibrations of mass are suddenly stopped, oscillation energy (or stored energy of spring) is dissipated as heat, causing rise of temperature.

So, conversation of energy gives

$$\frac{1}{2}kx_m^2 = (m_1s_1 + m_2s_2)\Delta T$$

where, x_m = amplitude of oscillation,

s_1 = specific heat of mass,

s_2 = specific heat of water

and $\quad \Delta T$ = rise in temperature.

Substituting values given in question, we have

$$\frac{1}{2}\times800\times(2\times10^{-2})^2$$

$$= \left(\left(\frac{500}{1000}\right)\times400 + 1\times4184\right)\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{16\times10^{-2}}{4384} = 3.65\times10^{-5}\text{ K}$$

3. When a system oscillates, the magnitude of restoring torque of system is given by

$$\tau = C\theta \quad \ldots(i)$$

where, C = constant that depends on system.

Also, $$\tau = I\alpha \quad \ldots(ii)$$

where, I = moment of inertia

and α = angular acceleration

From Eqs. (i) and (ii),

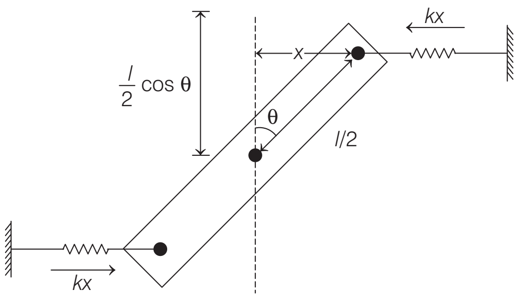
$$\alpha = \frac{C}{I}\cdot\theta \quad \ldots(iii)$$

and time period of oscillation of system will be

$$T = 2\pi\sqrt{\frac{I}{C}}$$

In given case, magnitude of torque is

τ = Force $\times$ perpendicular distance



$$\tau = 2kx \times \frac{l}{2}\cos\theta$$

For small deflection,

$$\tau = \left(\frac{kl^2}{2}\right)\theta \quad \ldots(iv)$$

$\because$ For small deflections, $\sin\theta = \dfrac{x}{(l/2)} \approx \theta$

$$\Rightarrow \quad x = \frac{l\theta}{2}$$

Also, $\cos\theta \approx 1$

comparing Eqs. (iv) and (i), we get

$$C = \frac{kl^2}{2} \Rightarrow \alpha = \frac{(kl^2/2)}{\left(\frac{1}{12}ml^2\right)}\cdot\theta$$

$$\Rightarrow \quad \alpha = \frac{6k}{m}\cdot\theta$$

Hence, time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{6k}}$$

Frequency of oscillation is given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

4. In a spring-block system, when a block is pulled with a constant force F, then its speed is maximum at the mean position. Also, it's acceleration will be zero.

In that case, force on the system is given as,

$$F = kx \quad \ldots(i)$$

where, x is the extension produced in the spring.

or $$x = \frac{F}{k}$$

Now we know that, for a system vibrating at its mean position, its maximum velocity is given as,

$$v_{\max} = A\omega$$

where, A is the amplitude and ω is the angular velocity.

Since, the block is at its mean position.

So, $$A = x = \frac{F}{k}$$

$$v_{\max} = \frac{F}{k}\sqrt{\frac{k}{m}} \quad \left[\because \omega = \sqrt{\frac{k}{m}}\right] = \frac{F}{\sqrt{km}}$$

Alternate Method

According to the work-energy theorem,

net work done = change in the kinetic energy

Here, net work done = work done due to external force (W_{ext}) + work done due to the spring (W_{spr}).

As, $$W_{\text{ext}} = F\cdot x$$

and $$W_{\text{spr}} = \frac{-1}{2}kx^2$$

$$\Rightarrow \quad \Delta\text{KE} = F\cdot x + \left(-\frac{1}{2}kx^2\right)$$

$$(\Delta\text{KE})_f - (\Delta\text{KE})_i = F\cdot x - \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}mv_{\max}^2 - \frac{1}{2}m(0)^2 = F\cdot\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2$$

[using Eq. (i)]

$$\Rightarrow \quad \frac{1}{2}mv_{\max}^2 = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k}$$

or $$v_{\max}^2 = \frac{F^2}{km}$$

$$\Rightarrow \quad v_{\max} = F/\sqrt{km}$$

5. t = time of flight of projectile

$$= \frac{2v\sin\theta}{g} \quad (\theta = 45°)$$

$$\therefore \quad v = \frac{gt}{2\sin\theta} = \frac{10\times 1}{2\times 1/\sqrt{2}} = \sqrt{50}\text{ m/s}$$

6. Frequency or time period of SHM depends on variable forces. It does not depend on constant external force. Constant external force can only change the mean position. For example, in the given question mean position is at natural length of spring in the absence of electric field.

Whereas in the presence of electric field mean position will be obtained after a compression of x_0. Where x_0 is given by

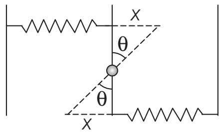
$$Kx_0 = QE \text{ or } x_0 = \frac{QE}{K}$$

$\therefore$ Correct answer is (a).

7. $x_1 + x_2 = A$ and $k_1x_1 = k_2x_2$ or $\dfrac{x_1}{x_2} = \dfrac{k_2}{k_1}$

Solving these equations, we get $x_1 = \left(\dfrac{k_2}{k_1 + k_2}\right) A$

8. $x = \dfrac{L}{2}\theta,$



Restoring torque $= -(2kx)\cdot\dfrac{L}{2}$

$$\alpha = -\frac{kL(L/2\theta)}{I} = -\left[\frac{kL^2/2}{ML^2/12}\right]\cdot\theta = -\left(\frac{6k}{M}\right)\theta$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

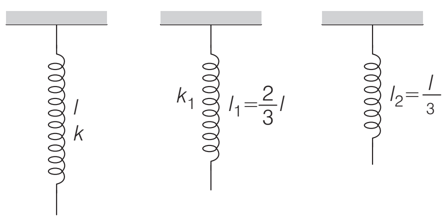
9. Angular frequency of the system, $\omega = \sqrt{\dfrac{k}{m+m}} = \sqrt{\dfrac{k}{2m}}$

Maximum acceleration of the system will be, $\omega^2 A$ or $\dfrac{kA}{2m}$.

This acceleration to the lower block is provided by friction.

Hence, $\quad f_{\max} = ma_{\max} = m\omega^2 A = m\left(\dfrac{kA}{2m}\right) = \dfrac{kA}{2}$

10. $l_1 = 2l_2 \qquad \therefore \; l_1 = \dfrac{2}{3}l$



$$\therefore \qquad k_1 = \frac{3}{2}k$$

Force constant $k \propto \dfrac{1}{\text{length of spring}}$

11. $$(v_M)_{\max} = (v_N)_{\max}$$

$$\therefore \qquad \omega_M A_M = \omega_N A_N$$

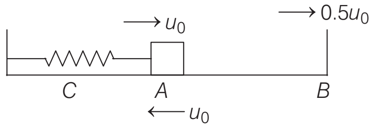
or $$\frac{A_M}{A_N} = \frac{\omega_N}{\omega_M} = \sqrt{\frac{k_2}{k_1}} \qquad \left(\because \omega = \sqrt{\frac{k}{m}}\right)$$

$\therefore$ Correct answer is (b).

12. At equilibrium ($t = 0$) particle has maximum velocity u_0. Therefore velocity at time t can be written as

$$u = u_{\max}\cos\omega t + u_0\cos\omega t$$

writing, $$u = 0.5u_0 = u_0\cos\omega t$$



$$\therefore \qquad \omega t = \frac{\pi}{3}$$

$$\therefore \qquad \frac{2\pi}{T}t = \frac{\pi}{3}$$

$$\therefore \qquad t = \frac{T}{6}$$

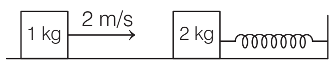
(b) $t = t_{AB} + t_{BA} = \dfrac{T}{6} + \dfrac{T}{6} = \dfrac{T}{3} = \dfrac{2\pi}{3}\sqrt{\dfrac{m}{k}}$

(c) $t = t_{AB} + t_{BA} + t_{AC} = \dfrac{T}{6} + \dfrac{T}{6} + \dfrac{T}{4} = \dfrac{7}{12}T = \dfrac{7\pi}{6}\sqrt{\dfrac{m}{k}}$

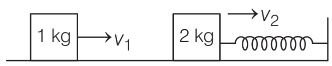
(d) $t = t_{AB} + t_{BA} + t_{AC} + t_{CA} = \dfrac{t}{6} + \dfrac{T}{6} + \dfrac{T}{4} + \dfrac{T}{4} = \dfrac{5}{6}T$

$$= \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

13. Just Before Collision,



Just After Collision



Let velocities of 1 kg and 2 kg blocks just after collision be v_1 and v_2 respectively.

From momentum conservation principle,

$$1 \times 2 = 1v_1 + 2v_2 \qquad \ldots\text{(i)}$$

Collision is elastic. Hence $e = 1$ or relative velocity of separation = relation velocity of approach.

$$v_2 - v_1 = 2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$v_2 = \frac{4}{3}\text{ m/s}, \; v_1 = \frac{-2}{3}\text{ m/s}$$

2 kg block will perform SHM after collision,

$$t = \frac{T}{2} = \pi\sqrt{\frac{m}{k}} = 3.14 \text{ s}$$

$$\text{Distance} = |v_1|t = \frac{2}{3} \times 3.14 = 2.093 = 2.09 \text{ m}$$

14. At distance y above the mean position velocity of the block.

$$v = \omega\sqrt{A^2 - y^2}$$

After detaching from the spring net downward acceleration of the block will be g.

Therefore, total height attained by the block above the mean position,

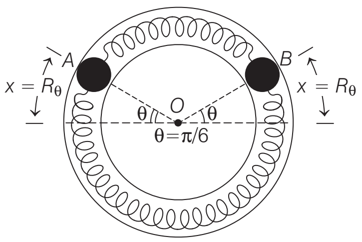
$$h = y + \frac{v^2}{2g} = y + \frac{\omega^2 (A^2 - y^2)}{2g}$$

For h to be maximum $dh/dy = 0$

Putting $\frac{dh}{dy} = 0$, we get $y = \frac{g}{\omega^2} = y_{\max}$

15. Given, mass of each block A and B, $m = 0.1\,\text{kg}$

Radius of circle, $R = 0.06\,\text{m}$



Natural length of spring $l_0 = 0.06\,\pi = \pi R$ (Half circle)

and spring constant, $k = 0.1\,\text{N/m}$

In the stretched position elongation in each spring

$$x = R\,\theta.$$

Let us draw FBD of A.

Spring in lower side is stretched by $2x$ and on upper side compressed by $2x$. Therefore, each spring will exert a force $2kx$ on each block. Hence, a restoring force, $F = 4kx$ will act on A in the direction shown in figure below .

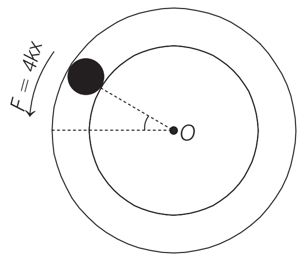
Restoring torque of this force about origin

$$\tau = -F \cdot R = -(4kx)R = -(4kR\theta)\,R$$

or $\quad \tau = -4kR^2 \cdot \theta \qquad \ldots\text{(i)}$

Since, $\tau \propto -\theta$, each ball executes angular SHM about origin O.

Eq. (i) can be rewritten as



$$I\alpha = -4kR^2\,\theta$$

or $\quad (mR^2)\,\alpha = -4kR^2\,\theta$

or $\quad \alpha = -\left(\frac{4k}{m}\right)\theta$

(a) Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\text{acceleration}}{\text{displacement}}\right|}$$

$$= \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

Substituting the values, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi}\,\text{Hz}$$

(b) In stretched position, potential energy of the system is

$$\text{PE} = 2\left\{\frac{1}{2}k\right\}\{2x\}^2 = 4kx^2$$

and in mean position, both the blocks have kinetic energy only. Hence, $\text{KE} = 2\left\{\frac{1}{2}mv^2\right\} = mv^2$

From energy conservation PE = KE

$\therefore \quad 4kx^2 = mv^2$

$\therefore \quad v = 2x\sqrt{\frac{k}{m}} = 2R\theta\sqrt{\frac{k}{m}}$

Substituting the values $v = 2\,(0.06)\,(\pi/6)\sqrt{\frac{0.1}{0.1}}$

or $\quad v = 0.0628\,\text{m/s}$

(c) Total energy of the system, E

= PE in stretched position

or = KE in mean position

$$E = mv^2 = (0.1)\,(0.0628)^2\ \text{J}$$

or $E = 3.9 \times 10^{-4}$ J

16. Between C and D block will move with constant speed of 120 cm/s. Therefore, period of oscillation will be (starting from C).

$$T = t_{CD} + \frac{T_2}{2} + t_{DC} + \frac{T_1}{2}$$

Here, $\quad T_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

and $\quad t_{CD} = t_{DC} = \frac{60}{120} = 0.5\,\text{s}$

$\therefore \quad T = 0.5 + \frac{2\pi}{2}\sqrt{\frac{0.2}{3.2}} + 0.5 + \frac{2\pi}{2}\sqrt{\frac{0.2}{1.8}}$

$(m = 200\,\text{g} = 0.2\,\text{kg})$

$$T = 2.82\,\text{s}$$

17. When m_1 is removed only m_2 is left. Therefore, angular frequency $\omega = \sqrt{k/m_2}$

Let x_1 be the extension when only m_2 is left. Then,

$$kx_1 = m_2 g \ \text{ or } \ x_1 = \frac{m_2 g}{k} \qquad \ldots\text{(i)}$$

Similarly, let x_2 be the extension in equilibrium when both m_1 and m_2 are suspended. Then,

$$(m_1 + m_2)g = kx_2$$

$\Rightarrow \quad x_2 = \frac{(m_1 + m_2)g}{k} \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii), amplitude of oscillation

$$A = x_2 - x_1 = \frac{m_1 g}{k}$$

18. $$T = 2\pi\sqrt{\frac{M}{K}} \quad \text{or} \quad T \propto \sqrt{M},\ \frac{T_2}{T_1} = \sqrt{\frac{M_2}{M_1}}$$

$\therefore \quad \frac{3}{2} = \sqrt{\frac{M+2}{M}} \qquad \ldots\text{(I)}$

Solving the equation (i), we get, $M = 1.6$ kg.

Topic 5 Miscellaneous Problems

1. We know that,
Time period of a pendulum is given by

$$T = 2\pi\sqrt{L / g_{\text{eff}}} \quad \ldots\text{(i)}$$

Here, L is the length of the pendulum and g_{eff} is the effective acceleration due to gravity in the respective medium in which bob is oscillating.

Initially, when bob is oscillating in air, $g_{\text{eff}} = g$.

So, initial time period, $T = 2\pi\sqrt{\frac{L}{g}}$ $\quad \ldots$(ii)

Let ρ_{bob} be the density of the bob.

When this bob is dipped into a liquid whose density is given as

$$\rho_{\text{liquid}} = \frac{\rho_{\text{bob}}}{16} = \frac{\rho}{16} \quad \text{(given)}$$

$\therefore$ Net force on the bob is

$$F_{\text{net}} = V\rho g - V \cdot \frac{\rho}{16} \cdot g \quad \ldots\text{(iii)}$$

(where, V = volume of the bob = volume of displaced liquid by the bob when immersed in it). If effective value of gravitational acceleration on the bob in this liquid is g_{eff}, then net force on the bob can also be written as

$$F_{\text{net}} = V\rho g_{\text{eff}} \quad \ldots\text{(iv)}$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{\text{eff}} = V\rho g - V\rho g / 16$$

$$\Rightarrow \quad g_{\text{eff}} = g - g/16 = \frac{15}{16}g \quad \ldots\text{(v)}$$

Substituting the value of g_{eff} from Eq. (v) in Eq. (i), the new time period of the bob will be

$$T' = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}} \Rightarrow T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}}$$

$$= \frac{4}{\sqrt{15}} \times T \quad \text{[using Eq. (ii)]}$$

2. Given, frequency of oscillations is $f = 5\,\text{osc s}^{-1}$

$\Rightarrow$ Time period of oscillations is $T = \frac{1}{f} = \frac{1}{5}\text{s}$

So, time for 10 oscillations is $= \frac{10}{5} = 2\text{s}$

Now, if A_0 = initial amplitude at $t = 0$ and γ = damping factor, then for damped oscillations, amplitude after t second is given as

$$A = A_0 e^{-\gamma t}$$

$\therefore$ After 2 s,

$$\frac{A_0}{2} = A_0 e^{-\gamma(2)} \quad \Rightarrow \quad 2 = e^{2\gamma}$$

$$\Rightarrow \quad \gamma = \frac{\log 2}{2} \quad \ldots\text{(i)}$$

Now, when amplitude is $\frac{1}{1000}$ of initial amplitude, i.e.

$$\frac{A_0}{1000} = A_0 e^{-\gamma t}$$

$$\Rightarrow \quad \log(1000) = \gamma t$$

$$\Rightarrow \quad \log(10^3) = \gamma t$$

$$3\log 10 = \gamma t$$

$$\Rightarrow \quad t = \frac{2 \times 3\log 10}{\log 2} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad t = 19.93\,\text{s}$$

$$\text{or} \quad t \approx 20\,\text{s}$$

3. We know that time period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, angular frequency $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$ $\quad$ ..(i)

Now, differentiate both side w.r.t g

$$\therefore \quad \frac{d\omega}{dg} = \frac{1}{2\sqrt{g}\sqrt{l}}$$

$$d\omega = \frac{dg}{2\sqrt{g}\sqrt{l}} \quad \text{...(ii)}$$

By dividing Eq. (ii) by Eq. (i), we get

$$\frac{d\omega}{\omega} = \frac{dg}{2g}$$

Or we can write

$$\frac{\Delta\omega}{\omega} = \frac{\Delta g}{2g} \quad \text{...(iii)}$$

As Δg is due to oscillation of support.

$$\therefore \quad \Delta g = 2\omega^2 A$$

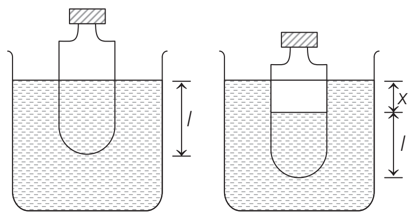
($\omega_1 \to 1\,\text{rad/s}$, support)

Putting value of Δg in Eq. (iii) we get

$$\frac{\Delta\omega}{\omega} = \frac{1}{2} \cdot \frac{2\omega_1^2 A}{g} = \frac{\omega_1^2 A}{g}; \; (A = 10^{-2}\,\text{m}^2)$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = \frac{1 \times 10^{-2}}{10} = 10^{-3}\ \text{rad/s}$$

4. In equilibrium condition bottle floats in water and its length 'l' inside water is same as the height of water upto which bottle is filled.



So, l = Volume of water in bottle/Area

$$= \frac{310}{\pi \times (2.5)^2} = 15.8\,\text{cm} = 0.158\,\text{m}$$

When bottle is slightly pushed inside by an amount x then, restoring force acting on the bottle is the upthrust of fluid displaced when bottle goes into liquid by amount x.

So, restoring force is;

$$F = -(\rho A x)\, g \quad \ldots(\text{i})$$

where ρ = density of water,

A = area of cross-section of bottle and

x = displacement from equilibriumposition

But $F = ma$ …(ii)

where, m = mass of water and bottle system

$= Al\rho$

From (i) and (ii) we have,

$$Al\rho\alpha = -\rho Axg \text{ or } a = -\frac{g}{l}x$$

As for SHM, $a = -\omega^2 x$

We have $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.158}} = \sqrt{63.29} \approx 8 \text{ rad s}^{-1}$

$\therefore$ No option is correct.

5 We know that in case of torsonal oscillation frequency

$$\nu = \frac{k}{\sqrt{I}}$$

where, I is moment of inertia and k is torsional constant.

$\therefore$ According to question, $\nu_1 = \dfrac{k}{\sqrt{\dfrac{M(2L)^2}{12}}}$

(As, MOI of a bar is $I = \dfrac{ML^2}{12}$)

or $$\nu_1 = \frac{k}{\sqrt{\dfrac{ML^2}{3}}} \quad \ldots(\text{i})$$

When two masses are attached at ends of rod. Then its moment of inertia is

$$\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2$$

So, new frequency of oscillations is,

$$\nu_2 = \frac{k}{\sqrt{\dfrac{M(2L)^2}{12} + 2m\left(\dfrac{L}{2}\right)^2}}$$

$$\nu_2 = \frac{k}{\sqrt{\dfrac{ML^2}{3} + \dfrac{mL^2}{2}}} \quad \ldots(\text{ii})$$

As, $\nu_2 = 80\%$ of $\nu_1 = 0.8\nu_1$

So, $$\frac{k}{\sqrt{\dfrac{ML^2}{3} + \dfrac{mL^2}{2}}} = \frac{0.8 \times k}{\sqrt{\dfrac{ML^2}{3}}}$$

After solving it, we get,

$$\frac{m}{M} = 0.37$$

6. Given, frequency, $f = 10^{12}$ /sec

Angular frequency, $\omega = 2\pi f = 2\pi \times 10^{12}$ /sec

Force constant, $k = m\omega^2 = \dfrac{108 \times 10^{-3}}{6.02 \times 10^{23}} \times 4\pi^2 \times 10^{24}$

$$k = 7.1 \text{ N/m}$$

7. Time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665 \text{ s}$$

Hence, time for 10 oscillations is $t = 6.65$ s.

8. $v = \omega\sqrt{A^2 - x^2}$ At, $x = \dfrac{2A}{3}$

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5}}{3}\omega A$$

As, velocity is trebled, hence $v' = \sqrt{5}A\omega$

This leads to new amplitude A'

$$\therefore \quad \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2} = \sqrt{5}A\omega$$

$$\Rightarrow \quad \omega^2\left[A'^2 - \frac{4A^2}{9}\right] = 5A^2\omega^2$$

$$\Rightarrow \quad A'^2 = 5A^2 + \frac{4}{9}A^2 = \frac{49}{9}A^2$$

$$A' = \frac{7}{3}A$$

9. In SHM, a particle starts from rest, we have

i.e. $x = A\cos\omega t$, at $t = 0, x = A$

When $t = \tau$, then $x = A - a$ ...(i)

When $t = 2\tau$, then

$$x = A - 3a \quad \ldots(\text{ii})$$

On comparing Eqs. (i) and (ii), we get

$$A - a = A\cos\omega\tau$$
$$A - 3a = A\cos 2\omega\tau$$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$

$$\Rightarrow \quad \frac{A - 3a}{A} = 2\left(\frac{A - a}{A}\right)^2 - 1$$

$$\Rightarrow \quad \frac{A - 3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$a^2 = 2aA$$

$$A = 2a$$

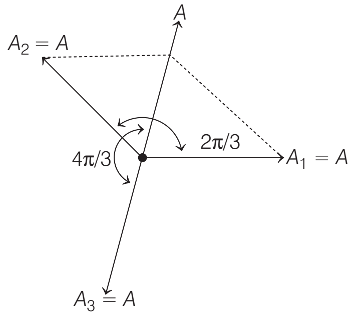
Now, $A - a = A\cos\omega\tau$

$\Rightarrow$ $\cos\omega\tau = 1/2$

$$\Rightarrow \qquad \frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$$\Rightarrow \qquad T = 6\pi$$

10.



Resultant amplitude of x_1 and x_2 is A at angle $\left(\frac{\pi}{3}\right)$ from A_1. To make resultant of x_1, x_2 and x_3 to be zero. A_3 should be equal to A at angle $\phi = \frac{4\pi}{3}$ as shown in figure.

$\therefore$ Correct answer is (b).

Alternate Solution.

It we substitute, $x_1 + x_2 + x_3 = 0$

or $A \sin \omega t + A \sin\left(\omega t + \frac{2\pi}{3}\right) + B \sin(\omega t + \phi) = 0$

Then by applying simple mathematics we can prove that

$$B = A \quad \text{and} \quad \phi = \frac{4\pi}{3}.$$

11. In SHM, velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme positions. Therefore, the time taken for the particle to go from O to $A/2$ will be less than the time taken to go it from A/2 to A, or $T_1 < T_2$.

NOTE

From the equations of SHM we can show that

$$t_1 = T_{o-A/2} = T/12 \quad \text{and} \quad t_2 = T_{A/2-A} = T/6$$

So, that $t_1 + t_2 = T_{o-A} = T/4$

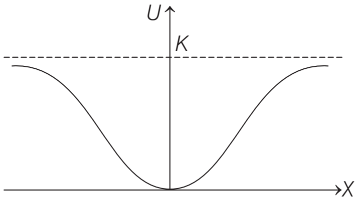
12. $U(x) = k(1 - e^{-x^2})$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore, U *versus* x graph will be as shown.

At origin.

Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is zero because

$$F = \frac{-dU}{dx} = -(\text{slope of } U\text{-}x \text{ graph}) = 0.$$



Therefore, origin is the stable equilibrium position. Hence, particle will oscillate simple harmonically about $x = 0$ for small displacements. Therefore, correct option is (d).

(a), (b) and (c) options are wrong due to following reasons.

(a) At equilibrium position $F = \frac{-dU}{dx} = 0$ i.e. slope of U-x graph should be zero and from the graph we can see that slope is zero at $x = 0$ and $x = \pm\infty$.

Now, among these equilibriums stable equilibrium position is that where U is minimum (Here $x = 0$). Unstable equilibrium position is that where U is maximum (Here none).

Neutral equilibrium position is that where U is constant (Here $x = \pm\infty$).

Therefore, option (a) is wrong.

(b) For any finite non-zero value of x, force is directed towards the origin because origin is in stable equilibrium position. Therefore, option (b) is incorrect.

(c) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, option (c) is also wrong.

13. $U(x) = k|x|^3$

$$\therefore \quad [k] = \frac{[U]}{[x^3]} = \frac{[ML^2 T^{-2}]}{[L^3]} = [ML^{-1} T^{-2}]$$

Now, time period may depend on

$$T \propto (\text{mass})^x (\text{amplitude})^y (k)^z$$

$$[M^0L^0T] = [M]^x [L]^y [ML^{-1} T^{-2}]^z$$

$$= [M^{x+z} L^{y-z} T^{-2z}]$$

Equating the powers, we get

$$-2z = 1 \quad \text{or} \quad z = -1/2$$

$$y - z = 0 \quad \text{or} \quad y = z = -1/2$$

Hence, $T \propto (\text{amplitude})^{-1/2}) \propto (a)^{-1/2}$

or $\quad T \propto \frac{1}{\sqrt{a}}$

14. If $E > V_0$, particle will escape. But simultaneously for oscillations, $E > 0$.

Hence, the correct answer is $V_0 > E > 0$ or the correct option is (c).

15. $[\alpha] = \left[\frac{PE}{x^4}\right] = \left[\frac{ML^2T^{-2}}{L^4}\right] = [ML^{-2}T^{-2}]$

$$\therefore \qquad \left[\frac{m}{\alpha}\right] = [L^2T^2]$$

$$\Rightarrow \qquad \left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = [T]$$

As dimensions of amplitude A is [L].

Hence, the correct option is (b).

16. For $|x| > X_0$, potential energy is constant. Hence, kinetic energy, speed or velocity will also remain constant.

$\therefore$ Acceleration will be zero.

Hence, the correct option is (d).

17. $A_1 = 10$ (directly)

For $A_2 : y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$

$$= 5\sin 3\pi t + 5\sqrt{3}\sin\left(3\pi t + \frac{\pi}{2}\right)$$

i.e. phase difference between two functions is $\frac{\pi}{2}$, so the resultant amplitude A_2 can be obtained by the vector method as under

$$A_2 = \sqrt{(5)^2 + (5\sqrt{3})^2} = 10$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{10}{10} = 1$$

18. (A) Potential energy is minimum at mean position.

(B) For $a = 0$, $s = vt \rightarrow$ option (q)

or $s = s_0 + vt \rightarrow$ option (r)

For $a =$ constant, $s = ut + \frac{1}{2}at^2 \rightarrow$ option (s)

(C) $R = \dfrac{v^2 \sin 2\theta}{g}$

$\therefore$ $R \propto v^2 \rightarrow$ option (s)

(D) $T = 2\pi\sqrt{\dfrac{l}{g}}$

$\therefore$ $T^2 \propto l \rightarrow$ option (q)

19. (A) Compare with the standard equation of SHM

$$v = \omega\sqrt{A^2 - x^2}$$

we see that the given motion is SHM with,

$$\omega = C_1 \text{ and } A^2 = C_2$$

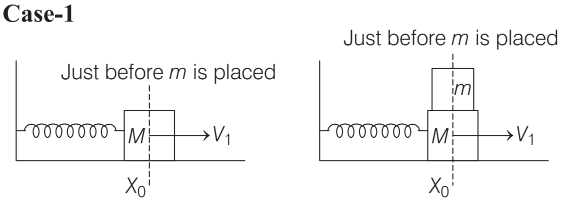
(B) The equation shows that the object does not change its direction and kinetic energy of the object keeps on decreasing.

(C) A pseudo force (with respect to elevator) will start acting on the object. Its means position is now changed and it starts SHM.

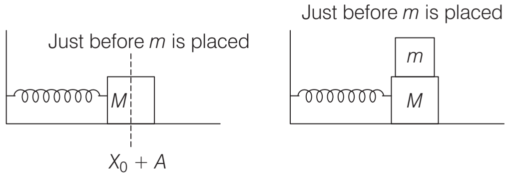
(D) The given velocity is greater than the escapevelocity

$= \left(\sqrt{\dfrac{2GM_e}{R_e}}\right)$. Therefore it keeps on moving towards infinity with decreasing speed.

20. Case-1



Case-2



In case-1,

$$Mv_1 = (M + m)v_2$$

$$v_2 = \left(\frac{M}{M + m}\right)v_1$$

$$\sqrt{\frac{k}{M + m}}\, A_2 = \left(\frac{M}{M + m}\right)\sqrt{\frac{k}{M}}\, A_1$$

$$A_2 = \sqrt{\frac{k}{M + m}}\, A_1$$

In case-2 $\quad A_2 - A_1$

$$T = 2\pi\sqrt{\frac{M + m}{k}} \text{ in both cases.}$$

Total energy decreases in first case whereas remain same in 2nd case. Instantaneous speed at x_0 decreases in both cases.

21. Ist Particle

$$P = 0 \text{ at } x = a$$

$\Rightarrow$ 'a' is the amplitude of oscillation 'A_1'.

At $x = 0$, $P = b$ (at mean position)

$$\Rightarrow \quad mv_{\max} = b$$

$$v_{\max} = \frac{b}{m}$$

$$E_1 = \frac{1}{2}mv_{\max}^2 = \frac{m}{2}\left[\frac{b}{m}\right]^2 = \frac{b^2}{2m}$$

$$A_1\omega_1 = v_{\max} = \frac{b}{m}$$

$$\Rightarrow \quad \omega_1 = \frac{b}{ma} = \frac{1}{mn^2} \quad \left(A_1 = a, \frac{a}{b} = n^2\right)$$

IInd Particle

$$P = 0 \text{ at } x = R$$

$$\Rightarrow \quad A_2 = R$$

At $\quad x = 0$, $P = R$

$$\Rightarrow \quad v_{\max} = \frac{R}{m}$$

$$E_2 = \frac{1}{2}mv_{\max}^2 = \frac{m}{2}\left[\frac{R}{m}\right]^2 = \frac{R^2}{2m}$$

$$A_2\omega_2 = \frac{R}{m}$$

$$\Rightarrow \quad \omega_2 = \frac{R}{mR} = \frac{1}{m}$$

(b) $\dfrac{\omega_2}{\omega_1} = \dfrac{1/m}{1/mn^2} = n^2$

(c) $\omega_1\omega_2 = \dfrac{1}{mn^2} \times \dfrac{1}{m} = \dfrac{1}{m^2n^2}$

(d) $\dfrac{E_1}{\omega_1} = \dfrac{b^2/2m}{1/mn^2} = \dfrac{b^2n^2}{2} = \dfrac{a^2}{2n^2} = \dfrac{R^2}{2}$

$$\frac{E_2}{\omega_2} = \frac{R^2/2m}{1/m} = \frac{R^2}{2}$$

$$\Rightarrow \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

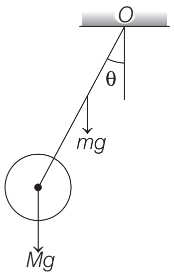
NOTE

It is not given that the second figure is a circle. But from the figure and as per the requirement of question, we consider it is a circle.

22. $\tau_A = \tau_B = \left(mg\dfrac{L}{2}\sin\theta + MgL\sin\theta \right)$

= Restoring torque about point O.

In case A, moment of inertia will be more. Hence, angular acceleration $(\alpha = \tau/I)$ will be less. Therefore, angular frequency will be less.



23. For $A = -B$ and $C = 2B$

$$X = B\cos 2\omega t + B\sin 2\omega t$$
$$= \sqrt{2}B\sin\left(2\omega t + \frac{\pi}{4}\right)$$

This is equation of SHM of amplitude $\sqrt{2}B$.

If $A = B$ and $C = 2B$, then $X = B + B\sin 2\omega t$

This is also equation of SHM about the point $X = B$. Function oscillates between $X = 0$ and $X = 2B$ with amplitude B.

24. From superposition principle

$$y = y_1 + y_2 + y_3$$
$$= a\sin\omega t + a\sin(\omega t + 45°) + a\sin(\omega t + 90°)$$
$$= a[\sin\omega t + \sin(\omega t + 90°)] + a\sin(\omega t + 45°)$$
$$= 2a\sin(\omega t + 45°)\cos 45° + a\sin(\omega t + 45°)$$
$$= (\sqrt{2} + 1)a\sin(\omega t + 45°) = A\sin(\omega t + 45°)$$

Therefore, resultant motion is simple harmonic of amplitude

$$A = (\sqrt{2} + 1)a$$

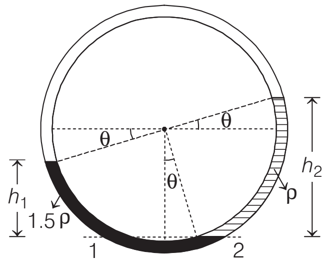
and which differ in phase by 45° relative to the first.

Energy in SHM $\propto$ (amplitude)^2 $\left[E = \dfrac{1}{2}mA^2\omega^2\right]$

$$\therefore \quad \frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2} + 1)^2 = (3 + 2\sqrt{2})$$

$$\therefore \quad E_{\text{resultant}} = (3 + 2\sqrt{2})E_{\text{single}}$$

25. (a) In equilibrium, pressure of same liquid at same level will be same.



Therefore, $p_1 = p_2$ or $p + (1.5\rho g h_1) = p + (\rho g h_2)$

(p = pressure of gas in empty part of the tube)

$$\therefore \quad 1.5h_1 = h_2$$

$$1.5[R\cos\theta - R\sin\theta] = \rho(R\cos\theta + R\sin\theta)$$

or $\quad 3\cos\theta - 3\sin\theta = 2\cos\theta + 2\sin\theta$

or $\quad 5\tan\theta = 1$

$$\theta = \tan^{-1}\left(\frac{1}{5}\right)$$

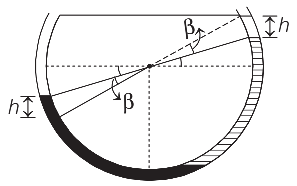
(b) When liquids are slightly disturbed by an angle β.

Net restoring pressure

$\Delta p = 1.5\rho g h + \rho g h = 2.5\rho g h$

This pressure will be equal at all sections of the liquid.

Therefore, net restoring torque on the whole liquid.



$$\tau = -(\Delta p)(A)(R)$$

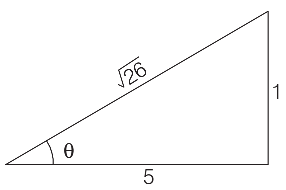
or $\tau = -2.5\rho g h AR$

$$= -2.5\rho g AR[R\sin(\theta + \beta) - R\sin\theta]$$
$$= -2.5\rho g AR^2[\sin\theta\cos\beta + \sin\beta\cos\theta - \sin\theta]$$

Assuming $\cos\beta \approx 1$ and $\sin\beta \approx \beta$ (as β is small)

$$\therefore \quad \tau = -(2.5\rho A g R^2\cos\theta)\beta$$

or $\quad I\alpha = -(2.5\rho A g R^2\cos\theta)\beta \quad \ldots\text{(i)}$



Here, $I = (m_1 + m_2)R^2$

$$= \left[\left(\frac{\pi R}{2}\cdot A\right)\rho + \left(\frac{\pi R}{2}\right)\cdot A(1.5\rho)\right]R^2$$

$$= (1.25\pi R^3\rho)A$$

and $$\cos\theta = \frac{5}{\sqrt{26}} = 0.98$$

Substituting in Eq. (i), we have $\alpha = -\frac{(6.11)\beta}{R}$

As angular acceleration is proportional to $-\beta$, motion is simple harmonic in nature.

$$T = 2\pi\sqrt{\left|\frac{\beta}{\alpha}\right|} = 2\pi\sqrt{\frac{R}{6.11}}$$

26. A torque will act on the rod, which tries to align the rod in the direction of electric field. This torque will be of restoring nature and has a magnitude $PE\sin\theta$. Therefore, we can write

$$\tau = -PE\sin\theta \quad \text{or} \quad I\alpha = -PE\sin\theta \qquad \ldots\text{(i)}$$

Here, $I = 2M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$ and $P = qL$

Further, since θ is small so, we can write, $\sin\theta \approx \theta$.

Substituting these values in Eq. (i), we have

$$\left(\frac{ML^2}{2}\right)\alpha = -(qL)(E)\theta$$

or $$\alpha = -\left(\frac{2qE}{ML}\right)\theta$$

As α is proportional to $-\theta$, motion of the rod is simple harmonic in nature, time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{ML}{2qE}}$$

The desired time will be,

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$$

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9

Properties of Matter

Topic 1 Elasticity

Objective Questions I (Only one correct option)

1. In an experiment, brass and steel wires of length 1 m each with areas of cross-section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress requires to produce a net elongation of 0.2 mm is

[Take, the Young's modulus for steel and brass are respectively $120 \times 10^9\text{ N/m}^2$ and $60 \times 10^9\text{ N/m}^2$] **(2019 Main, 10 April II)**

(a) $1.2 \times 10^6\text{ N/m}^2$ (b) $0.2 \times 10^6\text{ N/m}^2$
(c) $1.8 \times 10^6\text{ N/m}^2$ (d) $4.0 \times 10^6\text{ N/m}^2$

2. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod, if it is to support a 400 N load without exceeding its elastic limit? **(2019 Main, 10 April II)**

(a) 0.90 mm (b) 1.00 mm
(c) 1.16 mm (d) 1.36 mm

3. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to **(2019 Main, 8 April II)**

(a) 1.3 mm (b) 1.5 mm
(c) 1.9 mm (d) 1.7 mm

4. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi\text{ ms}^{-2}$, what will be the tensile stress that would be developed in the wire? **(2019 Main, 8 April I)**

(a) $6.2 \times 10^6\text{Nm}^{-2}$ (b) $5.2 \times 10^6\text{Nm}^{-2}$
(c) $3.1 \times 10^6\text{Nm}^{-2}$ (d) $4.8 \times 10^6\text{Nm}^{-2}$

5. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to **(Main 2019, 8 April I)**

(a) 10^6Nm^{-2} (b) 10^4Nm^{-2}
(c) 10^8Nm^{-2} (d) 10^3Nm^{-2}

6. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now, the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is **(2019 Main, 12 Jan II)**

(a) zero (b) 5.0 mm
(c) 4.0 mm (d) 3.0 mm

7. A rod of length L at room temperature and uniform area of cross-section A, is made of a metal having coefficient of linear expansion α /°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y for this metal is **(2019 Main, 9 Jan I)**

(a) $\dfrac{F}{2A\alpha\ \Delta T}$ (b) $\dfrac{F}{A\alpha(\Delta T - 273)}$
(c) $\dfrac{2F}{A\alpha\Delta T}$ (d) $\dfrac{F}{A\alpha\Delta T}$

8. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\dfrac{dr}{r}\right)$ is **(2018 Main)**

(a) $\dfrac{mg}{Ka}$ (b) $\dfrac{Ka}{mg}$
(c) $\dfrac{Ka}{3mg}$ (d) $\dfrac{mg}{3Ka}$

9. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y, then $\dfrac{1}{Y}$ is equal to (g = gravitational acceleration) **(2015 Main)**

(a) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

(c) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$

10. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is **(2013 Adv.)**

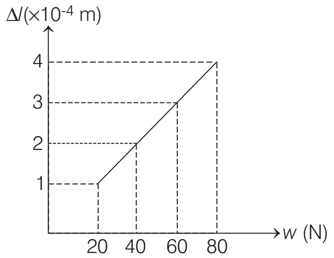
(a) 0.25 (b) 0.50 (c) 2.00 (d) 4.00

11. The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is **(2005, 2M)**

(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa

(c) 51.2×10^5 Pa (d) 1.55×10^5 Pa

12. The adjacent graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load w connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate from the graph the Young's modulus of the material of the wire. **(2003, 2M)**



(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2

(c) 3×10^{12} N/m^2 (d) 2×10^{13} N/m^2

13. A given quantity of an ideal gas is at pressure p and absolute temperature T. The isothermal bulk modulus of the gas is **(1998, 2M)**

(a) $\frac{2}{3}p$ (b) p (c) $\frac{3}{2}p$ (d) $2p$

14. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to **(1989, 2M)**

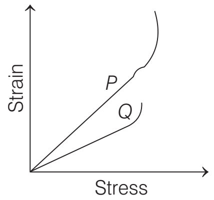
(a) 2 : 3 (b) 1 : 1 (c) 3 : 2 (d) 4 : 9

15. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ? **(1981, 2M)**

(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

Objective Question II (One or more correct option)

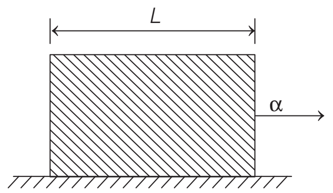
16. In plotting stress *versus* strain curves for two materials P and Q, a student by mistake puts strain on the Y-axis and stress on the X-axis as shown in the figure. Then the correct statements is/are **(2015 Adv.)**



(a) P has more tensile strength than Q
(b) P is more ductile than Q
(c) P is more brittle than Q
(d) The Young's modulus of P is more than that of Q

Fill in the Blanks

17. A uniform rod of length L and density ρ is being pulled along a smooth floor with a horizontal acceleration α (see figure). The magnitude of the stress at the transverse cross-section through the mid-point of the rod is ……… . **(1993, 1M)**



18. A solid sphere of radius R made of a material of bulk modulus k is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid the fractional change in the radius of the sphere, $\delta R/R$, is …… . **(1988, 2M)**

19. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is …… . **(1987, 2M)**

Analytical & Descriptive Questions

20. In Searle's experiment, which is used to find Young's modulus of elasticity, the diameter of experimental wire is $D = 0.05$ cm (measured by a scale of least count 0.001 cm) and length is $L = 110$ cm (measured by a scale of least count 0.1 cm). A weight of 50 N causes an extension of $l = 0.125$ cm (measured by a micrometer of least count 0.001 cm). Find maximum possible error in the values of Young's modulus. Screw gauge and meter scale are free from error. **(2004, 2M)**

21. A thin rod of negligible mass and area of cross-section 4×10^{-6} m^2, suspended vertically from one end, has a length of 0.5 m at 100°C. The rod is cooled to 0°C, but prevented from contracting by attaching a mass at the lower end. Find

(a) this mass and

(b) the energy stored in the rod, given for the rod. Young's modulus $= 10^{11}$ N/m^2, coefficient of linear expansion $= 10^{-5}$ K^{-1} and $g = 10$ m/s^2. **(1997 C, 5M)**

Topic 2 Ideal Fluids at Rest

Objective Questions I (Only one correct option)

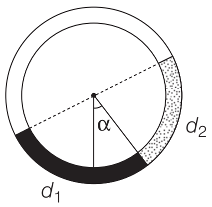
1. A submarine experience a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa, then $d_2 - d_1$ is approximately (density of water $= 10^3$ kg/ m^3 and acceleration due to gravity = 10 ms^{-2}) **(2019 Main, 10 Apr II)**

(a) 500 m (b) 400 m (c) 600 m (d) 300 m

2. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is **(2019 Main, 9 April II)**

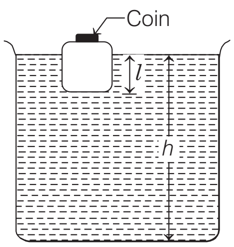
(a) 0.6 (b) 0.8 (c) 0.7 (d) 0.5

3. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio d_1 / d_2 is **(2014 Main)**



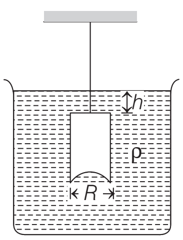
(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$ (c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

4. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance l and h are shown there. After sometime the coin falls into the water. Then **(2002,2M)**



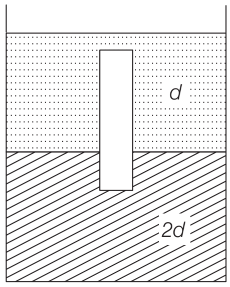
(a) l decreases and h increases
(b) l increases and h decreases
(c) Both l and h increase
(d) Both l and h decrease

5. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is **(2001, 2M)**



(a) Mg (b) $Mg - V\rho g$
(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g\,(V + \pi R^2 h)$

6. A homogeneous solid cylinder of length L and cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure p_0.



Then, density D of solid is given by **(1995, 2M)**

(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$ (c) $4d$ (d) $\frac{d}{5}$

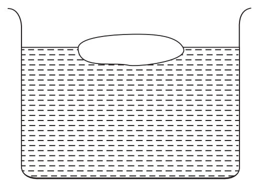
7. A vessel contains oil (density = 0.8 g/cm^3) over mercury (density = 13.6 g/cm^3). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in g/cm^3 is **(1988, 2M)**

(a) 3.3 (b) 6.4 (c) 7.2 (d) 12. 8

8. A U-tube of uniform cross-section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be **(1983, 1M)**

(a) 1.12 (b) 1.1 (c) 1.05 (d) 1.0

9. A body floats in a liquid contained in a beaker. The whole system as shown in figure falls freely under gravity. The upthrust on the body is **(1982, 3M)**



(a) zero
(b) equal to the weight of liquid displaced
(c) equal to the weight of the body in air
(d) equal to the weight of the immersed portion of the body

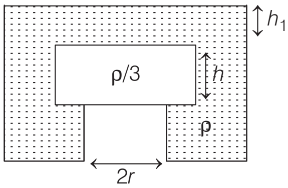
10. A metal ball immersed in alcohol weighs w_1 at 0°C and w_2 at 50°C. The coefficient of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that **(1980, 2M)**

(a) $w_1 > w_2$ (b) $w_1 = w_2$
(c) $w_1 < w_2$ (d) All of these

Passage Based Questions

Passage

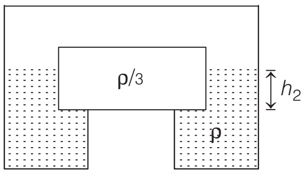
A wooden cylinder of diameter $4r$, height h and density $\rho/3$ is kept on a hole of diameter $2r$ of tank, filled with liquid of density ρ as shown in the figure.



11. Now level of the liquid starts decreasing slowly. When the level of liquid is at a height h_1 above the cylinder the block starts moving up. At what value of h_1, will the block rise ? **(2006, 5M)**

(a) $4h/9$ (b) $5h/9$
(c) $5h/3$ (d) Remains same

12. The block in the above question is maintained at the position by external means and the level of liquid is lowered. The height h_2 when this external force reduces to zero is **(2006, 5M)**



(a) $\frac{4h}{9}$ (b) $\frac{5h}{9}$
(c) Remains same (d) $2h/3$

13. If height h_2 of water level is further decreased, then **(2006, 5M)**

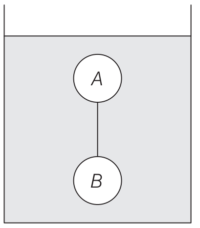
(a) cylinder will not move up and remains at its original position
(b) for $h_2 = h/3$, cylinder again starts moving up
(c) for $h_2 = h/4$, cylinder again starts moving up
(d) for $h_2 = h/5$, cylinder again starts moving up

Objective Questions II (One or more correct option)

14. A solid sphere of radius R and density ρ is attached to one end of a massless spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) **(2013 Adv.)**

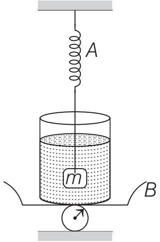
(a) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$
(b) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$
(c) the light sphere is partially submerged
(d) the light sphere is completely submerged

15. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if **(2011)**



(a) $d_A < d_F$
(b) $d_B > d_F$
(c) $d_A > d_F$
(d) $d_A + d_B = 2d_F$

16. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation **(1985, 2M)**

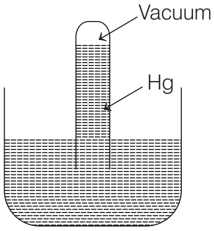


(a) the balance A will read more than 2 kg
(b) the balance B will read more than 5 kg
(c) the balance A will read less than 2 kg and B will read more than 5 kg
(d) the balances A and B will read 2 kg and 5 kg respectively

True / False

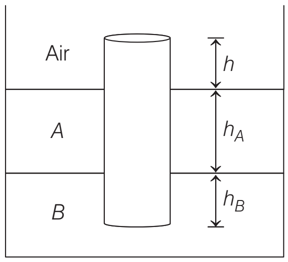
17. A block of ice with a lead shot embedded in it is floating on water contained in a vessel. The temperature of the system is maintained at 0°C as the ice melts. When the ice melts completely the level of water in the vessel rises. **(1986, 3M)**

18. A barometer made of a very narrow tube (see figure) is placed at normal temperature and pressure. The coefficient of volume expansion of mercury is 0.00018/°C and that of the tube is negligible. The temperature of mercury in the barometer is now raised by 1°C but the temperature of the atmosphere does not change. Then, the mercury height in the tube remains unchanged. **(1983, 2M)**



Analytical & Descriptive Questions

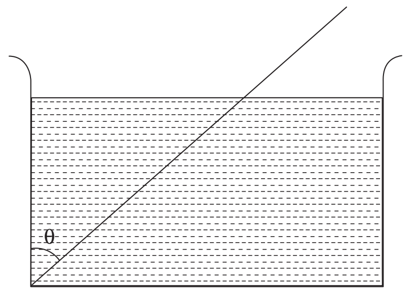
19. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm. **(2002, 5M)**



(a) Find the total force exerted by liquid A on the cylinder.
(b) Find h, the length of the part of the cylinder in air.
(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.

20. A wooden stick of length L, radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density $\sigma\ (>\rho)$. **(1999, 10M)**

21. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position (exclude the case $\theta = 0°$). **(1984, 8M)**



22. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? **(1979)**

23. A column of mercury of length 10 cm is contained in the middle of a horizontal tube of length 1 m which is closed at both the ends. The two equal lengths contain air at standard atmospheric pressure of 0.76 m of mercury. The tube is now turned to vertical position. By what distance will the column of mercury be displaced ? Assume temperature to be constant. **(1978)**

Topic 3 Ideal Fluids in Motion

Objective Questions I (Only one correct option)

1 A solid sphere of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2 when falling through the same fluid, the ratio (v_1 / v_2) equals **(2019 Main, 12 April II)**
(a) 9 (b) 1/27 (c) 1/9 (d) 27

2 Water from a tap emerges vertically downwards with an initial speed of $1.0\ \text{ms}^{-1}$. The cross-sectional area of the tap is $10^{-4}\ \text{m}^2$. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be [Take, $g = 10\ \text{ms}^{-2}$] **(2019 Main, 10 April II)**
(a) $2\times10^{-5}\ \text{m}^2$ (b) $1\times10^{-5}\ \text{m}^2$
(c) $5\times10^{-4}\ \text{m}^2$ (d) $5\times10^{-5}\ \text{m}^2$

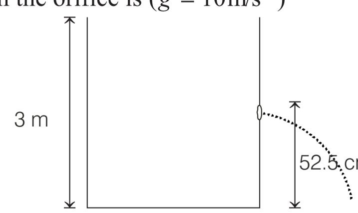
3 A long cylindrical vessel is half-filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides (in cm) will be **(2019 Main, 12 Jan II)**
(a) 0.1 (b) 1.2 (c) 0.4 (d) 2.0

4 A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected 25% losses all of its momentum and, 25% comes back with the same speed. The resultant pressure on the mesh will be **(2019 Main, 11 Jan I)**
(a) ρv^2 (b) $\frac{1}{2}\rho v^2$ (c) $\frac{1}{4}\rho v^2$ (d) $\frac{3}{4}\rho v^2$

5 Water flows into a large tank with flat bottom at the rate of $10^{-4}\ \text{m}^3\text{s}^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is **(2019 Main, 10 Jan I)**
(a) 4 cm (b) 2.9 cm (c) 5.1 cm (d) 1.7 cm

6 The top of a water tank is open to air and its water level is maintained. It is giving out $0.74\ \text{m}^3$ water per minute through a circular opening of 2 cm radius is its wall. The depth of the centre of the opening from the level of water in the tank is close to **(2019 Main, 9 Jan II)**
(a) 4.8 m (b) 6.0 m (c) 2.9 m (d) 9.6 m

7. Water is filled in a cylindrical container to a height of 3 m. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. The square of the speed of the liquid coming out from the orifice is ($g = 10\ \text{m/s}^2$) **(2005, 2M)**



(a) $50\ \text{m}^2/\text{s}^2$ (b) $50.5\ \text{m}^2/\text{s}^2$
(c) $51\ \text{m}^2/\text{s}^2$ (d) $52\ \text{m}^2/\text{s}^2$

8. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to **(2000, 2M)**
(a) $L/\sqrt{2\pi}$ (b) $2\pi L$ (c) L (d) $L/2\pi$

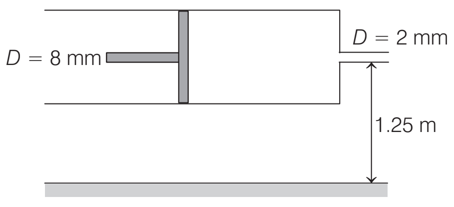
9. Water from a tap emerges vertically downwards with an initial speed of 1.0 m/s. The cross-sectional area of tap is 10^{-4} m^2. Assume that the pressure is constant throughout the steam of water and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is **(1998, 2M)**
(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-4} m^2
(c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

Fill in the Blank

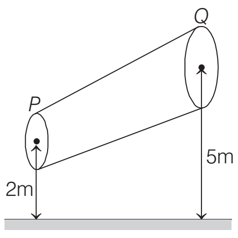
10. A horizontal pipeline carries water in a streamline flow. At a point along the pipe, where the cross-sectional area is 10 cm^2, the water velocity is 1ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2, is ......Pa. (Density of water = 10^3 kg-m^3) **(1994, 2M)**

Analytical & Descriptive Questions

11. Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. (Take $g = 10$ m/s^2). **(2004, 2M)**

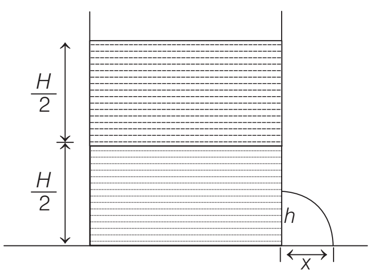


12. A non-viscous liquid of constant density 1000 kg/m^3 flows in streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross-section of the tube at two points P and Q at heights of 2 m and 5 m are respectively 4×10^{-3} m^2 and 8×10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. **(1997, 5M)**



13. A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $A/100$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$. Calculate **(1997 C, 5M)**
(a) the acceleration of the container and
(b) velocity of efflux when 75% of the liquid has drained out.

14. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in figure. The lower density liquid is open to the atmosphere having pressure p_0. **(1995, 5+5 M)**



(a) A homogeneous solid cylinder of length $L (L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid. Determine
(i) the density D of the solid,
(ii) the total pressure at the bottom of the container.
(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s ($s << A$) is punched on the vertical side of the container at a height h ($h < H/2$).
Determine :
(i) the initial speed of efflux of the liquid at the hole,
(ii) the horizontal distance x travelled by the liquid initially, and
(iii) the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m.
(Neglect the air resistance in these calculations)

Topic 4 Surface Tension and Viscosity

Objective Questions I (Only one correct option)

1 The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles with glass are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio (r_1 / r_2), is then close to **(2019 Main, 10 April I)**

(a) 3/5 (b) 2/3 (c) 2/5 (d) 4/5

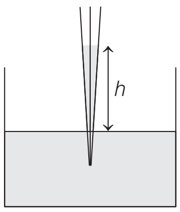
2 If M is the mass of water that rises in a capillary tube of radius r, then mass of water which will rise in a capillary tube of radius $2r$ is **(2019 Main, 9 April I)**

(a) $2M$ (b) $4M$ (c) $\frac{M}{2}$ (d) M

3. The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $d = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$m. Using $g = 9.80$ $\mathbf{m/s}^2$ and the simplified relation $T = \frac{rhg}{2} \times 10^3 \mathbf{N/m}$, the possible error in surface tension is closest to **(2017 Main)**

(a) 1.5% (b) 2.4% (c) 10% (d) 0.15%

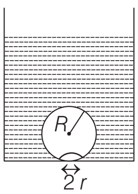
4. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross-sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross-section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity) **(2014 Adv.)**



(a) $\frac{2S}{b\rho g} \cos(\theta - \alpha)$ (b) $\frac{2S}{b\rho g} \cos(\theta + \alpha)$

(c) $\frac{2S}{b\rho g} \cos(\theta - \alpha / 2)$ (d) $\frac{2S}{b\rho g} \cos(\theta + \alpha / 2)$

5. On heating water, bubbles beings formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ_w) **(2014 Main)**

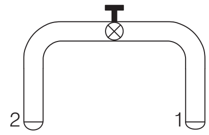


(a) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$ (c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

6. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, denstiy of liquid is ρ and L is its latent heat of vaporisation **(2013 Main)**

(a) $\frac{\rho L}{T}$ (b) $\sqrt{\frac{T}{\rho L}}$ (c) $\frac{T}{\rho L}$ (d) $\frac{2T}{\rho L}$

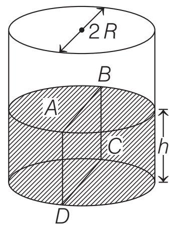
7. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherial soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure.



Just after opening the valve. **(2008, 3M)**

(a) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
(b) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(c) no change occurs
(d) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

8. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is p_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude **(2007, 3M)**



(a) $|2p_0Rh + \pi R^2 \rho gh - 2RT|$
(b) $|2p_0Rh + R\rho gh^2 - 2RT|$
(c) $|p_0\pi R^2 + R\rho gh^2 - 2RT|$
(d) $|p_0\pi R^2 + R\rho gh^2 + 2RT|$

Objective Questions II (Only one correct option)

9. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true ? **(2018 Adv.)**

(a) For a given material of the capillary tube, h decreases with increase in r
(b) For a given material of the capillary tube, h is independent of σ
(c) If this experiment is performed in a lift going up with a constant acceleration, then h decreases
(d) h is proportional to contact angle θ

Passage Based Question

Passage 1

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop.We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper. **(2010)**

10. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is

(a) $2\pi rT$ (b) $2\pi RT$ (c) $\dfrac{2\pi r^2 T}{R}$ (d) $\dfrac{2\pi R^2 T}{r}$

11. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kg m^{-3}, $g = 10\,\text{ms}^{-2}$, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m
(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m

12. After the drop detaches, its surface energy is

(a) 1.4×10^{-6} J (b) 2.7×10^{-6} J
(c) 5.4×10^{-6} J (d) 8.1×10^{-9} J

Integer Answer Type Questions

13. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \dfrac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$, then the value of α is **(2017 Adv.)**

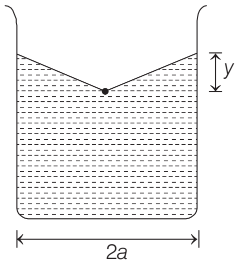
14. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is **(2016 Adv.)**

15. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 Nm^{-2}. The radii of bubbles A and B are 2 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 Nm^{-1}. Find the ratio $\dfrac{n_B}{n_A}$, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity] **(2009)**

Analytical & Descriptive Questions

16. A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity. **(2004, 2M)**

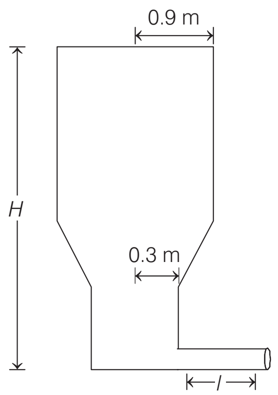
17. A container of width $2a$ is filled with a liquid. A thin wire of weight per unit length λ is gently placed over the liquid surface in the middle of the surface as shown in the figure. As a result, the liquid surface is depressed by a distance y ($y << a$). Determine the surface tension of the liquid. **(2004, 2M)**



18. A soap bubble is being blown at the end of very narrow tube of radius b. Air (density ρ) moves with a velocity v inside the tube and comes to rest inside the bubble. The surface tension of the soap solution is T. After sometime the bubble, having grown to radius r separates from the tube. Find the value of r. Assume that $r >> b$ so, that you can consider the air to be falling normally on the bubble's surface. **(2003, 4M)**

19. A liquid of density 900 kg/m^3 is filled in a cylindrical tank of upper radius 0.9 m and lower radius 0.3 m. A capillary tube of length l is attached at the bottom of the tank as shown in the figure. The capillary has outer radius 0.002 m and inner radius a. When pressure p is applied at the top of the tank volume flow rate of the liquid is 8×10^{-6} m^3/s and if capillary tube is detached, the liquid comes out from the tank with a velocity 10 m/s.

Determine the coefficient of viscosity of the liquid .
[Given, $\pi a^2 = 10^{-6}$ m^2 and $a^2/l = 2\times 10^{-6}$ m] **(2003, 4M)**



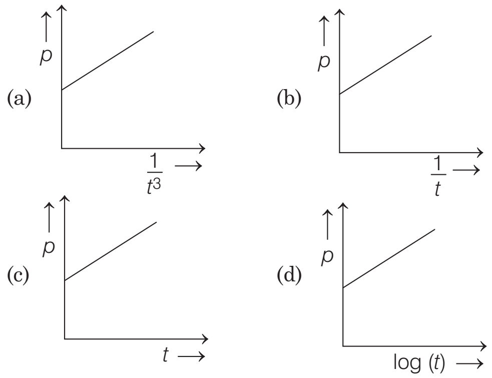
Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

1 Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (density of water = 1000kg/m^3, coefficient of viscosity of water = 1mPa s) **(2109 Main, 8 April I)**

(a) 10^3 (b) 10^4 (c) 10^2 (d) 10^6

2 A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by **(2019 Main, 12 Jan II)**



3. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of **(2017 Main)**

(a) $\frac{1}{9}$ (b) 81 (c) $\frac{1}{81}$ (d) 9

4. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists. **(2014 Adv.)**

	List I		List II
A	Lift is accelerating vertically up.	p	$d = 1.2$ m
B	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	q	$d > 1.2$ m
C	Lift is moving vertically up with constant speed.	r	$d < 1.2$ m
D	Lift is falling freely.	s	No water leaks out of the jar

Codes

(a) A-q, B-r, C-q, D-s (b) A-q, B-r, C-p, D-s
(c) A-p, B-p, C-p, D-s (d) A-q, B-r, C-p, D-p

5. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel, Young's modulus is $2 \times 10^{11} \text{Nm}^{-2}$ and coefficient of thermal expansion is $1.1 \times 10^{-5} \text{ K}^{-1}$) **(2014 Main)**

(a) 2.2×10^8 Pa (b) 2.2×10^9 Pa
(c) 2.2×10^7 Pa (d) 2.2×10^6 Pa

6. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) **(2014 Main)**

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

7. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is **(2013 Main)**

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{M}\right)$
(c) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1 + \frac{LA\sigma}{M}\right)$

8. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is **(2012)**

(a) more than half-filled if ρ_c is less than 0.5
(b) more than half-filled if ρ_c is more than 1.0
(c) half-filled if ρ_c is more than 0.5
(d) less than half-filled if ρ_c is less than 0.5

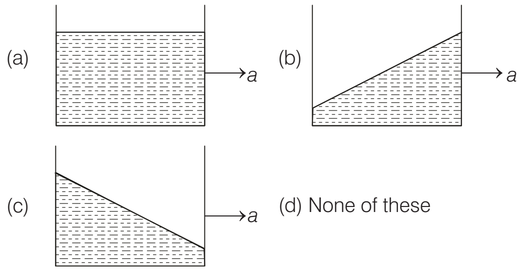
9. When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as **(2001, 2M)**

(a) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (b) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (c) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

10. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then, the pressure in the compartment is **(1999, 2M)**

(a) same everywhere (b) lower in front side
(c) lower in rear side (d) lower in upper side

11. A vessel containing water is given a constant acceleration a towards the right along a straight horizontal path. Which of the following diagrams represents the surface of the liquid ? **(1981, 2M)**



(d) None of these

Objective Questions II (One or more correct options)

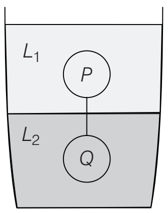
12. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true? **(2018 Adv.)**

(a) The resistive force of liquid on the plate is inversely proportional to h
(b) The resistive force of liquid on the plate is independent of the area of the plate
(c) The tangential (shear) stress on the floor of the tank increases with u_0
(d) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

13. A flat plane is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true? **(2017 Adv.)**

(a) At a later time the external force F balances the resistive force
(b) The plate will continue to move with constant non-zero acceleration, at all times
(c) The resistive force experienced by the plate is proportial to v
(d) The pressure difference between the leading and trailing faces of the plate is proportional to uv

14. Two spheres P and Q for equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\mathbf{v}_P$ and Q alone in L_1 has terminal velocity $\mathbf{v}_Q$, then **(2015 Adv.)**



(a) $\dfrac{|\mathbf{v}_P|}{|\mathbf{v}_Q|} = \dfrac{\eta_1}{\eta_2}$ (b) $\dfrac{|\mathbf{v}_P|}{|\mathbf{v}_Q|} = \dfrac{\eta_2}{\eta_1}$

(c) $\mathbf{v}_P \cdot \mathbf{v}_Q > 0$ (d) $\mathbf{v}_P \cdot \mathbf{v}_Q < 0$

Assertion and Reason

Mark your answer as

(a) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*
(b) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*
(c) *If Statement I is true; Statement II is false*
(d) *If Statement I is false; Statement II is true*

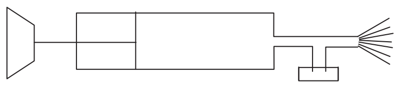
15. Statement I The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement II In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **(2008, 3M)**

Passage Based Questions

Passage 1

A spray gun is shown in the figure where a piston pushes air out of nozzle. A thin tube of uniform cross-section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out.



For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere. **(2014 Adv.)**

16. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

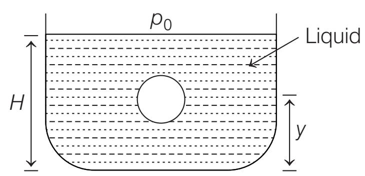
(a) 0.1 ms^{-1} (b) 1 ms^{-1} (c) 2 ms^{-1} (d) 8 ms^{-1}

17. If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(a) $\sqrt{\dfrac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$ (c) $\sqrt{\dfrac{\rho_l}{\rho_a}}$ (d) ρ_l

Passage 2

A small spherical monoatomic ideal gas bubble ($\gamma = 5/3$) is trapped inside a liquid of density ρ_l (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is p_0 (Neglect surface tension) **(2008, 4M)**



18. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it. **(2008, 4M)**

(a) Only the force of gravity

(b) The force due to gravity and the force due to the pressure of the liquid

(c) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(d) The force due to gravity and the force due to viscosity of the liquid

19. When the gas bubble is at a height y from the bottom, its temperature is **(2008, 4M)**

(a) $T_0\left(\frac{p_0+\rho_l gH}{p_0+\rho_l gy}\right)^{2/5}$ (b) $T_0\left(\frac{p_0+\rho_l g(H-y)}{p_0+\rho_l gH}\right)^{2/5}$

(c) $T_0\left(\frac{p_0+\rho_l gH}{p_0+\rho_l gy}\right)^{3/5}$ (d) $T_0\left(\frac{p_0+\rho_l g(H-y)}{p_0+\rho_l gH}\right)^{3/5}$

20. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant) **(2008, 4M)**

(a) $\rho_l nRgT_0\frac{(p_0+\rho_l gH)^{2/5}}{(p_0+\rho_l gy)^{2/5}}$

(b) $\frac{\rho_l nRgT_0}{(p_0+\rho_l gH)^{2/5}[p_0+\rho_l g(H-y)]^{3/5}}$

(c) $\rho_l nRgT_0\frac{(p_0+\rho_l gH)^{3/5}}{(p_0+\rho_l gy)^{8/5}}$

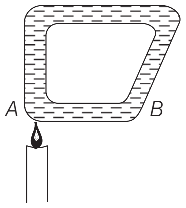
(d) $\frac{\rho_l nRgT_0}{(p_0+\rho_l gH)^{3/5}[p_0+\rho_l g(H-y)]^{2/5}}$

Fill in the Blank

21. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2 respectively. If the temperatures of both mercury and the metal are increased by an amount ΔT, the fraction of the volume of the metal submerged in mercury changes by the factor …… . **(1991, 2M)**

True / False

22. Water in a closed tube (see figure) is heated with one arm vertically placed above a lamp. Water will begin to circulate along the tube in counter-clockwise direction. **(1983, 2M)**



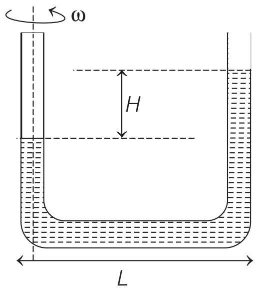
Integer Answer Type Question

23. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it upto height H. Now, the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. **(2009)**

[Take atmospheric pressure $= 1.0\times10^5\ \text{Nm}^{-2}$, density of water $= 1000\ \text{kg m}^{-3}$ and $g = 10\ \text{ms}^{-2}$. Neglect any effect of surface tension.]

Analytical & Descriptive Questions

24. A U-shaped tube contains a liquid of density ρ and it is rotated about the line as shown in the figure. Find the difference in the levels of liquid column. **(2005, 2M)**



25. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L. **(1992, 8 M)**

(a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.

(b) Is the motion of the ball simple harmonic ?

(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

26. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ . The height of the liquid in one vessel is h_1 and in the other is h_2. The area of either base is A. What is the work done by gravity in equalising the levels when the two vessels are connected ? **(1981, 4 M)**

Answers

Topic 1

1. (*) **2.** (c) **3.** (d) **4.** (c)
5. (a) **6.** (d) **7.** (a) **8.** (d)
9. (d) **10.** (c) **11.** (d) **12.** (a)
13. (b) **14.** (c) **15.** (a) **16.** (a, b)
17. $\frac{\rho L \alpha}{2}$ **18.** $\frac{Mg}{3AK}$ **19.** $\frac{1}{2}\left(\frac{YA}{L}\right)x^2$
20. 1.09×10^{10} N/m^2 **21.** (a) 40 kg (b) 0.1 J

Topic 2

1. (d) **2.** (a)
3. (c) **4.** (d) **5.** Most appropriate option is (d)
6. (a) **7.** (c) **8.** (b) **9.** (a)
10. (c) **11.** (c) **12.** (a) **13.** (a)
14. (a, d) **15.** (a, b, d) **16.** (b, c) **17.** F
18. F **19.** (a) Zero (b) 0.25 cm (c) $g/6$
20. $\pi R^2 L(\sqrt{\rho\sigma} - \rho)$ **21.** 45°
22. Level will come down. **23.** 2.95 cm

Topic 3

1. (a) **2.** (d) **3.** (d) **4.** (d)
5. (c) **6.** (a)
7. (a) **8.** (a) **9.** (c) **10.** 500
11. 2m **12.** 29025 J/m^3, – 29400 J/m^3
13. (a) $\frac{g}{50}$ (b) $\sqrt{\frac{gm_0}{2A\rho}}$ **14.** (a) (i) $\frac{5d}{4}$ (ii) $p = p_0 + \frac{dg(6H + L)}{4}$
(b) (i) $\sqrt{(3H - 4h)\frac{g}{2}}$ (ii) $\sqrt{h(3H - 4h)}$ (iii) At $h = \frac{3H}{8}; \frac{3}{4}H$

Topic 4

1. (c) **2.** (a)
3. (a)
4. (d) **5.** (*) **6.** (d) **7.** (b)
8. (b) **9.** (a, c)
10. (c) **11.** (a) **12.** (b) **13.** 6
14. 3 **15.** 6 **16.** $\frac{dQ}{dt} \propto r^5$
17. $\frac{\lambda a}{2y}$ **18.** $\frac{4T}{\rho v^2}$ **19.** $\frac{1}{720}$ N-s/m^2

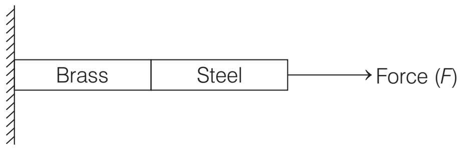
Topic 5

1. (b) **2.** (b)
3. (d) **4.** (c) **5.** (a) **6.** (a)
7. (c) **8.** (a) **9.** (a) **10.** (b)
11. (c) **12.** (a,c,d) **13.** (a, c, d) **14.** (a, d)
15. (a) **16.** (c) **17.** (a) **18.** (d)
19. (b) **20.** (b) **21.** $\frac{1 + \gamma_2 \Delta T}{1 + \gamma_1 \Delta T}$ **22.** F
23. 6 **24.** $H = \frac{\omega^2 L^2}{2g}$
25. (a) $\frac{t_1 d_L}{d_L - d}$ (b) no (c) remains same
26. $\frac{\rho A g}{4}(h_1 - h_2)^2$

Hints & Solutions

Topic 1 Elasticity

1. In given experiment, a composite wire is stretched by a force F.



Net elongation in the wire = elongation in brass wire + elongation in steel wire …(i)

Now, Young's modulus of a wire of cross-section (A) when some force (F) is applied, $Y = \frac{Fl}{A\Delta l}$

We have,

$$\Delta l = \text{elongation} = \frac{Fl}{AY}$$

So, from relation (i), we have

$$\Delta l_{\text{net}} = \Delta l_{\text{brass}} + \Delta l_{\text{steel}}$$

$$\Rightarrow \quad \Delta l_{\text{net}} = \left(\frac{Fl}{AY}\right)_{\text{brass}} + \left(\frac{Fl}{AY}\right)_{\text{steel}}$$

As wires are connected in series and they are of same area of cross-section, length and subjected to same force, so

$$\Delta l_{\text{net}} = \frac{F}{A}\left(\frac{l}{Y_{\text{brass}}} + \frac{l}{Y_{\text{steel}}}\right)$$

Here,

$$\Delta l_{\text{net}} = 0.2\,\text{mm} = 0.2 \times 10^{-3}\ \text{m}$$

and $$l = 1\,\text{m}$$

$$Y_{\text{brass}} = 60 \times 10^9\,\text{Nm}^{-2},\ Y_{\text{steel}} = 120 \times 10^9\,\text{Nm}^{-2}$$

On putting the values, we have

$$0.2 \times 10^{-3} = \frac{F}{A}\left(\frac{1}{60 \times 10^9} + \frac{1}{120 \times 10^9}\right)$$

$\Rightarrow \quad \text{Stress} = \dfrac{F}{A} = 8 \times 10^6 \text{Nm}^{-2}$

No options matches.

2. Let $d_{\min}$ = minimum diameter of brass.

Then, stress in brass rod is given by

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_{\min}^2} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

For stress not to exceed elastic limit, we have

$\sigma \leq 379\,\text{MPa}$

$$\Rightarrow \quad \frac{4F}{\pi d^2_{\min}} \leq 379 \times 10^6$$

Here, $F = 400\,\text{N}$

$$\therefore \quad d_{\min}^2 = \frac{1600}{\pi \times 379 \times 10^6}$$

$$\Rightarrow \quad d_{\min} = 1.16 \times 10^{-3}\text{ m} = 1.16\,\text{mm}$$

3. When a wire is stretched, then change in length of wire is $\Delta l = \dfrac{Fl}{\pi r^2 Y}$, where Y is its Young's modulus.

Here, for wires A and B,

$$l_A = 2\,\text{m}, l_B = 1.5\,\text{m},$$

$$\frac{Y_A}{Y_B} = \frac{7}{4}, r_B = 2\,\text{mm} = 2 \times 10^{-3}\text{ m and } \frac{F_A}{F_B} = 1$$

As, it is given that $\Delta l_A = \Delta l_B$

$$\Rightarrow \frac{F_A l_A}{\pi r_A^2 Y_A} = \frac{F_B l_B}{\pi r_B^2 Y_B}$$

$$\Rightarrow \quad r_A^2 = \frac{F_A}{F_B} \cdot \frac{l_A}{l_B} \cdot \frac{Y_B}{Y_A} \cdot r_B^2$$

$$= 1 \times \frac{2}{1.5} \times \frac{4}{7} \times 4 \times 10^{-6}\text{ m} = 3.04 \times 10^{-6}\text{ m}$$

or $\quad r_A = 1.7 \times 10^{-3}\text{ m}$

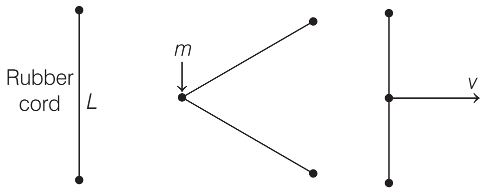
or $\quad r_A = 1.7\,\text{mm}$

4. Given, radius of wire, $r = 2\,\text{mm} = 2 \times 10^{-3}$ m Weight of load, $m = 4$ kg, $g = 3.1\,\pi\,\text{ms}^{-2}$

$$\therefore \quad \text{Tensile stress} = \frac{\text{Force}\,(F)}{\text{Area}\,(A)} = \frac{mg}{\pi r^2}$$

$$= \frac{4 \times 3.1 \times \pi}{\pi \times (2 \times 10^{-3})^2} = 3.1 \times 10^6 \text{Nm}^{-2}$$

5. When rubber cord is stretched, then it stores potential energy and when released, this potential energy is given to the stone as kinetic energy.



So, potential energy of stretched cord

= kinetic energy of stone

$$\Rightarrow \quad \frac{1}{2} Y \left(\frac{\Delta L}{L}\right)^2 A \cdot L = \frac{1}{2} mv^2$$

Here, $\Delta L = 20\,\text{cm} = 0.2\,\text{m}$, $L = 42\,\text{cm} = 0.42\,\text{m}$, $v = 20\,\text{ms}^{-1}$, $m = 0.02\,\text{kg}$, $d = 6\,\text{mm} = 6 \times 10^{-3}\,\text{m}$

$$\therefore \quad A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{6 \times 10^{-3}}{2}\right)^2$$

$$= \pi(3 \times 10^{-3})^2 = 9\pi \times 10^{-6}\text{ m}^2$$

On substituting values, we get

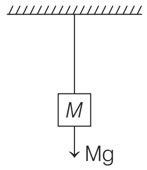
$$Y = \frac{mv^2 L}{A(\Delta L)^2} = \frac{0.02 \times (20)^2 \times 0.42}{9\pi \times 10^{-6} \times (0.2)^2} \approx 3.0 \times 10^6 \text{Nm}^{-2}$$

So, the closest value of Young's modulus is 10^6 Nm^{-2}.

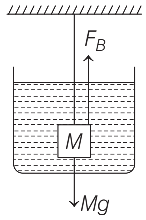
6. When load M is attached to wire, extension in length of wire is

$$\Delta l_1 = \frac{Mgl}{A.Y} \qquad \ldots\text{(i)}$$

where, Y is the Young's modulus of the wire.



when load is immersed in liquid of relative density 2, increase in length of wire as shown in the figure is



$$\Delta l_2 = \frac{(Mg - F_B)l}{A.Y}$$

where, F_B = Buoyant force

$$\therefore \quad \Delta l_2 = \frac{\left(Mg - Mg \cdot \dfrac{\rho_l}{\rho_b}\right)l}{A.Y} \qquad \left[\because F_B = V\rho_l g = \frac{Mg}{\rho_b}\rho_l g\right]$$

Here given that, $\dfrac{\rho_l}{\rho_b} = \dfrac{2}{8} = \dfrac{1}{4}$

$$\text{So,} \quad \Delta l_2 = \frac{\left(\dfrac{3}{4} Mg\right)l}{A.Y} \qquad \ldots\text{(ii)}$$

Dividing Eqs. (ii) by (i), we get

$$\frac{\Delta l_2}{\Delta l_1} = \frac{3}{4}$$

$$\Rightarrow \quad \Delta l_2 = \frac{3}{4} \times \Delta l_1$$

$$= \frac{3}{4} \times 4\text{ mm} = 3\text{ mm}$$

7. If a rod of length L and coefficient of linear expansion $\alpha/°C$, then with the rise in temperature by ΔT K, its change in length is given as,

$$\Delta L = L\alpha\,\Delta T \Rightarrow \frac{\Delta L}{L} = \alpha\,\Delta T \qquad \text{...(i)}$$

Also, when a rod is subjected to some compressive force (F), then its' Young's modulus is given as

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\frac{\Delta L}{L} = \frac{F}{YA} \qquad \text{...(ii)}$$

Since, it is given that the length of the rod does not change. So, from Eqs. (i) and (ii), we get

$$\alpha\,\Delta T = \frac{F}{YA} \Rightarrow Y = \frac{F}{A\alpha\,\Delta T}$$

8. $$\Delta P = \frac{F}{A} = \frac{mg}{a}$$

Bulk modulus, $K = \dfrac{-\Delta P}{\Delta V/V}$

Here, $\quad V = \dfrac{4}{3}\pi r^3$

and ΔV or $\quad dV = (4\pi r^3)dr$

$$\Rightarrow \quad K = -\frac{\frac{mg}{A}}{\frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3}} \quad \text{or} \quad \frac{dr}{r} = -\frac{mg}{3Ka}$$

9. $$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{...(i)}$$

$$T_M = 2\pi\sqrt{\frac{L+\Delta L}{g}}$$

Here, $\quad \Delta L = \dfrac{FL}{AY} = \dfrac{MgL}{AY}$

$$\therefore \quad T_M = 2\pi\sqrt{\frac{L + \frac{MgL}{AY}}{g}} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

10. $$\Delta l = \frac{FL}{AY} = \frac{FL}{(\pi r^2)Y} \Rightarrow \Delta l \propto \frac{L}{r^2}$$

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{L/R^2}{2L/(2R)^2} = 2$$

11. From the definition of bulk modulus, $B = \dfrac{-dp}{(dV/V)}$

Substituting the values, we have

$$B = \frac{(1.165 - 1.01)\times 10^5}{(10/100)} = 1.55\times 10^5\ \text{Pa}$$

12. $\Delta l = \left(\dfrac{l}{YA}\right).\,w$

i.e. graph is a straight line passing through origin (as shown in question also), the slope of which is $\dfrac{l}{YA}$.

$$\therefore \quad \text{Slope} = \left(\frac{l}{YA}\right)$$

$$\therefore \quad Y = \left(\frac{l}{A}\right)\left(\frac{1}{\text{slope}}\right) = \left(\frac{1.0}{10^{-6}}\right)\frac{(80-20)}{(4-1)\times 10^{-4}}$$

$$= 2.0\times 10^{11}\ \text{N/m}^2$$

13. In isothermal process

$$pV = \text{constant}$$

$$\therefore \quad pdV + Vdp = 0 \quad \text{or} \quad \left(\frac{dp}{dV}\right) = -\left(\frac{p}{V}\right)$$

$$\therefore \text{ Bulk modulus, } B = -\left(\frac{dp}{dV/V}\right) = -\left(\frac{dp}{dV}\right)V$$

$$\therefore \quad B = -\left[\left(-\frac{p}{V}\right)V\right] = p$$

$$\therefore \quad B = p$$

NOTE Adiabatic bulk modulus is given by $B = \gamma P$.

14. Thermal stress $\sigma = Y\alpha\,\Delta\theta$

Given, $\quad \sigma_1 = \sigma_2$

$$\therefore \quad Y_1\alpha_1\Delta\theta = Y_2\alpha_2\Delta\theta \quad \text{or} \quad \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

---

15. $$\Delta l = \frac{Fl}{AY} = \frac{Fl}{\left(\frac{\pi d^2}{4}\right)Y} \quad \text{or} \quad (\Delta l) \propto \frac{l}{d^2}$$

Now, $\dfrac{l}{d^2}$ is maximum in option (a).

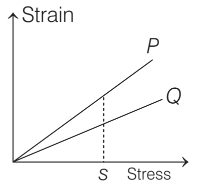
16. $\quad Y = \dfrac{\text{stress}}{\text{strain}}$

or $Y \propto \dfrac{1}{\text{strain}}$

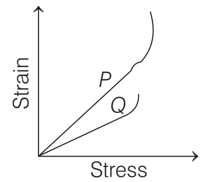
(for same stress say σ)

$(\text{strain})_Q < (\text{strain})_P$

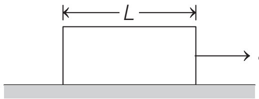
$\Rightarrow Y_Q > Y_P$



So, P is more ductile than Q. Further, from the given figure we can also see that breaking stress of P is more than Q. So, it has more tensile strength.

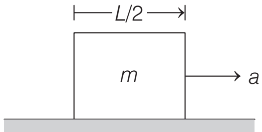


17. Let A be the area of cross-section of the rod. FBD of rod at mid-point



Mass m = volume × density

$$= \left(\frac{L}{2}.A\right)\rho$$



$$\therefore \quad T = m\alpha = \left(\frac{L}{2}A\rho\alpha\right)$$

$$\therefore \text{ Stress} = \frac{T}{A} = \frac{1}{2}\rho\alpha\, L$$

18. $\Delta p = \dfrac{Mg}{A} \quad \Rightarrow \quad \left|\dfrac{\Delta V}{V}\right| = \dfrac{\Delta p}{k} = \dfrac{Mg}{Ak}$

Now as, $\quad V = \dfrac{4}{3}\pi R^3$ or $V \propto R^3$

$$\therefore \quad \frac{\Delta V}{V} = 3\left(\frac{\Delta R}{R}\right)$$

$$\text{or} \quad \frac{\Delta R}{R} = \frac{1}{3}\left(\frac{\Delta V}{V}\right) \quad \ldots\text{(i)}$$

$\therefore$ From Eq. (i)

$$\frac{\Delta R}{R} = \frac{Mg}{3Ak}$$

19. $$W = \frac{1}{2}Kx^2$$

Here, $\quad K = \dfrac{YA}{L}$

$$\therefore \quad W = \frac{1}{2}\left(\frac{YA}{L}\right)x^2$$

20. Young's modulus of elasticity is given by

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{l/L} = \frac{FL}{lA} = \frac{FL}{l\left(\dfrac{\pi d^2}{4}\right)}$$

Substituting the values, we get

$$Y = \frac{50 \times 1.1 \times 4}{(1.25 \times 10^{-3}) \times \pi \times (5.0 \times 10^{-4})^2}$$

$$= 2.24 \times 10^{11} \text{ N/m}^2$$

$$\text{Now,} \quad \frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$$

$$= \left(\frac{0.1}{110}\right) + \left(\frac{0.001}{0.125}\right) + 2\left(\frac{0.001}{0.05}\right) = 0.0489$$

$$\Delta Y = (0.0489)Y = (0.0489) \times (2.24 \times 10^{11}) \text{ N/m}^2$$

$$= 1.09 \times 10^{10} \text{ N/m}^2$$

21. (a) The change in length due to decrease in temperature,

$$\Delta l_1 = L\alpha\, \Delta\theta = (0.5)(10^{-5})(0 - 100)$$

$$\Delta l_1 = -0.5 \times 10^{-3} \text{ m} \quad \ldots\text{(i)}$$

Negative sign implies that length is decreasing. Now, let M be the mass attached to the lower end. Then, change in length due to suspension of load is

$$\Delta l_2 = \frac{(Mg)L}{AY} = \frac{(M)(10)(0.5)}{(4 \times 10^{-6})(10^{11})}$$

$$\Delta l_2 = (1.25 \times 10^{-5})M \quad \ldots\text{(ii)}$$

$$\Delta l_1 + \Delta l_2 = 0$$

or $(1.25 \times 10^{-5})M = (0.5 \times 10^{-3})$

$$\therefore \quad M = \left(\frac{0.5 \times 10^{-3}}{1.25 \times 10^{-5}}\right)\text{kg}$$

or $\quad M = 40\,\text{kg}$

(b) Energy stored,

At 0°C the natural length of the wire is less than its actual length; but since a mass is attached at its lower end, an elastic potential energy is stored in it. This is given by

$$U = \frac{1}{2}\left(\frac{AY}{L}\right)(\Delta l)^2 \quad \ldots\text{(iii)}$$

Here, $\Delta l = |\Delta l_1| = \Delta l_2 = 0.5 \times 10^{-3}$ m

Substituting the values,

$$U = \frac{1}{2}\left(\frac{4 \times 10^{-6} \times 10^{11}}{0.5}\right)(0.5 \times 10^{-3})^2 = 0.1 \text{ J}$$

NOTE Comparing the equation

$$Y = \frac{F/A}{\Delta l/L} \quad \text{or} \quad F = \left(\frac{AY}{L}\right)\Delta l$$

with the spring equation $F = K \cdot \Delta x$, we find that equivalent spring constant of a wire is $k = \left(\dfrac{AY}{L}\right)$

Therefore, potential energy stored in it should be

$$U = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}\left(\frac{AY}{L}\right)(\Delta l)^2$$

Topic 2 Ideal Fluids at Rest

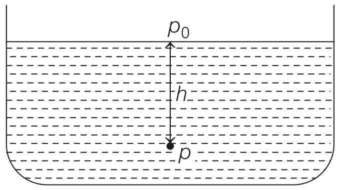
1. Pressure inside a fluid volume open to atmosphere is

$$p = p_0 + h\rho g$$

where, p = pressure at depth h, h = depth,

ρ = density of fluid

and $\quad g$ = acceleration due to gravity.



In problem given,

when $h = d_1$, pressure $p_1 = 5.05 \times 10^6$ Pa and when $h = d_2$, pressure $p_2 = 8.08 \times 10^6$ Pa

So, we have

$$p_1 = p_0 + d_1\rho g = 5.05 \times 10^6$$

and $$p_2 = p_0 + d_2\rho g = 8.08 \times 10^6$$

$$\Rightarrow \quad p_2 - p_1 = (d_2 - d_1)\rho g = 3.03 \times 10^6$$

$$\Rightarrow \quad d_2 - d_1 = \frac{3.03 \times 10^6}{\rho g}$$

Given, $\rho = 10^3 \text{ kgm}^{-3}$

and $g = 10 \text{ ms}^{-2}$

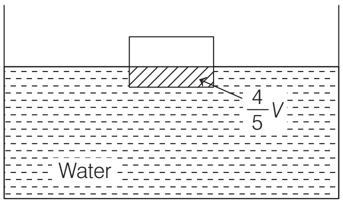
$$\therefore \quad d_2 - d_1 = \frac{3.03 \times 10^6}{10^3 \times 10}$$

$$= 303 \text{ m} \approx 300 \text{ m}$$

2. *(a)* For a floating body,

upthrust = weight of the part of object, i.e. submerged in the fluid.

In first situation,



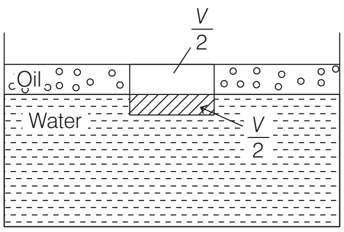
So, weight of block of volume V = weight of water of volume $\frac{4}{5}V \Rightarrow V\rho_b g = \frac{4}{5}V\rho_w g$

where, ρ_b = density of block

and ρ_w = density of water.

$$\Rightarrow \quad \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad \ldots \text{(i)}$$

In second situation,



So, weight of block of volume V = weight of oil of volume $\frac{V}{2}$ + weight of water of volume $\frac{V}{2}$.

$$\Rightarrow \quad V\rho_b g = \frac{V}{2}\rho_o g + \frac{V}{2}\rho_w g$$

where, ρ_o = density of oil.

$$\Rightarrow \quad 2\rho_b = \rho_o + \rho_w$$

$$\Rightarrow \quad \frac{2\rho_b}{\rho_w} = \frac{\rho_o}{\rho_w} + 1$$

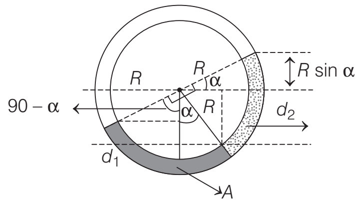
$$\Rightarrow \quad 2 \times \frac{4}{5} = \frac{\rho_o}{\rho_w} + 1 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \rho_0 / \rho_w = 8/5 - 1$$

$$= 3/5 = 0.6$$

3. Equating pressure at A, we get

$$R\sin\alpha\ d_2 + R\cos\alpha\ d_2 + R(1 - \cos\alpha)\ d_1$$
$$= R(1 - \sin\alpha)\ d_1$$



$$(\sin\alpha + \cos\alpha)\ d_2 = d_1(\cos\alpha - \sin\alpha)$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

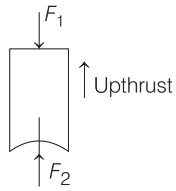
4. l will decrease, because the block moves up and h will decrease, because the coin will displace the volume of water (V_1) equal to its own volume when it is in the water whereas when it is on the block it will displace the volume of water (V_2) whose weight is equal to weight of coin and since density of coin is greater than the density of water, $V_1 < V_2$.

5. $F_2 - F_1$ = upthrust

$\therefore \quad F_2 = F_1$ + upthrust

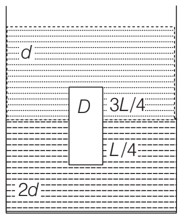
$$F_2 = (p_0 + \rho gh)\pi R^2 + V\rho g$$

$$= p_0\pi R^2 + \rho g\ (\pi R^2 h + V)$$



$\therefore$ Most appropriate option is (d).

6.



Considering vertical equilibrium of cylinder

Weight of cylinder = Upthrust due to upper liquid + upthrust due to lower liquid

$$\therefore \quad (A/5)(L)Dg = (A/5)(3L/4)(d)g + (A/5)(L/4)(2d)(g)$$

$$\therefore \quad D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$D = \frac{5}{4}d$$

7. Let density of material of sphere (in g/cm^3) be ρ.

Applying the condition of floatation,

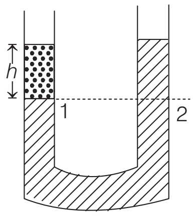
Weight = Upthrust

or $$V\rho g = \frac{V}{2}\rho_{\text{oil}}g + \frac{V}{2}\rho_{\text{Hg}}g$$

or $$\rho = \frac{\rho_{\text{oil}}}{2} + \frac{\rho_{\text{Hg}}}{2}$$

$$= \frac{0.8}{2} + \frac{13.6}{2} = 7.2 \text{ g/cm}^3$$

8.



$$p_1 = p_2$$
$$p_0 + \rho_I gh = p_0 + \rho_{II} gh$$
$$\therefore \quad \rho_I = \rho_{II} \Rightarrow \rho_{II} = 1.1$$

9. In a freely falling system $g_{eff} = 0$ and since,

$$\text{Upthrust} = V_i \rho_L g_{eff}$$

(V_i = immersed volume, ρ_L = density of liquid)

$\therefore$ Upthrust = 0

10. $$w_{app} = w_{actual} - \text{Upthrust}$$

Upthrust $F = V_S \rho_L g$

Here, V_S = volume of solid, ρ_L = density of liquid

At higher temperature $F' = V_S' \rho_L' g$

$$\therefore \quad \frac{F'}{F} = \frac{V_S'}{V_S} \cdot \frac{\rho_L'}{\rho_L} = \frac{(1+\gamma_S \Delta\theta)}{(1+\gamma_L \Delta\theta)}$$

Since, $\gamma_S < \gamma_L$ (given)

$$\therefore \quad F' < F \text{ or } w'_{app} > w_{app}$$

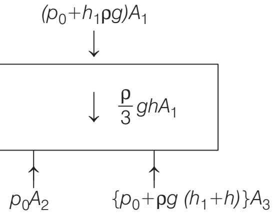
11. Let A_1 = Area of cross-section of cylinder = $4\pi r^2$

A_2 = Area of base of cylinder in air = πr^2

and A_3 = Area of base of cylinder in water

$= A_1 - A_2 = 3\pi r^2$

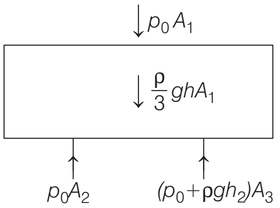
Drawing free body diagram of cylinder



Equating the net downward forces and net upward forces, we get, $h_1 = \dfrac{5}{3}h$.

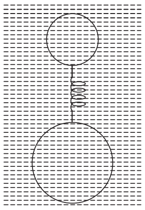
12. Again equating the forces, we get

$$h_2 = \frac{4h}{9}$$



13. For $h_2 < 4h/9$, buoyant force will further decrease. Hence, the cylinder remains at its original position.

14.



On small sphere

$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g \qquad \ldots\text{(i)}$$

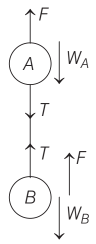
On second sphere (large)

$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx \qquad \ldots\text{(ii)}$$

By Eqs. (i) and (ii), we get

$$x = \frac{4\pi R^3 \rho g}{3k}$$

15. F = Upthrust $= Vd_F g$



Equilibrium of A,

$$Vd_F g = T + W_A = T + Vd_A g \qquad \ldots\text{(i)}$$

Equilibrium of B,

$$T + Vd_F g = Vd_B g \qquad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2d_f = d_A + d_B$$

$\therefore$ Option (d) is correct

From Eq. (i) we can see that $d_F > d_A$ (as $T > 0$)

$\therefore$ Option (a) is correct.

From Eq. (ii) we can see that, $d_B > d_F$

$\therefore$ Option (b) is correct.

$\therefore$ Correct options are (a), (b) and (d).

16. Liquid will apply an upthrust on m. An equal force will be exerted (from Newton's third law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg. Therefore, the correct options are (b) and (c).

17. When ice melts level of water does not change. In case of lead, it was initially floating i.e. it would had displaced the water equal to the weight of lead. So, volume of water displaced would be,

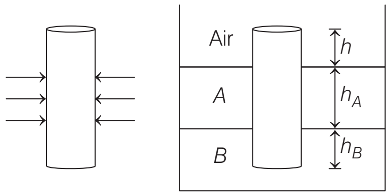
$$V_1 = \frac{m}{\rho_w} \ (m = \text{ mass of lead})$$

Now, when ice melts lead will sink and it would displace the water equal to the volume of lead itself. So, volume of water displaced in this case would be,

$V_2 = \dfrac{m}{\rho_l}$. Now, as $\rho_l > \rho_w, V_2 < V_1$ or level will fall.

18. On increasing the temperature of mercury its density will decrease. Hence, level of mercury in barometer tube will increase.

19. (a) Liquid A is applying the hydrostatic force on cylinder from all the sides. So, net force is zero.



(b) In equilibrium

Weight of cylinder = Net upthrust on the cylinder

Let s be the area of cross-section of the cylinder, then

weight $= (s)(h + h_A + h_B)\rho_{\text{cylinder}} g$

and upthrust on the cylinder

= upthrust due to liquid A + upthrust due to liquid B

$= sh_A\rho_A g + sh_B\rho_B g$

Equating these two,

$$s(h + h_A + h_B)\rho_{\text{cylinder}} g = sg(h_A\rho_A + h_B\rho_B)$$

or $\quad (h + h_A + h_B)\rho_{\text{cylinder}} = h_A\rho_A + h_B\rho_B$

Substituting,

$h_A = 1.2\,\text{cm}, h_B = 0.8\,\text{cm}$ and $\rho_A = 0.7\,\text{g/cm}^3$

$\rho_B = 1.2\ \text{g/cm}^3$ and $\rho_{\text{cylinder}} = 0.8\,\text{g/cm}^3$

In the above equation, we get $h = 0.25\text{cm}$

(c) Net upward force = extra upthrust $= sh\rho_B g$

$\therefore$ Net acceleration $a = \dfrac{\text{Force}}{\text{Mass of cylinder}}$

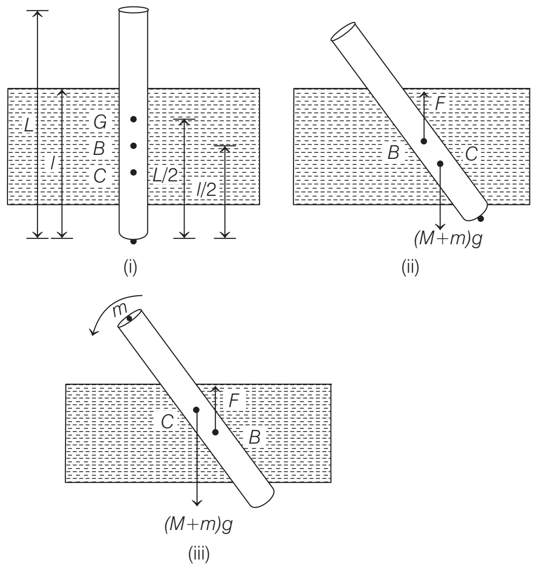
or $\quad a = \dfrac{sh\rho_B g}{s(h + h_A + h_B)\rho_{\text{cylinder}}}$

or $\quad a = \dfrac{h\rho_B g}{(h + h_A + h_B)\rho_{\text{cylinder}}}$

Substituting the values of h, h_A h_B, ρ_B and ρ_{cylinder},

we get, $a = \dfrac{g}{6}$ (upwards)

20. Let M = Mass of stick $= \pi R^2 \rho L$



l = Immersed length of the rod

G = CM of rod

B = Centre of buoyant force (F)

C = CM of rod + mass (m)

Y_{CM} = Distance of C from bottom of the rod

Mass m should be attached to the lower end because otherwise B will be below G and C will be above G and the torque of the couple of two equal and opposite forces F and $(M + m)g$ will be counter clockwise on displacing the rod leftwards. Therefore, the rod cannot be in rotational equilibrium. See the figure (iii).

Now, refer figures (i) and (ii).

For vertical equilibrium $Mg + mg = F$ (upthrust)

or $(\pi R^2 L)\rho g + mg = (\pi R^2\, l)\sigma\, g$

$$\therefore \qquad l = \left(\frac{\pi R^2 L\rho + m}{\pi R^2\sigma}\right)$$

Position of CM (of rod + m) from bottom

$$Y_{\text{CM}} = \frac{M.\frac{L}{2}}{M + m} = \frac{(\pi R^2 L\rho)\frac{L}{2}}{(\pi R^2 L\rho) + m}$$

Centre of buoyancy (B) is at a height of $\frac{l}{2}$ from the bottom.

We can see from figure (ii) that for rotational equilibrium of the rod, B should either lie above C or at the same level of B.

Therefore, $\qquad \dfrac{l}{2} \ge Y_{\text{CM}}$

or $\qquad \dfrac{\pi R^2 L\rho + m}{2\pi R^2\sigma} \ge \dfrac{(\pi R^2 L\rho)\frac{L}{2}}{(\pi R^2 L\rho) + m}$

or $\qquad m + \pi R^2 L\rho \ge \pi R^2 L\sqrt{\rho\sigma}$

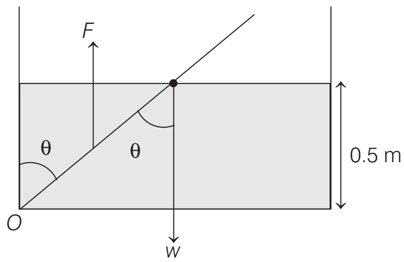
or $\qquad m \ge \pi R^2 L(\sqrt{\rho\sigma} - \rho)$

$\therefore$ Minimum value of m is $\pi R^2 L(\sqrt{\rho\sigma} - \rho)$.

21. Submerged length $= 0.5\sec\theta$, F = Upthrust, w = Weight

Three forces will act on the plank.

(a) Weight which will act at centre of plank.



(b) Upthrust which will act at centre of submerged portion.

(c) Force from the hinge at O.

Taking moments of all three forces about point O. Moment of hinge force will be zero.

$\therefore$ Moment of w (clockwise)

= Moment of F (anti-clockwise)

$$\therefore\quad (Alg\rho)\frac{l}{2}\sin\theta = A\,(0.5\sec\theta)(\rho_w)(g)\left(\frac{0.5\sec\theta}{2}\right)\sin\theta$$

$$\therefore\quad \cos^2\theta = \frac{(0.5)^2\,(1)}{(l^2)(\rho)}$$

$$= \frac{(0.5)^2}{(0.5)} = \frac{1}{2} \qquad (\text{as } l = 1\,\text{m})$$

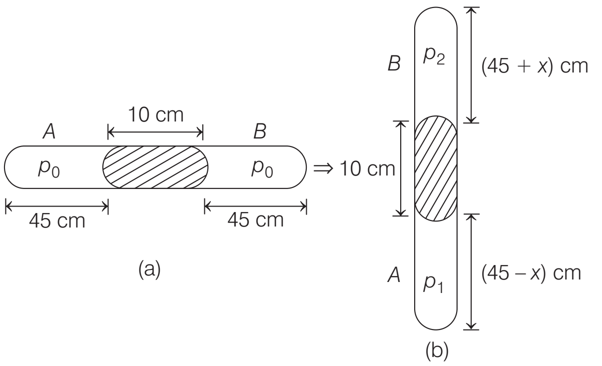
$$\therefore\quad \cos\theta = \frac{1}{\sqrt{2}}$$

or $\theta = 45^\circ$

22. When stones were floating with boat, they will be displacing water of volume (say V_1) whose weight should be equal to weight of stones. When the stones sink, they will displace water of volume (say V_2) whose volume is equal to the volume of stones. But since density of water is less than the density of stones.

$\therefore\quad V_1 > V_2$ or level will fall.

23. Let area of cross-section of the tube be A.



Applying $p_1V_1 = p_2V_2$ in A and B we have,

$$p_0(45)(A) = p_1\,(45 - x)\,A$$

or $$76 \times 45 = p_1(45 - x) \qquad \ldots\text{(i)}$$

$$p_0(45)(A) = p_2\,(45 + x)\,A$$

$\therefore$ $$76 \times 45 = p_2(45 + x) \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$(p_1 - p_2) = 76 \times 45\left(\frac{1}{45 - x} - \frac{1}{45 + x}\right)$$

From figure (b),

$(p_1 - p_2)\,A$ = Weight of 10 cm of Hg column

or $p_1 - p_2$ = Pressure equivalent to 10 cm of Hg column

$$76 \times 45\left(\frac{1}{45 - x} - \frac{1}{45 + x}\right) = 10$$

Solving this equation, we get

$$x = 2.95\,\text{cm}$$

Topic 3 Ideal Fluids in Motion

1. **Key Idea** Terminal speed of a sphere falling in a viscous fluid is

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_f)g$$

where, η = coefficient of viscosity of fluid,

ρ_0 = density of falling sphere

and ρ_f = density of fluid.

As we know, if other parameters remains constant, terminal velocity is proportional to square of radius of falling sphere.

i.e. $$v_T \propto r^2 \qquad \ldots\text{(i)}$$

Now, when sphere of radius R is broken into 27 identical solid sphere of radius r, then Volume of sphere of radius $R = 27 \times$ Volume of sphere of radius r

$$\Rightarrow\quad \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

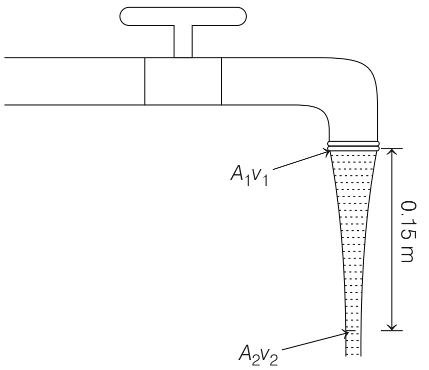
$$\Rightarrow\quad R = 3r$$

$$\Rightarrow\quad r = \frac{R}{3}$$

So, from Eq. (i), we have

$$\frac{v_1}{v_2} = \frac{R^2}{\left(\frac{R}{3}\right)^2} = 9$$

2 Given situation is as shown in the figure below



From equation of continuity,

$$A \propto \frac{1}{v}$$

where, A = area of flow

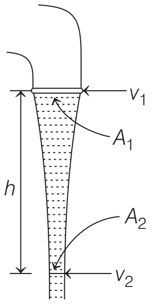
and v = velocity of flow.

$\therefore$ Increase in speed of flow causes a decrease in area of flow.

Here given that height of fall, $h = 0.15$ m

Area, $A = 10^{-4}\,\text{m}^2$

Initial speed, $v = 1\,\text{ms}^{-1}$



Velocity of water stream below h height is

$$v_2 = \sqrt{v_1^2 + 2gh} \qquad [\because v^2 - u^2 = 2gh]$$

Substituting the given values, we get

$$= \sqrt{1^2 + 2 \times 10 \times 0.15} = \sqrt{4} = 2\ \text{ms}^{-1}$$

Now, from equation of continuity, we have

$$A_1 v_1 = A_2 v_2$$

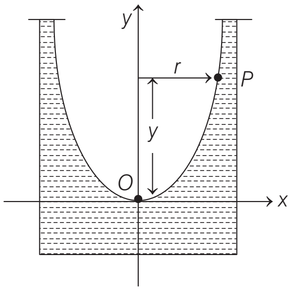
or $$A_2 = \frac{A_1 v_1}{v_2}$$

$$\therefore \quad A_2 = \frac{10^{-4} \times 1}{2}$$

$$= 0.5 \times 10^{-4}$$

$$= 5 \times 10^{-5}\ \text{m}^2$$

3 When liquid filled vessel is rotated the liquid profile becomes a paraboloid due to centripetal force, as shown in the figure below



Pressure at any point P due to rotation is

$$p_R = \frac{1}{2}\rho r^2 \omega^2$$

Gauge pressure at depth y is $p_G = -\rho g y$

If p_0 is atmospheric pressure, then total pressure at point P is

$$p = p_0 + \frac{1}{2}\rho r^2 \omega^2 - \rho g y$$

For any point on surface of rotating fluid,

$$p = p_0$$

Hence, for any surface point;

$$p_0 = p_0 + \frac{1}{2}\rho r^2 \omega^2 - \rho g y$$

or $$\frac{1}{2}\rho r^2 \omega^2 = \rho g y$$

or $$y = \frac{r^2 \omega^2}{2g} \qquad \ldots \text{(i)}$$

In the given case,

Angular speed, $\omega = 2\text{rps} = 2 \times 2\pi = 4\pi$ rad s^{-1}

Radius of vessel, $r = 5$ cm $= 0.05$ m

and $g = 10\ \text{ms}^{-2}$

Hence, substituting these values in Eq. (i), we get

$$y = \frac{\omega^2 r^2}{2g} = \frac{(4\pi)^2 (0.05)^2}{2 \times 10} = 0.02\ \text{m} = 2\ \text{cm}$$

4 Mass per unit time of a liquid flow is given by

$$\frac{dm}{dt} = \rho A v$$

where, ρ is density of liquid, A is area through which it is flowing and v is velocity.

$\therefore$ Rate of change in momentum of the 25% of liquid which loses all momentum is

$$\frac{dp_1}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right) v = \frac{1}{4}\rho A v^2 \qquad \ldots \text{(i)}$$

and the rate of change in momentum of the 25% of the liquid which comes back with same speed.

$$\frac{dp_2}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right) \times 2v = \frac{1}{2}\rho A v^2 \qquad \ldots \text{(ii)}$$

$[\because$ Net change in velocity is $= 2v]$

$\therefore$ Net pressure on the mesh is

$$p = \frac{F_{\text{net}}}{A} = \frac{(dp_1/dt + dp_2/dt)}{A} \qquad \left[\because F = \frac{dp}{dt}\right]$$

$\therefore$ From Eqs. (i) and (ii), we get

$$p = \frac{3}{4}\rho v^2 A / A = \frac{3}{4}\rho v^2$$

5 As, level of water in tank remains constant with time, so

(water inflow rate) = (outflow rate of water)

$\Rightarrow 10^{-4}\ \text{m}^3\text{s}^{-1}$ = Area of orifice × Velocity of outflow

$\Rightarrow 10^{-4}\ \text{m}^3\text{s}^{-1} = 10^{-4} \times \sqrt{2gh}$

where, h = Height of water above the orifice or hole.

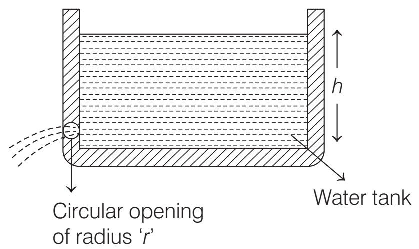
$\Rightarrow \sqrt{2gh} = 1$

or $2 \times 9.8 \times h = 1$

$$\Rightarrow \quad h = \frac{1}{19.6}\ \text{m} = \frac{100}{19.6}\text{cm}$$

or $h = 5.1$ cm

6 For the given condition, a water tank is open to air and its water level maintained.



Suppose the depth of the centre of the opening from level of water in tank is h and the radius of opening is r.

According to question, the water per minute through a circular opening flow rate $(Q) = 0.74\ m^3/min$

$$= \frac{0.74}{60}\ m^3/s$$

r = radius of circular opening = 2 cm

Here, the area of circular opening $= \pi(r^2)$

$$a = \pi \times (2 \times 10^{-2})^2 = 4\pi \times 10^{-4}\ m^2$$

Now flow rate through an area is given by

$$Q = \text{Area of circular opening} \times \text{Velocity of water}$$

$$Q = a \times v = \pi(r^2) \times v$$

$$\Rightarrow \quad \frac{0.74}{60} = (4\pi \times 10^{-4}) \times v \qquad \ldots\text{(i)}$$

According to Torricelli's law (velocity of efflux)

$$v = \sqrt{2gh} \qquad \ldots\text{(ii)}$$

Equation value of 'v' from (i) and (ii) we get,

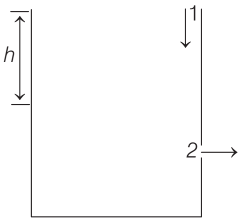
$$\sqrt{2gh} = \frac{0.74 \times 10^4}{60 \times 4\pi}$$

$$\Rightarrow \quad h = \left(\frac{0.74 \times 10^4}{60 \times 4\pi}\right)^2 \times \frac{1}{2g}$$

$$h \approx 4.8\ m$$

7. Applying continuity equation at 1 and 2, we have

$$A_1 v_1 = A_2 v_2 \qquad \ldots\text{(i)}$$



Further applying Bernoulli's equation at these two points, we have

$$p_0 + \rho g h + \frac{1}{2}\rho v_1^2 = p_0 + 0 + \frac{1}{2}\rho v_2^2 \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we have $v_2^2 = \dfrac{2gh}{1 - \dfrac{A_2^2}{A_1^2}}$

Substituting the values, we have

$$v_2^2 = \frac{2 \times 10 \times 2.475}{1 - (0.1)^2} = 50\ m^2/s^2$$

8. Velocity of efflux at a depth h is given by $v = \sqrt{2gh}$. Volume of water flowing out per second from both the holes are equal.

$$\therefore \quad a_1 v_1 = a_2 v_2$$

or $\quad (L^2)\sqrt{2g(y)} = \pi R^2 \sqrt{2g\ (4y)}$ or $R = \dfrac{L}{\sqrt{2\pi}}$

9. From conservation of energy

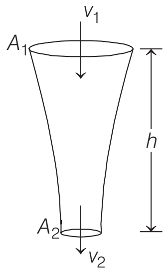
$$v_2^2 = v_1^2 + 2gh \qquad \ldots\text{(i)}$$

[can also be found by applying Bernoulli's theorem]

From continuity equation

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 \qquad \ldots\text{(ii)}$$



Substituting value of v_2 from Eq. (ii) in Eq. (i),

$$\frac{A_1^2}{A_2^2} \cdot v_1^2 = v_1^2 + 2gh \text{ or } A_2^2 = \frac{A_1^2\ v_1^2}{v_1^2 + 2gh}$$

$$\therefore \quad A_2 = \frac{A_1\ v_1}{\sqrt{v_1^2 + 2gh}}$$

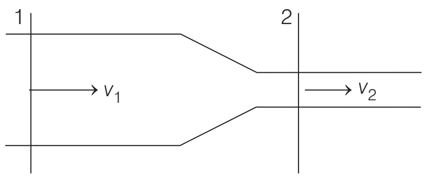
Substituting the given values

$$A_2 = \frac{(10^{-4})(1.0)}{\sqrt{(1.0)^2 + 2\,(10)\,(0.15)}}$$

$$A_2 = 5.0 \times 10^{-5}\ m^2$$

10. From continuity equation

$$A_1 v_1 = A_2 v_2$$



$$\therefore \quad v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{10}{5}\right)(1)$$

$$= 2\ m/s$$

Applying Bernoulli's theorem at 1 and 2

$$p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

$$\therefore \quad p_2 = p_1 + \frac{1}{2}\rho\ (v_1^2 - v_2^2)$$

$$= \left(2000 + \frac{1}{2} \times 10^3\ (1 - 4)\right)$$

$$\therefore \quad p_2 = 500\ Pa$$

11. From equation of continuity (Av = constant)

$$\frac{\pi}{4}(8)^2\ (0.25) = \frac{\pi}{4}(2)^2\ (v) \qquad \ldots\text{(i)}$$

Here, v is the velocity of water with which water comes out of the syringe (Horizontally).

Solving Eq. (i), we get $\quad v = 4\ m/s$

The path of water after leaving the syringe will be a parabola. Substituting proper values in equation of trajectory.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

According to question, we have,

$$-1.25 = R\tan 0° - \frac{(10)\,(R^2)}{(2)\,(4)^2\cos^2 0°}$$

(R = horizontal range)

Solving this equation, we get $R = 2\ m$

12. Given, $A_1 = 4 \times 10^{-3}\,\text{m}^2$, $A_2 = 8 \times 10^{-3}\,\text{m}^2$,

$h_1 = 2\,\text{m}$, $h_2 = 5\,\text{m}$,

$v_1 = 1\,\text{m/s}$ and $\rho = 10^3\,\text{kg/m}^3$

From continuity equation, we have

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

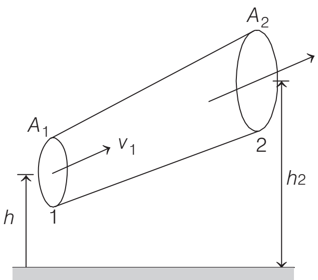
or $$v_2 = \left(\frac{4 \times 10^{-3}}{8 \times 10^{-3}}\right)(1\,\text{m/s})$$

$\Rightarrow$ $$v_2 = \frac{1}{2}\,\text{m/s}$$

Applying Bernoulli's equation at sections 1 and 2

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho\, v_2^2 + \rho g h_2$$

or $$p_1 - p_2 = \rho g\,(h_2 - h_1) + \frac{1}{2}\rho\,(v_2^2 - v_1^2) \qquad \ldots \text{(i)}$$



(a) Work done per unit volume by the pressure as the fluid flows from P to Q.

$$W_1 = p_1 - p_2$$
$$= \rho g\,(h_2 - h_1) + \frac{1}{2}\rho\,(v_2^2 - v_1^2) \qquad \text{[From Eq. (i)]}$$
$$= \left\{(10^3)(9.8)(5-2) + \frac{1}{2}(10^3)\left(\frac{1}{4} - 1\right)\right\}$$
$$= [29400 - 375] = 29025\,\text{J/m}^3$$

(b) Work done per unit volume by the gravity as the fluid flows from P to Q.

$$W_2 = \rho g\,(h_1 - h_2)$$
$$= \{(10^3)(9.8)(2-5)\}$$

or $$W_2 = -29400\,\text{J/m}^3$$

13. (a) Mass of water = (Volume) (density)

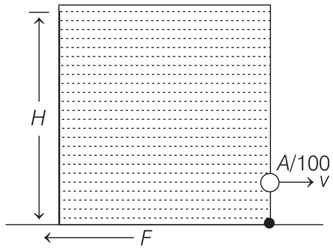
$\therefore$ $$m_0 = (AH)\rho$$

$\therefore$ $$H = \frac{m_0}{A\rho} \qquad \ldots\text{(i)}$$

Velocity of efflux,

$$v = \sqrt{2gH} = \sqrt{2g\frac{m_0}{A\rho}}$$
$$= \sqrt{\frac{2m_0 g}{A\rho}}$$

Thrust force on the container due to draining out of liquid from the bottom is given by,

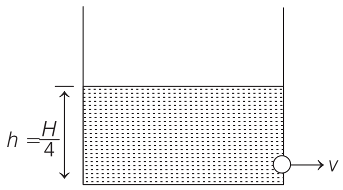


F = (density of liquid) (area of hole)(velocity of efflux)^2

$$F = \rho a v^2$$
$$F = \rho(A/100)v^2$$
$$= \rho(A/100)\left(\frac{2m_0 g}{A\rho}\right)$$
$$F = \frac{m_0 g}{50}$$

$\therefore$ Acceleration of the container, $a = F/m_0 = g/50$

(b) Velocity of efflux when 75% liquid has been drained out i.e. height of liquid, $h = \dfrac{H}{4} = \dfrac{m_0}{4A\rho}$



$$v = \sqrt{2gh} = \sqrt{2g\left(\frac{m_0}{4A\rho}\right)}$$
$$v = \sqrt{\frac{m_0 g}{2A\rho}}$$

14. (a) (i) Considering vertical equilibrium of cylinder

Weight of cylinder = upthrust due to upper liquid + upthrust due to lower liquid

$$\therefore \left(\frac{A}{5}\right)(L)D.g = \left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)(d)g + \left(\frac{A}{5}\right)\left(\frac{L}{4}\right)(2d)(g)$$

$$\therefore \quad D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$\Rightarrow \quad D = \frac{5}{4}d$$

(ii) Considering vertical equilibrium of two liquids and the cylinder.

$(p - p_0)A$ = weight of two liquids + weight of cylinder

$$\therefore \quad \frac{\text{weight of two liquids + weight of cylinder}}{A} \qquad \ldots\text{(i)}$$

Now, weight of cylinder

$$= \left(\frac{A}{5}\right)(L)(D)(g) = \left(\frac{A}{5}Lg\right)\left(\frac{5}{4}d\right) = \frac{ALdg}{4}$$

Weight of upper liquid $= \left(\dfrac{H}{2}\,Adg\right)$ and

$$\text{Weight of lower liquid} = \frac{H}{2} A\,(2d)g$$
$$= HAgd$$

$\therefore$ Total weight of two liquids $= \frac{3}{2} HAdg$

$\therefore$ From Eq. (i) pressure at the bottom of the container will be

$$p = p_0 + \frac{\left(\frac{3}{2}\right) HAdg + \frac{ALdg}{4}}{A}$$

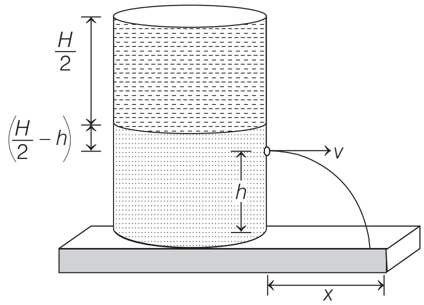
or $$p = p_0 + \frac{dg\,(6H + L)}{4}$$

(b) (i) Applying Bernoulli's theorem,

$$p_0 + dg\left(\frac{H}{2}\right) + 2dg\left(\frac{H}{2} - h\right) = p_0 + \frac{1}{2}(2d)\,v^2$$

Here, v is velocity of efflux at 2.
Solving this, we get

$$v = \sqrt{(3H - 4h)\frac{g}{2}}$$



(ii) Time taken to reach the liquid to the bottom will be

$$t = \sqrt{2h/g}$$

$\therefore$ Horizontal distance x travelled by the liquid is

$$x = vt = \sqrt{\left(3H - 4h\right)\frac{g}{2}}\left(\sqrt{\frac{2h}{g}}\right)$$

$$x = \sqrt{h\,(3H - 4h)}$$

(iii) For x to be maximum $\frac{dx}{dh} = 0$

or $$\frac{1}{2\sqrt{h(3H - 4h)}}(3H - 8h) = 0$$

or $$h = \frac{3H}{8}$$

Therefore, x will be maximum at $h = \frac{3H}{8}$

The maximum value of x will be

$$x_m = \sqrt{\left(\frac{3H}{8}\right)\left[3H - 4\left(\frac{3H}{8}\right)\right]}$$

$$x_m = \frac{3}{4}H$$

Topic 4 Surface Tension and Viscosity

1. Given,

$$\frac{T_{Hg}}{T_W} = 7.5,\ \frac{\rho_{Hg}}{\rho_W} = 13.6$$

and $$\frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135^\circ}{\cos 0^\circ} = -\frac{1}{\sqrt{2}}$$

Height of the fluid inside capillary tube is given by

$$h = \frac{2T\cos\theta}{\rho g r}$$

According to given situation, $h_w = h_{Hg}$

$$\therefore \quad \frac{2\,T_w \cos\theta_w}{\rho_w\, g\, r_w} = \frac{2T_{Hg}\cos\theta_{Hg}}{\rho_{Hg}\, g\, r_{Hg}}$$

$$\therefore \quad \frac{r_{Hg}}{r_w} = \left(\frac{T_{Hg}}{T_w}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_w}\right)\left(\frac{\rho_w}{\rho_{Hg}}\right)$$

Given, $r_{Hg} = r_1$ and $r_w = r_2$, then

Substituting the given values, we get

$$\frac{r_{Hg}}{r_w} = \frac{r_1}{r_2} = 7.5 \times \frac{1}{\sqrt{2}} \times \frac{1}{13.6}$$
$$= 0.4 = 2/5$$

2. Height of liquid rise in capillary tube,

$$h = \frac{2T\cos\theta_c}{\rho r g}$$

$$\Rightarrow \quad h \propto \frac{1}{r}$$

So, when radius is doubled, height becomes half.

$$\therefore \quad h' = h/2$$

Now, density $(\rho) = \frac{\text{mass }(M)}{\text{volume }(V)}$

$$\Rightarrow \quad M = \rho \times \pi r^2 h$$

$$\therefore \quad M' = \rho\pi r'^2 h'$$

$$\therefore \quad \frac{M'}{M} = \frac{r'^2 h'}{r^2 h} = \frac{(2r)^2 (h/2)}{r^2 h} = 2$$

$$\Rightarrow \quad M' = 2M$$

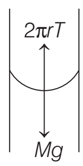
Alternate Solution

According to the given figure, force inside the capillary tube is

$2\pi rT = Mg \Rightarrow M \propto r$

When $r' = 2r$, then $M' = 2M$

3. By ascent formula, we have surface tension,



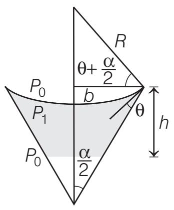
$$T = \frac{rhg}{2} \times 10^3\ \frac{\text{N}}{\text{m}}$$
$$= \frac{dhg}{4} \times 10^3\ \frac{\text{N}}{\text{m}}$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h} \qquad \text{[given, } g \text{ is constant]}$$

So, percentage $= \frac{\Delta T}{T} \times 100 = \left(\frac{\Delta d}{d} + \frac{\Delta h}{h}\right) \times 100$

$$= \left(\frac{0.01\times 10^{-2}}{1.25\times 10^{-2}} + \frac{0.01\times 10^{-2}}{1.45\times 10^{-2}} \right) \times 100$$
$$= 1.5\%$$
$$\therefore \quad \frac{\Delta T}{T} \times 100 = 1.5\%$$

4. Let R be the radius of the meniscus formed with a contact angle θ. By geometry, this radius makes an angle $\theta + \frac{\alpha}{2}$ with the horizontal and,



$$\cos\left(\theta + \frac{\alpha}{2}\right) = b/R \quad \text{...(i)}$$

Let P_0 be the atmospheric pressure and P_1 be the pressure just below the meniscus. Excess pressure on the concave side of meniscus of radius R is,

$$P_0 - P_1 = 2S/R \quad \text{...(ii)}$$

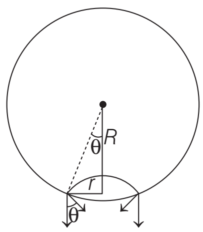
The hydrostatic pressure gives,

$$P_0 - P_1 = h\rho g \quad \text{...(iii)}$$

Eliminate $(P_0 - P_1)$ from second and third equations and substitute R from first equation to get,

$$h = \frac{2S}{\rho g R} = \frac{2S}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right)$$

5. The bubble will detach if,



$$\int \sin\theta T \times dl = T(2\pi r)\sin\theta$$

Buoyant force $\geq$ Surface tension force

$$\frac{4}{3}\pi R^3 \rho_w g \geq \int T \times dl \sin\theta$$
$$(\rho_w)\left(\frac{4}{3}\pi R^3\right) g \geq (T)(2\pi r)\sin\theta$$
$$\Rightarrow \quad \sin\theta = \frac{r}{R}$$

Solving, $r = \sqrt{\dfrac{2\rho_w R^4 g}{3T}} = R^2\sqrt{\dfrac{2\rho_w g}{3T}}$

No option matches with the correct answer.

6. Decrease in surface energy = heat required in vaporisation.

$$S = 4\pi r^2$$
$$\therefore \quad dS = 2(4\pi r)\,dr$$
$$\therefore \quad T(dS) = L(dm)$$
$$\therefore \quad T(2)(4\pi r)dr = L(4\pi r^2 dr)\rho$$
$$\therefore \quad r = \frac{2T}{\rho L}$$

7. $\Delta p_1 = \dfrac{4T}{r_1}$ and $\Delta p_2 = \dfrac{4T}{r_2}$

$$r_1 < r_2 \Rightarrow \Delta p_1 > \Delta p_2$$

$\therefore$ Air will flow from 1 to 2 and volume of bubble at end 1 will decrease.

Therefore, correct option is (b).

8. Force from right hand side liquid on left hand side liquid.

(i) Due to surface tension force $= 2RT$ (towards right)

(ii) Due to liquid pressure force

$$= \int_{x=0}^{x=h} (p_0 + \rho g h)(2R \cdot x)\,dx$$
$$= (2p_0Rh + R\rho g h^2) \text{ (towards left)}$$

$\therefore$ Net force is $|2p_0Rh + R\rho g h^2 - 2RT|$

9. $h = \dfrac{2\sigma\cos\theta}{r\rho g}$

(a) $\rightarrow h \propto \dfrac{1}{r}$

(b) h depends upon σ.

(c) If lift is going up with constant acceleration.

$$g_{\text{eff}} = (g+a) \Rightarrow h = \frac{2\sigma\cos\theta}{r\rho(g+a)}$$

It means h decreases.

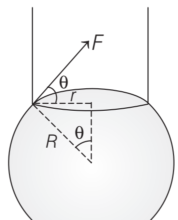
(d) h is proportional to $\cos\theta$.

10. Vertical force due to surface tension,

$$F_v = F\sin\theta = (T2\pi r)(r/R) = \frac{2\pi r^2 T}{R}$$

$\therefore$ Correct option is (c).

11. $\dfrac{2\pi r^2 T}{R} = mg = \dfrac{4}{3}\pi R^3 \cdot \rho \cdot g$



$$\therefore \quad R^4 = \frac{3r^2T}{2\rho g} = \frac{3\times(5\times 10^{-4})^2\,(0.11)}{2\times 10^3 \times 10}$$
$$= 4.125\times 10^{-12}\ \text{m}^4$$
$$\therefore \quad R = 1.425\times 10^{-3}\,\text{m} \approx 1.4\times 10^{-3}\ \text{m}$$

$\therefore$ Correct option is (a).

12. Surface energy, $E = (4\pi R^2)T$

$$= (4\pi)(1.4\times 10^{-3})^2\,(0.11)$$
$$= 2.7\times 10^{-6}\ \text{J}$$

$\therefore$ Correct option is (b).

13. From mass conservation,

$$\rho\cdot\frac{4}{3}\pi R^3 = \rho\cdot K\cdot\frac{4}{3}\pi r^3 \Rightarrow R = K^{1/3}r$$
$$\therefore \quad \Delta U = T\Delta A = T(K\cdot 4\pi r^2 - 4\pi R^2)$$
$$= T(K\cdot 4\pi R^2 K^{-2/3} - 4\pi R^2)$$
$$\Delta U = 4\pi R^2 T[K^{1/3} - 1]$$

Putting the values, we get

$$10^{-3} = \frac{10^{-1}}{4\pi}\times 4\pi\times 10^{-4}[K^{1/3} - 1]$$

$$100 = K^{1/3} - 1$$

$\Rightarrow \quad K^{1/3} \cong 100 = 10^2$

Given that $\quad K = 10^{\alpha}$

$\therefore \quad 10^{\alpha/3} = 10^2$

$\Rightarrow \quad \frac{\alpha}{3} = 2 \Rightarrow \alpha = 6$

14. Terminal velocity is given by

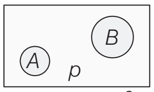
$$v_T = \frac{2}{9}\frac{r^2}{\eta}(d-\rho)g$$

$$\frac{v_P}{v_Q} = \frac{r_P^2}{r_Q^2} \times \frac{\eta_Q}{\eta_P} \times \frac{(d-\rho_P)}{(d-\rho_Q)}$$

$$= \left(\frac{1}{0.5}\right) \times \left(\frac{2}{3}\right) \times \frac{(8-0.8)}{(8-1.6)}$$

$$= 4 \times \frac{2}{3} \times \frac{7.2}{6.4} = 3$$

15. Although not given in the question, but we will have to assume that temperatures of A and B are same.



$p = 8\ \text{Nm}^{-2}$

$$\frac{n_B}{n_A} = \frac{p_B V_B / RT}{p_A V_A / RT} = \frac{p_B V_B}{p_A V_A}$$

$$= \frac{p + 4S / r_A \times 4/3\pi\, (r_A)^3}{(p + 4S / r_B) \times 4/3\pi\, (r_B)^3}$$

(S = surface tension)

Substituting the values, we get

$$\frac{n_B}{n_A} = 6$$

16. Terminal velocity $v_T = \dfrac{2r^2 g}{9\eta}(\rho_S - \rho_L)$

and viscous force $F = 6\pi\eta r v_T$

Rate of production of heat (power) : as viscous force is the only dissipative force.

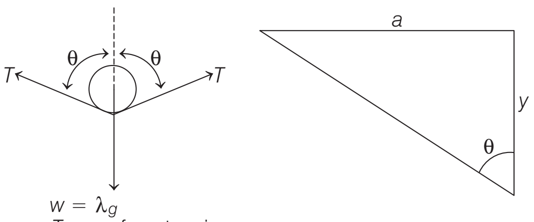
Hence,

$$\frac{dQ}{dt} = F v_T = (6\pi\eta r v_T)(v_T) = 6\pi\eta r v_T^2$$

$$= 6\pi\eta r \left\{\frac{2}{9}\frac{r^2 g}{\eta}(\rho_S - \rho_L)\right\}^2$$

$$= \frac{8\pi g^2}{27\eta}(\rho_S - \rho_L)^2 r^5 \quad \text{or} \quad \frac{dQ}{dt} \propto r^5$$

17. Free body diagram of the wire is as shown in figure.



$w = \lambda g$
T = surface tension

Considering the equilibrium of wire in vertical direction, we have

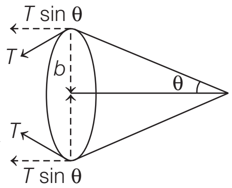
$$2Tl\cos\theta = \lambda l g \qquad \ldots\text{(i)}$$

For $\quad y << a, \cos\theta \approx \dfrac{y}{a}$

Substituting the values in Eq. (i), we get

$$T = \frac{\lambda a}{2y}$$

18. Surface Tension force
$= 2\pi b \times 2T\sin\theta$



Mass of the air per second entering the bubble $= \rho A v$

Momentum of air per second $=$ Force due to air $= \rho A v^2$

The bubble will separate from the tube when force due to moving air becomes equal to the surface tension force inside the bubble.

$$2\pi b \times 2T\sin\theta = \rho A v^2$$

putting $\sin\theta = \dfrac{b}{r}$, $A = \pi b^2$ and solving, we get

$$r = \frac{4T}{\rho v^2}$$

19. When the tube is not there,

$$p + p_0 + \frac{1}{2}\rho v_1^2 + \rho g H = \frac{1}{2}\rho v_2^2 + p_0$$

$$\therefore \quad p + \rho g H = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$A_1 v_1 = A_2 v_2$$

or $\quad v_1 = \dfrac{A_2 v_2}{A_1}$

$$\therefore \quad p + \rho g H = \frac{1}{2} \times \rho \left[v_2^2 - \left(\frac{A_2}{A_1} v_2\right)^2\right]$$

$$= \frac{1}{2} \times \rho \times v_2^2 \left[1 - \left(\frac{\pi\,(0.3)^2}{\pi\,(0.9)^2}\right)^2\right]$$

$$= \frac{1}{2} \times \rho \times (10)^2 \left[1 - \frac{1}{81}\right]$$

$$= \frac{4 \times 10^3\,\rho}{81}$$

$$= \frac{4 \times 10^3 \times 900}{81}$$

$$= \frac{4}{9} \times 10^5\ \text{N/m}^2$$

This is also the excess pressure Δp.

By Poiseuille's equation, the rate of flow of liquid in the capillary tube

$$Q = \frac{\pi(\Delta p)\,a^4}{8\eta l}$$

$$\therefore \quad 8\times10^{-6} = \frac{(\pi a^2)(\Delta p)}{8\eta}\left(\frac{a^2}{l}\right)$$

$$\therefore \quad \eta = \frac{(\pi a^2)(\Delta p)\left(\frac{a^2}{l}\right)}{8\times8\times10^{-6}}$$

Substituting the values, we have

$$\eta = \frac{(10^{-6})\left(\frac{4}{9}\times10^5\right)(2\times10^{-6})}{8\times8\times10^{-6}}$$

$$= \frac{1}{720}\,\text{N-s/m}^2$$

Topic 5 Miscellaneous Problems

1. Reynolds' number for flow of a liquid is given by

$$R_e = \frac{\rho v D}{\eta}$$

where, velocity of flow,

$$v = \frac{\text{volume flow rate}}{\text{area of flow}} = \frac{V/t}{A}$$

So, $R_e = \dfrac{\rho VD}{\eta At} = \dfrac{\rho V 2r}{\eta\times\pi r^2\times t} = \dfrac{2\rho V}{\eta\pi rt}$

Here, ρ = density of water = $1000\,\text{kgm}^{-3}$

$$\frac{V}{t} = \frac{100\times10^{-3}}{60}\ \text{m}^3\text{s}^{-1}$$

where, η = viscosity of water = 1×10^{-3} Pa-s

and r = radius of pipe = 5×10^{-2} m

$$R_e = \frac{2\times1000\times100\times10^{-3}}{1\times10^{-3}\times60\times3.14\times5\times10^{-2}}$$

$$= 212.3\times10^2 \approx 2.0\times10^4$$

So, order of Reynolds' number is of 10^4.

2 When soap bubble is being inflated and its temperature remains constant, then it follows Boyle's law, so

pV = constant (k)

$\Rightarrow p = \dfrac{k}{V}$

Differentiating above equation with time, we get

$$\frac{dp}{dt} = k.\frac{d}{dt}\left(\frac{1}{V}\right) \Rightarrow \frac{dp}{dt} = k\left(\frac{-1}{V^2}\right).\frac{dV}{dt}$$

It is given that, $\dfrac{dV}{dt} = c$ (a constant)

So, $$\frac{dp}{dt} = \frac{-kc}{V^2} \quad \ldots\text{(i)}$$

Now, from $\dfrac{dV}{dt} = c$; we get

$dV = cdt$

or $$\int dV = \int cdt \ \text{ or } V = ct \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{dp}{dt} = \frac{-kc}{c^2t^2} \ \text{ or } \ \frac{dp}{dt} = -\left(\frac{k}{c}\right)t^{-2} \Rightarrow dp = -\frac{k}{c}.t^{-2}dt$$

Integrating both sides, we get

$$\int dp = -\frac{k}{c}\int t^{-2}dt$$

$$p = -\frac{k}{c}.\left(\frac{t^{-2+1}}{-2+1}\right)$$

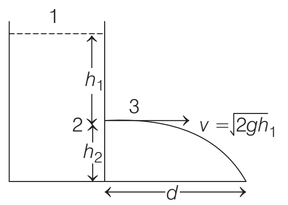
$$= -\frac{k}{c}.\frac{-1}{t} = \frac{k}{ct} \ \text{ or } p \propto \frac{1}{t}$$

Hence, p *versus* $\dfrac{1}{t}$ graph is a straight line, which is correctly represented in option (b).

3. Stress $= \dfrac{\text{Weight}}{\text{Area}} = \dfrac{9^3\times W_0}{9^2\times A_0} = 9\left(\dfrac{W_0}{A_0}\right)$

Hence, the stress increases by a factor of 9.

4. $d = 2\sqrt{h_1h_2} = \sqrt{4h_1h_2}$



This is independent of the value of g.

(A) $g_{\text{eff}} > g \quad d = \sqrt{4h_1h_2} = 1.2$ m

(B) $g_{\text{eff}} < g \quad d = \sqrt{4h_1h_2} = 1.2$ m

(C) $g_{\text{eff}} = g \quad d = \sqrt{4h_1h_2} = 1.2$ m

(D) $g_{\text{eff}} = 0$

No water leaks out of jar. As there will be no pressure difference between top of the container and any other point.

$$p_1 = p_2 = p_3 = p_0$$

5. If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.

According to Hooke's law i.e.

Young's modulus $(Y) = \dfrac{\text{Tensile stress}}{\text{Tensile strain}}$

So, $$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel wire (L) = 10 cm

Temperature (θ) = 100° C

As length is constant.

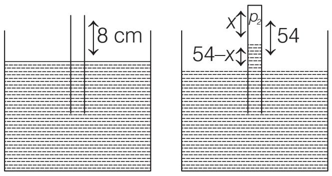
$\therefore$ Strain $= \dfrac{\Delta L}{L} = \alpha\,\Delta\theta$

Now, pressure = stress = $Y\times$ strain

[Given, $Y = 2\times10^{11}\,\text{N/m}^2$ and $\alpha = 1.1\times10^{-5}\,\text{K}^{-1}$]

$$= 2\times10^{11}\times1.1\times10^{-5}\times100 = 2.2\times10^8 \text{ Pa}$$

6. **Key Idea** In this question, the system is accelerating horizontally i.e. no component of acceleration in vertical direction. Hence, the pressure in the vertical direction will remain unaffected.



i.e. $\qquad p_1 = p_0 + \rho g h$

Again, we have to use the concept that the pressure in the same level will be same.

For air trapped in tube, $p_1 V_1 = p_2 V_2$

$$p_1 = p_{atm} = \rho g 76$$

$$V_1 = A \cdot 8 \qquad [A = \text{area of cross-section}]$$

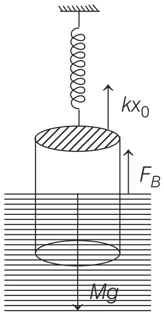
$$p_2 = p_{atm} - \rho g (54 - x) = \rho g (22 + x)$$

$$V_2 = A \cdot x \quad \rho g 76 \times 8A = \rho g (22 + x) A x$$

$$x^2 + 22x - 78 \times 8 = 0 \quad \Rightarrow x = 16 \text{ cm}$$

7. In equilibrium, Upward force = Downward force

$$kx_0 + F_B = mg$$



Here, kx_0 is restoring force of spring and F_B is buoyancy force.

$$kx_0 + \sigma \frac{L}{2} Ag = Mg$$

$$x_0 = \frac{Mg - \dfrac{\sigma L A g}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{\sigma L A}{2M}\right)$$

8. Let V_1 = total material volume of shell

V_2 = total inside volume of shell and

x = fraction of V_2 volume filled with water.

In floating condition,

Total weight = Upthrust

$$\therefore \quad V_1 \rho_c g + (xV_2)(1) g = \left(\frac{V_1 + V_2}{2}\right)(1) g$$

or $\qquad x = 0.5 + (0.5 - \rho_c)\dfrac{V_1}{V_2}$

From here we can see that, $x > 0.5$ if $\rho_c < 0.5$

9. $k_1 = \left(\dfrac{\rho_{Fe}}{\rho_{Hg}}\right)_{0°C}$ and $k_2 = \left(\dfrac{\rho_{Fe}}{\rho_{Hg}}\right)_{60°C}$.

Here, ρ = Density

$$\therefore \quad \frac{k_1}{k_2} = \frac{(\rho_{Fe})_{0°C}}{(\rho_{Hg})_{0°C}} \times \left(\frac{\rho_{Hg}}{\rho_{Fe}}\right)_{60°C} = \frac{(1 + 60\gamma_{Fe})}{(1 + 60\gamma_{Hg})}$$

NOTE In this problem two concepts are used

(i) When a solid floats in a liquid, then

Fraction of volume submerged $(k) = \dfrac{\rho_{solid}}{\rho_{liquid}}$

This result comes from the fact that

Weight = Upthrust

$$V\rho_{solid}\, g = V_{submerged}\, \rho_{liquid}\, g$$

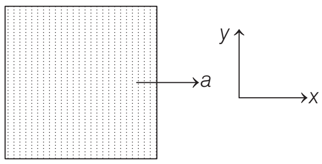
$$\therefore \quad \frac{V_{submerged}}{V} = \frac{\rho_{solid}}{\rho_{liquid}}$$

(ii) $\dfrac{\rho_{\theta°C}}{\rho_{0°C}} = \dfrac{1}{1 + \gamma \cdot \theta}$

This is because $\rho \propto \dfrac{1}{Volume}$ (mass remaining constant)

$$\therefore \quad \frac{\rho_{\theta°C}}{\rho_{0°C}} = \frac{V_{0°C}}{V_{\theta°C}} = \frac{V_{0°C}}{V_{0°C} + \Delta V} = \frac{V_{0°C}}{V_{0°C} + V_{0°C}\gamma\,\theta} = \frac{1}{1 + \gamma\,\theta}$$

10. If a fluid (gas or liquid) is accelerated in positive x-direction, then pressure decreases in positive x-direction. Change in pressure has following differential equation.

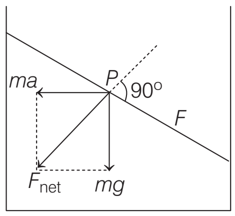


$$\frac{dp}{dx} = -\rho a$$

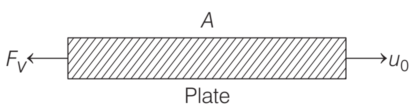
where, ρ is the density of the fluid. Therefore, pressure is lower in front side.

11. Net force on the free surface of the liquid in equilibrium (from accelerated frame) should be perpendicular to it.

Forces on a water particle P on the free surfaces have been shown in the figure. In the figure ma is the pseudo force.



12.



$$F_v = -\eta A\left(\frac{dv}{dy}\right)$$

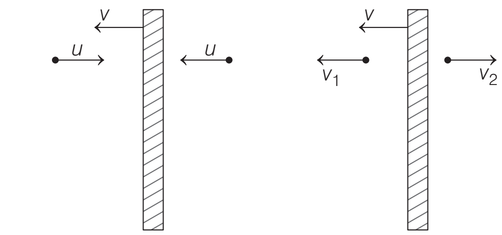
Since, height h of the liquid in tank is very small.

$$\Rightarrow \quad \frac{dv}{dy} = \frac{\Delta v}{\Delta y} = \left(\frac{u_0}{h}\right) \Rightarrow F_v = -(\eta) A\left(\frac{u_0}{h}\right)$$

$$F_v \propto \left(\frac{1}{h}\right), F_v \propto u_0, F \propto A, F_v \propto \eta$$

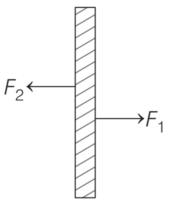
Statement a, c and are correct.

13.



Just before the collision   Just after the collision

$$v_1 = u + 2v$$
$$\Delta v_1 = (2u + 2v)$$
$$F_1 = \frac{dp_1}{dt}$$
$$= \rho A(u + v)(2u + 2v)$$
$$= 2\rho A(u + v)^2$$



$$v_2 = (u - 2v)$$
$$\Delta v_2 = (2u - 2v)$$
$$F_2 = \frac{dp_2}{dt}$$
$$= \rho A(u - v)(2u - 2v) = 2\rho A(u - v)^2$$

[ΔF is the net force due to the air molecules on the plate]

$$\Delta F = 2\rho A(4uv) = 8\rho Auv$$
$$P = \frac{\Delta F}{A} = 8\rho(uv)$$
$$F_{\text{net}} = (F - \Delta F) = ma$$

[m is mass of the plate]

$$F - (8\rho Au)v = ma$$

14. For floating, net weight of system = net upthrust

$\Rightarrow (\rho_1 + \rho_2)Vg = (\sigma_1 + \sigma_2)Vg$

Since string is taut, $\rho_1 < \sigma_1$ and $\rho_2 > \sigma_2$

$$\mathbf{v}_P = \frac{2r^2g}{2\eta_2}(\sigma_2 - \rho_1) \quad \text{(upward terminal velocity)}$$

$$\mathbf{v}_Q = \frac{2r^2g}{9\eta_1}(\rho_2 - \sigma_1) \quad \text{(downward terminal velocity)}$$

$$\left|\frac{\mathbf{v}_P}{\mathbf{v}_Q}\right| = \frac{\eta_1}{\eta_2}$$

Further, $\mathbf{v}_P \cdot \mathbf{v}_Q$ will be negative as they are opposite to each other.

15. From continuity equation, Av = constant

or $\quad A \propto \dfrac{1}{v}$

16. From continuity equation,

$$A_1v_1 = A_2v_2$$

Here, $\quad A_1 = 400A_2$

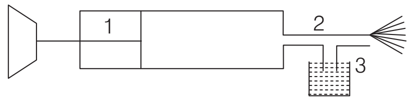
because $\quad r_1 = 20r_2$ and $A = \pi r^2$

$$\therefore \quad v_2 = \frac{A_1}{A_2}(v_1) = 400v_1$$
$$= 400(5)\text{mm/s} = 2000\text{ mm/s} = 2\text{m/s}$$

17. $$p_1 - p_2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow p_3 - p_2 = \frac{1}{2}\rho_l v_l^2$$

$$p_3 = p_1$$

$$\therefore \quad \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow v_l = \sqrt{\frac{\rho_a}{\rho_l}}\, v_a$$



$\therefore$ Volume flow rate $\propto \sqrt{\dfrac{\rho_a}{\rho_l}}$

18. As the bubble moves upwards, besides the buoyancy force (cause of which is pressure difference) only force of gravity and force of viscosity will act.

$\therefore$ Correct option is (d).

19. As there is no exhange of heat. Therefore, process is adiabatic. Applying,

$$Tp^{\frac{1-\gamma}{\gamma}} = \text{constant} \Rightarrow T_2p_2^{\frac{1-\gamma}{\gamma}} = T_1p_1^{\frac{1-\gamma}{\gamma}}$$

or $$T_2 = T_1\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Substituting the values, we have

$$T_2 = T_0\left[\frac{p_0 + \rho_l g(H - y)}{p_0 + \rho_l gH}\right]^{\frac{5/3-1}{5/3}}$$

$$= T_0\left[\frac{p_0 + \rho_l g(H - y)}{p_0 + \rho_l gH}\right]^{2/5}$$

$\therefore$ Correct option is (b).

20. Buoyancy force

$$F = \text{(volume of bubble)}(\rho_l)g = \left(\frac{nRT_2}{p_2}\right)\rho_l g$$

Here, $$T_2 = T_0\left[\frac{p_0 + \rho_e g\,(H - y)}{p_0 + \rho_l gh}\right]^{2/5}$$

and $$p_2 = p_0 + \rho_l g(H - y)$$

Substituting the values, we get

$$F = \frac{\rho_l nRgT_0}{(p_0 + \rho_l gH)^{2/5}[p_0 + \rho_l g(H - y)]^{3/5}}$$

$\therefore$ Correct option is (b)

21. The condition of floating is,

Weight = Upthrust

$$V\rho_1 g = V_i\rho_2 g$$

(ρ_1 = density of metal, ρ_2 = density of mercury)

$\therefore \qquad \dfrac{V_i}{V} = \dfrac{\rho_1}{\rho_2}$

= fraction of volume of metal submerged in mercury

= x (say)

Now, when the temperature is increased by ΔT.

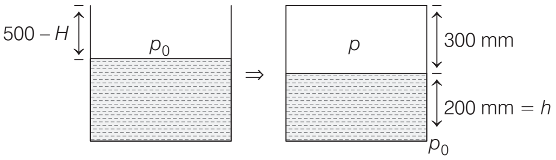
$$\rho_1' = \frac{\rho_1}{1+\gamma_1 \Delta T} \text{ and } \rho_2' = \frac{\rho_2}{1+\gamma_2 \Delta T}$$

$$\therefore \quad x' = \left(\frac{\rho_1}{1+\gamma_1 \Delta T}\right)\left(\frac{1+\gamma_2 \Delta T}{\rho_2}\right) = \frac{\rho_1}{\rho_2}\left(\frac{1+\gamma_2 \Delta T}{1+\gamma_1 \Delta T}\right)$$

$$\therefore \quad x' = x\left(\frac{1+\gamma_2\ \Delta T}{1+\gamma_1\ \Delta T}\right) \Rightarrow \frac{x'}{x} = \frac{1+\gamma_2\ \Delta T}{1+\gamma_1\ \Delta T}$$

22. When tube is heated the density of water at A will decrease, hence the water will rise up or it will circulate in clockwise direction.

23. In this question, we will have to assume that temperature of enclosed air about water is constant (or pV = constant)



$$p = p_0 - \rho gh \qquad \text{...(i)}$$

$$p_0[A(500 - H)] = p[A(300)] \qquad \text{...(ii)}$$

Solving these two equations, we get $H = 206$ mm

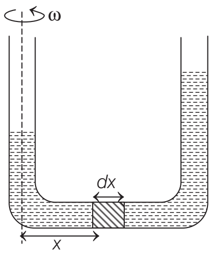
$\therefore$ Level fall = (206 − 200) mm = 6 mm

24. For circular motion of small element dx, we have

$$dF = (dm)\, x\, \omega^2$$

$$\therefore (dp)\, A = (\rho A dx) \cdot x\, \omega^2 \text{ or } dp = \rho\, \omega^2\, x \cdot dx$$

$$\therefore \int_{p_1}^{p_2} dp = \rho\, \omega^2 \int_0^L x\, dx \Rightarrow p_2 - p_1 = \frac{\rho\, \omega^2 L^2}{2}$$



$$\therefore \quad \rho g H = \frac{\rho \omega^2 L^2}{2} \Rightarrow H = \frac{\omega^2 L^2}{2g}$$

25. In elastic collision with the surface, direction of velocity is reversed but its magnitude remains the same. Therefore,

time of fall = time of rise or time of fall = $\dfrac{t_1}{2}$

Hence, velocity of the ball just before it collides with liquid is

$$v = g\frac{t_1}{2} \qquad \text{...(i)}$$

Retardation inside the liquid,

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}} = \frac{Vd_L g - Vdg}{Vd} = \left(\frac{d_L - d}{d}\right) g \quad \text{... (ii)}$$

Time taken to come to rest under this retardation will be

$$t = \frac{v}{a} = \frac{gt_1}{2a} = \frac{gt_1}{2\left(\dfrac{d_L - d}{d}\right)g} = \frac{dt_1}{2\,(d_L - d)}$$

Same will be the time to come back on the liquid surface. Therefore,

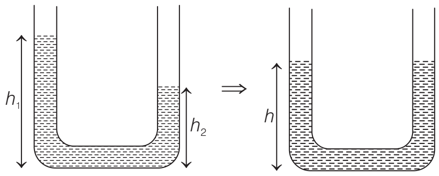
(a) t_2 = time the ball takes to came back to the position from where it was released

$$= t_1 + 2t = t_1 + \frac{dt_1}{d_L - d} = t_1\left[1 + \frac{d}{d_L - d}\right] \text{or } t_2 = \frac{t_1 d_L}{d_L - d}$$

(b) The motion of the ball is periodic but not simple harmonic because the acceleration of the ball is g in air and $\left(\dfrac{d_L - d}{d}\right) g$ inside the liquid which is not proportional to the displacement, which is necessary and sufficient condition for SHM.

(c) When $d_L = d$, retardation or acceleration inside the liquid becomes zero (upthrust = weight). Therefore, the ball will continue to move with constant velocity $v = gt_1/2$ inside the liquid.

26. Let h be the level in equilibrium. Equating the volumes, we have



$$Ah_1 + Ah_2 = 2Ah$$

$$\therefore \qquad h = \left(\frac{h_1 + h_2}{2}\right)$$

Work done by gravity = $U_i - U_f$

$$W = \left(m_1 g\frac{h_1}{2} + m_2 g\frac{h_2}{2}\right) - (m_1 + m_2)g\frac{h}{2}$$

$$= \frac{Ah_1\rho g h_1}{2} + \frac{Ah_2\rho g h_2}{2} - [Ah_1\rho + Ah_2\rho]g\left(\frac{h_1 + h_2}{4}\right)$$

Simplifying this, we get

$$W = \frac{\rho A g}{4}(h_1 - h_2)^2$$

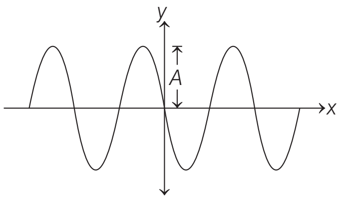
10

Wave Motion

Topic 1 Wave Pulse and Travelling Wave

Objective Questions I (Only one correct option)

1. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure. **(2019 Main, 12 April I)**



For this wave, the phase ϕ is

(a) $-\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

2. A submarine A travelling at 18 km/h is being chased along the line of its velocity by another submarine B travelling at 27 km/h. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to (Speed of sound in water = $1500\,\text{ms}^{-1}$) **(2019 Main, 12 April I)**

(a) 504 Hz (b) 507 Hz (c) 499 Hz (d) 502 Hz

3. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to **(2019 Main, 10 Jan I)**

(a) 16.6 cm (b) 33.3 cm (c) 10.0 cm (d) 20.0 cm

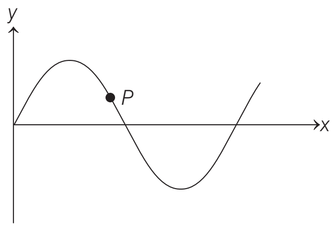
4. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m(m << M)$. When the car is at rest, the speed of transverse waves in the string is $60\,\text{ms}^{-1}$. When the car has acceleration a, the wave speed increases to $60.5\,\text{ms}^{-1}$. The value of a, in terms of gravitational acceleration g is closest to **(2019 Main, 9 Jan I)**

(a) $\frac{g}{20}$ (b) $\frac{g}{5}$ (c) $\frac{g}{30}$ (d) $\frac{g}{10}$

5. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (Take, $g = 10\,\text{ms}^{-2}$) **(2016 Main)**

(a) $2\pi\sqrt{2}$ s (b) 2 s
(c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s

6. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is **(2008)**



(a) $\frac{\sqrt{3}\pi}{50}\hat{\mathbf{j}}$ m/s (b) $-\frac{\sqrt{3}\pi}{50}\hat{\mathbf{j}}$ m/s (c) $\frac{\sqrt{3}\pi}{50}\hat{\mathbf{i}}$ m/s (d) $-\frac{\sqrt{3}\pi}{50}\hat{\mathbf{i}}$ m/s

7. A travelling wave in a stretched string is described by the equation ; $y = A \sin(kx - \omega t)$

The maximum particle velocity is **(1997, 1M)**

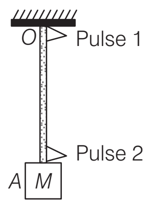
(a) $A\omega$ (b) ω/k (c) $d\omega/dk$ (d) x/ω

8. A transverse wave is described by the equation $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity if **(1984, 2M)**

(a) $\lambda = \pi y_0/4$ (b) $\lambda = \pi y_0/2$
(c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$Objective

Questions II (One or more correct option)

9. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(a) The time $T_{AO} = T_{OA}$ **(2017 Adv.)**
(b) The wavelength of Pulse 1 becomes longer when it reaches point A
(c) The velocity of any pulse along the rope is independent of its frequency and wavelength
(d) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the mid-point of rope

10. $Y(x, t) = \dfrac{0.8}{[(4x + 5t)^2 + 5]}$ represents a moving pulse where x and y are in metre and t is in second. Then, **(1999, 3M)**

(a) pulse is moving in positive x-direction
(b) in 2 s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a symmetric pulse

11. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10\,\text{m/s}$, then λ and f are given by **(1998, 2M)**

(a) $\lambda = 2\pi \times 10^{-2}$ m
(b) $\lambda = 10^{-3}$ m
(c) $f = \dfrac{10^3}{2\pi}$ Hz
(d) $f = 10^4$ Hz

12. A wave is represented by the equation ;

$$y = A \sin (10\,\pi x + 15\pi t + \pi/3)$$

where, x is in metre and t is in second. The expression represents **(1990, 2M)**

(a) a wave travelling in the positive x-direction with a velocity 1.5 m/s
(b) a wave travelling in the negative x-direction with a velocity 1.5 m/s
(c) a wave travelling in the negative x-direction with a wavelength 0.2 m
(d) a wave travelling in the positive x-direction with a wavelength 0.2 m

13. A wave equation which gives the displacement along the y-direction is given by : $y = 10^{-4} \sin (60\,t + 2\,x)$ where, x and y are in metre and t is time in second. This represents a wave **(1981, 3M)**

(a) travelling with a velocity of 30 m/s in the negative x-direction
(b) of wavelength (π) m
(c) of frequency $\left(\dfrac{30}{\pi}\right)$ Hz
(d) of amplitude 10^{-4} m

Fill in the Blanks

14. The amplitude of a wave disturbance travelling in the positive x-direction is given by $y = \dfrac{1}{(1 + x)^2}$ at time $t = 0$ and by $y = \dfrac{1}{[1 + (x - 1)^2]}$ at $t = 2$ s, where x and y are in metre. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is …… m/s. **(1990, 2M)**

15. A travelling wave has the frequency ν and the particle displacement amplitude A. For the wave the particle velocity amplitude is …… and the particle acceleration amplitude is …… . **(1983, 2M)**

Analytical & Descriptive Question

16. A harmonically moving transverse wave on a string has a maximum particle velocity and acceleration of 3m/s and 90 m/s^2 respectively. Velocity of the wave is 20 m/s. Find the waveform. **(2005, 2M)**

Topic 2 Standing Waves, Stretched Wire and Organ Pipes

Objective Questions I (Only one correct option)

1. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column $l_1 = 30$ cm and $l_2 = 70$ cm.

Then, v is equal to **(Main 2019, 12 April II)**

(a) 332 ms^{-1} (b) 384 ms^{-1} (c) 338 ms^{-1} (d) 379 ms^{-1}

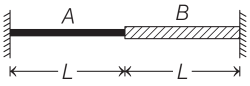
2. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is **(Main 2019, 9 April II)**

(a) 180 m/s, 80 Hz (b) 320 m/s, 80 Hz
(c) 320 m/s, 120 Hz (d) 180 m/s, 120 Hz

3. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is (All quantities are in SI units) **(Main 2019, 9 April I)**

(a) 60 m (b) 40 m (c) 80 m (d) 20 m

4. A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q, then the ratio $p : q$ is **(2019 Main, 8 April I)**



(a) 3 : 5 (b) 4 : 9 (c) 1 : 2 (d) 1 : 4

5. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark. near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment is close to **(Main 2019, 12 Jan II)**

(a) $328\ \text{ms}^{-1}$ (b) $341\ \text{ms}^{-1}$ (c) $322\ \text{ms}^{-1}$ (d) $335\ \text{ms}^{-1}$

6. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$, where distance and time are measured in SI units. The tension in the string is **(Main 2019, 11 Jan I)**

(a) 5 N (b) 12.5 N (c) 7.5 N (d) 10 N

7. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz) **(Main 2019, 10 Jan II)**

(a) 7 (b) 4 (c) 5 (d) 6

8. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to **(2019 Main, 10 Jan I)**

(a) 16.6 cm (b) 33.3 cm (c) 10.0 cm (d) 20.0 cm

9. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3\ \text{kg/m}^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **(2018 Main)**

(a) 7.5 kHz (b) 5 kHz (c) 2.5 kHz (d) 10 kHz

10. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water, so that half of it is in water. The fundamental frequency of the air column is now **(2016 Main)**

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

11. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **(2014 Main)**

(a) 12 (b) 8 (c) 6 (d) 4

12. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3\ \text{kg/m}^3$ and $2.2 \times 10^{11}\ \text{N/m}^2$ respectively? **(2013 Main)**

(a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

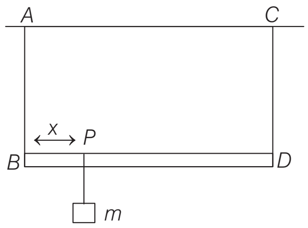
13. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38° C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is **(2012)**

(a) 14.0 cm (b) 15.2 cm (c) 16.4 cm (d) 17.6 cm

14. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is $320\ \text{ms}^{-1}$, the mass of the string is

(a) 5 g (b) 10 g (c) 20 g (d) 40 g **(2010)**

15. A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass m is suspended from point P such that BP is equal to x. If the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is **(2006)**



(a) $l/5$ (b) $l/4$ (c) $4l/5$ (d) $3l/4$

16. A tuning fork of 512 Hz is used to produce resonance in a resonance tube experiment. The level of water at first resonance is 30.7 cm and at second resonance is 63.2 cm. The error in calculating velocity of sound is **(2005, 2M)**

(a) 204.1 cm/s (b) 110 cm/s
(c) 58 cm/s (d) 280 cm/s

17. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option. **(2005, 2M)**

(a) $n = 3,\ f_2 = (3/4)\ f_1$ (b) $n = 3,\ f_2 = (5/4)\ f_1$
(c) $n = 5,\ f_2 = \frac{5}{4} f_1$ (d) $n = 5,\ f_2 = \frac{3}{4} f_1$

18. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open organ pipe is **(2004, 2M)**

(a) $\frac{L}{3}$ (b) $\frac{4L}{3}$ (c) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$ (d) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

19. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M. The wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **(2002, 2M)**

(a) 25 kg (b) 5 kg (c) 12.5 kg (d) 1/25 kg

20. Two vibrating strings of the same material but of lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes. The one of length L with frequency ν_1 and the other with frequency ν_2. The ratio ν_1/ν_2 is given by **(2000, 2M)**
(a) 2 (b) 4 (c) 8 (d) 1

21. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz then the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is **(1996, 2M)**
(a) 200 Hz (b) 300 Hz (c) 240 Hz (d) 480 Hz

22. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water, so that one half of its volume is submerged. The new fundamental frequency (in Hz) is **(1995, 2M)**
(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$
(c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$

23. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 and P_2 is **(1988, 2M)**
(a) 8/3 (b) 3/8 (c) 1/6 (d) 1/3

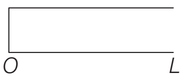
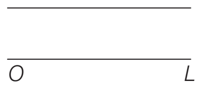
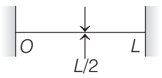
24. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superimposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is **(1988, 1M)**
(a) $a\sin(kx + \omega t)$ (b) $-a\cos(kx - \omega t)$
(c) $-a\cos(kx + \omega t)$ (d) $-a\sin(kx - \omega t)$

25. A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is opened at both ends the fundamental frequency that can be excited is (in Hz) **(1986, 2M)**
(a) 1024 (b) 512 (c) 256 (d) 128

26. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of its length is in water. The fundamental frequency of the air column is now **(1981, 2M)**
(a) $f/2$ (b) $3f/4$ (c) f (d) $2f$

Match the Columns

27. **Column I** shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves. **(2011)**

	Column I		Column II
(A)	Pipe closed at one end 	(p)	Longitudinal waves
(B)	Pipe open at both ends 	(q)	Transverse waves
(C)	Stretched wire clamped at both ends 	(r)	$\lambda_f = L$
(D)	Stretched wire clamped at both ends and at mid-point 	(s)	$\lambda_f = 2L$
		(t)	$\lambda_f = 4L$

Objective Questions II (One or more correct option)

28. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true?
(a) The speed of sound determined from this experiment is 332 m s^{-1}.
(b) The end correction in this experiment is 0.9 cm.
(c) The wavelength of the sound wave is 66.4 cm.
(d) The resonance at 50.7 cm corresponds to the fundamental harmonic.

29. One end of a taut string of length 3 m along the X-axis is fixed at $x = 0$. The speed of the waves in the string is $100\,\mathrm{ms^{-1}}$. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary wave is (are) **(2014 Adv.)**
(a) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$
(b) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$
(c) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$
(d) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

30. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\,\mathrm{m})\,[\sin(62.8\,\mathrm{m^{-1}})x]\cos[(628\,\mathrm{s^{-1}})t]$.
Assuming $\pi = 3.14$, the correct statement(s) is (are) **(2012)**

(a) the number of nodes is 5
(b) the length of the string is 0.25 m
(c) the maximum displacement of the mid-point of the string from its equilibrium position is 0.01 m
(d) the fundamental frequency is 100 Hz

31. Standing waves can be produced **(1999, 3M)**
(a) on a string clamped at both ends
(b) on a string clamped at one end and free at the other
(c) when incident wave gets reflected from a wall
(d) when two identical waves with a phase difference of π are moving in the same direction

32. The (x, y) coordinates of the corners of a square plate are $(0,0),(L,0,),(L,L)$ and $(0,L)$. The edges of the plate are clamped and transverse standing waves are set-up in it. If $u(x,y)$ denotes the displacement of the plate at the point (x,y) at some instant of time, the possible expression (s) for u is (are) (a = positive constant) **(1998, 2M)**
(a) $a\cos(\pi x/2L)\cos(\pi y/2L)$ (b) $a\sin(\pi x/L)\sin(\pi y/L)$
(c) $a\sin(\pi x/L)\sin(2\pi y/L)$ (d) $a\cos(2\pi x/L)\sin(\pi y/L)$

33. A wave disturbance in a medium is described by $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$, where x and y are in metre and t is in second. **(1995, 2M)**
(a) A node occurs at $x = 0.15$ m
(b) An antinode occurs at $x = 0.3$ m
(c) The speed of wave is 5 ms^{-1}
(d) The wavelength of wave is 0.2 m

34. Velocity of sound in air is 320m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency **(1989, 2M)**
(a) 80 Hz (b) 240 Hz (c) 320 Hz (d) 400 Hz

35. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz, if the length of the column in cm is (Speed of sound in air = 330 m/s) **(1985, 2M)**
(a) 31.25 (b) 62.50 (c) 93.75 (d) 125

Fill in the Blanks

36. A cylinder resonance tube open at both ends has fundamental frequency f in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is …… . **(1992, 2M)**

37. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become …… Hz. **(1987, 2M)**

38. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is …… m. Speed of sound = 330 m/s. **(1984, 2M)**

Integer Answer Type Questions

39. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is **(2010)**

40. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibration using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string. **(2009)**

Analytical & Descriptive Questions

41. A string of mass per unit length μ is clamped at both ends such that one end of the string is at $x = 0$ and the other is at $x = l$. When string vibrates in fundamental mode amplitude of the mid-point O of the string is a, and tension in the string is T. Find the total oscillation energy stored in the string. **(2003, 4M)**

42. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **(2002, 5M)**
(a) If the frequency to the second harmonic of pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.
(b) Now the open end of the pipe B is closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

43. A 3.6 m long pipe resonates with a source of frequency 212.5 Hz when water level is at certain heights in the pipe. Find the heights of water level (from the bottom of the pipe) at which resonances occur. Neglect end correction . Now the pipe is filled to a height H ($\approx$ 3.6m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively. Calculate the time interval between the occurrence of first two resonances. Speed of sound in air is 340 m/s and $g = 10\,\text{m/s}^2$. **(2000, 10M)**

44. The air column in a pipe closed at one end is made to vibrate in its second overtone by tuning fork of frequency 440 Hz. The speed of sound in air is 330 m/s. End corrections may be neglected. Let p_0 denote the mean pressure at any point in the pipe and Δp_0 the maximum amplitude of pressure variation. **(1998, 8M)**
(a) Find the length L of the air column.
(b) What is the amplitude of pressure variation at the middle of the column?
(c) What are the maximum and minimum pressures at the open end of the pipe?
(d) What are the maximum and minimum pressures at the closed end of the pipe?

45. A metallic rod of length 1 m is rigidly clamped at its mid-point. Longitudinal stationary waves are set-up in the rod in such a way that there are two nodes on either side of the mid-point. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the mid-point and those of the constituent waves in the rod. (Young's modulus of the material of the rod $= 2 \times 10^{11}\,\text{Nm}^{-2}$; density $= 8000\,\text{kg m}^{-3}$) **(1994, 6M)**

46. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$$

where, x and y are in cm and t is in second. **(1985,6M)**

(a) What is the maximum displacement of a point at $x = 5\,\text{cm}$?

(b) Where are the nodes located along the string?

(c) What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s?

(d) Write down the equations of the component waves whose superposition gives the above wave.

Topic 3 Wave Speed

Objective Questions I (Only one correct option)

1. The pressure wave $p = 0.01\sin[1000t - 3x]\,\text{Nm}^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is $0°$C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be $336\,\text{ms}^{-1}$. Approximate value of T is **(2019 Main, 9 April I)**

(a) $15°$C (b) $11°$C
(c) $12°$C (d) $4°$C

2. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin(50t + 2x)$, where x and y are in metre and t is in second. Which of the following is a correct statement about the wave? **(2019 Main, 12 Jan I)**

(a) The wave is propagating along the negative X-axis with speed $25\,\text{ms}^{-1}$.

(b) The wave is propagating along the positive X-axis with speed $25\,\text{ms}^{-1}$.

(c) The wave is propagating along the positive X-axis with speed $100\,\text{ms}^{-1}$.

(d) The wave is propagating along the negative X-axis with speed $100\,\text{ms}^{-1}$.

3. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light $= 3 \times 10^8\,\text{ms}^{-1}$) **(2017 Main)**

(a) 12.1 GHz (b) 17.3 GHz
(c) 15.3 GHz (d) 10.1 GHz

4. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer who is standing in air is **(2004, 2M)**

(a) 200 Hz (b) 3000 Hz
(c) 120 Hz (d) 600 Hz

5. Two monoatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to the gas 2 is given by **(2000, 2M)**

(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

6. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is **(1999, 2M)**

(a) $\sqrt{(2/7)}$ (b) $\sqrt{(1/7)}$ (c) $(\sqrt{3})/5$ (d) $(\sqrt{6})/5$

True / False

7. The ratio of the velocity of sound in hydrogen gas $\left(\gamma = \frac{7}{5}\right)$ to that in helium gas $\left(\gamma = \frac{5}{3}\right)$ at the same temperature is $\sqrt{21/5}$. **(1983, 2M)**

Analytical & Descriptive Questions

8. In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound in air at room temperature. **(2003, 2M)**

9. A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2 kg. The wire PQR is under a tension of 80 N. A sinusoidal wave pulse of amplitude 3.5 cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave pulse. Calculate **(1999, 10M)**

(a) the time taken by the wave pulse to reach the other end R and

(b) the amplitude of the reflected and transmitted wave pulse after the incident wave pulse crosses the joint Q.

10. A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area $10^{-6}\,m^2$ is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If transverse waves are set-up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. **(1984, 6M)**

Given : $Y_{steel} = 2 \times 10^{11}\,N/m^2$,

$\alpha_{steel} = 1.21 \times 10^{-5}/°C.$

11. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ? **(1984, 6M)**

12. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C.

Given, Young modulus of copper $= 1.3 \times 10^{11}\ N/m^2$.

Coefficient of linear expansion of copper $= 1.7 \times 10^{-5}\ °C^{-1}$.

Density of copper $= 9 \times 10^3\ kg/m^3$. **(1979, 4M)**

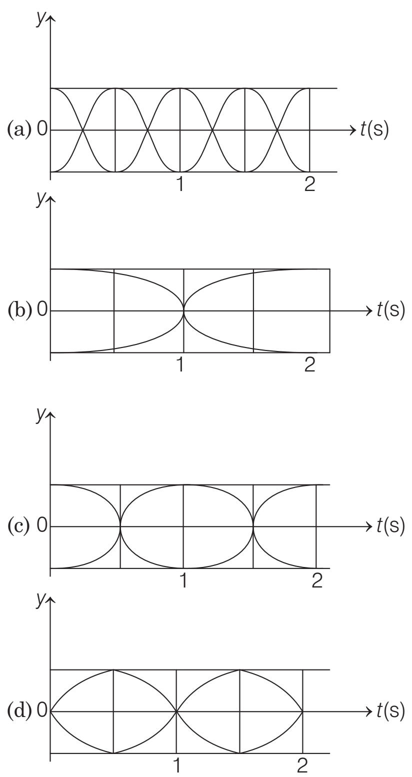
Topic 4 Beats and Doppler Effect

Objective Questions I (Only one correct option)

1. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equal to **(2019 Main, 12 April II)**

(a) 5.5 m/s (b) 15.0 m/s (c) 2.5 m/s (d) 10.0 m/s

2. The correct figure that shows schematically, the wave pattern produced by superposition of two waves of frequencies 9Hz and 11 Hz, is **(2019 Main, 10 April II)**



3. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take, velocity of sound in air is 350 m/s) **(2019 Main, 10 April II)**

(a) 807 Hz (b) 1143 Hz (c) 750 Hz (d) 857 Hz

4. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are in ms^{-1},

(Take, speed of sound = 300 m/s) **(2019 Main, 10 April I)**

(a) 12, 16 (b) 12, 18 (c) 16, 14 (d) 8, 18

5. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of $20\ ms^{-1}$ with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B ?

(speed of sound in air = $340\ ms^{-1}$) **(2019 Main, 9 April II)**

(a) 2060 Hz (b) 2250 Hz (c) 2300 Hz (d) 2150 Hz

6. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is **(2019 Main, 10 Jan I)**

(a) $\frac{19}{18}$ (b) $\frac{21}{20}$ (c) $\frac{20}{19}$ (d) $\frac{18}{17}$

7. A musician produce the sound of second harmonics from open end flute of 50 cm. The other person moves toward the musician with speed 10 km/h from the second end of room. If the speed of sound 330 m/s, the frequency heard by running person will be **(2019 Main, 9 Jan II)**

(a) 666 Hz (b) 500 Hz (c) 753 Hz (d) 333 Hz

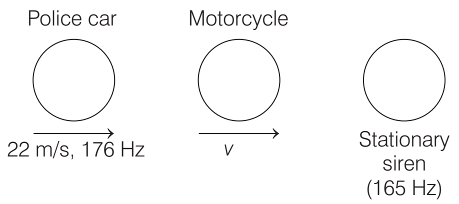
8. A train is moving on a straight track with speed $20\,ms^{-1}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is close to (speed of sound = $320\,ms^{-1}$) **(2015 Main)**

(a) 12% (b) 6% (c) 18% (d) 24%

9. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is
(a) 8.50 kHz (b) 8.25 kHz **(2011)**
(c) 7.75 kHz (d) 7.50 kHz

10. A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats/s when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is **(2008, 3M)**
(a) 344 (b) 336
(c) 117.3 (d) 109.3

11. A police car moving at 22 m/s chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that the motorcyclist does not observe any beats (speed of sound = 330 m/s). **(2003, 2M)**



(a) 33 m/s (b) 22 m/s (c) Zero (d) 11 m/s

12. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz, while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is
(a) 242/252 (b) 2 **(2002, 2M)**
(c) 5/6 (d) 11/6

13. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1/f_2 is **(2000, 2M)**
(a) 18/19 (b) 1/2 (c) 2 (d) 19/18

14. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer (in Hz) is (Speed of sound = 330 m/s) **(1997, 1M)**
(a) 409 (b) 429 (c) 517 (d) 500

Objective Questions II (One or more correct option)

15. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is v. The correct statement(s) is (are)
(a) If the wind blows from the observer to the source, $f_2 > f_1$ **(2013 Adv.)**
(b) If the wind blows from the source to the observer, $f_2 > f_1$
(c) If the wind blows from the observer to the source, $f_2 < f_1$
(d) If the wind blows from the source to the observer, $f_2 < f_1$

16. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c **(1995, 2M)**
(a) The number of waves striking the surface per second is $f\dfrac{(c+v)}{c}$
(b) The wavelength of reflected wave is $\dfrac{c\,(c-v)}{f\,(c+v)}$
(c) The frequency of the reflected wave is $f\dfrac{(c+v)}{(c-v)}$
(d) The number of beats heard by a stationary listener to the left of the reflecting surface is $\dfrac{vf}{c-v}$

17. Two identical straight wires are stretched so as to produce 6 beats/s when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension **(1991, 2M)**
(a) T_2 was decreased (b) T_2 was increased
(c) T_1 was decreased (d) T_1 was increased

Numerical Value

18. Two men are walking along a horizontal straight line in the same direction. The main in front walks at a speed $1.0\,\text{ms}^{-1}$ and the man behind walks at a speed $2.0\,\text{ms}^{-1}$. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air $330\,\text{ms}^{-1}$. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is .... **(2018 Adv.)**

Fill in the Blank

19. A bus is moving towards a huge wall with a velocity of $5\ \text{ms}^{-1}$. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be …… Hz. (Speed of sound in air = $342\ \text{ms}^{-1}$) **(1992, 2M)**

True / False

20. A source of sound with frequency 256 Hz is moving with a velocity v towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall he will hear beats. **(1985, 3M)**

Integer Answer Type Questions

21. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of $2\,\text{ms}^{-1}$. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is $330\ \text{ms}^{-1}$ and the car reflects the sound at the frequency it has received). **(2017 Adv.)**

22. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles $0, \frac{\pi}{3}, \frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is **(2015 Adv.)**

Analytical & Descriptive Questions

23. An observer standing on a railway crossing receives frequency of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train. **(2005, 2M)**
(The speed of the sound in air is 300 m/s.)

24. A boat is travelling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible. **(2001, 10M)**

(a) What will be the frequency detected by a receiver kept inside the river downstream?

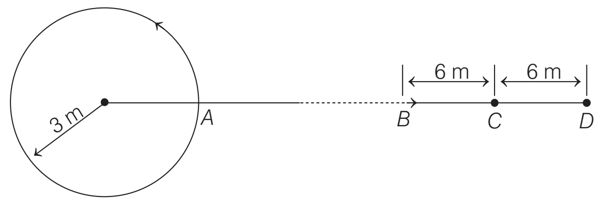
(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite to the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water $= 20°$ C ; Density of river water $= 10^3\ \text{kg/m}^3$;
Bulk modulus of the water $= 2.088 \times 10^9$ Pa ;
Gas constant $R = 8.31$ J/mol-K ;
Mean molecular mass of air $= 28.8 \times 10^{-3}$ kg / mol ;
C_p/C_V for air = 1.4)

25. A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound. Obtain an expression for the beat frequency heard by the motorist. **(1997, 5M)**

26. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad/s in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle. (Speed of sound = 330 m/s). **(1996, 3M)**

27. A source of sound is moving along a circular path of radius 3 m with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD (see figure) with an amplitude $BC = CD = 6$ m. The frequency of oscillation of the detector is $5/\pi$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. (Speed of sound = 340 m/s) **(1990, 7M)**



28. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. Speed of sound = 340 m/s. **(1986, 8M)**

29. A sonometer wire under tension of 64 N vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and mass of 1 g. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per second. Calculate the speed with which the tuning fork is moved, if the speed of sound in air is 300 m/s. **(1983, 6M)**

30. A string 25 cm long and having a mass of 2.5 g is under tension . A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats/s are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s find the tension in the string. **(1982, 7M)**

31. A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5 m/ s. How many beats per second will be heard by the observer on source itself if sound travels at a speed of 330 m/s ? **(1981, 4M)**

Topic 5 Miscellaneous Problems

Objective Questions I (Only one correct option)

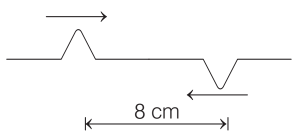
1. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Take, reference intensity of sound as 10^{-12} W/m^2 **(2019 Main, 12 April II)]**

(a) 40 cm (b) 20 cm (c) 10 cm (d) 30 cm

2. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction. **(2003, 2M)**

(a) 0.012 m (b) 0.025 m (c) 0.05 m (d) 0.024 m

3. Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s the total energy of the pulses will be **(2001, 2M)**



(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential

4. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment the displacement of the wire is $y_1 = A\sin\left(\frac{\pi x}{L}\right)\sin\omega t$ and energy is E_1 and in other experiment its displacement is $y_2 = A\sin\left(\frac{2\pi x}{L}\right)\sin 2\omega t$ and energy is E_2. Then **(2001, 1M)**

(a) $E_2 = E_1$ (b) $E_2 = 2E_1$
(c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

5. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt, which allows constructive interference between successive pulses, is **(1998, 2M)**

(a) 0.05 s (b) 0.10 s (c) 0.20 s (d) 0. 40 s

6. The extension in a string, obeying Hooke's law, is x. The speed of transverse wave in the stretched string is v. If the extension in the string is increased to 1.5 x, the speed of transverse wave will be **(1996, 2M)**

(a) 1.22 v (b) 0.61 v (c) 1.50 v (d) 0.75 v

7. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$

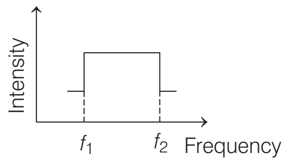
This expression may be considered to be a result of the superposition of .... independent harmonic motions. **(1992, 2M)**

(a) two (b) three
(c) four (d) five

Passage Based Questions

Passage 1

Two trains A and B are moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.

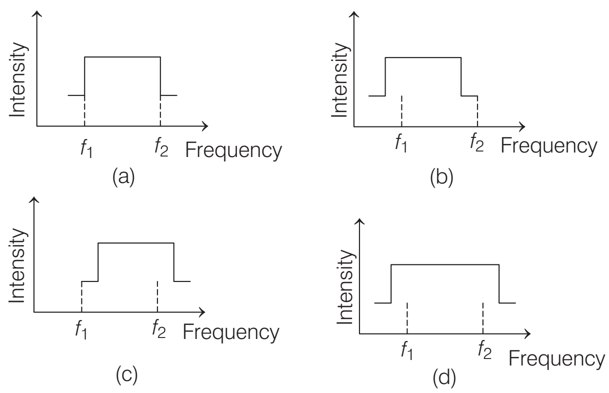


Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency–lowest frequency) is thus 320 Hz. The speed of sound in air is 340 m/s.

8. The speed of sound of the whistle is **(2007, 4M)**

(a) 340 m/s for passengers in A and 310 m/s for passengers in B
(b) 360 m/s for passengers in A and 310 m/s for passengers in B
(c) 310 m/s for passengers in A and 360 m/s for passengers in B
(d) 340 m/s for passengers in both the trains

9. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by **(2007, 4M)**



10. The spread of frequency as observed by the passengers in train B is **(2007, 4M)**

(a) 310 Hz (b) 330 Hz
(c) 350 Hz (d) 290 Hz

Passage 2

Two plane harmonic sound waves are expressed by the equations.

$$y_1(x,t) = A\cos(0.5\pi x - 100\pi t)$$
$$y_2(x,t) = A\cos(0.46\pi x - 92\pi t)$$

(All parameters are in MKS)

11. How many times does an observer hear maximum intensity in one second ? **(2006, 5M)**
(a) 4 (b) 10 (c) 6 (d) 8

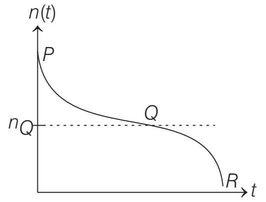
12. What is the speed of the sound ? **(2006, 5M)**
(a) 200 m/s (b) 180 m/s (c) 192 m/s (d) 96 m/s

13. At $x = 0$ how many times the amplitude of $y_1 + y_2$ is zero in one second ? **(2006, 5M)**
(a) 192 (b) 48 (c) 100 (d) 96

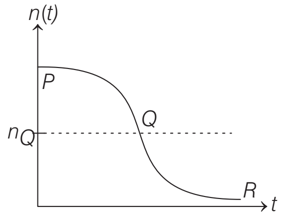
Objective Questions II (One or more correct option)

14. Two loudspeakers M and N are located 20m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car in initially at a point P, 1800 m away from the mid-point Q of the line MN and moves towards Q constantly at 60 km/h along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is (are) true regarding the sound heard by the person? **(2016 Adv.)**

(a) The plot below represents schematically the variation of beat frequency with time



(b) The rate of change in beat frequency is maximum when the car passes through Q
(c) $\nu_P + \nu_R = 2\nu_Q$
(d) The plot below represents schematically the variations of beat frequency with time



15. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is **(2014 Adv.)**

(**Useful information** : $\sqrt{167RT} = 640\ J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mole^{-1/2}$. The molar masses M in grams are given in the options. Take the value of $\sqrt{10/M}$ for each gas as given there.)
(a) Neon ($M = 20, \sqrt{10/20} = 7/10$)
(b) Nitrogen ($M = 28, \sqrt{10/28} = 3/5$)
(c) Oxygen ($M = 32, \sqrt{10/32} = 9/16$)
(d) Argon ($M = 36, \sqrt{10/36} = 17/32$)

16. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe, **(2012)**
(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

17. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then, **(2009)**
(a) the intensity of the sound heard at the first resonance was more than that at the second resonance.
(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube.
(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm.
(d) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air.

18. In a wave motion $y = a\sin(kx - \omega t)$, y can represent **(1999, 3M)**
(a) electric field (b) magnetic field
(c) displacement (d) pressure

19. As a wave propagates **(1999, 3M)**
(a) the wave intensity remains constant for a plane wave
(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times

20. The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion is (are) **(1987, 2M)**
(a) $\cos kx \sin \omega t$ (b) $k^2x^2 - \omega^2t^2$
(c) $\cos^2(kx + \omega t)$ (d) $\cos(k^2x^2 - \omega^2t^2)$

Fill in the Blank

21. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates along the negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6 m apart along the line of propagation is …… and the corresponding amplitude difference is …… m. **(1997, 1M)**

True / False

22. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming Snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. **(1984, 2M)**

Integer Answer Type Question

23. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. **(2010)**

Analytical & Descriptive Questions

24. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3$ Hz. A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2A^2$. **(1993, 4M)**

(a) Find the time interval between successive maxima of the intensity of the signal received by the detector.

(b) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

25. The displacement of the medium in a sound wave is given by the equation $y_i = A \cos (ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated a $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. **(1991, 8M)**

(a) What are the wavelength and frequency of incident wave ?

(b) Write the equation for the reflected wave.

(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

(d) Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave? What is the direction of propagation of travelling wave ?

26. A train approaching a hill at a speed of 40 km/h sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/h is blowing in the direction of motion of the train . Find **(1988, 5M)**

(a) the frequency of the whistle as heard by an observer on the hill,

(b) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.

(Velocity of sound in air = 1200 km/h)

27. The following equations represent transverse waves;

$$z_1 = A \cos (kx - \omega t);\ z_2 = A \cos (kx + \omega t);$$
$$z_3 = A \cos (ky - \omega t)$$

Identify the combination (s) of the waves which will produce (a) standing wave (s), (b) a wave travelling in the direction making an angle of 45 degrees with the positive X and positive Y-axes. In each case, find the position at which the resultant intensity is always zero. **(1987, 7M)**

Answers

Topic 1

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)
5. (c)	**6.** (a)	**7.** (a)	**8.** (b)
9. (a, c, d)	**10.** (b, d)	**11.** (a, c)	**12.** (b, c)
13. (a,b,c,d)	**14.** (*)	**15.** $2\pi\nu A$, $4\pi^2\nu^2 A$	

16. $y = (0.1 \text{ m}) \sin [(30 \text{ rad/s})t \pm (1.5 \text{ m}^{-1}) x + \phi]$

Topic 2

1. (b)	**2.** (b)	**3.** (c)	**4.** (c)
5. (a)	**6.** (b)	**7.** (d)	**8.** (d)
9. (b)	**10.** (d)	**11.** (c)	**12.** (b)
13. (b)	**14.** (b)	**15.** (a)	**16.** (d)
17. (c)	**18.** (c)	**19.** (a)	**20.** (d)
21. (a)	**22.** (a)	**23.** (c)	**24.** (c)
25. (a)	**26.** (c)		

27. A → p, t; B → p, s; C → q, s; D → q, r

28. (a,c)	**29.** (a, c, d)	**30.** (b, c)	**31.** (a, b, c)
32. (b, c)	**33.** (a, b, c, d)	**34.** (a, b, d)	**35.** (a, c)
36. f	**37.** 240	**38.** 0.125	**39.** (5)

40. (5) **41.** $\dfrac{\pi^2 a^2 T}{4l}$ **42.** (a) $\dfrac{400}{189}$ (b) $\dfrac{3}{4}$

43. 3.2 m, 2.4 m, 1.6 m, 0.8 m, $-\dfrac{dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$, 43 s

44. (a) $\dfrac{15}{16}$ m (b) $\pm \dfrac{\Delta p_0}{\sqrt{2}}$ (c) equal to mean pressure

(d) $p_0 + \Delta p_0$, $p_0 - \Delta p_0$

45. $y = 2 \times 10^{-6} \sin (0.1\pi) \sin (25000\pi t)$,

$y_1 = 10^{-6} \sin (25000\pi t - 5\pi x)$, $y_2 = 10^{-6} \sin (25000\pi t + 5\pi x)$

46. (a) $2\sqrt{3}$ cm (b) $x = 0$, 15 cm, 30 cm… etc

(c) Zero (d) $y_1 = 2 \sin \left(\dfrac{\pi x}{15} - 96\pi t\right)$

and $y_2 = 2 \sin \left(\dfrac{\pi x}{15} + 96\pi t\right)$

Topic 3

1. (d)	**2.** (a)	**3.** (b)	**4.** (d)
5. (b)	**6.** (c)	**7.** F	**8.** 336 m/s

9. (a) 0.14 s (b) $A_r = 1.5$ cm and $A_t = 2.0$ cm

10. 11 Hz **11.** 0.12 m **12.** 70.1 m/s

Topic 4

1. (c) **2.** (a) **3.** (c) **4.** (b)
5. (b) **6.** (a) **7.** (a) **8.** (a)
9. (a) **10.** (a) **11.** (b) **12.** (b)
13. (d) **14.** (d) **15.** (a, b) **16.** (a, b, c)
17. (b, c) **18.** 5 **19.** 6 **20.** F
21. 6 **22.** 3 **23.** $v_T = 30\,\text{m/s}$

24. (a) 1.0069×10^5 Hz (b) 1.0304×10^5 Hz

25. $\dfrac{2v_b(v + v_m)}{(v^2 - v_b^2)}$ **26.** 403.3 Hz, 484 Hz

27. 438.7 Hz to 257.3 Hz **28.** 1.5 m/s

29. 0.75 m/s **30.** 27.04 N **31.** 7.87 Hz

Topic 5

1. (a) **2.** (b) **3.** (b) **4.** (c) **5.** (b)
6. (a) **7.** (b) **8.** (b) **9.** (a)
10. (a) **11.** (a) **12.** (a) **13.** (c)
14. (b, c, d) **15.** (d) **16.** (a, b, d) **17.** (a, d)
18. (a, b, c, d) **19.** (a, c, d) **20.** (a, c) **21.** π, zero
22. T **23.** 7

24. (a) 6.28×10^{-3} s (b) 1.57×10^{-3} s

25. (a) $\dfrac{2\pi}{a}, \dfrac{b}{2\pi}$ (b) $y_r = -0.8\,A\cos(ax - bt)$ (c) $1.8\,Ab$, zero

(d) $y = -1.6\,A\sin ax \sin bt + 0.2A\cos(ax + bt)$. Antinodes are at $x = \left[n + \dfrac{(-1)^n}{2}\right]\dfrac{\pi}{a}$. Travelling wave is propagating in negative x-direction.

26. (a) 599.33 Hz (b) 0.935 km, 621.43 Hz

27. (a) z_1 and z_2; $x = (2n + 1)\dfrac{\pi}{2k}$ where $n = 0, \pm 1, \pm 2 \ldots$ etc. (b) z_1 and z_3, $x - y = (2n + 1)\dfrac{\pi}{k}$ where $n = 0, \pm 1, \pm 2 \ldots$ etc.

Hints & Solutions

Topic 1 Wave Pulse and Travelling Wave

1. From the given snapshot at $t = 0$,

$$y = 0 \text{ at } x = 0$$

and $y = -$ ve when x increases from zero.

Standard expression of any progressive wave is given by $y = A\sin(kx - \omega t + \phi)$

Here, ϕ is the phase difference, we need to get

at $t = 0$ $y = A\sin(kx + \phi)$

Clearly $\phi = \pi$, so that

$$y = A\sin(kx + \pi)$$
$$y = -A\sin(kx)$$

$\Rightarrow$ $y = 0$ at $x = 0$

and $y = -$ve at $x > 0$

2. Given, velocity of submarine (A),

$$v_A = 18\,\text{km/h} = \frac{18000}{3600}\,\text{m/s}$$

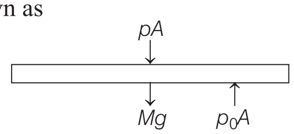
or $v_A = 5\,\text{m/s}$ ...(i)

and velocity of submarine (B),

$$v_B = 27\,\text{km/h} = \frac{27000}{3600}\,\text{m/s}$$

or $v_B = 7.5\,\text{m/s}$ ...(ii)

Signal sent by submarine (B) is detected by submarine (A) can be shown as



Frequency of the signal, $f_o = 500$ Hz

So, in this relative motion, frequency received by submarine (A) is

$$f_1 = \left(\frac{v_S - v_A}{v_S - v_B}\right) f_o = \left(\frac{1500 - 5}{1500 - 7.5}\right) 500\,\text{Hz}$$

$$\Rightarrow \quad f_1 = \frac{1495}{1492.5} \times 500\,\text{Hz}$$

The reflected frequency f_1 is now received back by submarine (B).

So, frequency received at submarine (B) is

$$f_2 = \left(\frac{v_S + v_B}{v_S + v_A}\right) f_1 = \left(\frac{1500 + 7.5}{1500 + 5}\right)\left(\frac{1495}{1492.5}\right) 500\,\text{Hz}$$

$$\Rightarrow \quad f_2 = \left(\frac{1507.5}{1505}\right)\left(\frac{1495}{1492.5}\right) 500\,\text{Hz}$$

$$\Rightarrow \quad f_2 = 1.00166 \times 1.00167 \times 500$$

$$\Rightarrow \quad f_2 = 501.67\,\text{Hz} \approx 502\,\text{Hz}$$

3. Velocity 'v' of the wave on the string $= \sqrt{\dfrac{T}{\mu}}$

where, T = tension and μ = mass per unit length. Substituting the given values, we get

$$v = \sqrt{\frac{8}{5} \times 1000} = 40\,\text{ms}^{-1}$$

Wavelength of the wave on the string,

$$\lambda = \frac{v}{f}$$

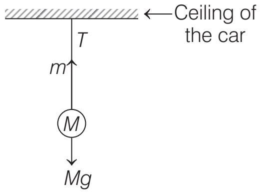
where, f = frequency of wave.

$$\Rightarrow \qquad \lambda = \frac{40}{100}\text{ m} = 40\text{ cm}$$

$\therefore$ Separation between two successive nodes is,

$$d = \frac{\lambda}{2} = \frac{40}{2} = 20.0\text{ cm}$$

4. When the car is at rest, then the situation can be shown in the figure below.



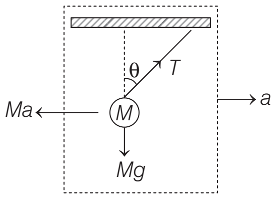
Since, $m << M$, then we can neglect the mass of the string.

So, the initial velocity of the wave in the string can be given as,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\frac{M}{l}}} = 60\text{ m/s} \qquad \text{...(i)}$$

where, T is the tension in the string and μ is the mass of the block per unit length (l).

Now, when the car has acceleration 'a', then the situation can be shown in the figure given below,



Resolving the components of 'T' along X-and Y-axis, we get

$$T\cos\theta = Mg \qquad \text{...(ii)}$$

$$T\sin\theta = Ma \qquad \text{...(iii)}$$

Squaring both sides of Eqs. (ii) and (iii) and adding them, we get

$$T^2(\sin^2\theta + \cos^2\theta) = M^2(g^2 + a^2)$$

$$\Rightarrow \qquad T = M(g^2 + a^2)^{\frac{1}{2}} \qquad \text{...(iv)}$$

Now, the velocity of the wave in the string would be given as,

$$v' = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M(g^2 + a^2)^{\frac{1}{2}}}{\frac{M}{l}}} = 60.5\text{ m/s} \qquad \text{...(v)}$$

Dividing Eq. (i) and Eq. (v), we get

$$\frac{v}{v'} = \frac{\sqrt{\frac{Mg}{\frac{M}{l}}}}{\sqrt{\frac{M(g^2+a^2)^{\frac{1}{2}}}{\frac{M}{l}}}} = \sqrt{\frac{g}{(g^2 + a^2)^{\frac{1}{2}}}} = \frac{60}{60.5}$$

$$\text{or} \qquad \frac{60.5}{60} = \sqrt{\frac{(g^2 + a^2)^{\frac{1}{2}}}{g}}$$

Squaring both the sides, we get

$$\frac{(60.5)^2}{(60)^2} = \frac{(g^2 + a^2)^{\frac{1}{2}}}{g}$$

$$\text{or} \qquad \left(1 + \frac{0.5}{60}\right)^2 = \sqrt{1 + \left(\frac{a}{g}\right)^2}$$

Using binomial expansion,

$$(1 + x)^n = 1 + \frac{nx}{1!} + \dots$$

On both sides, we get

$$1 + 2 \times \frac{0.5}{60} = 1 + \frac{1}{2}\cdot\frac{a^2}{g^2}$$

$$\Rightarrow 1 + \frac{1}{60} = 1 + \frac{1}{2}\cdot\frac{a^2}{g^2} \quad \text{or} \quad \frac{a}{g} = \frac{1}{\sqrt{30}}$$

$$\text{or} \qquad a = \frac{g}{\sqrt{30}} = \frac{g}{5.4} \simeq \frac{g}{5}$$

5. At distance x from the bottom

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(\frac{mgx}{L}\right)}{\left(\frac{m}{L}\right)}} = \sqrt{gx}$$

$$\therefore \qquad \frac{dx}{dt} = \sqrt{x}\sqrt{g}$$

$$\Rightarrow \qquad \int_0^L x^{-1/2}dx = \sqrt{g}\int_0^t dt$$

$$\Rightarrow \qquad \left[\frac{x^{1/2}}{(1/2)}\Bigg|_0^L\right] = \sqrt{g}\cdot t$$

$$\Rightarrow \qquad t = \frac{2\sqrt{L}}{\sqrt{g}}$$

$$\Rightarrow \qquad t = 2\sqrt{\frac{20}{10}} = 2\sqrt{2}\text{s}$$

6. Particle velocity $v_p = -v$ (slope of y-x graph)

Here, $v = +$ ve, as the wave is travelling in positive x-direction.

Slope at P is negative.

$\therefore$ Velocity of particle is in positive y (or $\hat{\mathbf{j}}$) direction.

$\therefore$ Correct option is (a).

7. This is an equation of a travelling wave in which particles of the medium are in SHM and maximum particle velocity in SHM is $A\omega$, where A is the amplitude and ω the angular velocity.

8. Wave velocity $v = \dfrac{\text{coefficient of } t}{\text{coefficient of } x} = \dfrac{2\pi f}{2\pi/\lambda} = \lambda f$

Maximum particle velocity $v_{pm} = \omega A = 2\pi f y_0$

Given, $\qquad v_{pm} = 4v$

or $\quad 2\pi f\, y_0 = 4\lambda\, f$

$\therefore \quad \lambda = \dfrac{\pi y_0}{2}$

9. $v = \sqrt{\dfrac{T}{\mu}}$, so speed at any position will be same for both pulses, therefore time taken by both pulses will be same.

$$\lambda f = v$$

$$\Rightarrow \quad \lambda = \frac{v}{f}$$

$$\Rightarrow \quad \lambda \propto v \propto T$$

since when pulse 1 reaches at A tension and hence speed decreases therefore λ decreases.

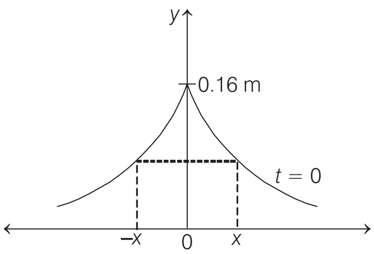
NOTE

If we refer velocity by magnitude only, then option (a, c, d) will be correct, else only (a, c) will be correct.

10. The shape of pulse at $x = 0$ and $t = 0$ would be as shown, in Fig. (a).

$$y(0,0) = \frac{0.8}{5} = 0.16\text{ m}$$

From the figure it is clear that $y_{\max} = 0.16\text{ m}$



(a)

Pulse will be symmetric (Symmetry is checked about $y_{\max}$) if at $t = 0$

$$y(x) = y(-x)$$

From the given equation,

and $\left.\begin{aligned} y(x) &= \dfrac{0.8}{16x^2 + 5} \\ y(-x) &= \dfrac{0.8}{16x^2 + 5} \end{aligned}\right\}$ at $t = 0$

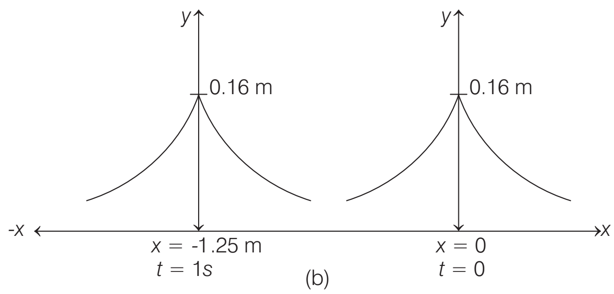
or $\quad y(x) = y(-x)$

Therefore, pulse is symmetric.

Speed of pulse

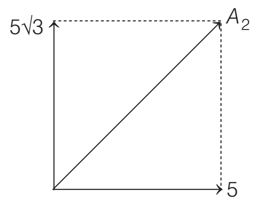
At $t = 1\text{ s}$ and $x = -1.25\text{ m}$

value of y is again 0.16 m, i.e. pulse has travelled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 m/s and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from Fig. (b).



(b)

11. Maximum speed of any point on the string $= a\omega = a(2\pi f)$



$\therefore \quad = \dfrac{v}{10} = \dfrac{10}{10} = 1 \qquad$ (Given, $v = 10$ m/s)

$\therefore \quad 2\pi a f = 1$

$\Rightarrow \quad f = \dfrac{1}{2\pi a}$

$a = 10^{-3}\text{ m} \qquad$ (Given)

$\therefore \quad f = \dfrac{1}{2\pi \times 10^{-3}} = \dfrac{10^3}{2\pi}$ Hz

Speed of wave $v = f\lambda$

$\therefore \quad (10\text{ m/s}) = \left(\dfrac{10^3}{2\pi}\text{s}^{-1}\right)\lambda$

$\Rightarrow \therefore \quad \lambda = 2\pi \times 10^{-2}\text{ m}$

12. $\omega = 15\pi$, $k = 10\pi$

Speed of wave, $\quad v = \dfrac{\omega}{k} = 1.5\text{ m/s}$

Wavelength of wave $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{10\pi} = 0.2\text{ m}$

$10\pi x$ and $15\pi t$ have the same sign. Therefore, wave is travelling in negative x-direction.

$\therefore$ Correct options are (b) and (c).

13. $y = 10^{-4}\sin(60t + 2x)$

$A = 10^{-4}\text{ m}$, $\omega = 60\text{ rad/s}$, $k = 2\text{m}^{-1}$

Speed of wave, $\quad v = \dfrac{\omega}{k} = 30\text{ m/s}$

Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{30}{\pi}$ Hz.

Wavelength $\lambda = \dfrac{2\pi}{k} = \pi\text{ m}$

Further, 60 t and $2x$ are of same sign. Therefore, the wave should travel in negative x-direction.

$\therefore$ All the options are correct.

14. In the question, it is given that the shape of the wave disturbance does not change, while in the opinion of author this is not true.

From the function, $y = \frac{1}{(1+x)^2}$, we can see that, at $x = -1$, $y = \infty$.

Whereas from the second function, $y = \frac{1}{[1+(x-1)^2]}$ we don't get any point where $y = \infty$.

So, how can we say that shape has not changed.

15. (a) Particle velocity amplitude means maximum speed

$$= \omega A = 2\pi \nu A$$

(b) Particle acceleration amplitude

$$= \omega^2 A = 4\pi^2\nu^2 A$$

16. Maximum particle velocity,

$$\omega A = 3\text{ m/s} \quad \ldots\text{(i)}$$

Maximum particle acceleration,

$$\omega^2 A = 90 \text{ m/s}^2 \quad \ldots\text{(ii)}$$

Velocity of wave, $\frac{\omega}{k} = 20\text{ m/s}$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\omega = 30\text{ rad/s} \Rightarrow A = 0.1\text{m} \text{ and } k = 1.5\text{ m}^{-1}$$

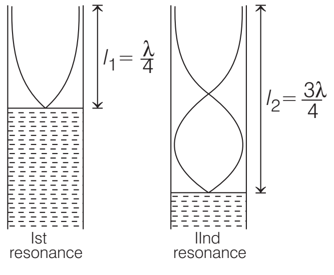
$\therefore$ Equation of waveform should be

$$y = A\sin(\omega t + kx + \phi)$$

$$y = (0.1\text{ m})\sin[(30\text{ rad/s})t \pm (1.5\text{m}^{-1})x + \phi]$$

Topic 2 Standing Waves, Stretched Wire and Organ Pipes

1. In a resonance tube apparatus, first and second resonance occur as shown



As in a stationary wave, distance between two successive nodes is $\frac{\lambda}{2}$ and distance of a node and an antinode is $\frac{\lambda}{4}$.

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$$

So, speed of sound, $v = f\lambda = f \times 2(l_2 - l_1)$

$$= 480 \times 2 \times (70 - 30) \times 10^{-2} = 384\text{ ms}^{-1}$$

2. Frequency of vibration of a string in nth harmonic is given by

$$f_n = n \cdot \frac{v}{2l} \quad \ldots\text{(i)}$$

where, v = speed of sound and l = length of string.

Here, $f_3 = 240$ Hz, $l = 2$ m and $n = 3$

Substituting these values in Eq. (i), we get

$$\therefore \quad 240 = 3 \times \frac{v}{2 \times 2} \Rightarrow v = \frac{4 \times 240}{3} = 320\text{ ms}^{-1}$$

Also, fundamental frequency is

$$f = \frac{f_n}{n} = \frac{f_3}{3} = \frac{240}{3} = 80\text{ Hz}$$

3. Given equation of stationary wave is

$$Y = 0.3\sin(0.157x)\cos(200\pi t)$$

Comparing it with general equation of stationary wave, i.e. $Y = a\sin kx\cos\omega t$, we get

$$k = \left(\frac{2\pi}{\lambda}\right) = 0.157$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{0.157} = 4\pi^2 \quad \left(\because \frac{1}{2\pi} \approx 0.157\right) \ldots\text{(i)}$$

$$\text{and} \quad \omega = 200\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{20}\text{ s}$$

As the possible wavelength associated with nth harmonic of a vibrating string, i.e. fixed at both ends is given as

$$\lambda = \frac{2l}{n} \quad \text{or} \quad l = n\left(\frac{\lambda}{2}\right)$$

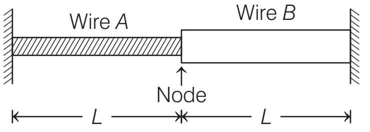
Now, according to question, string is fixed from both ends and oscillates in 4th harmonic, so

$$4\left(\frac{\lambda}{2}\right) = l \Rightarrow 2\lambda = l$$

or $l = 2 \times 4\pi^2 = 8\pi^2$ [using Eq. (i)]

Now, $\pi^2 \approx 10 \Rightarrow l \approx 80$ m

4.



Let mass per unit length of wires are μ_A and μ_B, respectively.

$\because$ For same material, density is also same.

So, $\mu_A = \frac{\rho\pi r^2 L}{L} = \mu$ and $\mu_B = \frac{\rho 4\pi r^2 L}{L} = 4\mu$

Tension (T) in both connected wires are same.

So, speed of wave in wires are

$$v_A = \sqrt{\frac{T}{\mu_A}} = \sqrt{\frac{T}{\mu}} [\because \mu_A = \mu \text{ and } \mu_B = 4\mu]$$

$$\text{and} \quad v_B = \sqrt{\frac{T}{\mu_B}} = \sqrt{\frac{T}{4\mu}}$$

So, nth harmonic in such wires system is

$$f_{n\text{th}} = \frac{pv}{2L}$$

$$\Rightarrow f_A = \frac{pv_A}{2L} = \frac{p}{2L}\sqrt{\frac{T}{\mu}} \quad \text{(for } p \text{ antinodes)}$$

Similarly, $f_B = \dfrac{qv_B}{2L} = \dfrac{q}{2L}\sqrt{\dfrac{T}{4\mu}} = \dfrac{1}{2}\left(\dfrac{q}{2L}\sqrt{\dfrac{T}{\mu}}\right)$

(for q antinodes)

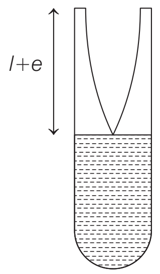
As frequencies f_A and f_B are given equal.

So, $f_A = f_B \Rightarrow \dfrac{p}{2L}\sqrt{\dfrac{T}{\mu}} = \dfrac{q}{2}\left[\dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}\right]$

$$\frac{p}{q} = \frac{1}{2} \Rightarrow p : q = 1 : 2$$

5. **Key idea** To overcome the error occured in measurement of resonant length.We introduce end correction factor e in length.

In first resonance, length of air coloumn $= \dfrac{\lambda}{4}$.



So, $l_1 + e = \dfrac{\lambda}{4}$ or $11 \times 4 + 4e = \lambda$

So, speed of sound is

$\Rightarrow \quad v = f_1\lambda = 512(44 + 4e) \quad \ldots\text{(i)}$

And in second case,

$$l_1' + e = \frac{\lambda'}{4} \text{ or } 27 \times 4 + 4e = \lambda'$$

$\Rightarrow \quad v = f_2\lambda' = 256\,(108 + 4e) \quad \ldots\text{(ii)}$

Dividing both Eqs. (i) and (ii), we get

$$1 = \frac{512\,(44 + 4e)}{256\,(108 + 4e)} \Rightarrow e = 5\,\text{cm}$$

Substituting value of e in Eq. (i), we get

Speed of sound $v = 512\,(44 + 4e)$

$= 512\,(44 + 4 \times 5)$

$= 512 \times 64\ \text{cm s}^{-1} = 327.68\ \text{ms}^{-1} \approx 328\ \text{ms}^{-1}$

6. Given, equation can be rewritten as,

$$y = 0.03 \sin 450\left(t - \frac{9x}{450}\right) \quad \ldots\text{(i)}$$

We know that the general equation of a travelling wave is given as,

$$y = A \sin \omega\,(t - x/v) \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\text{velocity, } v = \frac{450}{9} = 50\ \text{m/s}$$

and angular velocity, $\omega = 450$ rad/s

As, the velocity of wave on stretched string with tension (T) is given as $v = \sqrt{T/\mu}$

where, μ is linear density

$\therefore \quad T = \mu v^2 = 5 \times 10^{-3} \times 50 \times 50 = 12.5\,\text{N}$

($\because$ given, $\mu = 5\,\text{g/m} = 5 \times 10^{-3}\,\text{kg/m}$)

7. Fundamental frequency of closed organ pipe is given by $f_0 = v/4L$, where v is the velocity of sound in it and L is the length of the pipe.

Also, overtone frequencies are given by

$$f = (2n + 1)\frac{v}{4L} \quad \text{or} \quad f = (2n + 1)\,f_0$$

Given that, $f_0 = 1500\,\text{Hz}$ and $f_{\max} = 20000\,\text{Hz}$

This means, $f_{\max} > f$

So, $\quad f_{\max} > (2n + 1)\,f_0$

$\Rightarrow \quad 20000 > (2n + 1)\,1500$

$\Rightarrow \quad 2n + 1 < 13.33 \Rightarrow 2n < 13.33 - 1$

$\Rightarrow \quad 2n < 12.33 \quad \text{or} \quad n < 6.16$

or $n = 6$ (integer number)

Hence, total six overtones will be heard.

8. Velocity 'v' of the wave on the string $= \sqrt{\dfrac{T}{\mu}}$

where, T = tension and μ = mass per unit length.
Substituting the given values, we get

$$v = \sqrt{\frac{8}{5} \times 1000} = 40\ \text{ms}^{-1}$$

Wavelength of the wave on the string, $\lambda = \dfrac{v}{f}$

where, $\quad f$ = frequency of wave.

$$\Rightarrow \quad \lambda = \frac{40}{100}\,\text{m} = 40\,\text{cm}$$

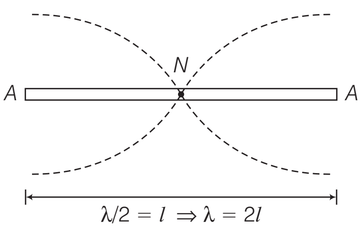
$\therefore$ Separation between two successive nodes is,

$$d = \frac{\lambda}{2} = \frac{40}{2} = 20.0\,\text{cm}.$$

9. Wave velocity $(v) = \sqrt{\dfrac{Y}{\rho}} = 5.86 \times 10^3$ m/s

For fundamental mode, $\lambda = 2l = 1.2$ m

$\therefore \quad$ Fundamental frequency $= \dfrac{v}{\lambda} = 4.88$ kHz ≈ 5 kHz



10. Fundamental frequency of open pipe.

$$f = \frac{v}{2l}$$

Now, after half filled with water it becomes a closed pipe of length $\frac{l}{2}$.

Fundamental frequency of this closed pipe,

$$f' = \frac{v}{4(l/2)} = \frac{v}{2l} = f$$

11. For closed organ pipe $= \frac{(2n+1)v}{4l}$ $[n = 0, 1, 2 ......]$

$$\frac{(2n+1)v}{4l} < 1250$$

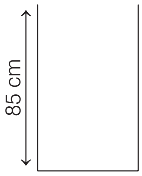
$$(2n+1) < 1250 \times \frac{4 \times 0.85}{340}$$

$$(2n+1) < 12.5 \; 2n < 11.50$$

$$n < 5.25$$

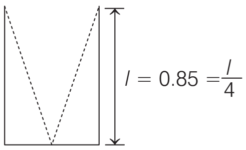
So, $n = 0, 1, 2, 3, ..., 5$

So, we have 6 possibilities.



Alternate method

In closed organ pipe, fundamental node



i.e. $\frac{\lambda}{4} = 0.85 \Rightarrow \lambda = 4 \times 0.85$

As we know, $\nu = \frac{c}{\lambda} \Rightarrow \frac{340}{4 \times 0.85} = 100\,\text{Hz}$

$\therefore$ Possible frequencies = 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz below 1250 Hz.

12. Fundamental frequency of sonometer wire

$$f = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{Ad}}$$

Here, μ = mass per unit length of wire.

Also, Young's modulus of elasticity $Y = \frac{Tl}{A\Delta l}$

$$\Rightarrow \frac{T}{A} = \frac{Y\Delta l}{l} \Rightarrow f = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{ld}} \Rightarrow l = 1.5\text{ m}, \frac{\Delta l}{l} = 0.01$$

$$d = 7.7 \times 10^3\ \text{kg/m}^3 \Rightarrow Y = 2.2 \times 10^{11}\ \text{N/m}^2$$

After substituting the values we get,

$$f \approx 178.2\text{ Hz}$$

13. With end correction,

$$f = n\left[\frac{v}{4(l+e)}\right], \quad (\text{where, } n = 1, 3, \ldots)$$

$$= n\left[\frac{v}{4(l+0.6r)}\right]$$

Because, $e = 0.6\,r$, where r is radius of pipe.

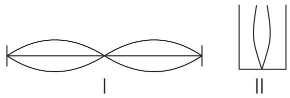
For first resonance, $n = 1$

$$\therefore \quad f = \frac{v}{4(l+0.6r)}$$

or $$l = \frac{v}{4f} - 0.6r = \left[\left(\frac{336 \times 100}{4 \times 512}\right) - 0.6 \times 2\right]\text{cm}$$

$$= 15.2\text{ cm}$$

14.



Given, 2nd harmonic of I = Fundamental of II

$$\therefore \quad 2\left(\frac{v_1}{2l_1}\right) = \frac{v_2}{4l_2} \Rightarrow \frac{T/\mu}{l_1} = \frac{v_2}{4l_2}$$

$$\Rightarrow \quad \mu = \frac{16\,T\,l_2^2}{v_2^2\,l_1^2} = \frac{16 \times 50 \times (0.8)^2}{(320)^2 \times (0.5)^2}$$

$$= 0.02\text{ kg/m}$$

$$\therefore \quad m_1 = \mu\, l_1 = (0.02)(0.2) = 0.01\text{ kg} = 10\text{ g}$$

15. $f \propto v \propto \sqrt{T} \Rightarrow f_{AB} = 2f_{CD}$

$\therefore$ $T_{AB} = 4T_{CD}$ ...(i)

Further $\Sigma\tau_p = 0$

$\therefore$ $T_{AB}(x) = T_{CD}(l - x)$

or $4x = l - x$ (as $T_{AB} = 4T_{CD}$)

or $x = l/5$

16. The question is incomplete, as speed of sound is not given. Let us assume speed of sound as 330 m/s. Then, method will be as under.

$$\frac{\lambda}{2} = (63.2 - 30.7)\,\text{cm} \quad \text{or} \quad \lambda = 0.65\,\text{m}$$

$\therefore$ Speed of sound observed,

$$v_0 = f\lambda = 512 \times 0.65 = 332.8\,\text{m/s}$$

$\therefore$ Error in calculating velocity of sound = 2.8 m/s

$$= 280\,\text{cm/s}$$

17. $f_1 = \frac{v}{l}$ (2nd harmonic of open pipe)

$f_2 = n\left(\frac{v}{4l}\right)$ (nth harmonic of closed pipe)

Here, n is odd and $f_2 > f_1$. It is possible when $n = 5$ because with $n = 5$

$$f_2 = \frac{5}{4}\left(\frac{v}{l}\right) = \frac{5}{4}f_1$$

18. $f_c = f_o$ (both first overtone)

or $$3\left(\frac{v_c}{4L}\right) = 2\left(\frac{v_o}{2l_o}\right)$$

$\therefore$ $$l_o = \frac{4}{3}\left(\frac{v_o}{v_c}\right)L = \frac{4}{3}\sqrt{\frac{\rho_1}{\rho_2}}L \quad \text{as } v \propto \frac{1}{\sqrt{\rho}}$$

19. Let $f_0 =$ frequency of tuning fork

Then, $f_0 = \frac{5}{2l}\sqrt{\frac{9g}{\mu}}$ (μ = mass per unit length of wire)

$= \frac{3}{2l}\sqrt{\frac{Mg}{\mu}}$

Solving this, we get $M = 25$ kg

In the first case, frequency corresponds to fifth harmonic while in the second case it corresponds to third harmonic.

20. Fundamental frequency is given by

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \quad \text{(with both the ends fixed)}$$

$\therefore$ Fundamental frequency

$$\nu \propto \frac{1}{l\sqrt{\mu}} \quad \text{(for same tension in both strings)}$$

where, $\mu =$ mass per unit length of wire

$= \rho . A$ (ρ = density)

$= \rho(\pi r^2)$ or $\sqrt{\mu} \propto r$

$$\therefore \quad \nu \propto \frac{1}{rl}$$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{l_2}{l_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$$

21. Length of the organ pipe is same in both the cases.

Fundamental frequency of open pipe is $f_1 = \frac{v}{2l}$ and

Frequency of third harmonic of closed pipe will be

$$f_2 = 3\left(\frac{v}{4l}\right)$$

Given that, $f_2 = f_1 + 100$

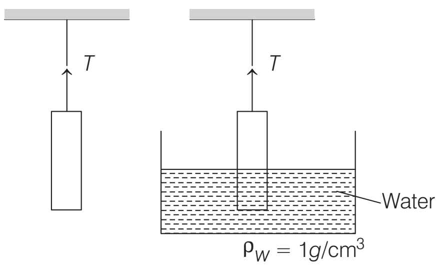
or $f_2 - f_1 = 100$ or $\frac{3}{4}\left(\frac{v}{l}\right) - \left(\frac{1}{2}\right)\left(\frac{v}{l}\right) = 100$

$\Rightarrow \frac{v}{4l} = 100$ Hz $\therefore \frac{v}{2l}$ or $f_1 = 200$ Hz

Therefore, fundamental frequency of the open pipe is 200 Hz.

22. The diagramatic representation of the given problem is shown in figure. The expression of fundamental frequency is

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$



In air $T = mg = (V\rho)g$

$$\therefore \quad \nu = \frac{1}{2l}\sqrt{\frac{A\rho g}{\mu}} \quad \text{...(i)}$$

When the object is half immersed in water

$$T' = mg - \text{upthrust} = V\rho g - \left(\frac{V}{2}\right)\rho_w g$$

$$= \left(\frac{V}{2}\right) g\,(2\rho - \rho_w)$$

The new fundamental frequency is

$$\nu' = \frac{1}{2l} \times \sqrt{\frac{T'}{\mu}} = \frac{1}{2l}\sqrt{\frac{(Vg/2)(2\rho - \rho_w)}{\mu}} \quad \text{...(ii)}$$

$$\therefore \quad \frac{\nu'}{\nu} = \left(\sqrt{\frac{2\rho - \rho_w}{2\rho}}\right)$$

$$\text{or} \quad \nu' = \nu\left(\frac{2\rho - \rho_w}{2\rho}\right)^{1/2}$$

$$= 300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{Hz}$$

23. First harmonic of closed = Third harmonic of open

$$\therefore \quad \frac{v}{4l_1} = 3\left(\frac{v}{2l_2}\right) \Rightarrow \therefore \quad \frac{l_1}{l_2} = \frac{1}{6}$$

24. For a stationary wave to form, two identical waves should travel in opposite direction. Further at $x = 0$, resultant y (from both the waves) should be zero at all instants.

25. $f_c = \frac{v}{4l} = 512$ Hz

$f_o = \frac{v}{2l} = 2f_c = 1024$ Hz

26. Initially the tube was open at both ends and then it is closed.

$$f_o = \frac{v}{2l_o} \text{ and } f_c = \frac{v}{4l_c}$$

Since, tube is half dipped in water, $l_c = \frac{l_o}{2}$

$$\therefore \quad f_c = \frac{v}{4\left(\frac{l_o}{2}\right)} = \frac{v}{2l_o} = f_o = f$$

27. In organ pipes, longitudinal waves are formed.

In string, transverse waves are formed. Open end of pipe is displacement antinode and closed end is displacement node. In case of string fixed end of a string is node.

Further, least distance between a node and an antinode is $\frac{\lambda}{4}$ and between two nodes is $\frac{\lambda}{2}$. Keeping these points in mind answer to this question is as under ; (A) $\rightarrow$ (p, t); (B) $\rightarrow$ (p, s); (C) $\rightarrow$ (q, s); (D) $\rightarrow$ (q, r)

28. Let nth harmonic is corresponding to 50.7 cm and $(n+1)$th harmonic is corresponding 83.9 cm.

$\therefore$ Their difference is $\frac{\lambda}{2}$.

$$\therefore \qquad \frac{\lambda}{2} = (83.9 - 50.7)\,\text{cm}$$

$$\text{or} \qquad \lambda = 66.4 \text{ cm}$$

$$\therefore \qquad \frac{\lambda}{4} = 16.6\,\text{cm}$$

Length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ and 50.7 cm must be an odd multiple of this length.

$16.6 \times 3 = 49.8$ cm. Therefore, 50.7 is 3rd harmonic.

If end correction is e, then

$$e + 50.7 = \frac{3\lambda}{4}$$

$$e = 49.8 - 50.7 = -0.9\,\text{cm}$$

$\therefore$ Speed of sound, $v = f\lambda$

$$\Rightarrow \qquad v = 500 \times 66.4 \text{ cm/s} = 332 \text{ m/s}$$

29. There should be a node at $x = 0$ and antinode at $x = 3$ m.

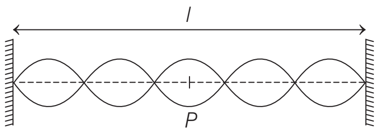
$$\text{Also,} \qquad v = \frac{\omega}{k} = 100\,\text{m/s}.$$

$$\therefore \qquad y = 0 \text{ at } x = 0$$

$$\text{and} \qquad y = \pm A \text{ at } x = 3\,\text{m}.$$

Only (a), (c) and (d) satisfy the condition.

30.



Number of nodes = 6

From the given equation, we can see that

$$k = \frac{2\pi}{\lambda} = 62.8 \text{ m}^{-1}$$

$$\therefore \qquad \lambda = \frac{2\pi}{62.8}\,\text{m} = 0.1 \text{ m}$$

$$l = \frac{5\lambda}{2} = 0.25 \text{ m}$$

The mid-point of the string is P, an antinode

$\therefore$ maximum displacement $= 0.01$ m

$$\omega = 2\pi f = 628 \text{ s}^{-1}$$

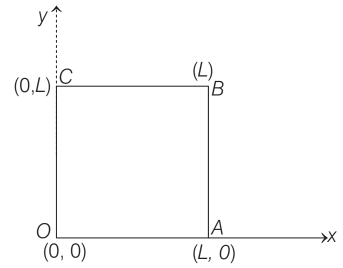
$$\therefore \qquad f = \frac{628}{2\pi} = 100\,\text{Hz}$$

But this is fifth harmonic frequency.

$\therefore$ Fundamental frequency $f_0 = \frac{f}{5} = 20$ Hz

31. Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.

32. Since, the edges are clamped, displacement of the edges $u(x, y) = 0$ for



Line,	OA i.e. $y = 0$		$0 \le x \le L$
	AB i.e. $x = L$		$0 \le y \le L$
	BC i.e. $y = L$		$0 \le x \le L$
	OC i.e. $x = 0$		$0 \le y \le L$

The above conditions are satisfied only in alternatives (b) and (c).

Note that $u(x, y) = 0$, for all four values *eg,* in alternative (d), $u(x, y) = 0$ for $y = 0$, $y = L$ but it is not zero for $x = 0$ or $x = L$. Similarly, in option (a) $u(x, y) = 0$ at $x = L$, $y = L$ but it is not zero for $x = 0$ or $y = 0$ while in options (b) and (c), $u(x, y) = 0$ for $x = 0$, $y = 0$ $x = L$ and $y = L$.

33. It is given that $y(x, t) = 0.02\cos(50\pi t + \pi/2)\cos(10\pi x)$

$$\cong A\cos(\omega t + \pi/2)\cos kx$$

Node occurs when $kx = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

$$10\pi x = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = 0.05\,\text{m}, 0.15\,\text{m} \qquad \text{option (a)}$$

Antinode occurs when $kx = \pi, 2\pi, 3\pi$ etc.

$$10\pi x = \pi, 2\pi, 3\pi \text{ etc.}$$

$$\Rightarrow \qquad x = 0.1\,\text{m}, 0.2\,\text{m}, 0.3\,\text{m} \qquad \text{option (b)}$$

Speed of the wave is given by,

$$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ m/s} \qquad \text{option (c)}$$

Wavelength is given by,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \left(\frac{1}{5}\right)\text{m} = 0.2\,\text{m} \qquad \text{option (d)}$$

34. For closed pipe, $f = n\left(\frac{v}{4l}\right)$ $n = 1, 3, 5\ldots$

$$\text{For } n = 1, \qquad f_1 = \frac{v}{4l} = \frac{320}{4 \times 1} = 80\,\text{Hz}$$

$$\text{For } n = 3, \qquad f_3 = 3f_1 = 240\,\text{Hz}$$

$$\text{For } n = 5, \qquad f_5 = 5f_1 = 400\,\text{Hz}$$

$\therefore$ Correct options are (a), (b) and (d).

35. For closed organ pipe,

$$f = n\left(\frac{v}{4l}\right) \text{ where, } n = 1, 3, 5\ldots\ldots$$

$$\therefore \qquad l = \frac{nv}{4f}$$

$$\text{For } n = 1, \qquad l_1 = \frac{(1)(330)}{4 \times 264} \times 100\,\text{cm} = 31.25\,\text{cm}$$

$$\text{For } n = 3, \qquad l_3 = 3l_1 = 93.75\,\text{cm}$$

$$\text{For } n = 5, \qquad l_5 = 5l_1 = 156.25\text{cm}$$

$\therefore$ Correct options are (a) and (c).

36. Fundamental frequency of open pipe $f = \dfrac{v}{2l}$

and fundamental frequency of closed pipe,

$$f' = \frac{v}{4(l/2)} = \frac{v}{2\,l} \quad \text{or} \quad f' = f$$

37. Fundamental frequency $f = \dfrac{v}{2l} = \dfrac{\sqrt{T/\mu}}{2l}$ or $f \propto \sqrt{T}$

$$\therefore \qquad \frac{f'}{f} = \sqrt{\frac{w-F}{w}}$$

Here, w = weight of mass and F = upthrust

$$\therefore \qquad f' = f\sqrt{\frac{w-F}{w}}$$

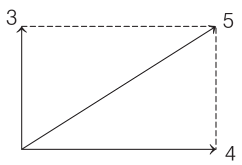
Substituting the values, we have

$$f' = 260\sqrt{\frac{(50.7)g - (0.0075)(10^3)g}{(50.7)g}} = 240\,\text{Hz}$$

38. Wall will be a node (displacement). Therefore, shortest distance from the wall at which air particles have maximum amplitude of vibration (displacement antinode) should be $\lambda/4$. Here, $\lambda = \dfrac{v}{f} = \dfrac{330}{660} = 0.5\,\text{m}$

$\therefore$ Desired distance is $\dfrac{0.5}{4} = 0.125\,\text{m}$

39. Phase difference between the two waves is 90°. Amplitudes are added by vector method.



$$A = \sqrt{3^2 + 4^2}$$

$\therefore$ Answer is 5.

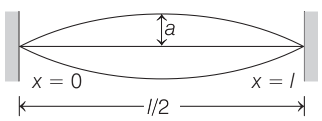
40. Distance between the successive nodes,

$$d = \frac{\lambda}{2} = \frac{v}{2f} = \frac{\sqrt{T/\mu}}{2f}$$

Substituting the values we get, $d = 5\,\text{cm}$

41. $l = \dfrac{\lambda}{2}$ or $\lambda = 2l$, $k = \dfrac{2\pi}{\lambda} = \dfrac{\pi}{l}$

The amplitude at a distance x from



$x = 0$ is given by $A = a \sin kx$

Total mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)\,A^2\,\omega^2$$

$$= \frac{1}{2}(\mu dx)\,(a \sin kx)^2\,(2\pi f)^2$$

or $\qquad dE = 2\pi^2\,\mu f^2\,a^2 \sin^2 kx\,dx \qquad$ ...(i)

Here, $\qquad f^2 = \dfrac{v^2}{\lambda^2} = \dfrac{\left(\dfrac{T}{\mu}\right)}{(4l^2)}$ and $k = \dfrac{\pi}{l}$

Substituting these values in Eq. (i) and integrating it from $x = 0$ to $x = l$, we get total energy of string

$$E = \frac{\pi^2\,a^2 T}{4l}.$$

42. (a) Frequency of second harmonic in pipe A

= frequency of third harmonic in pipe B

$$\therefore \qquad 2\left(\frac{v_A}{2l_A}\right) = 3\left(\frac{v_B}{4l_B}\right)$$

or $\qquad \dfrac{v_A}{v_B} = \dfrac{3}{4}$ or $\dfrac{\sqrt{\dfrac{\gamma_A RT_A}{M_A}}}{\sqrt{\dfrac{\gamma_B RT_B}{M_B}}} = \dfrac{3}{4} \qquad (\text{as } l_A = l_B)$

or $\qquad \sqrt{\dfrac{\gamma_A}{\gamma_B}}\sqrt{\dfrac{M_B}{M_A}} = \dfrac{3}{4}$ (as $T_A = T_B$) $\Rightarrow \dfrac{M_A}{M_B} = \dfrac{\gamma_A}{\gamma_B}\left(\dfrac{16}{9}\right)$

$$= \left(\frac{5/3}{7/5}\right)\left(\frac{16}{9}\right) \qquad \left(\gamma_A = \frac{5}{3} \text{ and } \gamma_B = \frac{7}{5}\right)$$

or $\qquad \dfrac{M_A}{M_B} = \left(\dfrac{25}{21}\right)\left(\dfrac{16}{9}\right) = \dfrac{400}{189}$

(b) Ratio of fundamental frequency in pipe A and in pipe B is

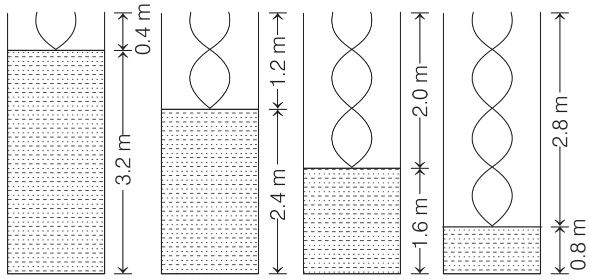
$$\frac{f_A}{f_B} = \frac{v_A/2l_A}{v_B/2l_B} = \frac{v_A}{v_B} \qquad (\text{as } l_A = l_B)$$

$$= \frac{\sqrt{\dfrac{\gamma_A\,RT_A}{M_A}}}{\sqrt{\dfrac{\gamma_B RT_B}{M_B}}} = \sqrt{\frac{\gamma_A}{\gamma_B}\cdot\frac{M_B}{M_A}} \qquad (\text{as } T_A = T_B)$$

Substituting $\dfrac{M_B}{M_A} = \dfrac{189}{400}$ from part (a), we get

$$\frac{f_A}{f_B} = \sqrt{\frac{25}{21} \times \frac{189}{400}} = \frac{3}{4}$$

43. Speed of sound $v = 340\,\text{m/s}$



Let l_0 be the length of air column corresponding to the fundamental frequency. Then,

$$\frac{v}{4l_0}=212.5 \quad\text{or}\quad l_0=\frac{v}{4(212.5)}=\frac{340}{4(212.5)}=0.4\text{ m}$$

In closed pipe only odd harmonics are obtained. Now let l_1, l_2, l_3, l_4, etc., be the lengths corresponding to the 3rd harmonic, 4th harmonic, 7th harmonic etc. Then,

$$3\left(\frac{v}{4l_1}\right)=212.5 \quad\Rightarrow\quad l_1=1.2\text{ m}$$

$$5\left(\frac{v}{4l_2}\right)=212.5 \quad\Rightarrow\quad l_2=2.0\text{ m}$$

and $$7\left(\frac{v}{4l_3}\right)=212.5 \quad\Rightarrow\quad l_3=2.8\text{ m}$$

$$9\left(\frac{v}{4l_4}\right)=212.5 \quad\Rightarrow\quad l_4=3.6\text{ m}$$

or heights of water level are $(3.6-0.4)$ m, $(3.6-1.2)$ m, $(3.6-2.0)$ m and $(3.6-2.8)$ m.

$\therefore$ Heights of water level are 3.2 m, 2.4m, 1.6m and 0.8 m.

Let A and a be the area of cross-sections of the pipe and hole respectively. Then,

$$A=\pi\,(2\times10^{-2})^2=1.26\times10^{-3}\text{ m}^2$$

and $$a=\pi\,(10^{-3})^2=3.14\times10^{-6}\text{ m}^2$$

Velocity of efflux, $v=\sqrt{2gH}$

Continuity equation at 1 and 2 gives

$$a\sqrt{2gH}=A\left(\frac{-dH}{dt}\right)$$

$\therefore$ Rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right)=\frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt}=\frac{3.14\times10^{-6}}{1.26\times10^{-3}}\sqrt{2\times10\times H}$$

or $$-\frac{dH}{dt}=(1.11\times10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$\therefore$ $$\frac{dH}{\sqrt{H}}=-(1.11\times10^{-2})\,dt$$

or $$\int_{3.2}^{2.4}\frac{dH}{\sqrt{H}}=-(1.11\times10^{-2})\int_0^t dt$$

or $2[\sqrt{2.4}-\sqrt{3.2}]=-(1.11\times10^{-2})\cdot t$ or $t\approx 43$ s

NOTE

- Rate of fall of level at a height h is
$$\left(\frac{-dh}{dt}\right)=\frac{a}{A}\sqrt{2gh}\propto\sqrt{H}$$
i.e., rate decreases as the height of water (or any other liquid) decreases in the tank. That is why, the time required to empty the first half of the tank is less than the time required to empty the rest half of the tank.

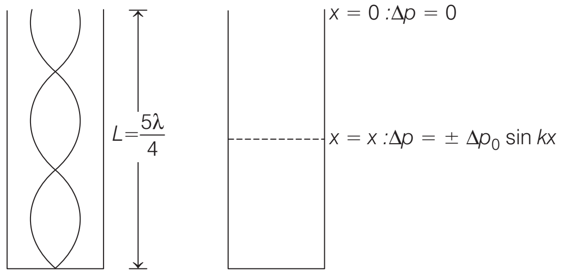
44. (a) Frequency of second overtone of the closed pipe

$$=5\left(\frac{v}{4L}\right)=440\ \Rightarrow\ L=\frac{5v}{4\times440}\text{ m}$$

Substituting v = speed of sound in air = 330 m/s

$$L=\frac{5\times330}{4\times440}=\frac{15}{16}\text{ m}$$

$$\lambda=\frac{4L}{5}=\frac{4\left(\frac{15}{16}\right)}{5}=\frac{3}{4}\text{ m}$$



(b) Open end is displacement antinode. Therefore, it would be a pressure node.

or at $\quad x=0;\ \Delta p=0$

Pressure amplitude at $x=x$, can be written as,

$$\Delta p=\pm\,\Delta p_0\sin kx$$

where, $$k=\frac{2\pi}{\lambda}=\frac{2\pi}{3/4}=\frac{8\pi}{3}\text{ m}^{-1}$$

Therefore, pressure amplitude at $x=\frac{L}{2}=\frac{15/16}{2}$ m or (15/32) m will be

$$\Delta p=\pm\,\Delta p_0\sin\left(\frac{8\pi}{3}\right)\left(\frac{15}{32}\right)$$

$$=\pm\,\Delta p_0\sin\left(\frac{5\pi}{4}\right)$$

$$\Delta p=\pm\frac{\Delta p_0}{\sqrt{2}}$$

(c) Open end is a pressure node i.e. $\Delta p=0$

Hence, $p_{\max}=p_{\min}$ = Mean pressure (p_0)

(d) Closed end is a displacement node or pressure antinode.

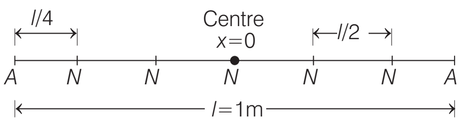
Therefore, $p_{\max}=p_0+\Delta p_0$

and $p_{\min}=p_0-\Delta p_0$

45. Speed of longitudinal travelling wave in the rod will be

$$v=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{2\times10^{11}}{8000}}=5000\text{ m/s}$$

Amplitude at antinode = $2A$ (Here, A is the amplitude of constituent waves)



$$= 2 \times 10^{-6} \text{m}$$

$\therefore$ $$A = 10^{-6} \text{ m} \Rightarrow l = \frac{5\lambda}{2}$$

$\Rightarrow$ $$\lambda = \frac{2l}{5} = \frac{(2)(1.0)}{5} \text{m} = 0.4 \text{ m}$$

Hence, the equation of motion at a distance x from the mid-point will be given by,

$$y = 2A \sin kx \sin \omega t$$

Here, $$k = \frac{2\pi}{0.4} = 5\pi$$

$$\omega = 2\pi f = 2\pi \frac{v}{\lambda}$$

$$= 2\pi \left(\frac{5000}{0.4}\right) \text{rad/s} = 25000\pi$$

$\therefore$ $$y = (2 \times 10^{-6}) \sin (5\pi x) \sin (25000\pi t)$$

Therefore, y at a distance $x = 2 \text{cm} = 2 \times 10^{-2} \text{m}$

is $y = 2 \times 10^{-6} \sin (5\pi \times 2 \times 10^{-2}) \sin (25000\pi t)$

or $y = 2 \times 10^{-6} \sin (0.1\pi) \sin (25000\pi t)$

The equations of constituent waves are

$$y_1 = A \sin (\omega t - kx) \text{ and } y_2 = A \sin (\omega t + kx)$$

or $$y_1 = 10^{-6} \sin (25000\pi t - 5\pi x)$$

and $$y_2 = 10^{-6} \sin (25000\pi t + 5\pi x)$$

46. (a) $y = 4 \sin \frac{\pi x}{15} \cos (96\pi t) = A_x \cos (96\pi t)$

Here, $A_x = 4 \sin \frac{\pi x}{15}$

at $x = 5$ cm, $A_x = 4 \sin \frac{5\pi}{15} = 4 \sin \frac{\pi}{3} = 2\sqrt{3}$ cm

This is the amplitude or maximum displacement at $x = 5$ cm.

(b) Nodes are located where $A_x = 0$

or $\frac{\pi x}{15} = 0, \pi, 2\pi \ldots\ldots$ or $x = 0, 15 \text{ cm}, 30 \text{ cm}$ etc.

(c) Velocity of particle,

$$v_p = \left|\frac{\partial y}{\partial t}\right|_{x = \text{constant}} = -384\pi \sin \left(\frac{\pi x}{15}\right) \sin (96\pi t)$$

At $x = 7.5$ cm and $t = 0.25$ s

$$v_p = -384\pi \sin \left(\frac{\pi}{2}\right) \sin (24\pi) = 0$$

(d) Amplitude of components waves is $A = \frac{4}{2} = 2$ cm

$$\omega = 96\pi \text{ and } k = \frac{\pi}{15}$$

$\therefore$ Component waves are,

$$y_1 = 2 \sin \left(\frac{\pi}{15} x - 96\pi t\right)$$

and $$y_2 = 2 \sin \left(\frac{\pi}{15} x + 96\pi t\right)$$

Topic 3 Wave Speed

1. Given, $p = 0.01 \sin (1000t - 3x) \text{ N/m}^2$

Comparing with the general equation of pressure wave of sound, i.e. $p_0 \sin (\omega t - kx)$,

we get

$$\omega = 1000 \text{ and } k = 3$$

Also, $$k = \frac{\omega}{v} \Rightarrow v = \omega / k$$

$\therefore$ Velocity of sound is $|v_1| = \frac{1000}{3}$

Or

Speed of sound wave can also be calculated as

$$v = -\frac{(\text{coefficient of } t)}{(\text{coefficient of } x)} = -\frac{1000}{(-3)} = \frac{1000}{3} \text{ m/s}$$

Now, relation between velocity of sound and temperature is

$$v = \sqrt{\frac{\gamma RT}{m}} \Rightarrow v \propto \sqrt{T}$$

or $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow T_2 = \frac{v_2^2}{v_1^2} \cdot T_1$$

Here, $v_2 = 336 \text{ m/s}, v_1 = 1000/3 \text{ m/s},$

$T_1 = 0°\text{C} = 273 \text{ K}$

$\therefore$ $$T_2 = \frac{(336)^2}{(1000/3)^2} \times 273 = 277.38 \text{ K}$$

$\therefore$ $T_2 = 4.38°\text{C} \simeq 4°\text{C}$

2. Wave equation is given by,

$$y = 10^{-3} \sin(50t + 2x)$$

Speed of wave is obtained by differentiating phase of wave.

Now, phase of wave from given equation is

$$\phi = 50t + 2x = \text{constant}$$

Differentiating 'ϕ' w.r.t 't', we get

$$\frac{d}{dx}(50t + 2x) = \frac{d}{dt}(\text{constant})$$

$\Rightarrow$ $$50 + 2\left(\frac{dx}{dt}\right) = 0$$

$\Rightarrow$ $$\frac{dx}{dt} = \frac{-50}{2} = -25 \text{ ms}^{-1}$$

So, wave is propagating in negative x–direction with a speed of 25 ms^{-1}.

Alternate Method

The general equation of a wave travelling in negative x direction is given as

$$y = a \sin(\omega t + kx) \quad \ldots\text{(i)}$$

Given equation of wave is

$$y = 10^{-3} \sin(50 + 2x) \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\omega = 50 \text{ and } k = 2$$

Velocity of the wave, $v = \frac{\omega}{k} = \frac{50}{2} = 25\,\text{m/s}$

3. As observer is moving with relativistic speed; formula $\frac{\Delta f}{f} = \frac{v_{\textbf{radial}}}{c}$, does not apply here.

Relativistic doppler's formula is

$$f_{\textbf{observed}} = f_{\textbf{actual}} \cdot \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

Here, $\frac{v}{c} = \frac{1}{2}$

So, $f_{\textbf{observed}} = f_{\textbf{actual}} \left(\frac{3/2}{1/2}\right)^{1/2}$

$\therefore \quad f_{\textbf{observed}} = 10 \times \sqrt{3} = 17.3\,\text{GHz}$

4. The frequency is a characteristic of source. It is independent of the medium. Hence, the correct option is (d).

5. Speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}},\ v \propto \frac{1}{\sqrt{M}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{m_2}{m_1}}$$

Here, $\gamma = \frac{C_p}{C_V} = \frac{5}{3}$ for both the gases $\left(\gamma_{\text{monoatomic}} = \frac{5}{3}\right)$

6. Speed of sound in an ideal gas is given by

$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow \therefore\ v \propto \sqrt{\frac{\gamma}{M}}$ (T is same for both the gases)

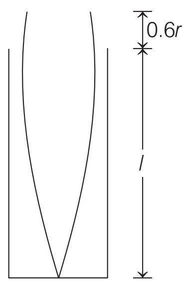
$$\therefore \quad \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \cdot \frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3}\left(\frac{4}{28}\right)} = \sqrt{3}/5$$

$$\gamma_{N_2} = \frac{7}{5} \quad \text{(Diatomic)}$$

$$\gamma_{He} = \frac{5}{3} \quad \text{(Monoatomic)}$$

7. $v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}} \Rightarrow \frac{v_{H_2}}{v_{He}} = \frac{\sqrt{\gamma_{H_2}/M_{H_2}}}{\sqrt{\gamma_{He}/M_{He}}} = \frac{\sqrt{(7/5)/2}}{\sqrt{(5/3)/4}} = \sqrt{\frac{42}{25}}$

8. Fundamental frequency, $f = \frac{v}{4(l + 0.6r)}$



$\therefore$ Speed of sound $v = 4f(l + 0.6r)$.

or $\quad v = (4)(480)[(0.16) + (0.6)(0.025)]$
$= 336\,\text{m/s}$

9. Amplitude of incident wave $A_i = 3.5\,\text{cm}$

Tension $T = 80\text{N}$

Amplitude of incident wave $A_i = 3.5\text{cm}$

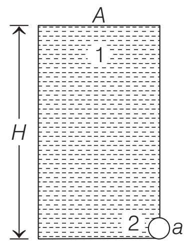
Mass per unit length of wire PQ is

$$m_1 = \frac{0.06}{4.8} = \frac{1}{80}\ \text{kg/m}$$

and mass per unit length of wire QR is

$$m_2 = \frac{0.2}{2.56} = \frac{1}{12.8}\ \text{kg/m}$$

(a) Speed of wave in wire PQ is



$$v_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{1/80}} = 80\,\text{m/s}$$

and speed of wave in wire

QR is $v_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{1/12.8}}$

$= 32\ \text{m/s}$

$\therefore$ Time taken by the wave pulse to reach from P to R is

$$t = \frac{4.8}{v_1} + \frac{2.56}{v_2} = \left(\frac{4.8}{80} + \frac{2.56}{32}\right)\text{s}$$

$t = 0.14\ \text{s}$

(b) The expressions for reflected and transmitted amplitudes (A_r and A_t) in terms of v_1, v_2 and A_i are as follows :

$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i \quad \text{and} \quad A_t = \frac{2v_2}{v_1 + v_2} A_i$$

Substituting the values, we get

$$A_r = \left(\frac{32 - 80}{32 + 80}\right)(3.5) = -1.5\ \text{cm}$$

i.e. the amplitude of reflected wave will be 1.5 cm. Negative sign of A_r indicates that there will be a phase change of π in reflected wave. Similarly,

$$A_t = \left(\frac{2 \times 32}{32 + 80}\right)(3.5) = 2.0\,\text{cm}$$

i.e. the amplitude of transmitted wave will be 2.0 cm.

NOTE The expressions of A_r *and* A_t are derived as below.

Derivation

Suppose the incident wave of amplitude A_i and angular frequency ω is travelling in positive x-direction with velocity v_1 then, we can write

$$y_i = A_i \sin w\left[t - \frac{x}{v_1}\right] \quad \text{...(i)}$$

In reflected as well as transmitted wave, ω will not change, therefore, we can write,

$$y_r = A_r \sin\omega\left[t + \frac{x}{v_1}\right] \quad \ldots\text{(ii)}$$

and
$$y_t = A_t \sin\omega\left[t - \frac{x}{v_2}\right] \quad \ldots\text{(iii)}$$

Now, as wave is continuous, so at the boundary $(x = 0)$.

Continuity of displacement requires

$$y_i + y_r = y_t \text{ for } (x = 0)$$

Substituting, we get

$$A_i + A_r = A_t \quad \ldots\text{(iv)}$$

Also at the boundary, slope of wave will be continuous *i.e.*,

$$\frac{\partial y_i}{\partial x} + \frac{\partial y_r}{\partial x} = \frac{\partial y_t}{\partial x}$$

for $x = 0$

Which gives,

$$A_i - A_r = \left(\frac{v_1}{v_2}\right) A_t \quad \ldots\text{(v)}$$

Solving Eqs. (iv) and (v) for A_r and A_t, we get the required equations i.e.

$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i$$

and
$$A_t = \frac{2v_2}{v_2 + v_1} A_i$$

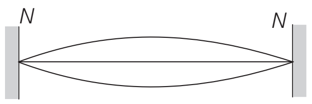
10. The temperature stress is $\sigma = Y\alpha\Delta\theta$

or tension in the steel wire $T = \sigma A = YA\alpha\Delta\theta$

Substituting the values, we have

$$T = (2\times10^{11})(10^{-6})(1.21\times10^{-5})(20) = 48.4\text{ N}$$

Speed of transverse wave on the wire, $v = \sqrt{\frac{T}{\mu}}$



$l\ /2 = l$
or $l\ = 2l$

Here, μ = mass per unit length of wire $= 0.1\text{ kg/m}$

$$\therefore \quad v = \sqrt{\frac{48.4}{0.1}} = 22\text{ m/s}$$

Fundamental frequency

$$f_o = \frac{v}{2l} = \frac{22}{2\times1} = 11\text{ Hz}$$

11. $v = \sqrt{T/\mu}$

$$\frac{v_{\text{top}}}{v_{\text{bottom}}} = \sqrt{\frac{T_{\text{top}}}{T_{\text{bottom}}}} = \sqrt{\frac{6+2}{2}} = 2 \quad \ldots\text{(i)}$$

Frequency will remain unchanged. Therefore, Eq. (i) can be written as,

$$\frac{f\lambda_{\text{top}}}{f\lambda_{\text{bottom}}} = 2$$

or $\lambda_{\text{top}} = 2\ (\lambda_{\text{bottom}}) = 2\times0.06 = 0.12\text{ m}$

12. Tension due to thermal stresses,

$$T = YA\,\alpha\cdot\Delta\theta \quad\Rightarrow\quad v = \sqrt{\frac{T}{\mu}}$$

Here, μ = mass per unit length. $= \rho A$

$$\therefore \quad v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{YA\alpha\cdot\Delta\theta}{\rho A}} = \sqrt{\frac{Y\alpha\ \Delta\theta}{\rho}}$$

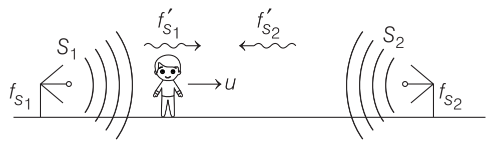
Substituting the values we have,

$$v = \sqrt{\frac{1.3\times10^{11}\times1.7\times10^{-5}\times20}{9\times10^{3}}}$$

$$= 70.1\text{ m/s}$$

Topic 4 Beats and Doppler Effect

1. When observer moves away from S_1 and towards S_2,



then due to Doppler's effect observed frequencies of sources by observer are

$$f'_{S_1} = \frac{v - v_o}{v}\cdot f_{S_1}$$

(observer moving away from source)

and
$$f'_{S_2} = \left(\frac{v + v_o}{v}\right)\cdot f_{S_2}$$

(observer moving towards source)

(where, v = speed to sound, v_o = speed of observer)

So, beat frequency heard by observer is

$$f_b = f'_{S_2} - f'_{S_1}$$

Here, $v_o = u$, $v = 330\text{ ms}^{-1}$

$$f_b = 10\text{ Hz},\ f_{S_1} = f_{S_2} = 660\text{ Hz}$$

On putting the values, we get

$$f_b = f'_{S_2} - f'_{S_1}$$
$$= \left(\frac{v+v_o}{v}\right)\cdot f_{S_2} - \left(\frac{v-v_o}{v}\right) f_{S_1}$$
$$= f_{S_1}\left\{\frac{v+v_o}{v} - \frac{v-v_o}{v}\right\} = f_{S_1}\cdot\frac{2v_o}{v}$$

$$\Rightarrow \quad 10 = \frac{660\times2u}{330} \quad [\because v_o = u]$$

$$\Rightarrow \quad u = \frac{330\times10}{2\times660} \Rightarrow u = 2.5\text{ ms}^{-1}$$

2. When waves of nearby frequencies overlaps, beats are produced.

Beat frequency is given by, $f_{\text{beat}} = f_1 - f_2$

Here, $f_1 = 11\text{ Hz}$ and $f_2 = 9\text{ Hz}$

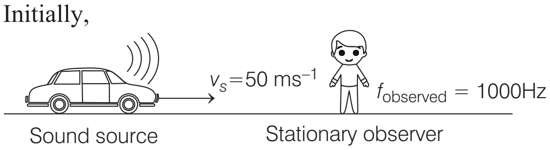
$\Rightarrow$ Beat frequency is, $f_{\text{beat}} = 11 - 9 = 2\,\text{Hz}$

Hence, time period of beats or time interval between beats,

$$T = \frac{1}{f_{\text{beat}}} \Rightarrow T = \frac{1}{2} = 0.5\,\text{s}$$

So, resultant wave has a time period of 0.5 s which is correctly depicted in option (a) only.

3. Initially,



After sometime,

When source is moving towards stationary observer, frequency observed is more than source frequency due to Doppler's effect, it is given by

$$f_{\text{observed}} = f\left(\frac{v}{v - v_s}\right)$$

where, f = source frequency,

f_o = observed frequency = 1000 Hz,

v = speed of sound in air = 350 ms^{-1}

and v_s = speed of source = 50 ms^{-1}

So, $$f = \frac{f_{\text{obs}}(v - v_s)}{v} = \frac{1000(350 - 50)}{350} = \frac{6000}{7}\,\text{Hz}$$

When source moves away from stationary observer, observed frequency will be lower due to Doppler's effect and it is given by

$$f_0 = f\left(\frac{v}{v + v_s}\right) = \frac{6000 \times 350}{7 \times (350 + 50)}$$

$$= \frac{6000 \times 350}{7 \times 400} = 750\,\text{Hz}$$

4. Given,

Frequency of sound source $(f_0) = 500$ Hz Apparent frequency heard by observer 1, $f_1 = 480$ Hz and apparent frequency heard by observer 2, $f_2 = 530$ Hz.

Let v_0 be the speed of sound.

When observer moves away from the source, Apparent

frequency, $$f_1 = f_0\left(\frac{v - v_0'}{v}\right) \quad \ldots \text{(i)}$$

When observer moves towards the source,

Apparent frequency, $$f_2 = f_0\left(\frac{v + v_0''}{v}\right) \quad \ldots \text{(ii)}$$

Substituting values in Eq. (i), we get

$$480 = 500\left(\frac{300 - v_0'}{300}\right)$$

$\Rightarrow$ $96 \times 3 = 300 - v_0'$

$\Rightarrow$ $v_0' = 12\,\text{m/s}$

Substituting values in Eq. (ii), we get

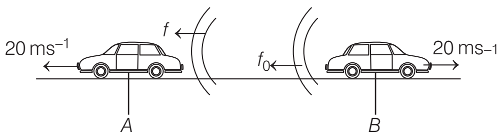
$$530 = 500\left(\frac{330 + v''}{300}\right)$$

$\Rightarrow$ $106 \times 3 = 300 + v''$

$\Rightarrow$ $v_0'' = 18\,\text{m/s}$

Thus, their respective speeds (in m/s) is 12 and 18.

5. The given condition can be shown below as,



Here, source and observer both are moving away from each other. So, by Doppler's effect, observed frequency is given by

$$f = f_0\left(\frac{v + v_o}{v - v_s}\right) \quad \ldots \text{(i)}$$

where, v = speed of sound = 340 ms^{-1},

v_o = speed of observer = $-$ 20 ms^{-1},

v_s = speed of source = $-$ 20 ms^{-1},

f_0 = true frequency

and f = apparent frequency = 2000 Hz

Substituting the given values in Eq. (i), we get

$$2000 = \left(\frac{340 - 20}{340 + 20}\right) \times f_0$$

$$\Rightarrow f_0 = \frac{2000 \times 360}{320} = 2250\,\text{Hz}$$

6. When a source is moving towards an stationary observer, observed frequency is given by

$$f_{\text{observed}} = f\left(\frac{v}{v + v_s}\right)$$

where, f = frequency of sound from the source,

v = speed of sound and v_s = speed of source.

Now applying above formula to two different conditions given in problem, we get

$$f_1 = \text{Observed frequency} = f\left(\frac{340}{340 - 34}\right)$$

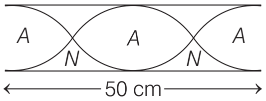
$$= f\left(\frac{340}{306}\right)$$

and f_2 = Observed frequency when speed of source is reduced

$$= f\left(\frac{340}{340 - 17}\right) = \frac{340}{323}$$

So, the ratio $f_1 : f_2$ is $\dfrac{f_1}{f_2} = \dfrac{323}{306} = \dfrac{19}{18}$.

7. According to the question, the musician uses a open flute of length 50 cm and produce second harmonic sound waves



When the flute is open from both ends and produce second harmonic, then

$$L = \lambda_2$$

$$\Rightarrow \qquad f_2 = \frac{v}{L}$$

where, λ_2 = wavelength for second harmonic,

f_2 = frequency for second harmonic

and v = speed of wave.

For given question $\Rightarrow f_2 = \frac{v}{L} = \frac{330}{50 \times 10^{-2}}$

$f_2 = 660\,\text{Hz}$ (frequency produce by source)

Now, a person runs towards the musician from another end of a hall

$v_{\text{observer}} = 10\,\text{km/h}$ (towards source)

There is apparant change in frequency, which heard by person and given by Doppler's effect formula

$$\Rightarrow \qquad \nu' = f' = \nu\left[\frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}}}\right]$$

$$f' = f_2\left[\frac{v_s + v_o}{v_{\text{sound}}}\right]$$

$$f' = 660\left[\frac{330 + \frac{50}{18}}{330}\right]$$

$$f' = 666\,\text{Hz}$$

Hence, option (a) is correct.

8. Observer is stationary and source is moving.

During approach, $f_1 = f\dfrac{v}{v - v_s}$

$$= 1000\left(\frac{320}{320 - 20}\right) = 1066.67\ \text{Hz}$$

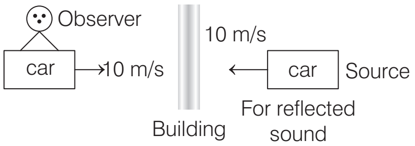
During recede, $f_2 = f\left(\dfrac{v}{v + v_s}\right)$

$$= 1000\left(\frac{320}{320 + 20}\right) = 941.18\ \text{Hz}$$

$$|\%\ \text{change in frequency}| = \left(\frac{f_1 - f_2}{f_1}\right) \times 100 \approx 12\%$$

9. $36\ \text{km/h} = 36 \times \dfrac{5}{18} = 10\ \text{m/s}$

Apparent frequency of sound heard by car driver (observer) reflected from the building will be



$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 8\left(\frac{320 + 10}{320 - 10}\right) = 8.5\,\text{kHz}$$

$\therefore$ Correct option is (a).

10. With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4.

$\therefore$ Frequency of tuning fork

= third harmonic frequency of closed pipe + 4

$$= 3\left(\frac{v}{4l}\right) + 4 = 3\left(\frac{340}{4 \times 0.75}\right) + 4$$

$$= 344\ \text{Hz}$$

$\therefore$ Correct option is (a).

11. The motorcyclist observes no beats. So, the apparent frequency observed by him from the two sources must be equal.

$$f_1 = f_2$$

$$\therefore \qquad 176\left(\frac{330 - v}{330 - 22}\right) = 165\left(\frac{330 + v}{330}\right)$$

Solving this equation, we get $v = 22\text{m/s}$

12. Using the formula $f' = f\left(\dfrac{v + v_0}{v}\right)$

we get, $$5.5 = 5\left(\frac{v + v_A}{v}\right) \qquad \ldots\text{(i)}$$

and $$6.0 = 5\left(\frac{v + v_B}{v}\right) \qquad \ldots\text{(ii)}$$

Here, v = speed of sound

v_A = speed of train A

v_B = speed of train B

Solving Eqs. (i) and (ii), we get $\dfrac{v_B}{v_A} = 2$

13. $f_1 = f\left(\dfrac{v}{v - v_s}\right) \Rightarrow f_1 = f\left(\dfrac{340}{340 - 34}\right) = f\left(\dfrac{340}{306}\right)$

and $$f_2 = f\left(\frac{340}{340 - 17}\right) = f\left(\frac{340}{323}\right)$$

$$\therefore \qquad \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$$

14. Source is moving towards the observer

$$f' = f\left(\frac{v}{v - v_s}\right) = 450\left(\frac{330}{330 - 33}\right)$$

$$f' = 500\ \text{Hz}$$

15. When wind blows from S to O

$$f_2 = f_1\left(\frac{v + w + u}{v + w - u}\right)$$

or $\qquad f_2 > f_1$

when wind blows from O to S

$$f_2 = f_1\left(\frac{v - w + u}{v - w - u}\right)$$

$\therefore \qquad f_2 > f_1$

16. Moving plane is like a moving observer. Therefore, number of waves encountered by moving plane,

$$f_1 = f\left(\frac{v + v_0}{v}\right) = f\left(\frac{c + v}{c}\right)$$

Frequency of reflected wave,

$$f_2 = f_1\left(\frac{v}{v - v_s}\right) = f\left(\frac{c + v}{c - v}\right)$$

Wavelength of reflected wave

$$\lambda_2 = \frac{v}{f_2} = \frac{c}{f_2} = \frac{c}{f}\left(\frac{c - v}{c + v}\right)$$

17. $\qquad T_1 > T_2$

$\therefore \qquad v_1 > v_2$

or $\qquad f_1 > f_2$

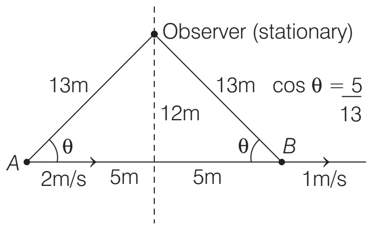
and $\qquad f_1 - f_2 = 6\,\text{Hz}$

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore, (d) option is wrong.

If T_1 is decreased, f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz.Therefore, (c) option is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. So, (b) is also correct. But (a) is wrong.

18.



$$f_A = 1430\left[\frac{330}{330 - 2\cos\theta}\right]$$

$$= 1430\left[\frac{1}{1 - \dfrac{2\cos\theta}{330}}\right] \approx 1430\left[1 + \frac{2\cos\theta}{330}\right]$$

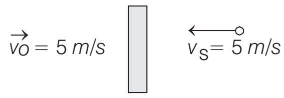
[from binomial expansion]

$$f_B = 1430\left[\frac{330}{330 + 1\cos\theta}\right] \approx 1430\left[1 - \frac{\cos\theta}{330}\right]$$

$$\text{Beat frequency} = f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

$$= 13\left(\frac{5}{13}\right) = 5.00\,\text{Hz}$$

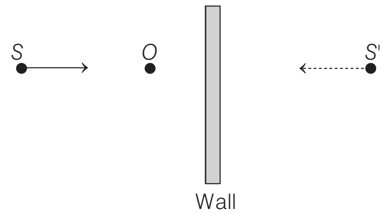
19. Frequency of refracted sound



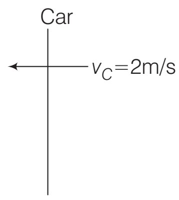
$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 200\left(\frac{342 + 5}{342 - 5}\right)\text{Hz} = 205.93\,\text{Hz}$$

$\therefore$ Beat frequency $= f' - f = (205.93 - 200)$
$= 5.93\,\text{Hz} \simeq 6\,\text{Hz}$

20. For reflected wave an image of source S' can assumed as shown. Since, both S and S' are approaching towards observer, no beats will be heard.



21.



Frequency observed at car

$$f_1 = f_0\left(\frac{v + v_C}{v}\right) \qquad (v = \text{speed of sound})$$

Frequency of reflected sound as observed at the source

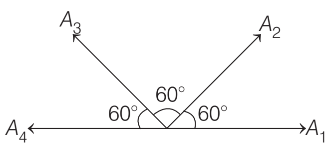
$$f_2 = f_1\left(\frac{v}{v - v_C}\right) = f_0\left(\frac{v + v_C}{v - v_C}\right)$$

$$\text{Beat frequency} = f_2 - f_0$$

$$= f_0\left[\frac{v + v_C}{v - v_C} - 1\right] = f_0\left[\frac{2v_C}{v - v_C}\right]$$

$$= 492 \times \frac{2 \times 2}{328} = 6\,\text{Hz}$$

22. Let individual amplitudes are A_0 each. Amplitudes can be added by vector method.



$$A_1 = A_2 = A_3 = A_4 = A_0$$

Resultant of A_1 and A_4 is zero. Resultant of A_2 and A_3 is

$$A = \sqrt{A_0^2 + A_0^2 + 2A_0A_0\cos 60^\circ} = \sqrt{3}A_0$$

This is also the net resultant.

Now, $\quad I \propto A^2$

$\therefore$ Net intensity will become $3I_0$.

$\therefore$ Answer is 3.

23. From the relation, $f' = f\left(\frac{v}{v \pm v_s}\right)$, we have

$$2.2 = f\left[\frac{300}{300 - v_T}\right] \quad \ldots\text{(i)}$$

and

$$1.8 = f\left[\frac{300}{300 + v_T}\right] \quad \ldots\text{(ii)}$$

Here, $v_T = v_s$ = velocity of source/train

Solving Eqs. (i) and (ii), we get

$$v_T = 30\,\text{m/s}$$

24. Velocity of sound in water is

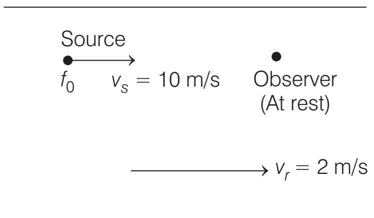
$$v_w = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}} = 1445 \text{ m/s}$$

Frequency of sound in water will be

$$f_0 = \frac{v_w}{\lambda_w} = \frac{1445}{14.45 \times 10^{-3}} \text{ Hz}$$

$$f_0 = 10^5 \text{ Hz}$$

(a) Frequency of sound detected by receiver (observer) at rest would be



$$f_1 = f_0\left(\frac{v_w + v_r}{v_w + v_r - v_s}\right) = (10^5)\left(\frac{1445 + 2}{1445 + 2 - 10}\right) \text{Hz}$$

$$f_1 = 1.0069 \times 10^5 \text{ Hz}$$

(b) Velocity of sound in air is

$$v_a = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4)(8.31)(20 + 273)}{28.8 \times 10^{-3}}}$$

$$= 344 \text{ m/s}$$

$\therefore$ Frequency does not depend on the medium. Therefore, frequency in air is also $f_0 = 10^5$ Hz.

$\therefore$ Frequency of sound detected by receiver (observer) in air would be

$$f_2 = f_0\left(\frac{v_a - w}{v_a - w - v_s}\right)$$

$$= 10^5\left[\frac{344 - 5}{344 - 5 - 10}\right] \text{Hz}$$

$$f_2 = 1.0304 \times 10^5 \text{ Hz}$$

25. Frequency received in case 1.

$$f_1 = f\left(\frac{v + v_m}{v + v_b}\right)$$

and in case 2, $f_2 = f\left(\frac{v + v_m}{v - v_b}\right)$

Obviously, $f_2 > f_1$

$\therefore$ Beat frequency,

$$f_b = f_2 - f_1 = f\left(\frac{v + v_m}{v - v_b}\right) - f\left(\frac{v + v_m}{v + v_b}\right)$$

or

$$f_b = \frac{2\,v_b(v + v_m)}{v^2 - v_b^2}$$

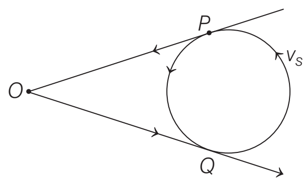
26. v_s = Speed of source (whistle) = $R\omega$ = (1.5) (20) m/s

$v_s = 30$ m/s

Maximum frequency will be heard by the observer in position P and minimum in position Q. Now,

$$f_{\max} = f\left(\frac{v}{v - v_s}\right)$$

where, $\quad v$ = speed of sound in air = 330 m/s



$$= (440)\left(\frac{330}{330 - 30}\right) \text{Hz}$$

$$f_{\max} = 484 \text{ Hz}$$

and

$$f_{\min} = f\left(\frac{v}{v + v_s}\right) = (440)\left(\frac{330}{330 + 30}\right)$$

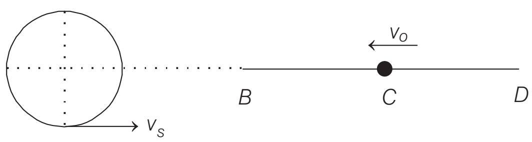
$$f_{\min} = 403.33 \text{ Hz}$$

Therefore, range of frequencies heard by observer is from 484 Hz to 403.33 Hz.

27. Angular frequency of detector

$$\omega = 2\pi f = 2\pi\left(\frac{5}{\pi}\right) = 10 \text{ rad/s}$$

Since, angular frequency of source of sound and of detector are equal, their time periods will also be equal.



Maximum frequency will be heard in the position shown in figure. Since, the detector is far away from the source, we can use,

$$f_{\max} = f\left(\frac{v+v_o}{v-v_s}\right)$$

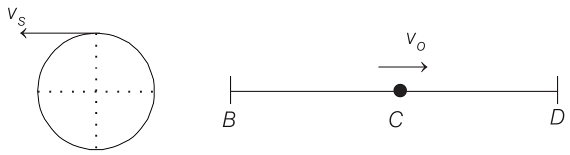
Here, v = speed of sound = 340 m/s (given)

$$v_s = R\omega = 30\text{ m/s}$$

$$v_o = \omega A = 60\text{ m/s}$$

$$\therefore \quad f_{\max} = 340\left[\frac{340+60}{340-30}\right]$$

$$= 438.7\text{ Hz}$$



Minimum frequency will be heard in the condition shown in figure. The minimum frequency will be

$$f_{\min} = f\left[\frac{v-v_o}{v+v_s}\right] = 340\left[\frac{340-60}{340+30}\right]$$

$$= 257.3\text{ Hz}$$

28. Given, $f_1 - f_2 = 3\text{Hz}$

or $$f\left(\frac{v}{v-v_s}\right) - f\left(\frac{v}{v+v_s}\right) = 3$$

or $$340\left[\frac{340}{340-v_s}\right] - 340\left[\frac{340}{340+v_s}\right] = 3$$

or $$340\left[\left(1-\frac{v_s}{340}\right)^{-1}\right] - 340\left[\left(1+\frac{v_s}{340}\right)^{-1}\right] = 3$$

As $v_s << 340\text{m/s}$

Using binomial expansion, we have

$$340\left(1+\frac{v_s}{340}\right) - 340\left(1-\frac{v_s}{340}\right) = 3$$

$$\therefore \quad \frac{2\times 340\times v_s}{340} = 3$$

$$\therefore \quad v_s = 1.5\text{ m/s}$$

29. Fundamental frequency of sonometer wire,

$$f = \frac{v}{2l} = \frac{\sqrt{T/\mu}}{2l} = \frac{1}{2\times 0.1}\sqrt{\frac{64\times 0.1}{10^{-3}}}$$

$$= 400\text{Hz}$$

Given beat frequency, $f_b = f - f' = 1\text{Hz}$

$$\therefore \quad f' = 399\text{Hz}$$

Using, $$f' = f\left(\frac{v}{v+v_s}\right)$$

or $$399 = 400\left(\frac{300}{300+v_s}\right)$$

$$\therefore \quad v_s = 0.75\text{ m/s}$$

30. By decreasing the tension in the string beat frequency is decreasing, it means frequency of string was greater than frequency of pipe. Thus,

First overtone frequency of string – Fundamental frequency of closed pipe = 8

$$\therefore \quad 2\left(\frac{v_1}{2l_1}\right) - \left(\frac{v_2}{4l_2}\right) = 8 \text{ or } v_1 = l_1\left[8+\frac{v_2}{4l_2}\right]$$

Substituting the value, we have

$$v_1 = 0.25\left[8+\frac{320}{4\times 0.4}\right] = 52\text{m/s}$$

Now, $$v_1 = \sqrt{\frac{T}{\mu}}$$

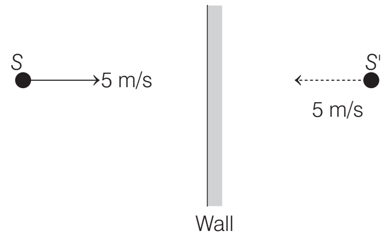
$$\therefore \quad T = \mu v_1^2 = \left(\frac{m}{l}\right)v_1^2 = \left(\frac{2.5\times 10^{-3}}{0.25}\right)(52)^2 = 27.04\text{ N}$$

31. Frequency heard by the observer due to S' (reflected wave)

$$f' = f\left(\frac{v+v_o}{v-v_s}\right) = 256\left(\frac{330+5}{330-5}\right)$$

$$= 263.87\text{ Hz}$$

$\therefore$ Beat frequency $f_b = f' - f = 7.87\text{ Hz}$



Topic 5 Miscellaneous Problems

1. Loudness of sound in decible is given by

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where, I = intensity of sound in W/m^2,

I_0 = reference intensity (= 10^{-12} W/m^2), chosen because it is near the lower limit of the human hearing range.

Here, $\beta = 120$ dB

So, we have $120 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$

$$\Rightarrow \quad 12 = \log_{10}\left(\frac{I}{10^{-12}}\right)$$

Taking antilog, we have

$$\Rightarrow \quad 10^{12} = \frac{I}{10^{-12}}$$

$$\Rightarrow \quad I = 1\text{ W/m}^2$$

This is the intensity of sound reaching the observer.

Now, intensity, $I = \dfrac{P}{4\pi r^2}$

where, r = distance from source,

P = power of output source.

Here, $P = 2$ W, we have

$$1 = \frac{2}{4\pi r^2} \Rightarrow r^2 = \frac{1}{2\pi} \Rightarrow r = \sqrt{\frac{1}{2\pi}}\ \text{m} = 0.398\ \text{m} \approx 40\ \text{cm}$$

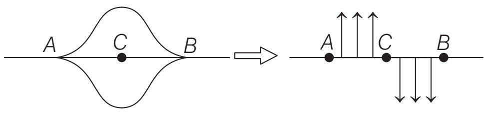
2. Let Δl be the end correction.

Given that, fundamental tone for a length 0.1m = first overtone for the length 0.35 m.

$$\frac{v}{4(0.1 + \Delta l)} = \frac{3v}{4(0.35 + \Delta l)}$$

Solving this equation, we get $\Delta l = 0.025\text{m} = 2.5\ \text{cm}$

3. After two seconds both the pulses will move 4 cm towards each other. So, by their superposition, the resultant displacement at every point will be zero. Therefore, total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half downwards.



4. Energy $E \propto (\text{amplitude})^2\ (\text{frequency})^2$

Amplitude (A) is same in both the cases, but frequency 2ω in the second case is two times the frequency (ω) in the first case.

Therefore, $E_2 = 4E_1$

5. Mass per unit length of the string,

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2}\text{kg/m}$$

$\therefore$ Velocity of wave in the string,

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}}$$

$$v = 8\ \text{m/s}$$

For constructive interference between successive pulses

$$\Delta t_{\min} = \frac{2l}{v} = \frac{(2)(0.4)}{8} = 0.10\ \text{s}$$

(After two reflections, the wave pulse is in same phase as it was produced, since in one reflection its phase changes by π, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

6. From Hooke's law

Tension in a string $(T) \propto$ extension (x)

and speed of sound in string

$$v = \sqrt{T/\mu}$$

or $$v \propto \sqrt{T}$$

Therefore, $$v \propto \sqrt{x}$$

x is increased to 1.5 times i.e. speed will increase by $\sqrt{1.5}$ times or 1.22 times. Therefore, speed of sound in new position will be 1.22 v.

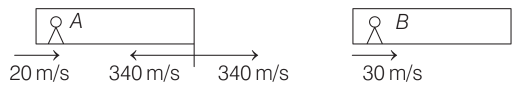
7. The given equation can be written as

$$y = 2\,(2\cos^2 \frac{t}{2})\sin\,(1000\,t)$$

$$y = 2\,(\cos\,t + 1)\sin\,(1000\,t)$$
$$= 2\cos t\,\sin\,1000\,t + 2\sin\,(1000\,t)$$
$$= \sin\,(1001\,t) + \sin\,(999\,t) + 2\sin\,(1000\,t)$$

i.e. the given expression is a result of superposition of three independent harmonic motions of angular frequencies 999, 1000 and 1001 rad/s.

8. $$v_{SA} = 340 + 20 = 360\ \text{m/s}$$
$$v_{SB} = 340 - 30 = 310\ \text{m/s}$$



9. For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20 m/s. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities. Therefore, the correct option is (a).

10. For the passengers in train B, observer is receding with velocity 30 m/s and source is approaching with velocity 20 m/s.

$\therefore$ $$f'_1 = 800\left(\frac{340 - 30}{340 - 20}\right) = 775\ \text{Hz}$$

and $$f'_2 = 1120\left(\frac{340 - 30}{340 - 20}\right) = 1085\ \text{Hz}$$

$\therefore$ Spread of frequency $= f'_2 - f'_1$
$= 310$ Hz

11. In one second number of maximas is called the beat frequency. Hence,

$$f_b = f_1 - f_2 = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4\text{Hz}$$

12. Speed of wave $v = \dfrac{\omega}{k}$

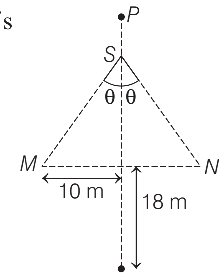
or $$v = \frac{100\pi}{0.5\,\pi} \text{ or } \frac{92\pi}{0.46\,\pi} = 200\ \text{m/s}$$

13. At $x = 0,\ y = y_1 + y_2 = 2A\cos 96\pi t \cos 4\pi t$

Frequency of $\cos\,(96\pi t)$ function is 48 Hz and that of $\cos\,(4\pi t)$ function is 2 Hz.

In one second, cos function becomes zero at $2f$ times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1 s.

14. Speed of car, $v = 60 \text{km/h} = \frac{500}{3} \text{m/s}$



At a point S, between P and Q

$$\nu'_M = \nu_M \left(\frac{C + v\cos\theta}{C}\right);$$

$$\nu'_N = \nu_N \left(\frac{C + v\cos\theta}{C}\right)$$

$$\Rightarrow \Delta v = (v_n - v_M)\left(1 + \frac{v\cos\theta}{C}\right)$$

Similarly, between Q and R

$$\Delta\nu = (\nu_N - \nu_M)\left(1 - \frac{v\cos\theta}{C}\right)$$

$$\frac{d(\Delta\nu)}{dt} = \pm(\nu_N - \nu_M)\frac{v}{C}\sin\theta\frac{d\theta}{dt}$$

$\theta = 0°$ at the P and R as they are large distance apart.

$\Rightarrow$ Slope of graph is zero.

at $Q, \theta = 90°$ $\sin\theta$ is maximum also value of $\frac{d\theta}{dt}$ is maximum

as $\frac{d\theta}{dt} = \frac{v}{r}$, where v is its velocity and r is the length of the line joining P and S and r is minimum at Q.

$\Rightarrow$ Slope is maximum at Q.

At P, $\nu_P = \Delta\nu = (\nu_N - \nu_M)\left(1 + \frac{v}{C}\right)(\theta \approx 0°)$

At R, $\nu_R = \Delta\nu = (\nu_N - \nu_M)\left(1 - \frac{v}{C}\right)(\theta \approx 0°)$

At Q, $\nu_Q = \Delta\nu = (\nu_N - \nu_M)\,(\theta = 90°)$

From these equations, we can see that

$$\nu_P + \nu_R = 2\nu_Q$$

15. Minimum length $= \frac{\lambda}{4}$

$$\lambda = 4l$$

Now, $v = f\lambda = (244) \times 4 \times l$

as $l = 0.350 \pm 0.005$

$\Rightarrow v$ lies between 336.7 m/s to 346.5 m/s

Now, $v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}}$, here M is molecular mass in gram

$$= \sqrt{100\gamma RT} \times \sqrt{\frac{10}{M}}.$$

For monoatomic gas, $\gamma = 1.67$

$$\Rightarrow \qquad v = 640 \times \sqrt{\frac{10}{M}}$$

For diatomic gas, $\gamma = 1.4$

$$\Rightarrow v = 590 \times \sqrt{\frac{10}{M}} \Rightarrow v_{Ne} = 640 \times \frac{7}{10} = 448 \text{ m/s}$$

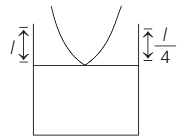
$$\Rightarrow v_{Ar} = 640 \times \frac{17}{32} = 340 \text{ m/s} \Rightarrow v_{O_2} = 590 \times \frac{9}{16} = 331.8 \text{ m/s}$$

$$\Rightarrow v_{N_2} = 590 \times \frac{3}{5} = 354 \text{ m/s}$$

$\therefore$ Only possible answer is Argon.

16. At open end phase of pressure wave changes by 180°. So, compression returns as rarefaction. At closed end there is no phase change. So, compression returns as compression and rarefaction as rarefaction.

17. $l < \frac{\lambda}{4}$

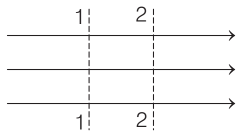


Further, larger the length of air column, feebler is the intensity.

18. In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

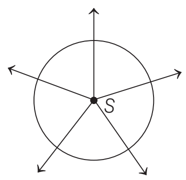
NOTE In general, y is any general physical quantity which is made to oscillate at one place and these oscillations are propagated to other places also.

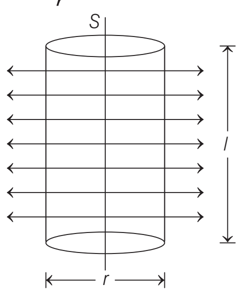
19. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.



But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$$I = \frac{P}{4\pi r^2} \quad \text{or} \quad I \propto \frac{1}{r^2}$$





NOTE For a line source $I \propto \frac{1}{r}$ because, $I = \frac{P}{\pi r l}$

20. (a) and (c) options satisfy the condition;

$$\frac{\partial^2 y}{\partial x^2} = (\text{constant})\frac{\partial^2 y}{\partial t^2}$$

21. $v = 300 \text{ m/s}, \quad f = 25 \text{Hz}$

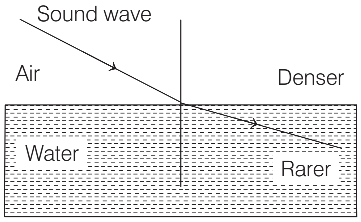
$\therefore$ Wavelength $\lambda = \frac{v}{f} = \frac{300}{25} = 12 \text{ m}$

(a) Phase difference $\Delta\phi = \frac{2\pi}{\lambda}$ (path difference)

$$= \frac{2\pi}{12} \times 6 = \pi$$

(b) In plane progressive wave amplitude does not change.

22. For sound wave water is rarer medium because speed of sound wave in water is more. When a wave travels from a denser medium to rarer medium it refracts away from the normal.



23. Firstly, car will be treated as an observer which is approaching the source. Then, it will be treated as a source, which is moving in the direction of sound.

Hence, $f_1 = f_0\left(\dfrac{v+v_1}{v-v_1}\right)$

$$f_2 = f_0\left(\frac{v+v_2}{v-v_2}\right)$$

$$\therefore \quad f_1 - f_2 = \left(\frac{1.2}{100}\right) f_0 = f_0\left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right]$$

or $\quad \left(\dfrac{1.2}{100}\right) f_0 = \dfrac{2v\,(v_1 - v_2)}{(v-v_1)(v-v_2)} f_0$

As v_1 and v_2 are very very less than v.

We can write, $(v - v_1)$ or $(v - v_2) \approx v$

$$\therefore \quad \left(\frac{1.2}{100}\right) f_0 = \frac{2\,(v_1 - v_2)}{v} f_0$$

or $\quad (v_1 - v_2) = \dfrac{v \times 1.2}{200} = \dfrac{330 \times 1.2}{200} = 1.98\ \text{ms}^{-1}$

$$= 7.128\ \text{kmh}^{-1}$$

$\therefore$ The nearest integer is 7.

24. (a) If the detector is at $x = 0$, the two radiowaves can be represented as

$$y_1 = A\sin\omega_1 t \quad \text{and} \quad y_2 = A\sin\omega_2 t$$

(Given, $A_1 = A_2 = A$)

By the principle of superposition

$$y = y_1 + y_2 = A\sin\omega_1 t + A\sin\omega_2 t$$

$$y = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\sin\left(\frac{\omega_1+\omega_2}{2}t\right)$$

$$= A_0 \sin\left(\frac{\omega_1+\omega_2}{2}t\right)$$

Here, $A_0 = 2A\cos\left(\dfrac{\omega_1 - \omega_2}{2}t\right)$

Since, $I \propto (A_0)^2 \propto 4A^2\cos^2\left(\dfrac{\omega_1-\omega_2}{2}t\right)$

So, intensity will be maximum when

$$\cos^2\left(\frac{\omega_1-\omega_2}{2}t\right) = \text{maximum} = 1$$

or $\quad \cos\left(\dfrac{\omega_1-\omega_2}{2}t\right) = \pm 1$

or $\quad \dfrac{\omega_1-\omega_2}{2}t = 0, \pi, 2\pi \ldots$

i.e., $t = 0, \dfrac{2\pi}{\omega_1-\omega_2}, \dfrac{4\pi}{\omega_1-\omega_2}, \ldots \dfrac{2n\pi}{\omega_1-\omega_2} \qquad n = 0, 1, 2 \ldots$

Therefore, time interval between any two successive maxima is $\dfrac{2\pi}{\omega_1-\omega_2} = \dfrac{2\pi}{10^3}$ s or 6.28×10^{-3} s.

(b) The detector can detect if resultant intensity $\geq 2A^2$, or the resultant amplitude $\geq \sqrt{2}A$.

Hence, $\quad 2A\cos\left(\dfrac{\omega_1-\omega_2}{2}t\right) \geq \sqrt{2}A$

$$\cos\left(\frac{\omega_1-\omega_2}{2}t\right) \geq \frac{1}{\sqrt{2}}$$

Therefore, the detector lies idle. When value of $\cos\left(\dfrac{\omega_1-\omega_2}{2}t\right)$ is between 0 and $1/\sqrt{2}$]

or when $\dfrac{\omega_1-\omega_2}{2}t$ is between $\dfrac{\pi}{2}$ and $\dfrac{\pi}{4}$

or t lies between

$$\frac{\pi}{\omega_1-\omega_2} \text{ and } \frac{\pi}{2(\omega_1-\omega_2)}$$

$$\therefore \quad t = \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)}$$

$$= \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2\times 10^3}$$

$$t = 1.57 \times 10^{-3}\ \text{s}$$

Hence, the detector lies idle for a time of 1.57×10^{-3} s in each cycle.

25. (a) Wavelength of incident wave $= \dfrac{2\pi}{a}$

and frequency of incident wave $= \dfrac{b}{2\pi}$

(b) Intensity of reflected wave has become 0.64 times. But since $I \propto A^2$ amplitude of reflected wave will become 0.8 times.

a and b will remain as it is. But direction of velocity of wave will become opposite. Further there will be a phase change of π, as it is reflected by an obstacle (denser medium). Therefore, equation of reflected wave would be

$$y_r = 0.8A\cos[ax - bt + \pi] = -0.8A\cos(ax - bt)$$

(c) The equation of resultant wave will be,

$$y = y_i + y_r = A\cos(ax + bt) - 0.8A\cos(ax - bt)$$

Particle velocity

$$v_p = \frac{\partial y}{\partial t} = -Ab\sin(ax+bt) - 0.8Ab\sin(ax - bt)$$

Maximum particle speed can be $1.8\,Ab$, where

$$\sin(ax+bt) = \pm 1 \text{ and } \sin(ax-bt) = \pm 1$$

and minimum particle speed can be zero, where

$$\sin(ax+bt) \text{ and } \sin(ax-bt) \text{ both are zero.}$$

(d) The resultant wave can be written as,

$$y = [0.8\,A\cos(ax+bt) - 0.8A\cos(ax-bt)] + 0.2A\cos(ax+bt)$$

$$= -1.6A\sin ax\sin bt + 0.2A\cos(ax+bt)$$

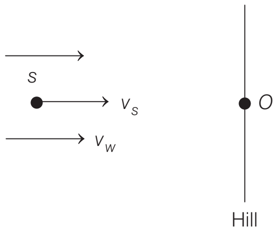
In this equation, $(-1.6A\sin ax\sin bt)$ is the equation of standing wave and $0.2\,A\cos(ax+bt)$ is the equation of travelling wave. The travelling wave is travelling in negative x-direction.

Antinodes are the points where,

$$\sin ax = \pm 1$$

or $\quad ax = \left[n\pi + (-1)^n \dfrac{\pi}{2}\right]$ or $x = \left[n + \dfrac{(-1)^n}{2}\right]\dfrac{\pi}{a}$

26. Given $v_s = v_w = 40$ km/h and $v = 1200$ km/h = speed of sound.

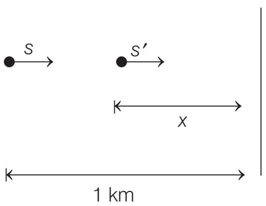


(a) Frequency observed by observer $f' = f\left(\dfrac{v+v_w}{v+v_w-v_s}\right)$

Substituting the values, we have

$$f' = 580\left[\frac{1200+40}{1200+40-40}\right] = 599.33\text{ Hz}$$

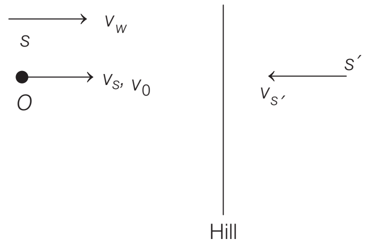
(b) Let x be the distance of the source from the hill at which echo is heard of the sound which was produced when source was at a distance 1 km from the hill. Then, time taken by the source to reach from s to s' = time taken by the sound to reach from s to hill and then from hill to s'. Thus,



$$\frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$$

Solving this equation, we get $x = 0.935$ km

Frequency heard by the driver of the reflected wave



$$f'' = f\left[\frac{v-v_w+v_o}{v-v_w-v_{s'}}\right] = 580\left[\frac{1200-40+40}{1200-40-40}\right] = 621.43\text{ Hz}$$

27. (a) For two waves to form a standing wave, they must be identical and should move in opposite directions. Therefore, z_1 and z_2 will produce a standing wave. The equation of standing wave in this case would be,

$$z = z_1 + z_2 = 2A\cos kx\cos\omega t = A_x\cos\omega t$$

Here, $\quad A_x = 2A\cos kx$

Resultant intensity will be zero, at the positions

where, $A_x = 0$

or $\quad kx = (2n+1)\dfrac{\pi}{2} \qquad$ where $n = 0, \pm 1, \pm 2$...... etc.

or $\quad x = (2n+1)\dfrac{\pi}{2k} \qquad$ where $n = 0, \pm 1, \pm 2$......etc.

(b) z_1 is a wave travelling in positive X-axis and z_3 is a wave travelling in positive Y-axis.

So, by their superposition a wave will be formed which will travel in positive x and positive y-axis. The equation of wave would be

$$z = z_1 + z_3 = 2A\cos\left[\frac{kx+ky}{2} - \omega t\right]\cos\left(\frac{kx-ky}{2}\right)$$

The resultant intensity is zero, where,

$$\cos k\left(\frac{x-y}{2}\right) = 0$$

or $\quad \dfrac{k(x-y)}{2} = (2n+1)\dfrac{\pi}{2}$

or $\quad (x-y) = (2n+1)\dfrac{\pi}{k}$

where $n = 0, \pm 1, \pm 2, \ldots$ etc.

11

Heat and Thermodynamics

Topic 1 Calorimetry

Objective Questions I (Only one correct option)

1. When M_1 gram of ice at $-10°C$ (specific heat = 0.5 cal $g^{-1}°C^{-1}$) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is **(2019 Main, 12 April I)**

(a) $\frac{50M_2}{M_1} - 5$ (b) $\frac{50M_1}{M_2} - 50$ (c) $\frac{50M_2}{M_1}$ (d) $\frac{5M_2}{M_1} - 5$

2. A thermally insulated vessel contains 150 g of water at 0°C. Then, the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to

(Latent heat of vaporisation of water = $2.10 \times 10^6 \text{ J kg}^{-1}$ and latent heat of fusion of water = $3.36 \times 10^5 \text{ J kg}^{-1}$) **(2019 Main, 8 April I)**

(a) 150 g (b) 20 g (c) 130 g (d) 35g

3. When 100 g of a liquid A at 100ºC is added to 50 g of a liquid B at temperature 75ºC, the temperature of the mixture becomes 90ºC. The temperature of the mixture, if 100 g of liquid A at 100ºC is added to 50 g of liquid B at 50ºC will be **(2019 Main, 11 Jan II)**

(a) 60ºC (b) 80ºC (c) 70ºC (d) 85ºC

4. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = k$, where k is a constant. In this process, the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (where, R is gas constant) **(2019 Main, 11 Jan II)**

(a) $\frac{1}{2} kR\Delta T$ (b) $\frac{2k}{3} \Delta T$ (c) $\frac{1}{2} R\Delta T$ (d) $\frac{3}{2} R\Delta T$

5. A metal ball of mass 0.1 kg is heated upto 500ºC and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30ºC. What is the approximate percentage increment in the temperature of the water? [Take, specific heat capacities of water and metal are respectively 4200 $\text{Jkg}^{-1}\text{K}^{-1}$ and 400 $\text{Jkg}^{-1}\text{K}^{-1}$] **(2019 Main, 11 Jan II)**

(a) 25% (b) 15% (c) 30% (d) 20%

6. Ice at $-20°C$ is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Take, specific heat of water = 4.2 J/g/°C specific heat of ice = 2.1 J/g/°C and heat of fusion of water at 0°C = 334 J/g) **(2019 Main, 11 Jan I)**

(a) 40 g (b) 50 g (c) 60 g (d) 100 g

7. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal, if water temperature stabilises at 21.5°C. (Take, specific heat of brass is 394 J $\text{kg}^{-1}\text{K}^{-1}$) **(2019 Main, 10 Jan II)**

(a) 916 J kg^{-1} K^{-1} (b) 654 J kg^{-1} K^{-1}
(c) 1232 J kg^{-1} K^{-1} (d) 458 J kg^{-1} K^{-1}

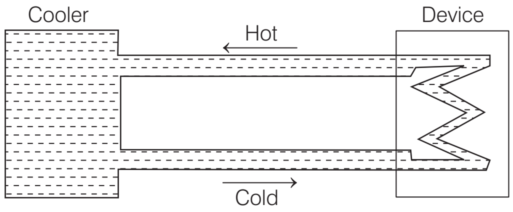
8. C_p and C_V are specific heats at constant pressure and constant volume, respectively. It is observed that $C_p - C_V = a$ for hydrogen gas $C_p - C_V = b$ for nitrogen gas. The correct relation between a and b is **(2017 Main)**

(a) $a = b$ (b) $a = 14b$ (c) $a = 28b$ (d) $a = \frac{1}{14} b$

9. A copper ball of mass 100 g is at a temperature T. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is (Given, room temperature = 30°C, specific heat of copper = 0.1 cal/g°C) **(2017 Main)**

(a) 885°C (b) 1250°C (c) 825°C (d) 800°C

10. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is **(2016 Adv.)**



(Specific heat of water is 4.2 kJ $kg^{-1}K^{-1}$ and the density of water is $1000 kg\ m^{-3}$)

(a) 1600 (b) 2067 (c) 2533 (d) 3933

11. Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1°C and it is defined under which of the following conditions ? **(2005, 2M)**

(a) From 14.5°C to 15.5°C at 760 mm of Hg

(b) From 98.5°C to 99.5°C at 760 mm of Hg

(c) From 13.5°C to 14.5°C at 76 mm of Hg

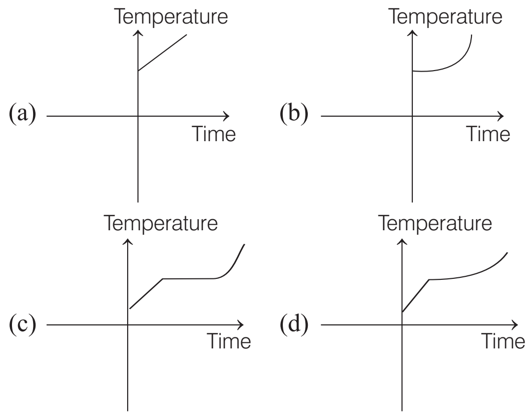
(d) From 3.5°C to 4.5°C at 76 mm of Hg

12. Water of volume 2 L in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at rate of 160 J/s. In how much time temperature will rise from 27°C to 77°C? **(2005, 2M)**

[Specific heat of water is 4.2 kJ/kg]

(a) 8 min 20 s (b) 6 min 2 s
(c) 7 min (d) 14 min

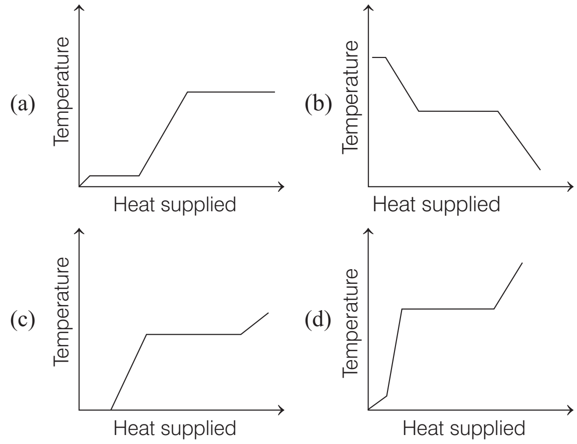
13. Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time? **(2004, 2M)**



14. 2 kg of ice at –20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg /°C and 0.5 kcal/kg /°C while the latent heat of fusion of ice is 80 kcal/kg **(2003, 2M)**

(a) 7 kg (b) 6 kg
(c) 4 kg (d) 2 kg

15. A block of ice at –10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomena qualitatively? **(2000, 2M)**



16. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kg is **(1986, 2M)**

(a) 0.130 (b) 0.065
(c) 0.260 (d) 0.135

17. 70 cal of heat are required to raise the temperature of 2 moles of an ideal diatomic gas at constant pressure from 30°C to 35°C. The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is **(1985, 2M)**

(a) 30 (b) 50
(c) 70 (d) 90

Integer Answer Type Question

18. A piece of ice (heat capacity = $2100\ Jkg^{-1}\ °C^{-1}$ and latent heat = $3.36 \times 10^5\ J\ kg^{-1}$) of mass *m* gram is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of *m* is **(2010)**

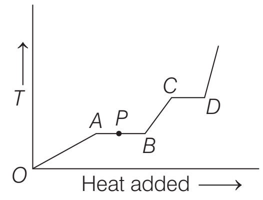
Fill in the Blanks

19. Earth receives 1400 W/m^2 of solar power. If all the solar energy falling on a lens of area 0.2 m^2 is focussed onto a block of ice of mass 280 g, the time taken to melt the ice will be …minutes. (Latent heat of fusion of ice = $3.3 \times 10^5 J/kg$) **(1997, 2M)**

20. A substance of mass *M* kg requires a power input of *P* watts to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time *t* seconds. The latent heat of fusion of the substance is …… . **(1992, 1M)**

21. 300 g of water at 25°C is added to 100 g of ice at 0°C. The final temperature of the mixture is ... °C. **(1989, 2M)**

22. The variation of temperature of a material as heat is given to it at a constant rate as shown in the figure. The material is in solid state at the point O. The state of the material at the point P is .... **(1985, 2M)**



Analytical & Descriptive Questions

23. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Find the final temperature of the mixture (in kelvin). **(2006, 6M)**

Given, $L_{\text{fusion}} = 80$ cal/g = 336 J/g,
$L_{\text{vaporisation}} = 540$ cal/g = 2268 J/g,
$S_{\text{ice}} = 2100$ J/kg, $K = 0.5$ cal/g-K
and $S_{\text{water}} = 4200$ J/kg, $K = 1$ cal/g-K.

24. An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation $S = A + BT$, where $A = 100$ cal/kg-K and $B = 2 \times 10^{-2}$ cal/kg-K^2. If the final temperature of the container is 27°C, determine the mass of the container.
(Latent heat of fusion for water $= 8 \times 10^4$ cal/kg, specific heat of water $= 10^3$ cal/kg-K). **(2001, 5M)**

25. The temperature of 100 g of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose. **(1996, 2M)**

26. A lead bullet just melts when stopped by an obstacle. Assuming that 25 per cent of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C. **(1981, 3M)**
(Melting point of lead = 327°C, specific heat of lead = 0.03 cal/g °C, latent heat of fusion of lead = 6 cal/g, $J = 4.2$ J/cal.)

Topic 2 Thermal Expansion

Objective Questions I (Only one correct option)

1 A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion of the material of the rod, is (nearly) equal to **(2019 Main, 12 April II)**
(a) $9F/(\pi r^2 YT)$ (b) $6F/(\pi r^2 YT)$
(c) $3F/(\pi r^2 YT)$ (d) $F/(3\pi r^2 YT)$

2 At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C, it regains its original length of 0.2 m. The value of M is close to
[Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2 respectively, $g = 10\,\text{ms}^{-2}$] **(2019 Main, 12 April I)**
(a) 9 kg (b) 0.5 kg (c) 1.5 kg (d) 0.9 kg

3 Two rods A and B of identical dimensions are at temperature 30ºC. If A is heated upto 180ºC and B upto TºC, then new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is **(2019 Main, 11 Jan II)**
(a) 230ºC (b) 270ºC (c) 200ºC (d) 250ºC

4. An external pressure p is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by **(2017 Main)**
(a) $\frac{p}{\alpha K}$ (b) $\frac{3\alpha}{pK}$ (c) $3pK\alpha$ (d) $\frac{p}{3\alpha K}$

5. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coefficient of linear expansion α of the metal of the pendulum shaft are, respectively. **(2016 Main)**
(a) 25°C, $\alpha = 1.85 \times 10^{-5}$/°C (b) 60°C, $\alpha = 1.85 \times 10^{-4}$/°C
(c) 30°C, $\alpha = 1.85 \times 10^{-3}$/°C (d) 55°C, $\alpha = 1.85 \times 10^{-2}$/°C

6. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially, each of the wire has a length of 1 m 10°C. Now, the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is **(2016 Adv.)**
(a) 0.78 mm (b) 0.90 mm (c) 1.56 mm (d) 2.34 mm

7. Two rods, one of aluminium and the other made of steel, having initial length l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperature are raised by t°C, then find the ratio $\frac{l_1}{l_1 + l_2}$ **(2003, 2M)**
(a) $\frac{\alpha_s}{\alpha_a}$ (b) $\frac{\alpha_a}{\alpha_s}$ (c) $\frac{\alpha_s}{(\alpha_a + \alpha_s)}$ (d) $\frac{\alpha_a}{(\alpha_a + \alpha_s)}$

Objective Question II (One or more correct option)

8. A bimetallic strip is formed out of two identical strips– one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then, R is **(1999, 3M)**

(a) proportional to ΔT
(b) inversely proportional to ΔT
(c) proportional to $|\alpha_B - \alpha_C|$
(d) inversely proportional to $|\alpha_B - \alpha_C|$

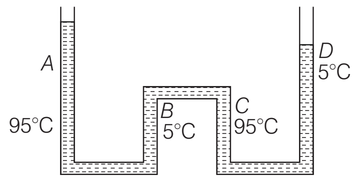
Integer Answer Type Question

9. Steel wire of length L at 40°C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length L. The coefficient of linear thermal expansion of the steel is $10^{-5}/°C$, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then the value of m in kg is nearly. **(2011)**

Analytical & Descriptive Questions

10. A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l showing all the steps. **(2004, 2M)**

11. The apparatus shown in figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95°C while the columns B and D are maintained at 5°C. The height of the liquid in A and D measured from the base line are 52.8 cm and 51 cm respectively. Determine the linear coefficient of thermal expansion of the liquid. **(1997, 5M)**



12. A composite rod is made by joining a copper rod, end to end, with a second rod of different material but of the same cross-section. At 25°C, the composite rod is 1 m in length, of which the length of the copper rod is 30 cm. At 125°C the length of the composite rod increases by 1.91 mm.

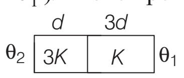
When the composite rod is not allowed to expand by holding it between two rigid walls, it is found that the length of the two constituents do not change with the rise of temperature. Find the Young's modulus and the coefficient of linear expansion of the second rod. (Given, Coefficient of linear expansion of copper $= 1.7 \times 10^{-5}$ per °C, Young's modulus of copper $= 1.3 \times 10^{11}$ N/m^2) **(1979)**

13. A sinker of weight w_0 has an apparent weight w_1 when placed in a liquid at a temperature T_1 and w_2 when weighed in the same liquid at a temperature T_2. The coefficient of cubical expansion of the material of the sinker is β. What is the coefficient of volume expansion of the liquid ? **(1978)**

Topic 3 Heat Transfer

Objective Questions I (Only one correct option)

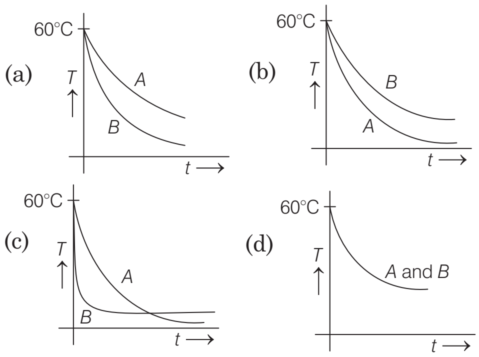
1. Two materials having coefficients of thermal conductivity '$3K$' and 'K' and thickness 'd' and '$3d$' respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, $(\theta_2 > \theta_1)$. The temperature at the interface is



(2019 Main, 9 April II)

(a) $\frac{\theta_2 + \theta_1}{2}$ (b) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$ (c) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$ (d) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$

2. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/ m^3 and specific heat of $2000\,\text{J kg}^{-1}\text{K}^{-1}$ while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1}K^{-1}. Which of the following best describes their temperature *versus* time graph schematically? (Assume the emissivity of both the beakers to be the same) **(2019 Main, 8 April I)**

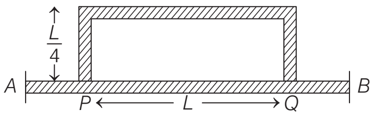


3. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius $2R$. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is **(2019 Main, 12 Jan I)**

(a) $\frac{K_1 + K_2}{2}$ (b) $\frac{K_1 + 3K_2}{4}$

(c) $\frac{2K_1 + 3K_2}{5}$ (d) $K_1 + K_2$

4. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$ is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to **(2019 Main, 9 Jan I)**



(a) 45° (b) 35°C
(c) 75°C (d) 60°C

5. Parallel rays of light of intensity $I = 912\ \text{Wm}^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan constant $\sigma = 5.7 \times 10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to **(2014 Main)**

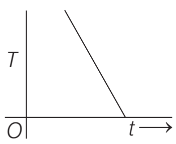
(a) 330 K (b) 660 K
(c) 990 K (d) 1550

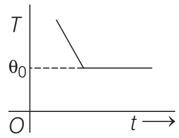
6. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod is 4 cm^2. End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively.

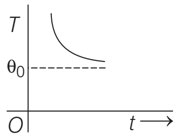
The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 in CGS units, respectively. Rate of heat flow through copper rod is **(2014 Main)**

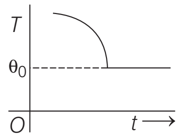
(a) 1.2 cal/s (b) 2.4 cal/s
(c) 4.8 cal/s (d) 6.0 cal/s

7. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0. The graph between the temperature T of the metal and time t will be closed to **(2013 Main)**

(a) 

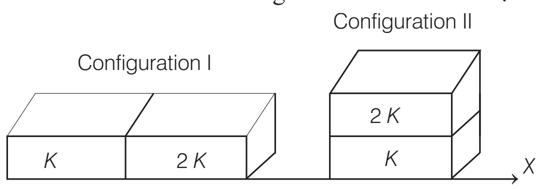
(b) 

(c) 

(d) 

8. Two rectangular blocks, having indentical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity K and the other $2K$.

The temperature difference between the ends along the X-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is **(2013 Adv.)**



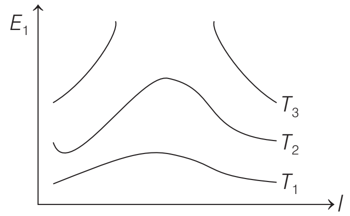
(a) 2.0 s (b) 3.0 s (c) 4.5 s (d) 6.0 s

9. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is **(2012)**

(a) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (b) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$

(c) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (d) $(97)^{\frac{1}{4}}\ T$

10. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following option is the correct match? **(2005, 2M)**



(a) Sun-T_1, tungsten filament-T_2, welding arc-T_3
(b) Sun-T_2, tungsten filament-T_1, welding arc-T_3
(c) Sun-T_3, tungsten filament-T_2, welding arc-T_1
(d) Sun-$T_{1,}$ tungsten filament-T_3, welding arc-T_2

11. In which of the following process, convection does not take place primarily? **(2005, 2M)**

(a) Sea and land breeze
(b) Boiling of water
(c) Warming of glass of bulb due to filament
(d) Heating air around a furnace

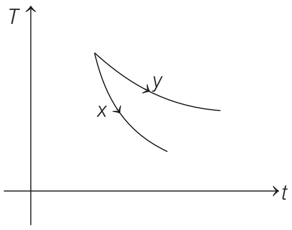
12. Three discs, A, B and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C respectively **(2004, 2M)**

(a) Q_A is maximum (b) Q_B is maximum
(c) Q_C is maximum (d) $Q_A = Q_B = Q_C$

13. Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 gram per second be the rate of melting of ice in the two cases respectively. The ratio $\frac{q_1}{q_2}$ is **(2004, 2M)**

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$ (c) $\frac{4}{1}$ (d) $\frac{1}{4}$

14. The graph, shown in the diagram, represents the variation of temperature (T) of the bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies **(2003, 2M)**

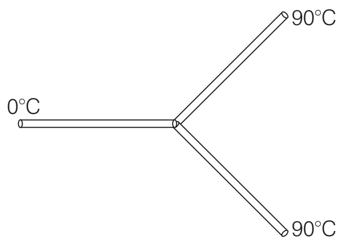


(a) $E_x > E_y$ and $a_x < a_y$
(b) $E_x < E_y$ and $a_x > a_y$
(c) $E_x > E_y$ and $a_x > a_y$
(d) $E_x < E_y$ and $a_x < a_y$

15. An ideal black body at room temperature is thrown into a furnace. It is observed that **(2002, 2M)**

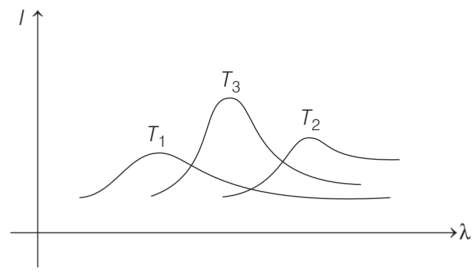
(a) initially it is the darkest body and at later times the brightest
(b) it is the darkest body at all times
(c) it cannot be distinguished at all times
(d) initially it is the darkest body and at later times it cannot be distinguished

16. Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of junction of the three rods will be **(2001, 2M)**



(a) 45°C (b) 60°C
(c) 30°C (d) 20°C

17. The plots of intensity *versus* wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are as shown. Their temperatures are such that **(2000, 2M)**



(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$
(c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

18. A black body is at a temperature of 2880 K. The energy of radiation emitted by this body with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant, $b = 2.88 \times 10^6$ nm-K. Then, **(1998, 2M)**

(a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$

19. A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be **(1997, 1M)**

(a) 225 (b) 450 (c) 900 (d) 1800

20. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the north star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperature of the sun and the north star is **(1997, 1M)**

(a) 1.46 (b) 0.69 (c) 1.21 (d) 0.83

21. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is **(1995, 2M)**

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{1}$ (d) $\left(\frac{1}{3}\right)^{1/3}$

22. Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC, right angled at B. The points A and B are maintained at temperatures T and $(\sqrt{2})T$ respectively. In the steady state, the temperature of the point C is T_c. Assuming that only heat conduction takes place, T_c/T is **(1995, 2M)**

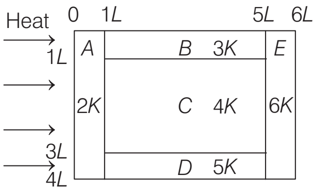
(a) $\frac{1}{2(\sqrt{2}-1)}$ (b) $\frac{3}{\sqrt{2}+1}$
(c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{(\sqrt{2}+1)}$

23. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is **(1988, 2M)**

(a) $K_1 + K_2$ (b) $K_1K_2/(K_1 + K_2)$
(c) $(K_1 + 3K_2)/4$ (d) $(3K_1 + K_2)/4$

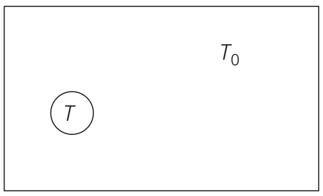
Objective Questions II (One or more correct option)

24. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat Q flows only from left to right through the blocks. Then, in steady state **(2011)**



(a) heat flow through A and E slabs are same
(b) heat flow through slab E is maximum
(c) temperature difference across slab E is smallest
(d) heat flow through C = heat flow through B + heat flow through D

25. A black body of temperature T is inside a chamber of temperature T_0. Now the closed chamber is slightly opened to sun such that temperature of black body (T) and chamber (T_0) remains constant **(2006, 3M)**



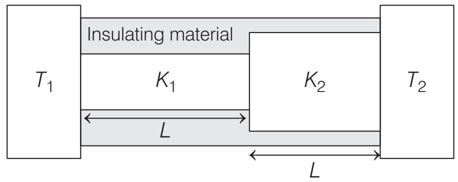
(a) black body will absorb more radiation
(b) black body will absorb less radiation
(c) black body emit more energy
(d) black body emit energy equal to energy absorbed by it

26. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A, by 1.00 μm. If the temperature of A is 5802 K **(1994, 2M)**

(a) the temperature of B is 1934 K
(b) $\lambda_B = 1.5\,\mu m$
(c) the temperature of B is 11604 K
(d) the temperature of B is 2901 K

Numerical Value

27. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300K$ and $T_2 = 100K$, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2, respectively. If the temperature at the junction of the cylinders in the steady state is 200K, then K_1/K_2 =.......... . **(2018 Adv.)**



Integer Answer Type Questions

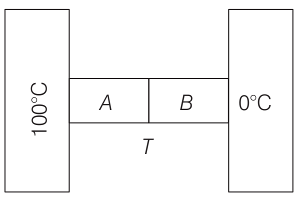
28. Two spherical stars A and B emit black body radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is **(2015 Adv.)**

29. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C? **(2016 Adv.)**

30. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? **(2010)**

Fill in the Blanks

31. Two metal cubes A and B of same size are arranged as shown in figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m °C and 200 W/m °C, respectively. After steady state is reached the temperature T of the interface will be …… . **(1996, 2M)**



32. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and inner surface of the shell is not to exceed T, the thickness of the shell should not be less than …… . **(1991, 1M)**

33. A solid copper sphere (density ρ and specific heat c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. The time required for the temperature of the sphere to drop to 100 K is …… . **(1991, 2M)**

34. The earth receives at its surface radiation from the sun at the rate of 1400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7×10^8 m. Treating the sun as a black body, it follows from the above data that its surface temperature is …… K. **(1989, 2M)**

True / False

35. Two spheres of the same material have radii 1 m and 4 m, temperature 4000 K and 2000 K respectively. The energy radiated per second by the first sphere is greater than that by the second. **(1988, 2M)**

Analytical & Descriptive Questions

36. A double-pane window used for insulating a room thermally from outside consists of two glass sheets each of area $1\,m^2$ and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room glass interface and the glass-outdoor interface are at constant temperatures of 27°C and 0°C respectively. Calculate the rate of heat flow through the window pane. Also, find the temperatures of other interfaces. Given, thermal conductivities of glass and air as 0.8 and $0.08\,W\,m^{-1}K^{-1}$ respectively. **(1997C,5M)**

37. A cylindrical block of length 0.4 m and area of cross-section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 W/mK and the specific heat capacity of the material of the disc is 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder. **(1992, 8M)**

38. An electric heater is used in a room of total wall area $137 m^2$ to maintain a temperature of +20°C inside it, when the outside temperature is –10°C. The walls have three different layers. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/m/°C respectively. **(1986, 8M)**

39. A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster? **(1982, 2M)**

40. A room is maintained at 20°C by a heater of resistance 20 Ω connected to 200 V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area $1\,m^2$ and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal $m^{-1}s^{-1}\,(°C)^{-1}$ and mechanical equivalent of heat is 4.2 $Jcal^{-1}$. **(1978)**

Topic 4 Kinetic Theory of Gases and Gas Equations

Objective Questions I (Only one correct option)

1. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process? **(2019 Main, 12 April II)**

(a) 25 J (b) 35 J (c) 30 J (d) 40 J

2. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume?

[Take, $R = 8.3$ J/mol-K] **(2019 Main, 12 April I)**

(a) 19.7 J/mol-K (b) 15.7 J/mol-K
(c) 17.4 J/mol-K (d) 21.6 J/mol-K

3. One mole of an ideal gas passes through a process, where pressure and volume obey the relation $p = p_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$.

Here, p_0 and V_0 are constants. Calculate the change in the temperature of the gas, if its volume changes from V_0 to $2V_0$. **(2019 Main, 10 April II)**

(a) $\frac{1}{2}\frac{p_0V_0}{R}$ (b) $\frac{1}{4}\frac{p_0V_0}{R}$ (c) $\frac{3}{4}\frac{p_0V_0}{R}$ (d) $\frac{5}{4}\frac{p_0V_0}{R}$

4. A $25 \times 10^{-3}\ m^3$ volume cylinder is filled with 1 mole of O_2 gas at room temperature (300 K). The molecular diameter of O_2 and its root mean square speed are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule? **(2019 Main, 10 April I)**

(a) $\sim 10^{10}$ (b) $\sim 10^{12}$ (c) $\sim 10^{11}$ (d) $\sim 10^{13}$

5. A cylinder with fixed capacity of 67.2 L contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is

[Take, $R = 8.31\ J\ mol^{-1}K^{-1}$] **(2019 Main, 10 April I)**

(a) 700 J (b) 748 J (c) 374 J (d) 350 J

6. The specific heats, C_p and C_V of a gas of diatomic molecules, A are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules B, has the corresponding values 30 and 21. If they are treated as ideal gases, then **(2019 Main, 9 April II)**
(a) A has a vibrational mode but B has none
(b) Both A and B have a vibrational mode each
(c) A has one vibrational mode and B has two
(d) A is rigid but B has a vibrational mode

7. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be **(2019 Main, 9 April I)**
(a) $\frac{m\bar{v}^2}{3k_B}$ (b) $\frac{m\bar{v}^2}{7k_B}$ (c) $\frac{m\bar{v}^2}{5k_B}$ (d) $\frac{m\bar{v}^2}{6k_B}$

8. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be **(2019 Main, 9 April I)**
(a) $100\sqrt{5}$ m/s (b) 80 m/s
(c) 100 m/s (d) $80\sqrt{5}$ m/s

9. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :
[Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K, Avogadro number $N_A = 6.02 \times 10^{26}$/kg, Radius of earth = 6.4×10^6 m, Gravitational acceleration on earth = 10 ms^{-2}] **(2019 Main, 8 Apri, II)**
(a) 10^4 K (b) 650 K (c) 3×10^5 K (d) 800K

10. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to **(2019 Main, 12 Jan II)**
(a) 4×10^{-8} s (b) 3×10^{-6} s
(c) 2×10^{-7} s (d) 0.5×10^{-8} s

11. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass m, will be given by
(where, R is universal gas constant and g is the acceleration due to gravity) **(2019 Main019, 12 Jan II)**
(a) $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1 l_2}\right]$ (b) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$
(c) $\frac{RT}{g}\left[\frac{2l_1 + l_2}{l_1 l_2}\right]$ (d) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1 l_2}\right]$

12. An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy of the gas is **(2019 Main, 12 Jan I)**
(a) 6×10^4 J (b) 10^8J (c) 9×10^6 J (d) 3×10^2 J

13. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is **(2019 Main, 11 Jan I)**
(a) 12 RT (b) 15 RT (c) 20 RT (d) 4 RT

14. 2 kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion ? **(2019 Main, 10 Jan II)**
(a) 10^6 J (b) 10^3 J (c) 10^4 J (d) 10^5 J

15. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled is about
(Take, R = 8.3 J/K-mol) **(2019 Main, 9 Jan II)**
(a) 10 kJ (b) 0.9 kJ (c) 14 kJ (d) 6 kJ

16. A mixture of 2 moles of helium gas (atomic mass = $4u$) and 1 mole of argon gas (atomic mass = $40u$) is kept at 300 K in a container. The ratio of their rms speeds
$\left[\frac{v_{\text{rms (helium)}}}{v_{\text{rms (argon)}}}\right]$ is close to **(2019 Main, 9 Jan I)**
(a) 0.32 (b) 2.24 (c) 3.16 (d) 0.45

17. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly **(2018 Main)**
(a) 2.35×10^4 N/m^2 (b) 2.35×10^3 N/m^2
(c) 4.70×10^2 N/m^2 (d) 2.35×10^2 N/m^2

18. The temperature of an open room of volume 30 **m**^{3} increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be **(2017 Main)**
(a) 1.38×10^{23} (b) 2.5×10^{25}
(c) -2.5×10^{25} (d) -1.61×10^{23}

19. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure p and volume V is given by pV^n = constant, then n is given by (Here, C_p and C_V are molar specific heat at constant pressure and constant volume, respectively) **(2016 Main)**
(a) $n = \frac{C_p}{C_V}$ (b) $n = \frac{C - C_p}{C - C_V}$
(c) $n = \frac{C_p - C}{C - C_V}$ (d) $n = \frac{C - C_V}{C - C_p}$

20. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is **(2013 Adv.)**
(a) 1 : 4 (b) 1 : 2
(c) 6 : 9 (d) 8 : 9

21. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{\text{rms}}\text{ (helium)}}{v_{\text{rms}}\text{ (argon)}}\right)$ is **(2012)**

(a) 0.32 (b) 0.45 (c) 2.24 (d) 3.16

22. A real gas behaves like an ideal gas if its **(2010)**

(a) pressure and temperature are both high
(b) pressure and temperature are both low
(c) pressure is high and temperature is low
(d) pressure is low and temperature is high

23. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is **(1999, 2M)**

(a) $4\,RT$ (b) $15\,RT$ (c) $9\,RT$ (d) $11\,RT$

24. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are found to be Δp and $1.5\,\Delta p$ respectively. Then **(1998, 2M)**

(a) $4\,m_A = 9\,m_B$ (b) $2\,m_A = 3\,m_B$
(c) $3\,m_A = 2\,m_B$ (d) $9\,m_A = 4\,m_B$

25. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is **(1998, 2M)**

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1
(d) depends on the moment of inertia of the two molecules

26. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is p. An identical vessel containing one mole of the gas (molar mass 4) at a temperature $2T$ has a pressure of **(1997, 1M)**

(a) $p/8$ (b) p (c) $2\,p$ (d) $8\,p$

27. The average translational kinetic energy of O_2 (molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (molar mass 28) molecules in eV at the same temperature is **(1997, 1M)**

(a) 0.0015 (b) 0.003 (c) 0.048 (d) 0.768

28. The average translational energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 m/s respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour) **(1997, 1M)**

(a) 12.42×10^{-21} J, 968 m/s (b) 8.78×10^{-21} J, 684 m/s
(c) 6.21×10^{-21} J, 968 m/s (d) 12.42×10^{-21} J, 684 m/s

29. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v, at 480 K it becomes **(1996, 2M)**

(a) $4\,v$ (b) $2\,v$ (c) $v/2$ (d) $v/4$

30. Three closed vessels A, B and C at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1, that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is **(1992, 2M)**

(a) $(v_1 + v_2)/2$ (b) v_1
(c) $(v_1\,v_2)^{1/2}$ (d) $\sqrt{3kT/M}$

where, M is the mass of an oxygen molecule.

31. If one mole of a monoatomic gas $(\gamma = 5/3)$ is mixed with one mole of a diatomic gas $(\gamma = 7/5)$, the value of γ for the mixture is **(1988, 2M)**

(a) 1.40 (b) 1.50
(c) 1.53 (d) 3.07

32. At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is

(a) H_2 (b) F_2 (c) O_2 (d) Cl_2

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) If Statement I is true; Statement II is false
(d) If Statement I is false; Statement II is true

33. Statement I The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

Statement II The molecules of a gas collide with each other and the velocities of the molecules change due to the collision. **(2007, 3M)**

Objective Questions II (One or more correct option)

34. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statements is/are **(2015 Adv.)**

(a) The average energy per mole of the gas mixture is $2RT$
(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{\frac{6}{5}}$
(c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{2}$
(d) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{\sqrt{2}}$

35. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m. Then, **(1998, 2M)**

(a) no molecule can have a speed greater than $\sqrt{2}v_{\text{rms}}$

(b) no molecule can have speed less than $v_p/\sqrt{2}$

(c) $v_p < \bar{v} < v_{\text{rms}}$

(d) the average kinetic energy of a molecule is $\frac{3}{4}mv_p^2$

36. From the following statements concerning ideal gas at any given temperature T, select the correct one (s). **(1995, 2M)**

(a) The coefficient of volume expansion at constant pressure is the same for all ideal gases

(b) The average translational kinetic energy per molecule of oxygen gas is 3 kT, k being Boltzmann constant

(c) The mean-free path of molecules increases with decrease in the pressure

(d) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different

Fill in the Blanks

37. A container of volume 1 m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300 K and the other part is vacuum. The whole system is thermally isolated from the surroundings. When the partition is removed, the gas expands to occupy the whole volume. Its temperature will now be …… **(1993, 1M)**

38. During an experiment, an ideal gas is found to obey an additional law p^2V = constant. The gas is initially at a temperature T and volume V. When it expands to a volume $2V$, the temperature becomes …… . **(1987, 2M)**

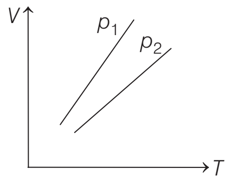
39. One mole of a monoatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is …… . **(1984, 2M)**

True / False

40. The root mean square (rms) speed of oxygen molecules (O_2) at a certain temperature T (degree absolute) is V. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged. **(1987, 2M)**

41. Two different gases at the same temperature have equal root mean square velocities. **(1982, 2M)**

42. The volume V *versus* temperature T graphs for a certain amount of a perfect gas at two pressure p_1 and p_2 are as shown in figure. It follows from the graphs that p_1 is greater than p_2. **(1982, 2M)**



43. The root mean square speeds of the molecules of different ideal gases, maintained at the same temperature are the same. **(1981, 2M)**

Analytical & Descriptive Questions

44. A cubical box of side 1 m contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 s, an atom travelling with the root mean square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take, $R = \frac{25}{3}$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K. **(2002, 5M)**

(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box.

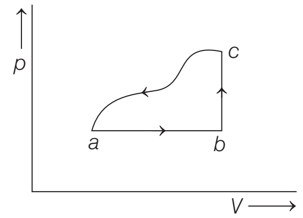
Topic 5 Thermodynamics

Objective Questions I (Only one correct option)

1. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are respectively, **(2019 Main, 12 April II)**

(a) 62°C, 124°C
(b) 99°C, 37°C
(c) 124°C, 62°C
(d) 37°C, 99°C

2. A sample of an ideal gas is taken through the cyclic process *abca* as shown in the figure. The change in the internal energy of the gas along the path *ca* is −180 J. The gas absorbs 250 J of heat along the path *ab* and 60 J along the path *bc*. The work done by the gas along the path *abc* is **(2019 Main, 12 April I)**



(a) 120 J (b) 130 J (c) 100 J (d) 140 J

3. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume, its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is **(2019 Main, 10 April II)**

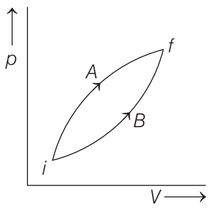
(a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$ (c) $\frac{3}{2}Q$ (d) $\frac{7}{5}Q$

4. n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is **(2019 Main, 10 April I)**

(a) $\frac{4nR}{C_V + nR}$ (b) $\frac{4nR}{C_V - nR}$

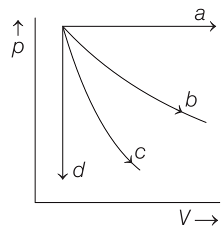
(c) $\frac{nR}{C_V - nR}$ (d) $\frac{nR}{C_V + nR}$

5. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies respectively, then **(2019 Main, 9 April I)**



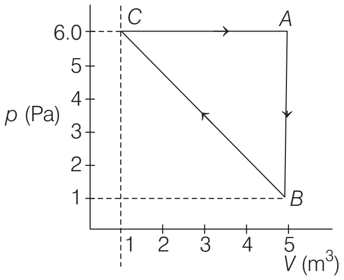
(a) $\Delta Q_A > \Delta Q_B$, $\Delta U_A > \Delta U_B$
(b) $\Delta Q_A < \Delta Q_B$, $\Delta U_A < \Delta U_B$
(c) $\Delta Q_A > \Delta Q_B$, $\Delta U_A = \Delta U_B$
(d) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$

6. The given diagram shows four processes, i.e. isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by **(2019 Main, 8 April II)**



(a) $d\ a\ b\ c$ (b) $a\ d\ b\ c$
(c) $d\ a\ c\ b$ (d) $a\ d\ c\ b$

7. For the given cyclic process CAB as shown for a gas, the work done is **(2019 Main, 12 Jan I)**



(a) 5 J (b) 10 J (c) 1 J (d) 30 J

8. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = k$, where k is a constant. In this process, the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (where, R is gas constant) **(2019 Main, 11 Jan II)**

(a) $\frac{1}{2}kR\Delta T$ (b) $\frac{2k}{3}\Delta T$ (c) $\frac{1}{2}R\Delta T$ (d) $\frac{3}{2}R\Delta T$

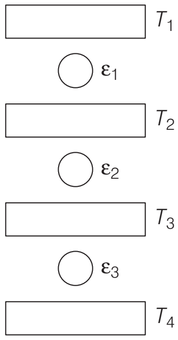
9. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is **(2019 Main, 11 Jan I)**

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

10. Half-mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to (Take, gas constant, $R = 8.31$ J/mol-K) **(Main 2019, 10 Jan II)**

(a) 291 J (b) 581 J
(c) 146 J (d) 73 J

11. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperatures T_2 and T_3, as shown with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if **(Main 2019, 10 Jan I)**

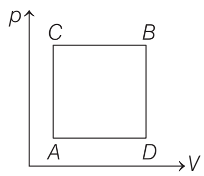


(a) $T_2 = (T_1^3 T_4)^{1/4}$; $T_3 = (T_1 T_4^3)^{1/4}$
(b) $T_2 = (T_1^2 T_4)^{1/3}$; $T_3 = (T_1 T_4^2)^{1/3}$
(c) $T_2 = (T_1 T_4)^{1/2}$; $T_3 = (T_1^2 T_4)^{1/3}$
(d) $T_2 = (T_1 T_4^2)^{1/3}$; $T_3 = (T_1^2 T_4)^{1/3}$

12. Two Carnot engines A and B are operated in series. The first one, A receives heat at $T_1 (= 600\text{ K})$ and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and in turn rejects to a heat reservoir at $T_3 (= 400\text{ K})$. Calculate the temperature T_2 if the work outputs of the two engines are equal. **(2019 Main, 9 Jan II)**

(a) 600 K (b) 500 K
(c) 400 K (d) 300 K

13. A gas can be taken from A to B *via* two different processes ACB and ADB. **(2019 Main, 9 Jan I)**



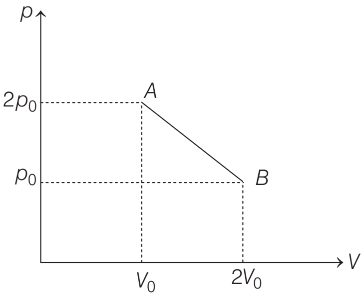
When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J the heat flow into the system in path ADB is

(a) 80 J (b) 40 J (c) 100 J (d) 20 J

14. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume $2V$. Calculate (i) the final temperature of the gas and (ii) change in its internal energy. **(2018 Main)**

(a) (i) 195 K (ii) 2.7 kJ (b) (i) 189 K (ii) 2.7 kJ
(c) (i) 195 K (ii) −2.7 kJ (d) (i) 189 K (ii) −2.7 kJ

15. n moles of an ideal gas undergoes a process A and B as shown in the figure. The maximum temperature of the gas during the process will be **(2016 Main)**

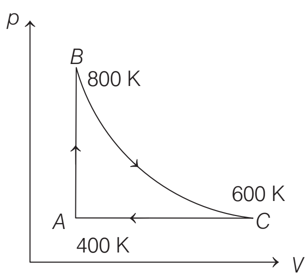


(a) $\frac{9}{4}\frac{p_0v_0}{nR}$ (b) $\frac{3}{2}\frac{p_0v_0}{nR}$ (c) $\frac{9}{2}\frac{p_0v_0}{nR}$ (d) $\frac{9p_0v_0}{nR}$

16. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $p_i = 10^5$ Pa and volume $V_1 = 10^{-3}\ \text{m}^3$ changes to a final state at $p_f = (1/32)\times 10^5$ Pa and $V_f = 8\times 10^{-3}\,\text{m}^3$ in an adiabatic quasi-static process, such that p^3V^5=constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps : an isobaric expansion at p_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately **(2016 Adv.)**

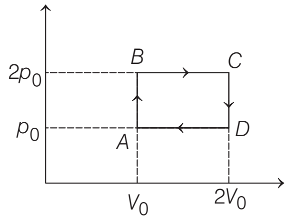
(a) 112 J (b) 294 J (c) 588 J (d) 813 J

17. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K, respectively. Choose the correct statement. **(2014 Main)**



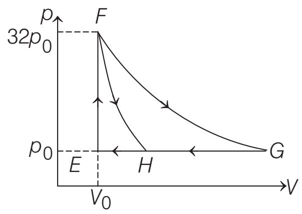
(a) The change in internal energy in whole cyclic process is 250 R
(b) The change in internal energy in the process CA is 700 R
(c) The change in internal energy in the process AB is $-350\,R$
(d) The change in internal energy in the process BC is $-500\,R$

18. The shown p-V diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is **(2013 Main)**



(a) p_0V_0 (b) $\left(\frac{13}{2}\right)p_0V_0$
(c) $\left(\frac{11}{2}\right)p_0V_0$ (d) $4\,p_0V_0$

19. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the p-V diagram. **(2013 Adv.)**



The processes involved are purely isochoric, isobaric, isothermal or adiabatic.

Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

	List I		List II
P.	$G \to E$	1.	$160\ p_0V_0 \ln 2$
Q.	$G \to H$	2.	$36\ p_0V_0$
R.	$F \to H$	3.	$24\ p_0V_0$
S.	$F \to G$	4.	$31\ p_0V_0$

Codes

	P	Q	R	S
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	1	2	4
(d)	1	3	2	4

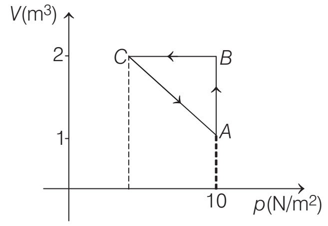
20. 5.6 L of helium gas at STP is adiabatically compressed to 0.7 L. Taking the initial temperature to be T_1, the work done in the process is **(2011)**

(a) $\frac{9}{8}RT_1$ (b) $\frac{3}{2}RT_1$ (c) $\frac{15}{8}RT_1$ (d) $\frac{9}{2}RT_1$

21. An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is p_1 and final pressure is p_3. The total work done is W. Then, **(2004, 2M)**

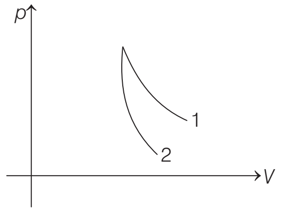
(a) $p_3 > p_1, W > 0$ (b) $p_3 < p_1, W < 0$
(c) $p_3 > p_1, W < 0$ (d) $p_3 = p_1, W = 0$

22. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is **(2002, 2M)**



(a) –5J (b) –10 J (c) –15 J (d) – 20 J

23. p-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to **(2001, 2M)**



(a) He and O_2 (b) O_2 and He
(c) He and Ar (d) O_2 and N_2

24. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways, the work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic, then **(2000, 2M)**

(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$
(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

25. A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by **(2000, 2M)**

(a) $(L_1/L_2)^{2/3}$ (b) (L_1/L_2)
(c) L_2/L_1 (d) $(L_2/L_1)^{2/3}$

26. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is **(1998, 2M)**

(a) 30 K (b) 18 K
(c) 50 K (d) 42 K

27. When an ideal diatomic gas is heated at constant pressure the fraction of the heat energy supplied which increases the internal energy of the gas is **(1990, 2M)**

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

28. An ideal monoatomic gas is taken round the cycle $ABCDA$ as shown in the p-V diagram (see figure). The work done during the cycle is **(1983, 1M)**

(a) pV (b) $2\,pV$ (c) $\frac{1}{2}\,pV$ (d) zero

Match the Columns

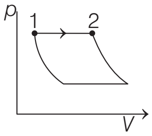
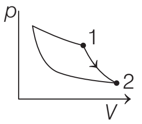
Directions (Q.Nos. 15-18) Matching the information given in the three columns of the following table.

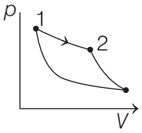
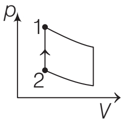
An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding p-V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standards notations and used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

29. One mole of a monoatomic ideal gas undergoes four thermodynamic processes as shown schematically in the pV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II. **(2018 Adv.)**

List-I		List-II	
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is $6p_0V_0$

(a) P $\to$ 4; Q $\to$ 3; R $\to$ 1; S $\to$ 2
(b) P $\to$ 1; Q $\to$ 3; R $\to$ 2; S $\to$ 4
(c) P $\to$ 3; Q $\to$ 4; R $\to$ 1; S $\to$ 2
(d) P $\to$ 3; Q $\to$ 4; R $\to$ 2; S $\to$ 1

	Column 1		Column 2		Column 3
(I)	$W_{1\to2} = \frac{1}{\gamma - 1}(p_2V_2 - p_1V_1)$	(i)	Isothermal	(P)	
(II)	$W_{1\to2} = -pV_2 + pV_1$	(ii)	Isochoric	(Q)	

(III)	$W_{1\to2} = 0$	(iii)	Isobaric	(R)	
(IV)	$W_{1\to2} = -nRT \ln\left(\frac{V_2}{V_1}\right)$	(iv)	Adiabatic	(S)	

30. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas? **(2017 Adv.)**

(a) (IV) (ii) (R) (b) (I) (ii) (Q)
(c) (I), (iv) (Q) (d) (III) (iv) (R)

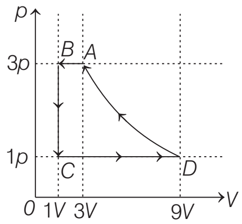
31. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - p\Delta V$? **(2017 Adv.)**

(a) (II) (iii) (S) (b) (II) (iii) (P)
(c) (III) (iii) (P) (d) (II) (iv) (R)

32. Which one of the following options is the correct combination? **(2017 Adv.)**

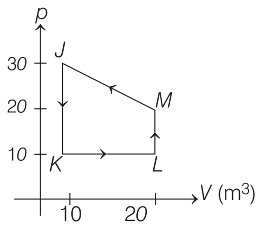
(a) (II) (iv) (P) (b) (III) (ii) (S)
(c) (II) (iv) (R) (d) (IV) (ii) (S)

33. One mole of a monatomic ideal gas is taken through a cycle $ABCDA$ as shown in the p-V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**. **(2011)**



	Column I		Column II
(A)	Process $A \to B$	(p)	Internal energy decreases
(B)	Process $B \to C$	(q)	Internal energy increases
(C)	Process $C \to D$	(r)	Heat is lost
(D)	Process $D \to A$	(s)	Heat is gained
		(t)	Work is done on the gas

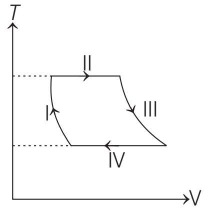
34. Match the following for the given process **(2006, 6M)**



	Column I		Column II
(A)	Process $J \to K$	(p)	$Q > 0$
(B)	Process $K \to L$	(q)	$W < 0$
(C)	Process $L \to M$	(r)	$W > 0$
(D)	Process $M \to J$	(s)	$Q < 0$

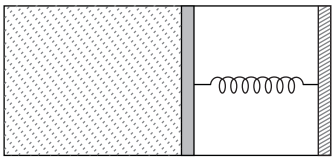
Objective Questions II (One or more correct option)

35. One mole of a monoatomic ideal gas undergoes a cyclic process as shown in the figure (where, V is the volume and T is the temperature). Which of the statements below is (are) true ? **(2018 Adv.)**



(a) Process I is an isochoric process
(b) In process II, gas absorbs heat
(c) In process IV, gas releases heat
(d) Processes I and III are not isobaric

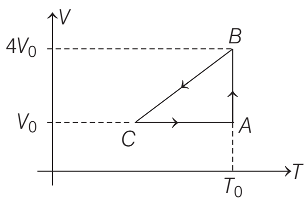
36. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure p_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure p_2 and volume V_2. During this process the piston moves out by a distance x. **(2015 Adv.)**



Ignoring the friction between the piston and the cylinder, the correct statements is/are

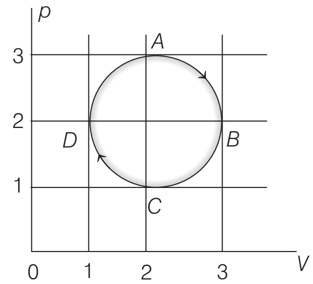
(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4} p_1V_1$
(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3p_1V_1$
(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3} p_1V_1$
(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is

37. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is p_0. Choose the correct option(s) from the following. **(2010)**



(a) Internal energies at A and B are the same
(b) Work done by the gas in process AB is p_0V_0 ln 4
(c) Pressure at C is $\frac{p_0}{4}$
(d) Temperature at C is $\frac{T_0}{4}$

38. The figure shows the p-V plot an ideal gas taken through a cycle $ABCDA$. The part ABC is a semi-circle and CDA is half of an ellipse. Then, **(2009)**



(a) the process during the path $A \to B$ is isothermal
(b) heat flows out of the gas during the path $B \to C \to D$
(c) work done during the path $A \to B \to C$ is zero
(d) positive work is done by the gas in the cycle $ABCDA$

39. During the melting of a slab of ice at 273 K at atmospheric pressure **(1998, 2M)**
(a) positive work is done by the ice-water system on the atmosphere
(b) positive work is done on the ice-water system by the atmosphere
(c) the internal energy of the ice-water increases
(d) the internal energy of the ice-water system decreases

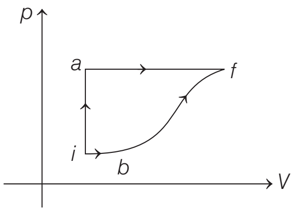
40. For an ideal gas **(1989, 2M)**
(a) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_V(T_2 - T_1)$, where C_V is the molar heat capacity at constant volume and n the number of moles of the gas
(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process
(c) the internal energy does not change in an isothermal process
(d) no heat is added or removed in an adiabatic process

Numerical Value

41. One mole of a monoatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0$ j mol^{-1} K^{-1}, the decrease in its internal energy in joule, is ……… . **(2018 Adv.)**

Integer Answer Type Questions

42. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf}/Q_{ib} is **(2014 Adv.)**



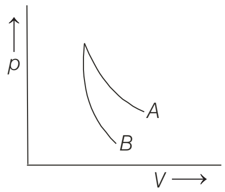
43. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in kelvin) and the final temperature is aT_i, the value of a is **(2010)**

Fill in the Blank

44. An ideal gas with pressure p, volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure p_i. If the same gas is expanded adiabatically to a volume $2V$, the final pressure is p_a. The ratio of the specific heats of the gas is 1.67. The ratio p_a/p_i is …… . **(1994, 2M)**

True / False

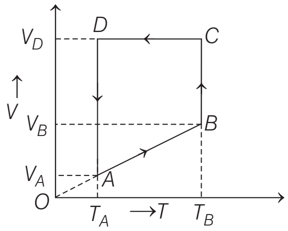
45. The curves A and B in the figure show p-V graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by the curve A. **(1985, 3M)**



Analytical & Descriptive Questions

46. A metal of mass 1 kg at constant atmospheric pressure and at initial temperature 20°C is given a heat of 20000 J. Find the following : **(2005, 6M)**
(a) change in temperature,
(b) work done and
(c) change in internal energy.
(Given, Specific heat = 400 J/kg/°C, coefficient of cubical expansion, $\gamma = 9 \times 10^{-5}$/°C, density $\rho = 9000$ kg/m^3, atmospheric pressure = 10^5 N/m^2)

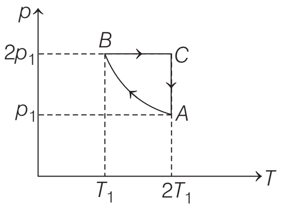
47. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The volume ratio are $\dfrac{V_B}{V_A} = 2$ and $\dfrac{V_D}{V_A} = 4$. If the temperature T_A at A is 27°C.



Calculate **(2001, 10M)**

(a) the temperature of the gas at point B,
(b) heat absorbed or released by the gas in each process,
(c) the total work done by the gas during the complete cycle.
Express your answer in terms of the gas constant R.

48. Two moles of an ideal monoatomic gas is taken through a cycle $ABCA$ as shown in the p-T diagram. During the process AB, pressure and temperature of the gas vary such that $pT =$ constant. If $T_1 = 300\,\text{K}$, calculate **(2000, 10M)**

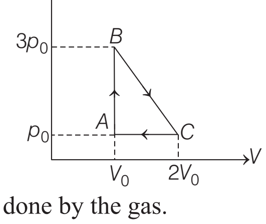


(a) the work done on the gas in the process AB and
(b) the heat absorbed or released by the gas in each of the processes.
Give answers in terms of the gas constant R.

49. Two moles of an ideal monoatomic gas initially at pressure p_1 and volume V_1 undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2. **(1999, 10M)**

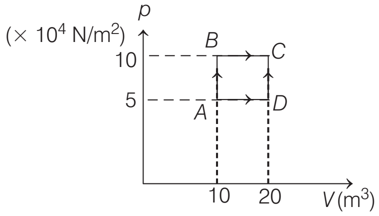
(a) Sketch the complete process on a p-V diagram.
(b) Find the total work done by the gas, the total change in internal energy and the final temperature of the gas.
(Give your answer in terms of p_1, V_1, V_2, Q and R)

50. One mole of an ideal monoatomic gas is taken round the cyclic process $ABCA$ as shown in figure. Calculate **(1998, 8M)**



(a) the work done by the gas.
(b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB.
(c) the net heat absorbed by the gas in the path BC.
(d) the maximum temperature attained by the gas during the cycle.

51. A sample of 2 kg monoatomic helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC (see fig). Given molecular mass of helium = 4. **(1997C, 5M)**



(a) What is the temperature of helium in each of the states A, B, C and D?
(b) Is there any way of telling afterwards which sample of helium went through the process ABC and which went through the process ADC? Write Yes or No.
(c) How much is the heat involved in the process ABC and ADC?

52. One mole of a diatomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A. The process $A \to B$ is an adiabatic compression. $B \to C$ is isobaric expansion, $C \to D$ an adiabatic expansion and $D \to A$ is isochoric. The volume ratio are $V_A / V_B = 16$ and $V_C / V_B = 2$ and the temperature at A is $T_A = 300\,\text{K}$. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle. **(1997, 5M)**

53. At 27°C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume $2V$.

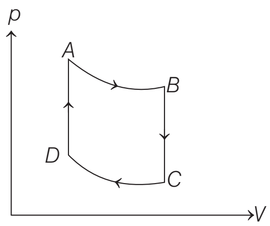
Calculate **(1996, 5M)**

(a) the final temperature of the gas,
(b) change in its internal energy,
(c) the work done by the gas during this process.

54. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960\,\text{J}, Q_2 = -5585\,\text{J}, Q_3 = -2980\,\text{J}$ and $Q_4 = 3645$ J respectively. The corresponding quantities of work involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4 respectively. **(1994, 6M)**

(a) Find the value of W_4.
(b) What is the efficiency of the cycle?

55. One mole of a monoatomic ideal gas is taken through the cycle shown in figure **(1993, 4+4+2M)**

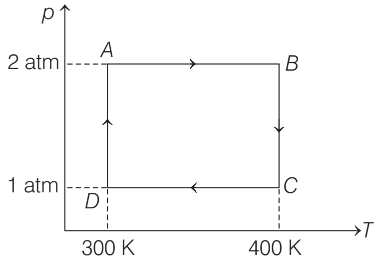


$A \to B$: adiabatic expansion
$B \to C$: cooling at constant volume
$C \to D$: adiabatic compression
$D \to A$: heating at constant volume.

The pressure and temperature at A, B, etc., are denoted by p_A, T_A, p_B, T_B etc., respectively. Given that, $T_A = 1000\,\text{K}$, $p_B = (2/3)p_A$ and $p_C = (1/3)p_A$, calculate the following quantities

(a) The work done by the gas in the process $A \rightarrow B$.
(b) The heat lost by the gas in the process $B \rightarrow C$.
(c) The temperature T_D. (Given : $(2/3)^{2/5} = 0.85$)

56. Two moles of helium gas undergo a cyclic process as shown in figure. Assuming the gas to be ideal, calculate the following quantities in this process. **(1992, 8M)**



(a) The net change in the heat energy.
(b) The net work done.
(c) The net change in internal energy.

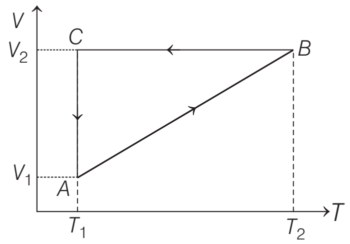
57. Three moles of an ideal gas ($C_p = \frac{7}{2}R$) at pressure, p_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally gas is compressed at constant volume to its original pressure p_A. **(1991, 4+4 M)**
(a) Sketch p-V and p-T diagrams for the complete process.
(b) Calculate the net work done by the gas, and net heat supplied to the gas during the complete process.

58. An ideal gas having initial pressure p, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $T/2$. **(1990, 7M)**
(a) How many degrees of freedom do gas molecules have?
(b) Obtain the work done by the gas during the expansion as a function of the initial pressure p and volume V.

59. An ideal gas has a specific heat at constant pressure $C_p = \frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m^3, at a temperature of 300 K and a pressure of $1.6 \times 10^6 \text{ N/m}^2$. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas. **(1987, 7M)**

60. Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 L respectively. The final volume of the gas is 2 L molar specific heat of the gas at constant volume is $3R/2$. **(1982, 8M)**

61. A cyclic process $ABCA$ shown in the V-T diagram is performed with a constant mass of an ideal gas. Show the same process on a p-V diagram. **(1981, 4M)**



(In the figure, line AB passes through origin).

Topic 6 Miscellaneous Problems

Objective Questions I (Only one correct option)

1 The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-ar^4}$. Then, the total number of molecules is proportional to **(2019 Main, 12 April II)**

(a) $n_0 \alpha^{-3/4}$ (b) $\sqrt{n_0}\,\alpha^{1/2}$ (c) $n_0 \alpha^{1/4}$ (d) $n_0 \alpha^{-3}$

2 If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of **(2019 Main, 8 April I)**

(a) 10^4 N/m^2 (b) 10^8 N/m^2
(c) 10^3 N/m^2 (d) 10^{16} N/m^2

3 A thermometer graduated according to a linear scale reads a value x_0, when in contact with boiling water and $x_0/3$, when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$? **(2019 Main, 11 Jan II)**

(a) 35 (b) 60 (c) 40 (d) 25

4 A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is $0.1 \text{ WK}^{-1}\text{m}^{-1}$, the energy flux through it in the steady state is **(2019 Main, 10 Jan I)**

(a) 90 Wm^{-2} (b) 65 Wm^{-2}
(c) 120 Wm^{-2} (d) 200 Wm^{-2}

5. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion, the relation between T and R is **(2015 Main)**

(a) $T \propto e^{-R}$ (b) $T \propto \frac{1}{R}$

(c) $T \propto e^{-3R}$ (d) $T \propto \frac{1}{R^3}$

6. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is $\left(\gamma = \frac{C_p}{C_V}\right)$ **(2015 Main)**

(a) $\frac{3\gamma + 5}{6}$ (b) $\frac{\gamma + 1}{2}$
(c) $\frac{3\gamma - 5}{6}$ (d) $\frac{\gamma - 1}{2}$

7. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases, body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively, is **(2015 Main)**

(a) ln 2, ln 2 (b) ln 2, 2 ln 2
(c) 2 ln 2, 8 ln 2 (d) ln 2, 4 ln 2

8. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is p_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **(2013 Main)**

(a) $\frac{1}{2\pi}\frac{A_\gamma p_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M p_0}{A^2\gamma}$
(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma p_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A_\gamma p_0}}$

9. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take $R = 8.31$ J/mol-K) **(2012)**

(a) 62 J (b) 104 J (c) 124 J (d) 208 J

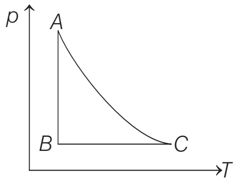
10. An ideal gas is expanding such that pT^2 = constant. The coefficient of volume expansion of the gas is **(2008, 3M)**

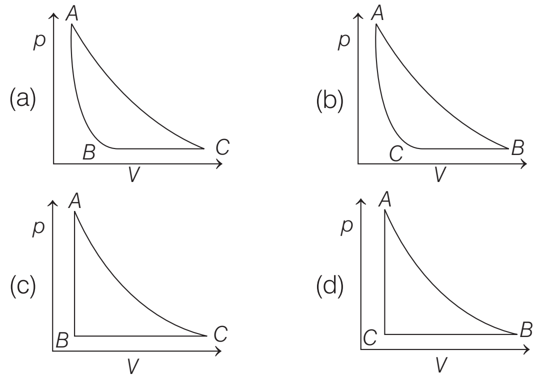
(a) $\frac{1}{T}$ (b) $\frac{2}{T}$
(c) $\frac{3}{T}$ (d) $\frac{4}{T}$

11. A body with area A and temperature T and emissivity $e = 0.6$ is kept inside a spherical black body. What will be the maximum energy radiated? **(2005, 2M)**

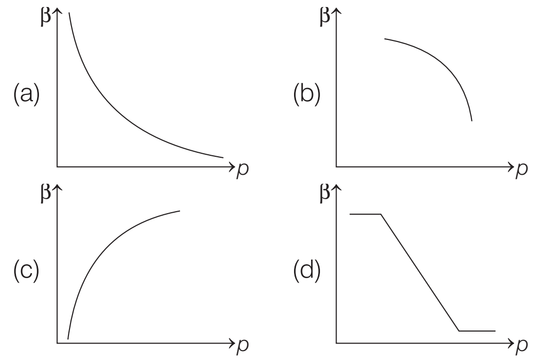
(a) 0.60 eAT^4 (b) 0.80 eAT^4
(c) 1.00 eAT^4 (d) 0.40 eAT^4

12. The p-T diagram for an ideal gas is shown in the figure, where AC is an adiabatic process, find the corresponding p-V diagram **(2003, 2M)**





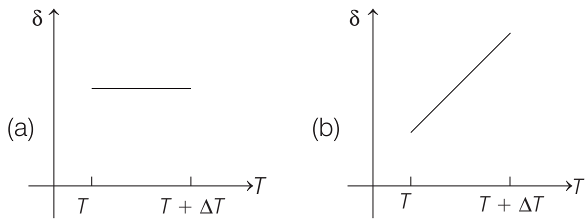
13. Which of the following graphs correctly represent the variation of $\beta = -\frac{dV/dp}{V}$ with p for an ideal gas at constant temperature? **(2002, 2M)**

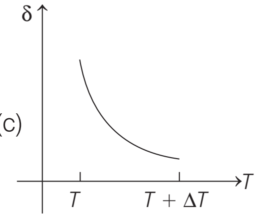


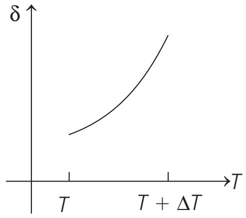
14. In a given process of an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas **(2001, S)**

(a) the temperature will decrease
(b) the volume will increase
(c) the pressure will remain constant
(d) the temperature will increase

15. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V/V\Delta T$ varies with temperature as **(2000, 2M)**



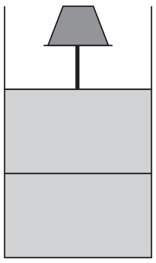
(c) 

(d) 

Passage Based Questions

Passage 1

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat.



The lower compartment of the container is filled with 2 moles of an ideal monoatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monoatomic gas are $C_V = \frac{3}{2}R, C_p = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_p = \frac{7}{2}R$. **(2014 Adv.)**

16. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be **(2014 Adv.)**

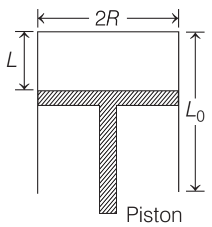
(a) 550 K (b) 525 K (c) 513 K (d) 490 K

17. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be **(2014 Adv.)**

(a) $250R$ (b) $200R$ (c) $100R$ (d) $-100R$

Passage 2

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is p_0.



18. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be **(2007, 4M)**

(a) p_0 (b) $\frac{p_0}{2}$

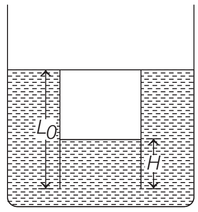
(c) $\frac{p_0}{2} + \frac{Mg}{\pi R^2}$ (d) $\frac{p_0}{2} - \frac{Mg}{\pi R^2}$

19. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is **(2007, 4M)**

(a) $\left(\frac{2p_0\pi R^2}{\pi R^2 p_0 + Mg}\right)(2L)$ (b) $\left(\frac{p_0\pi R^2 - Mg}{\pi R^2 p_0}\right)(2L)$

(c) $\left(\frac{p_0\pi R^2 + Mg}{\pi R^2 p_0}\right)(2L)$ (d) $\left(\frac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$

20. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies **(2007, 4M)**



(a) $\rho g(L_0 - H)^2 + p_0(L_0 - H) + L_0 p_0 = 0$

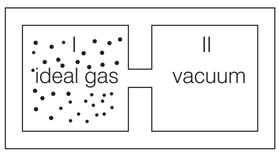
(b) $\rho g(L_0 - H)^2 - p_0(L_0 - H) - L_0 p_0 = 0$

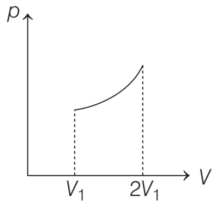
(c) $\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0$

(d) $\rho g(L_0 - H)^2 - p_0(L_0 - H) + L_0 p_0 = 0$

Match the Columns

21. **Column I** contains a list of processes involving expansion of an ideal gas. Match this with **Column II** describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **(2008, 7M)**

Column I	Column II
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened. 	(p) The temperature of the gas decreases
(B) An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^2}$, where V is the volume of the gas.	(q) The temperature of the gas increases or remains constant

Column I	Column II
(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^{4/3}}$, where V is its volume.	(r) The gas loses heat
(D) An ideal monoatomic gas expands such that its pressure p and volume V follows the behaviour shown in the graph. 	(s) The gas gains heat

22. **Column I** gives some devices and **Column II** gives some processes on which the functioning of these devices depend. Match the devices in **Column I** with the processes in **Column II.** **(2007, 6M)**

Column I	Column II
(A) Bimetallic strip	(p) Radiation from a hot body
(B) Steam engine	(q) Energy conversion
(C) Incandescent lamp	(r) Melting
(D) Electric fuse	(s) Thermal expansion of solids

Objective Questions II (One or more correct option)

23. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10K above the surrounding room temperature T_0. Take the room temperature to be $T_0 = 300$ K. For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460 \text{ Wm}^{-2}$ (where σ is the Stefan Boltzmann constant). Which of the following options is/are correct? **(2017 Adv.)**

(a) If the body temperature rises significantly, then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

(b) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(c) The amount of energy radiated by the body in 1s is close to 60 J

(d) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

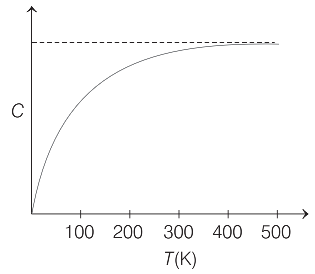
24. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to reasonable approximation.

(a) the rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T **(2013 Adv.)**

(b) heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K

(c) there is no change in the rate of heat absorbtion in the range 400-500 K

(d) the rate of heat absorption increases in the range 200-300 K



25. C_V and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then, **(2009)**

(a) $C_p - C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(b) $C_p + C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(c) $\frac{C_p}{C_V}$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(d) $C_p \cdot C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

26. An ideal gas is taken from the state A (pressure p, volume V) to the state B (pressure $p/2$, volume $2V$) along a straight line path in the p-V diagram. Select the correct statements from the following **(1993, 2M)**

(a) The work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along an isotherm

(b) In the T-V diagram, the path AB becomes a part of a parabola

(c) In the p-T diagram, the path AB becomes a part of a hyperbola

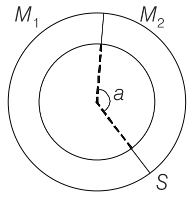
(d) In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases

Integer Answer Type Question

27. A metal rod AB of length $10x$ has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 calg^{-1} and latent heat of melting of ice is 80 calg^{-1}. If the point P is at a distance of λx from the ice end A, find the value of λ. (Neglect any heat loss to the surrounding.) **(2009)**

Fill in the Blanks

28. A ring shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α as shown in the figure is …… degrees. **(1997, 2M)**



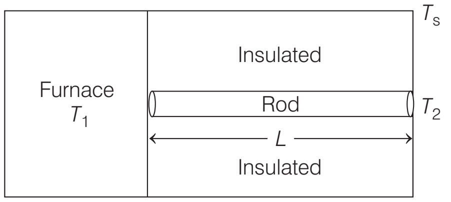
29. A gas thermometer is used as a standard thermometer for measurement of temperature. When the gas container of the thermometer is immersed in water at its triple point 273.16 K, the pressure in the gas thermometer reads $3.0 \times 10^4 \text{N/m}^2$. When the gas container of the same thermometer is immersed in another system, the gas pressure reads $3.5 \times 10^4 \text{N/m}^2$. The temperature of this system is therefore …… °C. **(1997, 1M)**

True / False

30. At a given temperature, the specific heat of a gas at a constant pressure is always greater than its specific heat at constant volume. **(1987, 2M)**

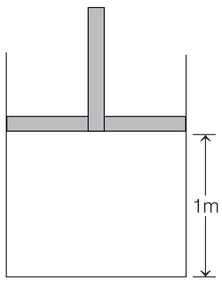
Analytical & Descriptive Questions

31. One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e.



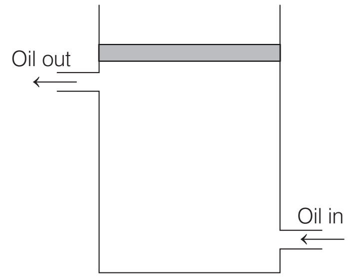
It is given that $T_2 = T_s + \Delta T$, where $\Delta T << T_s$, T_s being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_s)$, find the proportionality constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2. **(2004, 4M)**

32. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is 1 m^2. Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder.



If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas. You can leave the answer in fraction. **(2004, 2M)**

33. The top of an insulated cylindrical container is covered by a disc having emissivity 0.6 and conductivity 0.167 W/Km and thickness 1 cm. The temperature is maintained by circulating oil as shown. **(2003, 4M)**



(a) Find the radiation loss to the surroundings in W/m^2 if temperature of the upper surface of disc is 127°C and temperature of surroundings is 27°C.

(b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection.

(Given, $\sigma = \dfrac{17}{3} \times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$)

34. An insulated box containing a monoatomic gas of molar mass M moving with a speed v_0 is suddenly stopped. Find the increment in gas temperature as a result of stopping the box. **(2003, 2M)**

35. A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring losses. (For the steel wire : Young's modulus $= 2.1 \times 10^{11}$ Pa; Density = 7860 kg/m^3; Specific heat = 420 J/kg-K). **(2001, 5M)**

36. A solid body X of heat capacity C is kept in an atmosphere whose temperature is $T_A = 300$ K. At time $t = 0$, the temperature of X is $T_0 = 400$ K. It cools according to Newton's law of cooling. At time t_1 its temperature is found to be 350 K.

At this time (t_1) the body X is connected to a large body Y at atmospheric temperature T_A through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$. **(1998, 8M)**

37. A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of gas A with $\gamma = C_p/C_V = 5/3$ and another gas B with $\gamma = 7/5$ at a certain temperature T. The gram molecular weights of the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $pV^{19/13} = \text{c onstant}$, in adiabatic process. **(1995, 10M)**

(a) Find the number of gram moles of the gas B in the gaseous mixture.

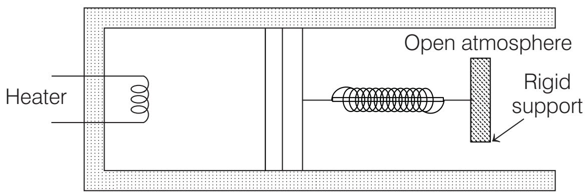
(b) Compute the speed of sound in the gaseous mixture at 300 K.

(c) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

(d) The mixture is compressed adiabatically to 1/5 of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.

38. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of 1×10^5 Nm^{-2}. The total mass of the mixture is 28 g. If the molar masses of neon and argon are 20 and 40 g mol^{-1} respectively, find the masses of the individual gases in the container assuming them to be ideal.
(Universal gas constant $R = 8.314$ J/mol-K). **(1994, 6M)**

39. An ideal monoatomic gas is confined in a cylinder by a spring-loaded piston of cross-section 8.0×10^{-3} m^2. Initially the gas is at 300 K and occupies a volume of 2.4×10^{-3} m^3 and the spring is in its relaxed (unstretched, uncompressed) state. The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m.



Calculate the final temperature of the gas and the heat supplied (in joules) by the heater. The force constant of the spring is 8000 N/m, and the atmospheric pressure 2.0×10^5 Nm^{-2}. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through the lead wires of the heater. The heat capacity of the heater coil is negligible. Assume the spring to the massless. **(1989, 8M)**

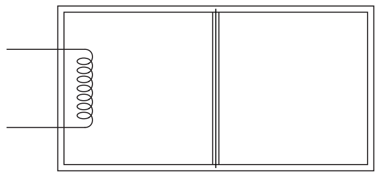
40. Two moles of helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 L. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value. **(1988, 6M)**

(a) Sketch the process on a p-V diagram.
(b) What are the final volume and pressure of the gas?
(c) What is the work done by the gas?

41. A thin tube of uniform cross-section is sealed at both ends. It lies horizontally, the middle 5 cm containing mercury and the two equal ends containing air at the same pressure p. When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46 cm and 44.5 cm respectively. Calculate the pressure p in centimetre of mercury. (The temperature of the system is kept at 30°C). **(1986, 6M)**

42. Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible. **(1985, 6M)**

43. The rectangular box shown in figure has a partition which can slide without friction along the length of the box.



Initially each of the two chambers of the box has one mole of a monoatomic ideal gas ($\gamma = 5/3$) at a pressure p_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands pushing the partition until the final pressure in both chambers becomes $243 p_0/32$. Determine (a) the final temperature of the gas in each chamber and (b) the work done by the gas in the right chamber. **(1984, 8M)**

44. One gram mole of oxygen at 27°C and one atmospheric pressure is enclosed in a vessel. (a) Assuming the molecules to be moving with v_{rms}, find the number of collisions per second which the molecules make with one square metre area of the vessel wall. (b) The vessel is next thermally insulated and moved with a constant speed v_0. It is then suddenly stopped. The process results in a rise of the temperature of the gas by 1°C. Calculate the speed v_0. **(1983, 8M)**

Answers

Topic 1

1. (a) **2.** (b) **3.** (b) **4.** (c)
5. (d) **6.** (a) **7.** (a)
8. (b) **9.** (a) **10.** (b) **11.** (a)
12. (a) **13.** (c) **14.** (b) **15.** (a)
16. (a) **17.** (b) **18.** 8 **19.** 5.5
20. $\dfrac{Pt}{M}$ **21.** $0°$ C
22. partly solid and partly liquid. **23.** 273K
24. 0.495 kg **25.** 12 g **26.** 409.8 m/s

Topic 2

1. (c) **2.** (a) **3.** (a)
4. (d) **5.** (a) **6.** (a) **7.** (c)
8. (b, d) **9.** 3 **10.** $\gamma_l = 2\alpha_s$
11. 6.7×10^{-5}/ °C
12. 1.105×10^{11} N/m^2, 2×10^{-5} per ° C
13. $\beta_l = \beta\left(\dfrac{w_0 - w_1}{w_0 - w_2}\right) + \dfrac{(w_2 - w_1)}{(w_0 - w_2)(T_2 - T_1)}$

Topic 3

1. (d) **2.** (b) **3.** (b) **4.** (a)
5. (a) **6.** (c) **7.** (c) **8.** (a)
9. (c) **10.** (c) **11.** (c) **12.** (b)
13. (c) **14.** (c) **15.** (a) **16.** (b)
17. (b) **18.** (d) **19.** (d) **20.** (b)
21. (d) **22.** (b) **23.** (c) **24.** (a, c, d)
25. (d) **26.** (a, b) **27.** 4 **28.** 2
29. 9 **30.** 9 **31.** 60°C **32.** $\dfrac{4\pi KTR^2}{P}$
33. 1.71 ρrc **34.** 5803 **35.** F
36. 41.6 W, 26.48 °C, 0.52 °C
37. 166.32 s **38.** 9091 W
39. Hollow sphere **40.** 15.24°C

Topic 4

1. (b) **2.** (c) **3.** (d) **4.** (*)
5. (b) **6.** (a) **7.** (b) **8.** (a)
9. (a) **10.** (a) **11.** (a) **12.** (c)
13. (b) **14.** (c) **15.** (a) **16.** (c)
17. (b) **18.** (c) **19.** (b) **20.** (d)
21. (d) **22.** (d) **23.** (d) **24.** (c)
25. (a) **26.** (c) **27.** (c) **28.** (d)
29. (b) **30.** (b) **31.** (b) **32.** (a)
33. (b)
34. (a, b, d) **35.** (c, d) **36.** (a, c) **37.** 300 K
38. $\sqrt{2}T$ **39.** 2 R **40.** F **41.** F
42. F **43.** F **44.** (a) 160 K (b) 3.312×10^{-21} J (c) 0.3 g

Topic 5

1. (b) **2.** (b) **3.** (d)
4. (d) **5.** (c) **6.** (a) **7.** (b)
8. (c) **9.** (a) **10.** (a) **11.** (b)
12. (b) **13.** (b) **14.** (d)
15. (a) **16.** (c) **17.** (d) **18.** (b)
19. (a) **20.** (a) **21.** (c) **22.** (a)
23. (b) **24.** (a) **25.** (d) **26.** (d)
27. (d) **28.** (a) **29.** (c) **30.** (c)
31. (b) **32.** (b)
33. (A) $\rightarrow$ p, r, t; (B) $\rightarrow$ p, r; (C) $\rightarrow$ q, s; (D) $\rightarrow$ r, t
34. (A) $\rightarrow$ s; (B) $\rightarrow$ p, r; (C) p; (D) $\rightarrow$ q,s
35. (b,c,d)
36. (a, b, c) **37.** (a, b) **38.** (b, d) **39.** (b, c)
40. (a, b, c, d) **41.** –900 J **42.** (2) **43.** 4
44. 0.628 **45.** T **46.** (a) 50°C (b) 0.05 J (c) 19999.95 J
47. (a) 600 K (b) 1500 R, 831.6 R, $-$900 R, $-$831.6 R (c) 600 R
48. (a) –1200 R (b) $Q_{AB} = -2100R$, $Q_{BC} = 1500R$, $Q_{CA} = 831.6R$
49. (b) (i) $W_{\text{Total}} = -\dfrac{3}{2} p_1V_1 \left[\left(\dfrac{V_1}{V_2}\right)^{2/3} - 1\right]$

(ii) $\Delta U_{\text{Total}} = \dfrac{3}{2} p_1V_1 \left[\left(\dfrac{V_1}{V_2}\right)^{2/3} - 1\right] + Q$

(iii) $T_{\text{Final}} = \dfrac{Q}{3R} + \dfrac{p_1V_1}{2R}\left(\dfrac{V_1}{V_2}\right)^{2/3}$

50. (a) p_0V_0 (b) $\dfrac{5}{2}p_0V_0$, $3p_0V_0$ (c) $\dfrac{p_0V_0}{2}$ (d) $\dfrac{25}{8}\,\dfrac{p_0V_0}{R}$
51. (a) $T_A = 120.34$ K, $T_B = 240.68$K, $T_C = 481.36$K, $T_D = 240.68$K
(b) No (c) $Q_{ABC} = 3.25 \times 10^6$ J, $Q_{ADC} = 2.75 \times 10^6$ J

52. $T_B = 909\text{K}, T_D = 791.4\text{ K}, 61.4\%$

53. (a) 189 K (b) – 2767 J (c) 2767 J

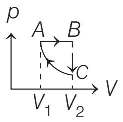
54. (a) 765 J (b) 10.82%

55. (a) 1869.75 J (b) 5297.6 J (c) 500 K

56. (a) 1152 J (b) 1152 J (c) zero **57.** (b) $0.58\ RT_A$

58. (a) $f = 5$ (b) $W = 1.23\ pV$ **59.** 675K, $3.6 \times 10^6\ \text{N/m}^2$

60. −972 J **61.**



Topic 6

1. (a)	**2.** (*)	**3.** (d)	**4.** (a)
5. (b)	**6.** (b)	**7.** (a)	**8.** (c)
9. (d)	**10.** (c)	**11.** Question is incomplete	
12. (none)	**13.** (a)	**14.** (a)	**15.** (c)
16. (d)	**17.** (d)	**18.** (a)	**19.** (d)
20. (c)	**21.** (A) $\rightarrow$ q; (B) $\rightarrow$ p, r; (C) $\rightarrow$ p, s; (D) $\rightarrow$ q, s		
22. (A) $\rightarrow$ s; (B) $\rightarrow$ q; (C) $\rightarrow$ p, q; (D) $\rightarrow$ q, r			**23.** (b, c, d)
24. (c, d)	**25.** (b, d)	**26.** (a, b, d)	**27.** 9

28. 192° **29.** 45.68°C **30.** T

31. Proportionality constant $= \dfrac{K}{4e\sigma LT_s^3 + K}$

32. $\dfrac{4}{3}$ m, 448.8K **33.** (a) 595 W/m^2 (b) 162.6 °C

34. $\Delta T = \dfrac{Mv_0^2}{3R}$ **35.** 4.568×10^{-3} °C

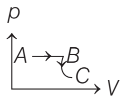
36. $[300 + 12.5\ e^{\frac{-2KAt_1}{CL}}]$ K

37. (a) 2 mol (b) 401 m/s (c) 0.167% (d) $-8.27 \times 10^{-5}\ V$

38. Mass of neon = 4.074 g, mass of argon = 23.926 g

39. 800 K, 720 J

40. (a)



(b) 113 L, $0.44 \times 10^5\ \text{N/m}^2$ (c) 12459 J

41. 75.4 cm of Hg

42. 83.83 cm of Hg

43. (a) $T_1 = 12.94\ T_0, T_2 = 2.25\ T_0$ (b) $-1.875\ RT_0$

44. (a) 1.96×10^{27}/ s (b) 36 m/s

Hints & Solutions

Topic 1 Calorimetry

1. **Key Idea** In such kind of heat transfer problems,
Heat given by water = Heat gained by ice
and $(\text{heat})_{\text{liquid}}$ = mass × specific heat × temperature $= \{m_l\ s_l\ \Delta T_l\}$
$(\text{Heat})_{\text{solid}}$ = (mass × latent heat) + (mass × specific heat × temperature)
$= (m_s \times L) + m_s \times s_s \times \Delta T_s$

Heat given by water is (specific heat of water is 1 cal g^{-1} °C^{-1})

$$(\Delta H)_{\text{water}} = M_2 \times 1 \times (50 - 0) = 50M_2 \quad \ldots\text{(i)}$$

Heat taken by ice is

$$(\Delta H)_{\text{ice}} = M_1 \times 0.5 \times [0 - (-10)] + M_1 \times L_{\text{ice}}$$

$$\Rightarrow \quad (\Delta H)_{\text{ice}} = 5M_1 + M_1 L_{\text{ice}} \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$(\Delta H)_{\text{water}} = (\Delta H)_{\text{ice}}$$

$$\therefore \quad 50M_2 = 5M_1 + M_1 L_{\text{ice}}$$

$$\Rightarrow \quad L_{\text{ice}} = \frac{50M_2 - 5M_1}{M_1}$$

$$L_{\text{ice}} = 50\frac{M_2}{M_1} - 5$$

2. Let x grams of water is evaporated.

According to the principle of calorimetry,

Heat lost by freezing water (that turns into ice) = Heat gained by evaporated water

Given, mass of water = 150 g

$$\Rightarrow \quad (150 - x) \times 10^{-3} \times 3.36 \times 10^5 = x \times 10^{-3} \times 2.10 \times 10^6$$

$$\Rightarrow \quad (150 - x) \times 3.36 = 21x$$

$$\Rightarrow \quad x = \frac{150}{7.25} = 20.6$$

$$\therefore \quad x \approx 20\text{ g}$$

3. In first case according to principle of calorimetry,

heat lost by liquid A = heat gained by liquid B

or $\quad m_A S_A \Delta T_A = m_B S_B \Delta T_B$

where, S_A is specific heat capacity of A and S_B is specific heat capacity of B

$$\Rightarrow \quad 100 \times S_A(100 - 90) = 50 \times S_B(90 - 75)$$

$$\Rightarrow \quad 1000 S_A = 50 \times 15 S_B$$

$$\text{or} \quad 4S_A = 3S_B \quad \ldots\text{(i)}$$

Similarly, in second case,

$$100 \times S_A(100 - T) = 50 \times S_B(T - 50)$$

where, T = Final temperature of the mixture.

$$\Rightarrow \quad 4S_A(100 - T) = 2S_B(T - 50)$$

Using Eq. (i),

$$3S_B(100 - T) = 2S_B(T - 50)$$

$$\text{or} \quad 300 - 3T = 2T - 100$$

$$\text{or} \quad 5T = 400$$

$$\text{or} \quad T = 80°\text{C}$$

4. Given, $VT = k$, (k is constant)

or $$T \propto \frac{1}{V} \quad \ldots\text{(i)}$$

Using ideal gas equation,

$$pV = nRT$$
$$pV \propto T$$

$\Rightarrow$ $$pV \propto \frac{1}{V}$$

or $$pV^2 = \text{constant} \quad \ldots\text{(ii)}$$

i.e a polytropic process with $x = 2$.

(Polytropic process means, pV^x = constant)

We know that, work done in a polytropic process is given by

$$\Delta W = \frac{p_2V_2 - p_1V_1}{1-x} \text{ (for } x \neq 1) \quad \text{...(iii)}$$

and, $$\Delta W = pV \ln\left(\frac{V_2}{V_1}\right) \text{(for } x = 1)$$

Here, $x = 2$,

$\therefore$ $$\Delta W = \frac{p_2V_2 - p_1V_1}{1-x} = \frac{nR(T_2 - T_1)}{1-x}$$

$\Rightarrow$ $$\Delta W = \frac{nR\Delta T}{1-`2} = -nR\Delta T \quad \text{...(iv)}$$

Now, for monoatomic gas change in internal energy is given by

$$\Delta U = \frac{3}{2}R\Delta T \quad \text{...(v)}$$

Using first law of thermodynamics, heat absorbed by one mole gas is

$$\Delta Q = \Delta W + \Delta U = \frac{3}{2}R\Delta T - R\Delta T$$

$\Rightarrow$ $$\Delta Q = \frac{1}{2}R\Delta T$$

5. Using heat lost or gained without change in state is $\Delta Q = ms\Delta T$, where s is specific heat capacity and $T =$ change in temperature

Let final temperature of ball be T.

Then heat lost by ball is,

$$\Delta Q = 0.1 \times 400(500 - T) \quad \text{...(i)}$$

This lost heat by ball is gained by water and vessel and given as

Heat gained by water,

$$\Delta Q_1 = 0.5 \times 4200(T - 30) \quad \text{...(ii)}$$

and heat gained by vessel is

$$\Delta Q_2 = \text{heat capacity} \times \Delta T$$
$$= 800 \times (T - 30) \quad \text{...(iii)}$$

According to principle of calorimetry, total heat lost = total heat gained

$\Rightarrow$ $$0.1 \times 400(500 - T)$$
$$= 0.5 \times 4200(T - 30) + 800(T - 30)$$

$\Rightarrow$ $$(500 - T) = \frac{(2100 + 800)(T - 30)}{40}$$

$\Rightarrow$ $$500 - T = 72.5(T - 30)$$

$\Rightarrow$ $500 + 217.5 = 72.5T$ or $T = 36.39$ K

So, percentage increment in temperature of water

$$\% = \frac{36.39 - 30}{30} \times 100 \approx 20\%$$

6. Let amount of ice be 'x' gm.

According to the principle of calorimeter,

heat lost by water = heat gained by ice

Here, heat lost by water

$$\Delta Q = ms_{\text{water}} \, \Delta T$$

Substituting the given values, we get

$$\Delta Q = 50 \times 4.2 \times 40$$

Heat gained by ice,

$$\Delta Q = x \, s_{\text{ice}} \, \Delta T + (x - 20) L$$
$$= x \times 2.1 \times 20 + (x - 20) \times 334$$
$$= 20\,x \times 2.1 + 334x - 6680$$

$\therefore$ $20\,x \times 2.1 + 334x - 6680 = 50 \times 4.2 \times 40$

$$42x + 334x - 6680 = 8400$$

$\Rightarrow$ $$376\,x = 15080$$

or $$x = 40.10 \text{ g}$$
$$x \simeq 40 \text{ g}$$

7.

> **Key Idea** The principle of calorimetry states that total heat lost by the hotter body equals to the total heat gained by colder body, provided that there is no exchange of heat with the surroundings.

Let specific heat of unknown metal is s and heat lost by this metal is ΔQ.

Heat lost and specific heat of a certain material/substance are related as

$$\Delta Q = ms\Delta T \quad \ldots\text{ (i)}$$

For unknown metal, $m = 192$ g and

$$\Delta T = (100 - 21.5)\,^\circ\text{C}$$

$\therefore$ $$\Delta Q' = 192(100 - 21.5) \times s \quad \text{...(ii)}$$

Now, this heat is gained by the calorimeter and water inside it.

As, heat gained by calorimeter can be calculated by Eq. (i).

So, for brass specific heat,

$$s = 394 \text{ J kg}^{-1} \text{ K}^{-1} \quad \text{(given)}$$
$$= 0.394 \text{ J g}^{-1} \text{ K}^{-1}$$

Mass of calorimeter, $m = 128$ g

Change in temperature, $\Delta T = (21.5 - 8.4)^\circ$C

So, using Eq. (i) for calorimeter, heat gained by brass

$$\Delta Q_1 = 128 \times 0.394 \times (21.5 - 8.4) \quad \ldots\text{(iii)}$$

Heat gained by water can be calculated as follows

mass of water, $m = 240$ g,

specific heat of water, $s = 4.18 \text{ J g}^{-1}\text{K}^{-1}$, change in temperature, $\Delta T = (21.5 - 8.4)^\circ C$

Using Eq. (i) for water also, we get

heat gained by water,

$$\Delta Q_2 = 240 \times 4.18 \times (21.5 - 8.4) \quad \ldots\text{(iv)}$$

Now, according to the principle of calorimeter, the total heat gained by the calorimeter and water must be equal to heat lost by unknown metal

$$\Delta Q' = \Delta Q_1 + \Delta Q_2$$

Using Eqs. (ii), (iii) and (iv), we get

$$\Rightarrow \quad = 192(100 - 21.5) \times s$$

$$= 128 \times 0.394 \times (21.5 - 8.4) + 240 \times 4.18 \times (21.5 - 8.4)$$

$$\Rightarrow \quad 15072\, s = 660.65 + 13142$$

$$\Rightarrow \quad s = 0.916 \text{ J g}^{-1} \text{ K}^{-1}$$

or $\quad s = 916 \text{ J Kg}^{-1}\text{K}^{-1}$.

8. By Mayor's relation, for 1 g mole of a gas,

$$C_p - C_V = R$$

So, when n gram moles are given,

$$C_p - C_V = \frac{R}{n}$$

As per given question,

$$a = C_p - C_V = \frac{R}{2}; \text{ for } H_2$$

$$b = C_p - C_V = \frac{R}{28}; \text{ for } N_2$$

$$a = 14b$$

9. Heat gained (water + calorimeter) = Heat lost by copper ball

$$\Rightarrow m_w s_w \Delta T + m_c s_c \Delta T = m_B s_B \Delta T$$

$$\Rightarrow 170 \times 1 \times 45 + 100 \times 0.1 \times 45 = 100 \times 0.1 \times (T - 75)$$

$$\therefore \quad T = 885^\circ C$$

10. Heat generated in device in 3 h

$$= \text{time} \times \text{power} = 3 \times 3600 \times 3 \times 10^3 = 324 \times 10^5 \text{ J}$$

Heat used to heat water

$$= ms\Delta\theta = 120 \times 1 \times 4.2 \times 10^3 \times 20\text{J}$$

Heat absorbed by coolant

$$= Pt = 324 \times 10^5 - 120 \times 1 \times 4.2 \times 10^3 \times 20\text{J}$$

$$Pt = (325 - 100.8) \times 10^5 \text{ J}$$

$$P = \frac{223.2 \times 10^8}{3600} = 2067 \text{ W}$$

11. 1 calorie is the heat required to raise the temperature of 1 g of water from 14.5°C to 15.5°C at 760 mm of Hg.
Hence, correct option is (a).

12. Energy gained by water (in 1 s)
= energy supplied − energy lost = 1000 J − 160 J = 840 J
Total heat required to raise the temperature of water from 27°C to 77°C is $ms\Delta\theta$.

Hence, the required time

$$t = \frac{ms\Delta\theta}{\text{rate by which energy is gained by water}}$$

$$= \frac{(2)(4.2 \times 10^3)(50)}{840}$$

$$= 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

13. Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen in gaseous state will further increase.

14. Heat released by 5 kg of water when its temperature falls from 20°C to 0°C is,

$$Q_1 = mc\Delta\theta = (5)(10^3)(20 - 0) = 10^5 \text{ cal}$$

when 2 kg ice at –20°C comes to a temperature of 0°C, it takes an energy

$$Q_2 = mc\Delta\theta = (2)(500)(20) = 0.2 \times 10^5 \text{ cal}$$

The remaining heat $Q = Q_1 - Q_2 = 0.8 \times 10^5$ cal will melt a mass m of the ice, where,

$$m = \frac{Q}{L} = \frac{0.8 \times 10^5}{80 \times 10^3} = 1 \text{kg}$$

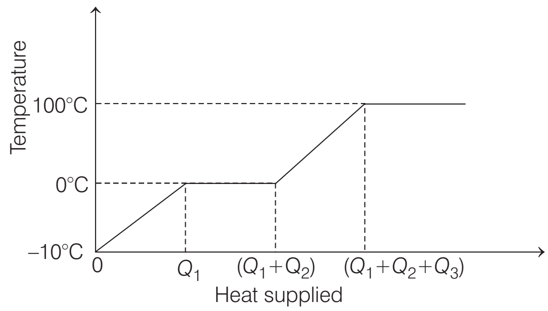
So, the temperature of the mixture will be 0°C, mass of water in it is 5 + 1 = 6 kg and mass of ice is 2 − 1 = 1 kg.

15. The temperature of ice will first increase from –10°C to 0°C. Heat supplied in this process will be

$$Q_1 = ms_i(10),$$

where, $\quad m$ = mass of ice

$\quad s_i$ = specific heat of ice



Then, ice starts melting. Temperature during melting will remain constant (0°C).
Heat supplied in this process will be
$Q_2 = mL$, L = latent heat of melting.

Now, the temperature of water will increase from 0°C to 100°C. Heat supplied will be

$$Q_3 = ms_w(100)$$

where, $\quad s_w$ = Specific heat of water.

Finally, water at 100°C will be converted into steam at 100°C and during this process temperature again remains constant. Temperature *versus* heat supplied graph will be as shown in above figure.

16. Heat required $Q_1 = (1.1 + 0.02) \times 10^3 \times 1 \times (80 - 15)$

$$= 72800 \text{ cal}$$

Heat given it m (in kg) steam is condensed :

$Q_2 = (m \times 540 \times 10^3) + m \times 1 \times 10^3 \times (100 - 80)$

Equating $Q_1 = Q_2$, we get

$$m = 0.130 \text{ kg}$$

17. $Q_1 = nC_p\,\Delta T, Q_2 = nC_V\,\Delta T, \dfrac{Q_2}{Q_1} = \dfrac{C_V}{C_p} = \dfrac{1}{\gamma}$

or $$Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50 \text{ cal}$$

18. Language of question is slightly wrong. As heat capacity and specific heat are two different physical quantities. Unit of heat capacity is J - kg^{-1} not J - kg^{-1} - $°C^{-1}$. The heat capacity given in the question is really the specific heat. Now applying the heat exchange equation.

$$420 = (m \times 10^{-3})(2100)(5) + (1 \times 10^{-3})(3.36 \times 10^5)$$

Solving this equation, we get

$$m = 8 \text{ g}$$

$\therefore$ The correct answer is 8.

19. Heat required to melt the block

$$= (0.28 \text{kg})(3.3 \times 10^5 \text{ J/kg}) = 9.24 \times 10^4 \text{ J}$$

Heat received per second $= (1400 \text{J/m}^2)(0.2\text{m}^2) = 280 \text{ J}$

$\therefore$ Time taken (in minutes) $= \dfrac{9.24 \times 10^4}{280 \times 60} = 5.5$

20. Heat lost in time $t = Pt = ML \Rightarrow \therefore L = \dfrac{Pt}{M}$

21. Heat liberated when 300 g water at 25°C goes to water at 0°C : $Q = ms\Delta\theta = (300)(1)(25) = 7500$ cal

From $Q = mL$, this much heat can melt mass of ice given by

$$m = \frac{Q}{L} = \frac{7500}{80} = 93.75 \text{ g}$$

i.e. whole ice will not melt.

Hence, the mixture will be at 0°C.

Mass of water in mixture

$$= 300 + 93.75$$
$$= 393.75 \text{ g and}$$

Mass of ice in mixture

$$= 100 - 93.75 = 6.25 \text{ g}$$

22. O to $A \longrightarrow$ material is in solid state (only temperature is increasing).

A to $B \longrightarrow$ material is partially solid and partially in liquid state.

B to $C \longrightarrow$ material is in liquid state.

C to $D \longrightarrow$ material is partially liquid and partially in vapour state.

23. 0.05 kg steam at 373 K $\xrightarrow{Q_1}$ 0.05 kg water at 373 K

0.05 kg water at 373 K $\xrightarrow{Q_2}$ 0.05 kg water at 273 K

0.45 kg ice at 253 K $\xrightarrow{Q_3}$ 0.45 kg ice at 273 K

0.45 kg ice at 273 K $\xrightarrow{Q_4}$ 0.45 kg water at 273 K

$Q_1 = (50)(540) = 27000 \text{ cal} = 27 \text{ kcal}$

$Q_2 = (50)(1)(100) = 5000 \text{ cal} = 5 \text{ kcal}$

$Q_3 = (450)(0.5)(20) = 4500 \text{ cal} = 4.5 \text{ kcal}$

$Q_4 = (450)(80) = 36000 \text{ cal} = 36 \text{ kcal}$

Now, since $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_3 + Q_4$ ice will come to 273 K from 253 K, but whole ice will not melt. Therefore, temperature of the mixture is 273 K.

24. Let m be the mass of the container.

Initial temperature of container,

$$T_i = (227 + 273) = 500 \text{ K}$$

and final temperature of container,

$$T_f = (27 + 273) = 300 \text{ K}$$

Now, heat gained by the ice cube = heat lost by the container

$\therefore \quad (0.1)(8 \times 10^4) + (0.1)(10^3)(27) = -m\int_{500}^{300}(A + BT)\,dT$

or $$10700 = -m\left[AT + \frac{BT^2}{2}\right]_{500}^{300}$$

After substituting the values of A and B and the proper limits, we get

$$m = 0.495 \text{ kg}$$

25. Let m be the mass of the steam required to raise the temperature of 100 g of water from 24°C to 90°C.

Heat lost by steam = Heat gained by water

$\therefore \quad m(L + s\Delta\theta_1) = 100 s\Delta\theta_2$ or $m = \dfrac{(100)(s)(\Delta\theta_2)}{L + s(\Delta\theta_1)}$

Here, s = specific heat of water = 1 cal/g-°C,

L = latent heat of vaporisation = 540 cal/g.

$$\Delta\theta_1 = (100 - 90) = 10°\text{C}$$

and $$\Delta\theta_2 = (90 - 24) = 66°\text{C}$$

Substituting the values, we have

$$m = \frac{(100)(1)(66)}{(540) + (1)(10)} = 12 \text{ g} \Rightarrow \therefore \quad m = 12 \text{ g}$$

26. 75% heat is retained by bullet

$$\frac{3}{4}\left[\frac{1}{2}mv^2\right] = ms\,\Delta\theta + mL \text{ or } v = \sqrt{\frac{(8s\Delta\theta + 8L)}{3}}$$

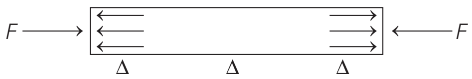
Substituting the values, we have

$$v = \sqrt{\frac{(8 \times 0.03 \times 4.2 \times 300) + (8 \times 6 \times 4.2)}{3 \times 10^{-3}}}$$

$$= 409.8 \text{ m/s}$$

Topic 2 Thermal Expansion

1. As length of rod remains unchanged,



Strain caused by compressive forces is equal and opposite to the thermal strain.
Now, compressive strain is obtained by using formula for Young's modulus,

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$$

Compressive strain,

$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi Y r^2} \quad \ldots(\text{i})$$

Also, thermal strain in rod is obtained by using formula for expansion in rod,

$$\Delta l = l \alpha \, \Delta T$$

$\Rightarrow$ Thermal strain,

$$\frac{\Delta l}{l} = \alpha \, \Delta T \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\frac{F}{\pi r^2 Y} = \alpha \, T \quad [\because \Delta T = T]$$

$$\Rightarrow \quad \alpha = \frac{F}{\pi r^2 Y T}$$

Hence, coefficient of volumetric expansion of rod is

$$\gamma = 3\alpha = \frac{3F}{\pi r^2 Y T}$$

2. Given, $T_1 = 40^\circ$ C and $T_2 = 20^\circ$ C

$$\Rightarrow \quad \Delta T = T_1 - T_2 = 40 - 20 = 20^\circ \text{ C}$$

Also, Young's modulus,

$$Y = 10^{11} \text{ N/m}^2$$

Coefficient of linear expansion,

$$\alpha = 10^{-5} /^\circ\text{C}$$

Area of the brass wire, $A = \pi \times (10^{-3})^2 \text{ m}^2$

Now, expansion in the wire due to rise in temperature is

$$\Delta l = l \alpha \Delta T \Rightarrow \frac{\Delta l}{l} = \alpha \Delta T \quad \ldots(\text{i})$$

We know that, Young's modulus is defined as

$$Y = \frac{Mgl}{A \Delta l} \Rightarrow M = \frac{YA\Delta l}{gl} \quad \ldots(\text{ii})$$

Using Eq. (i), we get

$$M = \frac{YA}{g} \times \alpha \Delta T = \frac{10^{11} \times 22 \times 10^{-6} \times 10^{-5} \times 20}{7 \times 10}$$

$$\Rightarrow \quad M = \frac{22 \times 20}{7 \times 10} = \frac{44}{7} = 6.28 \text{ kg}$$

Which is closest to 9, so option (a) is nearly correct.

3. Let initial length of identical rods is l_0 Thermal expansion in length of rod due to heating is given by the relation

$$\Delta l = l_0 \, \alpha \, (\Delta T) = l_0 \, \alpha \, (T_2 - T_1)$$

Here, α is coefficient of linear expansion.
So, change in length of rods are

$$\Delta l_1 = l_0 \, \alpha_1 (180 - 30)$$

$$\Delta l_2 = l_0 \alpha_2 (T - 30)$$

Because new lengths are same, so change in lengths of both rods are equal.

i.e. $\quad \Delta l_1 = \Delta l_2$

$$\Rightarrow \quad l_0 \, \alpha_1 \, (180 - 30) = l_0 \, \alpha_2 \, (T - 30)$$

$$\text{or} \quad \frac{\alpha_1}{\alpha_2} = \frac{(T - 30)}{150}$$

Given, $\alpha_1 : \alpha_2 = 4 : 3$

$$\therefore \quad \frac{T - 30}{150} = \frac{4}{3} \Rightarrow T - 30 = \frac{4}{3} \times 150 = 200$$

$$\text{or} \quad T = 200 + 30 = 230^\circ\text{C}$$

4. $K = \dfrac{p}{(-\Delta V / V)} \Rightarrow \dfrac{\Delta V}{V} = \dfrac{p}{K}$

$$\Rightarrow \quad -\Delta V = \frac{pV}{K} \Rightarrow \frac{pV}{K} = V (3\alpha) \, \Delta T$$

$$\therefore \quad \Delta T = \frac{p}{3\alpha K}$$

5.

$$T_0 = 2\pi \sqrt{\frac{L}{g}}$$

$$T' = T_0 + \Delta T = 2\pi \sqrt{\frac{L + \Delta L}{g}}$$

$$\therefore \quad T' = T_0 + \Delta T = 2\pi \sqrt{\frac{L(1 + \alpha \, \Delta\theta)}{g}}$$

$$= \left\{ 2\pi \sqrt{\frac{L}{g}} \right\} (1 + \alpha \, \Delta\theta)^{\frac{1}{2}} \approx T_0 \left(1 + \frac{\alpha \, \Delta\theta}{2} \right)$$

$$\therefore \quad \Delta T = T' - T_0 = \frac{\alpha \, \Delta\theta T_0}{2} \quad \ldots(\text{i})$$

$$\text{or} \quad \frac{\Delta T_1}{\Delta T_2} = \frac{\alpha \, \Delta\theta_1 T_0}{\alpha \, \Delta\theta_2 T_0}$$

$$\Rightarrow \quad \frac{12}{4} = \frac{40 - \theta}{\theta - 20}$$

$$\Rightarrow \quad 3(\theta - 20) = 40 - \theta$$

$$\Rightarrow \quad 4\theta = 100$$

$$\Rightarrow \quad \theta = 25^\circ \text{ C}$$

Time gained or lost is given by

$$\Delta T = \left(\frac{\Delta T}{T_0 + \Delta T} \right) t \approx \frac{\Delta t}{T_0} t$$

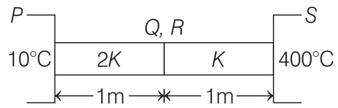
From Eq. (i), $\quad \dfrac{\Delta T}{T_0} = \dfrac{\alpha \Delta\theta}{2}$

$$\therefore \qquad \Delta t = \frac{\alpha (\Delta\theta) t}{2}$$

$$12 = \frac{\alpha (40 - 25)(24 \times 3600)}{2}$$

$$\therefore \qquad \alpha = 1.85 \times 10^{-5} /{}^\circ C$$

6.



Rate of heat flow from P to Q

$$\frac{dQ}{dt} = \frac{2KA(T-10)}{1}$$

Rate of heat flow from Q to S

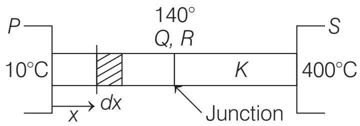
$$\frac{dQ}{dt} = \frac{KA(4000 - T)}{1}$$

At steady state rate of heat flow is same

$$\therefore \quad \frac{2KA(T-10)}{1} = KA(400 - T)$$

or $2T - 20 = 400 - T$ or $3T = 420$

$\therefore \qquad T = 140°$



Temperature of junction is 140°C

Temperature at a distance x from end P is $T_x = (130x + 10°)$

Change in length dx is suppose dy

Then, $dy = \alpha dx\,(T_x - 10)$

$$\int_0^{\Delta y} dy = \int_0^1 \alpha dx (130x + 10 - 10)$$

$$\Delta y = \left[\frac{\alpha x^2}{2} \times 130\right]_0^1$$

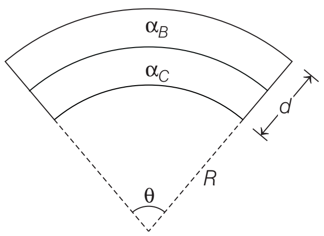
$$\Delta y = 1.2 \times 10^{-5} \times 65$$

$$\Delta y = 78.0 \times 10^{-5} \text{ m} = 0.78 \text{ mm}$$

7. Given $\Delta l_1 = \Delta l_2$ or $l_1 \alpha_a t = l_2 \alpha_s t$

$$\therefore \quad \frac{l_1}{l_2} = \frac{\alpha_s}{\alpha_a} \quad \text{or} \quad \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

8. Let l_0 be the initial length of each strip before heating. Length after heating will be



$$l_B = l_0(1 + \alpha_B \Delta T) = (R + d)\theta$$

and $\qquad l_C = l_0(1 + \alpha_C \Delta T) = R\,\theta$

$$\therefore \qquad \frac{R+d}{R} = \left(\frac{1 + \alpha_B \Delta T}{1 + \alpha_C \Delta T}\right)$$

$$\therefore \qquad 1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_C)\,\Delta T$$

[From binomial expansion]

$$\therefore \quad R = \frac{d}{(\alpha_B - \alpha_C)\Delta T} \quad \text{or} \quad R \propto \frac{1}{\Delta T} \propto \frac{1}{|\alpha_B - \alpha_C|}$$

9. $\Delta l_1 = \dfrac{FL}{AY} = \dfrac{mgL}{\pi r^2 Y}$ = Increase in length

$\Delta l_2 = L\alpha\ \Delta\theta$ = Decrease in length.

To regain its original length, $\Delta l_1 = \Delta l_2$

$$\therefore \quad \frac{mgL}{\pi r^2 Y} = L\alpha\ \Delta\theta \quad \Rightarrow \quad \therefore \quad m = \left(\frac{r^2 Y\ \alpha\ \Delta\theta}{g}\right)$$

Substituting the values we get, $m \simeq 3$ kg

$\therefore$ Answer is 3.

10. When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same.

Upthrust = Weight. Therefore, upthrust should not change

$$F = F'$$

$\therefore \qquad V_i \rho_L g = V_i' \rho'_L g \qquad$ (V_i = volume immersed)

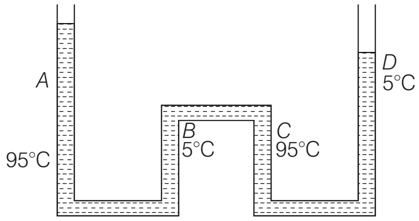
$$\therefore \quad (Ah_i)(\rho_L)(g) = A(1 + 2\alpha_s \Delta T)(h_i)\left(\frac{\rho_L}{1 + \gamma_l \Delta T}\right) g$$

Solving this equation, we get $\gamma_l = 2\alpha_s$

11. Density of a liquid varies with temperature as

$$\rho_{t°C} = \left(\frac{\rho_{0°C}}{1 + \gamma\ t}\right)$$

Here, γ is the coefficient of volume expansion of temperature.



In the figure

$$h_1 = 52.8 \text{ cm}, \ h_2 = 51 \text{ cm and } h = 49 \text{ cm}$$

Now, pressure at B = pressure at C

$$p_0 + h_1 \rho_{95°} g - h \rho_{5°} g = p_0 + h_2 \rho_{5°} g - h \rho_{95°} g$$

$$\Rightarrow \qquad \rho_{95°}(h_1 + h) = \rho_{5°}(h_2 + h)$$

$$\Rightarrow \qquad \frac{\rho_{95°}}{\rho_{5°}} = \frac{h_2 + h}{h_1 + h} \quad \Rightarrow \quad \frac{\dfrac{\rho_{0°}}{1 + 95\gamma}}{\dfrac{\rho_{0°}}{1 + 5\gamma}} = \frac{h_2 + h}{h_1 + h}$$

$$\Rightarrow \quad \frac{1+5\gamma}{1+95\gamma} = \frac{51+49}{52.8+49} = \frac{100}{101.8}$$

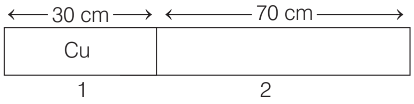
Solving this equation, we get

$$\gamma = 2 \times 10^{-4}/°\text{C}$$

$\therefore$ Coefficient of linear expansion of temperature,

$$\alpha = \frac{\gamma}{3} = 6.7 \times 10^{-5}/°\text{C}$$

12. $\Delta l = l\alpha\,\Delta\theta$

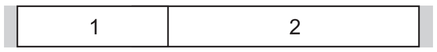


$$\Delta l = \Delta l_1 + \Delta l_2$$

$$(1.91 \times 10^{-3}) = 0.3 \times 1.7 \times 10^{-5} \times 100 + \alpha \times 0.7 \times 100$$

Solving this equation, we get, $\alpha = 2 \times 10^{-5}$ per $°$C

Length of two rods will not change if force on joint due to thermal stress from both sides is equal.



i.e. $\quad F_1 = F_2$

or $\quad Y_1 \times \left(\frac{\Delta l_1}{l_1}\right) \times A = Y_2 \times \left(\frac{\Delta l_2}{l_2}\right) \times A$

or $\quad Y_1 \times \alpha_1 \times \Delta\theta = Y_2 \times \alpha_2 \times \Delta\theta$

$$\therefore \quad Y_2 = \frac{\alpha_1 Y_1}{\alpha_2} = \frac{\alpha_{\text{Cu}}\, Y_{\text{Cu}}}{\alpha_2}$$

$$= \frac{(1.7 \times 10^{-5})(1.3 \times 10^{11})}{(2.0 \times 10^{-5})} = 1.105 \times 10^{11}\ \text{N/m}^2$$

13. Apparent weight = actual weight − upthrust

$$w_1 = w_0 - F_1$$

Here, F_1 = upthrust at temperature T_1

= (volume of sinker at temperature T_1) $\times$

(density of liquid at temperature T_1) $\times g$

$$= \frac{w_0}{g} \times \frac{1}{(\rho_s)_{T_1}} \times g \times (\rho_l)_{T_1} = w_0 \left[\frac{(\rho_l)_{T_1}}{(\rho_s)_{T_1}}\right]$$

$$\therefore \quad w_1 = w_0 - w_0 \left[\frac{(\rho_l)_{T_1}}{(\rho_s)_{T_1}}\right] \qquad \ldots\text{(i)}$$

Similarly, $$w_2 = w_0 - w_0 \left[\frac{(\rho_l)_{T_2}}{(\rho_s)_{T_2}}\right] \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{(\rho_l)_{T_1}}{(\rho_s)_{T_2}} = \frac{w_0 - w_1}{w_0} \quad \text{and} \quad \frac{(\rho_l)_{T_2}}{(\rho_s)_{T_2}} = \frac{w_0 - w_2}{w_0}$$

From these two equations we have,

$$\frac{(\rho_l)_{T_1}}{(\rho_l)_{T_2}} \times \frac{(\rho_s)_{T_2}}{(\rho_s)_{T_1}} = \frac{w_0 - w_1}{w_0 - w_2}$$

Using, $$\frac{\rho'}{\rho} = \frac{1}{1 + \beta\,\Delta T}$$

Here, β = thermal coefficient of volume expansion

$$\therefore \quad \frac{1 + \beta_l\,(T_2 - T_1)}{1 + \beta_s\,(T_2 - T_1)} = \frac{w_0 - w_1}{w_0 - w_2}$$

Given, $\beta_s = \beta$

$$\therefore \quad \left(\frac{w_0 - w_2}{w_0 - w_1}\right)[1 + \beta_l\,(T_2 - T_1)] = 1 + \beta\,(T_2 - T_1)$$

Solving this equation, we get

$$\beta_l = \frac{\{1 + \beta\,(T_2 - T_1)\}\left\{\frac{w_0 - w_1}{w_0 - w_2}\right\} - 1}{(T_2 - T_1)}$$

$$= \beta\left(\frac{w_0 - w_1}{w_0 - w_2}\right) + \frac{w_2 - w_1}{(w_0 - w_2)(T_2 - T_1)}$$

Topic 3 Heat Transfer

1. Let interface temperature in steady state conduction is θ, then assuming no heat loss through sides;

$$\left(\begin{array}{l}\text{Rate of heat}\\ \text{flow through}\\ \text{first slab}\end{array}\right) = \left(\begin{array}{l}\text{Rate of heat}\\ \text{flow through}\\ \text{second slab}\end{array}\right)$$

$$\Rightarrow \quad \frac{(3K)\,A(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d}$$

$$\Rightarrow \quad 9(\theta_2 - \theta) = \theta - \theta_1$$

$$\Rightarrow \quad 9\theta_2 + \theta_1 = 10\theta$$

$$\Rightarrow \quad \theta = \frac{9}{10}\theta_2 + \frac{1}{10}\theta_1$$

2. **Key Idea** From Newton's law of cooling, we have rate of cooling,

$$\frac{dQ}{dt} = \frac{h}{ms}(T - T_0)$$

where, h = heat transfer coefficient,
T = temperature of body,
T_0 = temperature of surrounding,
m = mass and s = specific heat.

We know, $m = V \cdot \rho$

where, V = volume and ρ = density.

So, we have

$$\frac{dQ}{dt} = \frac{h}{ms}(T - T_0) = \frac{h(T - T_0)}{V \cdot \rho s}$$

Since, h, $(T - T_0)$ and V are constant for both beaker.

$$\therefore \quad \frac{dQ}{dt} \propto \frac{1}{\rho s}$$

We have given that $\rho_A = 8 \times 10^2\ \text{kgm}^{-3}$,

$\rho_B = 10^3\ \text{kgm}^{-3}$,

$s_A = 2000\ \text{J kg}^{-1}\ \text{K}^{-1}$ and

$s_B = 4000\ \text{J kg}^{-1}\ \text{K}^{-1}$,

$$\rho_A s_A = 16 \times 10^5$$

and $$\rho_B s_B = 4 \times 10^6$$

So, $\rho_A < \rho_B$, $s_A < s_B$ and $\rho_A s_A < \rho_B s_B$

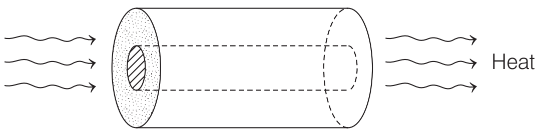
$$\Rightarrow \quad \frac{1}{\rho_A s_A} > \frac{1}{\rho_B s_B} \quad \Rightarrow \quad \frac{dQ_A}{dt} > \frac{dQ_B}{dt}$$

So, for container B, rate of cooling is smaller than the container A. Hence, graph of B lies above the graph of A and it is not a straight line (slope of A is greater than B).

3. Both the given cylinders are in parallel as heat flow is given along length. In parallel, equivalent thermal conductivity of system is

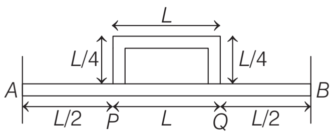
$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

So, in given system



$$K_{eq} = \frac{K_1(\pi R^2) + K_2[\pi(2R)^2 - \pi R^2]}{(\pi R^2) + (4\pi R^2 - \pi R^2)} \text{ or } K_{eq} = \frac{K_1 + 3K_2}{4}$$

4. According to the given question, the given figure with its length for each section is given as below



The above figure considering that every section has the same thermal conductivity, then in terms of thermal resistance is shown in the figure below,

Net resistance of the section $PQRS$ is $= \dfrac{R \times \dfrac{3R}{2}}{\dfrac{5R}{2}} = \dfrac{3R}{5}$ ...(i)

Total resistance of the net network, R_{net}

$$= \frac{R}{2} + \frac{R}{2} + \frac{3R}{5} = \frac{8R}{5}$$

$\therefore$ Thermal current, $I = \dfrac{\Delta T_{AB}}{R_{net}}$

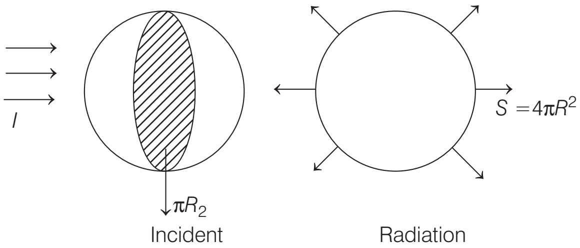
$$I = \frac{120 - 0}{\left(\frac{8R}{5}\right)} = \frac{120 \times 5}{8R}$$

Thus, the net temperature difference between point P and Q is

$$T_P - T_Q = I \times \frac{3R}{5} \quad \text{[using Eq. (i)]}$$

$$= \frac{120 \times 5}{8R} \times \frac{3R}{5} = 45°\text{C}$$

5. In steady state



Energy incident per second = Energy radiated per second

$$\therefore \; I\pi R^2 = \sigma(T^4 - T_0^4)\,4\pi R^2 \;\Rightarrow I = \sigma(T^4 - T_0^4)\,4$$

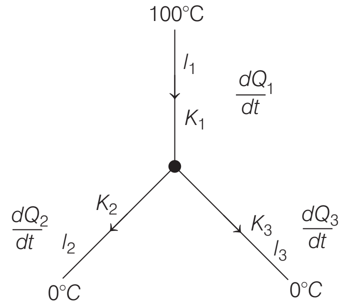
$$\Rightarrow \quad T^4 - T_0^4 = 40 \times 10^8 \;\Rightarrow\; T^4 - 81 \times 10^8 = 40 \times 10^8$$

$$\Rightarrow \quad T^4 = 121 \times 10^8 \;\Rightarrow\; T \approx 330\,\text{K}$$

6. In thermal conduction, it is found that in steady state the heat current is directly proportional to the area of cross-section A which is proportional to the change in temperature $(T_1 - T_2)$.

Then, $$\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{x}$$

According to thermal conductivity, we get



i.e. $$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_3}{dt}$$

$$\frac{0.92\,(100 - T)}{46} = \frac{0.26\,(T - 0)}{13} + \frac{0.12\,(T - 0)}{12}$$

$$\Rightarrow \quad T = 40°\text{ C}$$

$$\therefore \quad \frac{dQ_1}{dt} = \frac{0.92 \times 4\,(100 - 40)}{40}$$

$$= 4.8\,\text{cal/s}$$

7. According to Newton's cooling law, option (c) is correct answer.

8. $$R_{\text{I}} = R_1 + R_2 = \left(\frac{l}{KA}\right) + \left(\frac{l}{2KA}\right) = \frac{3}{2}\left(\frac{l}{KA}\right)$$

$$\frac{1}{R_{\text{II}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l}$$

or $$R_{\text{II}} = \frac{l}{3KA} = \frac{R_{\text{I}}}{4.5}$$

Since thermal resistance R_{II} is 4.5 times less than thermal resistance R_{I}.

$$\therefore \quad t_{\text{II}} = \frac{t_{\text{I}}}{4.5} = \frac{9}{4.5}\,\text{s} = 2\,\text{s}$$

9. Let temperature of middle plate in steady state is T_0

$Q_1 = Q_2$

$Q =$ net rate of heat flow

$\therefore\ \sigma A\,(3T)^4 - \sigma A T_0^4 = \sigma A T_0^4 - \sigma A(2T)^4$

Solving this equation, we get

$$T_0 = \left(\frac{97}{2}\right)^{1/4} T$$

10. $\lambda_m T =$ constant

From the graph $T_3 > T_2 > T_1$

Temperature of Sun will be maximum.

Therefore, (c) is the correct option.

NOTE Although graphs are not very clear.

11. Glass of bulb heats due to filament by radiation.

12. $Q \propto AT^4$ and $\lambda_m\, T =$ constant

Hence,

$$Q \propto \frac{A}{(\lambda_m)^4}$$

or $$Q \propto \frac{r^2}{(\lambda_m)^4}$$

$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$$

$$= \frac{4}{81} : \frac{1}{16} : \frac{36}{625} = 0.05 : 0.0625 : 0.0576$$

i.e. Q_B is maximum.

13. $$\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$$

or $$\frac{\text{Temperature difference}}{\text{Thermal resistance}} = L\left(\frac{dm}{dt}\right)$$

or $$\frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}}$$

$\Rightarrow$ $$q \propto \frac{1}{R}$$

In the first case rods are in parallel and thermal resistance is $\frac{R}{2}$ while in second case rods are in series and thermal resistance is $2R$.

$$\frac{q_1}{q_2} = \frac{2R}{R/2} = \frac{4}{1}$$

14. Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ emissivity (e)

From the graph,

$$\left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$$

$\therefore$ $$E_x > E_y$$

Further emissivity (E) = absorptive power (a)

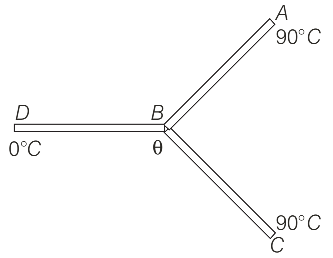
(good absorbers are good emitters also)

$\therefore$ $$a_x > a_y$$

15. Black body radiates maximum number of wavelength and maximum energy if all other conditions (e.g. temperature surface area etc.) are same. So, when the temperature of black body becomes equal to the temperature of the furnace, the black body will radiate maximum energy and it will be brightest of all. Initially, it will absorb all the radiant energy incident on it, so it is the darkest one.

16. Let θ be the temperature of the junction (say B). Thermal resistance of all the three rods is equal. Rate of heat flow through AB + Rate of heat flow through CB = Rate of heat flow through BD

$\therefore$ $$\frac{90° - \theta}{R} + \frac{90° - \theta}{R} = \frac{\theta - 0}{R}$$



Here, R = Thermal resistance

$\therefore$ $$3\theta = 180° \quad \text{or} \quad \theta = 60° \text{C}$$

NOTE

Rate of heat flow

$$(\text{H}) = \frac{\text{Temperature difference (TD)}}{\text{Thermal resistance (R)}}$$

where, $\text{R} = \frac{l}{\text{KA}}$

K = Thermal conductivity of the rod.

This is similar to the current flow through a resistance (R) where current (i) = Rate of flow of charge

$$= \frac{\text{Potential difference (PD)}}{\text{Electrical resistance (R)}}$$

Here, $R = \frac{l}{\sigma A}$ where σ = Electrical conductivity.

17. Wien's displacement law for a perfectly black body is

$\lambda_m T =$ constant $=$ Wien's constant b

Here, λ_m is the minimum wavelength corresponding to maximum intensity I.

or $$\lambda_m \propto \frac{1}{T}$$

From the figure $(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$

Therefore, $$T_1 > T_3 > T_2$$

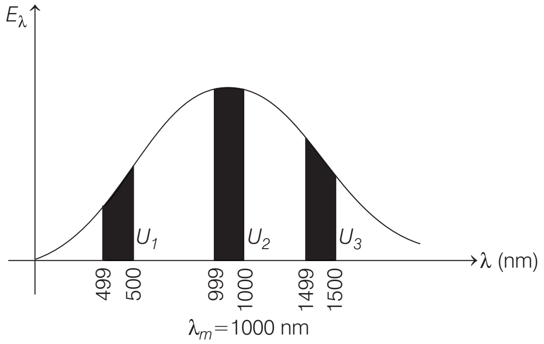
18. Wien's displacement law is

$$\lambda_m T = b \qquad (b = \text{Wien's constant})$$

$\therefore$ $$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm-K}}{2880\text{K}}$$

$\therefore$ $$\lambda = 1000 \text{ nm}$$

Energy distribution with wavelength will be as follows :



From the graph it is clear that

$$U_2 > U_1 \qquad \text{(In fact } U_2 \text{ is maximum)}$$

19. Power radiated ∝ (surface area) $(T)^4$. The radius is halved, hence, surface area will become $\frac{1}{4}$ times. Temperature is doubled, therefore, T^4 becomes 16 times.

New power $= (450)\left(\frac{1}{4}\right)(16) = 1800\,\text{W}$.

20. From Wien's displacement law

$$\lambda_m T = \text{constant}$$

or

$$T = \frac{1}{\lambda_m}$$

$$\therefore \quad \frac{T_{\text{sun}}}{T_{\text{north star}}} = \frac{(\lambda_m)_{\text{north star}}}{(\lambda_m)_{\text{sun}}}$$

$$= \frac{350}{510} \approx 0.69$$

21. The rate at which energy radiates from the object is

$$\frac{\Delta Q}{\Delta t} = e\sigma A T^4$$

Since, $\Delta Q = mc\Delta T$, we get

$$\frac{\Delta T}{\Delta t} = \frac{e\sigma A T^4}{mc}$$

Also, since $m = \frac{4}{3}\pi r^3 \rho$ for a sphere, we get

$$A = 4\pi r^2 = 4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}$$

Hence,

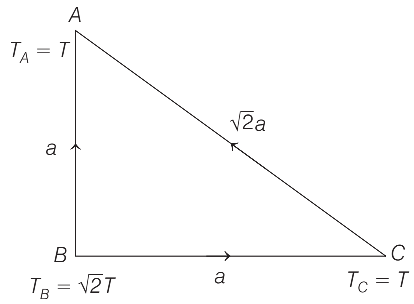
$$\frac{\Delta T}{\Delta t} = \frac{e\sigma T^4}{mc}\left[4\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right]$$

$$= K\left(\frac{1}{m}\right)^{1/3}$$

For the given two bodies

$$\frac{(\Delta T/\Delta t)_1}{(\Delta T/\Delta t)_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

22. The diagramatic representation of the given problem is shown in figure. Since, $T_B > T_A$ the heat will flow from B to A. Similarly, heat will also flow from B to C and C to A.



Applying the conduction formula

$$\frac{\Delta Q}{\Delta t} = \frac{KA}{l}(\Delta T)$$

$$\left(\frac{\Delta T}{\sqrt{2}a}\right)_{CA} = \left(\frac{\Delta T}{a}\right)_{BC} \Rightarrow \frac{T_C - T}{\sqrt{2}\,a} = \frac{\sqrt{2}T - T_C}{a}$$

$$3T = T_C\,(\sqrt{2}+1) \Rightarrow \frac{T_C}{T} = \frac{3}{(\sqrt{2}+1)}$$

23. Let R_1 and R_2 be the thermal resistances of inner and outer portions. Since, temperature difference at both ends is same, the resistances are in parallel. Hence,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad \frac{K(4\pi R^2)}{l} = \frac{K_1(\pi R^2)}{l} + \frac{K_2(3\pi R^2)}{l}$$

$$\therefore \quad K = \frac{3K_2 + K_1}{4}$$

24. Thermal resistance $R = \frac{l}{KA}$

$$\therefore \quad R_A = \frac{L}{(2K)(4Lw)} \qquad \text{(Here } w = \text{width)}$$

$$= \frac{1}{8Kw},$$

$$R_B = \frac{4L}{3K\,(Lw)} = \frac{4}{3Kw}$$

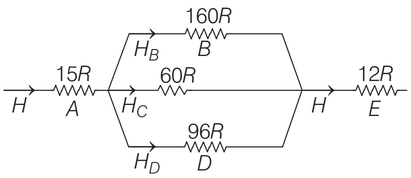
$$R_C = \frac{4L}{(4K)(2Lw)} = \frac{1}{2Kw}$$

$$R_D = \frac{4L}{(5K)(Lw)} = \frac{4}{5Kw}$$

$$R_E = \frac{L}{(6K)(Lw)} = \frac{1}{6Kw}$$

$$R_A : R_B : R_C : R_D : R_E$$
$$= 15 : 160 : 60 : 96 : 12$$

So, let us write, $R_A = 15\,R$, $R_B = 160\,R$ etc and draw a simple electrical circuit as shown in figure



H = Heat current = Rate of heat flow.

$$H_A = H_E = H \qquad \text{(let)}$$

In parallel current distributes in inverse ratio of resistance.

$$\therefore\quad H_B : H_C : H_D = \frac{1}{R_B} : \frac{1}{R_C} : \frac{1}{R_D}$$

$$= \frac{1}{160} : \frac{1}{60} : \frac{1}{96}$$

$$= 9 : 24 : 15$$

$$\therefore\quad H_B = \left(\frac{9}{9 + 24 + 15}\right) H = \frac{3}{16} H$$

$$H_C = \left(\frac{24}{9 + 24 + 15}\right) H = \frac{1}{2} H$$

and $$H_D = \left(\frac{15}{9 + 24 + 15}\right) H = \frac{5}{16} H$$

$$H_C = H_B + H_D$$

Temperature difference (let us call it T)
= (Heat current) × (Thermal resistance)

$$T_A = H_A R_A = (H)(15R) = 15\,HR$$

$$T_B = H_B R_B = \left(\frac{3}{16} H\right)(160\,R) = 30\,HR$$

$$T_C = H_C R_C = \left(\frac{1}{2} H\right)(60\,R) = 30HR$$

$$T_D = H_D R_D = \left(\frac{5}{16}\,H\right)(96\,R) = 30\,HR$$

$$T_E = H_E R_E = (H)(12\,R) = 12\,HR$$

Here, T_E is minimum. Therefore option (c) is also correct.

$\therefore$ Correct options are (a), (c) and (d).

25. Since, the temperature of black body is constant, total heat absorbed = total heat radiated.

26. Power radiated and surface area is same for both A and B.

Therefore, $\quad e_A \sigma T_A^4 A = e_B \sigma T_B^4 A$

$$\therefore\quad \frac{T_A}{T_B} = \left(\frac{e_B}{e_A}\right)^{1/4} = \left(\frac{0.81}{0.01}\right)^{1/4} = 3$$

$$\therefore\quad T_B = \frac{T_A}{3} = \frac{5802}{3}$$

$$= 1934\text{ K}$$

$$T_B = 1934\text{ K}$$

According to Wien's displacement law,

$$\lambda_m T = \text{constant}$$

$$\therefore\quad \lambda_A T_A = \lambda_B T_B$$

or $$\lambda_A = \lambda_B \left(\frac{T_B}{T_A}\right) = \frac{\lambda_B}{3}$$

Given, $$\lambda_B - \lambda_A = 1\,\mu\text{m}$$

$$\Rightarrow\quad \lambda_B - \frac{\lambda_B}{3} = 1\,\mu\text{m}$$

or $$\frac{2}{3}\lambda_B = 1\,\mu\text{m}$$

$$\Rightarrow\quad \lambda_B = 1.5\,\mu\text{m}$$

NOTE $\lambda_m T = b$ = Wien's constant value of this constant for perfectly black body is 2.89×10^{-3} m-K. For other bodies this constant will have some different value. In the opinion of author option (b) has been framed by assuming b to be constant for all bodies. If we take b different for different bodies. Option (b) is incorrect.

27. (*d*) Rate of heat flow will be same,

$$\therefore\quad \frac{300 - 200}{R_1} = \frac{200 - 100}{R_2} \qquad \left(\text{as } H = \frac{dQ}{dt} = \frac{T \cdot D}{R}\right)$$

$$\therefore\quad R_1 = R_2 \;\Rightarrow\; \frac{L_1}{K_1 A_1} = \frac{L_2}{K_2 A_2} \;\Rightarrow\; \frac{K_1}{K_2} = \frac{A_2}{A_1} = 4$$

28. Power, $P = (\sigma T^4 A) = \sigma T^4 (4\pi R^2)$

or, $\quad P \propto T^4 R^2 \qquad \ldots\text{(i)}$

According to Wien's law,

$$\lambda \propto \frac{1}{T}$$

(λ is the wavelength at which peak occurs)

$\therefore$ Eq. (i) will become,

$$P \propto \frac{R^2}{\lambda^4} \text{ or } \lambda \propto \left[\frac{R^2}{P}\right]^{1/4}$$

$$\Rightarrow\quad \frac{\lambda_A}{\lambda_B} = \left[\frac{R_A}{R_B}\right]^{1/2} \left[\frac{P_B}{P_A}\right]^{1/4} = [400]^{1/2} \left[\frac{1}{10^4}\right]^{1/4} = 2$$

29. $$\log_2 \frac{P_1}{P_0} = 1$$

Therefore, $\frac{P_1}{P_0} = 2$

According to Stefan's law,

$$P \propto T^2$$

$$\Rightarrow\quad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{2767 + 273}{487 + 273}\right)^4 = 4^4$$

$$\Rightarrow\quad \frac{P_2}{P_1} = \frac{P_2}{2P_0} = 4^4 \quad\Rightarrow\quad \frac{P_2}{P_0} = 2 \times 4^4$$

$$\log_2 \frac{P_2}{P_0} = \log_2 [2 \times 4^4] = \log_2 2 + \log_2 4^4$$

$$= 1 + \log_2 2^8 = 9$$

30. $\lambda_m \propto \dfrac{1}{T}$

$\therefore \quad \dfrac{\lambda_A}{\lambda_B} = \dfrac{T_B}{T_A} = \dfrac{500}{1500} = \dfrac{1}{3}$

$E \propto T^4 A$ (where, A = surface area = $4\pi R^2$)

$\therefore \quad E \propto T^4 R^2$

$$\frac{E_A}{E_B} = \left(\frac{T_A}{T_B}\right)^4 \left(\frac{R_A}{R_B}\right)^2 = (3)^4 \left(\frac{6}{18}\right)^2 = 9$$

$\therefore$ Answer is 9.

31. Thermal resistance, $R = \dfrac{l}{KA}$.

i.e. $R \propto \dfrac{1}{K}$, l and A being the same for both the blocks

$$\therefore \quad \frac{R_A}{R_B} = \frac{K_B}{K_A} = \frac{200}{300} = \frac{2}{3}$$

Rate of flow of heat (thermal current) along both the cubes will be equal (in series). Therefore, temperature difference across both the cubes will be in the ratio of their thermal resistances. Hence,

$$\frac{(\text{TD})_A}{(\text{TD})_B} = \frac{R_A}{R_B} = \frac{2}{3} \quad \text{or} \quad \frac{100^\circ\text{C} - T}{T - 0^\circ\text{C}} = \frac{2}{3}$$

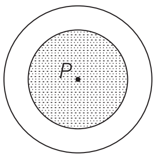
$\Rightarrow \quad 300^\circ\text{C} - 3T = 2T \Rightarrow T = 60^\circ\text{C}$

32. Thermal resistance $= \dfrac{l}{KA} = \dfrac{t}{K\,(4\pi R^2)}$ (t = thickness)

Now, rate of heat transfer $= \dfrac{\text{Temperature difference}}{\text{Thermal resistance}}$

$$= \frac{T}{t / 4\pi KR^2} = \frac{4\pi KTR^2}{t}$$

Equating this rate with the power of the source.



$\therefore \quad P = \dfrac{4\pi KTR^2}{t}$ or $t = \dfrac{4\pi KTR^2}{P}$

or thickness t should not exceed $\dfrac{4\pi KTR^2}{P}$.

33. Area of sphere, $A = 4\pi r^2$

Mass of sphere, $m = \left(\dfrac{4}{3}\pi\rho r^3\right)$

Now, energy radiated per second = $(\sigma A T^4)$

$$\therefore \quad mc\left(-\frac{dT}{dt}\right) = \sigma A T^4$$

or $$\int_0^t dt = \left(\frac{-mc}{\sigma A}\right)\int_{200}^{100} T^{-4} dT$$

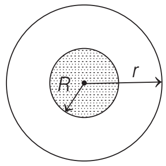
or $$t = \frac{mc}{3\sigma A}\left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]$$

$$= \frac{7mc}{3 \times 8\sigma A} \times 10^{-6} = \frac{7mc \times 10^{-6}}{24\sigma A}$$

Substituting the values,

$$t = \frac{\left(7 \times \frac{4}{3}\pi\rho r^3\right)(c) \times 10^{-6}}{3 \times 8 \times 5.67 \times 10^{-8} \times 4\pi r^2} = 1.71\rho rc$$

34. Given, $\dfrac{(\sigma T^4)(4\pi R^2)}{(4\pi r^2)} = 1400$



$$\therefore \quad T = \left(\frac{1400 \times r^2}{\sigma R^2}\right)^{1/4}$$

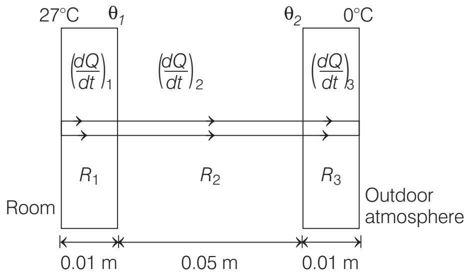
$$= \left[\frac{1400 \times (1.5 \times 10^{11})^2}{(5.67 \times 10^{-8}) \times (7 \times 10^8)^2}\right]^{1/4}$$

$= 5803\,\text{K}$

35. Energy radiated per second $\propto AT^4 \propto (R^2)T^4$

$$\frac{E_1}{E_2} = \left(\frac{1}{4}\right)^2 \left(\frac{4000}{2000}\right)^4 = 1 \text{ or } E_1 = E_2$$

36. Let θ_1 and θ_2 be the temperatures of the two interfaces as shown in figure.



Thermal resistance, $R = \dfrac{l}{KA}$

$$\therefore \quad R_1 = R_3 = \frac{(0.01)}{(0.8)(1)} = 0.0125\,\text{K/W or °C/W}$$

and $$R_2 = \frac{(0.05)}{(0.08)(1)} = 0.625^\circ\,\text{C/W}$$

Now, the rate of heat flow $\left(\dfrac{dQ}{dt}\right)$ will be equal from all the three sections and since rate of heat flow is given by

$$\frac{dQ}{dt} = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

and $$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 = \left(\frac{dQ}{dt}\right)_3$$

Therefore,

$$\frac{27 - \theta_1}{0.0125} = \frac{\theta_1 - \theta_2}{0.625} = \frac{\theta_2 - 0}{0.0125}$$

Solving this equation, we get

$$\theta_1 = 26.48^\circ\text{C} \quad \text{and} \quad \theta_2 = 0.52^\circ\text{C}$$

and $$\frac{dQ}{dt} = \frac{27 - \theta_1}{0.0125} \Rightarrow \frac{dQ}{dt} = \frac{(27 - 26.48)}{0.0125}$$

$= 41.6\,\text{W}$

37. Let at any time temperature of the disc be θ.
At this moment rate of heat flow,

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{l} = \frac{KA}{l}(\theta_0 - \theta) \quad \text{... (i)}$$

This heat is utilised in increasing the temperature of the disc. Hence,

$$\frac{dQ}{dt} = ms\frac{d\theta}{dt} \quad \text{...(ii)}$$

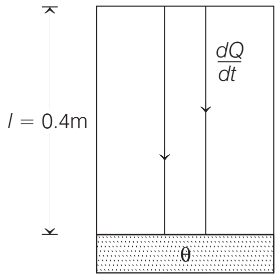
Equating Eqs. (i) and (ii), we have

$$ms\frac{d\theta}{dt} = \frac{KA}{l}(\theta_0 - \theta)$$

Therefore, $$\frac{d\theta}{\theta_0 - \theta} = \frac{KA}{msl}dt$$

or $$\int_{300\text{K}}^{350\text{K}} \frac{d\theta}{\theta_0 - \theta} = \frac{KA}{msl}\int_0^t dt$$

or $$[-\ln(\theta_0 - \theta)]_{300\text{K}}^{350\text{K}} = \frac{KA}{msl}t$$



$\therefore$ $$t = \frac{msl}{KA}\ln\left(\frac{\theta_0 - 300}{\theta_0 - 350}\right)$$

Substituting the values, we have

$$t = \frac{(0.4)(600)(0.4)}{(10)(0.04)}\ln\left(\frac{400 - 300}{400 - 350}\right)$$

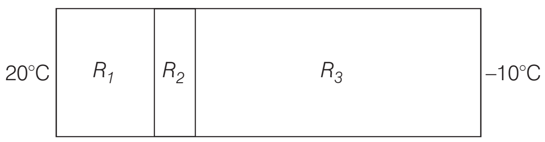
$$t = 166.32\text{ s}$$

38. Let R_1, R_2 and R_3 be the thermal resistances of wood, cement and brick. All the resistances are in series. Hence,

$$R = R_1 + R_2 + R_3$$

$$= \frac{2.5\times10^{-2}}{0.125\times137} + \frac{1.0\times10^{-2}}{1.5\times137} + \frac{25\times10^{-2}}{1.0\times137}\left(\text{as } R = \frac{l}{KA}\right)$$

$$= 0.33\times10^{-2}\ \text{°C/W}$$



$\therefore$ Rate of heat transfer,

$$\frac{dQ}{dt} = \frac{\text{temperature difference}}{\text{thermal resistance}} = \frac{30}{0.33\times10^{-2}}$$

$$\approx 9091\text{ W}$$

$\therefore$ Power of heater should be 9091 W.

39. Net rate of heat radiation $\left(\frac{dQ}{dt}\right)$ will be same in both the cases, as temperature and area are same.
Therefore, from equation

$$ms\left(-\frac{d\theta}{dt}\right) = \frac{dQ}{dt} \quad \text{or} \quad -\frac{d\theta}{dt} \propto \frac{1}{m}$$

The hollow sphere will cool faster as its mass is less.

40. Power produced by heater

= rate of heat flow through window

$\therefore$ $$\frac{V^2}{R} = \frac{\text{Temperature difference}}{\text{Thermal resistance}} = \frac{20-\theta}{(l/KA)}$$

$\therefore$ $$\theta = 20 - \frac{V^2 l}{KAR}$$

Substituting the values we have

$$\theta = 20 - \frac{(200)^2\ (0.2\times10^{-2})}{(0.2\times4.2)(1)(20)} = 15.24°\text{ C}$$

Topic 4 Kinetic Theory of Gases and Gas Equation

1. Work done by a gas is

$$W = p\Delta V = nR\Delta T = 10\text{J} \quad \text{(given) …(i)}$$

As the gas is diatomic,

C_p = specific heat of gas at constant pressure $= \frac{7}{2}R$

So, heat absorbed by gas at constant pressure is

$$\Delta Q = nC_p\ \Delta T = n\frac{7}{2}R\,\Delta T$$

$$= \frac{7}{2}(nR\ \Delta T)$$

$$= \frac{7}{2}\times 10 \quad \text{[From Eq. (i)]}$$

$$= 35\text{J}$$

2. Let molar specific heat of the mixture is C_V.

Total number of molecules in the mixture

$$= 3 + 2 = 5$$

$\therefore C_V$ can be determined using

$$n\,C_V\ dT = n_1\ C_{V_1}\ dT + n_2\ C_{V_2} dT$$

or $(n_1 + n_2)(C_V)_{\text{mix}} = n_1 C_{V_1} + n_2\ C_{V_2}$

[here, $n = n_1 + n_2$]

Here, $C_{V_1} = \frac{3R}{2}$ (for helium); $n_1 = 2$

$C_{V_2} = \frac{5R}{2}$ (for hydrogen); $n_2 = 3$

[For monoatomic gases, $C_V = \frac{3}{2}R$ and for diatomic gases, $C_V = \frac{5}{2}R$]

$$\therefore \quad 5 \times C_V = \left(2 \times \frac{3R}{2}\right) + \left(3 \times \frac{5R}{2}\right)$$

$$\Rightarrow \quad 5C_V = \frac{21R}{2}$$

$$\text{or} \quad C_V = \frac{21R}{10}$$

$$= \frac{21 \times 8.3}{10} = \frac{174.3}{10}$$

$$\text{or} \quad C_V = 17.4 \text{ J/mol-K}$$

3. Given process equation for 1 mole of an ideal gas is

$$p = p_0\left(1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right) \quad \ldots\text{(i)}$$

Also, for 1 mole of ideal gas,

$$pV = RT$$

$$\therefore \quad p = \frac{RT}{V} \quad \ldots\text{(ii)}$$

So, from Eqs. (i) and (ii), we have

$$\frac{RT}{V} = p_0\left(1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right)$$

$$\therefore \quad T = \frac{p_0V}{R}\left(1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right) \quad \ldots\text{(iii)}$$

When volume of gas is V_0, then by substituting $V = V_0$ in Eq. (iii), we get

Temperature of gas is

$$T_1 = \frac{p_0V_0}{R}\left(1 - \frac{1}{2}\left(\frac{V_0}{V_0}\right)^2\right) = \frac{p_0V_0}{2R}$$

Similarly, at volume, $V = 2V_0$

Temperature of gas is

$$T_2 = \frac{p_0(2V_0)}{R}\left(1 - \frac{1}{2}\left(\frac{V_0}{2V_0}\right)^2\right) = \frac{7}{4}\frac{p_0V_0}{R}$$

So, change in temperature as volume changes from V_0 to $2V_0$ is

$$\Delta T = T_2 - T_1 = \left(\frac{7}{4} - \frac{1}{2}\right)\frac{p_0V_0}{R} = \frac{5}{4}\frac{p_0V_0}{R}$$

4. Given,

Volume, $V = 25 \times 10^{-3} \text{ m}^3$

$N = 1$ mole of $O_2 = 6.023 \times 10^{23}$ atoms of O_2,

$T = 300$ K

Root mean square velocity of a gas molecule of O_2, $v_{\text{rms}} = 200$ m / s

Radius, $r = \frac{0.3}{2}$ nm $= \frac{0.3}{2} \times 10^{-9}$ m

Now, average time, $\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda}$

$$\text{where,} \quad \lambda = \frac{RT}{\sqrt{2}\, N\pi r^2 p}$$

$$\text{As,} \quad p = \frac{RT}{V} \Rightarrow \lambda = \frac{V}{\sqrt{2}\, N\pi r^2}$$

$\therefore$ Average collision per second,

$$\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda} = \frac{\sqrt{\frac{8}{3\pi}} \times v_{\text{rms}}}{\lambda}$$

$$\frac{1}{\tau} = \sqrt{\frac{8}{3\pi}} \times \frac{200 \times \sqrt{2} \times 6.023 \times 0^{23} \times \pi \times \frac{0.09}{4} \times 10^{-18}}{25 \times 10^{-3}}$$

$$\Rightarrow \frac{1}{\tau} = 4.4 \times 10^8 \text{ per second} \approx 10^8$$

$\therefore$ No option given is correct.

5. Given, capacity of cylinder is = 67.2 L and $\Delta T = 20°$ C

At STP volume = 22.4 L

$\therefore$ Number of moles $= \frac{67.2}{22.4} = 3$

Now, change in heat is given as

$$\Delta Q = nC_V\ \Delta T$$

Substituting the given values, we get

$$\Delta Q = 3 \times \frac{3R}{2} \times 20 \qquad (\because \text{ for He gas, } C_V = \frac{3}{2}R)$$

$$= 90\, R \text{ J}$$

Given, $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$

$$\therefore \quad \Delta Q = 90 \times 8.31 = 747.9 \text{ J} = 748 \text{ J}$$

6.

> **Key Idea** A diatomic gas molecule has 5 degrees of freedom, i.e. 3 translational and 2 rotational, at low temperature ranges (~ 250 K to 750 K). At temperatures above 750 K, molecular vibrations occurs and this causes two extra degrees of freedom.

Now, in given case,

For gas A, $C_p = 29$, $C_V = 22$

For gas B, $C_p = 30$, $C_V = 21$

By using

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$$

We have,

For gas A, $1 + \frac{2}{f} = \frac{29}{22} \approx 1.3 \Rightarrow f = 6.67 \approx 7$

So, gas A has vibrational mode of degree of freedom.

For gas B,

$$1 + \frac{2}{f} = \frac{30}{21} \approx 1.4 \Rightarrow f = 5$$

Hence, gas B does not have any vibrational mode of degree of freedom.

7.

> **Key Idea** According to the law of equipartition of energy, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{n}{2}k_BT$
> where, n is the degree of freedom.

Since, HCl is a diatomic molecule that has rotational, translational and vibrational motion.

So, $$n = 7 \Rightarrow \frac{1}{2}mv_{\text{rms}}^2 = \frac{7}{2}k_B T$$

Here, $$v_{\text{rms}}^2 = \bar{v} \Rightarrow T = \frac{m\bar{v}^2}{7k_B}$$

8. **Key Idea** For a gas molecule,

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\therefore \quad v_{\text{rms}} \propto \sqrt{T}$$

Let unknown rms speed be $v_{\text{rms, 2}}$

at $T_2 = 227°\text{C}$ (or 500 K)

and at $T_1 = 127°\text{C}$ (or 400 K)

$v_{\text{rms,1}} = 200$ m/s

$\therefore$ Using the relation $v_{\text{rms}} \propto \sqrt{T}$, we can write

$$\frac{v_{\text{rms,2}}}{v_{\text{rms,1}}} = \sqrt{\frac{T_2}{T_1}} \quad \ldots \text{(i)}$$

Substituting these given values in Eq. (i), we get

$$\therefore \quad v_{\text{rms, 2}} = \sqrt{\frac{500}{400}} \times 200 \text{ m/s}$$

$$= \frac{1}{2}\sqrt{5} \times 200 \text{ m/s} = 100\sqrt{5} \text{ m/s}$$

9. Root mean square velocity of hydrogen molecule is given as

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

Escape velocity of hydrogen molecule from the earth is given as

$$v_e = \sqrt{2gR_e}$$

Given, $$v_{\text{rms}} = v_e$$

or $$\sqrt{\frac{3k_B T}{m}} = \sqrt{2gR_e} \Rightarrow T = \frac{2gR_e m}{3 \times k_B}$$

Substituting the given values, we get

$$T = \frac{2 \times 10 \times 6.4 \times 10^6 \times 2}{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}} \approx 10^4 \text{ K}$$

Alternate Solution

At rms speed, average thermal kinetic energy of a hydrogen gas molecule is $= \frac{3}{2}k_B T$

and if v_e = escape velocity of the gas molecule from the earth, then its kinetic energy is $\text{KE} = \frac{1}{2}mv_e^2$, where m is the mass of the gas molecule.

Equating the above thermal and kinetic energies, we have

$$\frac{3}{2}k_B T = \frac{1}{2}mv_e^2 \text{ or } T = \frac{mv_e^2}{3k_B} \quad \ldots\text{(i)}$$

Here, $$k_B = 1.38 \times 10^{-23} \text{ JK}^{-1},$$

$$v_e = 11.2 \times 10^3 \text{ ms}^{-1},$$

$$m = \frac{2}{6.02 \times 10^{26}} = 0.332 \times 10^{-26} \text{ kg}$$

Substituting these value in Eq. (i), we get

$$T = \frac{0.332 \times 10^{-26} \times (11.2 \times 10^3)^2}{3 \times 1.38 \times 10^{-23}} \approx 10^4 \text{ K}$$

10. Mean time elapsed between two successive, collisions is $t = \frac{\lambda}{v}$

where, λ = mean free path length and

v = mean speed of gas molecule

$$\therefore \quad t = \frac{\left(\frac{k_B t}{\sqrt{2}\pi d^2 p}\right)}{\sqrt{\frac{8}{\pi} \cdot \frac{k_B T}{M}}} \Rightarrow t = \frac{C\sqrt{T}}{p}$$

where, $C = \frac{1}{4d^2}\sqrt{\frac{k_B M}{\pi}}$ = a constant for a gas.

So, $$\frac{t_2}{t_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} \cdot \left(\frac{p_1}{p_2}\right) \quad \ldots \text{(i)}$$

Here given, $\frac{p_1}{p_2} = \frac{1}{2}, \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{500}{300}} = \sqrt{\frac{5}{3}}$

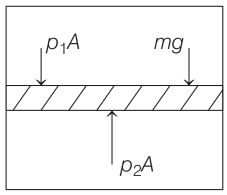
and $t_1 = 6 \times 10^{-8}$ s

Substituting there values in (i), we get

$$t_2 = 6 \times 10^{-8} \times \sqrt{\frac{5}{3}} \times \frac{1}{2} = 3.86 \times 10^{-8} \text{ s} \approx 4 \times 10^{-8} \text{ s}$$

11. **Key Idea** As piston is in equilibrium, so net force on piston is zero.

When the piston is stationary, i.e. on equilibrium as shown in the figure below,



then $$p_1 A + mg = p_2 A$$

$$\Rightarrow \quad mg = p_2 A - p_1 A$$

or $$mg = \left(\frac{nRTA}{V_2} - \frac{nRTA}{V_1}\right)$$

$\{\because pV = nRT$ (ideal gas equation)$\}$

$$= nRT\left(\frac{A}{Al_2} - \frac{A}{Al_1}\right) = nRT\left(\frac{l_1 - l_2}{l_1 l_2}\right)$$

or $$m = \frac{nRT}{g}\left(\frac{l_1 - l_2}{l_1 l_2}\right)$$

12. **Key Idea** Internal energy of 'n' moles of a gas with degree of freedom f (= 3 for an ideal gas), at temperature T is
$$E = \frac{f}{2} \cdot nRT = \frac{3}{2} nRT$$

For an ideal gas, internal energy, $E = \frac{3}{2} nRT$

$$= \frac{3}{2} p \cdot V \qquad (\because \text{ from } pV = nRT)$$

Substituting the given values, we get

$$= \frac{3}{2} \times 3 \times 10^6 \times 2 = 9 \times 10^6 \text{J}$$

13. Internal energy of a gas with f degree of freedom is

$U = \frac{n\,f\,R\,T}{2}$, where n is the number of moles.

Internal energy due to O_2 gas which is a diatomic gas is

$$U_1 = \frac{n_1 f_1 RT}{2} = 3 \times \frac{5}{2} RT$$

($\because n_1 = 3$ moles, degree of freedom for a diatomic gas $f_1 = 5$)

Internal energy due to Ar gas which is a monoatomic gas is

$$U_2 = \frac{n_2 f_2 RT}{2} = 5 \times \frac{3}{2} RT$$

($\because n_2 = 5$ moles, degree of freedom for a monoatomic gas $f_2 = 3$)

$\therefore$ Total internal energy $= U_1 + U_2$

$\Rightarrow \qquad U = 15\,RT$

14. Given, mass of gas, $m = 2$ kg

Pressure on gas, $p = 4 \times 10^4 \text{N/ m}^2$

Density of gas, $\rho = 8 \text{ kg/ m}^3$

$\Rightarrow$ Volume of gas $= \frac{\text{Mass}}{\text{Density}}$

$$\Rightarrow \qquad V = \frac{2}{8} = \frac{1}{4} \text{ m}^3 \qquad \ldots \text{(i)}$$

Using ideal gas equation,

$$pV = nRT$$

$$\Rightarrow \qquad nRT = 4 \times 10^4 \times \frac{1}{4}$$

$$nRT = 10^4 \qquad \ldots \text{(ii)}$$

Internal energy of n moles of a monoatomic gas is given by,

$$U = \frac{3}{2} nRT$$

$$\Rightarrow \qquad U = \frac{3}{2} \times 10^4 \text{ J} = 1.5 \times 10^4 \text{ J}$$

i.e. in order of 10^4 J.

15. We know that from kinetic theory of gases,

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ or } V_{\text{rms}} \propto \sqrt{T}$$

where, R is gas constant, T is temperature and M is molecular mass of a gas.

Here, to double the v_{rms}, temperature must be 4 times of the initial temperature.

$\therefore$ New temperature,

$$T_2 = 4 \times T_1$$

As, $\qquad T_1 = 27^\circ \text{ C} = 300 \text{ K}$

$\therefore \qquad T_2 = 4 \times 300 = 1200 \text{ K}$

As, gas is kept inside a closed vessel.

$$\therefore \qquad Q = nC_V \cdot dT$$

$$= \frac{15}{28} \times \frac{5R}{2} (1200 - 300)$$

$$\left[\text{As, } n = \frac{m}{M} = \frac{15}{28} \text{ for nitrogen}\right]$$

$$= \frac{15}{28} \times \frac{5}{2} \times 8.3 \times 900$$

[Given, $R = 8.3$ J/K-mole]

or $\qquad Q = 10004.4 \text{ J} \simeq 10 \text{ kJ}$

16. Root mean square (rms) velocity of the molecules of a gas is given as

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where, M is the atomic mass of the gas.

$$\Rightarrow \qquad v_{\text{rms}} \propto \sqrt{\frac{1}{M}}$$

$$\therefore \qquad \frac{v_{\text{rms (helium)}}}{v_{\text{rms (argon)}}} = \sqrt{\frac{M_{\text{argon}}}{M_{\text{helium}}}} \qquad \ldots \text{(i)}$$

Given, $\quad M_{\text{argon}} = 40u$ and $M_{\text{helium}} = 4u$

Substituting these values in Eq. (i), we get

$$\frac{v_{\text{rms (helium)}}}{v_{\text{rms (argon)}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

17. $\text{Pressure} = \frac{\text{Force}}{\text{Area}}$

$$= \frac{\frac{\text{Number of Collisions}}{\text{sec}} \times \frac{\text{Change in momentum}}{\text{collision}}}{\text{Area}}$$

$$= \frac{10^{23} \times 2mv\cos 45^\circ}{2 \times 10^{-4}} = 2.35 \times 10^3 \text{ N/m}^2$$

18. From $pV = nRT = \frac{N}{N_A} RT$

We have, $\quad n_f - n_i = \frac{pVN_A}{RT_f} - \frac{pVN_A}{RT_i}$

$$\Rightarrow \quad n_f - n_i = \frac{10^5 \times 30}{8.3} \times 6.02 \times 10^{23} \cdot \left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -2.5 \times 10^{25}$$

$$\therefore \qquad \Delta n = -2.5 \times 10^{25}$$

19. $\Delta Q = \Delta U + \Delta W$

In the process $pV^n =$ constant, molar heat capacity is given by

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-n} = C_V + \frac{R}{1-n}$$

$$C - C_V = \frac{R}{1-n} \Rightarrow 1 - n = \frac{C_p - C_V}{C - C_V}$$

$$\Rightarrow \quad n = 1 - \left(\frac{C_p - C_V}{C - C_V}\right)$$

$$= \frac{(C - C_V) - (C_p - C_V)}{C - C_V} = \frac{C - C_p}{C - C_V}$$

20. $\rho = \frac{pM}{RT} \quad \therefore \quad \rho \propto pM$

or $$\frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)\left(\frac{M_1}{M_2}\right) = \left(\frac{2}{3}\right)\left(\frac{4}{3}\right) = \frac{8}{9}$$

21. $$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \text{or} \quad v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$$

$$\therefore \quad \frac{(v_{\text{rms}})_{\text{He}}}{(v_{\text{rms}})_{\text{Ar}}} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$$

22. A real gas behaves like an ideal gas at low pressure and high temperature.

23. Internal energy of n moles of an ideal gas at temperature T is given by

$$U = n\left(\frac{f}{2}RT\right)$$

where, f = degrees of freedom.

$= 5$ for O_2 and 3 for Ar

Hence, $$U = U_{O_2} + U_{\text{Ar}} = 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11RT$$

24. Process is isothermal. Therefore, T = constant. $\left(p \propto \frac{1}{V}\right)$ volume is increasing, therefore, pressure will decrease.

In chamber A $\rightarrow$

$$-\Delta p = (p_A)_i - (p_A)_f = \frac{n_A RT}{V} - \frac{n_A RT}{2V}$$

$$= \frac{n_A RT}{2V} \qquad \text{...(i)}$$

In chamber B $\rightarrow$

$$-1.5\Delta p = (p_B)_i - (p_B)_f = \frac{n_B RT}{V} - \frac{n_B RT}{2V}$$

$$= \frac{n_B RT}{2V} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3} \quad \text{or} \quad \frac{m_A/M}{m_B/M} = \frac{2}{3}$$

or $$\frac{m_A}{m_B} = \frac{2}{3} \quad \text{or} \quad 3m_A = 2m_B$$

25. Average kinetic energy per molecule per degree of freedom $= \frac{1}{2}kT$. Since, both the gases are diatomic and at same temperature (300 K), both will have the same number of rotational degree of freedom i.e. two. Therefore, both the gases will have the same average rotational kinetic energy per molecule

$$= 2 \times \frac{1}{2}kT \text{ or } kT$$

$\Rightarrow$ Thus, ratio will be 1 : 1.

26. $$pV = nRT$$

or $$p = \frac{nRT}{V} \quad \text{or} \quad p \propto T$$

if V and n are same. Therefore, if T is doubled, pressure also becomes two times i.e. $2p$.

27. Average translational kinetic energy of an ideal gas molecule is $\frac{3}{2}kT$ which depends on temperature only. Therefore, if temperature is same, translational kinetic energy of O_2 and N_2 both will be equal.

28. The average translational KE $= \frac{3}{2}kT$ which is directly proportional to T, while rms speed of molecules is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ i.e. } v_{\text{rms}} \propto \sqrt{T}$$

When temperature of gas is increased from 300 K to 600 K (i.e. 2 times), the average translational KE will increase to 2 times and rms speed to $\sqrt{2}$ or 1.414 times.

$\therefore$ Average translational KE $= 2 \times 6.21 \times 10^{-21}$ J

$= 12.42 \times 10^{-21}$ J

and $$v_{\text{rms}} = (1.414)(484) \text{ m/s} \approx 684 \text{ m/s}$$

29. $$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

i.e. $$v_{\text{rms}} \propto \sqrt{T}$$

When temperature is increased from 120 K to 480 K (i.e. four times), the root mean square speed will become $\sqrt{4}$ or 2 times i.e. $2v$.

30. The average speed of molecules of an ideal gas is given by

$$\langle v \rangle = \sqrt{\frac{8RT}{\pi M}}$$

i.e. $\langle v \rangle \propto \sqrt{T}$ for same gas.

Since, temperatures of A and C are same, average speed of O_2 molecules will be equal in A and C i.e. v_1.

31. $\gamma_1 = \frac{5}{3}$ means gas is monoatomic or $C_{V_1} = \frac{3}{2}R$

$\gamma_2 = \frac{7}{5}$ means gas is diatomic or $C_{V_2} = \frac{5}{2}R$

C_V (of the mixture)

$$= \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}\right)R}{1+1} = 2R$$

C_p (of the mixture) $= C_V + R = 3R$

$\therefore \ \gamma_{\text{mixture}} = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$

32. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

Room temperature $T \approx 300\text{K}$

$$\therefore \quad 1930 = \sqrt{\frac{3 \times 8.31 \times 10^3 \times 300}{M}}$$

$\therefore \quad M = 2.0\,\text{g/mol}$ or the gas is H_2.

33. Total translational kinetic energy $= \frac{3}{2}nRT = \frac{3}{2}pV = 1.5pV$

34. (a) Total internal energy $U = \frac{f_1}{2}nRT + \frac{f_2}{2}nRT$

$$(U_{\text{ave}})_{\text{per mole}} = \frac{U}{2n} = \frac{1}{4}[5RT + 3RT] = 2RT$$

(b) $\gamma_{\text{mix}} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_2 C_{V_1} + n_2 C_{V_2}} = \frac{(1)\frac{7R}{2} + (1)\frac{5R}{2}}{(1)\frac{5R}{2} + (1)\frac{3R}{2}} = \frac{3}{2}$

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{M_1 + M_2}{2} = \frac{2+4}{2} = 3$$

Speed of sound $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{\frac{\gamma}{M}}$

$$\frac{V_{\text{mix}}}{V_{\text{He}}} = \sqrt{\frac{\gamma_{\text{mix}}}{\gamma_{\text{He}}} \times \frac{M_{\text{He}}}{M_{\text{mix}}}} = \sqrt{\frac{3/2}{5/3} \times \frac{4}{3}} = \sqrt{\frac{6}{5}}$$

(d) $V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \Rightarrow V_{\text{rms}} \propto \frac{1}{\sqrt{M}}$,

$$\frac{V_{\text{He}}}{V_{\text{H}}} = \sqrt{\frac{M_{\text{H}}}{M_{\text{He}}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

35. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}},\ \bar{v} = \sqrt{\frac{8}{\pi}.\frac{RT}{M}} \approx \sqrt{\frac{2.5RT}{M}}$

and $\quad v_p = \sqrt{\frac{2RT}{M}}$

From these expressions we can see that,

$$v_p < \bar{v} < v_{\text{rms}}$$

Secondly, $\quad v_{\text{rms}} = \sqrt{\frac{3}{2}}v_p$

and average kinetic energy of a gas molecule

$$= \frac{1}{2}mv_{\text{rms}}^2 = \frac{1}{2}m\left(\sqrt{\frac{3}{2}}v_p\right)^2 = \frac{3}{4}mv_p^2$$

36. For one mole of an ideal gas

$$pV = RT$$

The coefficient of volume expansion at constant pressure is given by,

$$\left(\frac{\Delta V}{\Delta T}\right)_p = \frac{R}{p} = \text{constant} \qquad \text{[option (a)]}$$

The average translational kinetic energy per molecule is $(3/2)kT$ and not $3\ kT$. With decrease of pressure, volume of the gas increases so its mean free path. [option (c)] The average translational kinetic energy of the molecules is independent of their nature, so each component of the gaseous mixture will have the same value of average translational kinetic energy.

37. Since, the system is insulated, $Q = 0$. Other part is vacuum, therefore, work done by the gas W is also zero. Hence, from first law of thermodynamics, $\Delta U = 0$ i.e. temperature remains constant.

38. $\quad Vp^2 =$ constant

Putting, $p = \frac{nRT}{V}$, we have $\frac{T^2}{V} =$ constant

or $\quad T \propto \sqrt{V}$

So, if V is doubled, T becomes $\sqrt{2}$ times.

39. $C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}R\right)}{1+1} = 2R$

40. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ or $v_{\text{rms}} \propto \sqrt{\frac{T}{M}} \qquad$ ...(i)

When oxygen gas dissociates into atomic oxygen its atomic mass M will remain half. Temperature is doubled. So, from Eq. (i) v_{rms} will become two times.

41. $\quad v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

So, v_{rms} not only depends on T but also on M.

42. $\quad pV = nRT$

$$\therefore \quad V = \left(\frac{nR}{p}\right)T$$

Comparing this with $y = mx$, V-T graph is a straight line passing through origin with slope $= \frac{nR}{p}$.

or $\quad$ slope $\propto \frac{1}{p} \quad (\text{slope})_1 > (\text{slope})_2$

$\therefore \quad p_1 < p_2$

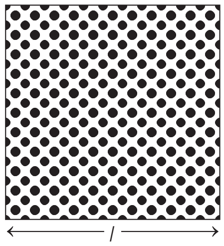
43. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

It not only depends on T but also on M.

44. Volume of the box = $1\,m^3$

Pressure of the gas = $100\,N/m^2$

Let T be the temperature of the gas. Then,



(a) Time between two consecutive collisions with one wall $=\frac{1}{500}$ s. This time should be equal to $\frac{2l}{v_{rms}}$, where l is the side of the cube.

$$\frac{2l}{v_{rms}}=\frac{1}{500}$$

or $$v_{rms}=1000\,\text{m/s} \quad (\text{as } l = 1\,\text{m})$$

or $$\sqrt{\frac{3RT}{M}}=1000$$

$$\therefore \quad T=\frac{(1000)^2 M}{3R}=\frac{(10)^6(4\times10^{-3})}{3(25/3)}=160\,K$$

(b) Average kinetic energy per atom

$$=\frac{3}{2}kT=\frac{3}{2}(1.38\times10^{-23})(160)\,J$$
$$=3.312\times10^{-21}\,J$$

(c) From $pV=nRT=\frac{m}{M}RT$

We get mass of helium gas in the box, $m=\frac{pVM}{RT}$

Substituting the values, we get

$$m=\frac{(100)(1)(4)}{(25/3)(160)}=0.3\,g$$

Topic 5 Thermodynamics

1. Efficiency of a Carnot engine working between source of temperature T_1 and sink of temperature T_2 is given by

$$\eta = 1-\frac{T_2}{T_1}$$

Here, T_2 and T_1 are absolute temperatures.

Initially,

$$\eta=\frac{1}{6}$$

$$\therefore \quad \frac{1}{6}=1-\frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1}=\frac{5}{6}$$

Finally, efficiency is doubled on reducing sink temperature by 62°C.

$$\therefore \quad \eta=\frac{2}{6},\ T_{sink}=T_2'=T_2-62$$

So, $$\eta = 1-\frac{T'_2}{T'_1}$$

$$\Rightarrow \quad \frac{2}{6}=1-\frac{T_2-62}{T_1} \Rightarrow \frac{T_2-62}{T_1}=\frac{4}{6}$$

$$\Rightarrow \quad \frac{T_2}{T_1}-\frac{62}{T_1}=\frac{4}{6}$$

$$\Rightarrow \quad \frac{5}{6}-\frac{62}{T_1}=\frac{4}{6} \qquad \left[\because \frac{T_2}{T_1}=\frac{5}{6}\right]$$

$$\Rightarrow T_1=6\times62=372\,K=372-273=99°\,C$$

$$\Rightarrow \quad T_2=\frac{5}{6}\times T_1\simeq 310\,K$$
$$=310-273=37°\,C$$

2.

> **Key Idea** In p-V curve, work done dW, change in internal energy ΔU and heat absorbed ΔQ are connected with first law of thermodynamics, i.e.
>
> $$\Delta Q=\Delta U+dW \qquad \ldots(i)$$
>
> and total change in internal energy in complete cycle is always zero. Using this equation in different part of the curve, we can solve the given problem.

In Process $a \to b$

Given, $\Delta Q_{ab}=250\,J$

$$\therefore \quad 250\,J=\Delta U_{ab}+dW_{ab} \qquad \ldots(ii)$$

In Process $b \to c$

Given, $\Delta Q_{bc}=60\,J$

Also, V is constant, so $dV=0$

$$\Rightarrow \quad dW_{bc}=p(dV)_{bc}=0 \qquad \ldots(iii)$$

$$\therefore \quad 60\,J=\Delta U_{bc}+0$$

$$\Rightarrow \quad \Delta U_{bc}=60\,J$$

In Process $c \to a$

Given, $$\Delta U_{ca}=-180\,J \qquad \ldots(iv)$$

Now, for complete cycle,

$$\Delta U_{abca}=\Delta U_{ab}+\Delta U_{bc}+\Delta U_{ca}=0 \qquad \ldots(v)$$

From Eqs. (iii), (iv) and (v), we get

$$\Delta U_{ab}=-\Delta U_{bc}-\Delta U_{ca}$$
$$\Delta U_{ab}=-60+180=120\,J \qquad \ldots(vi)$$

From Eq. (ii), we get

$$250\,J=120\,J+dW_{ab}$$

$$\Rightarrow \quad dW_{ab}=130\,J \qquad \ldots(vii)$$

From Eqs. (i) and (vii), we get

Work done by the gas along the path abc,

$$dW_{abc}=dW_{ab}+dW_{bc}$$
$$=130\,J+0K$$

$$\Rightarrow \quad dW_{abc}=130\,J$$

3. Given, heat supplied at constant volume is Q.

$$\therefore \quad Q=nC_V\Delta T$$

For same change in temperature, if heat supplied at constant pressure is Q, then

$$Q'=nC_p\Delta T$$

So, we have

$$\frac{Q'}{Q}=\frac{nC_p\Delta T}{nC_V\Delta T} \Rightarrow \frac{Q'}{Q}=\frac{C_p}{C_V}$$

$$\Rightarrow \qquad Q' = Q\gamma \qquad \left[\because \gamma = \frac{C_p}{C_V}\right]$$

As given gas is diatomic, so $\gamma = \dfrac{7}{5}$

$$\therefore \qquad Q' = \frac{7}{5}Q$$

4. For isobaric process, work done is given as
$W = nR\Delta T$
Heat supplied, $\Delta Q = nC_p\Delta T$ and

$$C_p - \frac{C_V}{n} = R$$

$$\Rightarrow \qquad C_p = R + C_V / n$$

$$\therefore \qquad \frac{W}{\Delta Q} = \frac{nR\Delta T}{nC_p\Delta T} = \frac{R}{C_p} = \frac{R}{R + \dfrac{C_V}{n}}$$

$$\Rightarrow \qquad \frac{W}{\Delta Q} = \frac{nR}{nR + C_V}$$

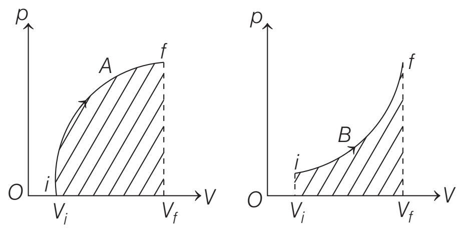
5. According to the first law of thermodynamics,
Heat supplied (ΔQ) = Work done (W) + Change in internal energy of the system (ΔU)

$$\Delta Q_A = \Delta U_A + W_A$$

Similarly, for process B,

$$\Delta Q_B = \Delta U_B + W_B$$

Now, we know that,
work done for a process = area under it's p-V curve
Here,



Thus, it is clear from the above graphs,

$$W_A > W_B \qquad \text{...(i)}$$

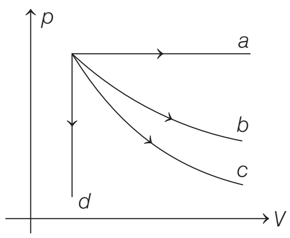
Also, since the initial and final state are same in both process, so

$$\Delta U_A = \Delta U_B \qquad \text{...(ii)}$$

So, from Eqs. (i) and (ii), we can conclude that

$$\Delta Q_A > \Delta Q_B$$

6. Given processes are



For process a, pressure is constant.
$\therefore$ a is isobaric.
For process d, volume is constant.
$\therefore$ d is isochoric.
Also, as we know that, slope of adiabatic curve in p-V diagram is more than that of isothermal curve.
$\therefore$ b is isothermal and c is adiabatic.

7. **Key Idea** In a cyclic thermodynamic process work done = area under $p - V$ diagram.
Also in clockwise cycle, work done is positive.

In the given cyclic process,
work done $= \oint pdV$ = area enclosed by the cycle

$= \dfrac{1}{2} \times$ base $\times$ height of triangle (CAB) made by cycle

$$= \frac{1}{2}(V_2 - V_1)(p_2 - p_1)$$

From graph, given

$V_2 = 5\,\text{m}^3, V_1 = 1\,\text{m}^3,$
$p_2 = 6\,\text{Pa}, \quad p_1 = 1\,\text{Pa}$

$$= \frac{1}{2}(5-1)(6-1) = \frac{1}{2} \times 4 \times 5 = 10\,\text{J}$$

8. Given, $VT = k$, (k is constant)

$$\text{or} \qquad T \propto \frac{1}{V} \qquad \text{...(i)}$$

Using ideal gas equation,

$$pV = nRT$$

$$pV \propto T \Rightarrow pV \propto \frac{1}{V}$$

$$\text{or} \qquad pV^2 = \text{constant} \qquad \text{...(ii)}$$

i.e a polytropic process with $x = 2$.
(Polytropic process means, pV^x = constant)
We know that, work done in a polytropic process is given by

$$\Delta W = \frac{p_2V_2 - p_1V_1}{1 - x} \ (\text{for } x \neq 1) \qquad \text{...(iii)}$$

$$\text{and,} \qquad \Delta W = pV \ln\left(\frac{V_2}{V_1}\right) (\text{for } x = 1)$$

Here, $x = 2$,

$$\therefore \qquad \Delta W = \frac{p_2V_2 - p_1V_1}{1 - x} = \frac{nR(T_2 - T_1)}{1 - x}$$

$$\Rightarrow \qquad \Delta W = \frac{nR\Delta T}{1 - 2} = -nR\Delta T \qquad \text{...(iv)}$$

Now, for monoatomic gas change in internal energy is given by

$$\Delta U = \frac{3}{2}R\Delta T \qquad \text{...(v)}$$

Using first law of thermodynamics, heat absorbed by one mole gas is

$$\Delta Q = \Delta W + \Delta U = \frac{3}{2}R\Delta T - R\Delta T \Rightarrow \Delta Q = \frac{1}{2}R\Delta T$$

9. **Key Idea** For an ideal gas undergoing an adiabatic process at room temperature,

$$pV^{\gamma} = \text{constant or } TV^{\gamma-1} = \text{constant}$$

For a diatomic gas, degree of freedom, $f = 5$

$$\therefore \qquad \gamma = 1 + 2/f = 1 + \frac{2}{5} = \frac{7}{5}$$

As for adiabatic process, $TV^{\gamma-1} = \text{constant}$ …(i)

and it is given that, here $TV^{x} = \text{constant}$ …(ii)

Comparing Eqs. (i) and (ii), we get

$$\gamma - 1 = x \Rightarrow \frac{7}{5} - 1 = x$$

or $\qquad x = 2/5$

10. Work done by gas during heat process at constant pressure is given by

$$\Delta W = p\Delta V$$

Using ideal gas equation,

$$pV = nRT$$

$$\Rightarrow \qquad p\Delta V = nR\Delta T$$

So, $\qquad \Delta W = nR\Delta T$ … (i)

Now, it is given that, $n = \frac{1}{2}$

and $\qquad \Delta T = 90^\circ\text{C} - 20^\circ\text{C}$

$= 363\text{ K} - 293\text{ K} = 70\text{ K}$

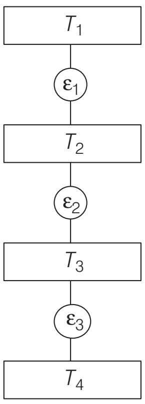
and R (gas constant) $= 8.31$ J/mol-K

Substituting these values in Eq. (i), we get

$$\Delta W = \frac{1}{2} \times 8.31 \times 70 = 290.85\text{ J}$$

$$\Delta W \simeq 291\text{ J}$$

11. Given, Carnot engines operates as,



As, efficiency of a Carnot's engine is given by

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

We have,

$$\eta_1 = \text{efficiency of engine } \varepsilon_1 = 1 - \frac{T_2}{T_1}$$

$$\eta_2 = \text{efficiency of engine } \varepsilon_2 = 1 - \frac{T_3}{T_2}$$

$$\eta_3 = \text{efficiency of engine } \varepsilon_3 = 1 - \frac{T_4}{T_3}$$

For equal efficiencies,

$$\eta_1 = \eta_2 = \eta_3$$

$$\Rightarrow \qquad 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \qquad \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow \qquad T_2^2 = T_1T_3 \text{ and } T_3^2 = T_2T_4$$

$$\Rightarrow \qquad T_2^4 = T_1^2T_3^2$$

$$T_2^4 = T_1^2T_2T_4$$

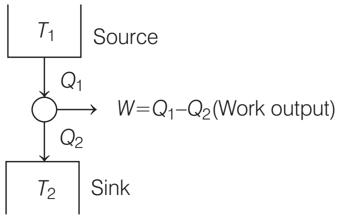
$$T_2^3 = T_4T_1^2$$

or $\qquad T_2 = (T_1^2\,T_4)^{\frac{1}{3}}$

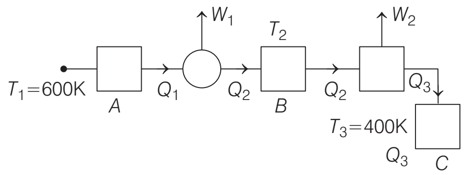
Also, $\qquad T_3^4 = T_2^2\,T_4^2 = T_1T_3T_4^2$

$$T_3^3 = T_1T_4^2 \text{ or } T_3 = (T_1T_4^2)^{\frac{1}{3}}$$

12. **Key idea** In a Carnot engine the heat flow from higher temperature source (at T_1) to lower temperature sink (at T_2) and give the work done equal to the $W = Q_1 - Q_2$.



For the given condition, Carnot engine A and B are operated in series as shown below



where,

Q_1 = heat rejected by engine A at T_1K,

Q_2 = heat received by engine B at T_2K and

Q_3 = heat rejected by engine B to source C at T_3K.

According to Carnot engine principle,

$W_1 = Q_1 - Q_2$ (work-output from source A and B)

$W_2 = Q_2 - Q_3$ (work output from source B and C)

As per the given condition, if the work outputs of the two engines are equal, then

$$Q_1 - Q_2 = Q_2 - Q_3$$

$$\Rightarrow \qquad Q_1 + Q_3 = 2Q_2$$

$$\frac{Q_1}{Q_2} + \frac{Q_3}{Q_2} = 2 \quad \ldots(i)$$

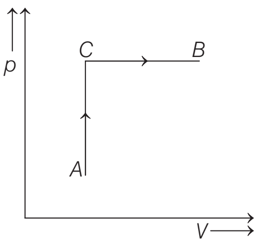
Therefore $\qquad \dfrac{T_1}{T_2} + \dfrac{T_3}{T_2} = 2$

So, $\qquad \dfrac{T_1 + T_3}{T_2} = 2$

$$\Rightarrow \quad T_2 = \frac{T_1 + T_3}{2} = \frac{600 + 400}{2}$$

$$T_2 = 500\,\text{K}$$

13. For the ABC as shown in the figure below,



According to the first law of thermodynamics, heat supplied, ΔQ = work done (ΔW) + internal energy (ΔU)

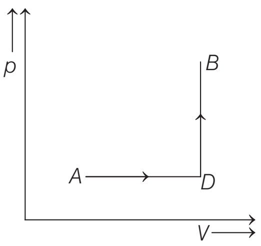
$$\Rightarrow \quad \Delta Q_{CB} = \Delta W_{ACB} + (U_B - U_A)$$

[where, $\Delta U = U_B - U_A$]

Substituting the given values,

$$U_B - U_A = 60 - 30 = 30\,\text{J} \quad \ldots(i)$$

Similarly for the ADB as shown in the figure below,



$$\Delta Q_{ADB} = \Delta W_{ADB} + (U_B - U_A)$$

$$\Rightarrow \Delta Q_{ADB} = 10 + 30 \quad \text{[using Eq. (i)]}$$

$$= 40\,\text{J}$$

14. **For adiabatic process,**

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1} \quad \left(\because \gamma = \frac{5}{3}\right)$$

$$\Rightarrow \quad 300 (V)^{\frac{2}{3}} = T_2 (2V)^{\frac{2}{3}} \Rightarrow T_2 = \frac{300}{2^{\frac{2}{3}}} \approx 189\text{K}$$

$$\Delta U = \frac{f}{2} nR\Delta T = \left(\frac{3}{2}\right)(2)\left(\frac{25}{3}\right)(189 - 300)$$

$$= -2.7\ \text{kJ}$$

15. p-V equation for path AB

$$p = -\left(\frac{p_0}{V_0}\right) V + 3p_0$$

$$\Rightarrow \quad pV = 3p_0 V - \frac{p_0}{V_0} V^2$$

or $\qquad T = \dfrac{pV}{nR} = \dfrac{1}{nR}\left(3p_0 V - \dfrac{p_0}{V_0} V^2\right)$

For maximum temperature,

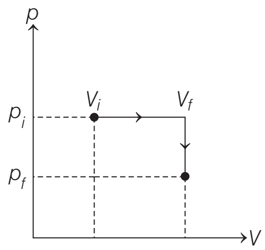
$$\frac{dT}{dV} = 0 \Rightarrow 3p_0 - \frac{2p_0 V}{V_0} = 0$$

$$\Rightarrow \quad V = \frac{3}{2} V_0 \text{ and } p = 3p_0 - \frac{p_0}{V_0} = \frac{3p_0}{2}$$

Therefore, at these values :

$$\therefore \quad T_{\max} = \frac{\left(\frac{3p_0}{2}\right)\left(\frac{3V_0}{2}\right)}{nR} = \frac{9p_0 V_0}{4nR}$$

16. In the first process : $p_i V_i^{\gamma} = p_f V_f^{\gamma}$



$$\Rightarrow \quad \frac{p_i}{p_f} = \left(\frac{V_f}{V_i}\right)^{\gamma} \Rightarrow 32 = 8^{\gamma}$$

$$\gamma = \frac{5}{3} \quad \ldots(i)$$

For the two step process

$$W = p_i (V_f - V_i) = 10^5 (7 \times 10^{-3}) = 7 \times 10^2\,\text{J}$$

$$\Delta U = \frac{f}{2}(p_f V_f - p_i V_i) = \frac{1}{\gamma - 1}\left(\frac{1}{4} \times 10^2 - 10^2\right)$$

$$\Delta U = -\frac{3}{2} \cdot \frac{3}{4} \times 10^2 = -\frac{9}{8} \times 10^2\,\text{J}$$

$$Q - W = \Delta U$$

$$\Rightarrow \quad Q = 7 \times 10^2 - \frac{9}{8} \times 10^2 = \frac{47}{8} \times 10^2\,\text{J} = 588\,\text{J}$$

17. According to first law of thermodynamics, we get

(i) Change in internal energy from A to B i.e. ΔU_{AB}

$$\Delta U_{AB} = nC_V (T_B - T_A) = 1 \times \frac{5R}{2}(800 - 400) = 1000R$$

(ii) Change in internal energy from B to C

$$\Delta U_{BC} = nC_V (T_C - T_B) = 1 \times \frac{5R}{2}(600 - 800)$$

$$= -500R$$

(iii) $\Delta U_{\text{isothermal}} = 0$

(iv) Change in internal energy from C to A i.e. ΔU_{CA}

$$\Delta U_{CA} = nC_V (T_A - T_C)$$

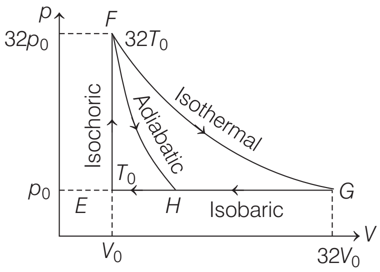
$$= 1 \times \frac{5R}{2}(400 - 600) = -500R$$

18. Heat is extracted from the source means heat is given to the system (or gas) or Q is positive. This is positive only along the path ABC.

Heat supplied

$$\therefore \quad Q_{ABC} = \Delta U_{ABC} + W_{ABC}$$
$$= nC_V\ (T_f - T_i) + \text{Area under } p\text{-}V \text{ graph}$$
$$= n\left(\frac{3}{2}R\right)(T_C - T_A) + 2p_0V_0$$
$$= \frac{3}{2}(nRT_C - nRT_A) + 2p_0V_0$$
$$= \frac{3}{2}(p_CV_C - p_AV_A) + 2p_0V_0$$
$$= \frac{3}{2}(4p_0V_0 - p_0V_0) + 2p_0V_0 = \frac{13}{2}\ p_0V_0$$

19.



In $F \to G$ work done in isothermal process is

$$nRT\ \ln\left(\frac{V_f}{V_i}\right) = 32\ p_0V_0\ \ln\left(\frac{32\,V_0}{V_0}\right)$$
$$= 32\ p_0V_0\ \ln 2^5 = 160\,p_0\,V_0\ \ln 2$$

In $G \to E, \Delta W = p_0\Delta V = p_0(31V_0) = 31\ p_0V_0$

In $G \to H$ work done is less than $31\ p_0V_0$ i.e. $24\ p_0V_0$

In $F \to H$ work done is $36\ p_0V_0$

20. At STP, 22.4 L of any gas is 1 mole.

$$\therefore \quad 5.6\text{ L} = \frac{5.6}{22.4} = \frac{1}{4}\text{ moles} = n$$

In adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\therefore \quad T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1} \text{ or } T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\gamma = \frac{C_p}{C_V} = \frac{5}{3} \text{ for monoatomic He gas.}$$

$$\therefore \quad T_2 = T_1\left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1} = 4\ T_1$$

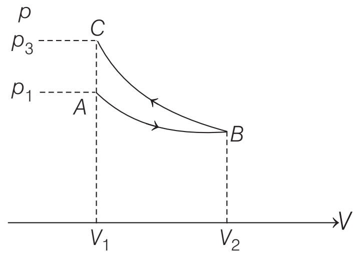
Further in adiabatic process, $Q = 0$

$$\therefore \quad W + \Delta U = 0$$

or $$W = -\Delta U = -nC_V\Delta T = -n\left(\frac{R}{\gamma-1}\right)(T_2 - T_1)$$

$$= -\frac{1}{4}\left(\frac{R}{\frac{5}{3}-1}\right)(4T_1 - T_1) = -\frac{9}{8}RT_1$$

21. Slope of adiabatic process at a given state (p, V, T) is more than the slope of isothermal process. The corresponding p-V graph for the two processes is as shown in figure.



In the graph, AB is isothermal and BC is adiabatic.

W_{AB} = positive (as volume is increasing)

and W_{BC} = negative (as volume is decreasing) plus,

$|W_{BC}| > |W_{AB}|$, as area under p-V graph gives the work done.

Hence, $\quad W_{AB} + W_{BC} = W < 0$

From the graph itself, it is clear that $p_3 > p_1$.

NOTE At point B, slope of adiabatic (process BC) is greater than the slope of isothermal (process AB).

22. $\Delta W_{AB} = p\Delta V = (10)(2-1) = 10\text{ J}$

$\Delta W_{BC} = 0 \qquad$ (as V = constant)

From first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U$$

$\Delta U = 0 \qquad$ (process $ABCA$ is cyclic)

$$\therefore \quad \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$$
$$\therefore \quad \Delta W_{CA} = \Delta Q - \Delta W_{AB} - \Delta W_{BC}$$
$$= 5 - 10 - 0 = -5\text{ J}$$

23. In adiabatic process

slope of p-V graph, $\quad \dfrac{dp}{dV} = -\gamma\dfrac{p}{V}$

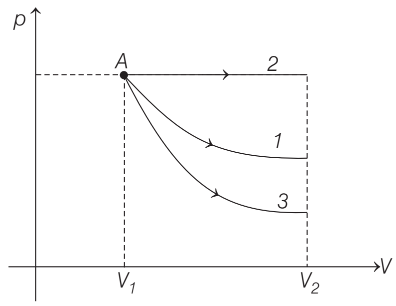
slope $\propto \gamma \qquad$ (with negative sign)

From the given graph,

$$(\text{slope})_2 > (\text{slope})_1$$
$$\therefore \quad \gamma_2 > \gamma_1$$

Therefore, 1 should correspond to O_2 ($\gamma = 1.4$) and 2 should correspond to He ($\gamma = 1.67$).

24. The corresponding p-V graphs (also called indicator diagram) in three different processes will be as shown :



Area under the graph gives the work done by the gas.

$$(\text{Area})_2 > (\text{Area})_1 > (\text{Area})_3$$

$$\therefore \quad W_2 > W_1 > W_3$$

25. During adiabatic expansion, we know

$$TV^{\gamma-1} = \text{constant} \quad \text{or} \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

For a monoatomic gas, $\gamma = \frac{5}{3}$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{AL_2}{AL_1}\right)^{(5/3)-1}$$

(A = Area of cross-section of piston)

$$= \left(\frac{L_2}{L_1}\right)^{2/3}$$

26. A is free to move, therefore, heat will be supplied at constant pressure

$$\therefore \quad dQ_A = nC_p dT_A \quad \text{...(i)}$$

B is held fixed, therefore, heat will be supplied at constant volume.

$$\therefore \quad dQ_B = nC_V dT_B \quad \text{...(ii)}$$

But $\quad dQ_A = dQ_B \quad$ (given)

$$\therefore \quad nC_p dT_A = nC_V dT_B$$

$$\therefore \quad dT_B = \left(\frac{C_p}{C_V}\right) dT_A$$

$$= \gamma\,(dT_A) \quad [\gamma = 1.4 \text{ (diatomic)}]$$

$$(dT_A = 30\,\text{K})$$

$$= (1.4)(30\text{K})$$

$$\therefore \quad dT_B = 42\,\text{K}$$

27. The desired fraction is

$$f = \frac{\Delta U}{\Delta Q} = \frac{nC_V\Delta T}{nC_p\Delta T} = \frac{C_V}{C_p} = \frac{1}{\gamma} \quad \text{or} \quad f = \frac{5}{7} \qquad \left(\text{as}\,\gamma = \frac{7}{5}\right)$$

28. Work done in a cyclic process = area between the cycle

$$= AB \times BC = (2p - p) \times (2V - V) = pV$$

NOTE If cycle is clockwise (p on Y-axis and V on X-axis) work done is positive and if it is anti-clockwise work done is negative.

29. $\left(\frac{dp}{dV}\right)_{\text{adiabatic}} = \gamma\left(\frac{dp}{dV}\right)_{\text{isothermal}}$

List-I

(P) Process I $\Rightarrow$ Adiabatic $\Rightarrow Q = 0$

(Q) Process II $\Rightarrow$ Isobaric

$\therefore \quad W = p\Delta V = 3p_0[3V_0 - V_0] = 6p_0V_0$

(R) Process III $\Rightarrow$ Isochoric $\Rightarrow W = 0$

(S) Process (IV) $\Rightarrow$ Isothermal $\Rightarrow$ Temperature = Constant

30. $v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$. As the sound wave propagates, the air in a chamber undergoes compression and rarefaction very fastly, hence undergo a adiabatic process. So, curves are steeper than isothermal.

$$\left(\frac{dp}{dV}\right)_{\text{Adi}} = -\gamma\left(\frac{p}{V}\right) \quad \text{...(i)}$$

$$\left(\frac{dp}{dV}\right)_{\text{Iso}} = -\left(\frac{p}{V}\right) \quad \text{...(ii)}$$

Graph 'Q' satisfies Eq. (i)

31. $$\Delta U = \Delta Q - p\Delta V$$

$$\Delta U + p\Delta V = \Delta Q$$

As $\Delta U \neq 0, W \neq 0, \Delta Q \neq 0$. The process represents, isobaric process

$$W_{\text{gas}} = -p(\Delta V) = -p(V_2 - V_1) = -pV_2 + pV_1$$

Graph 'P' satisfies isobaric process.

32. Work done in isochoric process is zero.

$W_{12} = 0$ as $\Delta V = 0$

Graph 'S' represents isochoric process.

33. Internal energy $\propto T \propto pV$

This is because $U = \frac{nf}{2}RT = \frac{f}{2}(pV)$

Here, n = number of moles, f = degree of freedom.

$\therefore$ If the product pV increases then internal energy will increase and if product decreases the internal energy will decrease.

Further, work is done on the gas, if volume of gas decreases.

For heat exchange,

$$Q = W + \Delta U$$

Work done is area under p-V graph. If volume increases work done by gas is positive and if volume decreases work done by gas is negative. Further ΔU is positive if product of pV is increasing and ΔU is negative if product of pV is decreasing.

If heat is taken by the gas Q is positive and if heat is lost by the gas Q is negative. Keeping the above points in mind the answer to this question is as under.

(A) $\to$ (p, r, t) ; (B) $\to$ (p, r); (C) $\to$ (q, s) ; (D) $\to$ (r, t)

34. **In process $J \to K$** V is constant whereas p is decreasing. Therefore, T should also decrease.

$$\therefore \quad W = 0, \Delta U = -\text{ve and } Q < 0$$

In process $K \to L$ p is constant while V is increasing. Therefore, temperature should also increase.

$$\therefore \quad W > 0, \Delta U > 0 \text{ and } Q > 0$$

In process $L \to M$ This is inverse of process $J \to K$.

$$\therefore \quad W = 0, \Delta U > 0 \text{ and } Q > 0$$

In process $M \to J$

V is decreasing. Therefore, $W < 0$

$$(pV)_J < (pV)_M$$

$$\therefore \quad T_J < T_M$$

or $\quad \Delta U < 0$

Therefore, $\quad Q < 0.$

35. Process-II is isothermal expansion,

$$\Delta U = 0\,,\, W > 0$$
$$\Delta Q = W > 0$$

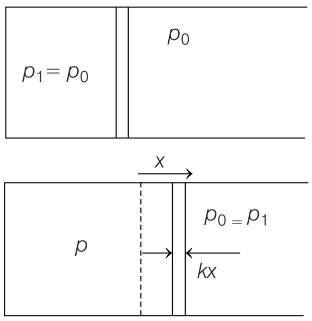
Process-IV is isothermal compression,

$$\Delta U = 0\,,\, W < 0$$
$$\Delta Q = W < 0$$

Process-I and III are not isobaric because in isobaric process $T \propto V$ hence, T-V graph should be a straight line passing through origin.

Option b, c and d are true.

36. **Note** This question can be solved if right hand side hand side chamber is assumed open, so that its pressure remains constant even if the piston shifts towards right.



(a) $$pV = nRT$$
$$p \propto \frac{T}{V}$$

Temperature is made three times and volume is doubled

$$\Rightarrow \quad p_2 = \frac{3}{2}\, p_1$$

Further $\quad x = \frac{\Delta V}{A} = \frac{V_2 - V_1}{A} = \frac{2V_1 - V_1}{A} = \frac{V_1}{A}$

$$p_2 = \frac{3p_1}{2} = p_1 + \frac{kx}{A} \Rightarrow kx = \frac{p_1 A}{2}$$

Energy of spring

$$\frac{1}{2}\, kx^2 = \frac{p_1 A}{4}\, x = \frac{p_1 V_1}{4}$$

(b) $\Delta U = nc_v\, \Delta T = n\left(\frac{3}{2} R\right)\Delta T$

$$= \frac{3}{2}\,(p_2 V_2 - p_1 V_1)$$
$$= \frac{3}{2}\left[\left(\frac{3}{2}\, p_1\right)(2V_1) - p_1 V_1\right] = 3\, p_1 V_1$$

(c) $p_2 = \frac{4\,p_1}{3}$

$$\Rightarrow \quad p_2 = \frac{4}{3}\, p_1 = p_1 + \frac{kx}{A}$$
$$\Rightarrow \quad kx = \frac{p_1 A}{3}$$
$$\Rightarrow \quad x = \frac{\Delta V}{A} = \frac{2V_1}{A}$$

$$W_{\text{gas}} = (p_0\, \Delta V + W_{\text{spring}}) = \left(p_1 Ax + \frac{1}{2}\, kx \cdot x\right)$$
$$= + \left(p_1 A \cdot \frac{2V_1}{A} + \frac{1}{2} \cdot \frac{p_1 A}{3} \cdot \frac{2V_1}{A}\right)$$
$$= 2p_1 V_1 + \frac{p_1 V_1}{3} = \frac{7p_1 V_1}{3}$$

(d) $\Delta Q = W + \Delta U$

$$= \frac{7p_1V_1}{3} + \frac{3}{2}\,(p_2 V_2 - p_1 V_1)$$
$$= \frac{7p_1 V_1}{3} + \frac{3}{2}\left(\frac{4}{3}\, p_1 \cdot 3V_1 - p_1 V_1\right)$$
$$= \frac{7p_1 V_1}{3} + \frac{9}{2}\, p_1\, V_1 = \frac{41 p_1 V_1}{6}$$

NOTE $\Delta U = \frac{3}{2}(p_2\, V_2 - p_1\, V_1)$ *has been obtained in part (b).*

37. $T_A = T_B \quad \Rightarrow \quad \therefore \quad U_A = U_B$

$$W_{AB} = (1)\,(R)\,T_0 \,\ln\left(\frac{V_f}{V_i}\right)$$
$$= RT_0 \,\ln\left(\frac{4V_0}{V_0}\right) = p_0 V_0 \,\ln\,(4)$$

Information regarding p and T at C can not be obtained from the given graph. Unless it is mentioned that line BC passes through origin or not.

Hence, the correct options are (a) and (b).

38. (A) p-V graph is not rectangular hyperbola. Therefore, process A-B is not isothermal.

(B) In process BCD, product of pV (therefore temperature and internal energy) is decreasing. Further, volume is decreasing. Hence, work done is also negative. Hence, Q will be negative or heat will flow out of the gas.

(C) W_{ABC} = positive

(D) For clockwise cycle on p-V diagram with P on Y-axis, net work done is positive.

39. There is a decrease in volume during melting on an ice slab at 273 K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence, option (b) is correct. Secondly heat is absorbed during melting (*i.e., dQ* is positive) and as we have seen, work done by ice-water system is negative (dW is negative). Therefore, from first law of thermodynamics $dU = dQ - dW$

change in internal energy of ice-water system, dU will be positive or internal energy will increase.

40. (a) $\Delta U = nC_V\;\; \Delta T = nC_V\,(T_2 - T_1)$ in all processes

(b) In adiabatic process $\Delta Q = 0$

$\therefore \quad \Delta U = -\,\Delta W \quad$ or $\quad |\Delta U| = |\Delta W|$

(c) In isothermal process $\Delta T = 0$

$\therefore \qquad \Delta U = 0 \qquad$ (as $\Delta U = nC_V\, \Delta T$)

(d) In adiabatic process $\Delta Q = 0$

$\therefore$ All the options are correct.

41. Given, $n = 1, \gamma = \frac{5}{3}$

T-V equation in adiabatic process is

$$TV^{\gamma - 1} = \text{constant}$$

$$\therefore \quad T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

$$\Rightarrow \quad T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$= 100 \times \left(\frac{1}{8}\right)^{\frac{2}{3}}$$

$$\Rightarrow \quad T_2 = 25\text{ K}$$

$C_V = \frac{3}{2} R$ for monoatomic gas

$$\therefore \quad \Delta U = nC_V \Delta T = n \times \left(\frac{3R}{2}\right)(T_2 - T_1)$$

$$= 1 \times \frac{3}{2} \times 8 \times (25 - 100)$$

$$= -900\text{ J}$$

$\therefore$ Decrease in internal energy = 900 J

42. $W_{ibf} = W_{ib} + W_{bf} = 50\text{ J} + 100\text{ J} = 150\text{ J}$

$$W_{iaf} = W_{ia} + W_{af} = 0 + 200\text{ J} = 200\text{ J}$$

$$Q_{iaf} = 500\text{ J}$$

So $\quad \Delta U_{iaf} = Q_{iaf} - W_{iaf}$

$$= 500\text{ J} - 200\text{ J}$$

$$= 300\text{ J} = U_f - U_i$$

So, $\quad U_f = U_{iaf} + U_i$

$$= 300\text{ J} + 100\text{ J} = 400\text{ J}$$

$$\Delta U_{ib} = U_b - U_i$$

$$= 200\text{ J} - 100\text{ J} = 100\text{ J}$$

$$Q_{ib} = \Delta U_{ib} + W_{ib}$$

$$= 100\text{ J} + 50\text{ J} = 150\text{ J}$$

$$Q_{ibf} = \Delta U_{ibf} + W_{ibf} = \Delta U_{iaf} + W_{ibf}$$

$$= 300\text{ J} + 150\text{ J} = 450\text{ J}$$

So, the required ratio

$$\frac{Q_{bf}}{Q_{ib}} = \frac{Q_{ibf} - Q_{ib}}{Q_{ib}}$$

$$= \frac{450 - 150}{150} = 2$$

43. In adiabatic process,

$$TV^{\gamma - 1} = \text{constant}$$

$$\therefore \quad T_i V_i^{0.4} = T_f V_f^{0.4} \quad (\text{as } \gamma = 1.4 \text{ for diatomic gas})$$

or $\quad T_i V_i^{0.4} = (q\, T_i)\left(\frac{V_i}{32}\right)^{0.4}$

or $\quad q\,(32)^{0.4} = 4$

$\therefore$ The correct answer is 4.

44. Under isothermal conditions, $pV = \text{constant}$

$$\therefore \quad (p_i)(2V) = pV$$

$$\Rightarrow \quad p_i = p/2$$

Under adiabatic conditions,

$$pV^{\gamma} = \text{constant where } \gamma = 1.67$$

$$\therefore \quad (p_a)(2V)^{1.67} = pV^{1.67}$$

$$\Rightarrow \quad p_0 = p/(2)^{1.67}$$

Therefore, the ratio $p_a/p_i = (2/2^{1.67}) = 0.628$

45. In p-V graph slope of adiabatic = y (slope of isothermal)

or slope of adiabatic > slope of isothermal.

46. (a) From $\Delta Q = ms\Delta T$

$$\Delta T = \frac{\Delta Q}{ms} = \frac{20000}{1 \times 400} = 50^\circ\text{C}$$

(b) $\Delta V = V\gamma\, \Delta T = \left(\frac{1}{9000}\right)(9 \times 10^{-5})(50)$

$$= 5 \times 10^{-7}\text{ m}^3$$

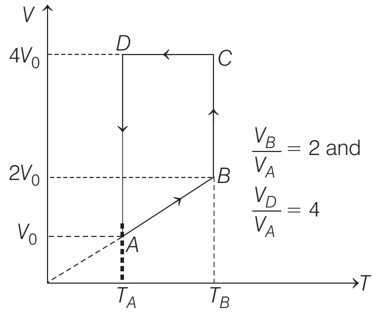
$$\therefore \quad W = p \cdot \Delta V = (10^5)(5 \times 10^{-7}) = 0.05\text{J}$$

(c) $\Delta U = \Delta Q - W = (20000 - 0.05)\text{ J}$

$$= 19999.95\text{ J}$$

47. Given,

Number of moles, $n = 2$



$$C_V = \frac{3}{2}R \text{ and } C_p = \frac{5}{2}R \qquad \text{(monoatomic)}$$

$$T_A = 27^\circ\text{C} = 300\text{ K}$$

Let $V_A = V_0$ then $V_B = 2V_0$

and $V_D = V_C = 4V_0$

(a) **Process $A \to B$**

$$V \propto T$$

$$\Rightarrow \quad \frac{T_B}{T_A} = \frac{V_B}{V_A}$$

$$\therefore \quad T_B = T_A\left(\frac{V_B}{V_A}\right) = (300)(2) = 600\text{ K}$$

$$\therefore \quad T_B = 600\text{ K}$$

(b) **Process $A \to B$**

$$V \propto T$$

$$\Rightarrow \quad p = \text{constant}$$

$$\therefore \quad Q_{AB} = nC_p dT = nC_p(T_B - T_A)$$

$$= (2)\left(\frac{5}{2}R\right)(600 - 300)$$

$Q_{AB} = 1500\,R$ (absorbed)

Process $B \to C$

$$T = \text{constant}$$

$\therefore \quad dU = 0$

$$\therefore \quad Q_{BC} = W_{BC} = nRT_B \ln\left(\frac{V_C}{V_B}\right)$$

$$= (2)(R)(600)\ln\left(\frac{4V_0}{2V_0}\right)$$

$$= (1200R)\ln(2) = (1200R)(0.693)$$

or $\quad Q_{BC} \approx 831.6\,R$ (absorbed)

Process $C \to D$ V = constant

$$\therefore \quad Q_{CD} = nC_V\,dT = nC_V(T_D - T_C)$$

$$= n\left(\frac{3}{2}R\right)(T_A - T_B)$$

$(T_D = T_A$ and $T_C = T_B)$

$$= (2)\left(\frac{3}{2}R\right)(300 - 600)$$

$Q_{CD} = -900\,R$ (released)

Process $D \to A$ T = constant

$\Rightarrow \quad \Delta U = 0$

$$\therefore \quad Q_{DA} = W_{DA} = nRT_D \ln\left(\frac{V_A}{V_D}\right)$$

$$= (2)(R)(300)\ln\left(\frac{V_0}{4V_0}\right)$$

$$= 600R\ln\left(\frac{1}{4}\right)$$

$Q_{DA} \approx -831.6\,R$ (released)

(c) **In the complete cycle** $\Delta U = 0$

Therefore, from conservation of energy

$$W_{\text{net}} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$

$$W_{\text{net}} = 1500\,R + 831.6\,R - 900\,R - 831.6\,R$$

or $\quad W_{\text{net}} = W_{\text{total}} = 600\,R$

48. (a) Number of moles, $n = 2$, $T_1 = 300\,\text{K}$

During the process $A \to B$

pT = constant or p^2V = constant $= K$ (say)

$$\therefore \quad p = \frac{\sqrt{K}}{\sqrt{V}}$$

$$\therefore \quad W_{A\to B} = \int_{V_A}^{V_B} p.dV = \int_{V_A}^{V_B} \frac{\sqrt{K}}{\sqrt{V}}\,dV$$

$$= 2\sqrt{K}[\sqrt{V_B} - \sqrt{V_A}]$$

$$= 2[\sqrt{KV_B} - \sqrt{KV_A}]$$

$$= 2[\sqrt{(p_B^2V_B)V_B} - \sqrt{(p_A^2V_A)V_A}]$$

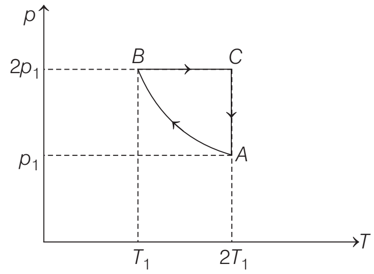
$$= 2[p_BV_B - p_AV_A]$$

$$= 2[nRT_B - nRT_A] = 2\,nR\,[T_1 - 2T_1]$$

$$= (2)(2)(R)[300 - 600] = -1200\,R$$

$\therefore$ Work done on the gas in the process AB is $1200\,R$.

Alternate solution



$$pV = nRT$$

$$\therefore \quad pdV + Vdp = nRdT$$

or $$pdV + \frac{(nRT)}{p}.dp = nRdT \quad \text{... (i)}$$

From the given condition

$$pT = \text{constant}$$

$$pdT + Tdp = 0 \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$pdV = 2nRdT$$

$$\therefore W_{A\to B} = \int pdV = 2nR\int_{T_A}^{T_B} dT = 2nR(T_B - T_A)$$

$$= 2nR(T_1 - 2T_1) = (2)(2)(R)(300 - 600)$$

or $\quad W_{A\to B} = -1200\,R$

(b) **Heat absorbed/released in different processes.**

Since, the gas is monoatomic.

Therefore, $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$ and $\gamma = \frac{5}{3}$

Process $A \to B$ $\Delta U = nC_V\Delta T$

$$= (2)\left(\frac{3}{2}R\right)(T_B - T_A)$$

$$= (2)\left(\frac{3}{2}R\right)(300 - 600) = -900\,R$$

$$\therefore \quad Q_{A\to B} = W_{A\to B} + \Delta U$$

$$= (-1200\,R) - (900\,R)$$

$Q_{A\to B} = -2100\,R$ (released)

Alternate solution

In the process pV^x = constant

Molar heat capacity, $C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$

Here the process is p^2V = constant

or $\quad pV^{1/2}$ = constant

i.e. $\quad x = \frac{1}{2}$

$$\therefore \qquad C = \frac{R}{\frac{5}{3} - 1} + \frac{R}{1 - \frac{1}{2}}$$

$$\therefore \qquad C = 3.5\,R$$

$$\therefore \qquad Q_{A \to B} = nC\Delta T$$
$$= (2)(3.5\,R)(300 - 600)$$

or $\qquad Q_{A \to B} = -2100\,R$

Process $B \to C$ Process is isobaric.

$$Q_{B \to C} = nC_p\Delta T = (2)\left(\frac{5}{2}R\right)(T_C - T_B)$$
$$= 2\left(\frac{5}{2}R\right)(2T_1 - T_1)$$
$$= (5R)(600 - 300)$$
$$Q_{B \to C} = 1500\,R \text{ (absorbed)}$$

Process $C \to A$ Process is isothermal.

$$\therefore \qquad \Delta U = 0$$

and $\quad Q_{C \to A} = W_{C \to A} = nRT_C \ln\left(\frac{p_C}{p_A}\right)$

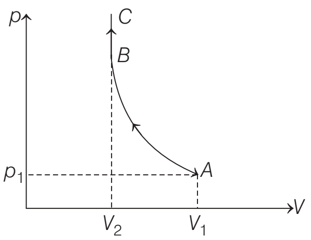
$$= nR(2T_1)\ln\left(\frac{2p_1}{p_1}\right) = (2)(R)(600)\ln(2)$$

$$Q_{C \to A} = 831.6\,R \text{ (absorbed)}$$

NOTE

- In first law of thermodynamics, $(dQ = dU + dW)$ we come across three terms dQ, dU and dW.
  $dU = nC_V dT$ for all the processes whether it is isobaric, isochoric or else and $dQ = nCdT$ where
  $$C = \frac{R}{g - 1} + \frac{R}{1 - X}$$
  in the process $pV^r =$ constant.
- In both the terms we require dT $(= T_f - T_i)$ only. The third term dW is obviously $dQ - dU$. Therefore if in any process change in temperature (dT) and p-V relation is known, then the above method is the simplest one. Note that even if we have V-T or T-p relation, it can be converted into p-V relation by the equation
  $$pV = nRT$$

49. (a) The p-V diagram for the complete process will be as shown below :



Process $A \to B$ is adiabatic compression and
Process $B \to C$ is isochoric.

(b) **(i) Total work done by the gas**

Process $A \to B$

$$W_{AB} = \frac{p_A V_A - p_B V_B}{\gamma - 1}$$

$$W_{\text{adiabatic}} = \frac{p_i V_i - p_f V_f}{\gamma - 1} = \frac{p_1 V_1 - p_2 V_2}{\frac{5}{3} - 1}$$

$\gamma = 5/3$ for monoatomic gas

$$= \frac{p_1 V_1 - p_1\left(\frac{V_1}{V_2}\right)^{\gamma} V_2}{2/3} \qquad \left[\begin{aligned} p_1 V_1^{\gamma} &= p_2 V_2^{\gamma} \\ \therefore\ p_2 &= p_1\left(\frac{V_1}{V_2}\right)^{\gamma}\end{aligned}\right]$$

$$= \frac{3}{2}p_1V_1\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right] = -\frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{5/3 - 1} - 1\right]$$

$$= -\frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Process $B \to C$ $W_{BC} = 0$ (V = constant)

$$\therefore \qquad W_{\text{Total}} = W_{AB} + W_{BC}$$
$$= -\frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

(ii) **Total change in internal energy**

Process $A \to B$ $Q_{AB} = 0$ (Process is adiabatic)

$$\therefore \quad \Delta U_{AB} = -W_{AB} = \frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Process $B \to C$ $W_{BC} = 0$

$$\therefore \qquad \Delta U_{BC} = Q_{BC} = Q \qquad \text{(Given)}$$

$$\therefore \qquad \Delta U_{\text{Total}} = \Delta U_{AB} + \Delta U_{BC}$$
$$= \frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q$$

(iii) **Final temperature of the gas**

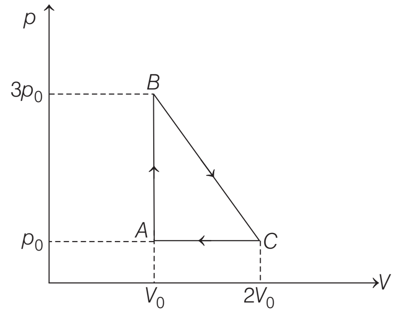
$$\Delta U_{\text{Total}} = nC_V\Delta T = 2\left(\frac{R}{\gamma - 1}\right)(T_C - T_A)$$

$$\therefore \quad \frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q = \frac{2R}{5/3 - 1}\left(T_C - \frac{p_A V_A}{2R}\right)$$

or $\quad \frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q = 3R\left(T_C - \frac{p_1V_1}{2R}\right)$

$$\therefore \quad T_C = \frac{Q}{3R} + \frac{p_1V_1}{2R}\left(\frac{V_1}{V_2}\right)^{2/3} = T_{\text{final}}$$

50. (a) $ABCA$ is a clockwise cyclic process.



$\therefore$ Work done by the gas

$$W = + \text{ Area of triangle } ABC$$

$$= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2V_0 - V_0)(3p_0 - p_0)$$

$$W = p_0V_0$$

(b) Number of moles $n = 1$ and gas is monoatomic, therefore

$$C_V = \frac{3}{2}R \quad \text{and} \quad C_p = \frac{5}{2}R$$

$$\Rightarrow \quad \frac{C_V}{R} = \frac{3}{2} \quad \text{and} \quad \frac{C_p}{R} = \frac{5}{2}$$

(i) **Heat rejected in path *CA*** (Process is isobaric)

$$\therefore \quad dQ_{CA} = C_p dT = C_p(T_f - T_i)$$

$$= C_p\left(\frac{p_fV_f}{R} - \frac{p_iV_i}{R}\right) = \frac{C_p}{R}(p_fV_f - p_iV_i)$$

Substituting the values

$$dQ_{CA} = \frac{5}{2}(p_0V_0 - 2p_0V_0) = -\frac{5}{2}p_0V_0$$

Therefore, heat rejected in the process CA is $\frac{5}{2}p_0V_0$.

(ii) **Heat absorbed in path *AB*** (Process is isochoric)

$$\therefore \quad dQ_{AB} = C_V dT = C_V(T_f - T_i)$$

$$= C_V\left(\frac{p_fV_f}{R} - \frac{p_iV_i}{R}\right)$$

$$= \frac{C_V}{R}(p_fV_f - p_iV_i)$$

$$= \frac{3}{2}(p_fV_f - p_iV_i)$$

$$= \frac{3}{2}(3p_0V_0 - p_0V_0)$$

$$dQ_{AB} = 3p_0V_0$$

$\therefore$ Heat absorbed in the process AB is $3\,p_0V_0$.

(c) Let dQ_{BC} be the heat absorbed in the process BC :

Total heat absorbed,

$$dQ = dQ_{CA} + dQ_{AB} + dQ_{BC}$$

$$dQ = \left(-\frac{5}{2}p_0V_0\right) + (3p_0V_0) + dQ_{BC}$$

$$dQ = dQ_{BC} + \frac{p_0V_0}{2}$$

Change in internal energy, $dU = 0$

$$\therefore \quad dQ = dW$$

$$\therefore \quad dQ_{BC} + \frac{p_0V_0}{2} = p_0V_0 \;\therefore\; dQ_{BC} = \frac{p_0V_0}{2}$$

$\therefore$ Heat absorbed in the process BC is $\frac{p_0V_0}{2}$.

(d) Maximum temperature of the gas will be somewhere between B and C. Line BC is a straight line. Therefore, p-V equation for the process BC can be written as

$$p = -mV + c \qquad (y = mx + c)$$

Here, $m = \frac{2p_0}{V_0}$ and $c = 5p_0$

$$\therefore \quad p = -\left(\frac{2p_0}{V_0}\right)V + 5p_0$$

Multiplying the equation by V,

$$pV = -\left(\frac{2p_0}{V_0}\right)V^2 + 5p_0V \qquad (pV = RT \text{ for } n = 1)$$

$$RT = -\left(\frac{2p_0}{V_0}\right)V^2 + 5p_0V$$

$$\text{or } T = \frac{1}{R}\left[5p_0V - \frac{2p_0}{V_0}V^2\right] \qquad \text{...(i)}$$

For T to be maximum, $\frac{dT}{dV} = 0$

$$\Rightarrow \quad 5p_0 - \frac{4p_0}{V_0}V = 0 \Rightarrow V = \frac{5V_0}{4}$$

i.e. at $V = \frac{5V_0}{4}$, (on line BC), temperature of the gas is maximum. From Eq. (i) this maximum temperature will be

$$T_{\max} = \frac{1}{R}\left[5p_0\left(\frac{5V_0}{4}\right) - \frac{2p_0}{V_0}\left(\frac{5V_0}{4}\right)^2\right]$$

$$T_{\max} = \frac{25}{8}\frac{p_0V_0}{R}$$

51. Number of gram moles of He,

$$n = \frac{m}{M} = \frac{2 \times 10^3}{4} = 500$$

(a) $V_A = 10\text{ m}^3$, $p_A = 5 \times 10^4 \text{ N/m}^2$

$$\therefore \quad T_A = \frac{p_AV_A}{nR} = \frac{(10)(5 \times 10^4)}{(500)(8.31)}\text{ K}$$

or $\quad T_A = 120.34$ K

Similarly, $V_B = 10\text{ m}^3$, $p_B = 10 \times 10^4 \text{ N/m}^2$

$$\therefore \quad T_B = \frac{(10)(10 \times 10^4)}{(500)(8.31)}\text{ K}$$

$$\therefore \quad T_B = 240.68\text{ K}$$

$V_C = 20\text{ m}^3$, $p_C = 10 \times 10^4 \text{ N/m}^2$

$$\therefore \quad T_C = \frac{(20)(10 \times 10^4)}{(500)(8.31)}\text{ K}$$

$$T_C = 481.36\text{K}$$

and $V_D = 20\text{m}^3,\ p_D = 5 \times 10^4\ \text{N/m}^2$

$$\therefore \quad V_D = \frac{(20)(5 \times 10^4)}{(500)(8.31)}\text{K}$$

$$T_D = 240.68\text{K}$$

(b) No, it is not possible to tell afterwards which sample went through the process ABC or ADC. But during the process if we note down the work done in both the processes, then the process which require more work goes through process ABC.

(c) In the process ABC

$$\Delta U = nC_V\ \Delta T = n\left(\frac{3}{2}R\right)(T_C - T_A)$$

$$= (500)\left(\frac{3}{2}\right)(8.31)(481.36 - 120.34)\ \text{J}$$

$$\Delta U = 2.25 \times 10^6\ \text{J}$$

and ΔW = Area under BC

$$= (20 - 10)(10) \times 10^4\ \text{J} = 10^6\ \text{J}$$

$$\therefore \Delta Q_{ABC} = \Delta U + \Delta W = (2.25 \times 10^6 + 10^6)\ \text{J}$$

$$\Delta Q_{ABC} = 3.25 \times 10^6\ \text{J}$$

In the process ADC ΔU will be same (because it depends on initial and final temperatures only)

ΔW = Area under AD

$$= (20 - 10)(5 \times 10^4)\ \text{J} = 0.5 \times 10^6\ \text{J}$$

$$\Delta Q_{ADC} = \Delta U + \Delta W = (2.25 \times 10^6 + 0.5 \times 10^6)\ \text{J}$$

$$\Delta Q_{ADC} = 2.75 \times 10^6\ \text{J}$$

52. The corresponding p-V diagram is as shown

Given $T_A = 300$ K, $n = 1$, $\gamma = 1.4$, $V_A / V_B = 16$ and $V_C / V_B = 2$

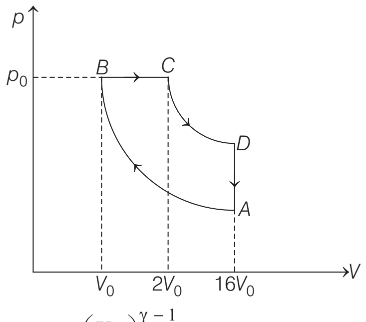
Let $\quad V_B = V_0$ and $p_B = p_0$

Then, $\quad V_C = 2V_0$ and $V_A = 16V_0$

Temperature at B

Process A-B is adiabatic.

Hence, $\quad T_A V_A^{\gamma - 1} = T_B V_B^{\gamma - 1}$



or $$T_B = T_A\left(\frac{V_A}{V_B}\right)^{\gamma - 1} = (300)(16)^{1.4 - 1}$$

$$T_B = 909\ \text{K}$$

Temperature at D

$B \to C$ is an isobaric process (p = constant)

$$\therefore \quad T \propto V, V_C = 2V_B$$

$$\therefore \quad T_C = 2T_B = (2)(909)\ \text{K}$$

$$T_C = 1818\ \text{K}$$

Now, the process $C \to D$ is adiabatic.

Therefore, $$T_D = T_C\left(\frac{V_C}{V_D}\right)^{\gamma - 1} = (1818)\left(\frac{2}{16}\right)^{1.4 - 1}$$

$$T_D = 791.4\ \text{K}$$

Efficiency of cycle

Efficiency of cycle (in percentage) is defined as

$$\eta = \frac{\text{Net work done in the cycle}}{\text{Heat absorbed in the cycle}} \times 100$$

or $$\eta = \frac{W_{\text{Total}}}{Q_{+\text{ve}}} \times 100$$

$$= \frac{Q_{+\text{ve}} - Q_{-\text{ve}}}{Q_{+\text{ve}}} \times 100 = \left(1 - \frac{Q_1}{Q_2}\right) \times 100 \qquad \text{... (i)}$$

where, Q_1 = negative heat in the cycle (heat released)

and $\quad Q_2$ = positive heat in the cycle (heat absorbed)

In the cycle

$$Q_{AB} = Q_{CD} = 0 \qquad \text{(Adiabatic process)}$$

$$Q_{DA} = nC_V\ \Delta T = (1)\left(\frac{5}{2}R\right)(T_A - T_D)$$

$$\left(C_V = \frac{5}{2}R \text{ for a diatomic gas}\right)$$

$$= \frac{5}{2} \times 8.31(300 - 791.4)\ \text{J}$$

or $$Q_{DA} = -10208.8\ \text{J}$$

and $$Q_{BC} = nC_p\Delta T = (1)\left(\frac{7}{2}R\right)(T_C - T_B)$$

$$\left(C_p = \frac{7}{2}R \text{ for a diatomic gas}\right)$$

$$= \left(\frac{7}{2}\right)(8.31)(1818 - 909)\ \text{J}$$

or $$Q_{BC} = 26438.3\ \text{J}$$

Therefore, substituting $Q_1 = 10208.8$ J and $Q_2 = 26438.3$ J in Eq. (i), we get

$$\therefore \quad \eta = \left\{1 - \frac{10208.8}{26438.3}\right\} \times 100 \quad \text{or} \quad \eta = 61.4\%$$

53. Given, $T_1 = 27^\circ\text{C} = 300\ \text{K}, V_1 = V, V_2 = 2V$

(a) Final temperature

In adiabatic process, $TV^{\gamma - 1}$ = constant

$$\therefore \quad T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma - 1} = 300\left(\frac{V}{2V}\right)^{5/3 - 1}$$

$$\gamma = \frac{5}{3} \text{ for a monoatomic gas } T_2 \approx 189\text{ K}$$

(b) Change in internal energy,

$$\Delta U = nC_V \Delta T$$

$$\Delta U = (2)\left(\frac{3}{2}R\right)(T_2 - T_1)$$

$$\Delta U = 2\left(\frac{3}{2}\right)(8.31)(189 - 300)\text{ J}$$

$$\Delta U = -2767\text{ J}$$

(c) Work done

Process is adiabatic, therefore $\Delta Q = 0$

and from first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

$$\Delta W = -\Delta U = -(-2767\text{ J})$$

$$\Delta W = 2767\text{ J}$$

54. (a) In a cyclic process $\Delta U = 0$

Therefore, $\quad Q_{\text{net}} = W_{\text{net}}$

or $Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$

Hence, $W_4 = (Q_1 + Q_2 + Q_3 + Q_4) - (W_1 + W_2 + W_3)$

$$= \{(5960 - 5585 - 2980 + 3645) - (2200 - 825 - 1100)\}$$

or $\quad W_4 = 765\text{ J}$

(b) Efficiency,

$$\eta = \frac{\text{Total work done in the cycle}}{\text{Heat absorbed (positive heat) by the gas during the cycle}} \times 100$$

$$= \left(\frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4}\right) \times 100$$

$$= \left\{\frac{(2200 - 825 - 1100 + 765)}{5960 + 3645}\right\} \times 100$$

$$= \frac{1040}{9605} \times 100$$

$$\eta = 10.82\%$$

NOTE

- From energy conservation

$W_{\text{net}} = Q_{+\text{ve}} - Q_{-\text{ve}}$ (in a cycle)

$$\therefore \quad \eta = \frac{W_{\text{net}}}{Q_{+\text{ve}}} \times 100 = \frac{(Q_{+\text{ve}} - Q_{-\text{ve}})}{Q_{+\text{ve}}} \times 100 = \left(1 - \frac{Q_{-\text{ve}}}{Q_{+\text{ve}}}\right) \times 100$$

In the above question

$$Q_{-\text{ve}} = |Q_2| + |Q_3| = (5585 + 2980)\text{ J} = 8565\text{ J}$$

and $\quad Q_{+\text{ve}} = Q_1 + Q_4 = (5960 + 3645)\text{ J} = 9605\text{ J}$

$$\therefore \quad \eta = \left(1 - \frac{8565}{9605}\right) \times 100$$

$$\eta = 10.82\%$$

55. Given, $T_A = 1000\text{ K}$, $p_B = \frac{2}{3}p_A$, $p_C = \frac{1}{3}p_A$

Number of moles, $n = 1$, $\gamma = \frac{C_p}{C_V} = \frac{5}{3}$ (monoatomic)

(a) $A \to B$ is an adiabatic process, therefore

$$p_A^{1-\gamma}T_A^{\gamma} = p_B^{1-\gamma}T_B^{\gamma}$$

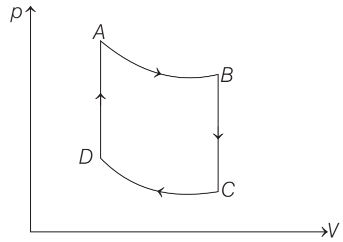
$$\therefore \quad T_B = T_A\left(\frac{p_A}{p_B}\right)^{\frac{1-\gamma}{\gamma}}$$

$$= (1000)\left(\frac{3}{2}\right)^{\frac{1-5/3}{5/3}} = (1000)\left(\frac{3}{2}\right)^{-2/5}$$

$$= (1000)\left(\frac{2}{3}\right)^{2/5}$$

$$T_B = (1000)(0.85)$$

$$\therefore \quad T_B = 850\text{ K}$$



Now, work done in the process $A \to B$ will be

$$W_{AB} = \frac{R}{1-\gamma}(T_B - T_A)$$

$$= \frac{8.31}{1 - 5/3}(850 - 1000) \quad \text{or} \quad W_{AB} = 1869.75\text{ J}$$

(b) $B \to C$ is an isochoric process (V = constant)

$$\therefore \quad \frac{T_B}{T_C} = \frac{p_B}{p_C}$$

$$\therefore \quad T_C = \left(\frac{p_C}{p_B}\right)T_B$$

$$= \left(\frac{(1/3)p_A}{(2/3)p_A}\right)850\text{ K}$$

$$T_C = 425\text{ K}$$

Therefore, $Q_{BC} = nC_V\Delta T = (1)\left(\frac{3}{2}R\right)(T_C - T_B)$

$$= \left(\frac{3}{2}\right)(8.31)(425 - 850)$$

$$Q_{BC} = -5297.6\text{ J}$$

Therefore, heat lost in the process BC is 5297.6 J.

(c) $C \to D$ and $A \to B$ are adiabatic processes. Therefore,

$$p_C^{1-\gamma}T_C^{\gamma} = p_D^{1-\gamma}T_D^{\gamma} \Rightarrow \frac{p_C}{p_D} = \left(\frac{T_D}{T_C}\right)^{\frac{\gamma}{1-\gamma}} \quad \text{...(i)}$$

$$p_A^{1-\gamma}T_A^{\gamma} = p_B^{1-\gamma}T_B^{\gamma}$$

$$\Rightarrow \quad \frac{p_A}{p_B} = \left(\frac{T_B}{T_A}\right)^{\frac{\gamma}{1-\gamma}} \quad \text{...(ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$\frac{p_C\, p_A}{p_D\, p_B} = \left(\frac{T_D T_B}{T_C T_A}\right)^{\frac{\gamma}{1-\gamma}} \quad ...(iii)$$

Processes $B \to C$ and $D \to A$ are isochoric. (V = constant)

Therefore, $\frac{p_C}{p_B} = \frac{T_C}{T_B}$ and $\frac{p_A}{p_D} = \frac{T_A}{T_D}$

Multiplying these two equations, we get

$$\frac{p_C\ p_A}{p_D\ p_B} = \frac{T_C\ T_A}{T_B\ T_D} \quad ... (iv)$$

From Eqs. (iii) and (iv), we have

$$\left(\frac{T_D\ T_B}{T_C\ T_A}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_C\ T_A}{T_B\ T_D}\right)$$

or $$\left(\frac{T_C\ T_A}{T_D\ T_B}\right)^{\frac{1-\gamma}{\gamma}} = \left(\frac{T_C\ T_A}{T_B\ T_D}\right)$$

$\Rightarrow$ $$\frac{T_C\ T_A}{T_D\ T_B} = 1$$

or $$T_D = \frac{T_C\ T_A}{T_B} = \frac{(425)(1000)}{850}$$

or $$T_D = 500\,\text{K}$$

56. Given number of moles $n = 2$

Process AB and CD are isobaric.

Hence, $Q_{AB} = -Q_{CD}$

[Because $(\Delta T)_{AB} = +100\,\text{K}$ whereas $(\Delta T)_{CD} = -100\,\text{K}$ and $Q_{\text{isobaric}} = nC_p\Delta T$]

or $$Q_{AB} + Q_{CD} = 0$$

Process BC is isothermal $(\Delta U = 0)$

$\therefore$ $$Q_{BC} = W_{BC} = nRT_B \ln\left(\frac{p_B}{p_C}\right)$$

$$= (2)(8.31)(400)\ln\left(\frac{2}{1}\right)$$

$$Q_{BC} = 4608\,\text{J}$$

Similarly, process DA is also isothermal hence,

$$Q_{DA} = W_{DA} = nRT_D \ln\left(\frac{p_D}{p_A}\right)$$

$$= (2)(8.31)(300)\ln\left(\frac{1}{2}\right)$$

$$Q_{BC} = -3456\,\text{J}$$

(a) Net heat exchange in the process

$$Q = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} = (4608 - 3456)\,\text{J}$$

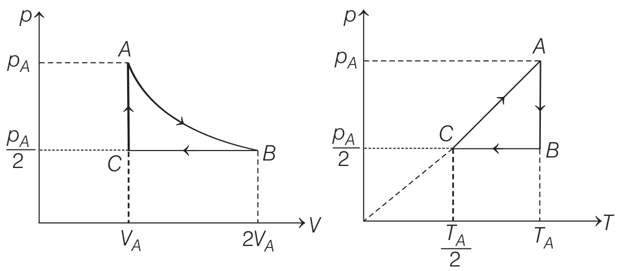
$$Q = 1152\,\text{J}$$

(b) From first law of thermodynamics, $\Delta U = 0$ (in complete cycle) $Q_{\text{net}} = W_{\text{net}}$

Hence, $W = Q = 1152\,\text{J}$

(c) Since, $T_i = T_f$, therefore net change in internal energy, $dU = 0$.

57. (a) The p-V and p-T diagrams are shown below



$$(pV)_C = \frac{(pV)_A}{2} \Rightarrow \therefore\ T_C = \frac{T_A}{2}$$

(b) **Process A-B** T = constant

$\therefore$ $p \propto \frac{1}{V}$, V is doubled. Therefore, p will become half.

Further, $$V_A = \frac{nRT_A}{p_A} = \frac{3RT_A}{p_A}$$

$$\Delta U_{AB} = 0$$

$\therefore$ $$Q_{AB} = W_{AB} = nRT_A \ln\left(\frac{2V_A}{V_A}\right)$$

$$= 3RT_A \ln(2) = 2.08RT_A$$

Process B-C

$$Q_{BC} = nC_p(T_C - T_B)$$

$$= (3)\left(\frac{7}{2}R\right)\left(\frac{T_A}{2} - T_A\right) = -\frac{21}{4}RT_A$$

$$= -5.25RT_A$$

Process C-A V = constant

$\therefore$ $$W_{CA} = 0$$

or $$Q_{CA} = \Delta U_{CA} = nC_V(T_A - T_C)$$

$$= (3)\left(\frac{5}{2}R\right)\left(T_A - \frac{T_A}{2}\right) = 3.75\,RT_A$$

In a cyclic process,

$$\Delta U = 0$$

$\therefore$ $$Q_{\text{net}} = W_{\text{net}} = 0.58\,RT_A$$

58. (a) In adiabatic process $TV^{\gamma-1}$ = constant

$\therefore$ $$TV^{\gamma-1} = \left(\frac{T}{2}\right)(5.66V)^{\gamma-1}$$

or $$(5.66)^{\gamma-1} = 2$$

$\therefore$ $(\gamma - 1)\ln(5.66) = \ln(2)$

$\therefore$ $\gamma - 1 = 0.4$

or $\gamma = 1.4$

i.e. degree of freedom, $f = 5$ as $\gamma = 1 + \frac{2}{f}$

(b) Using, pV^{γ} = constant

$$pV^{1.4} = p_f(5.66V)^{1.4}$$

$\therefore$ $$p_f = 0.09\,p$$

Now, work done in adiabatic process

$$W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

$$= \frac{(pV) - (0.09p)(5.66V)}{1.4 - 1}$$

$$= 1.23\ pV$$

59. Vessel is closed. Therefore, $\Delta W = 0$

or $\quad \Delta Q = \Delta U = nC_V\ \Delta T$

$$\Delta T = \frac{\Delta Q}{nC_V} = \frac{\Delta Q}{\left(\frac{pV}{RT}\right)(C_p - R)}$$

$$= \frac{(\Delta Q)(RT)}{pV\,(C_p - R)}$$

Substituting the values, we have

$$\Delta T = \frac{(2.49 \times 10^4)(300)}{(1.6 \times 10^6)(0.0083)(3/2)} = 375\,\text{K}$$

$\therefore \quad T_f = \Delta T + T = 675\ \text{K}$

Further at constant volume

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

$$\therefore \quad p_2 = \frac{T_2}{T_1}\ p_1$$

$$= \left(\frac{675}{300}\right)(1.6 \times 10^6)$$

$$= 3.6 \times 10^6 \text{N/m}^2$$

60. In adiabatic process

$$p_i V_i^\gamma = p_f V_f^\gamma$$

$$p_f = \left(\frac{V_i}{V_f}\right)^\gamma p_i \qquad \ldots\text{(i)}$$

Further, $\quad C_V = \dfrac{3R}{2}$

$\therefore \quad C_p = C_V + R = \dfrac{5R}{2}$

or $\quad \gamma = \dfrac{C_p}{C_V} = \dfrac{5}{3}$

$$\therefore \quad p_f = \left(\frac{6}{2}\right)^{5/3} (10)^5 = 6.24 \times 10^5\ \text{N/m}^2$$

Now, work done in adiabatic process is given by

$$W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

$$= \frac{10^5 \times 6 \times 10^{-3} - 6.24 \times 10^5 \times 2 \times 10^{-3}}{\left(\frac{5}{3}\right) - 1}$$

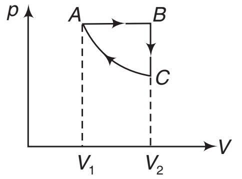
$$= -\,972\ \text{J}$$

NOTE Work done is negative because volume of the gas is decreasing.

61.

Name of the process	Information obtained from the graph	Pressure	Volume	Nature of p-V graph
$A \to B$	$V \propto T$ $\therefore$ p = constant V and T both are increasing	Constant	Increasing	Straight line parallel to V-axis as p = constant
$B \to C$	V = constant $\therefore$ $p \propto T$ T is decreasing. $\therefore$ p will decrease	Decreasing	Constant	Straight line parallel to p-axis, as V = constant
$C \to A$	T = constant $\therefore$ $p \propto \frac{1}{V}$ V is decreasing $\therefore$ p should increase	Increasing	Decreasing	Rectangular hyperbola as $p \propto \frac{1}{V}$

The corresponding graph is shown in figure given below



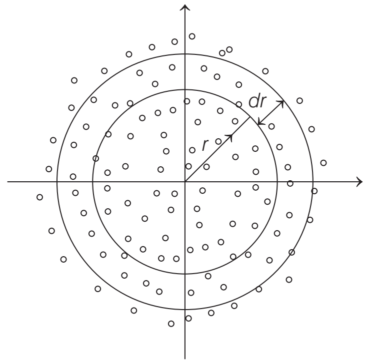
Topic 6 Miscellaneous Problems

1. Number density of gas molecules,

$$n = \frac{\text{Number of molecules}}{\text{Volume of gas}}$$

$\Rightarrow$ Number of molecules,

$N = n \times$ volume of gas



Now, consider a shell of radius r and thickness dr with centre at $r = 0$.

Volume of shell of differentiable thickness (dr), dV = surface area $\times$ thickness $= 4\pi r^2\ dr$

Now, number of molecules in this shell is

$$dN = n(r) \cdot dV = n_0 e^{-\alpha r^4} \cdot 4\pi r^2\ dr$$

So, total number of molecules present in given volume (extending from $r = 0$ to $r = \infty$) is

$$N = \int_0^\infty n(r) \cdot dV = \int_0^\infty n_0\ e^{-\alpha r^4} \cdot 4\pi r^2\ dr$$

$$= \int_0^\infty 4\pi n_0 \, e^{-\alpha r^4} \cdot r^2 \, dr \quad \ldots(i)$$

Here, we take

$$\alpha r^4 = t \Rightarrow r = t^{\frac{1}{4}} \cdot \alpha^{\frac{-1}{4}}$$

$$\Rightarrow \quad 4\alpha r^3 dr = dt$$

$$\Rightarrow \quad r^2 \, dr = \frac{dt}{4\alpha r} = \frac{dt}{4\alpha \, t^{\frac{1}{4}} \cdot \alpha^{-\frac{1}{4}}} = \frac{dt}{4\,\alpha^{3/4}\, t^{\frac{1}{4}}}$$

Also, when $r = 0, t = 0$ and when $r = \infty, t = \infty$ substituting in Eq. (i), we get

$$N = \int_0^\infty 4\pi n_0 e^{-t} \cdot \frac{dt}{4\,\alpha^{\frac{3}{4}}\, t^{\frac{1}{4}}}$$

$$N = \pi\alpha^{-\frac{3}{4}} \cdot n_0 \cdot \int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} \, dt$$

As value of definite integral $\int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} \, dt$ is a constant (= k let), we have

$$N = \pi k n_0 \alpha^{-\frac{3}{4}} \Rightarrow N \propto n_0 \alpha^{-\frac{3}{4}}$$

2. Momentum imparted to the surface in one collision,

$$\Delta p = (p_i - p_f) = mv - (-mv) = 2mv \quad \ldots(i)$$

Force on the surface due to n collision per second,

$$F = \frac{n}{t}(\Delta p) = n\Delta p \quad (\because t = 1\,\text{s})$$

$$= 2\,mnv \quad \text{[from Eq. (i)]}$$

So, pressure on the surface,

$$p = \frac{F}{A} = \frac{2mnv}{A}$$

Here, $m = 10^{-26}$ kg, $n = 10^{22}\ \text{s}^{-1}$,

$v = 10^4\ \text{ms}^{-1}$, $A = 1\,\text{m}^2$

$$\therefore \text{Pressure,} \quad p = \frac{2 \times 10^{-26} \times 10^{22} \times 10^4}{1} = 2\,\text{N/m}^2$$

So, pressure exerted is order of 10^0.

3. *(d)* By principle of thermometry for any liner temperature scale,

$$\frac{T - T_{\text{LFP}}}{T_{\text{UFP}} - T_{\text{LFP}}} = \text{a (constant)}$$

where,

T = temperature measured

T_{LFP} = temperature of melting ice or lower fixed point.

T_{UFP} = temperature of boling water or upper fixed point.

If, T = temperature of given object.

Then we have,

$$\frac{T - 0°\text{C}}{100°\text{C} - 0°\text{C}} = \frac{\frac{x_0}{2} - \frac{x_o}{3}}{x_0 - \frac{x_0}{3}} \text{ or } \frac{T}{100} = \frac{1}{4} \quad \text{or } T = 25°\text{C}$$

4. Energy flux is the rate of heat flow per unit area through the rod.

Also, rate of flow of heat per unit time through a material of area of cross-section A and thermal conductivity k between the temperatures T_1 and T_2 $(T_1 > T_2)$ is given as,

$$\frac{\Delta Q}{\Delta t} = \frac{kA(T_1 - T_2)}{l} \quad \ldots(i)$$

Energy flux using Eq. (i), we get

$$= \frac{1}{A} \cdot \frac{\Delta Q}{\Delta t} = \frac{k(T_1 - T_2)}{l}$$

Here, $k = 0.1\,\text{W K}^{-1}\text{m}^{-1}$, $l = 1$ m

$T_1 = 1000\,\text{K}$ and $T_2 = 100\,\text{K}$

$$\therefore \quad \text{Energy flux} = \frac{0.1(1000 - 100)}{1}$$

$$= 90\ \text{Wm}^{-2}.$$

5. Given, $\frac{U}{V} \propto T^4$

$$\frac{U}{V} = \alpha T^4 \quad \ldots(i)$$

It is also given that, $P = \frac{1}{3}\left(\frac{U}{V}\right)$

$$\Rightarrow \quad \frac{nR_0 T}{V} = \frac{1}{3}(\alpha T^4) \quad (R_0 = \text{Gas constant})$$

$$\text{or} \quad VT^3 = \frac{3nR_0}{\alpha} = \text{constant}$$

$$\therefore \quad \left(\frac{4}{3}\pi R^3\right)T^3 = \text{constant or } RT = \text{constant}$$

$$\therefore \quad T \propto \frac{1}{R}$$

6. Average time between two collisions is given by

$$\tau = \frac{1}{\sqrt{2}\pi n v_{\text{rms}}\, d^2} \quad \ldots(i)$$

Here, n = number of molecules per unit volume = $\frac{N}{V}$

$$\text{and} \quad v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Substituting these values in Eq.(i) we have,

$$\tau \propto \frac{V}{\sqrt{T}} \quad \ldots(ii)$$

For adiabatic process, $TV^{\gamma - 1}$ = constant

substituting in Eq. (ii), we have $\tau \propto \frac{V}{\sqrt{\left(\frac{1}{V^{\gamma - 1}}\right)}}$

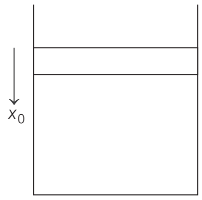
$$\text{or} \quad \tau \propto V^{1 + \left(\frac{\gamma - 1}{2}\right)} \quad \text{or} \quad \tau \propto V^{\left(\frac{1 + \gamma}{2}\right)}$$

7. Entropy is a state functions. Therefore in both cases answer should be same.

8. In equilibrium,

$$p_0 A = Mg \qquad \ldots(\text{i})$$

when slightly displaced downwards,



$$dp = -\gamma\left(\frac{p_0}{V_0}\right)dV\left(\text{As in adiabatic process}, \frac{dp}{dV} = -\gamma\,\frac{p}{V}\right)$$

$\therefore$ Restoring force,

$$F = (dp)A = -\left(\frac{\gamma\, p_0}{V_0}\right)(A)(Ax)$$

$$F \propto -x$$

Therefore, motion is simple harmonic comparing with $F = -kx$ we have

$$k = \frac{\gamma\, p_0 A^2}{V_0}$$

$$\therefore \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\gamma\, p_0 A^2}{MV_0}}$$

9. The process may be assumed to be isobaric.

$$\therefore \qquad Q = n\,C_p\,\Delta T = (2)\left(\frac{5}{2}R\right)(5)$$

$$= 5 \times 8.31 \times 5 = 207.75 \text{ J} \approx 208 \text{ J}$$

10.. pT^2 = constant

$$\therefore \quad \left(\frac{nRT}{V}\right)T^2 = \text{constant} \quad \text{or} \quad T^3V^{-1} = \text{constant}$$

Differentiating the equation, we get

$$\frac{3T^2}{V}.dT - \frac{T^3}{V^2}dV = 0 \quad \text{or} \quad 3dT = \frac{T}{V}.dV \qquad \ldots(\text{i})$$

From the equation

$$dV = V\gamma\, dT$$

$$\gamma = \text{coefficient of volume expansion of gas} = \frac{dV}{V.dT}$$

From Eq. (i) $\gamma = \dfrac{dV}{V.dT} = \dfrac{3}{T}$

11. This question is incomplete.

12. Out of the alternatives provided, none appears completely correct.

AB is an isothermal process.

(T = constant, $p \propto \frac{1}{V}$). So, p-V graph should be rectangular hyperbola with p decreasing and V increasing.

BC is an isobaric process.

(p = constant, $V \propto T$). Temperature is increasing. Hence, volume should also increase.

CA is an adiabatic process (pV^γ = constant). Pressure is increasing. So, volume should decrease. At point A, an isotherm AB and an adiabatic curve AC are meeting. We know that

(slope of an adiabatic graph in p-V diagram) = γ (slope of an isothermal graph in the same diagram) with $\gamma > 1$ or $(\text{Slope})_{\text{adiabatic}} > (\text{Slope})_{\text{isothermal}}$.

None of the given examples fulfill all the above requirements.

13.

$$\beta = -\frac{dV/dp}{V} = \text{compressibility of gas}$$

$$= \frac{1}{\text{Bulk modulus of elasticity}}$$

and $\beta = \dfrac{1}{p}$ under isothermal conditions.

Thus, β *versus* p graph will be a rectangular hyperbola.

14. From first law of thermodynamics,

$$dQ = dU + dW$$

$$dQ = dU, \text{ if } dW = 0$$

Since, $\qquad dQ < 0$

Therefore, $\qquad dU < 0$

or $\qquad U_{\text{final}} < U_{\text{initial}}$

or temperature will decrease.

NOTE Internal energy U of an ideal gas depends only on the temperature of the gas. Internal energy of n moles of an ideal gas is given by

$$U = n(f/2)RT$$

$$U \propto T$$

Here, f is the degree of freedom of the gas.

15. For an ideal gas, $pV = nRT$

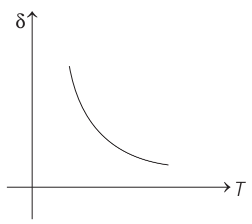
For $\qquad p$ = constant

$$p\Delta V = nR\Delta T$$

$$\therefore \qquad \frac{\Delta V}{\Delta T} = \frac{nR}{p} = \frac{nR}{\frac{nRT}{V}} = \frac{V}{T}$$

$$\therefore \qquad \frac{\Delta V}{V\Delta T} = \frac{1}{T} \text{ or } \delta = \frac{1}{T}$$

Therefore, δ is inversely proportional to temperature T. i.e. when T increases, δ decreases and *vice-versa*.



Hence, δ-T graph will be a rectangular hyperbola as shown in the above figure.

16. Let final equilibrium temperature of gases is T
Heat rejected by gas by lower compartment

$$= nC_V \Delta T = 2 \times \frac{3}{2} R\ (700 - T)$$

Heat received by the gas in above compartment

$$= nC_p \Delta T = 2 \times \frac{7}{2} R\ (T - 400)$$

Equating the two, we get

$$2100 - 3T = 7T - 2800$$

$$\Rightarrow \quad T = 490\ \text{K}$$

17. $\Delta W_1 + \Delta U_1 = \Delta Q_1$ ...(i)

$\Delta W_2 + \Delta U_2 = \Delta Q_2$ ...(ii)

$\Delta Q_1 + \Delta Q_2 = 0$

$\therefore (nC_p \Delta T)_1 + (nC_p \Delta T)_2 = 0$

But $\quad n_1 = n_2 = 2$

$$\therefore \quad \frac{5}{2} R\ (T - 700) + \frac{7}{2} R\ (T - 400) = 0$$

Solving, we get $T = 525\ \text{K}$
Now, from Eqs. (i) and (ii), we get

$$\Delta W_1 + \Delta W_2 = -\Delta U_1 - \Delta U_2$$

as $\quad \Delta Q_1 + \Delta Q_2 = 0$

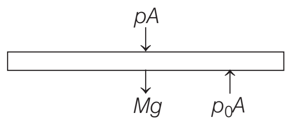
$$\therefore \quad \Delta W_1 + \Delta W_2 = -[(nC_V \Delta T)_1 + (nC_V \Delta T)_2]$$

$$= -\left[2 \times \frac{3}{2} R \times (525 - 700) + 2 \times \frac{5}{2} R \times (525 - 400)\right]$$

$$= -100R$$

18. Since it is open from top, pressure will be p_0.

19. Let p be the pressure in equilibrium.



Then, $\quad pA = p_0 A - Mg$

$$p = p_0 - \frac{Mg}{A} = p_0 - \frac{Mg}{\pi R^2}$$

Applying, $\quad p_1 V_1 = p_2 V_2$

$$\therefore \quad p_0(2AL) = (p)(AL')$$

$$\therefore \quad L' = \frac{2p_0 L}{p} = \left(\frac{p_0}{p_0 - \dfrac{Mg}{\pi R^2}}\right)(2L)$$

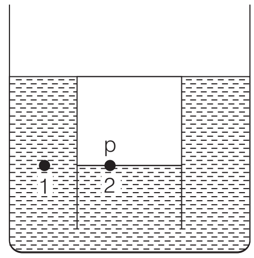
$$= \left(\frac{p_0 \pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$$

$\therefore$ Correct option is (d).

20.

$$p_1 = p_2$$

$$p_0 + \rho g (L_0 - H) = p \qquad \text{...(i)}$$



Now, applying $p_1 V_1 = p_2 V_2$ for the air inside the cylinder,

we have $\quad p_0(L_0) = p(L_0 - H)$

$$\therefore \quad p = \frac{p_0 L_0}{L_0 - H}$$

Substituting in Eq. (i), we have

$$p_0 + \rho g (L_0 - H) = \frac{p_0 L_0}{L_0 - H}$$

or $\quad \rho g (L_0 - H)^2 + p_0 (L_0 - H) - p_0 L_0 = 0$

$\therefore$ Correct option is (c).

21. (A) In case of free expansion under adiabatic conditions, change in internal energy $\Delta U = 0$.

$\therefore$ Internal energy and temperature will remain constant.

(B) $p \propto \dfrac{1}{V^2}$

$\Rightarrow \quad \therefore \quad pV^2 = \text{constant}$ ... (i)

or $\left(\dfrac{nRT}{V}\right).V^2 = \text{constant} \quad \Rightarrow \quad \therefore \quad T \propto \dfrac{1}{V}$ ... (ii)

If volume is doubled, temperature will decrease as per Eq. (ii).

Further, molar heat capacity in process $pV^x = $ constant is

$$C = C_V + \frac{R}{1 - x}$$

From Eq. (i), $\quad x = 2$

$$\therefore \quad C = \frac{3}{2} R + \frac{R}{1 - 2} = + \frac{R}{2}$$

Since, molar heat capacity is positive, according to $Q = nC\Delta T$, Q will be negative if ΔT is negative. Or gas loses heat if temperature is decreasing.

(C) $p \propto \dfrac{1}{V^{4/3}}, \quad pV^{4/3} = \text{constant}$

$$\therefore \quad \left(\frac{nRT}{V}\right) V^{4/3} = \text{constant}$$

$$\therefore \quad T \propto \frac{1}{V^{1/3}}$$

Further, with increase in volume temperature will decrease.

Here, $\quad x = \dfrac{4}{3} \Rightarrow C = \dfrac{3}{2} R + \dfrac{R}{1 - 4/3} = -1.5R$

As molar heat capacity is negative, Q will be positive if ΔT is negative. Or gas gains heat with decrease in temperature.

(D) $T \propto pV$

In expansion from V_1 to $2V_1$, product of pV is increasing. Therefore, temperature will increase. Or $\Delta U = +$ve. Further, in expansion work done is also positive.

Hence, $Q = W + \Delta U = +$ve or, gas gains heat.

23. Assumption $e = 1$ [black body radiation]

$$P = \sigma A(T^4 - T_0^4)$$

(c) $P_{\text{rad}} = \sigma A T^4 = \sigma \cdot 1 \cdot (T_0 + 10)^4$

$$= \sigma \cdot T_0^4 \left(1 + \frac{10}{T_0}\right)^4 \qquad [T_0 = 300\text{K given}]$$

$$= \sigma \cdot (300)^4 \cdot \left(1 + \frac{40}{300}\right) \approx 460 \times \frac{17}{15} \approx 520 \text{ J}$$

$$P_{\text{net}} = 520 - 460 \approx 60 \text{ W}$$

$\Rightarrow$ Energy radiated in 1 s = 60 J

(b) $$P = \sigma A(T^4 - T_0^4)$$

$$dP = \sigma A(0 - 4T_0^3 \cdot dT)$$

and $$dT = -\Delta T$$

$\Rightarrow$ $$dP = 4\sigma A T_0^3 \Delta T$$

(d) If surface area decreases, then energy radiation also decreases.

NOTE

While giving answer (b) and (c) it is assumed that energy radiated refers the net radiation. If energy radiated is taken as only emission, then (b) and (c) will not be included in answer.

24. $Q = mCT \quad \Rightarrow \quad \dfrac{dQ}{dt} = mc\dfrac{dT}{dt}$

R = rate of absorption of heat $= \dfrac{dQ}{dt} \propto C$

(i) in $0 - 100$ K C increase, so R increases but not linearly

(ii) $\Delta Q = mC\Delta T$ as C is more in (400 K-500 K) then $(0 - 100\text{ K})$ so heat is increasing

(iii) C remains constant so there no change in R from $(400\text{ K} - 500\text{ K})$

(iv) C is increases so R increases in range(200 K -300K)

25. For monoatomic gas, $C_p = \dfrac{5}{2}R$

and $$C_V = \frac{3}{2}R$$

For diatomic gas, $$C_p = \frac{7}{2}R$$

and $$C_V = \frac{5}{2}R$$

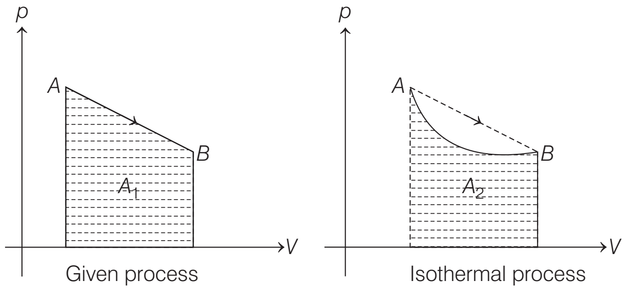
26. (a) Work done = Area under p-V graph

$$A_1 > A_2$$

$$\therefore \quad W_{\text{given process}} > W_{\text{isothermal process}}$$

(b) In the given process p-V equation will be of a straight line with negative slope and positive intercept *i.e.,* $p = -\alpha V + \beta$ (Here α and β are positive constants)

$\Rightarrow$ $$pV = -\alpha V^2 + \beta V$$



$\Rightarrow$ $$nRT = -\alpha V^2 + \beta V$$

$\Rightarrow$ $$T = \frac{1}{nR}(-\alpha V^2 + \beta V) \qquad \text{... (i)}$$

This is an equation of parabola in T and V.

(d) $\dfrac{dT}{dV} = 0 = \beta - 2\alpha V$

$\Rightarrow$ $$V = \frac{B}{2\alpha}$$

Now, $$\frac{d^2T}{dV^2} = -2\alpha = -\text{ve}$$

ie, T has some maximum value.

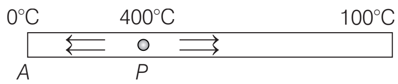
Now, $T \propto pV$

and $(pA)_A = (pV)_B$

$\Rightarrow$ $T_A = T_B$

We conclude that temperatures are same at A and B and in between temperature has a maximum value. Therefore, in going from A to B, T will first increase to a maximum value and then decrease.

27.



Heat will flow both sides from point P.

$$L_1 \frac{dm_1}{dt} = \left(\frac{\text{Temperature difference}}{\text{Thermal resistance}}\right)_1$$

$$= \frac{400}{(\lambda x)/kA} \qquad \text{...(i)}$$

$$L_1 \frac{dm_2}{dt} = \frac{400 - 100}{(100 - \lambda)x/kA} \qquad \text{...(ii)}$$

In about two equations, $\dfrac{dm_1}{dt} = \dfrac{dm_2}{dt}$ (given)

$L_1 = 80 \text{ calg}^{-1}$ and $L_2 = 540 \text{ calg}^{-1}$

Solving these two equations, we get $\lambda = 9$.

28. Pressure on both sides will be equal

i.e. $$p_1 = p_2$$

$$\frac{n_1RT}{V_1} = \frac{n_2RT}{V_2} \qquad \left(n = \frac{m}{M}\right)$$

or $$\frac{m}{M_1V_1} = \frac{m}{M_2V_2}$$

or $$\frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7}$$

$$\therefore \quad \alpha = \left(\frac{360°}{8+7}\right) \times 8 = 192°$$

29. The gas thermometer works at constant volume. Therefore,

$$T \propto p$$

or $$T_2/T_1 = p_2/p_1$$

or $$T_2 = T_1(p_2/p_1)$$

or $$T_2 = (273.16)\left(\frac{3.5 \times 10^4}{3 \times 10^4}\right) \text{K}$$

$$= 318.68 \text{ K}$$

or temperature at given pressure will be

$$(318.68 - 273)° \text{C}$$

or $$45.68° \text{C}$$

30. $$C_p = C_V + R$$

$$\Rightarrow \therefore \quad C_p > C_V$$

31. Rate of heat conduction through rod = rate of the heat lost from right end of the rod.

$$\therefore \quad \frac{KA(T_1 - T_2)}{L} = eA\sigma\,(T_2^4 - T_s^4) \quad \text{... (i)}$$

Given that $$T_2 = T_s + \Delta T$$

$$\therefore \quad T_2^4 = (T_s + \Delta T)^4$$

$$= T_s^4\left(1 + \frac{\Delta T}{T_s}\right)^4$$

Using binomial expansion, we have

$$T_2^4 = T_s^4\left(1 + 4\frac{\Delta T}{T_s}\right) \quad (\text{as } \Delta T << T_s)$$

$$\therefore \quad T_2^4 - T_s^4 = 4(\Delta T)(T_s^3)$$

Substituting in Eq. (i), we have

$$\frac{K(T_1 - T_s - \Delta T)}{L} = 4e\sigma T_s^3 . \Delta T$$

or $$\frac{K(T_1 - T_s)}{L} = \left(4e\sigma T_s^3 + \frac{K}{L}\right)\Delta T$$

$$\therefore \quad \Delta T = \frac{K(T_1 - T_s)}{(4e\sigma L T_s^3 + K)}$$

Comparing with the given relation, proportionality constant

$$= \frac{K}{4e\sigma L T_s^3 + K}$$

32. At constant pressure, $V \propto T$

or $$\frac{V_2}{V_1} = \frac{T_2}{T_1} \text{ or } \frac{Ah_2}{Ah_1} = \frac{T_2}{T_1}$$

$$\therefore \quad h_2 = h_1\left(\frac{T_2}{T_1}\right) = (1.0)\left(\frac{400}{300}\right) \text{m} = \frac{4}{3} \text{ m}$$

As there is no heat loss, process is adiabatic.
For adiabatic process,

$$T_f V_f^{\gamma - 1} = T_i V_i^{\gamma - 1}$$

$$\therefore \quad T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma - 1} = (400)\left(\frac{h_i}{h_f}\right)^{1.4 - 1}$$

$$= 400\left(\frac{4}{3}\right)^{0.4} = 448.8 \text{ K}$$

33. (a) Rate of heat loss per unit area due to radiation

$$I = e\sigma\,(T^4 - T_0^4)$$

Here, $T = 127 + 273 = 400 \text{ K}$

and $T_0 = 27 + 273 = 300 \text{ K}$

$$\therefore \quad I = 0.6 \times \frac{17}{3} \times 10^{-8}[(400)^4 - (300)^4]$$

$$= 595 \text{ W/m}^2$$

(b) Let θ be the temperature of the oil.

Then, rate of heat flow through conduction = rate of heat loss due to radiation

$$\therefore \quad \frac{\text{Temperature difference}}{\text{Thermal resistance}} = (595)A$$

$$\frac{(\theta - 127)}{\left(\frac{l}{KA}\right)} = (595)A$$

Here, A = area of disc ; K = thermal conductivity and l = thickness (or length) of disc

$$\therefore (\theta - 127)\frac{K}{l} = 595$$

$$\therefore \theta = 595\left(\frac{l}{K}\right) + 127 = \frac{595 \times 10^{-2}}{0.167} + 127 = 162.6°\text{C}$$

34. Decrease in kinetic energy = increase in internal energy of the gas

$$\therefore \quad \frac{1}{2}mv_0^2 = nC_V\Delta T = \left(\frac{m}{M}\right)\left(\frac{3}{2}R\right)\Delta T$$

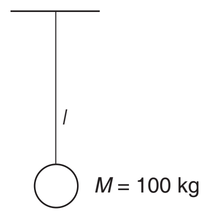
$$\therefore \quad \Delta T = \frac{Mv_0^2}{3R}$$

35. Given,

Length of the wire, $l = 5 \text{ m}$

Radius of the wire, $r = 2 \times 10^{-3}$ m

Density of wire, $\rho = 7860 \text{kg/m}^3$



Young's modulus,

$$Y = 2.1 \times 10^{11} \text{ N/m}^2$$

and specific heat, $s = 420$ J/kg-K

Mass of wire, m = (density) (volume)

$$= (\rho)(\pi r^2 l)$$

$$= (7860)(\pi)(2\times10^{-3})^2(5)\,\text{kg} = 0.494\text{ kg}$$

Elastic potential energy stored in the wire,

$$U = \frac{1}{2}\text{(stress) (strain) (volume)}$$

$$\left[\because \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\times\text{stress}\times\text{strain}\right]$$

or $$U = \frac{1}{2}\left(\frac{Mg}{\pi r^2}\right)\left(\frac{\Delta l}{l}\right)(\pi r^2 l)$$

$$= \frac{1}{2}(Mg)\cdot\Delta l \qquad \left(\Delta l = \frac{Fl}{AY}\right)$$

$$= \frac{1}{2}(Mg)\frac{(Mgl)}{(\pi r^2)Y} = \frac{1}{2}\frac{M^2g^2l}{\pi r^2 Y}$$

Substituting the values, we have

$$U = \frac{1}{2}\frac{(100)^2(10)^2(5)}{(3.14)(2\times10^{-3})^2(2.1\times10^{11})}\text{ J}$$

$$= 0.9478\text{ J}$$

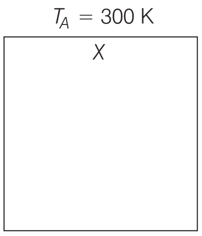
When the bob gets snapped, this energy is utilised in raising the temperature of the wire.

So, $$U = ms\,\Delta\theta$$

$\therefore$ $$\Delta\theta = \frac{U}{ms} = \frac{0.9478}{0.494\,(420)}\,^\circ\text{C or K}$$

$$\Delta\theta = 4.568\times10^{-3}\,^\circ\text{C}$$

36. In the first part of the question ($t \le t_1$)



At $t = 0, T_X = T_0 = 400$ K and at $t = t_1, T_X = T_1 = 350$K

Temperature of atmosphere, $T_A = 300$ K (constant)

This cools down according to Newton's law of cooling. Therefore, rate of cooling $\propto$ temperature difference.

$\therefore$ $$\left(-\frac{dT}{dt}\right) = k(T - T_A)$$

$\Rightarrow$ $$\frac{dT}{T - T_A} = -k\,dt$$

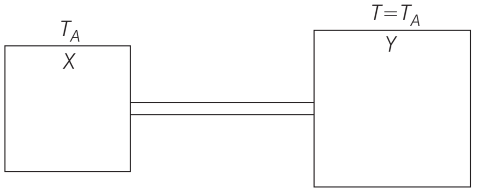
$\Rightarrow$ $$\int_{T_0}^{T_1}\frac{dT}{T - T_A} = -k\int_0^{t_1}dt$$

$\Rightarrow$ $$\ln\left(\frac{T_1 - T_A}{T_0 - T_A}\right) = -kt_1$$

$\Rightarrow$ $$kt_1 = -\ln\left(\frac{350 - 300}{400 - 300}\right)$$

$\Rightarrow$ $$kt_1 = \ln(2) \qquad \text{... (i)}$$

In the second part ($t > t_1$), body X cools by radiation (according to Newton's law) as well as by conduction.



Therefore, rate of cooling

= (cooling by radiation) + (cooling by conduction)

$\therefore$ $$\left(-\frac{dT}{dt}\right) = k(T - T_A) + \frac{KA}{CL}(T - T_A) \qquad \text{... (ii)}$$

In conduction, $\dfrac{dQ}{dt} = \dfrac{KA(T - T_A)}{L} = C\left(-\dfrac{dT}{dt}\right)$

$\therefore$ $$\left(-\frac{dT}{dt}\right) = \frac{KA}{LC}(T - T_A)$$

where, C = heat capacity of body X

$$\left(-\frac{dT}{dt}\right) = \left(k + \frac{KA}{CL}\right)(T - T_A) \qquad \text{... (iii)}$$

Let at $t = 3t_1$, temperature of X becomes T_2

Then from Eq. (iii)

$$\int_{T_1}^{T_2}\frac{dT}{T - T_A} = -\left(k + \frac{KA}{LC}\right)\int_{t_1}^{3t_1}dt$$

$$\ln\left(\frac{T_2 - T_A}{T_1 - T_A}\right) = -\left(k + \frac{KA}{LC}\right)(2t_1)$$

$$= -\left(2kt_1 + \frac{2KA}{LC}t_1\right)$$

or $$\ln\left(\frac{T_2 - 300}{350 - 300}\right) = -2\ln(2) - \frac{2KAt_1}{LC};$$

$$kt_1 = \ln(2)\text{ from Eq. (i).}$$

This equation gives

$$T_2 = \left(300 + 12.5e^{\frac{-2KAt_1}{CL}}\right)\text{K}$$

37. (a) Number of moles of gas A are $n_A = 1$ (given)

Let the number of moles of gas B be $n_B = n$

For a mixture

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \qquad \text{...(i)}$$

Substituting the values in Eq. (i), we have

$$\frac{1 + n}{(19/13) - 1} = \frac{1}{(5/3) - 1} + \frac{n}{(7/5) - 1}$$

Solving this, we get $n = 2$.

(b) Molecular weight of the mixture will be given by,

$$M = \frac{n_A M_A + n_B M_B}{n_A + n_B} = \frac{(1)(4) + 2(32)}{1+2}$$

$$M = 22.67$$

Speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Therefore, in the mixture of the gas

$$v = \sqrt{\frac{(19/13)(8.31)(300)}{22.67 \times 10^{-3}}} \text{ m/s}$$

$$v \approx 401 \text{ m/s}$$

(c) $$v \propto \sqrt{T}$$

or $$v = KT^{1/2} \qquad \text{... (ii)}$$

$$\Rightarrow \frac{dv}{dT} = \frac{1}{2}KT^{-1/2} \quad \Rightarrow \quad dv = K\left(\frac{dT}{2\sqrt{T}}\right)$$

$$\Rightarrow \frac{dv}{v} = \frac{K}{v}\cdot\left(\frac{dT}{2\sqrt{T}}\right) \Rightarrow \frac{dv}{v} = \frac{1}{\sqrt{T}}\left(\frac{dT}{2\sqrt{T}}\right) = \frac{1}{2}\left(\frac{dT}{T}\right)$$

$$\Rightarrow \frac{dv}{v} \times 100 = \frac{1}{2}\left(\frac{dT}{T}\right) \times 100 = \frac{1}{2}\left(\frac{1}{300}\right) \times 100$$

$$= 0.167$$

Therefore, percentage change in speed is 0.167%.

(d) Compressibility $= \dfrac{1}{\text{Bulk modulus}} = \beta$ (say)

Adiabatic bulk modulus is given by

$$B_{\text{adi}} = \gamma\ p \qquad \left(B = -\frac{dp}{dV/V}\right)$$

$\therefore$ Adiabatic compressibility will be given by

$$\beta_{\text{adi}} = \frac{1}{\gamma\ p} \text{ and } \beta'_{\text{adi}} = \frac{1}{\gamma p'} = \frac{1}{\gamma p(5)^\gamma} \quad (pV^\gamma = \text{constant})$$

$$[pV^\gamma = p'(V/5)^\gamma \quad \Rightarrow \quad p' = p(5)^\gamma]$$

$$\therefore\ \Delta\beta = \beta'_{\text{adi}} - \beta_{\text{adi}} = \frac{-1}{\gamma\ p}\left[1 - \left(\frac{1}{5}\right)^\gamma\right]$$

$$= \frac{-V}{\gamma\ (n_A + n_B)RT}\left[1 - \left(\frac{1}{5}\right)^\gamma\right] \qquad \left(\because p = \frac{nRT}{V}\right)$$

$$= \frac{-V}{\left(\frac{19}{13}\right)(1+2)(8.31)(300)}\left[1 - \left(\frac{1}{5}\right)^{\frac{19}{13}}\right]$$

$$\left(\gamma = \gamma_{\text{mixture}} = \frac{19}{13}\right)$$

$$\Delta\beta = -\,8.27 \times 10^{-5}\ V$$

38. Given, temperature of the mixture, $T = 27°\text{C} = 300$ K

Let m be the mass of the neon gas in the mixture. Then, mass of argon would be $(28 - m)$

Number of gram moles of neon, $n_1 = \dfrac{m}{20}$

Number of gram moles of argon, $n_2 = \dfrac{(28 - m)}{40}$

From Dalton's law of partial pressures.

Total pressure of the mixture (p) = Pressure due to neon (p_1) + Pressure due to argon (p_2)

or $$p = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = (n_1 + n_2)\frac{RT}{V}$$

Substituting the values

$$1.0 \times 10^5 = \left(\frac{m}{20} + \frac{28 - m}{40}\right)\frac{(8.314)(300)}{0.02}$$

Solving this equation, we get

$$28 - m = 23.926 \text{ g} \Rightarrow m = 4.074 \text{ g}$$

Therefore, in the mixture, 4.074 g neon is present and the rest i.e. 23.926 g argon is present.

39. Final pressure $= p_0 + \dfrac{kx}{A}$

$$= 1.0 \times 10^5 + \frac{(8000)(0.1)}{8 \times 10^{-3}} = 2 \times 10^5 \text{ N/m}^2$$

Final volume $= 2.4 \times 10^{-3} + (0.1)(8 \times 10^{-3})$

$$= 3.2 \times 10^{-3} \text{ m}^3$$

Applying, $$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

we have, $$T_f = \left(\frac{p_f V_f}{p_i V_i}\right) T_i$$

$$= \frac{(2 \times 10^5)(3.2 \times 10^{-3})}{(1 \times 10^5)(2.4 \times 10^{-3})} \times 300 = 800 \text{ K}$$

Heat supplied $Q = W_{\text{gas}} + \Delta U$

$$= p_0\ (\Delta V) + \frac{1}{2}kx^2 + nC_V\,\Delta T \qquad \left(\text{as } n = \frac{p_i V_i}{RT_i}\right)$$

$$= (10)^5\ (3.2 - 2.4) \times 10^{-3} + \frac{1}{2} \times 8000 \times (0.1)^2$$

$$+ \frac{10^5 \times 2.4 \times 10^{-3}}{8.31 \times 300} \times \frac{3}{2} \times 8.31 \times (800 - 300)$$

$$= 80 + 40 + 600 = 720 \text{J}$$

40. **(a) and (b) Process *AB*** p = constant

$\therefore$ $$V \propto T$$

i.e. if V is doubled, T also becomes two times.

$$T_A = 300 \text{ K} \quad \Rightarrow \quad \therefore \quad T_B = 600 \text{ K}$$

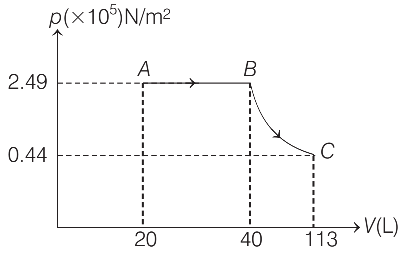
$$V_A = 20 \text{ L} \quad \Rightarrow \quad \therefore \quad V_B = 40 \text{ L}$$

$$p_A = \frac{n\,RT_A}{V_A} = \frac{(2)(8.31)(300)}{20 \times 10^{-3}}$$

$$= 2.49 \times 10^5 \text{ N/m}^2$$

Process 2 Process is adiabatic. So, applying

$$T^\gamma p^{1-\gamma} = \text{constant}$$



or $\left(\frac{600}{300}\right)^{5/3} = \left(\frac{2.49\times10^5}{p_c}\right)^{5/3-1} = \left(\frac{2.49\times10^5}{p_c}\right)^{2/3}$

$\therefore\ p_c = (2.49\times10^5)\left(\frac{300}{600}\right)^{5/2} \Rightarrow p_c = 0.44\times10^5\ \text{N/m}^2$

Similarly, using pV^γ = constant we can find that $V_c = 113$ L

The corresponding p-V graph is shown above

(c) W_{AB} = Area under p-V graph

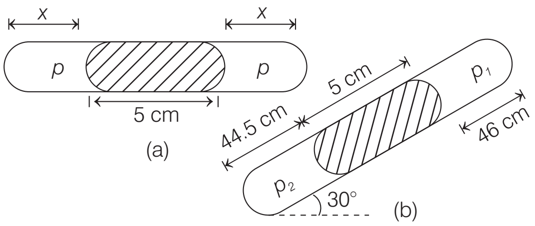
$= (2.49\times10^5)(40-20)\times10^{-3} = 4980$ J

$W_{BC} = -\Delta U = nC_V\ (T_B - T_C)$

$= (2)\left(\frac{3}{2}\times 8.31\right)(600-300) = 7479$ J

$\therefore\ W_{\text{net}} = (4980 + 7479)$ J $= 12459$ J

41. From the two figures we can see that



$2x + 5 = 44.5 + 5 + 46 \Rightarrow x = 45.25$ cm

Let A be the area of cross-section of the tube. Process is given isothermal. Hence, apply pV = constant in two sides of mercury column.

$pAx = p_2A\,(44.5)$ or $p\,(45.25) = p_2(44.5)$ …(i)

Similarly, $pAx = p_1A(46)$

or $p\,(45.25) = p_1(46)$ …(ii)

From figure (b), $p_2 = p_1 + 5\sin 30°$ …(iii)

Solving these three equations, we get

$p = 75.4$ cm of Hg

42. Let x moles transfer from bulb of higher temperature to lower temperature.

Applying $pV = nRT$ for both the bulbs initially and finally

$76\times V = nR\times 273$ …(i)

$p'\times V = (n+x)R\times 273$ …(ii)

$p'\times V = (n-x)R\times 335$ …(iii)

Solving these equations, we get $n = \frac{602}{62}x$

Now, dividing Eq. (ii) with Eq. (i), we get

$$\frac{p'}{76} = \frac{n+x}{n} = 1+\frac{x}{n} = 1+\frac{62}{602} = 1.103$$

$\therefore\ p' = 83.83$ cm of Hg

43. (a) Applying $\frac{pV}{T}$ = constant for both the chambers.

$$\frac{p_0V_0}{T_0} = \frac{pV_1}{T_1} = \frac{pV_2}{T_2} \qquad \left(\text{Here, } p = \frac{243p_0}{32}\right)$$

$\therefore\ V_1 = \frac{32}{243}\left(\frac{T_1}{T_0}\right)V_0$ and $V_2 = \left(\frac{32}{243}\right)\left(\frac{T_2}{T_0}\right)V_0$

Further, $V_1 + V_2 = 2V_0$

or $\left(\frac{16}{243}\right)(T_1+T_2) = T_0$ or $T_1 + T_2 = \frac{243}{16}T_0$ …(i)

Applying the p-T equation of adiabatic process to the right chamber,

$$\left(\frac{T_0}{T_2}\right)^{5/3} = \left(\frac{243p_0}{32p_0}\right)^{1-5/3}$$

Solving this equation, we get

$T_2 = 2.25\,T_0$

From Eq. (i), $T_1 = 12.94T_0$

(b) Work done by the gas in right chamber

($\Delta Q = 0$, adiabatic process)

$\Delta W = -\Delta U = nC_V\ (T_i - T_f)$

$= (1)\left(\frac{3}{2}R\right)(T_0 - 2.25T_0) = -1.875\ RT_0$

44. (a) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times8.31\times300}{32\times10^{-3}}} = 483.4$ m/s

Given, $p_0 = 1.01\times10^5$ N/m^2 = Force per unit area.

Let n molecules of oxygen strike the wall per second per m^2 and recoil with same speed. Change in momentum is ($2nmv_{\text{rms}}$). The change in momentum per unit time is the force.

Hence, $p_0 = 2nmv_{\text{rms}}$

$$\therefore\ n = \frac{p_0}{2mv_{\text{rms}}} = \frac{1.01\times10^5}{2\left[\frac{32}{6.02\times10^{26}}\right](483.4)}$$

$= 1.96\times10^{27}$/s

(b) $\frac{1}{2}(m_{\text{gas}})v_0^2 = nC_V\Delta T$

$$\therefore\ v_0 = \sqrt{\frac{2nC_V\Delta t}{m_{\text{gas}}}} = \sqrt{\frac{(2)(n)\left(\frac{5}{2}\times8.31\right)(1)}{(n)(32\times10^{-3})}} = 36\ \text{m/s}$$

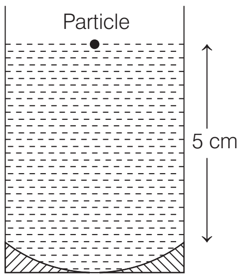
12

Optics

Topic 1 Reflection of Light

Objective Questions I (Only one correct option)

1. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to



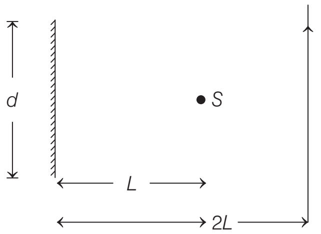
[Refractive index of water = 1.33] **(2019 Main, 12 April I)**

(a) 6.7 cm (b) 13.4 cm (c) 8.8 cm (d) 11.7 cm

2. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is **(2019 Main, 9 April I)**

(a) 0.16 m (b) 1.60 m (c) 0.32 m (d) 0.24 m

3. A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below. The distance over which the man can see the image of the light source in the mirror is **(2019 Main, 12 Jan I)**



(a) $\frac{d}{2}$ (b) d (c) $3d$ (d) $2d$

4. The plane mirrors (M_1 and M_2) are inclined to each other such that a ray of light incident on mirror M_1 and parallel to the mirror M_2 is reflected from mirror M_2 parallel to the mirror M_1. The angle between the two mirror is **(2019 Main, 9 Jan II)**

(a) 45° (b) 75° (c) 90° (d) 60°

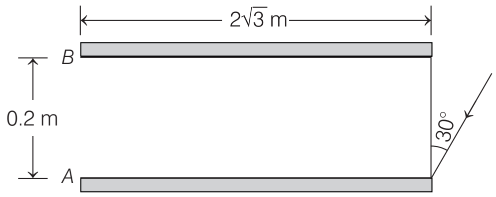
5. A ray of light travelling in the direction $\frac{1}{2}(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}})$. The angle of incidence is

(a) 30° (b) 45° **(2013 Adv.)**
(c) 60° (d) 75°

6. In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then **(2007, 3M)**

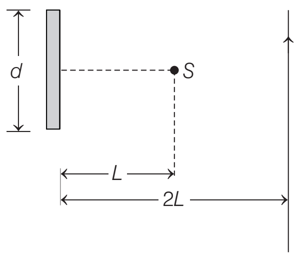
(a) $x < f$ (b) $f < x < 2f$ (c) $x = 2f$ (d) $x > 2f$

7. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is **(2002, 2M)**



(a) 28 (b) 30
(c) 32 (d) 34

8. A point source of light S, placed at a distance L in front of the centre of a plane mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown. The greatest distance over which he can see the image of the light source in the mirror is **(2000, 2M)**



(a) $d/2$ (b) d (c) $2d$ (d) $3d$

9. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to

(a) $b\left(\frac{u-f}{f}\right)^{1/2}$ (b) $b\left(\frac{f}{u-f}\right)^{1/2}$

(c) $b\left(\frac{u-f}{f}\right)$ (d) $b\left(\frac{f}{u-f}\right)^{2}$

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.

(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(c) If Statement I is true; Statement II is false.

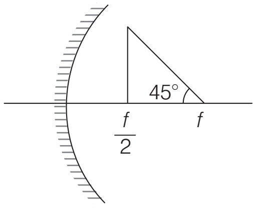
(d) If Statement I is false; Statement II is true.

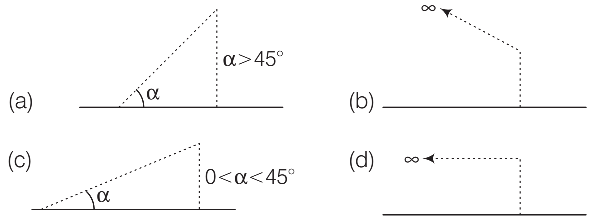
10. **Statement I** The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature. **(2007, 3M)**

Statement II Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

Objective Question II (One or more correct option)

11. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.) **(2018 Adv.)**





12. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are) **(2009)**

(a) (42, 56)
(b) (48, 48)
(c) (66, 33)
(d) (78, 39)

Fill in the Blank

13. A thin rod of length $f/3$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. The magnification is ...... **(1991, 1M)**

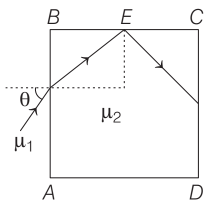
Integer Answer Type Question

14. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 s. What is the speed of the object in km h^{-1}? **(2010)**

Topic 2 Refraction of Light and TIR

Objective Questions I (Only one correct option)

1. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1 (\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC.

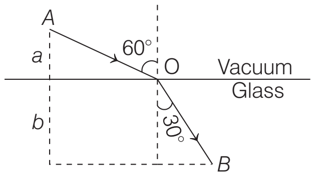


Then, θ must satisfy **(2019 Main, 12 April II)**

(a) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (b) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

(c) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$ (d) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

2. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is **(2019 Main, 11 April I)**



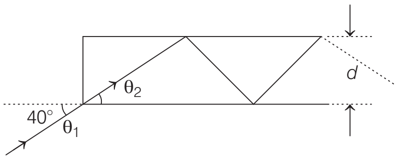
(a) $\frac{2\sqrt{3}}{a} + 2b$

(b) $2a + \frac{2b}{3}$

(c) $2a + 2b$

(d) $2a + \frac{2b}{\sqrt{3}}$

3. In figure, the optical fibre is $l = 2$ m long and has a diameter of $d = 20\,\mu$m. If a ray of light is incident on one end of the fibre at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to (refractive index of fibre is 1.31 and $\sin 40° = 0.64$) **(Main 2019, 8 April I)**

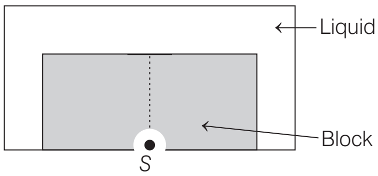


(a) 55000 (b) 66000 (c) 45000 (d) 57000

4. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement. **(2014 Main)**

(a) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal
(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium
(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium
(d) The entire spectrum of visible light will come out of the water at various angles to the normal

5. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is **(2014 Adv.)**



(a) 1.21 (b) 1.30 (c) 1.36 (d) 1.42

6. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is $\frac{4}{3}$. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as **(2009)**

(a) 9 ms^{-1} (b) 12 ms^{-1} (c) 16 ms^{-1} (d) 21.33 ms^{-1}

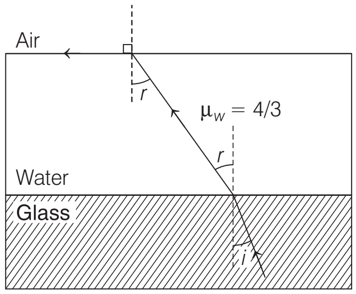
7. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be **(2007, 3M)**

(a) only a reflected ray and no refracted ray
(b) only a refracted ray and no reflected ray
(c) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

8. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is **(2004, 2M)**

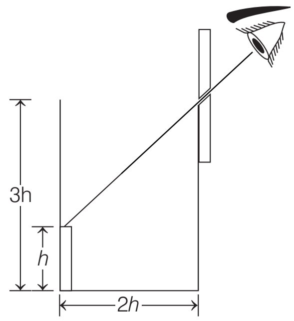
(a) 2 cm (b) 4 cm
(c) 6 cm (d) 12 cm

9. A ray of light is incident at the glass-water interface at an angle i, it emerges finally parallel to the surface of water, then the value of μ_g would be **(2003, 2M)**



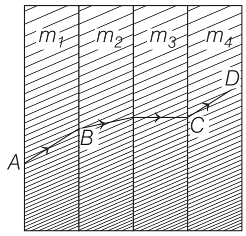
(a) (4/3) sin i (b) 1/(sin i) (c) 4/3 (d) 1

10. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is **(2002, 2M)**



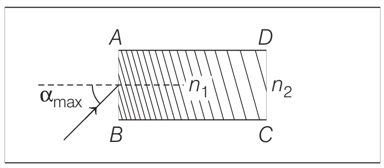
(a) $\frac{5}{2}$ (b) $\sqrt{\frac{5}{2}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

11. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have **(2001, 2M)**



(a) $\mu_1 = \mu_2$ (b) $\mu_2 = \mu_3$ (c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$

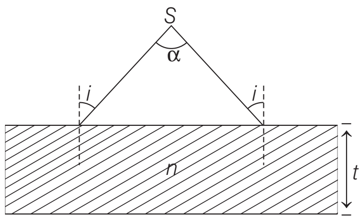
12. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index $n_2 (n_1 > n_2)$. A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max}, such that the ray comes out only from the other surface CD, is given by **(2000, 2M)**



(a) $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$ (b) $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\frac{1}{n_2}\right)\right]$

(c) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$ (d) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

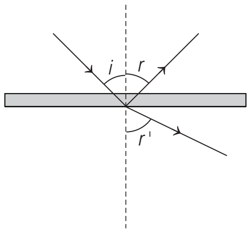
13. A diverging beam of light from a point source S having divergence angle α falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and its refractive index is n, then the divergence angle of the emergent beam is



(a) zero (b) α **(2000,2M)**
(c) $\sin^{-1}(1/n)$ (d) $2\sin^{-1}(1/n)$

14. A spherical surface of radius of curvature R, separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and $PO = OQ$. The distance PO is equal to **(1998, 2M)**
(a) $5R$ (b) $3R$ (c) $2R$ (d) $1.5R$

15. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i (see figure). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. The critical angle is **(1983, 1M)**

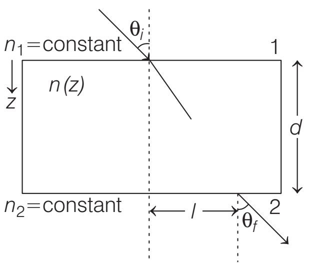


(a) $\sin^{-1}(\tan r)$ (b) $\sin^{-1}(\cot i)$
(c) $\sin^{-1}(\tan r')$ (d) $\tan^{-1}(\sin i)$

16. When a ray of light enters a glass slab from air **(1980, 1M)**
(a) its wavelength decreases
(b) its wavelength increases
(c) its frequency increases
(d) neither its wavelength nor its frequency changes

Objective Questions II (One or more correct option)

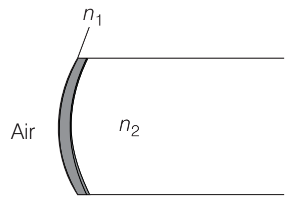
17. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here, z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2 (> n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l. **(2016 Main)**



Which of the following statement(s) is (are) true?

(a) l is independent on $n(z)$
(b) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$
(c) $n_1 \sin\theta_i = n_2 \sin\theta_f$
(d) l is independent of n_2

18. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then **(2014 Adv.)**

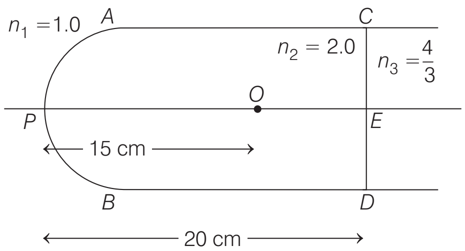


(a) $|f_1| = 3R$ (b) $|f_1| = 2.8R$
(c) $|f_2| = 2R$ (d) $|f_2| = 1.4R$

19. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value (s) of n from the following **(1998, 2M)**
(a) 1.3 (b) 1.4
(c) 1.5 (d) 1.6

Fill in the Blanks

20. A slab of material of refractive index 2 shown in figure has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is air and on the right of CD is water with refractive indices as given in the figure. An object O is placed at a distance of 15 cm from the pole P as shown. The distance of the final image of O from P, as viewed from the left is ...... **(1991, 2M)**



21. A monochromatic beam of light of wavelength 6000Å in vacuum enters a medium of refractive index 1.5. In the medium its wavelength is ......, and its frequency is ...... **(1985, 2M)**

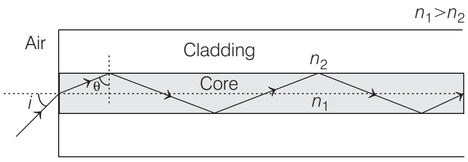
22. A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1.5. In the medium the velocity of the light wave is ...... and its wavelength is ...... **(1983, 2M)**

Passage Based Questions

Passage 1

Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.

23. For two structures namely S_1 with $n_1 = \frac{\sqrt{45}}{4}$ and $n_2 = \frac{3}{2}$, and S_2 with $n_1 = \frac{8}{5}$ and $n_2 = \frac{7}{5}$ and taking the refractive index of water to be $\frac{4}{3}$ and that to air to be 1, the correct options is/are **(2015 Adv.)**



(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(b) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(c) NA of S_1 placed in air is the same as that S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(d) NA of S_1 placed in air is the same as that of S_2 placed in water

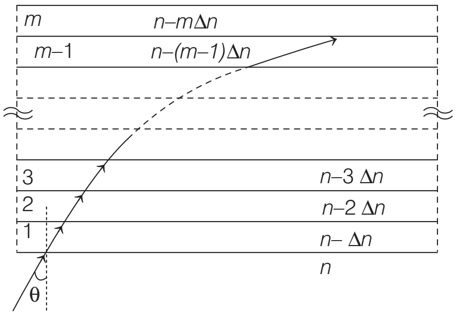
24. If two structures of same cross-sectional area, but different numerical apertures NA_1 and $NA_2 (NA_2 < NA_1)$ are joined longitudinally, the numerical aperture of the combined structure is **(2015 Adv.)**

(a) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$

(c) NA_1 (d) NA_2

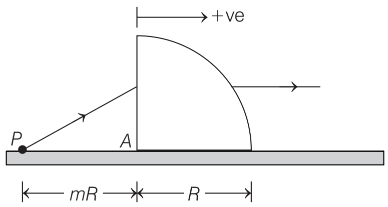
Integer Answer Type Question

25. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)$th and mth slabs from the right side of the stack. What is the value of m? **(2017 Adv.)**



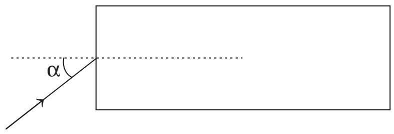
Analytical & Descriptive Questions

26. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in figure. **(1999, 5M)**



27. The x-y plane is the boundary between two transparent media. Medium-1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium-2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium-1 given by vector $\mathbf{A} = 6\sqrt{3}\hat{\mathbf{i}} + 8\sqrt{3}\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium-2. **(1999, 10 M)**

28. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. Determine the least value of n so that the light entering the rod does not emerge from the curved surface of the rod irrespective of the value of α. **(1992,8M)**



29. A parallel beam of light travelling in water (refractive index $= 4/3$) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial. **(1988, 6M)**

(a) Find the position of the image due to refraction at the first surface and the position of the final image.

(b) Draw a ray diagram showing the positions of both the images.

30. A right angled prism is to be made by selecting a proper material and the angles A and B $(B \leq A)$, as shown in figure. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two internal reflections. **(1987, 7M)**

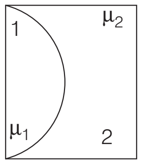
(a) What should be the minimum refractive index n for this to be possible?

(b) For $n = 5/3$ is it possible to achieve this with the angle B equal to 30 degrees?

Topic 3 Lens Theory

Objective Questions I (Only one correct option)

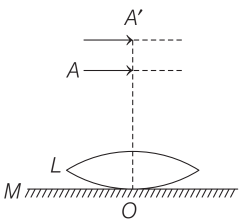
1. One plano-convex and one plano-concave lens of same radius of curvature R but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is **(2019 Main, 10 April I)**



(a) $\dfrac{2R}{\mu_1 - \mu_2}$ (b) $\dfrac{R}{2-(\mu_1 - \mu_2)}$

(c) $\dfrac{R}{2(\mu_1 - \mu_2)}$ (d) $\dfrac{R}{\mu_1 - \mu_2}$

2. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that $OA = 18$ cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A', such that $OA' = 27$ cm, to get its inverted real image at A' itself. The value of μ_l will be **(2019 Main, 9 April II)**



(a) $\sqrt{3}$ (b) $\sqrt{2}$

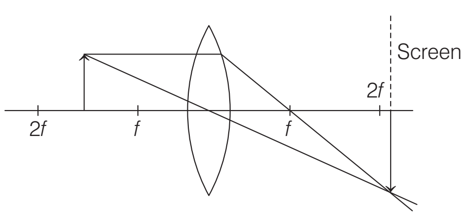
(c) $\dfrac{4}{3}$ (d) $\dfrac{3}{2}$

3. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 $(x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is **(2019 Main, 9 April II)**

(a) 5 : 3 (b) 2 : 1

(c) 4 : 3 (d) 3 : 1

4. Formation of real image using a biconvex lens is shown below. If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen? **(2019 Main, 12 Jan II)**



(a) No Change (b) Magnified image

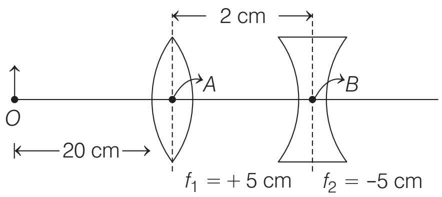
(c) Image disappears (d) Erect real image

5. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be **(2019 Main, 12 Jan II)**

(a) $f_1 - f_2$ (b) $\dfrac{R}{\mu_2 - \mu_1}$

(c) $f_1 + f_2$ (d) $\dfrac{2f_1 f_2}{f_1 + f_2}$

6. What is the position and nature of image formed by lens combination shown in figure? (where, f_1 and f_2 are focal lengths) **(2019 Main, 12 Jan I)**



(a) $\dfrac{20}{3}$ cm from point B at right, real

(b) 70 cm from point B at right, real

(c) 40 cm from point B at right, real

(d) 70 cm from point B at left, virtual

7. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be **(2019 Main, 11 Jan I)**

(a) 3.22×10^{-3} m/s towards the lens

(b) 0.92×10^{-3} m/s away from the lens

(c) 2.26×10^{-3} m/s away from the lens

(d) 1.16×10^{-3} m/s towards the lens

8. A plano-convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano-concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as **(2019 Main, 10 Jan I)**

(a) $3\mu_2 - 2\mu_1 = 1$ (b) $2\mu_2 - \mu_1 = 1$

(c) $2\mu_1 - \mu_2 = 1$ (d) $\mu_1 + \mu_2 = 3$

9. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now, a glass block (refractive index is 1.5) of 1.5 cm thickness is placed in contact with the light source.

To get the sharp image again, the screen is shifted by a distance d. Then, d is **(2019 Main, 9 Jan I)**

(a) 0

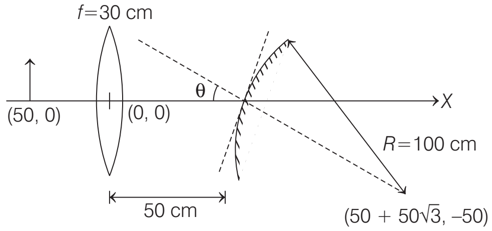
(b) 1.1 cm away from the lens

(c) 0.55 cm away from the lens

(d) 0.55 cm towards the lens

10. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is **(2017 Main)**
(a) virtual and at a distance of 40 cm from convergent lens
(b) real and at a distance of 40 cm from the divergent lens
(c) real and at a distance of 6 cm from the convergent lens
(d) real and at a distance of 40 cm from convergent lens

11. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure. **(2016 Adv.)**

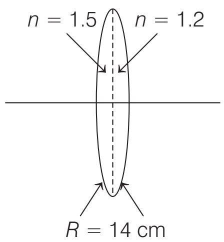


If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are
(a) $(125/3, 25/\sqrt{3})$ (b) $(50-25\sqrt{3}, 25)$
(c) (0, 0) (d) $(25, 25\sqrt{3})$

12. A thin convex lens made from crown glass $(\mu = 3/2)$ has focal length f. When it is measured in two different liquids having refractive indices 4/3 and 5/3. It has the focal lengths f_1 and f_2, respectively. The correct relation between the focal length is **(2014 Main)**
(a) $f_1 = f_2 < f$
(b) $f_1 > f$ and f_2 becomes negative
(c) $f_2 > f$ and f_1 becomes negative
(d) f_1 and f_2 both become negative

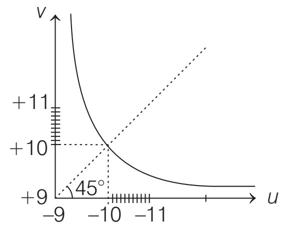
13. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is 2/3 times the wavelength in free space. The radius of the curved surface of the lens is **(2013 Adv.)**
(a) 1 m (b) 2 m (c) 3 m (d) 6 m

14. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm.



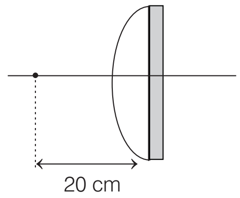
For this bi-convex lens, for an object distance of 40 cm, the image distance will be **(2012)**
(a) -280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

15. The graph between object distance u and image distance v for a lens is given below. The focal length of the lens is **(2006, 3M)**



(a) 5 ± 0.1 (b) 5 ± 0.05
(c) 0.5 ± 0.1 (d) 0.5 ± 0.05

16. A point object is placed at a distance of 20 cm from a thin plano-convex lens of focal length 15 cm. The plane surface of the lens is now silvered. The image created by the system is at **(2006, 3M)**

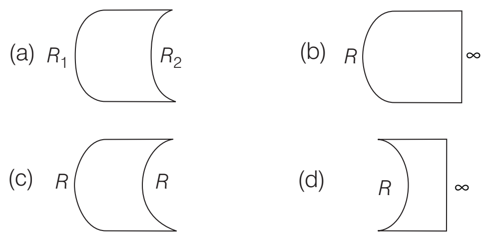


(a) 60 cm to the left of the system
(b) 60 cm to the right of the system
(c) 12 cm to the left of the system
(d) 12 cm to the right of the system

17. A convex lens is in contact with concave lens. The magnitude of the ratio of their focal length is $\frac{3}{2}$. Their equivalent focal length is 30 cm. What are their individual focal lengths ? **(2005, 2M)**
(a) –75, 50 (b) –10, 15 (c) 75, 50 (d) –15, 10

18. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image. **(2003, 2M)**
(a) 1.25 cm (b) 2.5 cm (c) 1.05 cm (d) 2 cm

19. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams **(2002, 2M)**



20. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refracting indices n_1 and n_2 respectively ($n_2 > n_1 > 1$). The lens will diverge a parallel beam of light if it is filled with **(2000, 2M)**
(a) air and placed in air
(b) air and immersed in L_1
(c) L_1 and immersed in L_2
(d) L_2 and immersed in L_1

21. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a **(1999, 2M)**
(a) convergent lens of focal length 3.5 R
(b) convergent lens of focal length 3.0 R
(c) divergent lens of focal length 3.5 R
(d) divergent lens of focal length 3.0 R

22. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration **(1998, 2M)**
(a) is absent
(b) is smaller if the curved surface of the lens faces the object
(c) is smaller if the plane surface of the lens faces the object
(d) is the same whichever side of the lens faces the object

23. An eye specialist prescribes spectacles having combination of convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination in diopters is **(1997, 2M)**
(a) + 1.5 (b) – 1.5 (c) + 6.67 (d) – 6.67

24. Spherical aberration in a thin lens can be reduced by **(1994, 2M)**
(a) using a monochromatic light
(b) using a doublet combination
(c) using a circular annular mark over the lens
(d) increasing the size of the lens

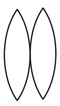
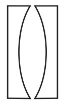
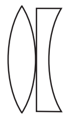
25. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of the combination is **(1982, 3M)**
(a) – 1.5 D (b) – 6.5 D (c) + 6.5 D (d) + 6.67 D

Objective Question II (One or more correct option)

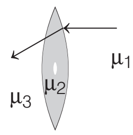
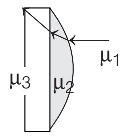
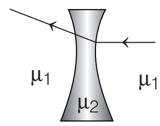
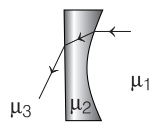
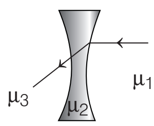
26. A plano-convex lens is made of material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is (are) true? **(2016 Adv.)**
(a) The refractive index of the lens is 2.5
(b) The radius of curvature of the convex surface is 45 cm
(c) The faint image is erect and real
(d) The focal length of the lens is 20 cm

Match the Columns

27. Four combinations of two thin lenses are given in Column I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in Column I with their focal length in Column II and select the correct answer using the codes given below the lists. **(2014 Adv.)**

	Column I		Column II
A.		p.	$2r$
B.		q.	$r/2$
C.		r.	$-r$
D.		s.	r

28. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **Column II**. A ray traversing these media is also shown in the figures. In **Column I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in **Column II**. **(2010)**

	Column I		Column II
(A)	$\mu_1 < \mu_2$	(p)	
(B)	$\mu_1 > \mu_2$	(q)	
(C)	$\mu_2 = \mu_3$	(r)	
(D)	$\mu_2 > \mu_3$	(s)	
		(t)	

Fill in the Blanks

29. Two thin lenses, when in contact, produce a combination of power + 10 D. When they are 0.25 m apart, the power reduces to + 6 D. The focal length of the lenses are …… m and ……m. **(1997, 2M)**

30. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index 4/3, its focal length will become …… cm. **(1987, 2M)**

31. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as parallel beam, then d is equal to …… cm. **(1985, 2M)**

True / False

32. A parallel beam of white light fall on a combination of a concave and a convex lens, both of the same material. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the same side of the lens system, one sees coloured patterns with violet colour nearer to the lens. **(1988, 2M)**

33. A convex lens of focal length 1 m and a concave lens of focal length 0.25 m are kept 0.75 m apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. **(1983, 2M)**

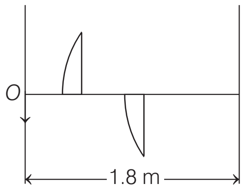
Integer Answer Type Question

34. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is **(2010)**

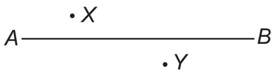
Analytical & Descriptive Questions

35. An object is approaching a thin convex lens of focal length 0.3 m with a speed of 0.01 m/s. Find the magnitudes of the rates of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens. **(2004, 4M)**

36. A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis. The separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and separation between the halves. Draw the ray diagram for image formation. **(1996, 5M)**

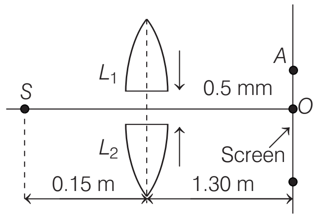


37. An image Y is formed of point object X by a lens whose optic axis is AB as shown in figure.



Draw a ray diagram to locate the lens and its focus. If the image Y of the object X is formed by a concave mirror (having the same optic axis as AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagrams. **(1994, 6M)**

38. In given figure, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO. **(1993, 5+1M)**



(a) If the third intensity maximum occurs at the point A on the screen, find the distance OA.

(b) If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease, or remain the same.

39. A plano-convex lens has a thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. Find the focal length of the lens. Assume thickness to be negligible while finding its focal length. **(1984, 6M)**

40. The radius of curvature of the convex face of a planoconvex lens is 12 cm and its $\mu = 1.5$. **(1979)**

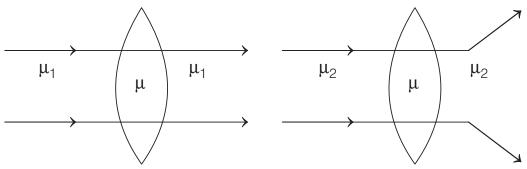
(a) Find the focal length of the lens. The plane face of the lens is now silvered.

(b) At what distance from the lens will parallel rays incident on the convex surface converge ?

(c) Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.

(d) Calculate the image distance when the object is placed as in (c)

41. What is the relation between the refractive indices μ_1 and μ_2? If the behaviour of light rays is as shown in the figure. **(1979)**



42. A pin is placed 10 cm in front of a convex lens of focal length 20 cm and made of a material of refractive index 1.5. The convex surface of the lens farther away from the pin is silvered and has a radius of curvature of 22 cm. Determine the position of the final image. Is the image real or virtual ? **(1978)**

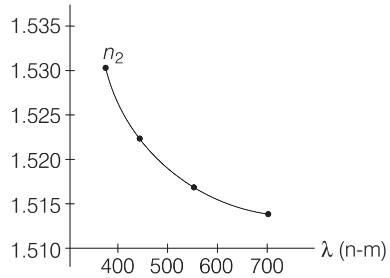
Topic 4 Prism

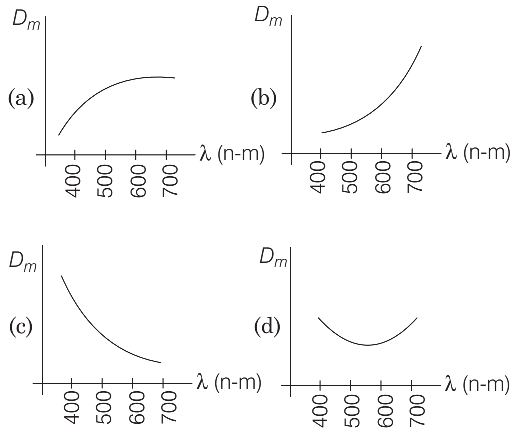
Objective Questions I (Only one correct option)

1. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is **(2019 Main, 11 Jan II)**

(a) 45º (b) 90º (c) 60º (d) 30º

2. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation ? **(2019 Main, 11 Jan I)**

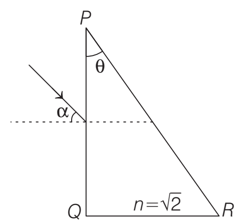




3. In an experiment for determination of refractive index of glass of a prism by i-δ plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index? **(2016 Main)**

(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8

4. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n=\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°.



The angle θ of the prism is **(2016 Adv.)**

(a) 15° (b) 22.5° (c) 30° (d) 45°

5. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided **(2015 Main)**

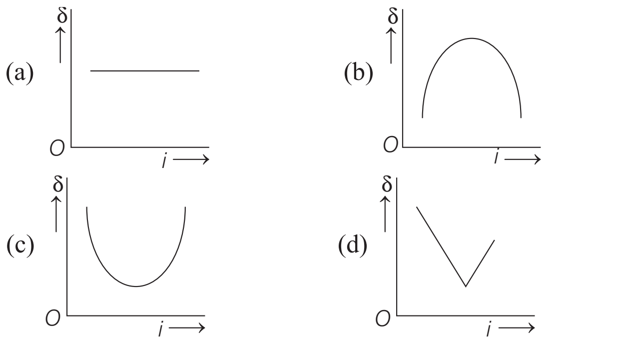
(a) $\theta < \cos^{-1}\left[\mu \sin\left\{A + \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

(b) $\theta < \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

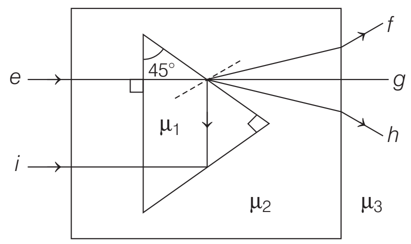
(c) $\theta > \cos^{-1}\left[\mu \sin\left\{A + \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

(d) $\theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

6. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **(2013 Main)**



7. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure, A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'. **(2013 Adv.)**



Match the paths in Column I with conditions of refractive indices in Column II and select the correct answer using the codes given below the lists.

Column I		Column II	
P.	$e \rightarrow f$	1.	$\mu_1 > \sqrt{2}\mu_2$
Q.	$e \rightarrow g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \rightarrow h$	3.	$\mu_1 = \mu_2$
S.	$e \rightarrow i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

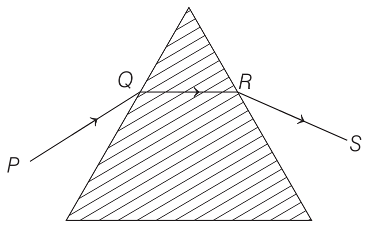
Codes

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

8. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be **(2008, 3M)**

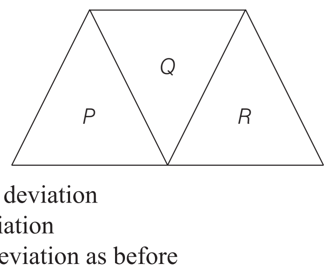
(a) 30° for both the colours
(b) greater for the violet colour
(c) greater for the red colour
(d) equal but not 30° for both the colours

9. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true ? **(2004, 2M)**



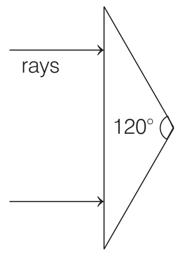
(a) PQ is horizontal
(b) QR is horizontal
(c) RS is horizontal
(d) Either PQ or RS is horizontal

10. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will suffer **(2001,2M)**



(a) greater deviation
(b) no deviation
(c) same deviation as before
(d) total internal reflection

11. An isosceles prism of angle 120° has a refractive index 1.44. Two parallel rays of monochromatic light enter the prism parallel to each other in air as shown. The rays emerging from the opposite face **(1995, 2M)**

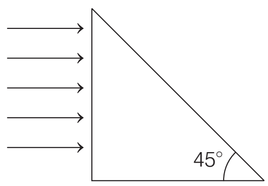


(a) are parallel to each other
(b) are diverging
(c) make an angle $2[\sin^{-1}(0.72) - 30°]$ with each other
(d) make an angle $2\sin^{-1}(0.72)$ with each other

12. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is **(1990, 2M)**

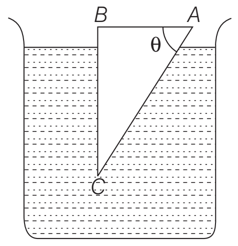
(a) 5.33° (b) 4°
(c) 3° (d) 2.6°

13. A beam of light consisting of red, green and blue colours is incident on a right-angled prism. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will **(1989, 2M)**



(a) separate the red colour from the green and blue colours
(b) separate the blue colour from the red and green colours
(c) separate all the three colours from one another
(d) not separate even partially any colour from the other two colours.

14. A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam incident normally on the face AB is totally reflected to reach the face BC if **(1981, 3M)**



(a) $\sin\theta > 8/9$ (b) $2/3 < \sin\theta < 8/9$
(c) $\sin\theta < 2/3$ (d) None of these

Objective Question II (One or more correct option)

15. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct? **(2017 Adv.)**

(a) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(b) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = \left(\frac{i_i}{2}\right)$

(c) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1} - \cos A\right]$

(d) For this prism, the refractive index μ and the angle prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

Fill in the Blanks

16. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is …… degrees. **(1997, 2M)**

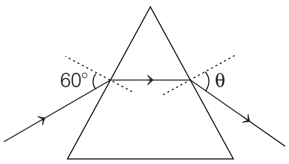
17. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is …… **(1992, 1M)**

True / False

18. A beam of white light passing through a hollow prism give no spectrum. **(1983, 2M)**

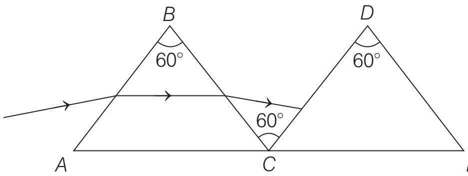
Integer Answer Type Question

19. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is **(2015 Adv.)**



Analytical & Descriptive Questions

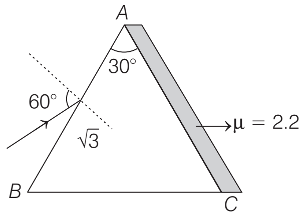
20. A ray of light is incident on a prism ABC of refractive index $\sqrt{3}$ as shown in figure. **(2005, 4M)**



(a) Find the angle of incidence for which the deviation of light ray by the prism ABC is minimum.

(b) By what angle the second identical prism must be rotated, so that the final ray suffers net minimum deviation.

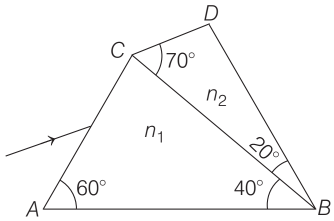
21. A prism of refracting angle 30° is coated with a thin film of transparent material of refractive index 2.2 on face AC of the prism. A light of wavelength 6600 Å is incident on face AB such that angle of incidence is 60°. Find **(2003, 4M)**



(a) the angle of emergence and

(b) the minimum value of thickness of the coated film on the face AC for which the light emerging from the face has maximum intensity. (Given refractive index of the material of the prism is $\sqrt{3}$)

22. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together with a gap as shown in the figure. The angles of the prism are as shown. n_1 and n_2 depend on λ, the wavelength of light according to :



$$n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2} \text{ and}$$

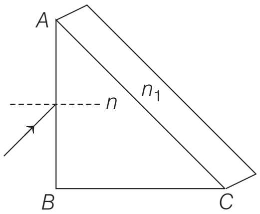
$$n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$$

where λ is in nm. **(1998, 8M)**

(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(b) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum.

23. A right angled prism (45°-90°-45°) of refractive index n has a plane of refractive index n_1 $(n_1 < n)$ cemented to its diagonal face. The assembly is in air. The ray is incident on AB. **(1996, 3M)**



(a) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

(b) Assuming $n = 1.352$, calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated.

24. A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the lens. **(1978)**

Topic 5 Optical Instruments

Objective Questions I (Only one correct option)

1. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000Å is used, the minimum separation between two points, to be seen as distinct, will be **(2019 Main, 12 April I)**

(a) $0.24\,\mu m$ (b) $0.38\,\mu m$ (c) $0.12\,\mu m$ (d) $0.48\,\mu m$

2. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm coming from a distant object, the limit of resolution of the telescope is close to **(2019 Main, 9 April II)**

(a) 3.0×10^{-7} rad (b) 2.0×10^{-7} rad

(c) 1.5×10^{-7} rad (d) 4.5×10^{-7} rad

3. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star. **(2019 Main, 8 April II)**

(a) 610×10^{-9} rad (b) 305×10^{-9} rad

(c) 457.5×10^{-9} rad (d) 152.5×10^{-9} rad

4. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To observer the tree appears **(2016 Main)**

(a) 10 times taller (b) 10 times nearer

(c) 20 times taller (d) 20 times nearer

5. The box of a pin hole camera of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when **(2016 Main)**

(a) $a=\dfrac{\lambda^2}{L}$ and $b_{min}=\left(\dfrac{2\lambda^2}{L}\right)$

(b) $a=\sqrt{\lambda L}$ and $b_{min}=\left(\dfrac{2\lambda^2}{L}\right)$

(c) $a=\sqrt{\lambda L}$ and $b_{min}=\sqrt{4\lambda L}$

(d) $a=\dfrac{\lambda^2}{L}$ and $b_{min}=\sqrt{4\lambda L}$

6. In a compound microscope, the intermediate image is **(2000, 2M)**

(a) virtual, erect and magnified

(b) real, erect and magnified

(c) real, inverted and magnified

(d) virtual, erect and reduced

7. The focal lengths of the objective and the eyepiece of a compound microscope are 2.0 cm and 3.0 cm respectively. The distance between the objective and the eyepiece is 15.0 cm. The final image formed by the eyepiece is at infinity. The two lenses are thin. The distance in cm of the object and the image produced by the objective, measured from the objective lens, are respectively **(1995,2M)**

(a) 2.4 and 12.0 (b) 2.4 and 15.0

(c) 2.0 and 12.0 (d) 2.0 and 3.0

8. An astronomical telescope has an angular magnification of magnitude 5 for far objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of the eyepiece are **(1989, 2M)**

(a) $f_o = 45$ cm and $f_e = -9$ cm

(b) $f_o = 50$ cm and $f_e = 10$ cm

(c) $f_o = 7.2$ cm and $f_e = 5$ cm

(d) $f_o = 30$ cm and $f_e = 6$ cm

Objective Question II (One or more correct option)

9. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm **(1992, 2M)**

(a) the distance between the objective and the eyepiece is 16.02 m

(b) the angular magnification of the planet is –800

(c) the image of the planet is inverted

(d) the objective is larger than the eyepiece

Fill in the Blank

10. The resolving power of electron microscope is higher than that of an optical microscope because the wavelength of electrons is …… than the wavelength of visible light. **(1992, 1M)**

Topic 6 Wave Optics

Objective Questions I (Only one correct option)

1. A system of three polarisers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarised light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarisers is I. The ratio (I_0 / I) equals (nearly) **(2019 Main, 12 April II)**

(a) 5.33 (b) 16.00 (c) 10.67 (d) 1.80

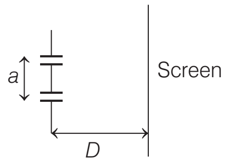
2. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) **(2019 Main, 12 April I)**

(a) $\dfrac{2\lambda}{(\mu-1)}$ (b) $\dfrac{\lambda}{2(\mu-1)}$ (c) $\dfrac{\lambda}{(\mu-1)}$ (d) $\dfrac{\lambda}{(2\mu-1)}$

3. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be **(2019 Main, 10 April II)**

(a) 4 : 1 (b) 25 : 9
(c) 9 : 1 (d) $(\sqrt{3}+1)^4 : 16$

4. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be **(2019 Main, 9 April I)**



(a) $\dfrac{2nD\lambda}{a(\mu-1)}$ (b) $\dfrac{2D\lambda}{a(\mu-1)}$ (c) $\dfrac{D\lambda}{a(\mu-1)}$ (d) $\dfrac{nD\lambda}{a(\mu-1)}$

5. In an interference experiment, the ratio of amplitudes of coherent waves is $\dfrac{a_1}{a_2}=\dfrac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be **(2019 Main, 8 April I)**

(a) 2 (b) 18 (c) 4 (d) 9

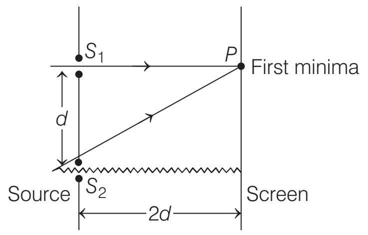
6. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μ-m and a width of 4.05 μ-m. The number of bright fringes between the first and the second diffraction minima is **(2019 Main, 11 Jan II)**

(a) 5 (b) 10 (c) 9 (d) 4

7. In a Young's double slit experiment, the path difference at a certain point on the screen between two interfering waves is $\dfrac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to **(2019 Main, 11 Jan I)**

(a) 0.80 (b) 0.74 (c) 0.94 (d) 0.85

8. Consider a Young's double slit experiment as shown in figure



What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)? **(2019 Main, 10 Jan II)**

(a) $\dfrac{\lambda}{2(5-\sqrt{2})}$ (b) $\dfrac{\lambda}{(5-\sqrt{2})}$
(c) $\dfrac{\lambda}{2(\sqrt{5}-2)}$ (d) $\dfrac{\lambda}{(\sqrt{5}-2)}$

9 In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\dfrac{1}{40}$ rad by using light of wavelength λ_1. When the light of the wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 n-m to 740 n-m), their values are **(2019 Main, 10 Jan I)**

(a) 380 n-m, 525 n-m (b) 400 n-m, 500 n-m
(c) 380 n-m, 500 n-m (d) 625 n-m, 500 n-m

10 In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ n-m is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is **(2019 Main, 9 Jan II)**

(a) 320 (b) 321
(c) 640 (d) 641

11 Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio **(2019 Main, 9 Jan I)**

(a) 16 : 9 (b) 5 : 3 (c) 25 : 9 (d) 4 : 1

12. Unpolarised light of intensity I passes through an ideal polariser A. Another identical polariser B is placed behind A. The intensity of light beyond B is found to be $\dfrac{I}{2}$. Now, another identical polariser C is placed between A and B. The intensity beyond B is now found to be $\dfrac{1}{8}$. The angle between polariser A and C is **(2018 Main)**

(a) 60° (b) 0° (c) 30° (d) 45°

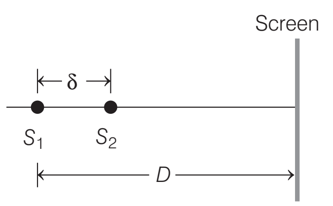
13. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(i.e. distance between the centres of each slit.) **(2018 Main)**

(a) 100 μm (b) 25 μm (c) 50 μm (d) 75 μm

14. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide, is **(2017 Main)**

(a) 7.8 mm (b) 9.75 mm
(c) 15.6 mm (d) 1.56 mm

15. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be **(2013 Main)**



(a) points (b) straight lines
(c) semi-circle (d) concentric circles

16. In the Young's double slit experiment using a monochromatic light of wavelength λ the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is **(2013 Adv.)**

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$

(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

17. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is

(a) I_0 (b) $I_0/2$ **(2013 Main)**

(c) $I_0/4$ (d) $I_0/8$

18. Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively.

Then, **(2012)**

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$

(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

19. A biconvex lens of focal length f forms a circular image of radius r of sun in focal plane. Then which option is correct ?

(a) $\pi r^2 \propto f$ **(2006, 3M)**

(b) $\pi r^2 \propto f^2$

(c) If lower half part is convered by black sheet, then area of the image is equal to $\pi r^2/2$

(d) If f is doubled, intensity will increase

20. In Young's double slit experiment intensity at a point is (1/4) of the maximum intensity. Angular position of this point is

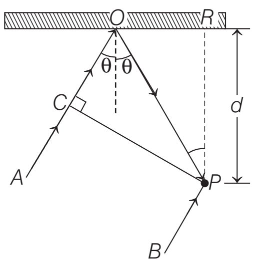
(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$ **(2005, 2M)**

(c) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

21. In a YDSE bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is **(2004, 2M)**

(a) 4 mm (b) 5.6 mm (c) 14 mm (d) 28 mm

22. In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition of θ for constructive interference at P between the ray BP and reflected ray OP **(2003, 2M)**



(a) $\cos\theta = \frac{3\lambda}{2d}$ (b) $\cos\theta = \frac{\lambda}{4d}$

(c) $\sec\theta - \cos\theta = \frac{\lambda}{d}$ (d) $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

23. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is **(2002, 2M)**

(a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

24. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by **(2001, 2M)**

(a) 12 (b) 18 (c) 24 (d) 30

25. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between resultant intensities at A and B is **(2001, 2M)**

(a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$

26. In a double slit experiment instead of taking slits of equal widths, one slit is made twice as wide as the other, then in the interference pattern **(2000, 2M)**

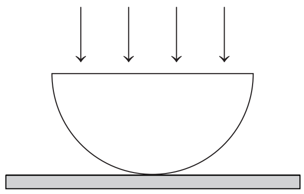
(a) the intensities of both the maxima and the minima increases

(b) the intensity of the maxima increases and the minima has zero intensity

(c) the intensity of maxima decreases and that of minima increases

(d) the intensity of maxima decreases and the minima has zero intensity

27. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate as shown. The observed interference fringes from this combination shall be



(a) straight **(1999, 2M)**

(b) circular

(c) equally spaced

(d) having fringe spacing which increases as we go outwards

28. Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal **(1999, 2M)**

(a) that the central maximum is narrower

(b) more number of fringes

(c) less number of fringes

(d) no diffraction pattern

29. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is **(1998, 2M)**

(a) zero (b) $\pi / 2$ (c) π (d) 2π

30. A narrow slit of width 1 mm is illuminated by monochromatic light of wavelength 600 nm. The distance between the first minima on either side of a screen at a distance of 2 m is **(1994, 1M)**

(a) 1.2 cm (b) 1.2 mm
(c) 2.4 cm (d) 2.4 mm

31. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities i n the resulting beam are **(1988, 1M)**

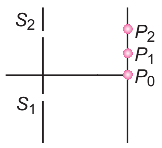
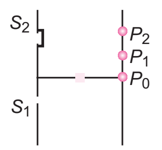
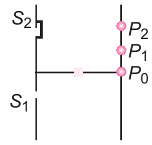
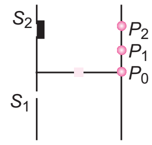
(a) $5I$ and I
(b) $5I$ and $3I$
(c) $9I$ and I
(d) $9I$ and $3I$

32. In Young's double slit experiment, the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is **(1981, 2M)**

(a) unchanged (b) halved
(c) doubled (d) quadrupled

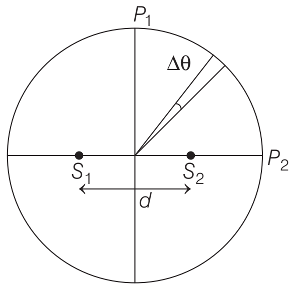
Match the Column

33. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \dfrac{\lambda}{4}$ and $S_1P_2 - S_2P_2 = \lambda / 2$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thickness of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in Column I with the statement(s) in Column II valid for that situation. **(2009)**

Column I		Column II
(A)		(p) $\delta(P_0) = 0$
(B) $(\mu - 1)t = \dfrac{\lambda}{4}$		(q) $\delta(P_1) = 0$
(C) $(\mu - 1)t = \dfrac{\lambda}{2}$		(r) $I(P_1) = 0$
(D) $(\mu - 1)t = \dfrac{3\lambda}{4}$		(s) $I(P_0) > I(P_1)$
		(t) $I(P_2) > I(P_1)$

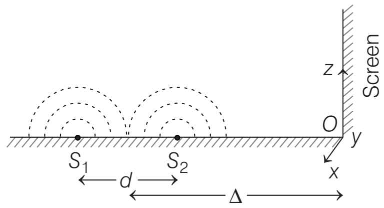
Objective Questions II (One or more correct option)

34. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct? **(2017 Adv.)**



(a) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant
(b) A dark spot will be formed at the point P_2
(c) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000
(d) At P_2 the order of the fringe will be maximum

35. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 mm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance $D = 3$m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the source $d = 0.6003$ mm . The origin O is at the intersection of the screen and the line joining S_1S_2. **(2016 Adv.)**



Which of the following is (are) true of the intensity pattern on the screen?

(a) Semi circular bright and dark bands centered at point O
(b) The region very close to the point O will be dark
(c) Straight bright and dark bands parallel to the X-axis
(d) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

36. A light source, which emits two wavelengths $\lambda_1 = 400\,\text{nm}$ and $\lambda_2 = 600\,\text{nm}$, is used in a Young's double-slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then **(2014 Adv.)**

(a) $\beta_2 > \beta_1$
(b) $m_1 > m_2$
(c) from the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(d) the angular separation of fringes of λ_1 is greater than λ_2

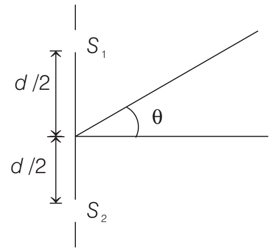
37. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0° **(2013 Adv.)**

(a) the absolute error in d remains constant
(b) the absolute error in d increases
(c) the fractional error in d remains constant
(d) the fractional error in d decreases

38. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice (s). **(2008, 4M)**

(a) If $d = \lambda$, the screen will contain only one maximum
(b) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
(c) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
(d) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

39. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance $d = 150.0\,\text{m}$. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by **(1995, 2M)**



(a) $I(\theta) = I_0 / 2$ for $\theta = 30°$
(b) $I(\theta) = I_0 / 4$ for $\theta = 90°$
(c) $I(\theta) = I_0$ for $\theta = 0°$
(d) $I(\theta)$ is constant for all values of θ

40. White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is b and the screen is at a distance d ($>> b$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are **(1984, 2M)**

(a) $\lambda = b^2/d$ (b) $\lambda = 2b^2/d$
(c) $\lambda = b^2/3d$ (d) $\lambda = 2b^2/3d$

41. In the Young's double slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that **(1982, 2M)**

(a) the intensities at the screen due to the two slits are 5 units and 4 units respectively
(b) the intensities at the screen due to the two slits are 4 units and 1 unit respectively
(c) the amplitude ratio is 3
(d) the amplitude ratio is 2

Fill in the Blanks

42. A slit of width d is placed in front of a lens of focal length 0.5 m and is illuminated normally with light of wavelength 5.89×10^{-7} m. The first diffraction minima on either side of the central diffraction maximum are separated by 2×10^{-3} m. The width d of the slit is …… m. **(1997, 1M)**

43. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same set-up the two slits are sources of equal amplitude A and wavelength λ, but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is …… **(1986, 2M)**

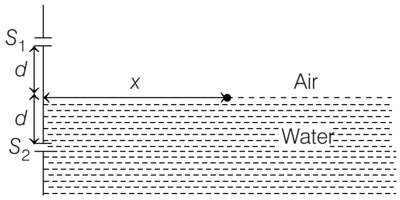
True / False

44. In a Young's double slit experiment performed with a source of white light, only black and white fringes are observed. **(1987, 2M)**

45. Two slits in a Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed on the screen. **(1984, 2M)**

Integer Answer Type Question

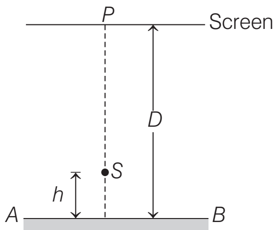
46. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index $= 4/3$) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index $= 1$), $2d$ is the separation between the slits and m is an integer. The value of p is **(2015 Adv.)**



Analytical & Descriptive Questions

47. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? Take $D/d = 10^3$. Symbols have their usual meanings. **(2004, 4M)**

48. A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it. **(2002,5M)**

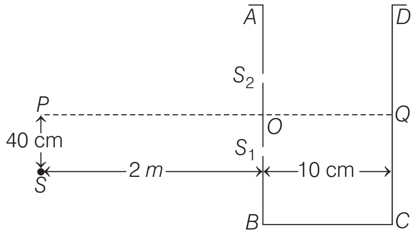


(a) What is the shape of the interference fringes on the screen?

(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

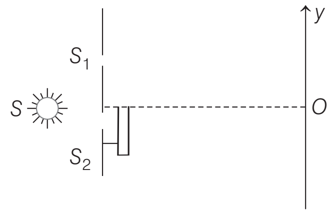
(c) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

49. A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure alongside. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now, a liquid is poured into the vessel and filled upto OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid. **(2001, 5M)**



50. A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of t for which the rays interfere constructively. **(2000, 4M)**

51. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in the figure.



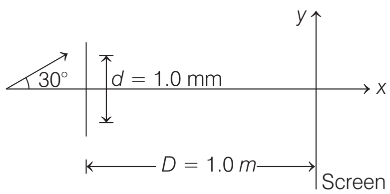
(a) Find the location of central maximum (bright fringe with zero path difference) on the y-axis.

(b) Find the light intensity of point O relative to the maximum fringe intensity.

(c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O.

(All wavelengths in the problem are for the given medium of refractive index 4/3. Ignore dispersion) **(1999, 10M)**

52. A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in the figure. **(1998, 8M)**



(a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.

(b) If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in figure), find the y-coordinates of the first minima on either side of the central maximum.

53. In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4, while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen, where the central maximum ($n = 0$) fall before the glass plates were inserted, now has 3/4 the original intensity. It is further observed that what used to be the fifth maximum earlier lies below the point P while the sixth minima lies above P. Calculate the thickness of glass plate. (Absorption of light by glass plate may be neglected). **(1997, 5M)**

54. In Young's experiment, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for green light. Also estimate the change in fringe width due to the change in wavelength. **(1997C, 5M)**

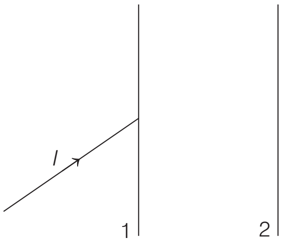
55. A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å . **(1996, 3M)**

(a) Calculate the fringe width.

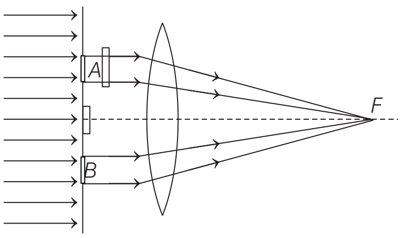
(b) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum as the axis.

56. Angular width of central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find refractive index of the liquid. **(1996, 2M)**

57. A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one-and parallel to it. Each glass plate reflects 25 per cent of the light incident on it and transmits the remaining. Find the ratio of the minimum and maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate. **(1990, 7M)**



58. In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $(10/\pi)$ Wm^{-2} is incident normally on two apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000Å is placed in front of aperture A (see figure). Calculate the power (in W) received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot. **(1989, 8M)**



59. A beam of light consisting of two wavelengths, 6500Å and 5200 Å is used to obtain interference fringe in a Young's double slit experiment. **(1985, 6M)**

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 6500 Å.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

60. In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment. **(1983, 6M)**

Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be **(2019 Main, 8 April II)**

(a) 25 cm (b) 20 cm (c) 10 cm (d) 30 cm

2. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be **(2019 Main, 8 April I)**

(a) 20 cm from the convergent mirror, same size as the object
(b) 40 cm from the convergent mirror, same size as the object
(c) 40 cm from the convergent lens, twice the size of the object
(d) 20 cm from the convergent mirror,twice size of the object

3. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propogating in the glass medium will be **(2019 Main, 12 Jan I)**

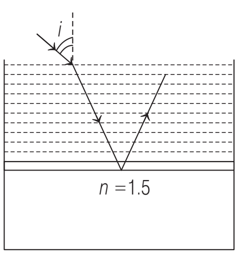
(a) 30 V/m (b) 6 V/m (c) 10 V/m (d) 24 V/m

4. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. **(2019 Main, 10 Jan II)**

(a) 4.0 cm (b) 2 cm (c) 3.1 cm (d) 1 cm

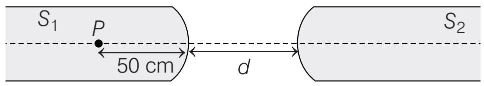
5. Consider a tank made of glass (refractive index is 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarised.

For this to happen, the minimum value of μ is **(2019 Main, 9 Jan I)**



(a) $\dfrac{3}{\sqrt{5}}$ (b) $\dfrac{5}{\sqrt{3}}$ (c) $\dfrac{4}{3}$ (d) $\sqrt{\dfrac{5}{3}}$

6. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is **(2015 Adv.)**



(a) 60 cm (b) 70 cm
(c) 80 cm (d) 90 cm

7. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is **(2015 Main)**

(a) 30 μm (b) 1 μm
(c) 100 μm (d) 300 μm

8. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens principle leads us to conclude that as it travels, the light beam **(2015 Main)**

(a) becomes narrower
(b) goes horizontally without any deflection
(c) bends upwards
(d) bends downwards

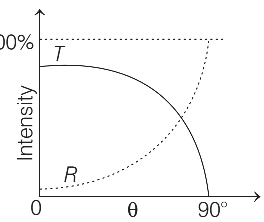
9. Two beams, A and B, of plane polarised light with mutually perpendicular planes of polarisation are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A / I_B equals **(2014 Main)**

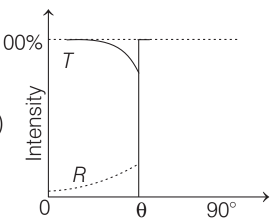
(a) 3 (b) 3/2 (c) 1 (d) 1/3

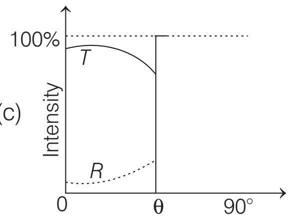
10. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **(2013 Main)**

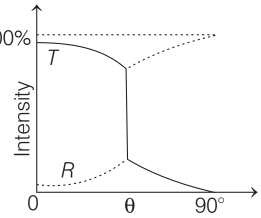
(a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

11. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is **(2011)**

(a)


(b)


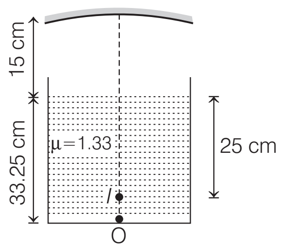
(c) 

(d) 

12. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is **(2010)**
(a) virtual and at a distance of 16 cm from the mirror
(b) real and at a distance of 16 cm from the mirror
(c) virtual and at a distance of 20 cm from the mirror
(d) real and at a distance of 20 cm from the mirror

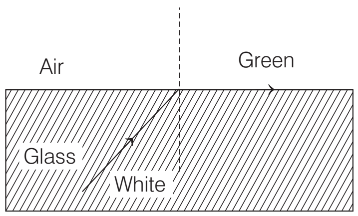
13. A light beam is travelling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are $n_0, \frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is **(2008, 3M)**
(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$ (c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

14. A container is filled with water ($\mu = 1.33$) up to a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. The focal length of the mirror is **(2005, 2M)**



(a) 10 cm (b) 15 cm
(c) 20 cm (d) 25 cm

15. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains. **(2004, 2M)**

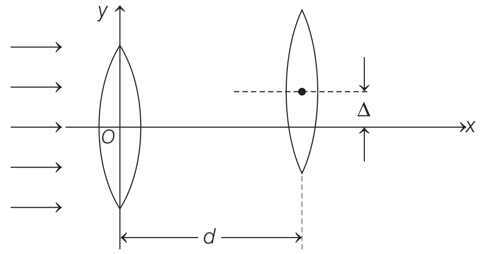


(a) yellow, orange, red (b) violet, indigo, blue
(c) all colours (d) all colours except green

16. A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be
(a) real and will remain at C **(1998, 2M)**
(b) real and located at a point between C and ∞
(c) virtual and located at a point between C and O
(d) real and located at a point between C and O

17. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by placing
(a) a plane mirror **(1995, 2M)**
(b) a convex mirror of suitable focal length
(c) a convex lens of focal length less than 0.25 m
(d) a concave lens of suitable focal length

18. Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d < f_1, d < f_2$) and their centres are displaced by a vertical separation Δ as shown in the figure. **(1993)**



Taking the origin of coordinates, O, at the centre of the first lens, the x and y-coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by **(1993; 2M)**

(a) $x = \frac{f_1 f_2}{f_1 + f_2}, \; y = \Delta$

(b) $x = \frac{f_1(f_2 + d)}{f_1 + f_2 - d}, \; y = \frac{\Delta}{f_1 + f_2}$

(c) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, \; y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(d) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, \; y = 0$

Numerical Value

19. Sunlight of intensity $1.3\,\text{kWm}^{-2}$ is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is .... **(2018 Adv.)**

Passage Based Questions

Passage 1

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **(2012)**

20. Choose the correct statement.

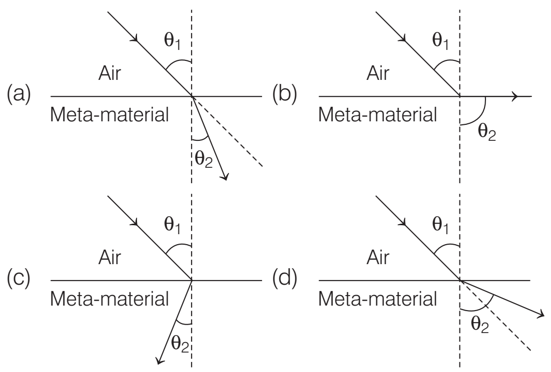
(a) The speed of light in the meta-material is $v = c\,|n|$

(b) The speed of light in the meta-material is $v = \dfrac{c}{|n|}$

(c) The speed of light in the meta-material is $v = c$

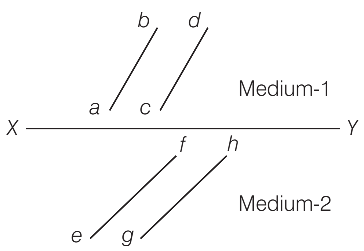
(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{\text{air}}\,|n|$, where λ_{air} is the wavelength of the light in air.

21. For light incident from air on a meta-material, the appropriate ray diagram is



Passage 2

The figure shows a surface XY separating two transparent media, medium-1 and medium-2. The lines ab and cd represent wavefronts of a light wave travelling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



22. Light travels as a **(2007, 4M)**

(a) parallel beam in each medium

(b) convergent beam in each medium

(c) divergent beam in each medium

(d) divergent beam in one medium and convergent beam in the other medium

23. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$ **(2007, 4M)**

(a) ϕ_c cannot be equal to ϕ_d

(b) ϕ_d can be equal to ϕ_e

(c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$

(d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

24. Speed of light is **(2007, 4M)**

(a) the same in medium-1 and medium-2

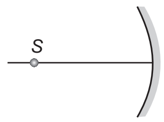
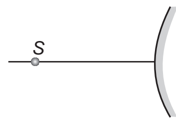
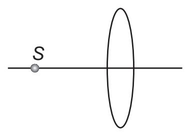
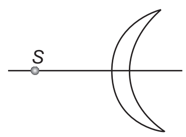
(b) larger in medium-1 than in medium-2

(c) larger in medium-2 than in medium-1

(d) different at b and d

Match the Columns

25. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. **(2008, 7M)**

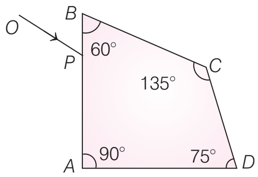
	Column I		Column II
(A)		(p)	Real image
(B)		(q)	Virtual image
(C)		(r)	Magnified image
(D)		(s)	Image at infinity

26. Some laws/processes are given in Column I. Match these with the physical phenomena given in Column II. **(2006, 4M)**

	Column I		Column II
(A)	Intensity of light received by lens	(p)	radius of aperture (R)
(B)	Angular magnification	(q)	dispersion of lens
(C)	Length of telescope	(r)	focal length f_o, f_e
(D)	Sharpness of image	(s)	spherical aberration

Objective Questions II (One or more correct option)

27. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? **(2010)**



(a) The ray gets totally internally reflected at face CD
(b) The ray comes out through face AD
(c) The angle between the incident ray and the emergent ray is 90°
(d) The angle between the incident ray and the emergent ray is 120°

28. Which of the following form(s) a virtual and erect image for all positions of the object ? **(1996, 2M)**
(a) Convex lens (b) Concave lens
(c) Convex mirror (d) Concave mirror

29. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen **(1986, 2M)**
(a) half of the image will disappear
(b) complete image will be formed
(c) intensity of the image will increase
(d) intensity of the image will decrease

Fill in the Blanks

30. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameters is ……… **(1992, 1M)**

31. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance 9 m and 25 m respectively from the source. The ratio of amplitudes of the waves at P and Q is …… **(1989, 2M)**

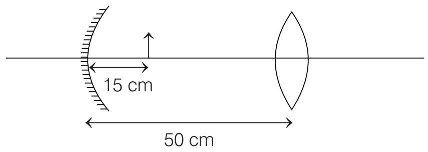
True / False

32. The intensity of light at a distance r from the axis of a long cylindrical source is inversely proportional to r. **(1981, 2M)**

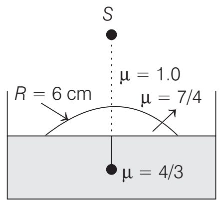
Integer Answer Type Questions

33. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. **(2015 Adv.)**

When the set-up is kept in a medium of refractive index $\frac{7}{6}$, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



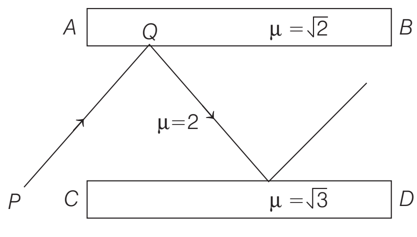
34. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature $R = 6$ cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then x is. **(2011)**



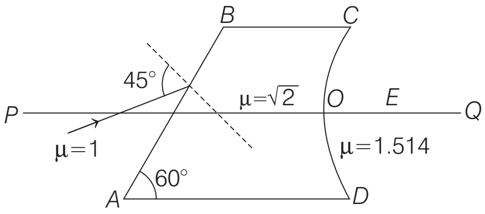
35. A large glass slab $\left(\mu = \frac{5}{3}\right)$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **(2010)**

Analytical & Descriptive Questions

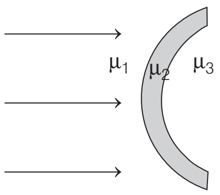
36. AB and CD are two slabs. The medium between the slabs has refractive index 2. Find the minimum angle of incidence of Q, so that the ray is totally reflected by both the slabs. **(2005, 2M)**



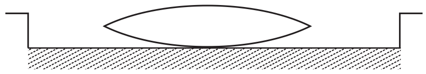
37. Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown in the figure. After refraction it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE upto two places of decimal. **(2004, 2M)**



38. In the figure, light is incident on a thin lens as shown. The radius of curvature for both the surfaces is R. Determine the focal length of this system. **(2003, 2M)**



39. A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid. **(2001, 5M)**

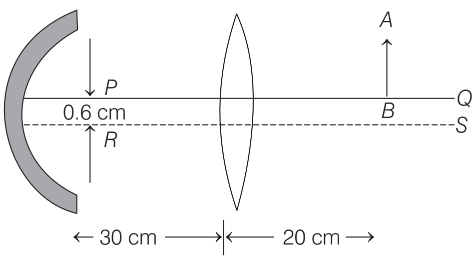


40. The refractive indices of the crown glass for blue and red light are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light.

(a) Determine the angle of the flint glass prism.

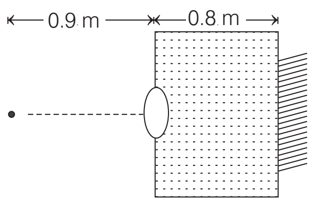
(b) Calculate the net dispersion of the combined system. **(2001, 5M)**

41. A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axis PQ and RS parallel but separated in vertical direction by 0.6 cm as shown.



The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optic axis PQ of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and the reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also locate positions of A' and B' with respect to the optic axis RS. **(2000, 6M)**

42. A thin equiconvex lens of glass of refractive index $\mu = 3/2$ and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $\mu = 4/3$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in figure. The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank in front of lens. Find the position (relative to the lens) of the image of the object formed by the system **(1997C, 5M)**



43. A ray of light travelling in air is incident at grazing angle (Incident angle = 90°) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m. The point of incidence is the origin $A(0,0)$. The medium has a variable index of refraction $n(y)$ given by

$$n(y) = [ky^{3/2} + 1]^{1/2} \text{ where } k = 1.0\,(\text{m})^{-3/2}.$$

The refractive index of air is 1.0. **(1995, 10 M)**

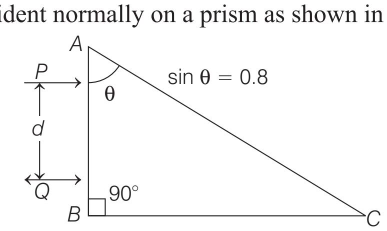
(a) Obtain a relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incident angle at that point.

(b) Obtain an equation for the trajectory $y(x)$ of the ray in the medium.

(c) Determine the coordinates (x_1, y_1) of the point P, where the ray intersects the upper surface of the slab-air boundary.

(d) Indicate the path of the ray subsequently.

44. Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in figure.



The refractive index of the prism as a function of wavelength is given by the relation, $\mu(\lambda) = 1.20 + \dfrac{b}{\lambda^2}$ where λ is in Å and b is positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wavelength and is not satisfied for the other. **(1991, 2+2+4M)**

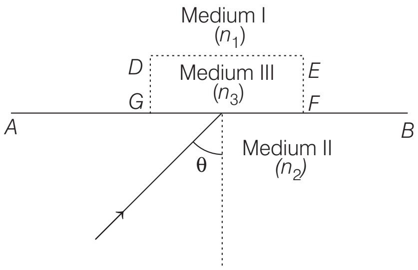
(a) Find the value of b.

(b) Find the deviation of the beams transmitted through the face AC.

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and the lower beams immediately after transmission from the face AC, are $4I$ and I respectively, find the resultant intensity at the focus.

45. Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 ($n_2 > n_1$) at an angle of incidence θ as shown in the figure.

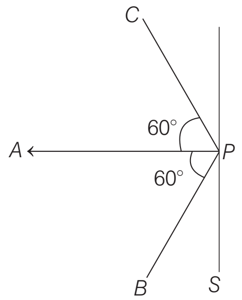
The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab $DEFG$ of uniform thickness and of refractive index n_3 is introduced on the interface (as shown in the figure), show that for any value of n_3 all light will ultimately be reflected back again into medium II.



Consider separately the cases : **(1986, 6M)**

(a) $n_3 < n_1$ and (b) $n_3 > n_1$

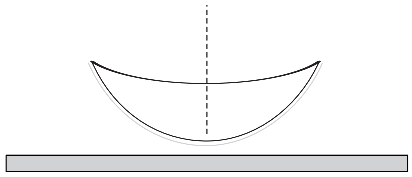
46. Screen S is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards point P on the screen (see figure). Line AP is normal to the screen and the lines AP, BP and CP are in one plane. The distances AP, BP and CP are in one plane. The radiant powers of sources A and B are 90 W and 180 W respectively. The beam from C is of intensity 20 W/m^2. Calculate intensity at P on the screen. **(1982, 5M)**



47. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. **(1981, 2M)**

(a) Where should a pin be placed on the optic axis such that its image is formed at the same place?

(b) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved, so that the image of the pin again coincides with the pin.



Answers

Topic 1

1. (c) **2.** (c) **3.** (c) **4.** (d)
5. (a) **6.** (b) **7.** (b) **8.** (d)
9. (d) **10.** (c) **11.** (d) **12.** (c, d)
13. (–1.5) **14.** 3

Topic 2

1. (c) **2.** (c) **3.** (d)
4. (d) **5.** (c) **6.** (c) **7.** (c)
8. (c) **9.** (b) **10.** (b) **11.** (d)
12. (a) **13.** (b) **14.** (a)
15. (a) **16.** (a) **17.** (a, c, d) **18.** (a, c)
19. (c, d) **20.** 30 cm to the right of P. Image will be virtual
21. 4000Å, 5×10^{14} Hz
22. 2×10^8 m/s, 4×10^{-7}m
23. (a,c) **24.** (d) **25.** 8
26. $\frac{4}{3}$ **27.** $\frac{1}{5\sqrt{2}}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$
28. $\sqrt{2}$ **29.** (a) – 6 mm, –5mm
30. (a) $\sqrt{2}$ (b) No

Topic 3

1. (d) **2.** (c) **3.** (d) **4.** (d) **5.** (c)
6. (b) **7.** (d) **8.** (c) **9.** (c)
10. (b) **11.** (d) **12.** (b) **13.** (c)
14. (b) **15.** (b) **16.** (c) **17.** (d)
18. (b) **19.** (c) **20.** (d) **21.** (a)
22. (b) **23.** (b) **24.** (c) **25.** (a)
26. (a, d) **27.** (A) $\rightarrow$ p; (B) $\rightarrow$ s; (C) $\rightarrow$ r; (D) $\rightarrow$ p
28. (A) $\rightarrow$ p, r; (B) $\rightarrow$ q, s, t; (C) $\rightarrow$ p, r, t; (D) $\rightarrow$ q, s
29. 0.125, 0.5 **30.** 60 **31.** 15
32. T **33.** F **34.** 6
35. 0.09 m/s, 0.3/s **36.** 0.4 m, 0.6 m **37.** See the hints

38. (a) 1 mm (b) Increase **39.** 75 cm

40. (a) + 24 cm (b) at 12 cm (d) $v = -30$ cm

41. $\mu_1 < \mu_2$ **42.** at a distance of 11 cm, virtual

Topic 4

1. (c) **2.** (c)

3. (a) **4.** (a) **5.** (d) **6.** (c)
7. (d) **8.** (a) **9.** (b) **10.** (c)
11. (c) **12.** (c) **13.** (a) **14.** (a)
15. (a, b, c) **16.** 15° **17.** 30° **18.** T
19. (2) **20.** (a) 60° (b) 60°

21. (a) zero (b) 1500Å

22. (a) 600 nm (b) $\sin^{-1}\left[\frac{3}{4}\right]$

23. (a) $i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2} - n_1\right)\right\}$

(b) 73° **24.** $\mu = \sqrt{3}$

Topic 5

1. (a) **2.** (a) **3.** (b)

4. (d) **5.** (c) **6.** (c) **7.** (a)
8. (d) **9.** (a, b, c, d) **10.** smaller

Topic 6

1. (c) **2.** (c) **3.** (c) **4.** (*)

5. (c) **6.** (a) **7.** (d) **8.** (c)

9. (d) **10.** (d) **11.** (c) **12.** (d)

13. (b) **14.** (a) **15.** (d) **16.** (b)
17. (c) **18.** (d) **19.** (b) **20.** (c)
21. (d) **22.** (b) **23.** (a) **24.** (b)
25. (b) **26.** (a) **27.** (a) **28.** (d)
29. (d) **30.** (d) **31.** (c) **32.** (d)

33. (A) $\to$ p, s; (B) $\to$ q; (C) $\to$ t; (D) $\to$ r, s, t

34. (c, d) **35.** (a, b) **36.** (a, b, c) **37.** (d)
38. (a, b) **39.** (a, c) **40.** (a, c) **41.** (b, d)
42. 2.945×10^{-4} **43.** 2 **44.** F **45.** T

46. (3) **47.** 3.5 mm

48. (a) circular (b) $\frac{1}{16}$ (c) 300 nm

49. 2 cm above point Q on side CD, $\mu = 1.0016$

50. $3.6t = \left(n - \frac{1}{2}\right)\lambda$ with $n = 1, 2, 3\ldots$, 90 nm

51. (a) 4.33 mm (b) $I = \frac{3I_{max}}{4}$ (c) 650 nm; 433.33nm

52. (a) ± 0.26 m, ± 1.13 m (b) 0.26 m, 1.13 m

53. 9.3 μm

54. 7×10^{-6}m, 1.6, -5.71×10^{-5}m

55. (a) 0.63 mm (b) 1.579 μm

56. 4200 Å, 1.43 **57.** $\frac{1}{49}$ **58.** 7×10^{-6} W

59. (a) 1.17 mm (b) 1.56 mm **60.** 5892Å

Topic 7

1. (c) **2.** (*) **3.** (d) **4.** (c)

5. (a) **6.** (b) **7.** (a) **8.** (c)
9. (d) **10.** (c) **11.** (c) **12.** (b)
13. (b) **14.** (c) **15.** (a) **16.** (d)
17. (c) **18.** (c) **19.** (130.0) **20.** (b)

21. (c) **22.** (a) **23.** (c) **24.** (b)

25. (A) $\to$ p, q, r, s; (B) $\to$ q; (C) $\to$ p, q, r, s; (D) $\to$ p, q, r, s

26. (A) $\to$ p; (B) $\to$ r; (C) $\to$ r; (D) $\to$ p, q, r

27. (a, b, c) **28.** (b, c) **29.** (b, d) **30.** $\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$

31. $\frac{25}{9}$ **32.** T **33.** (7) **34.** (2)

35. 6 **36.** 60° **37.** 6.06 m

38. $\frac{\mu_3 R}{\mu_3 - \mu_1}$ **39.** 1.6 **40.** (a) 4° (b) – 0.04°

41. 15 cm, $-3/2$

42. 0.9 m from the lens (rightwards) or 0.1 m behind the mirror

43. (a) Slope = cot i (b) $4y^{1/4} = x$ (c) (4m, 1m)
(d) the ray will emerge grazingly

44. (a) $b = 8 \times 10^5 (\text{Å})^2$ (b) $\delta_{4000\text{Å}} = 37°, \delta_{5000\text{Å}} = 27.13°$ (c) $9I$

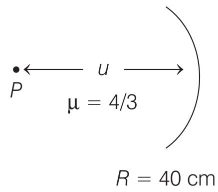
46. 13.97 W/m^2

47. (a) 15cm (b) 1.16 cm (downwards) reflection of light

Hints & Solutions

Topic 1 Reflection of Light

1. In the given case,



$u = -5\,\text{cm}$

Focal length,

$$f = \frac{-R}{2} = \frac{-40}{2} = -20\,\text{cm}$$

Now, using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= -\frac{1}{20} + \frac{1}{5} = +\frac{3}{20}$$

$$\Rightarrow \quad v = +\frac{20}{3}\,\text{cm}$$

For the light getting refracted at water surface, this image will act as an object.

So, distance of object, $d = 5\,\text{cm} + \frac{20}{3}\,\text{cm} = \frac{35}{3}\,\text{cm}$

(below the surface). Let's assume final image at distance d after refraction.

$$\frac{d'}{d} = \frac{\mu_2}{\mu_1} \Rightarrow d' = d\left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{35}{3}\,\text{cm}\right)\left(\frac{1}{\frac{4}{3}}\right)$$

$$= \frac{35}{3} \times \frac{3}{4}\,\text{cm} = \frac{35}{4}\,\text{cm} = 8.75\,\text{cm} \approx 8.8\,\text{cm}$$

2. Given, focal length of concave mirror,

$$f = -\,0.4\text{ m}$$

Magnification = 5

We know that, magnification produced by a mirror,

$$m = -\frac{\text{image distance}}{\text{object distance}}$$

$$\Rightarrow \quad \frac{v}{u} = -\,5 \text{ or } v = -\,5u$$

Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Substituting the given values in the above equation, we get

$$\frac{1}{-\,5u} + \frac{1}{u} = -\frac{1}{0.4}$$

$$\Rightarrow \quad \frac{4}{5u} = -\frac{1}{0.4}$$

$$\Rightarrow \quad u = -\frac{1.6}{5} = -\,0.32\text{ m}$$

Alternate Solution

Magnification produced by a mirror can also be given as

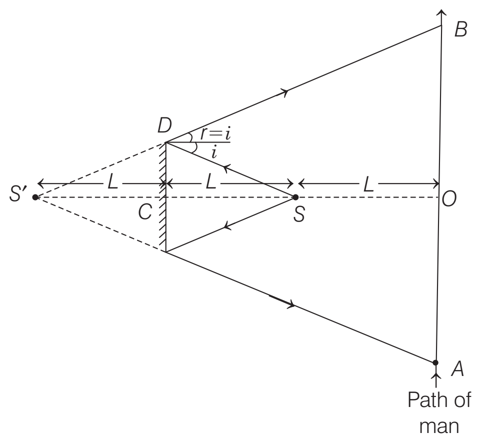
$$m = \frac{f}{f - u}$$

Substituting the given values, we get

$$5 = \frac{-0.4}{-0.4 - u}$$

or $\quad u = -\,0.32\text{ m}$

3. Light from mirror is reflected in a straight line and It is appear to come from its image formed at same distance (as that of source) behind the mirror as shown in the ray diagram below.



From ray diagram in similar triangles,

$\Delta S'CD$ and $\Delta S'OB$, we have

$$\frac{S'C}{S'O} = \frac{CD}{OB}$$

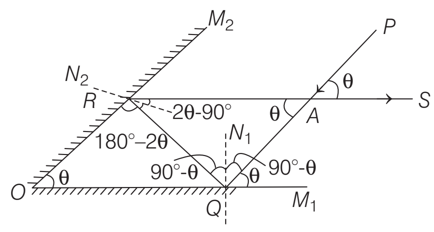
So, $\quad OB = \frac{CD \times S'O}{S'C} = \frac{\frac{d}{2} \times 3L}{L} = \frac{3d}{2}$

Also, $\quad OA = \frac{3d}{2}$

So, distance over which man can see the image S' is

$$\frac{3d}{2} + \frac{3d}{2} = 3d$$

4. The given condition is shown in the figure given below, where two plane mirror inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from second mirror (M_2) parallel to the first mirror.



where, PQ = incident ray parallel to the mirror M_2,

QR = reflected ray from the mirror M_1,

RS = reflected ray from the mirror M_2 which is parallel to the M_1 and θ = angle between M_1 and M_2.

According to geometry,

$\angle PAS = \angle PQM_1 = \theta$ (angle on same line)

$\angle AQN_1$ = angle of incident = $90 - \theta$

$\angle N_1QR$ = angle of reflection = $(90 - \theta)$.

Therefore, for triangle ΔORQ, (according to geometry)

$$\angle\theta + \angle\theta + \angle ORQ = 180^\circ$$

$$\angle ORQ = 180^\circ - 2\theta \qquad \ldots\text{(i)}$$

For normal N_2,

angle of incident = angle of reflection

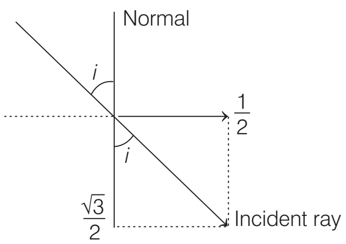
$$= 2\theta - 90^\circ \qquad \ldots\text{(ii)}$$

Therefore, for the triangle ΔRAQ

$$\Rightarrow \quad 4\theta - 180^\circ + 180^\circ - 2\theta + \theta = 180$$

$$3\theta = 180^\circ \Rightarrow \theta = 60^\circ$$

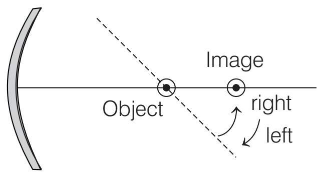
5.



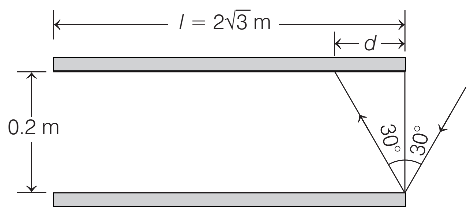
$$\tan i = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad i = 30^\circ$$

6. Since object and image move in opposite directions, the positioning should be as shown in the figure. Object lies between focus and centre of curvature $f < x < 2f$.



7. $d = 0.2 \tan 30^\circ = \dfrac{0.2}{\sqrt{3}}$

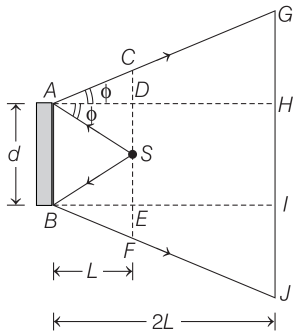


$$\frac{l}{d} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$$

Therefore, maximum number of reflections are 30.

NOTE Answer of this question may be 30 or 31.

8. The ray diagram will be as follows :



$$HI = AB = d$$

$$DS = CD = \frac{d}{2}$$

Since, $AH = 2AD$

$$\therefore \quad GH = 2CD = 2\frac{d}{2} = d$$

Similarly, $\quad IJ = d$

$$\therefore \quad GJ = GH + HI + IJ$$

$$= d + d + d = 3d$$

9. From the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad (f = \text{constant}) \ \ldots\text{(i)}$$

$$-v^{-2}dv - u^{-2}du = 0$$

or $$|dv| = \left|\frac{v^2}{u^2}\right| |du| \qquad \ldots\text{(ii)}$$

Here, $|dv|$ = size of image

$|du|$ = size of object (short) lying along the axis $= b$

Further, from Eq. (i), we can find

$$\frac{v^2}{u^2} = \left(\frac{f}{u - f}\right)^2$$

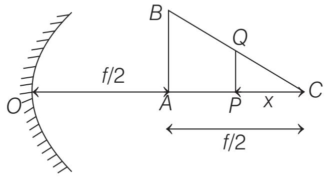
Substituting these values in Eq. (ii), we get

$$\text{Size of image } = b\left(\frac{f}{u - f}\right)^2$$

$\therefore$ Correct option is (d).

10. Laws of reflection can be applied to any type of surface.

11. Image of point A



$$\frac{PQ}{x} = \frac{AB}{f/2} \Rightarrow PQ = \frac{2(AB)x}{f}$$

For A :

$$\frac{1}{v} + \frac{1}{[-(f/2)]} = \frac{1}{-f} \Rightarrow v = f$$

$$\Rightarrow \qquad \frac{I_{AB}}{AB} = -\frac{v}{u} = -\frac{f}{\left(-\frac{f}{2}\right)}$$

$$\Rightarrow \qquad I_{AB} = 2\,AB$$

For height of PQ,

$$\frac{1}{v} + \frac{1}{-[(f-x)]} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{(f-x)} - \frac{1}{f} \Rightarrow v = \frac{f(f-x)}{x}$$

$$\Rightarrow \qquad \frac{I_{PQ}}{PQ} = -\frac{v}{u} = \frac{f(f-x)}{x[(f-x)]} = \left(\frac{f}{x}\right)$$

$$\Rightarrow \qquad I_{PQ} = \frac{f}{x} PQ = \left(\frac{f}{x}\right)\left(\frac{2\,(AB)x}{f}\right) \left[\because PQ = \frac{2(AB)x}{f}\right]$$

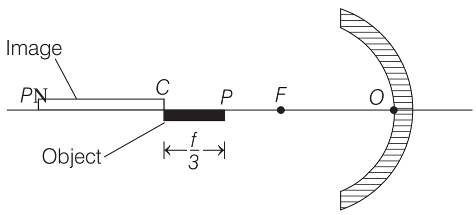
$$I_{PQ} = 2AB$$

(Size of image is independent of x. So, final image will be of same height terminating at infinity.)

12. Values of options (c) and (d) don't match with the mirror formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

13. F = focus, C = centre of curvature



When the object lies between F and C, image is real, elongated and inverted. As one end of rod just touches its image, this end should lie at C. Because image of object at C is at C itself.

Let P' be the image of other end of rod P.

For P

$$u = -(2f - f/3) = -\frac{5f}{3}$$

Applying the mirror formula : $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

or $\quad \dfrac{1}{v} - \dfrac{3}{5f} = \dfrac{1}{-f} \quad \Rightarrow \quad \dfrac{1}{v} = \dfrac{3}{5f} - \dfrac{1}{f}$

or $\quad v = -\dfrac{5f}{2} \quad$ or $\quad OP' = \dfrac{5f}{2}$

$\therefore$ Length of image of rod

$$CP' = OP' - OC = \frac{5f}{2} - 2f = \frac{f}{2}$$

$$\therefore \qquad \text{Magnification} = -\left(\frac{f/2}{f/3}\right) = -1.5$$

Here, negative sign implies that image is inverted.

14. Using mirror formula twice,

$$\frac{1}{+25/3} + \frac{1}{-u_1} = \frac{1}{+10}$$

or $\quad \dfrac{1}{u_1} = \dfrac{3}{25} - \dfrac{1}{10}$

or $\quad u_1 = 50$ m and $\dfrac{1}{(+50/7)} + \dfrac{1}{-u_2} = \dfrac{1}{+10}$

$\therefore \quad \dfrac{1}{u_2} = \dfrac{7}{50} - \dfrac{1}{10}$ or $u_2 = 25$ m

Speed of object $= \dfrac{u_1 - u_2}{\text{time}}$

$$= \frac{25}{30}\ \text{ms}^{-1} = 3\ \text{kmh}^{-1}$$

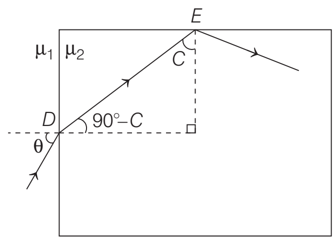
$\therefore$ Answer is 3.

Topic 2 Refraction of Light and TIR

1. **Key Idea** The critical angle is defined as the angle of incidence that provides an angle of refraction of 90°.

So, $\qquad \theta_c = \sin^{-1}\dfrac{\mu_2}{\mu_1}$

For total internal reflection, angle of incidence(i) at medium interface must be greater than critical angle (C).



where,

$$\sin C = \frac{\mu_1}{\mu_2} \qquad \ldots\text{(i)}$$

Now, in given arrangement,

at point D,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \qquad \text{(Snell's law)}$$

$$\Rightarrow \qquad \frac{\sin\theta}{\sin(90° - C)} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\sin\theta}{\cos C} = \frac{\mu_2}{\mu_1}$$

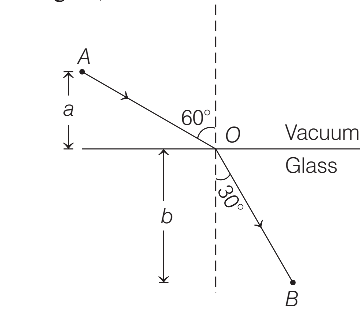
$$\Rightarrow \qquad \sin\theta = \frac{\mu_2}{\mu_1}\cdot\cos C = \frac{\mu_2}{\mu_1}\sqrt{1-\sin^2 C} \qquad \text{[from Eq. (i)]}$$

$$= \frac{\mu_2}{\mu_1}\sqrt{1-\frac{\mu_1^2}{\mu_2^2}} = \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1} \Rightarrow \theta = \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

For TIR at E, $i > C$

$$\Rightarrow \qquad \theta < \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

2. From the figure,



$$\cos 60° = \frac{a}{AO}$$

$\Rightarrow$ $$AO = \frac{a}{\cos 60°} = 2a \quad \text{...(i)}$$

and $$\cos 30° = \frac{b}{BO}$$

or $$BO = \frac{b}{\cos 30°} = \frac{2}{\sqrt{3}}b \quad \text{... (ii)}$$

Optical path length of light ray

$$= AO + \mu\ (BO) \quad \text{... (iii)}$$

Here, μ can be determined using Snell's law, i.e.

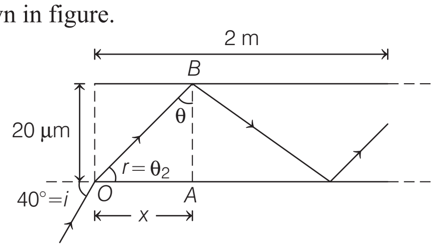
$$\mu = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \quad \text{... (iv)}$$

Substituting the values from Eqs. (i), (ii) and (iv) in Eq. (iii), we get

$\therefore$ Optical path $= 2a + (\sqrt{3} \times \frac{2}{\sqrt{3}}b)$

$$= 2a + 2b$$

3. Total internal reflection occurs through given glass rod as shown in figure.



From Snell's law, $n_1 \sin i = n_2 \sin r$

where, $n_1 = 1$, $n_2 = 1.31$ and $i = 40°$

So, we get

$1 \sin 40° = 1.31 \sin r \Rightarrow \sin r = \frac{0.64}{1.31} = 0.49 \approx 0.5$

So, $r = 30°$

From ΔOAB, $\theta = 90 - r = 60°$

Now, $\tan\theta = \frac{x}{20\,\mu\text{m}}$

$\Rightarrow$ $x = 20\sqrt{3}\ \mu\text{m}$ $\quad [\because \tan 60° = \sqrt{3}]$

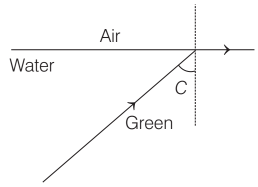
One reflection occurs in $20\sqrt{3}\ \mu$m.

$\therefore$ Total number of reflections occurring in 2m

$$= n = \frac{2\text{m}}{20\sqrt{3}\ \mu\text{m}} = \frac{2}{20\sqrt{3} \times 10^{-6}}$$

$$= 57735 \text{ reflections} \approx 57000 \text{ reflections}$$

4. For total internal reflection of light take place, following conditions must be obeyed.



(*i*) The ray must travel from denser to rarer medium.

(*ii*) Angle of incidence (θ) must be greater than or equal to critical angle (C)

i.e. $$C = \sin^{-1}\left[\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right]$$

Here, $\sin C = \frac{1}{n_{\text{water}}}$ and $n_{\text{water}} = a + \frac{b}{\lambda^2}$

If frequency is less $\Rightarrow \lambda$ is greater and hence, RI $n_{(\text{water})}$ is less and therefore, critical angle increases. So, they do not suffer reflection and come out at angle less than 90°.

5. At point Q angle of incidence is critical angle θ_C, where

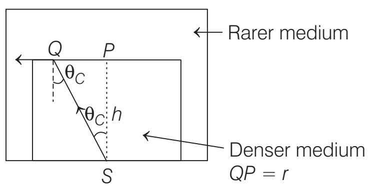
$$\sin\theta_C = \frac{\mu_l}{\mu_{\text{block}}}$$

In ΔPQS, $\sin\theta_C = \frac{r}{\sqrt{r^2 + h^2}}$

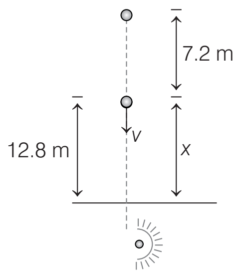
$\therefore$ $$\frac{\mu_l}{\mu_{\text{block}}} = \frac{r}{\sqrt{r^2 + h^2}}$$

$\Rightarrow$ $$\mu_l = \frac{r}{\sqrt{r^2 + h^2}} \times 2.72$$

$$= \frac{5.77}{11.54} \times 2.72 = 1.36$$



6. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 7} = 12\ \text{ms}^{-1}$



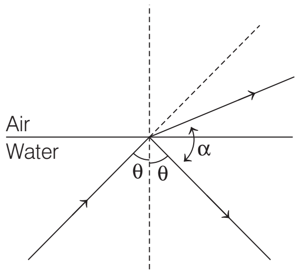
In this case when eye is inside water,

$$x_{\text{app.}} = \mu x$$

$$\therefore \qquad \frac{dx_{\text{app.}}}{dt} = \mu \cdot \frac{dx}{dt}$$

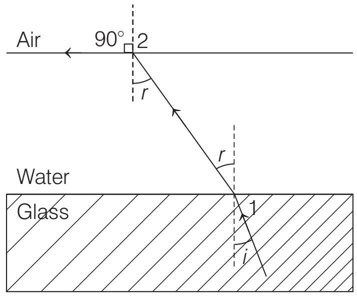
or $$v_{\text{app.}} = \mu v = \frac{4}{3} \times 12 = 16 \text{ ms}^{-1}$$

7. Since $\theta < \theta_c$, both reflection and refraction will take place. From the figure we can see that angle between reflected and refracted rays α is less than $180° - 2\theta$.



8. When the object is placed at the centre of the glass sphere, the rays from the object fall normally on the surface of the sphere and emerge undeviated.

9. Applying Snell's law (μ sin i = constant) at 1 and 2, we have



$$\mu_1 \sin i_1 = \mu_2 \sin i_2$$

Here, $$\mu_1 = \mu_{\text{glass}}, \; i_1 = i$$
$$\mu_2 = \mu_{\text{air}} = 1 \text{ and } i_2 = 90°$$

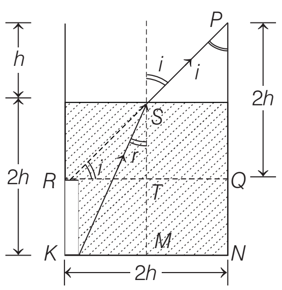
$$\therefore \qquad \mu_g \sin i = (1)(\sin 90°) \text{ or } \mu_g = \frac{1}{\sin i}$$

10. $$PQ = QR = 2h \Rightarrow \angle i = 45°$$

$$\therefore \qquad ST = RT = h = KM = MN$$

So, $$KS = \sqrt{h^2 + (2h)^2} = h\sqrt{5}$$

$$\therefore \qquad \sin r = \frac{h}{h\sqrt{5}} = \frac{1}{\sqrt{5}}$$



$$\therefore \qquad \mu = \frac{\sin i}{\sin r} = \frac{\sin 45°}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}$$

11. Applying Snell's law at B and C,

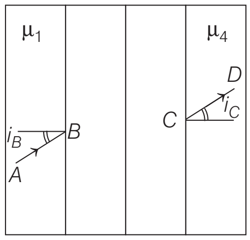
$\mu \sin i$ = constant or

$$\mu_1 \sin i_B = \mu_4 \sin i_C$$

But $AB \parallel CD$

$\therefore \qquad i_B = i_C$

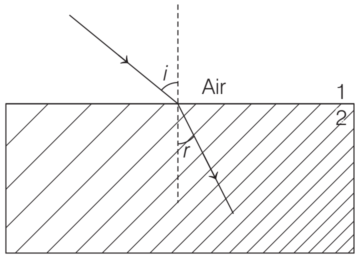
or $\qquad \mu_1 = \mu_4$



NOTE The Snell's law, which the students read in their plus two syllabus $\mu = \dfrac{\sin i}{\sin r}$ is derived from

$\mu \sin i$ = constant or $\mu_1 \sin i_1 = \mu_2 \sin i_2$

Here : $\mu_1 = 1, \mu_2 = \mu$, or $\sin i = \mu \sin r$ or $\mu = \dfrac{\sin i}{\sin r}$



12. Rays come out only from CD, means rays after refraction from AB get total internally reflected at AD.

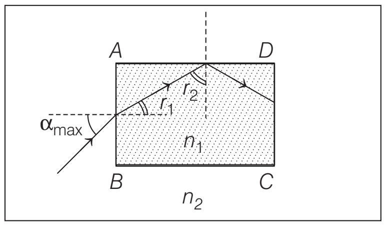
From the figure

$$r_1 + r_2 = 90°$$

$$\therefore \qquad r_1 = 90° - r_2$$

$$(r_1)_{\max} = 90° - (r_2)_{\min} \text{ and } (r_2)_{\min} = \theta_C$$

(for total internal reflection at AD)



where, $$\sin \theta_C = \frac{n_2}{n_1}$$

or $$\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\therefore \qquad (r_1)_{\max} = 90° - \theta_C$$

Now, applying Snell's law at face AB

$$\frac{n_1}{n_2} = \frac{\sin \alpha_{\max}}{\sin (r_1)_{\max}}$$

$$= \frac{\sin \alpha_{\max}}{\sin (90° - \theta_C)}$$

$$= \frac{\sin \alpha_{max}}{\cos \theta_C}$$

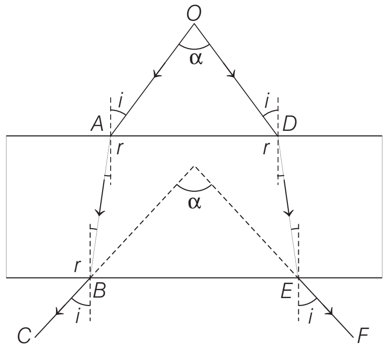
or $$\sin \alpha_{max} = \frac{n_1}{n_2} \cos \theta_C$$

$\therefore$ $$\alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2} \cos \theta_C\right]$$

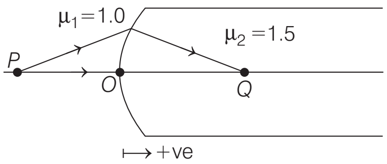
$$= \sin^{-1}\left[\frac{n_1}{n_2} \cos\left(\sin^{-1} \frac{n_2}{n_1}\right)\right]$$

13. Divergence angle will remain unchanged because in case of a glass slab every emergent ray is parallel to the incident ray. However, the rays are displaced slightly towards outer side.

(In the figure $OA \parallel BC$ and $OD \parallel EF$)



14. Let us say $PO = OQ = X$



Applying $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values with sign

$$\frac{1.5}{+X} - \frac{1.0}{-X} = \frac{1.5 - 1.0}{+R}$$

(Distances are measured from O and are taken as positive in the direction of ray of light)

$\therefore$ $$\frac{2.5}{X} = \frac{0.5}{R}$$

$\therefore$ $$X = 5R$$

15. $r + r' + 90° = 180°$

$\therefore$ $$r' = 90° - r$$

Further, $$i = r$$

Applying Snell's law, $\mu_D \sin i = \mu_R \sin r'$

or $\mu_D \sin r = \mu_R \sin(90° - r) = \mu_R \cos r$

$\therefore$ $$\frac{\mu_R}{\mu_D} = \tan r, \theta_C = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right) = \sin^{-1}(\tan r)$$

16. $\lambda = \dfrac{v}{f}$

In moving from air to glass, f remains unchanged while v decreases. Hence, λ should decrease.

17. From Snell's law,

$$n \sin \theta = \text{constant}$$

$\therefore$ $n_1 \sin \theta_i = n_2 \sin \theta_f$

Further, l will depend on n_1 and $n(z)$. But it will be independent of n_2.

18. $\dfrac{1}{f_{film}} = (n_1 - 1)\left(\dfrac{1}{R} - \dfrac{1}{R}\right) \Rightarrow f_{film} = \infty$ (infinite)

$\therefore$ There is no effect of presence of film.

From Air to Glass

Using the equation $$\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}$$

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{R} \Rightarrow v = 3R$$

$\therefore$ $$f_1 = 3R$$

From Glass to Air Again using the same equation

$$\frac{1}{v} - \frac{n_2}{u} = \frac{1 - n_2}{-R} \Rightarrow \frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-R} \Rightarrow v = 2R$$

$\therefore$ $$f_2 = 2R$$

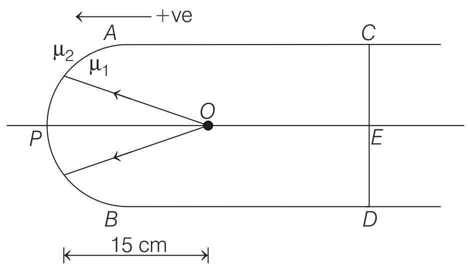
19. For total internal reflection to take place
Angle of incidence, $i >$ critical angle, θ_c

or $\sin i > \sin \theta_c$ or $\sin 45° > 1/n$

or $\dfrac{1}{\sqrt{2}} > \dfrac{1}{n}$ or $n > \sqrt{2}$ or $n > 1.414$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.

20. Rays starting from O will suffer single refraction from spherical surface APB. Therefore, applying



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.0}{v} - \frac{2.0}{-15} = \frac{1.0 - 2.0}{-10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{7.5}$$

or $$v = -30 \text{ cm}$$

Therefore, image of O will be formed at 30 cm to the right of P. Note that image will be virtual. There will be no effect of CED.

21. $\lambda_{\text{medium}} = \dfrac{\lambda_{\text{air}}}{\mu} = \dfrac{6000}{1.5} = 4000$ Å

$$f = \frac{v_{\text{air}}}{\lambda_{\text{air}}} = \frac{v_{\text{medium}}}{\lambda_{\text{medium}}} = \frac{3.0\times10^8}{6.0\times10^{-7}} = 5.0\times10^{14}\ \text{Hz}$$

Frequency remains unchanged.

22. $v = \dfrac{c}{\mu} = \dfrac{3\times10^8}{1.5} = 2.0\times10^8\ \text{m/s}$

$$\lambda = \frac{v}{f} = \frac{2.0\times10^8}{5.0\times10^{14}} = 4.0\times10^{-7}\ \text{m}$$

23. $\dfrac{4}{3}\sin i = \dfrac{\sqrt{45}}{4}\sin(90-\theta_c) = \dfrac{\sqrt{45}}{4}\cos\theta_c$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\therefore \qquad \cos\theta_c = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$\Rightarrow \qquad \frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\frac{3}{\sqrt{45}}$$

$$\sin i = \frac{9}{16}$$

In second case,

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{7}{8} \Rightarrow \cos\theta_c = \frac{\sqrt{15}}{8}$$

$$\frac{16}{3\sqrt{15}}\sin i = \frac{8}{5}\sin(90-\theta_c)$$

Simplifying we get,

$$\sin i = \frac{9}{16}$$

Same approach can be adopted for other options. Correct answers are (a) and (c).

24. (1) $\sin i_m = n_1 \sin(90° - \theta_c)$

$\Rightarrow \sin i_m = n_1 \cos\theta_c$

$\Rightarrow \quad NA = n_1\sqrt{1-\sin^2\theta_c}$

$$= n_1\sqrt{1-\frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$$

Substituting the values we get,

$$NA_1 = \frac{3}{4} \text{ and } NA_2 = \frac{\sqrt{15}}{5} = \sqrt{\frac{3}{4}}$$

$$NA_2 < NA_1$$

Therefore, the numerical aperture of combined structure is equal to the lesser of the two numerical aperture, which is NA_2.

25. But this value of refractive index is not possible.

$$1.6\sin\theta = (n - m\Delta n)\sin 90°$$
$$1.6\sin\theta = n - m\Delta n$$
$$1.6\times\frac{1}{2} = 1.6 - m(0.1)$$
$$0.8 = 1.6 - m\,(0.1)$$
$$m\times 0.1 = 0.8$$
$$m = 8$$

26. Applying $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2-\mu_1}{R}$

First on plane surface

$$\frac{1.5}{AI_1} - \frac{1}{-mR} = \frac{1.5-1}{\infty} = 0 \qquad (R=\infty)$$

$\therefore \qquad AI_1 = -(1.5\,mR)$

Then, on curved surface

$$\frac{1}{\infty} - \frac{1.5}{-(1.5\,mR + R)} = \frac{1-1.5}{-R}$$

[$v = \infty$, because final image is at infinity]

$$\Rightarrow \qquad \frac{1.5}{(1.5\,m+1)R} = \frac{0.5}{R}$$

$$\Rightarrow \qquad 3 = 1.5m + 1$$

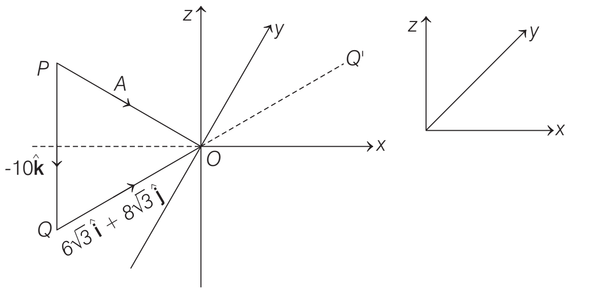
$$\Rightarrow \qquad \frac{3}{2}m = 2$$

$$\text{or} \qquad m = \frac{4}{3}$$

27. Incident ray $\mathbf{A} = 6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$

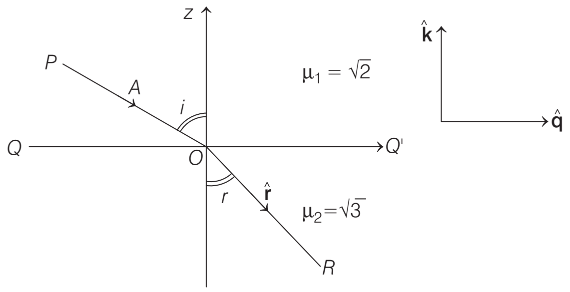
$$= (6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}}) + (-10\hat{\mathbf{k}})$$

$$= \mathbf{QO} + \mathbf{PQ} \qquad \text{(As shown in figure)}$$



Note that **QO** is lying on x-y plane.

Now, QQ' and Z-axis are mutually perpendicular. Hence, we can show them in two-dimensional figure as below.



Vector **A** makes an angle i with z-axis, given by

$$i = \cos^{-1}\left\{\frac{10}{\sqrt{(10)^2 + (6\sqrt{3})^2 + (8\sqrt{3})^2}}\right\} = \cos^{-1}\left\{\frac{1}{2}\right\}$$

$$i = 60^\circ$$

Unit vector in the direction of QOQ' will be

$$\hat{\mathbf{q}} = \frac{6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}}}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}}$$

$$= \frac{1}{5}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$$

Snell's law gives

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin r}$$

$$\therefore \quad \sin r = \frac{\sqrt{3}/2}{\sqrt{3}/\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad r = 45^\circ$$

Now, we have to find a unit vector in refracted ray's direction OR. Say it is $\hat{\mathbf{r}}$ whose magnitude is 1. Thus,

$$\hat{\mathbf{r}} = (1\sin r)\hat{\mathbf{q}} - (1\cos r)\hat{\mathbf{k}}$$

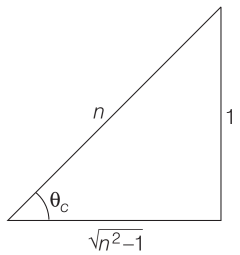
$$= \frac{1}{\sqrt{2}}[\hat{\mathbf{q}} - \hat{\mathbf{k}}] = \frac{1}{\sqrt{2}}\left[\frac{1}{5}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) - \hat{\mathbf{k}}\right]$$

$$\hat{\mathbf{r}} = \frac{1}{5\sqrt{2}}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}).$$

28. $\sin\theta_c = \frac{1}{n}$ (θ_c = critical angle)

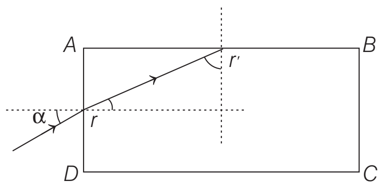
$$r' = 90^\circ - r \quad \Rightarrow \quad (r')_{\min} = 90^\circ - (r)_{\max}$$

and $$n = \frac{\sin (i)_{\max}}{\sin (r)_{\max}} = \frac{\sin 90^\circ}{\sin (r)_{\max}} \qquad (\because\ i_{\max} = 90^\circ)$$



Then, $$\sin (r)_{\max} = \frac{1}{n} = \sin\theta_c$$

$$(r)_{\max} = \theta_c \quad \text{or} \quad (r')_{\min} = (90^\circ - \theta_c)$$



Now, if minimum value of r' i.e. $90^\circ - \theta_c$ is greater than θ_c, then obviously all values of r' will be greater than θ_c *i.e.*, total internal reflection will take place at face AB in all conditions. Therefore, the necessary condition is

$$(r')_{\min} \geq \theta_c$$

or $$(90^\circ - \theta_c) \geq \theta_c$$

or $$\sin (90^\circ - \theta_c) \geq \sin\theta_c$$

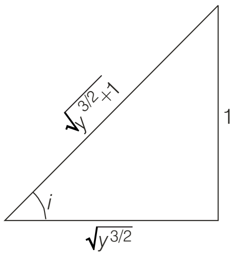
or $$\cos\theta_c \geq \sin\theta_c$$

or $$\cot\theta_c \geq 1$$

or $$\sqrt{n^2 - 1} \geq 1$$

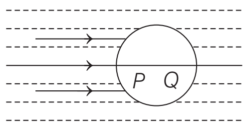
or $$n^2 \geq 2$$

or $$n \geq \sqrt{2}$$



Therefore, minimum value of n is $\sqrt{2}$.

29. (a)



Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, one by one on two spherical surfaces.

First on left surface

$$\frac{1}{v_1} - \frac{4/3}{\infty} = \frac{1 - 4/3}{+2}$$

or $$\frac{1}{v_1} = -\frac{1}{6}$$

or $$v_1 = -6 \text{ mm}$$

i.e. first image will be formed at 6 mm towards left of P

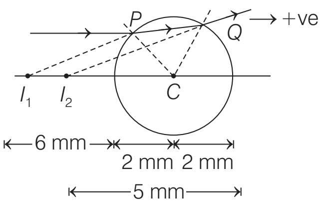
Second on right surface Now, distance of first image I_1 from Q will be 10 mm (towards left).

$$\frac{4/3}{v_2} - \frac{1}{-10} = \frac{4/3 - 1}{-2}$$

or $$\frac{4}{3v_2} = -\frac{1}{6} - \frac{1}{10} = -\frac{4}{15}$$

or $$v_2 = -5 \text{mm}$$

(b) The ray diagram is shown in figure.

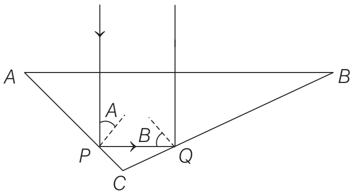


NOTE

- At P and Q both normal will pass through C.
- At P ray of light is travelling from a denser medium (water) to rarer medium (air) therefore, ray of light will bend away from the normal and on extending meet at I_1. Similarly at Q, ray of light bends towards the normal.
- Both the images I_1 and I_2 are virtual.

30. (a) At P, angle of incidence $i_A = A$

and at Q, angle of incidence $i_B = B$



If TIR satisfies for the smaller angle of incidence than for larger angle of incidence is automatically satisfied.

$$B \le A \quad \therefore \quad i_B \le i_A$$

Maximum value of B can be 45°. Therefore, if condition of TIR is satisfied, then condition of TIR will be satisfied for all value of i_A and i_B

Thus, $45° \ge \theta_c$

or $\sin 45° \ge \sin\theta_c$

or $\frac{1}{\sqrt{2}} \ge \frac{1}{\mu}$ or $\mu \ge \sqrt{2}$

$\therefore$ Minimum value of μ or n is $\sqrt{2}$.

(b) For $n = \frac{5}{3}, \sin\theta_c = \frac{1}{n} = \sin^{-1}\left(\frac{3}{5}\right) \approx 37°$

If $B = 30°$, then $i_B = 30°$

then $A = 60°$ or $i_A = 60°$

$$i_A > \theta_c \text{ but } i_B < \theta_c$$

i.e. TIR will take place at A but not at B.

or we write : $\sin i_B < \sin\theta_c < \sin i_A$

or $\sin 30° < \frac{3}{5} < \sin 60°$

or $0.5 < 0.6 < 0.86$

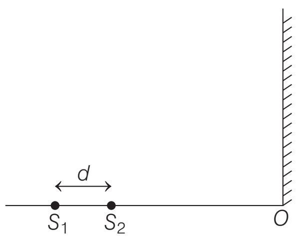
Topic 3 Lens Theory

1. Focal length of a lens is given as

$$\frac{1}{f} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ Focal length of plano-convex lens, i.e. lens 1,

$R_1 = \infty$ and $R = -R$



$\Rightarrow \quad \frac{1}{f_1} = \frac{\mu_1 - 1}{R}$

or $\quad f_1 = \frac{R}{(\mu_1 - 1)}$ ... (i)

Similarly, focal length of plano-concave lens, i.e. lens 2,

$$R_1 = -R \text{ and } R_2 = \infty$$

$\Rightarrow \quad \frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R}$

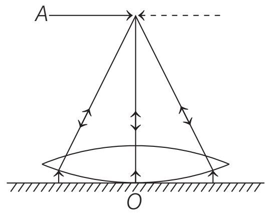
or $\quad f_2 = \frac{-R}{(\mu_2 - 1)}$ ... (ii)

From Eqs. (i) and (ii), net focal length is

$$\frac{1}{f} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{\mu_1 - \mu_2}{R}$$

$\Rightarrow \quad f = \frac{R}{\mu_1 - \mu_2}$

2. Light from plane mirror is reflected back on it's path, so that image of A coincides with A itself.



This would happen when rays refracted by the convex lens falls normally on the plane mirror, i.e. the refracted rays form a beam parallel to principal axis of the lens. Hence, the object would then be considered at the focus of convex lens.

$\therefore$ Focal length of curvature of convex lens is, $f_1 = 18$ cm

With liquid between lens and mirror, image is again coincides with object, so the second measurement is focal length of combination of liquid lens and convex lens.

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} \Rightarrow \frac{1}{18} + \frac{1}{f_2} = \frac{1}{27} \Rightarrow f_2 = -54 \text{ cm}$$

For convex lens by lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) \Rightarrow \frac{1}{18} = 0.5 \times \frac{2}{R} \Rightarrow R = 18 \text{ cm}$$

and for plano-convex liquid lens, we have

$$\frac{1}{f} = (\mu_l - 1)\left(\frac{-1}{R}\right) \Rightarrow -\frac{1}{54} = (\mu_l - 1)\left(\frac{-1}{18}\right)$$

$$\Rightarrow \mu_l = 1 + \frac{1}{3} = \frac{4}{3}$$

3. Since, image formed by a convex lens can be real or virtual in nature.

Thus, $\quad m = +2$ or -2.

First let us take image to be real in nature, then

$m = -2 = \frac{v}{u}$, where v is image distance and u is object distance.

$\Rightarrow \quad v = -2x_1 \quad$ [Taking $u = x_1$]

Now, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-2x_1} - \frac{1}{x_1} = \frac{1}{20} \Rightarrow \frac{-3}{2x_1} = \frac{1}{20}$$

$\Rightarrow \quad x_1 = -30$ cm

Now, let us take image to be virtual in nature, then

$$m = 2 = \frac{v}{u} \Rightarrow v = 2x_2 \qquad [\text{Taking } u = x_2]$$

Again, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{2x_2} - \frac{1}{x_2} = \frac{1}{20}$$

$$\Rightarrow \qquad \frac{-1}{2x_2} = \frac{1}{20} \Rightarrow x_2 = -10 \text{ cm}$$

So, the ratio of x_1 and x_2 is

$$\frac{x_1}{x_2} = \frac{-30}{-10} = 3:1$$

Alternate Solution

Magnification for a lens can also be written as

$$m = \left(\frac{f}{f+u}\right)$$

When $m = -2$ (for real image)

$$-2 = \frac{f}{f + x_1}$$

$$\Rightarrow \qquad -2f + (-2)x_1 = f \text{ or } x_1 = -\frac{3f}{2}$$

Similarly, when $m = +2$ (for virtual image)

$$+2 = \frac{f}{f + x_2} \Rightarrow 2f + 2x_2 = f \text{ or } x_2 = \frac{-f}{2}$$

Now, the ratio of x_1 and x_2 is

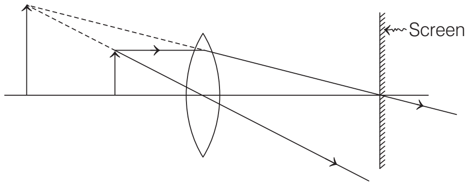
$$\frac{x_1}{x_2} = \frac{\frac{-3f}{2}}{-f/2} = \frac{3}{1}$$

4. When given set up is immersed in water, focal length of lens increases,

Note
$$\left\{\begin{array}{l} \dfrac{f_{\text{liquid}}}{f_{\text{air}}} = \dfrac{n_{ga} - 1}{n_{gl} - 1} = \dfrac{n_{ga} - 1}{\left(\dfrac{n_{ga}}{n_{la}} - 1\right)} \\ \text{Now, } n_{ga} = \dfrac{3}{2} \text{ and } n_{la} = \dfrac{4}{3} \\ \therefore f_{\text{liquid}} = f_{\text{air}}\left(\dfrac{\frac{1}{2}}{\frac{1}{8}}\right) = 4 f_{\text{air}} \end{array}\right.$$

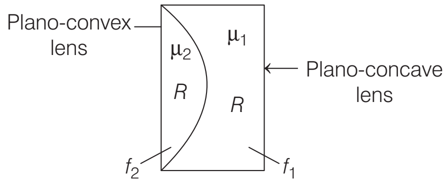
As focal length increases there is no focussing of image on screen

$\therefore$ image will disappear



Actually a virtual image is formed on same side of object.

5. Given combination is as shown below



As lenses are in contact, equivalent focal length of combination is

$$\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{(1-\mu_1)}{R}$$

and $$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{(\mu_2 - 1)}{R}$$

$$\therefore \qquad \frac{1}{f_{\text{eq}}} = \left(\frac{\mu_2 - 1}{R}\right) + \left(\frac{1-\mu_1}{R}\right)$$

$$= \frac{\mu_2 - 1 + 1 - \mu_1}{R} = \frac{\mu_2 - \mu_1}{R}$$

So, $$f_{\text{eq}} = \frac{R}{\mu_2 - \mu_1}.$$

6. For lens A, object distance, $u = -20$ cm

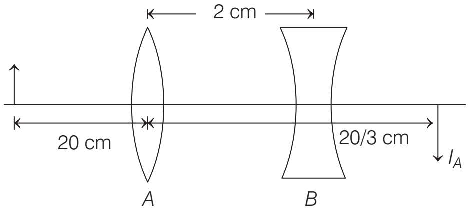
Focal length, $f = +5$ cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

We have, $$\frac{1}{v} = \frac{1}{5} - \frac{1}{20}$$

$$\frac{1}{v} = \frac{20-5}{20 \times 5} = \frac{15}{100}$$

$$v = \frac{20}{3} \text{ cm}$$



For lens B, image of A is object for B.

$$\therefore \qquad u = \frac{20}{3} - 2 = +\frac{14}{3} \text{ cm}$$

$$f = -5 \text{ cm}$$

Now, from lens formula, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{-5}+\frac{3}{14}$$

$$\frac{1}{v}=\frac{15-14}{5\times 14}$$

$$v=70\,\text{cm}$$

Hence, image is on right of lens B and is real in nature.

7. Lens formula is given as

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \qquad \ldots \text{(i)}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$

$$\Rightarrow \quad \frac{uf}{u+f}=v$$

$$\Rightarrow \quad \frac{v}{u}=\frac{f}{u+f} \qquad \ldots \text{(ii)}$$

Now, by differentiating Eq. (i), we get

$$0=-\frac{1}{v^2}\cdot\frac{dv}{dt}+\frac{1}{u^2}\cdot\frac{du}{dt}$$

[$\because$ f (focal length of a lens is constant)]

$$\text{or} \quad \frac{dv}{dt}=\frac{v^2}{u^2}\,du/dt$$

$$\Rightarrow \quad \frac{dv}{dt}=\left(\frac{f}{u+f}\right)^2\cdot\frac{du}{dt} \qquad \text{[using Eq. (ii)]}$$

Given, $f=0.3$ m, $u=-20$ m, $du/dt=5$ m / s

$$\therefore \quad \frac{dv}{dt}=\left(\frac{0.3}{0.3-20}\right)^2\times 5=\left(\frac{3}{197}\right)^2\times 5$$

$$=1.16\times 10^{-3}\ \text{m / s}$$

Thus, the image is moved with a speed of 1.16×10^{-3} m / s towards the lens.

8. Given, $f_1=2f_2$

$$\Rightarrow \quad \frac{1}{|f_1|}=\frac{1}{|2f_2|} \qquad \ldots \text{(i)}$$

Using lens Maker's formula, we get

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)$$

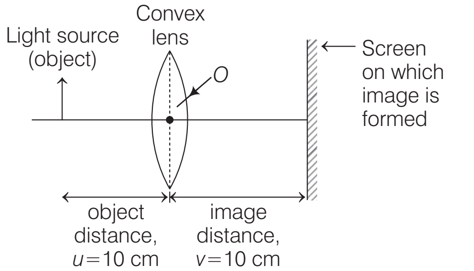
$$\Rightarrow \quad \left|(\mu_1-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)\right|=\left|\frac{1}{2}(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)\right|$$

[$\because$ usingEq. (i)]

$$\Rightarrow \quad \frac{\mu_1-1}{R}=\frac{\mu_2-1}{2R}$$

$$\Rightarrow \quad 2\mu_1-\mu_2=1$$

9. Initially, when a light source (i.e. an object) is placed at 10 cm from the convex mirror and an image is form on the screen as shown in the figure below,



Since, $u=v$ which can only be possible in the situation when the object is place at '$2f$' of the lens.

So, that the image can form at '$2f$' only on the other side of the lens.

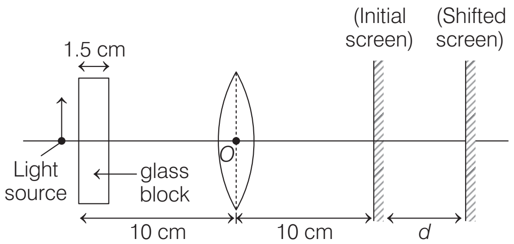
Thus, the distance from the optical centre (O) of the lens and $2f$ is

$$2\times \text{focal length } (f)=\text{object distance}$$

$$\Rightarrow \quad 2\times f=10$$

$$\text{or} \quad f=5\,\text{cm}$$

Now, when a glass block is placed in contact with the light source i.e., object, then the situation is shown in the figure given below



Then due to the block, the position of the object in front of the lens would now be shifted due to refraction of the light source rays through the block.

The shift in the position of the object is given as

$$x=\left(1-\frac{1}{\mu}\right)t$$

where, μ is the refractive index of the block and t is its thickness.

$$\Rightarrow \quad x=\left(1-\frac{1}{1.5}\right)1.5=\left(1-\frac{2}{3}\right)\frac{3}{2}$$

$$=\frac{1}{3}\times\frac{3}{2}=\frac{1}{2}=0.5\,\text{cm}$$

$\therefore$ The new object distance of the light source in front of the lens will be

$$u'=10-0.5=9.5\,\text{cm}$$

Since, the focal length of the lens is 5 cm.

Therefore, the image distance of the light source now can be given as,

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'} \qquad \text{(using lens formula)}$$

Substituting the values, we get

$$\frac{1}{v'} = \frac{1}{5} + \left(\frac{1}{-9.5}\right) = \frac{+9.5 - 5}{47.5} = \frac{4.5}{47.5}$$

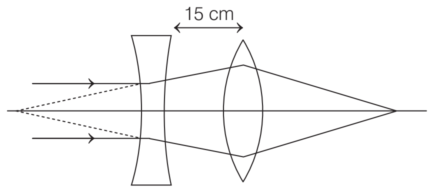
or $v' = 10.55\ \text{cm}$

$\therefore$ The value of $d = v' - v = 10.55 - 10$

$= 0.55\ \text{cm}$, away from the lens

Focal length in the above question can be calculated by using lens formula i.e. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

10.



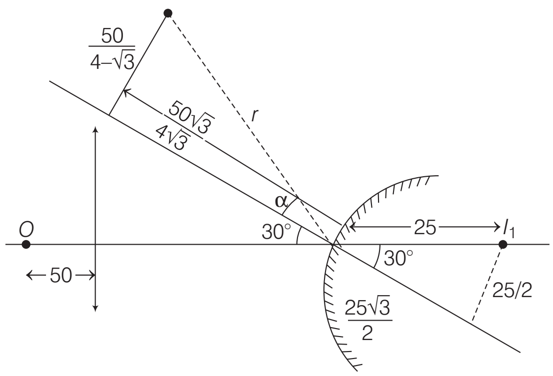
Here, $f_1 = -25\ \text{cm}$, $f_2 = 20\ \text{cm}$

For diverging lens, $v = -25\ \text{cm}$

For converging lens, $u = -(15 + 25) = -40\ \text{cm}$

$$\therefore \quad \frac{1}{v} - \frac{1}{-40} = \frac{1}{+20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40} \Rightarrow v = 40\ \text{cm}$$

11.



For Lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u+f}$$

$$\Rightarrow \quad v = \frac{(-50)(30)}{-50+30} = 75$$

For Mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u-f}$$

$$\Rightarrow \quad v = \frac{\left(\frac{25\sqrt{3}}{2}\right)(50)}{\frac{25\sqrt{3}}{2} - 50} = \frac{-50\sqrt{3}}{4-\sqrt{3}}$$

$$\Rightarrow \quad m = -\frac{v}{u} = \frac{h_2}{h_1} \Rightarrow h_2 = -\left(\frac{\frac{-50\sqrt{3}}{4-\sqrt{3}}}{\frac{25\sqrt{3}}{2}}\right)\cdot\frac{25}{2}$$

$$\Rightarrow \quad h_2 = \frac{+50}{4-\sqrt{3}}$$

The x-coordinate of the images

$= 50 - v\cos 30 + h_2 \cos 60 \approx 25$

The y-coordinate of the images

$$= v\sin 30 + h_2 \sin 60 \approx 25\sqrt{3}$$

12. It is based on lens maker's formula and its magnification.

i.e. $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

According to lens maker's formula, when the lens in the air.

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \frac{1}{2x} \Rightarrow f = 2x$$

Here, $$\left(\frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}\right)$$

In case of liquid, where refractive index is $\frac{4}{3}$ and $\frac{5}{3}$, we get

Focal length in first liquid

$$\frac{1}{f_1} = \left(\frac{\mu_s}{\mu_{l_1}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_1} = \left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)\frac{1}{x}$$

$\Rightarrow f_1$ is positive.

$$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f}$$

$$\Rightarrow \quad f_1 = 4f$$

Focal length in second liquid

$$\frac{1}{f_2} = \left(\frac{\mu_s}{\mu_{l_2}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = \left(\frac{\frac{3}{2}}{\frac{5}{3}} - 1\right)\left(\frac{1}{x}\right)$$

$\Rightarrow f_2$ is negative.

13. $$\mu = \frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}} = \frac{1}{(2/3)} = \frac{3}{2}$$

Further, $$|m| = \frac{1}{3} = \left|\frac{v}{u}\right|$$

$$\therefore \quad |v| = \frac{|u|}{3}$$

$\Rightarrow \quad u = -24\ \text{m} \qquad$ (Real object)

$\therefore \quad v = +8\ \text{m} \qquad$ (Real image)

Now, $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=(\mu-1)\left(\frac{1}{+R}-\frac{1}{\infty}\right)$$

$\therefore$ $$\frac{1}{8}+\frac{1}{24}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R}\right)$$

$\therefore$ $$R = 3 \text{ m}$$

14. Using the lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \text{ or } \frac{1}{v}=\frac{1}{u}+\frac{1}{f_1}+\frac{1}{f_2}$$

$$=\frac{1}{u}+(n_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+(n_2-1)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)$$

Substituting the values, we get

$$\frac{1}{v}=\frac{1}{-40}+(1.5-1)\left(\frac{1}{14}-\frac{1}{\infty}\right)+(1.2-1)\left(\frac{1}{\infty}-\frac{1}{-14}\right)$$

Solving this equations, we get

$$v = +40 \text{ cm}$$

15. From the lens formula,

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \text{ we have,}$$

$$\frac{1}{f}=\frac{1}{10}-\frac{1}{-10}$$

or $$f = +5$$

Further, $\Delta u = 0.1$

and $\Delta v = 0.1$ (from the graph)

Now, differentiating the lens formula, we have

$$\frac{\Delta f}{f^2}=\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}$$

or $$\Delta f=\left(\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}\right)f^2$$

Substituting the values, we have

$$\Delta f=\left(\frac{0.1}{10^2}+\frac{0.1}{10^2}\right)(5)^2 = 0.05$$

$\therefore$ $$f \pm \Delta f = 5 \pm 0.05$$

16. Refraction from lens : $\dfrac{1}{v_1}-\dfrac{1}{-20}=\dfrac{1}{15}$

$\therefore$ $v = 60 \text{ cm}$ $\xrightarrow{\hspace{2em}}$ + ve direction

i.e. first image is formed at 60 cm to the right of lens system.

Reflection from mirror

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens

$$\frac{1}{v_3}-\frac{1}{60}=\frac{1}{15} \quad \longleftarrow + \text{ ve direction}$$

or $$v_3 = 12 \text{ cm}$$

Therefore, the final image is formed at 12 cm to the left of the lens system.

17. Let focal length of convex lens is $+f$, then focal length of concave lens would be $-\dfrac{3}{2}f$.

From the given condition,

$$\frac{1}{30}=\frac{1}{f}-\frac{2}{3f}=\frac{1}{3f}$$

$\therefore$ $$f = 10 \text{ cm}$$

Therefore, focal length of convex lens = + 10 cm

and that of concave lens = – 15 cm

18. Image formed by convex lens at I_1 will act as a virtual object for concave lens. For concave lens

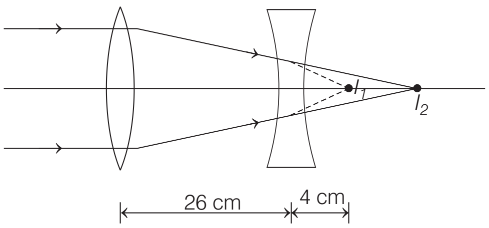
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

or $$\frac{1}{v}-\frac{1}{4}=\frac{1}{-20}$$

or $$v = 5 \text{ cm}$$

Magnification for concave lens

$$m=\frac{v}{u}=\frac{5}{4}=1.25$$



As size of the image at I_1 is 2 cm. Therefore, size of image at I_2 will be $2 \times 1.25 = 2.5$ cm.

19. $$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For no dispersion, $d\left\{\dfrac{1}{f}\right\}=0$

or $$R_1 = R_2$$

20. The lens makers' formula is

$$\frac{1}{f}=\left(\frac{n_L}{n_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

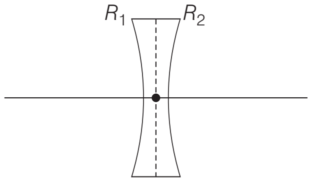
where, n_L = Refractive index of lens and

n_m = Refractive index of medium.

In case of double concave lens, R_1 is negative and R_2 is positive. Therefore, $\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$ will be negative.

For the lens to be diverging in nature, focal length f should be negative or $\left(\dfrac{n_L}{n_m}-1\right)$ should be positive or $n_L > n_m$ but since $n_2 > n_1$ (given), therefore the lens should be filled with L_2 and immersed in L_1.

21. $R_1 = -R,\ R_2 = +R,\ \mu_g = 1.5$ and $\mu_m = 1.75$



$$\therefore \qquad \frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

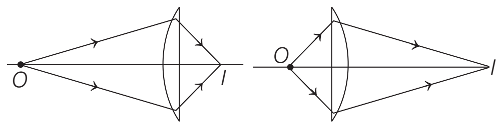
Substituting the values, we have

$$\frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(\frac{1}{-R} - \frac{1}{R}\right) = \frac{1}{3.5R}$$

$$\therefore \qquad f = +3.5R$$

Therefore, in the medium it will behave like a convergent lens of focal length $3.5R$. It can be understood as, $\mu_m > \mu_g$, the lens will change its behaviour.

22. In general spherical aberration is minimum when the total deviation produced by the system is equally divided on all refracting surfaces. A plano-convex lens is used for this purpose. In order that the total deviation be equally divided on two surfaces, it is essential that more parallel beam (or the incident and refracted) be incident on the convex side. Thus, when the object is far away from the lens, incident rays will be more parallel than the refracted rays, therefore, the object should face the convex side,but if the object is near the lens, the object should face the plane side. This has been shown in figure.



23. The focal length of combination is given by

$$\frac{1}{F} = \frac{1}{40} - \frac{1}{25} \qquad \left(\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}\right)$$

$$\text{or} \qquad F = -\frac{200}{3}\text{ cm}$$

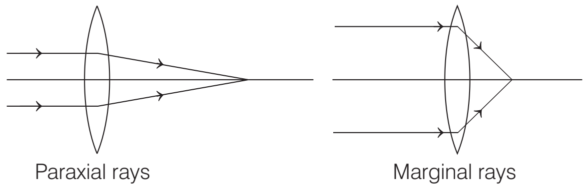
$$= -\frac{2}{3}\text{ m}$$

$\therefore$ Power of the combination in dioptres,

$$P = -\frac{3}{2} \qquad \left[P = \frac{1}{F\,(\text{m})}\right]$$

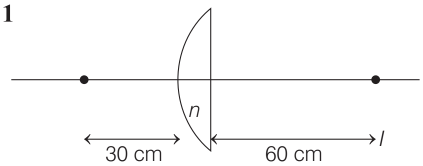
$$= -1.5$$

24. Spherical aberration is caused due to spherical nature of lens. Paraxial and marginal rays are focused at different places on the axis of the lens. Therefore, image so formed is blurred. This aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mark over the lens.



25. $P = P_1 + P_2 = \dfrac{1}{f_1\,(\text{m})} + \dfrac{1}{f_2\,(\text{m})} = \dfrac{1}{0.4} - \dfrac{1}{0.25} = -1.5\text{ D}$

26. **Case 1**



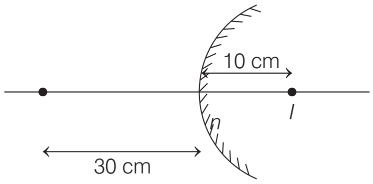
Using lens formula,

$$\frac{1}{60} + \frac{1}{30} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1} = \frac{1}{60} + \frac{2}{60}$$

$$\Rightarrow \qquad f_1 = 20\text{ cm}$$

Further, $\dfrac{1}{f_1} = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right) \Rightarrow f_1 = \dfrac{R}{n-1} = +20\text{ cm}$

Case 2



Using mirror formula

$$\frac{1}{10} - \frac{1}{30} = \frac{1}{f_2}$$

$$\frac{3}{30} - \frac{1}{30} = \frac{1}{f_2} = \frac{2}{30}$$

$$f_2 = 15 = \frac{R}{2} \Rightarrow R = 30$$

$$R = 30\text{ cm}$$

$$\frac{R}{n-1} + 20\text{ cm} = \frac{30}{n-1}$$

$$\Rightarrow \qquad = 2n - 2 = 3 \Rightarrow f_1 = +20\text{ cm}$$

Refractive index of lens is 2.5.

Radius of curvature of convex surface is 30 cm.

Faint image is erect and virtual focal length of lens is 20 cm.

27. (P)  $\dfrac{1}{f} = \left(\dfrac{3}{2} - 1\right)\left(\dfrac{1}{r} + \dfrac{1}{r}\right) = \dfrac{1}{r} \Rightarrow f = r$

 $\Rightarrow \dfrac{1}{f_{\text{eq}}} = \dfrac{1}{f} + \dfrac{1}{f} = \dfrac{2}{r} \Rightarrow f_{\text{eq}} = \dfrac{r}{2}$

(Q)  $\dfrac{1}{f} = \left(\dfrac{3}{2} - 1\right)\left(\dfrac{1}{r}\right) \Rightarrow f = 2r$

 $\Rightarrow \dfrac{1}{\phi} + \dfrac{1}{\phi} = \dfrac{2}{\phi} = \dfrac{1}{\rho} \Rightarrow f_{\text{eq}} = r$

(R) $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(-\frac{1}{r}\right) = -\frac{1}{2r} \Rightarrow f = -2r$

$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = -\frac{2}{2r} \Rightarrow f_{eq} = -r$

(S) $\Rightarrow \frac{1}{f_{eq}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \Rightarrow f_{eq} = 2r$

28. (A) $\rightarrow$ since $\mu_1 < \mu_2$, the ray of light will bend towards normal after first refraction.

(B) $\rightarrow$ $\mu_1 > \mu_2$, the ray of light will bend away from the normal after first refraction.

(C) $\rightarrow$ since $\mu_2 = \mu_3$ means in second refraction there will be no change in the path of ray of light.

(D) $\rightarrow$ Since $\mu_2 > \mu_3$, ray of light will bend away from the normal after second refraction.

Therefore the correct options are as under.

(A) $\rightarrow$ p, r (B) $\rightarrow$ q, s, t (C) $\rightarrow$ p, r, t (D) $\rightarrow$ q, s

29. When the lenses are in contact, the power of the system is

$$P = P_1 + P_2 \text{ or } P_1 + P_2 = 10 \quad \ldots\text{(i)}$$

When lenses are separated by a distance $d = 0.25 \text{ m} = \frac{1}{4}\text{m}$

The power is $P = P_1 + P_2 - d\,P_1P_2$

or $$P_1 + P_2 - \frac{P_1P_2}{4} = 6 \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we can find that $P_1 = 8$ D and $P_2 = 2$ D

$\therefore$ $$f_1 = \frac{1}{8}\text{m} = 0.125\text{m}$$

$\Rightarrow$ $$f_2 = \frac{1}{2}\text{m} = 0.5\text{m}$$

30. $$\frac{1}{f_{air}} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots\text{(i)}$$

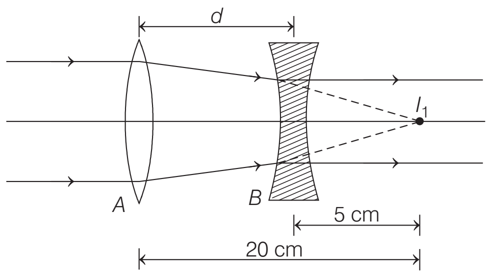
$$\frac{1}{f_{medium}} = \left(\frac{1.5}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

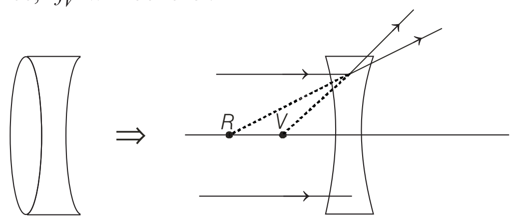
$$\frac{f_{medium}}{f_{air}} = 4$$

$\therefore$ $$f_{medium} = 4 f_{air} = 4 \times 15 = 60\,\text{cm}$$

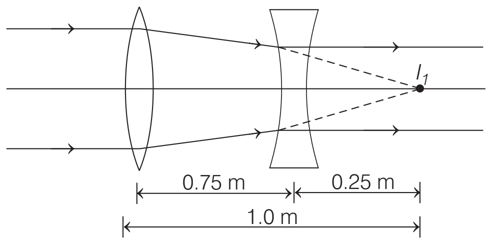
31. At I_1, second focus of convex lens should coincide the first focus of concave lens.



32. Focal length of concave is less, i.e. power of concave lens will be more. Hence, the combination will behave like a concave lens. Further, μ_V is greater than all other colours. Hence, f_V will be least.



33. At I_1, second focus of convex lens coincides with first focus of concave lens. Hence, rays will become parallel to the optic axis after refraction from both the lenses.



34. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{u}{v} - 1 = \frac{u}{f}$ or $\frac{u}{v} = \left(\frac{u+f}{f}\right)$

$\therefore$ $m = \frac{v}{u} = \left(\frac{f}{u+f}\right) \Rightarrow \frac{m_{25}}{m_{50}} = \frac{\left(\frac{20}{-25+20}\right)}{\left(\frac{20}{-50+20}\right)} = 6$

$\therefore$ Answer is 6.

35. Differentiating the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with respect to time, we get

$$-\frac{1}{v^2}.\frac{dv}{dt} + \frac{1}{u^2}.\frac{du}{dt} = 0 \quad \text{(as } f = \text{constant)}$$

$\therefore$ $$\left(\frac{dv}{dt}\right) = \left(\frac{v^2}{u^2}\right).\frac{du}{dt} \quad \ldots\text{(i)}$$

Further, substituting proper values in lens formula, we have

$$\frac{1}{v} + \frac{1}{0.4} = \frac{1}{0.3} \quad (u = -0.4 \text{ m}, f = 0.3 \text{ m})$$

or $$v = 1.2\,\text{m}$$

Putting the values in Eq. (i), we get

Magnitude of rate of change of position of image $= 0.09$ m/s

Lateral magnification, $m = \frac{v}{u}$

$\therefore$ $$\frac{dm}{dt} = \frac{u.\frac{dv}{dt} - v.\frac{du}{dt}}{u^2} = \frac{(-0.4)(0.09) - (1.2)(0.01)}{(0.4)^2}$$

$$= -0.3\,/\text{s}$$

$\therefore$ Magnitude of rate of change of lateral magnification

$$= 0.3\,/\text{s}$$

36. For both the halves, position of object and image is same. Only difference is of magnification. Magnification for one of the halves is given as 2(> 1). This can be for the first one, because for this, $|v| > |u|$. Therefore, magnification, $|m| = |v/u| > 1$. So, for the first half

$$|v/u| = 2 \quad \text{or} \quad |v| = 2|u|$$

Let $u = -x$ then $v = +2x$ and $|u| + |v| = 1.8\,\text{m}$

i.e. $3x = 1.8$ m or $x = 0.6$ m

Hence, $u = -0.6$ m and $v = +1.2$ m.

Using, $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$$

$\therefore$ $$f = 0.4 \text{ m}$$

For the second half

$$\frac{1}{f} = \frac{1}{1.2 - d} - \frac{1}{-(0.6 + d)}$$

or $$\frac{1}{0.4} = \frac{1}{1.2 - d} + \frac{1}{0.6 + d}$$

Solving this, we get $d = 0.6$ m.

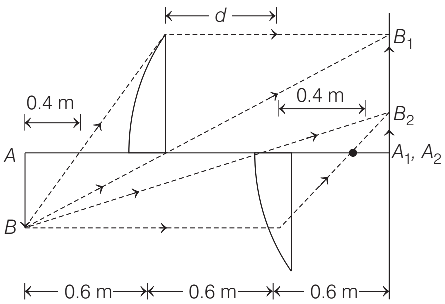
Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

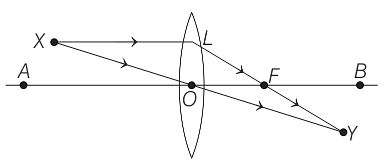
and magnification for the first half is

$$m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$$

The ray diagram is as follows :



37. Steps I In case of a lens

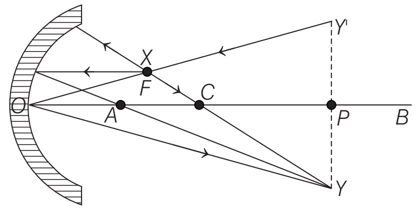


(a) Join X and Y. The point O, where the line XY cuts the optic axis AB, is the optical centre of the lens.

(b) Draw a line parallel to AB from point X. Let it cuts the lens at L. Join L and Y. The point F where the line LY cuts the optic axis AB is the focus of the lens F.

NOTE As the image is inverted, lens should be a convex because a concave lens always forms a virtual and erect image.

Step II In case of a concave mirror



(a) Draw a line YY' perpendicular to AB from point Y. Let it cuts the line AB at point P. Locate a point Y' such that $PY = PY'$.

(b) Extend the line XY'. Let it cuts the line AB at point O. Then O is the pole of the mirror.

(c) Join X and Y. The point C, where the line XY cuts the optic axis AB, is the centre of curvature of the mirror.

(d) The centre point F of OC is the focus of the mirror.

38. (a) For the lens, $u = -0.15$ m ; $f = +0.10$ m

Therefore, using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{(-0.15)} + \frac{1}{(0.10)} \quad \text{or} \quad v = 0.3\,\text{m}$$

Linear magnification, $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images S_1 and S_2 of S will be formed at 0.3 m from the lens as shown in figure. Image S_1 due to part 1 will be formed at 0.5 mm above its optic axis ($m = -2$). Similarly, S_2 due to part 2 is formed 0.5 mm below the optic axis of this part as shown.

Hence, d = distance between S_1 and S_2 = 1.5 mm

$$D = 1.30 - 0.30 = 1.0 \text{ m} = 10^3 \text{ mm}$$

$$\lambda = 500 \text{ nm} = 5 \times 10^{-4} \text{ mm}$$

Therefore, fringe width,

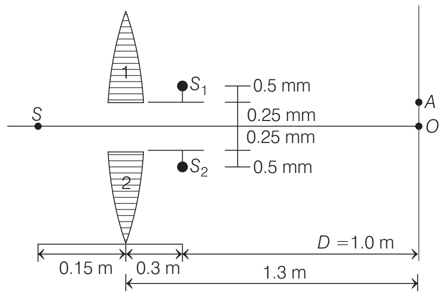
$$\omega = \frac{\lambda D}{d} = \frac{(5 \times 10^{-4})(10^3)}{(1.5)} = \frac{1}{3} \text{ mm}$$

Now, as the point A is at the third maxima

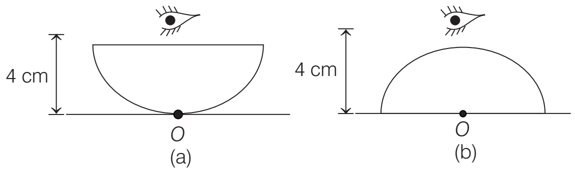
$OA = 3\omega = 3(1/3)$ mm or $OA = 1$ mm

NOTE The language of the question is slightly confusing. The third intensity maximum may be understood as second order maximum (zero order, first order and the second order). In that case $OA = 2\omega = 2(1/3)$ mm $= 0.67$ mm.

(b) If the gap between L_1 and L_2 is reduced, d will decrease. Hence, the fringe width ω will increase or the distance OA will increase.



39. **Refer figure (a)**



In this case refraction of the rays starting from O takes place from a plane surface. So, we can use

$$d_{\text{app}} = \frac{d_{\text{actual}}}{\mu} \quad \text{or} \quad 3 = \frac{4}{\mu} \quad \text{or} \quad \mu = \frac{4}{3}$$

Refer figure (b) In this case refraction takes place from a spherical surface. Hence, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

we have, $$\frac{1}{(-25/8)} - \frac{4/3}{-4} = \frac{1-4/3}{-R}$$

or $$\frac{1}{3R} = \frac{1}{3} - \frac{8}{25} = \frac{1}{75}$$

$\therefore$ $$R = 25\,\text{cm}$$

Now, to find the focal length we will use the lens Maker's formula

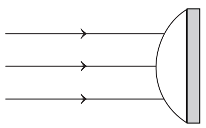
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-25}\right) = \frac{1}{75}$$

$\therefore$ $f = 75\,\text{cm}$

40. (a) $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{12} - \frac{1}{\infty}\right) = \frac{1}{24}$$

$\therefore$ $f = +24\,\text{cm}$

(b) The system will behave like a mirror of power given by



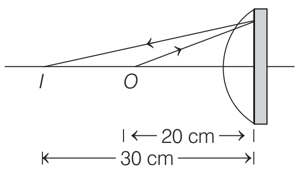
$$P = 2P_L + P_M$$

$$\therefore \quad -\frac{1}{F(\text{m})} = 2\left(\frac{1}{0.24}\right) + 0$$

$$\therefore \quad F = -0.12\,\text{m} = -12\,\text{cm}$$

Hence, the system will behave like a concave mirror of focal length 12 cm. Therefore parallel rays will converge at a distance of 12 cm (to the left) of the system.

(c)



(d) Using mirror formula

$$\frac{1}{v} - \frac{1}{20} = \frac{-1}{12}$$

Solving, we get $v = -30\,\text{cm}$.

Therefore the image will be formed at a distance of 30 cm to the left of system.

41. From first figure it is clear that

$$\mu = \mu_1$$

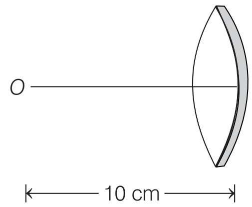
From second figure it is clear that

$$\mu < \mu_2$$

$$\therefore \quad \mu_1 < \mu_2$$

42. The given system behaves like a mirror of power given by

$$P = 2P_L + P_M$$



or $$-\frac{1}{F} = 2\left(\frac{1}{0.2}\right) + \left(\frac{-2}{0.22}\right)$$

As $$P_L = \frac{1}{f(\text{m})}$$

and $$P_M = \frac{-1}{f(\text{m})} = \frac{-2}{R(\text{m})}$$

Solving this equation, we get

$$F = -1.1\,\text{m} = -110\,\text{cm}$$

i.e. the system behaves as a concave mirror of focal length 18.33 cm.

Using the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ we have}$$

$$\frac{1}{v} - \frac{1}{10} = -\frac{1}{110}$$

or $$v = 11\,\text{cm}$$

i.e. virtual image will be formed at a distance of 11 cm.

Topic 4 Prism

1. Given, refractive index of material of prism $n = \sqrt{3}$, prism angle $A = 60°$

Method 1

Using prism formula,

$$n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \sqrt{3} = \frac{\sin\left(\frac{60+\delta}{2}\right)}{\sin 30°}$$

$$\Rightarrow \quad \sin\left(\frac{60+\delta}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin\left(\frac{60+\delta}{2}\right) = \sin 60°$$

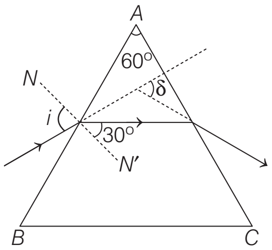
or $$\frac{60+\delta}{2} = 60$$

or angle of minimum deviation $\delta = 60°$

Incident angle, $\quad i = \dfrac{60+\delta}{2} = 60°$

Method 2 For minimum deviation, ray should pass symmetrically (i.e. parallel to the base of the equilateral prism)

$\Rightarrow$ From geometry of given figure, we have, $r = 30°$



Using Snell's law,

$$n = \frac{\sin i}{\sin r}$$

$$\sin i = n \sin r = \sqrt{3} \sin 30°$$

$$\Rightarrow \qquad \sin i = \frac{\sqrt{3}}{2} \text{ or } i = 60°$$

2. For a crown glass thin prism i.e., prism with small angle, the angle of minimum deviation is given as,

$$D_m = (n-1)A$$

where, A is prism angle and n is refractive index.

$\Rightarrow D_m \propto n$ … (i)

Since from the given graph, the value of 'n' decrease with the increase in 'λ'. Thus, from relation (i), we can say that, D_m will also decrease with the increase in 'λ'.

$\therefore$ Hence, option (c) is correct.

3.
$$\delta = (i_1 + i_2) - A$$

$$\Rightarrow \qquad 40° = (35° + 79°) - A$$

$$\Rightarrow \qquad A = 74°$$

Now, we know that

$$\mu = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

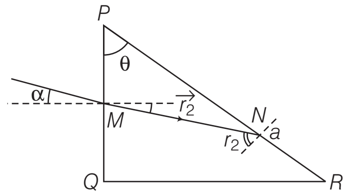
It we take the given deviation as the minimum deviation then,

$$\mu = \frac{\sin\left(\dfrac{74°+40°}{2}\right)}{\sin\left(\dfrac{74°}{2}\right)} = 1.51$$

The given deviation may or may not be the minimum deviation. Rather it will be less than this value. Therefore, μ will be less than 1.51.

Hence, maximum possible value of refractive index is 1.51.

4.



Applying Snell's law at M,

$$n = \frac{\sin \alpha}{\sin r_1} \Rightarrow \sqrt{2} = \frac{\sin 45°}{\sin r_1}$$

$$\Rightarrow \qquad \sin r_1 = \frac{\sin 45°}{\sqrt{2}} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$$

$$r_1 = 30°$$

$$\sin \theta_c = \frac{1}{n} = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45°$$

Let us take $r_2 = \theta_c = 45°$ for just satisfying the condition of TIR.

In ΔPNM,

$$\theta + 90 + r_1 + 90 - r_2 = 180°$$

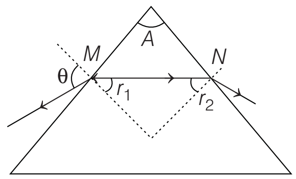
$$\text{or} \qquad \theta = r_2 - r_1 = 45° - 30° = 15°$$

NOTE If $\alpha > 45°$ (the given value). Then, $r_1 > 30°$ (the obtained value)

$\therefore \qquad r_2 - r_1 = \theta$ or $r_2 = \theta + r_1$

or TIR will take place. So, for taking TIR under all conditions α should be greater than 45° or this is the minimum value of α.

5.



Applying Snell's law at M,

$$\mu = \frac{\sin \theta}{\sin r_1}$$

$$\therefore \qquad r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) \text{ or } \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\text{Now,} \qquad r_2 = A - r_1 = A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

Ray of light would get transmitted form face AC if

$$r_2 < \theta_c \quad \text{or} \quad A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \theta_c$$

$$\text{where,} \qquad \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

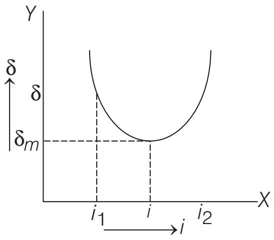
$$\therefore \qquad \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) > A - \theta_c$$

or $$\frac{\sin\theta}{\mu} > \sin(A-\theta_c)$$

$\therefore$ $$\theta > \sin^{-1}[\mu\sin(A-\theta_c)]$$

or $$\theta > \sin^{-1}\left[\mu\sin\left\{A-\sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

6. We know that the angle of deviation depends upon the angle of incidence. If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot i (on-X-axis) and δ (on-Y-axis), we get a curve as shown in figure.



Clearly if angle of incidence is gradually increased, from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase.

7. For $e \to i$

$\Rightarrow$ $$45° > \theta_c$$

$\Rightarrow$ $$\sin 45° > \sin\theta_c$$

$\Rightarrow$ $$\frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1}$$

$\Rightarrow$ $$\mu_1 > \sqrt{2}\mu_2$$

For $e \to f$

angle of refraction is lesser than angle of incidence, so $\mu_2 > \mu_1$ and then $\mu_2 > \mu_3$

For $e \to g$, $\mu_1 = \mu_2$

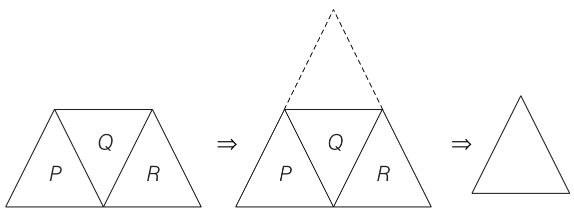
for $e \to h, \mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

8. At minimum deviation $(\delta = \delta_m)$:

$$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30° \quad \text{(For both colours)}$$

9. During minimum deviation the ray inside the prism is parallel to the base of the prism in case of an equilateral prism.

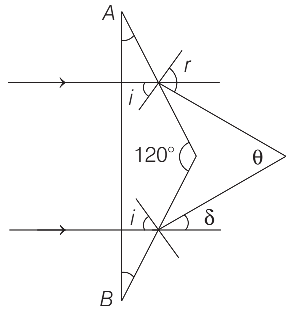
10. Figure (a) is part of an equilateral prism of figure (b) as shown in figure which is a magnified image of figure (c). Therefore, the ray will suffer the same deviation in figure (a) and figure (c).



(a) (b) (c)

NOTE Questions are often asked based on part of a prism. For example, section shown in figure (a) is part of a prism shown in figure (c).

11. The diagramatic representation of the given problem is shown in figure.



From figure it follows that $\angle i = \angle A = 30°$

From Snell's law, $n_1 \sin i = n_2 \sin r$

or $$\sin r = \frac{1.44\sin 30°}{1} = 0.72$$

Now, $$\angle\delta = \angle r - \angle i = \sin^{-1}(0.72) - 30°$$

$\therefore$ $$\theta = 2\,(\angle\delta) = 2\,\{\sin^{-1}(0.72) - 30°\}$$

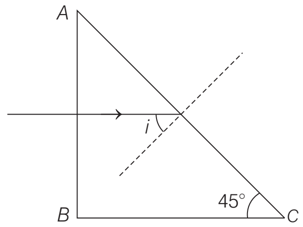
12. Deviation $\delta = (\mu - 1)A$

Given, $\delta_{\text{net}} = 0$

$\therefore$ $$(\mu_1 - 1)A_1 = (\mu_2 - 1)\,A_2$$

or $$A_2 = \frac{(\mu_1-1)}{(\mu_2-1)}A_1 = \frac{(1.54-1)}{(1.72-1)}(4°) = 3°$$

13. The colours for which $i > \theta_c$, will get total internal reflection : $i > \theta_c$ or $\sin i > \sin\theta_c$



$i = 45°$

or $\sin 45° > \dfrac{1}{\mu}$ or $\dfrac{1}{\sqrt{2}} > \dfrac{1}{\mu}$

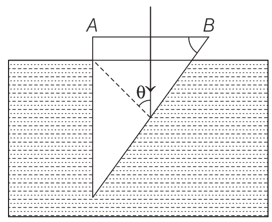
or for which $\mu > \sqrt{2}$ or $\mu > 1.414$

Hence, the rays for which $\mu > 1.414$ will get TIR.

For green and blue $\mu > 1.414$, so they will suffer TIR on face AC. Only red comes out from this face.

14. Let θ_c be the critical angle at face BC, then

$$\sin\theta_c = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$



Angle of incidence at face BC is $i = \theta$

Total internal reflection (TIR) will take place on this surface if,

$$i > \theta_c \text{ or } \theta > \theta_c \text{ or } \sin\theta > \sin\theta_c \text{ or } \sin\theta > \frac{8}{9}$$

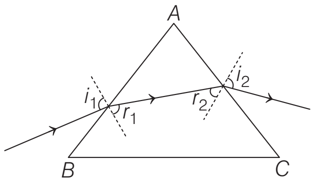
15 The minimum deviation produced by a prism

$$\delta_m = 2i - A = A$$

$\therefore$ $i_1 = i_2 = A$ and

$r_1 = r_2 = A/2$

$\therefore$ $r_1 = i_1/2$



Now, using Snell's law

$$\sin A = \mu \sin A/2$$

$$\Rightarrow \quad \mu = 2\cos(A/2)$$

For this prism when the emergent ray at the second surface is tangential to the surface

$$i_2 = \pi/2 \Rightarrow r_2 = \theta_c \Rightarrow r_1 = A - \theta_c$$

so, $\sin i_1 = \mu \sin(A - \theta_c)$

so, $$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$$

For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism if $\angle B = \angle C$.

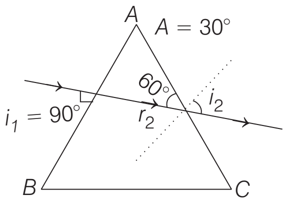
But it is not necessarily parallel to the base if,

$$\angle A = \angle B \text{ or } \angle A = \angle C$$

16. Ray falls normally on the face AB. Therefore, it will pass undeviated through AB.

$\therefore$ $r_2 = 90° - 60° = 30°$

$$\mu = \sqrt{2} = \frac{\sin i_2}{\sin r_2}$$



$i_2 = 45°$

Deviation $= i_2 - r_2 = 45° - 30° = 15°$

(Deviation at face AC only)

17. Let δ_m be the angle of minimum deviation. Then

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin(A/2)} \quad (A = 60° \text{ for an equilateral prism})$$

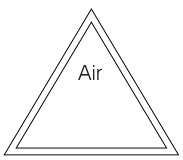
$$\therefore \quad \sqrt{2} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

Solving this we get $\delta_m = 30°$

The given deviation is also 30° (i.e. δ_m)

Under minimum deviation, the ray inside the prism is parallel to base for an equilateral prism.

18. Through a thin glass slab ray of light almost passes undeviated. A hollow prism can be assumed to be made up of three thin glass slabs as shown in figure.



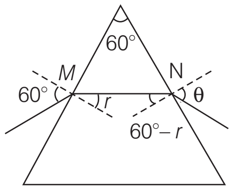
19. Applying Snell's law at M and N,

$$\sin 60° = n \sin r \qquad \text{...(i)}$$

$$\sin\theta = n\sin(60 - r) \qquad \text{...(ii)}$$

Differentiating we get

$$\cos\theta\frac{d\theta}{dn} = -n\cos(60 - r)\frac{dr}{dn} + \sin(60 - r)$$



Differentiating Eq. (i),

$$n\cos r\frac{dr}{dn} + \sin r = 0$$

or $$\frac{dr}{dn} = -\frac{\sin r}{n\cos r} = \frac{-\tan r}{n}$$

$$\Rightarrow \quad \cos\theta\frac{d\theta}{dn} = -n\cos(60 - r)\left(\frac{-\tan r}{n}\right) + \sin(60 - r)$$

$$\frac{d\theta}{dn} = \frac{1}{\cos\theta}[\cos(60 - r)\tan r + \sin(60 - r)]$$

Form Eq. (i), $\quad r = 30°$ for $n = \sqrt{3}$

$$\Rightarrow \frac{d\theta}{dn} = \frac{1}{\cos 60}(\cos 30 \times \tan 30 + \sin 30) = 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

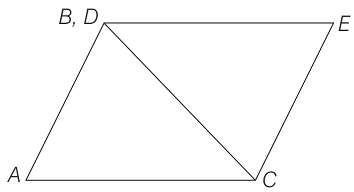
20. (a) At minimum deviation, $r_1 = r_2 = 30°$

$\therefore$ From Snell's law

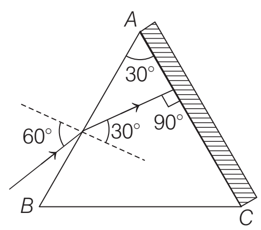
$$\mu = \frac{\sin i_1}{\sin r_1} \quad \text{or} \quad \sqrt{3} = \frac{\sin i_1}{\sin 30°}$$

$$\therefore \quad \sin i_1 = \frac{\sqrt{3}}{2} \quad \text{or} \quad i_1 = 60°$$

(b) In the position shown net deviation suffered by the ray of light should be minimum. Therefore, the second prism should be rotated by 60° (anti-clockwise).



21. (a) $\sin i_1 = \mu \sin r_1$



or $\quad \sin 60° = \sqrt{3} \sin r_1$

$\therefore \quad \sin r_1 = \frac{1}{2} \quad$ or $\quad r_1 = 30°$

Now, $\quad r_1 + r_2 = A$

$\therefore \quad r_2 = A - r_1 = 30° - 30° = 0°$

Therefore, ray of light falls normally on the face AC and angle of emergence $i_2 = 0°$.

(b) Multiple reflections occur between surface of film. Intensity will be maximum if constructive interference takes place in the transmitted wave.

For maximum thickness

$$\Delta x = 2\mu t = \lambda \qquad (t = \text{thickness})$$

$$\therefore \quad t = \frac{\lambda}{2\mu} = \frac{6600}{2 \times 2.2} = 1500 \text{ Å}$$

22. $n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2}$ and $n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$

Here, λ is in nm.

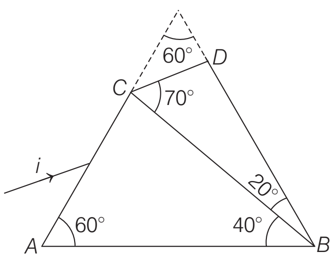
(a) The incident ray will not deviate at BC if $n_1 = n_2$

$$\Rightarrow 1.20 + \frac{10.8 \times 10^4}{\lambda_0^2} = 1.45 + \frac{1.80 \times 10^4}{\lambda_0^2}$$

$$\Rightarrow \frac{9 \times 10^4}{\lambda_0^2} = 0.25 \quad \text{or} \quad \lambda_0 = \frac{3 \times 10^2}{0.5}$$

or $\quad \lambda_0 = 600 \text{ nm}$

(b) The given system is a part of an equilateral prism of prism angle 60° as shown in figure.



At minimum deviation,

$$r_1 = r_2 = \frac{60°}{2} = 30° = r \text{ (say)}$$

$$\therefore \quad n_1 = \frac{\sin i}{\sin r}$$

$$\therefore \quad \sin i = n_1 . \sin 30°$$

$$\sin i = \left\{1.20 + \frac{10.8 \times 10^4}{(600)^2}\right\}\left(\frac{1}{2}\right) = \frac{1.5}{2} = \frac{3}{4}$$

or $\quad i = \sin^{-1}(3/4)$

23. (a) Critical angle θ_c at face AC will be given by

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n}\right)$$

or $\quad \sin\theta_c = \frac{n_1}{n}$

Now, it is given that $r_2 = \theta_c$

$\therefore \quad r_1 = A - r_2 = (45° - \theta_c)$

Applying Snell's law at face AB, we have

$$n = \frac{\sin i_1}{\sin r_1} \quad \text{or} \quad \sin i_1 = n \sin r_1$$

$\therefore \quad i_1 = \sin^{-1}(n \sin r_1)$

Substituting value of r_1, we get

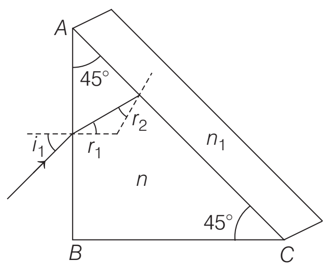
$$i_1 = \sin^{-1}\{n \sin (45° - \theta_c)\}$$

$$= \sin^{-1}\{n(\sin 45° \cos\theta_c - \cos 45° \sin\theta_c)\}$$

$$= \sin^{-1}\left\{\frac{n}{\sqrt{2}}(\sqrt{1 - \sin^2\theta_c} - \sin\theta_c)\right\}$$

$$= \sin^{-1}\left\{\frac{n}{\sqrt{2}}\left(\sqrt{1 - \frac{n_1^2}{n^2}} - \frac{n_1}{n}\right)\right\}$$

$$i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}(\sqrt{n^2 - n_1^2} - n_1)\right\}$$



Therefore, required angle of incidence (i_1) at face AB for which the ray strikes at AC at critical angle is

$$i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}(\sqrt{n^2 - n_1^2} - n_1)\right\}$$

(b) The ray will pass undeviated through face AC when either $n_1 = n$ or $r_2 = 0°$ i.e. ray falls normally on face AC.

Here $n_1 \neq n$ (because $n_1 < n$ is given)

$\therefore \quad r_2 = 0°$

or $\quad r_1 = A - r_2 = 45° - 0° = 45°$

Now applying Snell's law at face AB, we have $n = \frac{\sin i_1}{\sin r_1}$

or $\quad 1.352 = \frac{\sin i_1}{\sin 45°}$

$\therefore \quad \sin i_1 = (1.352)\left(\frac{1}{\sqrt{2}}\right)$,

$\sin i_1 = 0.956$

$\therefore \quad i_1 = \sin^{-1}(0.956) \approx 73°$

Therefore, required angle of incidence is

$$i = 73°$$

24. Given $i_1 = 60°$, $A = 30°$, $\delta = 30°$

From the relation

$$\delta = (i_1 + i_2) - A$$

we have, $\quad i_2 = 0°$

i.e. the ray is perpendicular to the face from which it emerges.

Further, $i_2 = 0°$

$\therefore \quad r_2 = 0°$

$$r_1 + r_2 = A$$

$$r_1 = A = 30°$$

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

Topic 5 Optical Instruments

1. **Key Idea** Numerical Aperture (NA) of the microscope is given by $NA = \frac{0.61\lambda}{d}$

Here, d = minimum separation between two points to be seen as distinct and λ = wavelength of light.

Given, $\lambda = 5000 Å = 5000 \times 10^{-10}$ m and NA = 1.25

Now, $$d = \frac{0.61\lambda}{NA} = \frac{0.61 \times 5000 \times 10^{-10}}{1.25}$$

or $$d = \frac{3.05}{1.25} \times 10^{-7} \text{ m}$$

$$= 2.4 \times 10^{-7} \text{ m}$$

or $$d = 0.24 \mu\text{m}$$

2 Limit of resolution for a telescope from Rayleigh's criteria is

$$\theta_R = \frac{1.22\lambda}{D}$$

Here, $D = 250 \text{ cm} = 250 \times 10^{-2}$ m

and $\lambda = 600 \text{ nm} = 600 \times 10^{-9}$ m

So, limit of resolution is

$$\theta_R = \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}}$$

$$= 2.93 \times 10^{-7} \text{ rad}$$

$$\approx 3 \times 10^{-7} \text{ rad}$$

3 For a telescope, limit of resolution is

given by $\Delta\theta = \frac{1.22\lambda}{D}$

Here, $\lambda = 500 \text{ nm} = 500 \times 10^{-9}$ m,

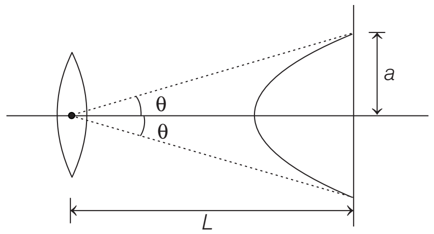
$D = 200 \text{ cm} = 200 \times 10^{-2}$ m

So, limit of resolution is $\Delta\theta = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}}$

$$= 305 \times 10^{-9} \text{ rad}$$

4. Telescope resolves and brings objects closer. Hence, telescope with magnifying power of 20, the tree appears 20 times nearer.

5.

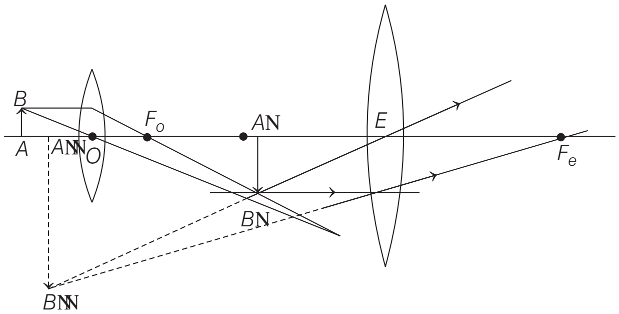


$$a \sin\theta \approx \lambda$$

$$a\left(\frac{a}{L}\right) \approx \lambda \Rightarrow a = \sqrt{\lambda L}$$

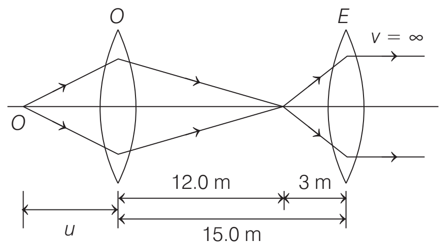
$$\text{Spread} = 2a = \sqrt{4\lambda L}$$

6. The ray diagram is as follows



From the figure it is clear that image formed by objective (or the intermediate image) is real, inverted and magnified.

7. Since, the final image is formed at infinity, the image formed by the objective will be at the focal point of the eyepiece, which is 3.0 cm. The image formed by the objective will be at a distance of 12.0 cm (= 15.0 cm – 3.0 cm) from the objective.



If u is the distance of the object from the objective, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{u} + \frac{1}{12.0} = \frac{1}{2.0}$$

$$\Rightarrow \quad u = \frac{(12.0)(2.0)}{12.0 - 2.0}$$

$$= \frac{24.0}{10.0} = 2.4 \text{cm}$$

8. Image formed by objective (I_1) is at second focus of it because objective is focussed for distant objects. Therefore,

$$P_1 I_1 = f_o$$

Further I_1 should lie at first focus of eyepiece because final image is formed at infinity.

$\therefore \quad P_2 I_1 = f_e$

Given $\quad P_1 P_2 = 36\,\text{cm}$

$\therefore \quad f_o + f_e = 36 \quad \ldots\text{(i)}$

Further angular magnification is given as 5. Therefore,

$$\frac{f_o}{f_e} = 5 \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$f_o = 30\,\text{cm} \text{ and } f_e = 6\,\text{cm}$$

9. Distance between objective and eyepiece

$$L = f_o + f_e = (16 + 0.02)\,\text{m} = 16.02\,\text{m}$$

Angular magnification

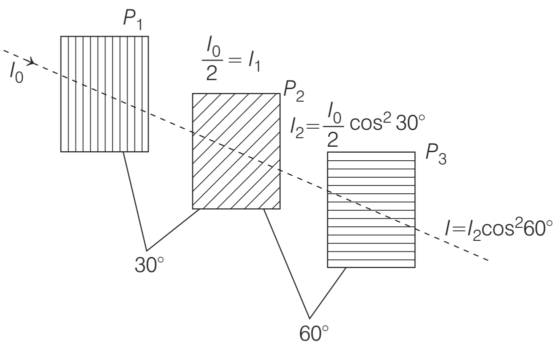
$$M = -f_o/f_e = -16/0.02 = -800$$

Image is inverted and objective is larger than the eyepiece.

10. The resolving power of a microscope is inversely proportional to the wavelength of the wave used. de-Broglie matter wave is used in case of an electron microscope whose wavelength is less than the wavelength of visible light used in optical microscope.

Topic 6 Wave Optics

1.



When unpolarised light pass through polaroid P_1, intensity obtained is

$$I_1 = \frac{I_0}{2}$$

where, I_0 = intensity of incident light.

Now, this transmitted light is polarised and it pass through polariser P_2. So, intensity I_2 transmitted is obtained by Malus law.

$$\Rightarrow \quad I_2 = I_1 \cos^2\theta$$

As angle of pass axis of P_1 and P_3 is 90° and angle of pass axis of P_2 and P_3 is 60°, so angle between pass axis of P_1 and P_2 is (90°− 60°) = 30°. So,

$$I_2 = \frac{I_0}{2}\cos^2 30° = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

When this light pass through third polariser P_3, intensity I transmitted is again obtained by Malus law.

So, $\quad I = I_2 \cos^2 60° = \left(\frac{3}{8} I_0\right)\cos^2 60°$

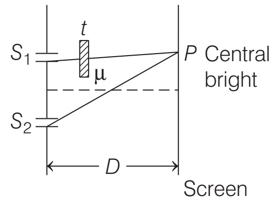
$$= \frac{3}{8} I_0 \times \left(\frac{1}{2}\right)^2 = \frac{3}{32} I_0$$

So, ratio $\frac{I_0}{I} = \frac{32}{3} = 10.67$

2 As we know,

Path difference introduced by thin film,

$$\Delta = (\mu - 1)t \quad \ldots\text{(i)}$$



and if fringe pattern shifts by one frings width, then path difference,

$$\Delta = 1 \times \lambda = \lambda \quad \ldots\text{(ii)}$$

So, from Eqs. (i) and (ii), we get

$$(\mu - 1)t = \lambda \Rightarrow t = \frac{\lambda}{\mu - 1}$$

Alternate Solution

Path difference introduced by the thin film of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)t$$

$\therefore$ Position of the fringe is

$$x = \frac{\Delta D}{d} = \frac{(\mu - 1)tD}{d} \quad \ldots\text{(i)}$$

Fringe width of one fringe is given by

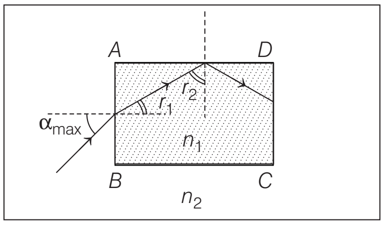
$$\beta = \frac{\lambda D}{d} \quad \ldots\text{(ii)}$$

Given that $x = \beta$, so from Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{(\mu - 1)\,tD}{d} = \frac{\lambda D}{d}$$

$$\Rightarrow \quad (\mu - 1)t = \lambda \text{ or } t = \frac{\lambda}{(\mu - 1)}$$

3.



Key Idea Amplitude of light is directly proportional to area through which light is passing.

For same length of slits,

$$\text{amplitude} \propto (\text{width})^{1/2}$$

Also, $\quad \text{intensity} \propto (\text{amplitude})^2$

In YDSE, ratio of intensities of maxima and minima is given by

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

where, I_1 and I_2 are the intensities obtained from two slits, respectively.

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

where, a_1 and a_2 are light amplitudes from slits 1 and 2, respectively.

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{W_1} + \sqrt{W_2})^2}{(\sqrt{W_1} - \sqrt{W_2})^2}$$

where, W_1 and W_2 are the widths of slits, respectively.

Here, $\left(\frac{W_1}{W_2}\right) = \left(\frac{a_1}{a_2}\right)^2 = \frac{4}{1} \Rightarrow \sqrt{\frac{W_1}{W_2}} = \frac{2}{1}$

So, we have

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{W_1} + \sqrt{W_2}}{\sqrt{W_1} - \sqrt{W_2}}\right)^2 = \left(\frac{\sqrt{\frac{W_1}{W_2}} + 1}{\sqrt{\frac{W_1}{W_2}} - 1}\right)$$

$$= \left(\frac{2+1}{2-1}\right)^2 = 9:1$$

4. Path difference introduced by a slab of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)\, t$$

Position of the fringe is $x = \frac{\Delta D}{d} = \frac{(\mu - 1)tD}{d}$

Also, fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

According to the question,

$$n\beta = x$$

$$\Rightarrow \quad \frac{n\lambda D}{d} = (\mu - 1)t\frac{D}{d}$$

$$\Rightarrow n\lambda = (\mu - 1)t \;\text{ or }\; t = \frac{n\lambda}{(\mu - 1)}$$

Alternate Solution

Path difference, $\Delta = \frac{xd}{D} = (\mu - 1)t$

or $\quad t = \frac{xd}{(\mu - 1)D}$

Also, $\quad \beta = \frac{\lambda D}{d} \Rightarrow \frac{d}{D} = \frac{\lambda}{\beta}$

$$\therefore \quad t = \frac{x\lambda}{(\mu - 1)\beta} = \frac{n\lambda D \times \lambda}{d(\mu - 1)\beta} \qquad [\because x = n\beta]$$

$$\Rightarrow \quad t = \frac{n\lambda}{(\mu - 1)}$$

So, no option is correct.

5. In Young's double slit experiment, ratio of maxima and minima intensity is given by

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2$$

As, intensity $(I) \propto [\text{amplitude } (a)]^2$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

So, $\quad \frac{I_{\max}}{I_{\min}} = \left(\frac{\frac{1}{3} + 1}{\frac{1}{3} - 1}\right)^2 = 4:1$

6. Here, wavelength of light used $(\lambda) = 5303\,\text{Å}$.

Distance between two slit $(d) = 19.44\ \mu$-m

Width of single slit (a)= $4.05\,\mu$m

Here, angular width between first and second diffraction minima

$$\theta \simeq \frac{\lambda}{a}$$

and angular width of a fringe due to double slit is

$$\theta' = \frac{\lambda}{d}$$

$\therefore$ Number of fringes between first and second diffraction minima, $n = \frac{\theta}{\theta'} = \frac{\frac{\lambda}{a}}{\frac{\lambda}{d}} = \frac{d}{a} = \frac{19.44}{4.05} = 4.81$ or $n \simeq 5$

$\therefore$ 5 interfering bright fringes lie between first and second diffraction minima.

7. Let intensity of each wave is I_0.

Then, intensity at the centre of bright fringe will be $4I_0$.

Given, path difference, $\Delta x = \lambda / 8$

$\therefore$ Phase difference, $\phi = \Delta x \times \frac{2\pi}{\lambda}$

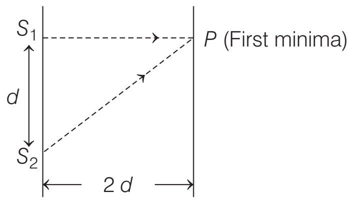
$$\Rightarrow \quad \phi = \frac{\lambda}{8} \times \frac{2\pi}{\lambda} \text{ or } \phi = \pi/4$$

Intensity of light at this point,

$$I' = I_0 + I_0 + 2I_0 \cos(\pi/4) = 2I_0 + \sqrt{2}\, I_0 = 3.41\, I_0$$

Now, $\quad \frac{I'}{4I_0} = \frac{3.41}{4} = 0.85$

8. In the given case, figure for first minima will be as shown below



We know that condition for minima in Young's double slit experiment is path difference,

$$\Delta x = (2n - 1)\lambda / 2$$

For first minima, $n = 1$

$\Rightarrow$ $$\Delta x = \lambda / 2 \quad \text{... (i)}$$

Path difference between the rays coming from virtual sources S_1 and S_2 at point 'P' will be

$$\Delta x = S_2P - S_1P \quad \text{... (ii)}$$

From triangle S_1S_2P,

$$S_1P = 2d \quad \text{...(iii)}$$

and $$(S_2P^2) = (S_1S_2)^2 + (S_1P)^2 = d^2 + (2d)^2$$

$\Rightarrow$ $$(S_2P^2) = 5d^2 \text{ or } S_2P = \sqrt{5}\, d \quad \text{...(iv)}$$

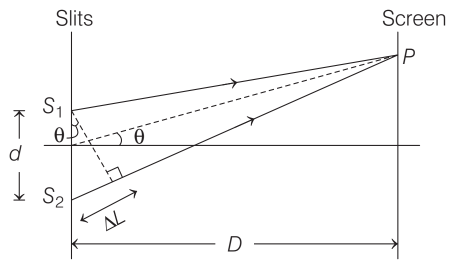
Substituting the values from Eqs. (iii) and (iv) in Eq. (ii), we get

$$\Delta x = \sqrt{5}\, d - 2d \quad \text{...(v)}$$

From Eqs. (i) and (v), we get

$$\sqrt{5}\, d - 2d = \lambda / 2 \Rightarrow d = \frac{\lambda}{2(\sqrt{5} - 2)}$$

9. **Key Idea** In a YDSE, path difference between 2 rays, reaching at some common point P located at angular position 'θ' as shown in the figure below is



$$\Delta L = d \sin\theta$$

For small value of θ; $\sin\theta \approx \theta$

So, path difference $= \Delta L = d\theta$

For a bright fringe at same angular position 'θ', both of the rays from slits S_1 and S_2 are in phase.

Hence, path difference is an integral multiple of wavelength of light used.

i.e., $$\Delta L = n\lambda$$

or $$d\theta = n\lambda \Rightarrow \lambda = \frac{d\theta}{n}$$

Here, $\theta = \frac{1}{40}$ rad, $d = 0.1$ mm

Hence, $$\lambda = \frac{0.1}{40n} \text{ mm} = \frac{0.1 \times 10^{-3} \text{ m}}{40n}$$

$$= \frac{0.1 \times 10^{-3} \times 10^{9}}{40n} \text{ n-m}$$

$\Rightarrow$ $$\lambda = \frac{2500}{n} \text{ n-m}$$

So, with light of wavelength λ_1 we have

$$\lambda_1 = \frac{2500}{n_1} \text{ (n-m)}$$

and with light of wavelength λ_2, we have

$$\lambda_2 = \frac{2500}{n_2} \text{ (n-m)}$$

Now, choosing different integral values for n_1 and n_2, (i.e., $n_1, n_2 = 1, 2, 3 \ldots$ etc) we find that for

$$n_1 = 4, \lambda_1 = \frac{2500}{4} = 625 \text{ n-m}$$

and for $n_2 = 5$,

$$\lambda_2 = \frac{2500}{5} = 500 \text{ n-m}$$

These values lie in given interval 500 n-m to 625 n-m.

10. In Young's double slit experiment, the condition of bright fringe and dark fringe are,

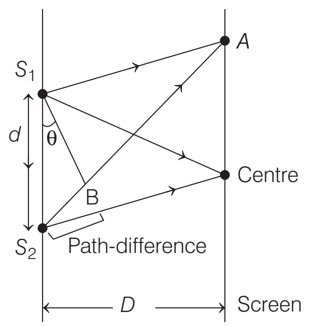
for bright fringes (maxima), path difference $= n\lambda$

$$d \sin\theta = n\lambda$$

for dark fringes (minima),

path-difference $= (2n - 1)\frac{\lambda}{2}$

$$d \sin\theta = (2n - 1)\frac{\lambda}{2}$$



For the given question, distance between slits

$$(d) = 0.320 \text{ mm}$$

Wavelength of light used $(\lambda) = 500$ n-m

Angular range for bright fringe $(\theta) = -30°$ to $30°$

Hence, for bright-fringe,

$$n\lambda = d \sin\theta$$

$$n = \frac{d \sin\theta}{\lambda} = \frac{0.320 \times 10^{-3} \times \sin 30°}{500 \times 10^{-9}}$$

$$n_{max} = 320$$

$\therefore$ Total number of maxima between the two lines are

$$n = (n_{max} \times 2) + 1$$

Here, $$n = (320 \times 2) + 1$$

$$n = 641$$

11. Let the intensity of two coherent sources be I_1 and I_2, respectively. It is given that,

$$\frac{\text{maximum intensity}}{\text{minimum intensity}} = \frac{I_{max}}{I_{min}} = \frac{16}{1}$$

Since, we know

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

and $$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$\therefore$ We can write,

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = 16$$

$$\Rightarrow \quad \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{\sqrt{16}}{1} = \frac{4}{1}$$

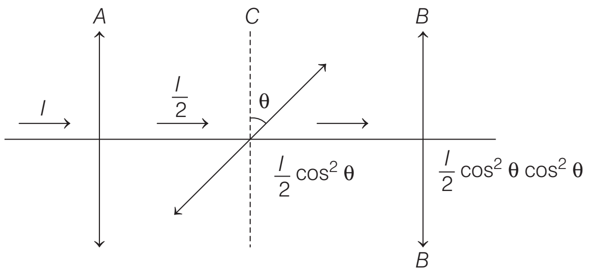
$$\sqrt{I_1} + \sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2}$$

$$\Rightarrow \quad 5\sqrt{I_2} = 3\sqrt{I_1} \Rightarrow \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{5}{3}$$

Squaring on both the sides, we get

$$\frac{I_1}{I_2} = \frac{25}{9}$$

12.



Using the relation, $I = I_0 \cos^2 \theta$

We have,

$$\frac{I}{2}(\cos^2\theta)^2 = \frac{I}{8} \Rightarrow \cos^2\theta = \frac{1}{2}$$

or $$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$

13. In single slit diffraction pattern, $\lambda = b \sin\theta$

At $\theta = 30°$,

$$\lambda = \frac{b}{2} = \frac{1 \times 10^{-6}}{2} = 5 \times 10^{-7}\,\text{m}$$

In YDSE,

fringe width, $$\omega = \frac{\lambda D}{d} \Rightarrow d = \frac{\lambda D}{\omega} = \frac{5 \times 10^{-7} \times 0.5}{1 \times 10^{-2}}$$

$$d = 25 \times 10^{-6}\ \text{m} = 25\,\mu\text{m}$$

14. Let nth bright fringes coincides, then

$$y_{n_1} = y_{n_2}$$

$$\Rightarrow \quad \frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d} \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{520}{650} = \frac{4}{5}$$

Hence, distance from the central maxima is

$$y = \frac{n_1 \lambda_1 D}{d} = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} = 7.8\,\text{mm}$$

15. A fringe is a locus of points having constant path difference from the two coherent sources S_1 and S_2. It will be concentric circle.

16. $$I = I_{max} \cos^2 \frac{\phi}{2} \qquad \text{...(i)}$$

Given, $$I = \frac{I_{max}}{2} \qquad \text{...(ii)}$$

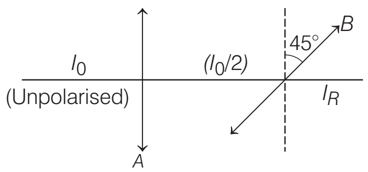
$\therefore$ From Eqs. (i) and (ii), we have

$$\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

Or path difference, $\Delta x = \left(\frac{\lambda}{2\pi}\right) \cdot \phi$

$$\therefore \quad \Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} .., \left(\frac{2n+1}{4}\right)\lambda$$

17. Relation between intensities is



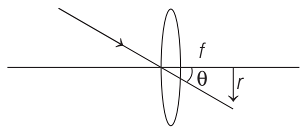
$$I_R = \left(\frac{I_0}{2}\right)\cos^2(45°) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

18. Fringe width $\beta = \frac{\lambda D}{d}$ or $\beta \propto \lambda$

Now, $$\lambda_R > \lambda_G > \lambda_B$$

$$\therefore \quad \beta_R > \beta_G > \beta_B$$

19.



$$r = f \tan\theta$$

or $$r \propto f$$

$$\therefore \quad \pi r^2 \propto f^2$$

20. $$I = I_{max} \cos^2\left(\frac{\phi}{2}\right)$$

$$\therefore \quad \frac{I_{max}}{4} = I_{max} \cos^2 \frac{\phi}{2}$$

$$\cos\frac{\phi}{2} = \frac{1}{2}$$

or $\dfrac{\phi}{2} = \dfrac{\pi}{3}$

$\therefore$ $\phi = \dfrac{2\pi}{3} = \left(\dfrac{2\pi}{\lambda}\right)\Delta x$ ...(i)

where $\Delta x = d\sin\theta$

Substituting in Eq. (i), we get

$$\sin\theta = \frac{\lambda}{3d} \text{ or } \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

21. Let nth minima of 400 nm coincides with mth minima of 560 nm, then

$$(2n-1)\left(\frac{400}{2}\right) = (2m-1)\left(\frac{560}{2}\right)$$

or $\dfrac{2n-1}{2m-1} = \dfrac{7}{5} = \dfrac{14}{10} = \dots$

i.e. 4th minima of 400 nm coincides with 3rd minima of 560 nm.

Location of this minima is,

$$Y_1 = \frac{(2\times 4-1)(1000)(400\times 10^{-6})}{2\times 0.1}$$

$= 14$ mm

Next 11th minima of 400 nm will coincide with 8th minima of 560 nm.

Location of this minima is,

$$Y_2 = \frac{(2\times 11-1)(1000)(400\times 10^{-6})}{2\times 0.1} = 42\text{ mm}$$

$\therefore$ Required distance $= Y_2 - Y_1 = 28$ mm

22. $PR = d$

$\therefore$ $PO = d\sec\theta$

and $CO = PO\cos 2\theta = d\sec\theta\cos 2\theta$

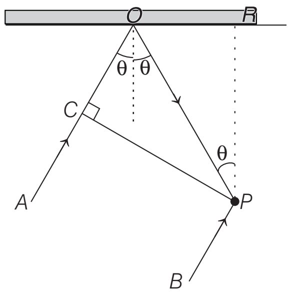
path difference between the two rays is,

$$\Delta x = PO + OC = (d\sec\theta + d\sec\theta\cos 2\theta)$$

phase difference between the two rays is

$\Delta\phi = \pi$ (one is reflected, while another is direct)

Therefore, condition for constructive interference should be



$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2} \dots$$

or $d\sec\theta\,(1+\cos 2\theta) = \dfrac{\lambda}{2}$

or $\left(\dfrac{d}{\cos\theta}\right)(2\cos^2\theta) = \dfrac{\lambda}{2}$ or $\cos\theta = \dfrac{\lambda}{4d}$

23. Path difference due to slab should be integral multiple of λ or

$$\Delta x = n\lambda \quad \text{or} \quad (\mu-1)t = n\lambda \quad n = 1, 2, 3, \dots$$

or $t = \dfrac{n\lambda}{\mu-1}$

For minimum value of t, $n = 1$

$\therefore$ $t = \dfrac{\lambda}{\mu-1} = \dfrac{\lambda}{1.5-1} = 2\lambda$

24. Fringe width, $\omega = \dfrac{\lambda D}{d} \propto \lambda$

When the wavelength is decreased from 600 nm to 400 nm, fringe width will also decrease by a factor of $\dfrac{4}{6}$ or $\dfrac{2}{3}$ or the number of fringes in the same segment will increase by a factor of 3/2.

Therefore, number of fringes observed in the same segment

$$= 12\times\frac{3}{2} = 18$$

NOTE Since $\omega \propto \lambda$, therefore, if YDSE apparatus is immersed in a liquid of refractive index μ, the wavelength λ and thus the fringe width will decrease μ times.

25. $I(\phi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$ ...(i)

Here, $I_1 = I$ and $I_2 = 4I$

At point A, $\phi = \dfrac{\pi}{2}$

$\therefore$ $I_A = I + 4I = 5I$

At point B, $\phi = \pi$

$\therefore$ $I_B = I + 4I - 4I = I$

$\therefore$ $I_A - I_B = 4I$

NOTE Eq. (i) for resultant intensity can be applied only when the sources are coherent. In the question it is given that the rays interfere. Interference takes place only when the sources are coherent. That is why we applied equation number (i). When the sources are incoherent, the resultant intensity is given by $I = I_1 + I_2$

26. In interference we know that

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

and $I_{\min} = (\sqrt{I_1} \sim \sqrt{I_2})^2$

Under normal conditions (when the widths of both the slits are equal)

$$I_1 \approx I_2 = I \text{ (say)}$$

$\therefore$ $I_{\max} = 4I$ and $I_{\min} = 0$

When the width of one of the slits is increased. Intensity due to that slit would increase, while that of the other will remain same. So, let :

$I_1 = I$

and $I_2 = \eta I$ $(\eta > 1)$

Then, $I_{\max} = I\,(1+\sqrt{\eta})^2 > 4I$

and $I_{\min} = I\,(\sqrt{\eta}-1)^2 > 0$

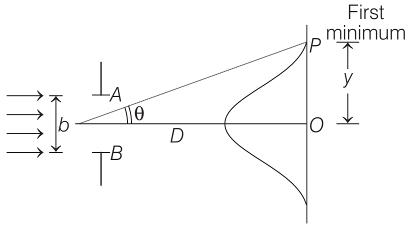
$\therefore$ Intensity of both maxima and minima is increased.

27. Locus of equal path difference are the lines running parallel to the axis of the cylinder. Hence, straight fringes are obtained.

NOTE Circular rings (also called Newton's rings) are observed in interference pattern when a plano-convex lens of large focal length is placed with its convex surface in contact with a plane glass plate because locus of equal path difference in this case is a circle.

28. Diffraction is obtained when the slit width is of the order of wavelength of light (or any electromagnetic wave) used. Here, wavelength of X-rays (1-100 Å) << slit width (0.6 mm). Therefore, no diffraction pattern will be observed.

29. At first minima, $b \sin\theta = \lambda$



or $\quad b\,\theta = \lambda \quad$ or $\quad b\left(\dfrac{y}{D}\right) = \lambda$

or $\quad y = \dfrac{\lambda D}{b}$

or $\quad \dfrac{yb}{D} = \lambda \quad$ ... (i)

Now, at P (First minima) path difference between the rays reaching from two edges (A and B) will be

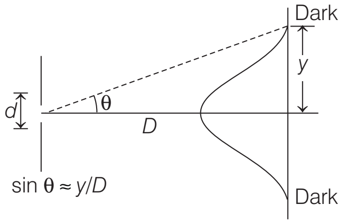
$$\Delta x = \frac{yb}{D} \qquad \left(\text{Compare with } \Delta x = \frac{yd}{D} \text{ in YDSE}\right)$$

or $\quad \Delta x = \lambda \quad$ [From Eq. (i)]

Corresponding phase difference (ϕ) will be

$$\phi = \left(\frac{2\pi}{\lambda}\right).\Delta x,\ \phi = \frac{2\pi}{\lambda}.\lambda = 2\pi$$

30. For first dark fringe on either side $d \sin\theta = \lambda$



or $\quad \dfrac{dy}{D} = \lambda$

$\therefore \quad y = \dfrac{\lambda D}{d}$

Therefore, distance between two dark fringes on either side

$$= 2y = \frac{2\lambda D}{d}$$

Substituting the values, we have

$$\text{Distance} = \frac{2(600 \times 10^{-6} \text{ mm})(2 \times 10^{3} \text{ mm})}{(1.0 \text{ mm})} = 2.4 \text{ mm}$$

31. $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{4I} + \sqrt{I})^2 = 9I$

$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{4I} - \sqrt{I})^2 = I$

32. $\omega = \dfrac{\lambda D}{d}$

d is halved and D is doubled

$\therefore$ Fringe width ω will become four times.

$\therefore$ Correct option is (d).

33. (A) $\rightarrow$ (p, s) $\rightarrow$ Intensity at P_0 is maximum. It will continuously decrease from P_0 towards P_2

(B) $\rightarrow$ (q) $\rightarrow$ Path difference due to slap will be compensated by geometrical path difference. Hence, $\delta(P_1) = 0$.

(C) $\rightarrow$ (t) $\rightarrow$ $\delta(P_0) = \dfrac{\lambda}{2}, \delta(P_1) = \dfrac{\lambda}{2} - \dfrac{\lambda}{4} = \dfrac{\lambda}{4}$ and

$\delta(P_2) = \dfrac{\lambda}{2} - \dfrac{\lambda}{3} = \dfrac{\lambda}{6}$. When path difference increases from 0 to $\dfrac{\lambda}{2}$, intensity will decrease from maximum to zero. Hence, in this case,

$$I(P_2) > I(P_1) > I(P_0)$$

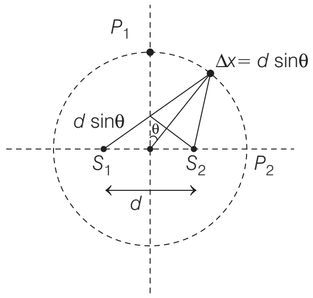
(D) $\rightarrow$ (r, s, t,)

$$\delta(P_0) = \frac{3\lambda}{4}, \delta(P_1) = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$$

and $\quad \delta(P_2) = \dfrac{3\lambda}{4} - \dfrac{\lambda}{3} = \dfrac{5\lambda}{12}.$

In this case $I(P_1) = 0$..

34.



$\lambda = 600$ nm

at $P_1 \quad \Delta x = 0$

at $P_2 \quad \Delta x = 1.8 \text{ mm} = n\lambda$

Number of maximas will be $n = \dfrac{\Delta x}{\lambda} = \dfrac{1.8 \text{ mm}}{600 \text{ nm}} = 3000$

at $P_2 \quad \Delta x = 3000\lambda$

Hence, bright fringe will be formed.

At P_2, 3000th maxima is formed.

For (a) option

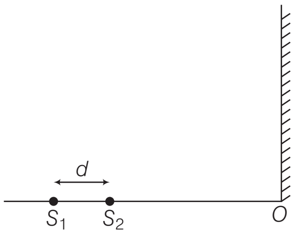
$$\Delta x = d\sin\theta \Rightarrow d\Delta x = d\cos d\theta$$

$$R\lambda = d\cos\theta R d\theta \Rightarrow R d\theta = \frac{R\lambda}{d\cos\theta}$$

As we move from P_1 to P_2

$\theta \uparrow \cos\theta \downarrow R d\theta \uparrow$

35.



Path difference at point $O = d = 0.6003$ mm
$= 600300$ nm.

This path difference is equal to $\left(1000\lambda + \dfrac{\lambda}{2}\right)$.

$\Rightarrow$ Minima is formed at point O.

Line S_1S_2 and screen are $\perp$ to each other so fringe pattern is circular (semi-circular because only half of screen is available)

36. Fringe width $\beta = \dfrac{\lambda D}{d}$

or $\quad \beta \propto \lambda$

$\therefore \quad \lambda_2 > \lambda_1$

So $\quad \beta_2 > \beta_1$

Number of fringes in a given width

$$m = \frac{y}{\beta} \quad \text{or} \quad m \propto \frac{1}{\beta}$$

$\Rightarrow \quad m_2 < m_1$ as $\beta_2 > \beta_1$

Distance of 3rd maximum of λ_2 from central maximum

$$= \frac{3\lambda_2 D}{d} = \frac{1800D}{d}$$

Distance of 5th minimum of λ_1 from central maximum

$$= \frac{9\lambda_1 D}{2d} = \frac{1800D}{d}$$

So, 3rd maximum of λ_2 will overlap with 5th minimum of λ_1.

Angular separation (or angular fringe width) $= \dfrac{\lambda}{d} \propto \lambda$

$\Rightarrow$ Angular separation for λ_1 will be lesser.

37. $d = \dfrac{\lambda}{2\sin\theta}$ ln $d = \ln\lambda - \ln 2 - \ln\sin\theta$

$$\frac{\Delta(d)}{d} = 0 - 0 - \frac{1}{\sin\theta} \times \cos\theta\,(\Delta\theta)$$

Fractional error $= \left|\dfrac{\Delta d}{d}\right| = (\cot\theta)\,\Delta\theta$

Absolute error $\Delta d = (d\cot\theta)\,\Delta\theta$

$$= \left(\frac{\lambda}{2\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)\Delta\theta$$

Now, given that $\Delta\theta$ = constant

As θ increases, $\sin\theta$ increases, $\cos\theta$ and $\cot\theta$ decrease.

$\therefore$ Both fractional and absolute errors decrease.

38. For $d = \lambda$, there will be only one, central maxima.

For $\lambda < d < 2\lambda$, there will be three maximas on the screen corresponding to path difference, $\Delta x = 0$ and $\Delta x = \pm\lambda$.

39. The intensity of light is $I(\theta) = I_0\cos^2\left(\dfrac{\delta}{2}\right)$

where, $\quad \delta = \dfrac{2\pi}{\lambda}(\Delta x) = \left(\dfrac{2\pi}{\lambda}\right)(d\sin\theta)$

(a) For $\theta = 30°$

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{10^6} = 300 \text{ m and } d = 150 \text{ m}$$

$$\delta = \left(\frac{2\pi}{300}\right)(150)\left(\frac{1}{2}\right) = \frac{\pi}{2}$$

$\therefore \quad \dfrac{\delta}{2} = \dfrac{\pi}{4}$

$\therefore \quad I(\theta) = I_0\cos^2\left(\dfrac{\pi}{4}\right) = \dfrac{I_0}{2}$ [option (a)]

(b) For $\quad \theta = 90°$

$$\delta = \left(\frac{2\pi}{300}\right)(150)(1) = \pi$$

or $\quad \dfrac{\delta}{2} = \dfrac{\pi}{2}$ and $I(\theta) = 0$

(c) For $\theta = 0°, \delta = 0$ or $\dfrac{\delta}{2} = 0$

$\therefore \quad I(\theta) = I_0$ [option (c)]

40. At P (directly infront of S_1) $y = b/2$

$\therefore$ Path difference,

$$\Delta X = S_2P - S_1P = \frac{y\cdot(b)}{d} = \frac{\left(\frac{b}{2}\right)(b)}{d} = \frac{b^2}{2d}$$

Those wavelengths will be missing for which

$$\Delta X = \frac{\lambda_1}{2}, \frac{3\lambda_2}{2}, \frac{5\lambda_3}{2} \ldots$$

$\therefore \quad \lambda_1 = 2\Delta x = \dfrac{b^2}{d}$

$$\lambda_2 = \frac{2\Delta x}{3} = \frac{b^2}{3d}$$

$$\lambda_3 = \frac{2\Delta x}{5} = \frac{b^2}{5d}$$

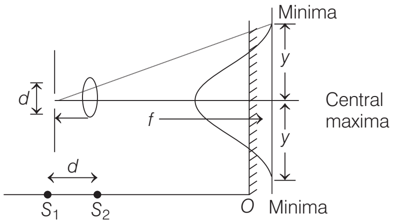
41. $\dfrac{I_{max}}{I_{min}} = \dfrac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \left(\dfrac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2 = 9$ (Given)

Solving this, we have $\dfrac{I_1}{I_2} = 4$ but $I \propto A^2$

$\therefore \quad \dfrac{A_1}{A_2} = 2$

$\therefore$ Correct options are (b) and (d).

42. Given



$$2y = 2 \times 10^{-3} \text{ m}$$
$$y = 1 \times 10^{-3} \text{ m}$$

First minima is obtained at

$$d \sin\theta = \lambda \text{ but } \sin\theta \approx \tan\theta = \frac{y}{f}$$

$$\therefore \quad d\left(\frac{y}{f}\right) = \lambda$$

$$d = \frac{\lambda f}{y} = \frac{5.89 \times 10^{-7} \times 0.5}{1 \times 10^{-3}}$$
$$= 2.945 \times 10^{-4} \text{ m}$$

43. In case of YDSE, at mid-point intensity will be $I_{\max} = 4I_0$

In the second case when sources are incoherent, the intensity will be $I = I_0 + I_0 = 2I_0$

Therefore, the desired ratio is

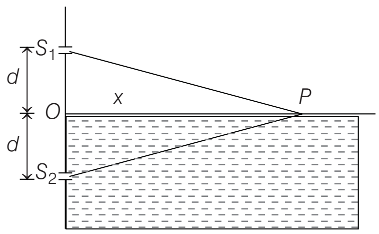
$$\frac{4I_0}{2I_0} = 2$$

Here, I_0 is the intensity due to one slit.

44. With white light we get coloured fringes (not only black and white) with centre as white.

45. To obtain interference, sources must be coherent. Two different light sources can never be coherent.

46.



$$\mu(S_2P) - S_1P = m\lambda$$

$$\Rightarrow \quad \mu\sqrt{d^2 + x^2} - \sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow \quad (\mu - 1)\sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow \quad \left(\frac{4}{3} - 1\right)\sqrt{d^2 + x^2} = m\lambda$$

or $$\sqrt{d^2 + x^2} = 3m\lambda$$

Squaring this equation we get,

$$x^2 = 9m^2\lambda^2 - d^2$$

$$\Rightarrow \quad p^2 = 9 \quad \text{or} \quad p = 3$$

47. Let n_1 bright fringe corresponding to wavelength $\lambda_1 = 500$ nm coincides with n_2 bright fringe corresponding to wavelength $\lambda_2 = 700$ nm.

$$\therefore \quad n_1 \frac{\lambda_1 D}{d} = n_2 \frac{\lambda_2 D}{d}$$

or $$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

This implies that 7th maxima of λ_1 coincides with 5th maxima of λ_2. Similarly 14th maxima of λ_1 will coincide with 10th maxima of λ_2 and so on.

$$\therefore \text{ Minimum distance} = \frac{n_1\lambda_1 D}{d} = 7 \times 5 \times 10^{-7} \times 10^3$$
$$= 3.5 \times 10^{-3} \text{ m} = 3.5 \text{ mm}$$

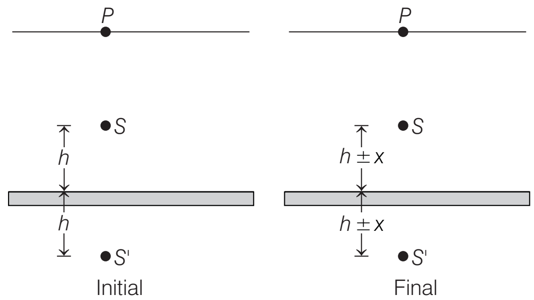
48. (a) Shape of the interference fringes will be circular.

(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is $I_2 = 36\%$ of $I_0 = 0.36I_0$

$$\therefore \quad \frac{I_1}{I_2} = \frac{I_0}{0.36I_0} = \frac{1}{0.36} \text{ or } \sqrt{\frac{I_1}{I_2}} = \frac{1}{0.6}$$

$$\therefore \quad \frac{I_{\min}}{I_{\max}} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$

(c) Initially path difference at P between two waves reaching from S and S' is $2h$.



Therefore, for maximum intensity at P :

$$2h = \left(n - \frac{1}{2}\right)\lambda \qquad \dots \text{(i)}$$

Now, let the source S is path difference will be $2h + 2x$ or $2h - 2x$. So, for displaced by x (away or towards mirror) then nemaximum intensity at P

$$2h + 2x = \left[n + 1 - \frac{1}{2}\right]\lambda \qquad \dots \text{(ii)}$$

or $$2h - 2x = \left[n - 1 - \frac{1}{2}\right]\lambda \qquad \dots \text{(iii)}$$

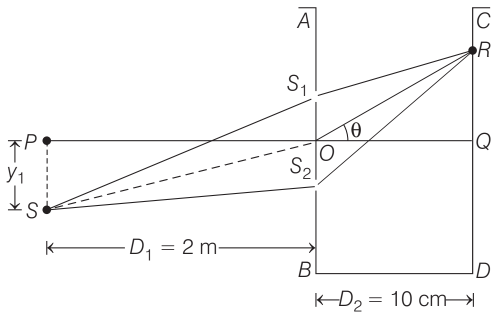
Solving Eqs. (i) and (ii) or Eqs. (i) and (iii), we get

$$x = \frac{\lambda}{2} = \frac{600}{2} = 300 \text{ nm}$$

NOTE Here, we have taken the condition of maximum intensity at P as : Path difference $\Delta x = \left(n - \frac{1}{2}\right)\lambda$

Because the reflected beam suffers a phase difference of π.

49. Given $y_1 = 40\,\text{cm}$, $D_1 = 2\text{ m} = 200\,\text{cm}$, $D_2 = 10\,\text{cm}$



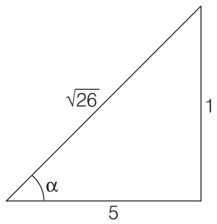
$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5} \Rightarrow \therefore\ \alpha = \tan^{-1}(1/5)$$

$$\sin\alpha = \frac{1}{\sqrt{26}} \approx \frac{1}{5} = \tan\alpha$$

Path difference between SS_1 and SS_2 is

$$\Delta X_1 = SS_1 - SS_2 \text{ or } \Delta X_1 = d\sin\alpha = (0.8\text{mm})\left(\frac{1}{5}\right)$$

or $\quad \Delta X_1 = 0.16\,\text{mm} \quad$ ...(i)



Now, let at point R on the screen, central bright fringe is observed (*i.e.,* net path difference = 0).

Path difference between S_2R and S_1R would be

$$\Delta X_2 = S_2R - S_1R$$

or $\quad \Delta X_2 = d\sin\theta \quad$ ...(ii)

Central bright fringe will be observed when net path difference is zero.

or $\quad \Delta X_2 - \Delta X_1 = 0$

$$\Delta X_2 = \Delta X_1$$

or $\quad d\sin\theta = 0.16$

or $\quad (0.8)\sin\theta = 0.16$

or $\quad \sin\theta = \frac{0.16}{0.8} = \frac{1}{5}$

$$\tan\theta = \frac{1}{\sqrt{24}} \approx \sin\theta = \frac{1}{5}$$

Hence, $\quad \tan\theta = \frac{y_2}{D_2} = \frac{1}{5}$

$\therefore \quad y_2 = \frac{D_2}{5} = \frac{10}{5} = 2\,\text{cm}$

Therefore, central bright fringe is observed at 2 cm above point Q on side CD.

Alternate solution

ΔX at R will be zero if $\Delta X_1 = \Delta X_2$

or $\quad d\sin\alpha = d\sin\theta$

or $\quad \alpha = \theta$

or $\quad \tan\alpha = \tan\theta$

$\Rightarrow \quad \frac{y_1}{D_1} = \frac{y_2}{D_2}$

or $\quad y_2 = \frac{D_2}{D_1}\cdot y_1 = \left(\frac{10}{200}\right)(40)\,\text{cm}$

or $\quad y_2 = 2\,\text{cm}$

(b) The central bright fringe will be observed at point Q. If the path difference created by the liquid slab of thickness $t = 10\,\text{cm}$ or 100 mm is equal to ΔX_1, so that the net path difference at Q becomes zero.

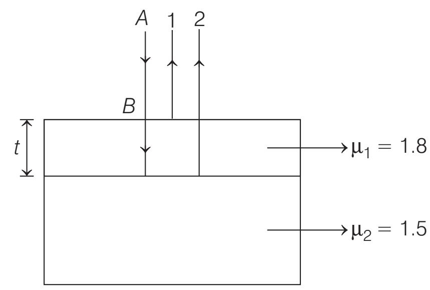
So, $\quad (\mu - 1)\,t = \Delta X_1$

or $\quad (\mu - 1)(100) = 0.16$

or $\quad \mu - 1 = 0.0016$

or $\quad \mu = 1.0016$

50. Incident ray AB is partly reflected as ray 1 from the upper surface and partly reflected as ray 2 from the lower surface of the layer of thickness t and refractive index $\mu_1 = 1.8$ as shown in figure. Path difference between the two rays would by



$$\Delta x = 2\mu_1\, t = 2(1.8)\,t = 3.6\,t$$

Ray 1 is reflected from a denser medium, therefore, it undergoes a phase change of π, whereas the ray 2 gets reflected from a rarer medium, therefore, there is no change in phase of ray 2.

Hence, phase difference between rays 1 and 2 would be $\Delta\phi = \pi$. Therefore, condition of constructive interference will be

$$\Delta x = \left(n - \frac{1}{2}\right)\lambda \text{ where } n = 1, 2, 3\ldots \text{ or } 3.6\,t = \left(n - \frac{1}{2}\right)\lambda$$

Least value of t is corresponding to $n = 1$ or

$$t_{\min} = \frac{\lambda}{2\times 3.6}$$

or $\quad t_{\min} = \frac{648}{7.2}\,\text{nm}$

or $\quad t_{\min} = 90\,\text{nm}$

NOTE

- For a wave (whether it is sound or electromagnetic), a medium is denser or rarer is decided from the speed of wave in that medium. In denser medium speed of wave is less. For example, water is rarer for sound, while denser for light compared to air because speed of sound in water is more than in air, while speed of light is less.
- In transmission/refraction, no phase change takes place. In reflection, there is a change of phase of π when it is reflected by a denser medium and phase change is zero if it is reflected by a rarer medium.
- If two waves in phase interfere having a path difference of Δx; then condition of maximum intensity would be $\Delta x = n\lambda$ where $n = 0, 1, 2, \ldots$
- But if two waves, which are already out of phase (a phase difference of π) interfere with path difference Δx, then condition of maximum intensity will be $\Delta x = \left(n - \frac{1}{2}\right)\lambda$ where $n = 1, 2, \ldots$

51. Given, $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m,

$d = 0.45 \text{ mm} = 0.45 \times 10^{-3} \text{ m}$,

$D = 1.5 \text{ m}$.

Thickness of glass sheet, $t = 10.4\ \mu\text{m} = 10.4 \times 10^{-6}$ m

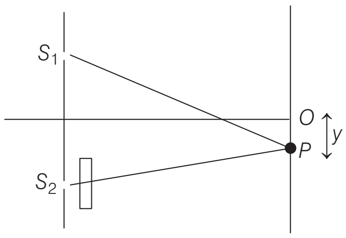
Refractive index of medium, $\mu_m = 4/3$

and refractive index of glass sheet, $\mu_g = 1.5$

(a) Let central maximum is obtained at a distance y below point O. Then $\Delta x_1 = S_1P - S_2P = \dfrac{yd}{D}$

Path difference due to glass sheet

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$



Net path difference will be zero when

$$\Delta x_1 = \Delta x_2$$

or
$$\frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$\therefore$
$$y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

Substituting the values, we have

$$y = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4 \times 10^{-6}(1.5)}{0.45 \times 10^{-3}}$$

$$y = 4.33 \times 10^{-3} \text{ m}$$

or we can say $y = 4.33$ mm.

(b) At O, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\dfrac{\mu_g}{\mu_m} - 1\right)t$

$\therefore$ Net path difference, $\Delta x = \Delta x_2$

Corresponding phase difference, $\Delta\phi$

or simple $\phi = \dfrac{2\pi}{\lambda}.\Delta x$

Substituting the values, we have

$$\phi = \frac{2\pi}{6 \times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6})$$

$$\phi = \left(\frac{13}{3}\right)\pi$$

Now, $$I(\phi) = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

$\therefore$ $$I = I_{\max}\cos^2\left(\frac{13\pi}{6}\right)$$

$$I = \frac{3}{4}I_{\max}$$

(c) At O, path difference is $\Delta x = \Delta x_2 = \left(\dfrac{\mu_g}{\mu_m} - 1\right)t$

For maximum intensity at O

$\Delta x = n\lambda$ (Here $n = 1, 2, 3, \ldots$)

$\therefore$ $\lambda = \dfrac{\Delta x}{1}, \dfrac{\Delta x}{2}, \dfrac{\Delta x}{3} \ldots$ and so on

$$\Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6} \text{ m})$$

$$= \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{3} \text{ nm})$$

$$\Delta x = 1300 \text{ nm}$$

$\therefore$ Maximum intensity will be corresponding to

$$\lambda = 1300 \text{ nm}, \frac{1300}{2} \text{ nm}, \frac{1300}{3} \text{ nm}, \frac{1300}{4} \text{ nm} \ldots$$

$= 1300$ nm, 650 nm, 433.33 nm, 325 nm ...

The wavelengths in the range 400 to 700 nm are 650 nm and 433.33 nm.

52. Given, $\lambda = 0.5$ mm, $d = 1.0$ mm, $D = 1$ m

(a) When the incident beam falls normally :

Path difference between the two rays S_2P and S_1P is

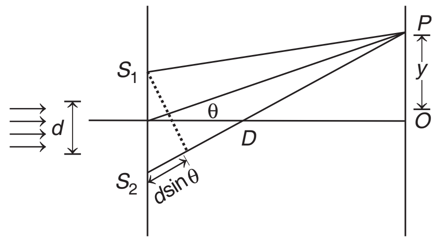
$$\Delta x = S_2P - S_1P \approx d\sin\theta$$

For minimum intensity,

$$d\sin\theta = (2n - 1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots$$

or
$$\sin\theta = \frac{(2n-1)\lambda}{2d}$$

$$= \frac{(2n-1)0.5}{2 \times 1.0} = \frac{2n-1}{4}$$



As $\sin\theta \le 1$ therefore $\dfrac{(2n-1)}{4} \le 1$ or $n \le 2.5$

So, n can be either 1 or 2.

When $n = 1$, $\sin\theta_1 = \dfrac{1}{4}$ or $\tan\theta_1 = \dfrac{1}{\sqrt{15}}$

$$n = 2, \sin\theta_2 = \frac{3}{4}$$

or $\quad \tan\theta_2 = \dfrac{3}{\sqrt{7}}$

$\therefore \quad y = D\tan\theta = \tan\theta \qquad (D = 1\text{ m})$

So, the position of minima will be

$$y_1 = \tan\theta_1 = \frac{1}{\sqrt{15}}\text{ m} = 0.26\text{ m}$$

$$y_2 = \tan\theta_2 = \frac{3}{\sqrt{7}}\text{ m} = 1.13\text{ m}$$

And as minima can be on either side of centre O.

Therefore there will be four minimas at positions ± 0.26 m and ± 1.13 m on the screen.

(b) When $\alpha = 30°$, path difference between the rays before reaching S_1 and S_2 is

$$\Delta x_1 = d\sin\alpha = (1.0)\sin 30° = 0.5\text{ mm} = \lambda$$

So, there is already a path difference of λ between the rays.

Position of central maximum Central maximum is defined as a point where net path difference is zero. So,

$$\Delta x_1 = \Delta x_2$$

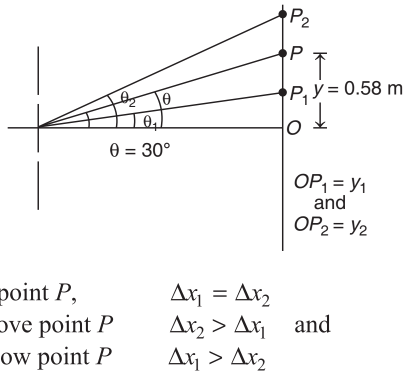
or $\quad d\sin\alpha = d\sin\theta$

or $\quad \theta = \alpha = 30°$

or $\quad \tan\theta = \dfrac{1}{\sqrt{3}} = \dfrac{y_0}{D}$

$\therefore \quad y_0 = \dfrac{1}{\sqrt{3}}\text{ m} \qquad (D = 1\text{ m})$

$$y_0 = 0.58\text{ m}$$



At point P, $\quad \Delta x_1 = \Delta x_2$

Above point $P \quad \Delta x_2 > \Delta x_1$ and

Below point $P \quad \Delta x_1 > \Delta x_2$

Now, let P_1 and P_2 be the minimas on either side of central maxima. Then, for P_2

$$\Delta x_2 - \Delta x_1 = \frac{\lambda}{2}$$

or $\quad \Delta x_2 = \Delta x_1 + \dfrac{\lambda}{2} = \lambda + \dfrac{\lambda}{2} = \dfrac{3\lambda}{2}$

or $\quad d\sin\theta_2 = \dfrac{3\lambda}{2}$

or $\quad \sin\theta_2 = \dfrac{3\lambda}{2d} = \dfrac{(3)(0.5)}{(2)(1.0)} = \dfrac{3}{4}$

$\therefore \quad \tan\theta_2 = \dfrac{3}{\sqrt{7}} = \dfrac{y_2}{D}$

or $\quad y_2 = \dfrac{3}{\sqrt{7}} = 1.13$ m

Similarly by for P_1

$\Delta x_1 - \Delta x_2 = \dfrac{\lambda}{2}$ or $\quad \Delta x_2 = \Delta x_1 - \dfrac{\lambda}{2} = \lambda - \dfrac{\lambda}{2} = \dfrac{\lambda}{2}$

or $\quad d\sin\theta_1 = \dfrac{\lambda}{2}$

or $\quad \sin\theta_1 = \dfrac{\lambda}{2d} = \dfrac{(0.5)}{(2)(1.0)} = \dfrac{1}{4}$

$\therefore \quad \tan\theta_1 = \dfrac{1}{\sqrt{15}} = \dfrac{y_1}{D}$

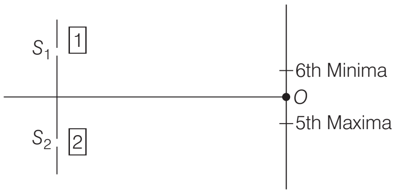
or $\quad y_1 = \dfrac{1}{\sqrt{15}}$ m $= 0.26$ m

Therefore, y-coordinates of the first minima on either side of the central maximum are $y_1 = 0.26$ m and $y_2 = 1.13$ m.

NOTE In this problem $\sin\theta \approx \tan\theta \approx \theta$ is not valid as θ is large.

53. $\mu_1 = 1.4$ and $\mu_2 = 1.7$ and let t be the thickness of each glass plates.

Path difference at O, due to insertion of glass plates will be



$$\Delta x = (\mu_2 - \mu_1)t = (1.7 - 1.4)t = 0.3t \qquad \text{... (i)}$$

Now, since 5th maxima (earlier) lies below O and 6th minima lies above O.

This path difference should lie between 5λ and $5\lambda + \dfrac{\lambda}{2}$

So, let $\quad \Delta x = 5\lambda + \Delta \qquad \text{... (ii)}$

where $\quad \Delta < \dfrac{\lambda}{2}$

Due to the path difference Δx, the phase difference at O will be

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(5\lambda + \Delta)$$

$$= \left(10\pi + \frac{2\pi}{\lambda}.\Delta\right) \qquad \text{...(iii)}$$

Intensity at O is given $\dfrac{3}{4}I_{\max}$ and since

$$I(\phi) = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\therefore \quad \frac{3}{4} I_{max} = I_{max} \cos^2\left(\frac{\phi}{2}\right)$$

or $$\frac{3}{4} = \cos^2\left(\frac{\phi}{2}\right) \qquad \ldots \text{(iv)}$$

From Eqs. (iii) and (iv), we find that

$$\Delta = \frac{\lambda}{6}$$

ie, $$\Delta x = 5\lambda + \frac{\lambda}{6} = \frac{31}{6}\lambda = 0.3\, t$$

$$\therefore \quad t = \frac{31\lambda}{6(0.3)} = \frac{(31)(5400 \times 10^{-10})}{1.8}$$

or $$t = 9.3 \times 10^{-6} \text{ m} = 9.3\,\mu\text{m}$$

54. (a) Path difference due to the glass slab,

$$\Delta x = (\mu - 1)t = (1.5 - 1)\, t = 0.5t$$

Due to this slab, 5 red fringes have been shifted upwards. Therefore,

$$\Delta x = 5\lambda_{red}$$

or $$0.5t = (5)(7 \times 10^{-7} \text{m})$$

$\therefore$ t = thickness of glass slab = 7×10^{-6} m

(b) Let μ' be the refractive index for green light then

$$\Delta x' = (\mu' - 1)\, t$$

Now the shifting is of 6 fringes of red light. Therefore,

$$\Delta x' = 6\lambda_{red}$$

$$\therefore \quad (\mu' - 1)\, t = 6\lambda_{red}$$

$$\therefore \quad (\mu' - 1) = \frac{(6)(7 \times 10^{-7})}{7 \times 10^{-6}} = 0.6$$

$$\therefore \quad \mu' = 1.6$$

(c) In part (a), shifting of 5 bright fringes was equal to 10^{-3} m. Which implies that

$$5\omega_{red} = 10^{-3} \text{m} \qquad (\text{Here, } \omega = \text{Fringe width})$$

$$\therefore \quad \omega_{red} = \frac{10^{-3}}{5} \text{m} = 0.2 \times 10^{-3} \text{m}$$

Now since $\omega = \dfrac{\lambda D}{d}$

or $$\omega \propto \lambda$$

$$\therefore \quad \frac{\omega_{green}}{\omega_{red}} = \frac{\lambda_{green}}{\lambda_{red}}$$

$$\therefore \quad \omega_{green} = \omega_{red} \frac{\lambda_{green}}{\lambda_{red}} = (0.2 \times 10^{-3})\left(\frac{5 \times 10^{-7}}{7 \times 10^{-7}}\right)$$

$$\omega_{green} = 0.143 \times 10^{-3} \text{m}$$

$$\therefore \quad \Delta\omega = \omega_{green} - \omega_{red} = (0.143 - 0.2) \times 10^{-3} \text{m}$$

$$\Delta\omega = -5.71 \times 10^{-5} \text{ m}$$

55. Given, $\mu = 1.33$, $d = 1$ mm, $D = 1.33$ m,

$$\lambda = 6300 \text{ Å}$$

(a) Wavelength of light in the given liquid :

$$\lambda' = \frac{\lambda}{\mu} = \frac{6300}{1.33} \text{Å} \approx 4737 \text{ Å}$$

$$= 4737 \times 10^{-10} \text{ m}$$

$\therefore$ Fringe width, $\omega = \dfrac{\lambda' D}{d}$

$$\omega = \frac{(4737 \times 10^{-10} \text{ m})(1.33 \text{ m})}{(1 \times 10^{-3} \text{ m})} = 6.3 \times 10^{-4} \text{ m}$$

$$\omega = 0.63 \text{ mm}$$

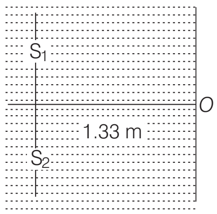
(b) Let t be the thickness of the glass slab.

Path difference due to glass slab at centre O.

$$\Delta x = \left(\frac{\mu_{glass}}{\mu_{liquid}} - 1\right) t = \left(\frac{1.53}{1.33} - 1\right) t$$

or $$\Delta x = 0.15\, t$$

Now, for the intensity to be minimum at sO, this path difference should be equal to $\dfrac{\lambda'}{2}$

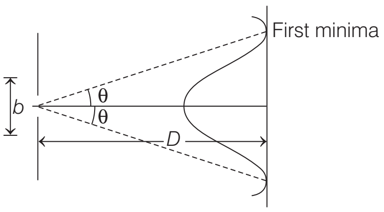


$$\therefore \quad \Delta x = \frac{\lambda'}{2} \quad \text{or } 0.15\, t = \frac{4737}{2} \text{Å}$$

$$\therefore \quad t = 15790 \text{ Å} \quad \text{or} \quad t = 1.579\,\mu\text{m}$$

56. (a) Given, $\lambda = 6000$ Å

Let b be the width of slit and D the distance between screen and slit.



First minima is obtained at $b \sin\theta = \lambda$

or $b\,\theta = \lambda$ as $\sin\theta \approx \theta$ or $\theta = \dfrac{\lambda}{b}$

Angular width of first maxima $= 2\theta = \dfrac{2\lambda}{b} \propto \lambda$

Angular width will decrease by 30% when λ is also decreased by 30%.

Therefore, new wavelength

$$\lambda' = \left\{(6000) - \left(\frac{30}{100}\right) 6000\right\} \text{ Å}$$

$$\lambda' = 4200 \text{ Å}$$

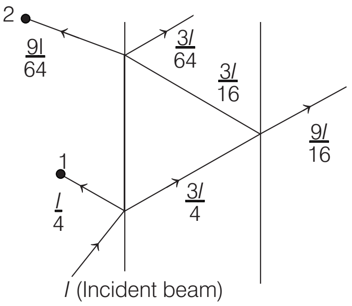
(b) When the apparatus is immersed in a liquid of refractive index μ, the wavelength is decreased μ times. Therefore,

$$4200\ \text{Å} = \frac{6000\ \text{Å}}{\mu}$$

$$\therefore \qquad \mu = \frac{6000}{4200}$$

or $\qquad \mu = 1.429 \simeq 1.43$

57. Each plate reflects 25% and transmits 75%.



Incident beam has an intensity I. This beam undergoes multiple reflections and refractions. The corresponding intensity after each reflection and refraction (transmission) are shown in figure.

Interference pattern is to take place between rays 1 and 2.

$$I_1 = \frac{I}{4} \quad \text{and} \quad I_2 = 9I/64$$

$$\therefore \qquad \frac{I_{\min}}{I_{\max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \frac{1}{49}$$

58. Power received by aperture A,

$$P_A = I\,(\pi r_A^2) = \frac{10}{\pi}(\pi)(0.001)^2 = 10^{-5}\ \text{W}$$

Power received by aperture B,

$$P_B = I\,(\pi r_B^2) = \frac{10}{\pi}(\pi)(0.002)^2$$

$$= 4 \times 10^{-5}\ \text{W}$$

Only 10% of P_A and P_B goes to the original direction. Hence, 10% of $P_A = 10^{-6} = P_1$ (say)

and 10% of $P_B = 4 \times 10^{-6} = P_2$ (say)

Path difference created by slab

$$\Delta x = (\mu - 1)\,t$$

$$= (1.5 - 1)(2000) = 1000\text{Å}$$

Corresponding phase difference,

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{6000} \times 1000 = \frac{\pi}{3}$$

Now, resultant power at the focal point

$$P = P_1 + P_2 + 2\sqrt{P_1 P_2}\cos\phi$$

$$= 10^{-6} + 4 \times 10^{-6} + 2\sqrt{(10^{-6})(4 \times 10^{-6})}\cos\frac{\pi}{3}$$

$$= 7 \times 10^{-6}\ \text{W}$$

59. (a) The desired distance will be

$$y_1 = 3\omega_1 = 3\left(\frac{\lambda_1 D}{d}\right) = \frac{(3)(6500 \times 10^{-10})(1.2)}{(2 \times 10^{-3})}$$

$$= 11.7 \times 10^{-4}\text{m} = 1.17\ \text{mm}$$

(b) Let n_1 bright fringe of λ_1 coincides with n_2 bright fringe of λ_2. Then,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d} \quad \text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{5200}{6500} = \frac{4}{5}$$

Therefore, 4th bright of λ_1 coincides with 5th bright of λ_2. Similarly, 8th bright of λ_1 will coincide with 10th bright of λ_2 and so on. The least distance from the central maximum will therefore corresponding to 4th bright of λ_1 (or 5th bright of λ_2.) Hence,

$$Y_{\min} = \frac{4\lambda_1 D}{d} = \frac{4(6500 \times 10^{-10})(1.2)}{(2 \times 10^{-3})}$$

$$= 15.6 \times 10^{-4}\ \text{m} = 1.56\ \text{mm}$$

60. Shifting of fringes due to introduction of slab in the path of one of the slits, comes out to be,

$$\Delta y = \frac{(\mu - 1)tD}{d} \qquad \text{...(i)}$$

Now, the distance between the screen and slits is doubled.

Hence, the new fringe width will become.

$$\omega' = \frac{\lambda(2D)}{d} \qquad \text{...(ii)}$$

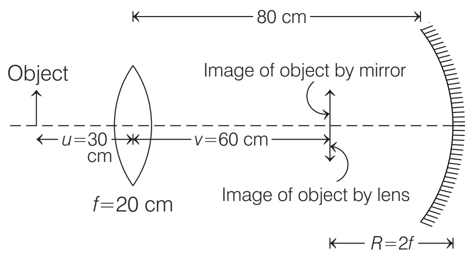
Given, $\Delta y = \omega'$ or $\dfrac{(\mu - 1)\,tD}{d} = \dfrac{2\lambda D}{d}$

$$\therefore \qquad \lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6 - 1)(1.964 \times 10^{-6})}{2}$$

$$= 0.5892 \times 10^{-6}\ \text{m} = 5892\ \text{Å}$$

Topic 7 Miscellaneous Problems

1. The given situation can be drawn as shown below



For lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Substituting given values, we get

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20}$$

$$\Rightarrow \qquad v = 60\ \text{cm}$$

So, this image is at a distance of

$80 - 60 = 20$ cm from the mirror.

As, the image formed by the mirror coincides with image formed by the lens. This condition is only possible, if any object that has been placed in front of concave mirror is at centre of curvature, i.e. at $2f$.

So, radius of curvature of mirror is $R = 20\,\text{cm}$

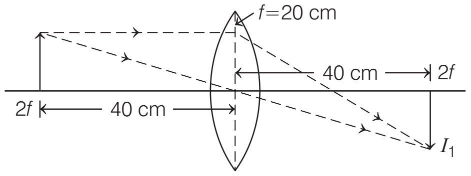
$\therefore$ Focal length of mirror, $f = \dfrac{R}{2} = 10\,\text{cm}$

As, for virtual image, the object is to kept between pole and focus of the mirror.

$\therefore$ The maximum distance of the object for which this concave mirror by itself produce a virtual image would be 10 cm.

2. In given system of lens and mirror, position of object O in front of lens is at a distance $2f$.

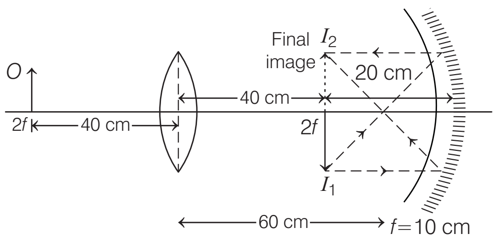
i.e. $u = 2f = 40\,\text{cm}$



So, image (I_1) formed is real, inverted and at a distance, $v = 2f = 2 \times 20 = 40$ cm, (behind lens) magnification, $m_1 = \dfrac{v}{u} = \dfrac{40}{40} = 1$

Thus, size of image is same as that of object.

This image (I_1) acts like a real object for mirror.



As object distance for mirror is $u = C = 2f = -20\,\text{cm}$

where, C = centre of curvature.

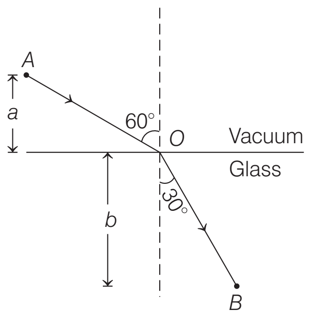
So, image (I_2) formed by mirror is at $2f$.

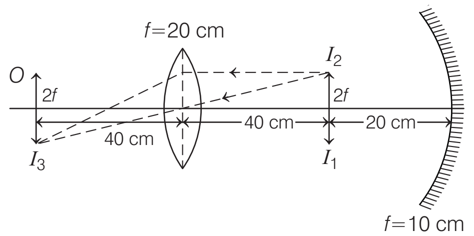
$\therefore$ For mirror $v = 2f = 2(-10) = -20\,\text{cm}$

Magnification, $m_2 = -\dfrac{v}{u} = -\dfrac{(-20)}{(-20)} = -1$

Thus, image size is same as that of object.

The image I_2 formed by the mirror will act like an object for lens.





As the object is at $2f$ distance from lens, so image (I_3) will be formed at a distance $2f$ or 40 cm. Thus, magnification,

$$m_3 = \frac{v}{u} = \frac{40}{40} = 1$$

So, final magnification, $m = m_1 \times m_2 \times m_3 = -1$

Hence, final image (I_3) is real, inverted of same size as that of object and coinciding with object.

3. Energy of a light wave $\propto$ Intensity of the light wave

Since, intensity $= \varepsilon v A^2$

where, ε is the permittivity of the medium in which light is travelling with velocity v and A is its amplitude.

Since, only 4% of the energy of the light gets reflected.

$\therefore$ 96% of the energy of the light is transmitted.

$E_{\text{transmitted}}(E_t) = 96\%$ of $E_{\text{incident}}(E_i)$

$$\varepsilon_0 \varepsilon_r v A_t^2 = \frac{96}{100} \times \varepsilon_0 \times c \times A_1^0$$

$$A_t^2 = \frac{96}{100} \cdot \frac{\varepsilon_0}{\varepsilon_r} \cdot \frac{c}{v} A_i^2$$

$$A_t^2 = \frac{96}{100} \cdot \frac{v^2}{c^2} \cdot \frac{c}{v} A_i^2 \qquad \left[\because \sqrt{\frac{\varepsilon_0}{\varepsilon_r}} = \frac{v}{c}\right]$$

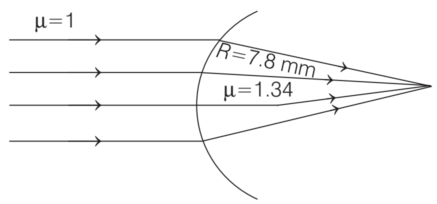
$$A_t^2 = \frac{96}{100} \cdot \frac{v}{c} \cdot A_i^2$$

$$= \frac{96}{100} \times \frac{1}{3/2} \times 30^2 \qquad \left[\because \mu = \frac{c}{v} = 1.5\right]$$

$$A_t = \sqrt{\frac{64}{100} \times 30^2}$$

$$A_t = 24 \text{ V/m}$$

4. The given condition is shown in the figure below



where, a parallel beam of light is coming from air ($\mu = 1$) to a spherical surface (eye) of refractive index 1.34.

Radius of curvature of this surface is 7.8 mm.

From the image formation formula for spherical surface, i.e. relation between object, image and radius of curvature.

$$\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R} \qquad \ldots \text{(i)}$$

Given, $\mu_r = 1.34$, $\mu_i = 1$, $u = \infty$ (– ve) and

$$R = 7.8$$

Substituting the given values, we get

$$\frac{1.34}{v} + \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

or $$\frac{1.34}{v} = \frac{0.34}{7.8}$$

$\Rightarrow$ $$v = \frac{1.34 \times 7.8}{0.34} \text{ mm}$$

$\Rightarrow$ $$v = \frac{4}{3} \times 3 \times 7.8 \text{ mm}$$

($\because$ approximately $1.34 = 4/3$ and $0.34 = 1/3$)

$\Rightarrow$ $$v = 31.2 \text{ mm or } 3.12 \text{ cm}$$

5. **Key Idea** When a beam of unpolarised light is reflected from a transparent medium of refractive index μ, then the reflected light is completely plane polarised at a certain angle of incidence i_B, which is known as Brewster's angle.

In the given condition, the light reflected irrespective of an angle of incidence is never completely polarised. So,

$$i_C > i_B$$

where, i_C is the critical angle.

$\Rightarrow$ $$\sin i_C < \sin i_B \qquad \ldots \text{(i)}$$

From Brewster's law, we know that

$$\tan i_B = {}^w\mu_g = \frac{\mu_{\text{glass}}}{\mu_{\text{water}}} = \frac{1.5}{\mu} \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{\mu} < \frac{1.5}{\sqrt{(1.5)^2 + (\mu)^2}}$$

$\Rightarrow$ $$\sqrt{(1.5)^2 + \mu^2} < 1.5\,\mu$$

$$\mu^2 + (1.5)^2 < (1.5\,\mu)^2 \quad \text{or} \quad \mu < \frac{3}{\sqrt{5}}$$

$\therefore$ The minimum value of μ should be $\frac{3}{\sqrt{5}}$.

6. $R = 10$ cm

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ two times

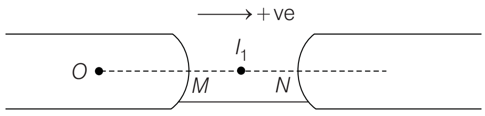
$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-10}$$

$$\frac{1}{v} + \frac{1.5}{50} = \frac{0.5}{10}$$

$$\frac{1}{v} = \frac{0.5}{10} - \frac{1.5}{50} = \frac{2.5 - 1.5}{50} \Rightarrow v = 50$$

$$MN = d,\ MI_1 = 50 \text{ cm},$$

$$NI_1 = (d - 50) \text{ cm}$$

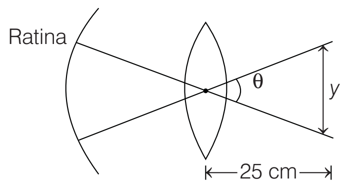


Again, $$\frac{1.5}{\infty} - \frac{1}{-(d - 50)} = \frac{1.5 - 1}{10}$$

$$\frac{1}{d - 50} = \frac{1}{20}$$

$$d = 70$$

7.



Resolving angle of naked eye is given by

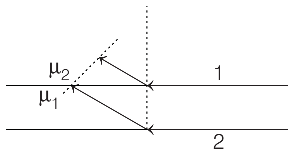
$$\theta = \frac{1.22\lambda}{D}$$

$\therefore$ $$\frac{y}{25 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{0.25 \times 2 \times 10^{-2}}$$

$\therefore$ $$y = 30 \times 10^{-6} \text{ m}$$

$$= 30 \mu\text{m}$$

8.



$$\mu_2 > \mu_1$$

Dotted line is the normal.

According to Huygen's principle, each point on wavefront behaves as a point source of light.

Ray 2 will travel faster than 1 as $\mu_2 > \mu_1$. So, beam will bend upwards.

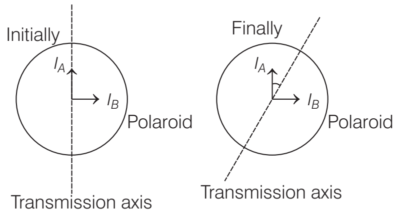
9. By law of Malus i.e. $I = I_0 \cos^2 \theta$

Now, $$I_{A'} = I_A \cos^2 30°$$

$$I_{B'} = I_B \cos^2 60°$$

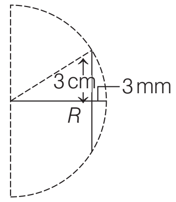
As, $$I'_A = I'_B$$

$$I_A \cos^2 30° = I_B \cos^2 60°$$



$\Rightarrow$ $$I_A \frac{3}{4} = I_B \frac{1}{4} \quad \Rightarrow \quad \frac{I_A}{I_B} = \frac{1}{3}$$

10. By Pythagoras theorem



$$R^2 = (3)^2 + (R - 0.3)^2 \Rightarrow R \approx 15\,\text{cm}$$

Refractive index of material of lens $\mu = \dfrac{c}{v}$

Here c = speed of light in vacuum $= 3 \times 10^8$ m/s

v = speed of light in material of lens $= 2 \times 10^8$ m/s

$$= \frac{3 \times 10^8}{2 \times 10^8} = \frac{3}{2}$$

From lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

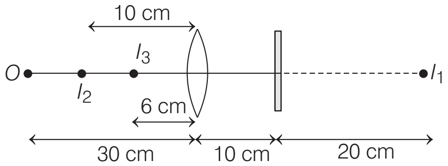
Here, $R_1 = R$ and $R_2 = \infty$ (For plane surface)

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$\Rightarrow$ $f = 30\,\text{cm}$

11. After critical angle reflection will be 100% and transmission is 0%. Options (b) and (c) satisfy this condition. But option (c) is the correct option. Because in option (b) transmission is given 100% at $\theta = 0°$, which is not true.

12. Object is placed at distance $2f$ from the lens. So first image I_1 will be formed at distance $2f$ on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.



Now applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\therefore$ $$\frac{1}{v} - \frac{1}{+10} = \frac{1}{+15}$$

or $v = 6$ cm

Therefore, the final image is at distance 16 cm from the mirror. But, this image will be real.

This is because ray of light is travelling from right to left.

13. Critical angle from region III to region IV

$$\sin\theta_C = \frac{n_0/8}{n_0/6} = \frac{3}{4}$$

Now, applying Snell's law in region I and region III

$$n_0 \sin\theta = \frac{n_0}{6}\sin\theta_C$$

or $$\sin\theta = \frac{1}{6}\sin\theta_C$$

$$= \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8}$$

$\therefore$ $$\theta = \sin^{-1}\left(\frac{1}{8}\right)$$

14. Distance of object from mirror

$$= 15 + \frac{33.25}{1.33} = 40\,\text{cm}$$

Distance of image from mirror $= 15 + \dfrac{25}{1.33}$

$= 33.8\,\text{cm}$

For the mirror, $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$\therefore$ $$\frac{1}{-33.8} + \frac{1}{-40} = \frac{1}{f}$$

$\therefore$ $f = -18.3\,\text{cm}$

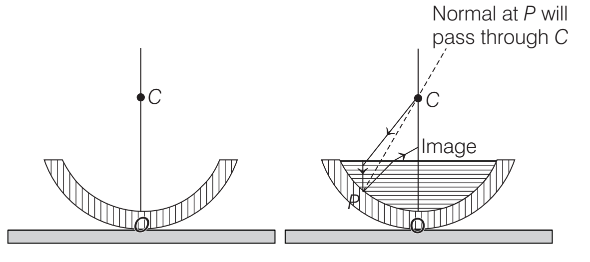
$\therefore$ Most suitable answer is (c).

15. Critical angle $\theta_C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

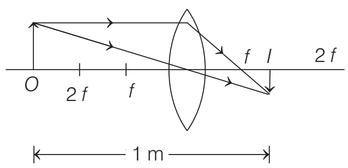
Wavelength increases in the sequence of VIBGYOR. According to Cauchy's formula refractive index (μ) decreases as the wavelength increases. Hence the refractive index will increase in the sequence of ROYGBIV. The critical angle θ_C will thus increase in the same order VIBGYOR.

For green light the incidence angle is just equal to the critical angle. For yellow, orange and red the critical angle will be greater than the incidence angle. So, these colours will emerge from the glass air interface.

16. The ray diagram is shown in figure. Therefore, the image will be real and between C and O.



17. Image can be formed on the screen if it is real. Real image of reduced size can be formed by a concave mirror or a convex lens.

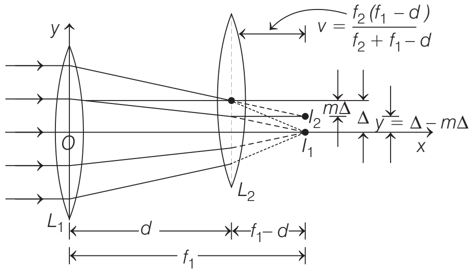


A diminished real image is formed by a convex lens when the object is placed beyond $2f$ and the image of such object is formed between f and $2f$ on other side.

Thus, $\quad d > (2f + 2f)$

or $\quad 4f < 0.1\,\text{m}$ or $\quad f < 0.25$ m

18. From the first lens parallel beam of light is focused at its focus i.e. at a distance f_1 from it. This image I_1 acts as virtual object for second lens L_2. Therefore, for L_2



$$u = +(f_1 - d),\ f = +f_2$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{f_2} + \frac{1}{f_1 - d}$$

Hence, $\quad v = \dfrac{f_2(f_1 - d)}{f_2 + f_1 - d}$

Therefore, x-coordinate of its focal point will be

$$x = d + v = d + \frac{f_2(f_1 - d)}{f_2 + f_1 - d}$$

$$= \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$$

Linear magnification for L_2

$$m = \frac{v}{u} = \frac{f_2(f_1 - d)}{f_2 + f_1 - d} \cdot \frac{1}{f_1 - d}$$

$$= \frac{f_2}{f_2 + f_1 - d}$$

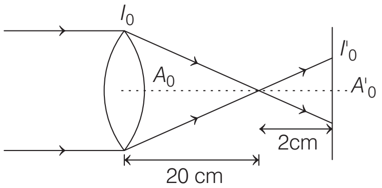
Therefore, second image will be formed at a distance of $m\Delta$ or $\left(\dfrac{f_2}{f_2 + f_1 - d}\right).\Delta$ below its optic axis.

Therefore, y-coordinate of the focus of system will be

$$y = \Delta - \left(\frac{f_2\,\Delta}{f_2 + f_1 - d}\right)$$

or $\quad y = \dfrac{(f_1 - d).\Delta}{f_2 + f_1 - d}$

19.



$$\frac{A_0'}{A_0} = \left(\frac{2}{20}\right)^2 = \frac{1}{100}$$

$$\Rightarrow \quad A_0' = \frac{A_0}{100}$$

$$P = I_0 A_0 = I_0' A_0'$$

$$\Rightarrow \quad I_0' = \frac{I_0 A_0}{\dfrac{A_0}{100}}$$

$$\Rightarrow \quad = 100 I_0 = 130\,\text{kW/m}^2$$

20. Since value of n in meta-material is negative.

$$\therefore \quad v = \frac{c}{|n|}$$

21. According to the paragraph, refracted ray in meta-material should be on same side of normal.

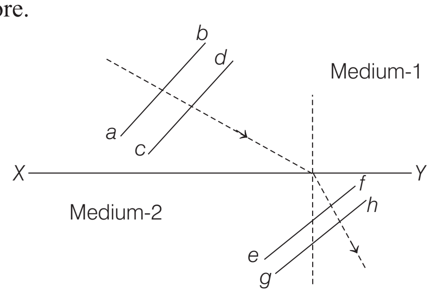
22. Wavefronts are parallel in both media. Therefore, light which is perpendicular to wavefront travels as a parallel beam in each medium.

23. All points on a wavefront are at the same phase.

$\therefore \quad \phi_d = \phi_c$ and $\phi_f = \phi_e$

$\therefore \quad \phi_d - \phi_f = \phi_c - \phi_e$

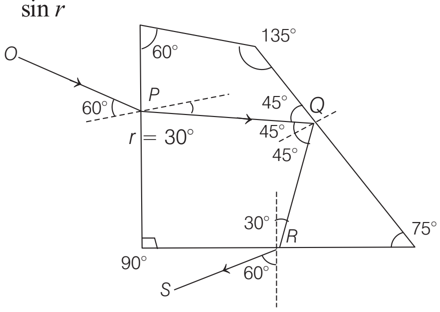
24. In medium-2 wavefront bends away from the normal after refraction. Therefore, ray of light which is perpendicular to wavefront bends towards the normal in medium-2 during refraction. So, medium-2 is denser or its speed in medium-1 is more.



25. **(A)**, **(C)** and **(D)** In case of concave mirror or convex lens image can be real, virtual, diminished magnified or of same size.

(B) In case of convex mirror image is always virtual (for real object).

27. $\sqrt{3} = \dfrac{\sin 60°}{\sin r}$



$\therefore \qquad r = 30°$

$$\theta_C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{ or } \sin\theta_C = \frac{1}{\sqrt{3}}$$

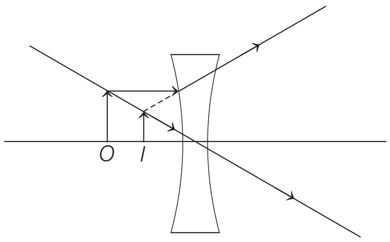
$$= 0.577$$

At point Q, angle of incidence inside the prism is $i = 45°$.

Since $\sin i = \dfrac{1}{\sqrt{2}}$ is greater than $\sin\theta_C = \dfrac{1}{\sqrt{2}}$, ray gets totally internally reflected at face CD. Path of ray of light after point Q is shown in figure.

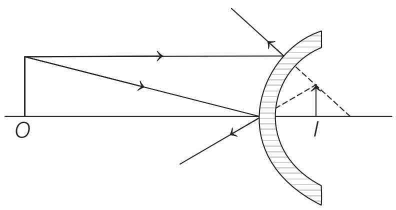
From the figure, we can see that angle between incident ray OP and emergent ray RS is 90°.

28. For a lens $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$, i.e. $\dfrac{1}{v} = \dfrac{1}{f} + \dfrac{1}{u}$



For a concave lens, f and u are negative, i.e. v will always be negative and image will always be virtual.

For a mirror :



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ i.e. } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Here, f is positive and u is negative for a convex mirror. Therefore, v is always positive and image is always virtual.

29. When upper half of the lens is covered, image is formed by the rays coming from lower half of the lens. Or image will be formed by less number of rays. Therefore, intensity of image will decrease. But complete image will be formed.

30. Speed of light in vacuum, $c = \dfrac{1}{\sqrt{\varepsilon_0\mu_0}}$

and speed of light in medium, $v = \dfrac{1}{\sqrt{\varepsilon\mu}}$

Therefore, refractive index of the medium is

$$\mu = \frac{c}{v} = \frac{1/\sqrt{\varepsilon_0\mu_0}}{1/\sqrt{\varepsilon\mu}} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

31. $I \propto \dfrac{1}{r^2}$ (in case of point source)

and $\quad I \propto A^2$

$\Rightarrow \qquad A \propto \dfrac{1}{r}$

or $\qquad \dfrac{A_1}{A_2} = \dfrac{r_2}{r_1} = \dfrac{25}{9}$

32. At a distance r from a line source of power P and length l, the intensity will be,

$$I = \frac{P}{S} = \frac{P}{2\pi rl}$$

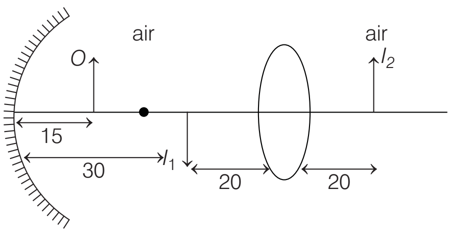
or $\qquad I \propto \dfrac{1}{r}$

33. Case I

Reflection from mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \Rightarrow \quad \frac{1}{-10} = \frac{1}{v} + \frac{1}{-15}$$

$\Rightarrow \qquad v = -30$



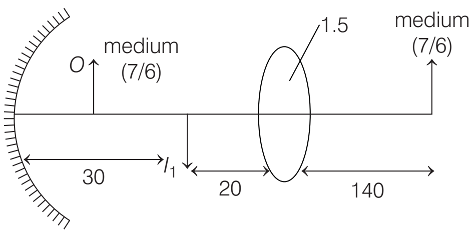
For lens $\qquad \dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-20}$$

$$v = 20$$

$$|M_1| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{20}{20}\right) = 2 \times 1 = 2 \qquad \text{(in air)}$$

Case II For mirror, there is no change.



For lens, $\qquad \dfrac{1}{f_{air}} = \left(\dfrac{3/2}{1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$$\frac{1}{f_{medium}} = \left(\frac{3/2}{7/6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

with $\qquad f_{air} = 10\,\text{cm}$

We get $\qquad \dfrac{1}{f_{medium}} = \dfrac{4}{70}\,\text{cm}^{-1}$

$$\frac{1}{v} - \frac{1}{-20} = \frac{4}{70}$$

$$\frac{1}{v} + \frac{1}{20} = \left(\frac{2}{7}\right)\left(\frac{2}{10}\right) = \frac{4}{70}$$

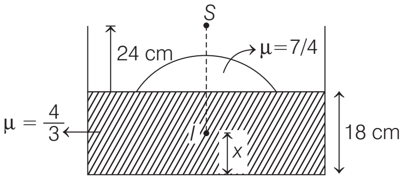
$$\frac{1}{v} = \frac{4}{70} - \frac{1}{20} \quad \Rightarrow \quad v = 140,$$

$$|\text{M}_2| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{140}{20}\right),$$

$$= (2)\left(\frac{140}{20}\right) = 14$$

$$\Rightarrow \qquad \left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7$$

34.



Two refractions will take place, first from spherical surface and the other from the plane surface.

So, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

two times with proper sign convention.

Ray of light is travelling downwards. Therefore, downward direction is taken as positive direction.

$$\frac{7/4}{v} - \frac{1.0}{-24} = \frac{7/4 - 1.0}{+6} \qquad \text{...(i)}$$

$$\frac{4/3}{(18 - x)} - \frac{7/4}{v} = \frac{4/3 - 7/4}{\infty} \qquad \text{...(ii)}$$

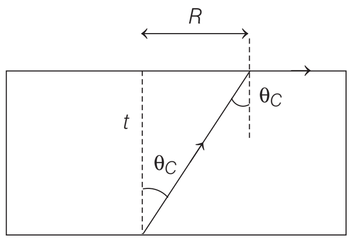
Solving these equations, we get, $x = 2\,\text{cm}$

∴ Answer is 2.

35. $\frac{R}{t} = \tan\theta_C$ or $R = t\,(\tan\theta_C)$

But, $\sin\theta_C = \frac{1}{\mu} = \frac{3}{5}$

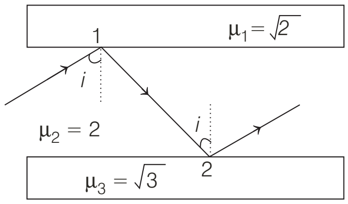
∴ $\tan\theta_C = \frac{3}{4}$



∴ $R = \frac{3}{4}t = \frac{3}{4}(8\,\text{cm}) = 6\text{ cm}$

Hence the answer is 6.

36. Critical angles at 1 and 2



$$\theta_{C_1} = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

$$\theta_{C_2} = \sin^{-1}\left(\frac{\mu_3}{\mu_2}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$$

Therefore, minimum angle of incidence for total internal reflection to take place on both slabs should be 60°.

$$i_{\min} = 60°$$

37. Applying Snell's law on face AB,

$$(1)\sin 45° = (\sqrt{2})\sin r$$

∴ $\sin r = \frac{1}{2}$

or $r = 30°$

i.e. ray becomes parallel to AD inside the block.

Now applying,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \text{ on face } CD,$$

$$\frac{1.514}{OE} - \frac{\sqrt{2}}{\infty} = \frac{1.514 - \sqrt{2}}{0.4}$$

Solving this equation, we get $OE = 6.06\,\text{m}$

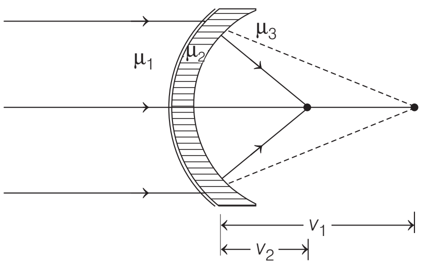
38. For refraction at first surface,

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \qquad \text{...(i)}$$

For refraction at second surface,

$$\frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \qquad \text{... (ii)}$$

Adding Eqs. (i) and (ii), we get



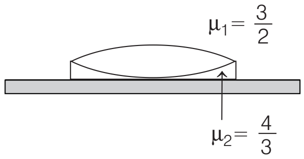
$$\frac{\mu_3}{v_2} = \frac{\mu_3 - \mu_1}{R}$$

or $$v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

Therefore, focal length of the given lens system is

$$\frac{\mu_3 R}{\mu_3 - \mu_1}.$$

39. Let R be the radius of curvature of both the surfaces of the equi-convex lens. In the first case :



Let f_1 be the focal length of equi-convex lens of refractive index μ_1 and f_2 the focal length of plano-concave lens of refractive index μ_2. The focal length of the combined lens system will be given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$= \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) + \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R}\right)$$

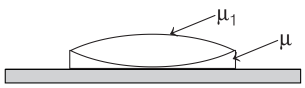
$$= \frac{1}{R} - \frac{1}{3R} = \frac{2}{3R} \quad \text{or} \quad F = \frac{3R}{2}$$

Now, image coincides with the object when ray of light retraces its path or it falls normally on the plane mirror. This is possible only when object is at focus of the lens system.

Hence, $F = 15\,\text{cm}$ (Distance of object $= 15$ cm)

or $\quad \frac{3R}{2} = 15\,\text{cm} \quad$ or $\quad R = 10\,\text{cm}$

In the second case, let μ be the refractive index of the liquid filled between lens and mirror and let F' be the focal length of new lens system. Then,



$$\frac{1}{F'} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

or $\quad \frac{1}{F'} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) - \frac{(\mu - 1)}{R}$

or $\quad = \frac{1}{R} - \frac{\mu - 1}{R} = \frac{(2 - \mu)}{R}$

$\therefore \quad F' = \frac{R}{2 - \mu} = \frac{10}{2 - \mu} \qquad (\because R = 10\,\text{cm})$

Now, the image coincides with object when it is placed at 25 cm distance.

Hence, $\quad F' = 25$

or $\quad \frac{10}{2 - \mu} = 25$

or $\quad 50 - 25\mu = 10$

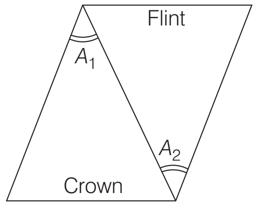
or $\quad 25\mu = 40$

$\therefore \quad \mu = \frac{40}{25} = 1.6$

or $\quad \mu = 1.6$

40. (a) When angle of prism is small and angle of incidence is also small, the deviation is given by $\delta = (\mu - 1)A$.

Net deviation by the two prisms is zero. So,



$$\delta_1 + \delta_2 = 0$$

or $\quad (\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0 \qquad \dots \text{(i)}$

Here, μ_1 and μ_2 are the refractive indices for crown and flint glasses respectively.

Hence, $\quad \mu_1 = \frac{1.51 + 1.49}{2} = 1.5$

and $\quad \mu_2 = \frac{1.77 + 1.73}{2} = 1.75$

$A_1 =$ Angle of prism for crown glass $= 6°$

Substituting the values in Eq. (i), we get

$$(1.5 - 1)(6°) + (1.75 - 1)A_2 = 0$$

This gives $A_2 = -4°$

Hence, angle of flint glass prism is 4° (Negative sign shows that flint glass prism is inverted with respect to the crown glass prism.)

(b) Net dispersion due to the two prisms is

$$= (\mu_{b_1} - \mu_{r_1})A_1 + (\mu_{b_2} - \mu_{r_2})A_2$$
$$= (1.51 - 1.49)(6°) + (1.77 - 1.73)(-4°) = -0.04°$$

$\therefore$ Net dispersion is – 0.04°

41. (a) Rays coming from object AB first refract from the lens and then reflect from the mirror.

Refraction from the lens

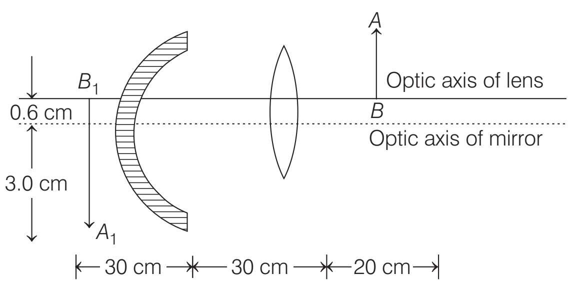
$$u = -20\,\text{cm}, \; f = +15\,\text{cm}$$

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-20} = \frac{1}{15}$

$\therefore \quad v = +60\,\text{cm}$

and linear magnification, $m_1 = \frac{v}{u} = \frac{+60}{-20} = -3$

i.e. first image formed by the lens will be at 60 cm from it (or 30 cm from mirror) towards left and 3 times magnified but inverted. Length of first image A_1B_1 would be $1.2 \times 3 = 3.6\,\text{cm}$ (inverted).



Reflection from mirror Image formed by lens (A_1B_1) will behave like a virtual object for mirror at a distance of 30 cm from it as shown. Therefore $u = +30\,\text{cm}$, $f = -30\,\text{cm}$.

Using mirror formula, $\dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f}$ or $\dfrac{1}{v}+\dfrac{1}{30}=-\dfrac{1}{30}$

$\therefore \qquad v = -15\,\text{cm}$

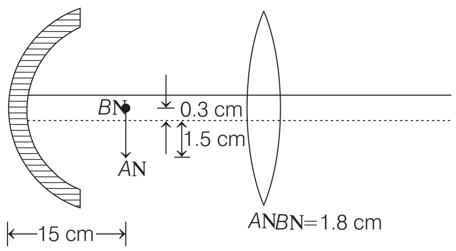
and linear magnification,

$$m_2 = -\frac{v}{u} = -\frac{-15}{+30} = +\frac{1}{2}$$

i.e. final image $A'B'$ will be located at a distance of 15 cm from the mirror (towards right) and since magnification is $+\dfrac{1}{2}$, length of final image would be $3.6\times\dfrac{1}{2} = 1.8\,\text{cm}$.

$\therefore \qquad A'B' = 1.8\,\text{cm}$

Point B_1 is 0.6 cm above the optic axis of mirror, therefore, its image B' would be $(0.6)\left(\dfrac{1}{2}\right) = 0.3$ cm above optic axis. Similarly, point A_1 is 3 cm below the optic axis, therefore, its image A' will be $3\times\dfrac{1}{2} = 1.5\,\text{cm}$ below the optic axis as shown below



Total magnification of the image,

$$m = m_1 \times m_2 = (-3)\left(+\frac{1}{2}\right) = -\frac{3}{2}$$

$$\therefore \quad A'B' = (m)(AB) = \left(-\frac{3}{2}\right)(1.2) = -1.8\,\text{cm}$$

Note that, there is no need of drawing the ray diagram if not asked in the question.

NOTE With reference to the pole of an optical instrument (whether it is a lens or a mirror) the coordinates of the object (X_o, Y_o) are generally known to us. The corresponding coordinates of image (X_i, Y_i) are found as follows

X_i is obtained using $\dfrac{1}{v}\pm\dfrac{1}{u}=\dfrac{1}{f}$

Here, v is actually X_i and u is X_o ie, the above formula can be written as $\dfrac{1}{X_i}\pm\dfrac{1}{X_o}=\dfrac{1}{f}$

Similarly, Y_i is obtained from $m=\dfrac{I}{O}$

Here, I is Y_i and O is Y_o *i.e.,* the above formula can be written as $m = Y_i / Y_o$ or $Y_i = mY_o$.

42. From lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

we have $\qquad \dfrac{1}{0.3} = \left(\dfrac{3}{2}-1\right)\left(\dfrac{1}{R}-\dfrac{1}{-R}\right)$

(Here, $R_1 = R$ and $R_2 = -R$)

$R = 0.3$

Now applying $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$ at air glass surface, we get

$$\frac{3/2}{v_1}-\frac{1}{-(0.9)}=\frac{3/2-1}{0.3}$$

$\therefore \qquad v_1 = 2.7\,\text{m}$

i.e. first image I_1 will be formed at 2.7 m from the lens. This will act as the virtual object for glass water surface. Therefore, applying $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$ at glass water surface, we have

$$\frac{4/3}{v_2}-\frac{3/2}{2.7}=\frac{4/3-3/2}{-0.3}$$

$\therefore \qquad v_2 = 1.2\,\text{m}$

i.e. second image I_2 is formed at 1.2 m from the lens or 0.4 m from the plane mirror. This will act as a virtual object for mirror. Therefore, third real image I_3 will be formed at a distance of 0.4 m in front of the mirror after reflection from it. Now this image will work as a real object for water-glass interface. Hence, applying

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

we get $\quad \dfrac{3/2}{v_4}-\dfrac{4/3}{-(0.8-0.4)}=\dfrac{3/2-4/3}{0.3}$

$\therefore \qquad v_4 = -0.54\,\text{m}$

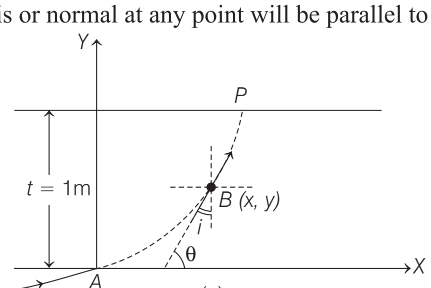
i.e. fourth image is formed to the right of the lens at a distance of 0.54 m from it. Now finally applying the same formula for glass-air surface,

$$\frac{1}{v_5}-\frac{3/2}{-0.54}=\frac{1-3/2}{-0.3}$$

$\therefore \qquad v_5 = -0.9\,\text{m}$

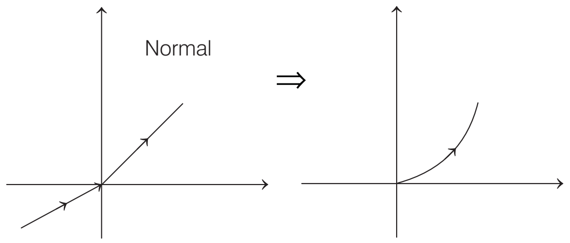
i.e. position of final image is 0.9 m relative to the lens (rightwards) or the image is formed 0.1 m behind the mirror.

43. (a) Refractive index is a function of y. It varies along Y-axis i.e. the boundary separating two media is parallel to X-axis or normal at any point will be parallel to Y-axis.



(a)

Secondly, refractive index increases as y is increased. Therefore, ray of light is travelling from rarer to denser medium i.e. it will bend towards the normal and shape of its trajectory will be as shown below.



(b)

Now, refer to figure (a)

Let i be the angle of incidence at any point B on its path

$$\theta = 90° - i \quad \text{or} \quad \tan\theta = \tan(90° - i) = \cot i$$

or $\quad$ slope $= \cot i$

(b) but $\tan\theta = \dfrac{dy}{dx} \quad \Rightarrow \quad \therefore \quad \dfrac{dy}{dx} = \cot i \qquad \text{... (i)}$

Applying Snell's law at A and B

$n_A \sin i_A = n_B \sin i_B \;\Rightarrow\; n_A = 1$ because $y = 0$

$\sin i_A = 1$ because $i_A = 90°$ $\qquad$ (Grazing incidence)

$$n_B = \sqrt{ky^{3/2} + 1} = \sqrt{y^{3/2} + 1}$$

because $\;k = 1.0(\text{m})^{-3/2}$

$$\therefore \quad (1)(1) = \sqrt{(y^{3/2} + 1)}\sin i \;\Rightarrow\; \sin i = \frac{1}{\sqrt{y^{3/2} + 1}}$$

$$\therefore \quad \cot i = \sqrt{y^{3/2}} \quad \text{or} \quad y^{3/4} \qquad \text{... (ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{dy}{dx} = y^{3/4} \quad \text{or} \quad y^{-3/4}dy = dx$$

$$\text{or} \int_0^y y^{-3/4}dy = \int_0^x dx \quad \text{or} \quad 4y^{1/4} = x \qquad \text{... (iii)}$$

The required equation of trajectory is $4y^{1/4} = x$.

(c) At point P, where the ray emerges from the slab

$$y = 1.0\,\text{m}$$

$$\therefore \quad x = 4.0\,\text{m} \qquad \text{[From Eq. (iii)]}$$

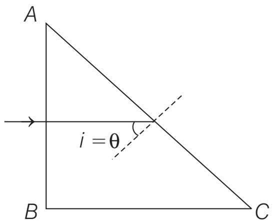
Therefore, coordinates of point P are

$$P = (4.0\,\text{m}, 1.0\,\text{m})$$

(d) As $n_A \sin i_A = n_P \sin i_p$ and as $n_A = n_P = 1$

Therefore, $i_P = i_A = 90°$ i.e. the ray will emerge parallel to the boundary at P i.e. at grazing emergence.

44. (a) Total internal reflection (TIR) will take place first for that wavelength for which critical angle is small or μ is large.



From the given expression of μ, it is more for the wavelength for which value of λ is less.

Thus, condition of TIR is just satisfied for 4000 Å.

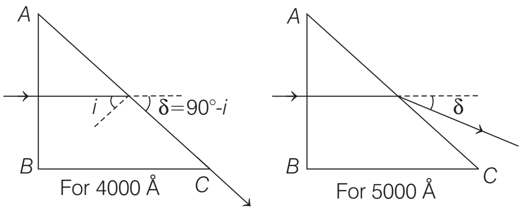
or $\quad i = \theta_c$ for 4000Å $\quad$ or $\quad \theta = \theta_c \quad$ or $\quad \sin\theta = \sin\theta_c$

or $\quad 0.8 = \dfrac{1}{\mu}$ $\qquad$ (for 4000Å)

$$\text{or} \quad 0.8 = \frac{1}{1.20 + \dfrac{b}{(4000)^2}}$$

Solving this equation, we get $b = 8.0 \times 10^5 (\text{Å})^2$

(b)



For, 4000Å condition of TIR is just satisfied. Hence, it will emerge from AC, just grazingly.

or $\delta_{4000\text{Å}} = 90° - i = 90° - \sin^{-1}(0.8) \approx 37°$

$$\text{For 5000Å} \quad \mu = 1.2 + \frac{b}{\lambda^2} = 1.2 + \frac{8.0 \times 10^5}{(5000)^2} = 1.232$$

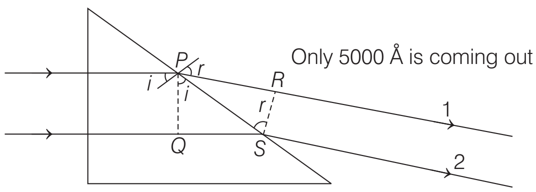
$$\text{Applying} \quad \mu = \frac{\sin i_{\text{air}}}{\sin i_{\text{medium}}} \quad \text{or} \quad 1.232 = \frac{\sin i_{\text{air}}}{\sin\theta}$$

$$= \frac{\sin i_{\text{air}}}{0.8} \;\Rightarrow\; i_{\text{air}} = 80.26°$$

$$\therefore \quad \delta_{5000\text{Å}} = i_{\text{air}} - i_{\text{medium}}$$

$$= 80.26° - \sin^{-1}(0.8) = 27.13°$$

(c)



Path difference between rays 1 and 2

$$\Delta x = \mu(QS) - PR \qquad \text{...(i)}$$

$$\text{Further,} \quad \frac{QS}{PS} = \sin i \;\Rightarrow\; \frac{PR}{PS} = \sin r$$

$$\therefore \quad \frac{PR/PS}{QS/PS} = \frac{\sin r}{\sin i} = \mu \;\Rightarrow\; \mu\,(QS) = PR$$

Substituting in Eq. (i), we get $\Delta x = 0$.

$\therefore$ Phase difference between rays 1 and 2 will be zero.

Or these two rays will interfere constructively. So, maximum intensity will be obtained from their interference.

$$\text{or} \quad I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{4I} + \sqrt{I})^2 = 9I$$

NOTE In this question we have written,

$$\mu = \frac{\sin r}{\sin i} \text{ not } \frac{\sin i}{\sin r}$$

because in medium angle with normal is i and in air angle with normal is r.

$$\text{or} \quad \mu = \frac{\sin i_{\text{air}}}{\sin i_{\text{medium}}}$$

45. Given θ is slightly greater than $\sin^{-1}\left(\frac{n_1}{n_2}\right)$

(a) **When $n_3 < n_1$**

i.e. $n_3 < n_1 < n_2$

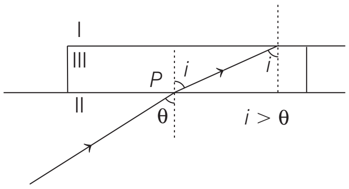
or $\frac{n_3}{n_2} < \frac{n_1}{n_2}$ or $\sin^{-1}\left(\frac{n_3}{n_2}\right) < \sin^{-1}\left(\frac{n_1}{n_2}\right)$

Hence, critical angle for III and II will be less than the critical angle for II and I. So, if TIR is taking place between I and II, then TIR will definitely take place between I and III.

(b) **When $n_3 > n_1$** Now two cases may arise :

Case 1 $n_1 < n_3 < n_2$

In this case there will be no TIR between II and III but TIR will take place between III and I. This is because



Ray of light first enters from II to III *ie,* from denser to rarer.

$\therefore$ $i > \theta$

Applying Snell's law at P

$$n_2 \sin\theta = n_3 \sin i \text{ or } \sin i = \left(\frac{n_2}{n_3}\right)\sin\theta$$

Since, $\sin\theta$ is slightly greater than $\frac{n_1}{n_2}$

$\therefore$ $\sin i$ is slightly greater than $\frac{n_2}{n_3} \times \frac{n_1}{n_2}$ or $\frac{n_1}{n_3}$

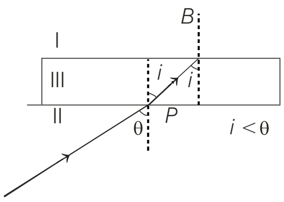
but $\frac{n_1}{n_3}$ is nothing but $\sin(\theta_c)_{\text{I, III}}$

$\therefore$ $\sin(i)$ is slightly greater than $\sin(\theta_c)_{\text{I, III}}$

Or TIR will now take place on I and III and the ray will be reflected back.

Case 2 $n_1 < n_2 < n_3$

This time while moving from II to III, ray of light will bend towards normal. Again applying Snell's law at P



$$n_2 \sin\theta = n_3 \sin i$$

$\Rightarrow$ $\sin i = \frac{n_2}{n_3}\sin\theta$

Since, $\sin\theta$ slightly greater than $\frac{n_1}{n_2}$

$\sin i$ will be slightly greater than $\frac{n_2}{n_3} \times \frac{n_1}{n_2}$ or $\frac{n_1}{n_3}$

but $\frac{n_1}{n_3}$ is $\sin(\theta_c)_{\text{I, III}}$

i.e. $\sin i > \sin(\theta_c)_{\text{I, III}}$ or $i > (\theta_c)_{\text{I, III}}$

Therefore, TIR will again take place between I and III and the ray will be reflected back.

NOTE Case I and case 2 of $n_3 > n_1$ can be explained by one equation only. But two cases are deliberately formed for better understanding of refraction, Snell's law and total internal reflection (TIR).

46. Resultant intensity at P

$$I_P = I_A + I_B + I_C$$
$$= \frac{P_A}{4\pi(PA)^2} + \frac{P_B}{4\pi(PB)^2}\cos 60° + I_C \cos 60°$$
$$= \frac{90}{4\pi(3)^2} + \frac{180}{4\pi\,(1.5)^2}\cos 60° + 20\cos 60°$$
$$= 0.79 + 3.18 + 10 = 13.97 \text{ W/m}^2$$

47. (a) Image of object will coincide with it if ray of light after refraction from the concave surface fall normally on concave mirror so formed by silvering the convex surface. Or image after refraction from concave surface should form at centre of curvature of concave mirror or at a distance of 20 cm on same side of the combination. Let x be the distance of pin from the given optical system.

Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

With proper signs, $\frac{1.5}{-20} - \frac{1}{-x} = \frac{1.5 - 1}{-60}$

or $\frac{1}{x} = \frac{3}{40} - \frac{1}{120} = \frac{8}{120} \Rightarrow x = \frac{120}{8} = 15 \text{ cm}$

(b) Now, before striking with the concave surface, the ray is first refracted from a plane surface. So, let x be the distance of pin, then the plane surface will form its image at a distance $\frac{4}{3}x$ ($h_{\text{app.}} = \mu h$) from it.

Now, using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper signs,

we have $\frac{1.5}{-20} - \frac{4/3}{-\frac{4x}{3}} = \frac{1.5 - 4/3}{-60}$

or $\frac{1}{x} = \frac{3}{40} - \frac{1}{360}$ or $x = 13.84$ cm

$\therefore$ $\Delta x = x_1 - x_2 = 15 \text{ cm} - 13.84 \text{ cm}$

$= 1.16 \text{ cm}$ (downwards)

Download Chapter Test
http://tinyurl.com/y248rq8u *or*

13

Current Electricity

Topic 1 Equivalent Resistance, Drift Velocity, Resistivity and Conductivity

Objective Questions I (Only one correct option)

1. Space between two concentric conducting spheres of radii a and $b(b > a)$ is filled with a medium of resistivity ρ. The resistance between the two spheres will be **(2019 Main, 10 April II)**

(a) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (b) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

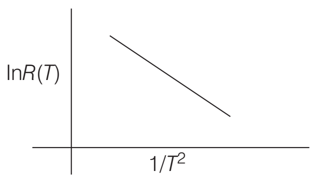
(c) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (d) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

2. A current of 5 A passes through a copper conductor (resistivity $= 1.7 \times 10^{-8}$ Ω-m) of radius of cross-section 5 mm. Find the mobility of the charges, if their drift velocity is 1.1×10^{-3} m/s. **(2019 Main, 10 April I)**

(a) 1.5 m^2 / V-s (b) 1.3 m^2 / V-s

(c) 1.0 m^2 / V-s (d) 1.8 m^2 / V-s

3. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line. **(2019 Main, 10 April I)**



One may conclude that

(a) $R(T) = R_0 e^{-T^2/T_0^2}$ (b) $R(T) = R_0 e^{T^2/T_0^2}$

(c) $R(T) = R_0 e^{-T_0^2/T^2}$ (d) $R(T) = \frac{R_0}{T^2}$

4. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60º at the centre, the equivalent resistance between these two points will be **(Main 2019, 9 April II)**

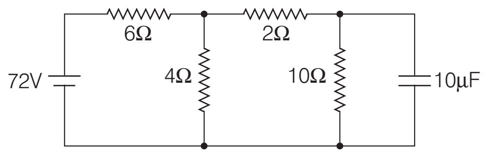
(a) $\frac{7}{2}\Omega$ (b) $\frac{5}{2}\Omega$ (c) $\frac{12}{5}\Omega$ (d) $\frac{5}{3}\Omega$

5. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25 fs (femto second), it's approximate resistivity is (Take, $m_e = 9.1 \times 10^{-31}$ kg) **(2019 Main, 9 April II)**

(a) 10^{-7} Ω-m (b) 10^{-5} Ω-m

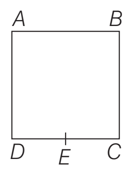
(c) 10^{-6} Ω-m (d) 10^{-8} Ω-m

6. Determine the charge on the capacitor in the following circuit **(2019 Main, 9 April I)**



(a) 2μC (b) 200μC (c) 10 μC (d) 60 μC

7. A wire of resistance R is bent to form a square $ABCD$ as shown in the figure. The effective resistance between E and C is [E is mid-point of arm CD] **(2019 Main, 9 April I)**



(a) $\frac{7}{64}R$ (b) $\frac{3}{4}R$

(c) R (d) $\frac{1}{16}R$

8. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is **(2019 Main, 10 Jan I)**

(a) 12 Ω (b) 8 Ω

(c) 2 Ω (d) 4 Ω

9. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance, if its volume remains unchanged is **(2019 Main, 9 Jan I)**

(a) 2.0% (b) 1.0% (c) 0.5% (d) 2.5%

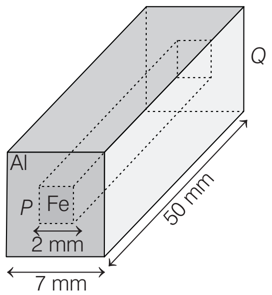
10. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If for an n - type semiconductor, the density of electrons is $10^{19} m^{-3}$ and their mobility is $1.6 \, m^2$ (V-s), then the resistivity of the semiconductor (since, it is an n-type semiconductor contribution of holes is ignored) is close to **(2019 Main, 9 Jan I)**

(a) 2 Ω-m (b) 0.2 Ω-m (c) 0.4 Ω-m (d) 4Ω-m

11. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross-section 5 mm^2 is v. If the electron density in copper is $9 \times 10^{28} / m^3$, the value of v in mm/s is close to (Take, charge of electron to be = 1.6×10^{-19} C) **(201 9Main, 9 Jan I)**

(a) 0.02 (b) 0.2 (c) 2 (d) 3

12. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ωm and 1.0×10^{-7} Ωm, respectively. The electrical resistance between the two faces P and Q of the composite bar is **(2015 Adv.)**



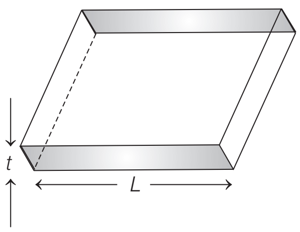
(a) $\frac{2475}{64} \mu \Omega$ (b) $\frac{1875}{64} \mu \Omega$ (c) $\frac{1875}{49} \mu \Omega$ (d) $\frac{2475}{132} \mu \Omega$

13. When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is $2.5 \times 10^{-4} ms^{-1}$. If the electron density in the wire is $8 \times 10^{28} \, m^{-3}$ the resistivity of the material is close to **(2015 Main)**

(a) 1.6×10^{-8} Ωm (b) 1.6×10^{-7} Ωm

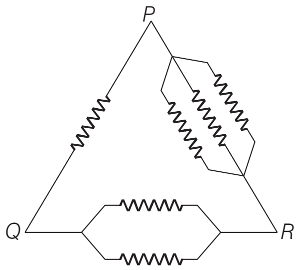
(c) 1.6×10^{-5} Ωm (d) 1.6×10^{-6} Ωm

14. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is **(2010)**



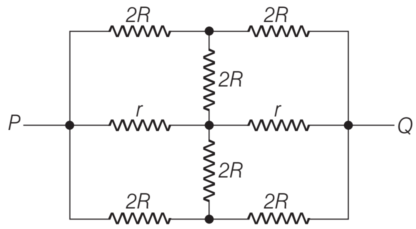
(a) directly proportional to L
(b) directly proportional to t
(c) independent of L
(d) independent of t

15. Six equal resistances are connected between points P,Q and R as shown in the figure. Then, the net resistance will be maximum between **(2004, 2M)**



(a) P and Q
(b) Q and R
(c) P and R
(d) any two points

16. The effective resistance between points P and Q of the electrical circuit shown in the figure is **(2002, 2M)**



(a) $\frac{2Rr}{R + r}$ (b) $\frac{8R(R + r)}{3R + r}$

(c) $2r + 4R$ (d) $\frac{5R}{2} + 2r$

17. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities constant along the length of the conductor is/are **(1997, 1M)**

(a) current, electric field and drift speed
(b) drift speed only
(c) current and drift speed
(d) current only

18. Read the following statements carefully **(1993, 2M)**

Y : The resistivity of semiconductor decreases with increase of temperature.

Z : In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature.

Select the correct statement (s) from the following

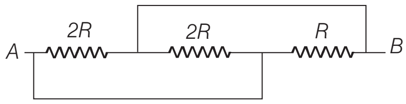
(a) Y is true but Z is false
(b) Y is false but Z is true\
(c) Both Y and Z are true
(d) Y is true and Z is the correct reason for Y

19. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of **(1988, 1M)**

(a) each of them increases
(b) each of them decreases
(c) copper increases and germanium decreases
(d) copper decreases and germanium increases

Fill in the Blank

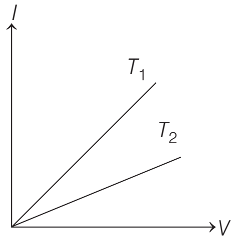
20. The equivalent resistance between points A and B of the circuit given below is ……. . **(1997, 2M)**



True/False

21. The current-voltage graphs for a given metallic wire at two different temperatures T_1 and T_2 are shown in the figure.

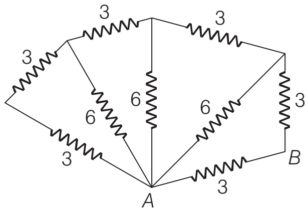
The temperature T_2 is greater than T_1 **(1985, 3M)**



22. Electrons in a conductor have no motion in the absence of a potential difference across it. **(1982, 2M)**

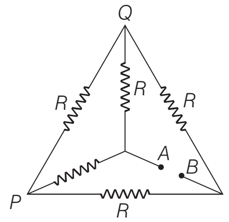
Analytical & Descriptive Questions

23. All resistances in the diagram are in ohm. **(1980)**



Find the effective resistance between the points A and B.

24. If each of the resistances in the network shown in the figure is R, what is the resistance between the terminals A and B ? **(1978)**

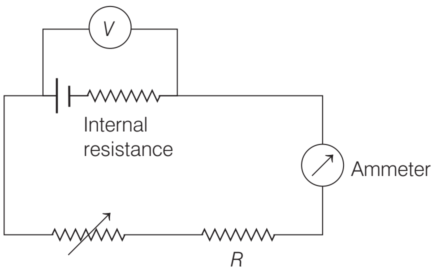


25. A copper wire is stretched to make it 0.1% longer. What is the percentage change in its resistance ? **(1978)**

Topic 2 Kirchhoff's Laws and Combination of Batteries

Objective Questions I (Only one correct option)

1. To verify Ohm's law, a student connects the voltmeter across the battery as shown in the figure. The measured voltage is plotted as a function of the current and the following graph is obtained **(2019, Main, 12 April I)**



If V_0 is almost zero, then identify the correct statement.

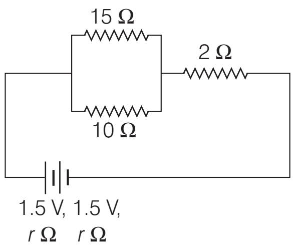
(a) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

(b) The value of the resistance R is 1.5 Ω

(c) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

(d) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

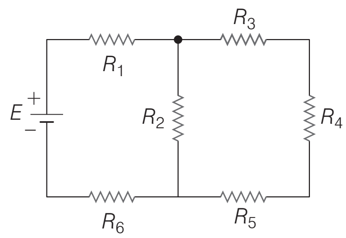
2. In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2 V. The internal resistance r, of each cell is **(2019 Main, 10 April I)**



(a) 1.5 Ω (b) 0.5 Ω (c) 1 Ω (d) 0 Ω

3. In the figure shown, what is the current (in ampere) drawn from the battery? You are given :

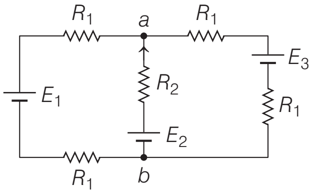
$R_1 = 15\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 20\,\Omega$, $R_4 = 5\,\Omega$, $R_5 = 25\,\Omega$, $R_6 = 30\,\Omega$, $E = 15\,V$ **(2019 Main, 8 April II)**



(a) 13/24 (b) 7/18

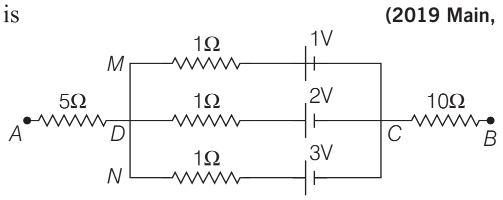
(c) 20/3 (d) 9/32

4. For the circuit shown with $R_1 = 1.0\Omega$, $R_2 = 2.0\Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points a and b is approximately (in volt) **(2019 Main, 8 April I)**



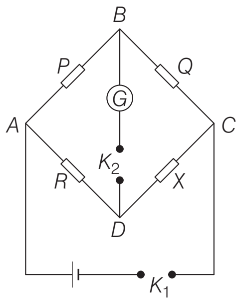
(a) 2.7 (b) 2.3 (c) 3.7 (d) 3.3

5. In the circuit shown, the potential difference between A and B is **(2019 Main, 11 Jan II)**



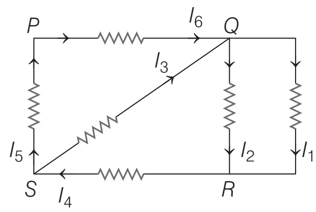
(a) 3 V (b) 1 V (c) 6 V (d) 2 V

6. In a Wheatstone bridge (see figure), resistances P and Q are approximately equal. When $R = 400\Omega$, the bridge is balanced. On interchanging P and Q, the value of R for balance is 405Ω. The value of X is close to **(2019 Main, 11 Jan I)**



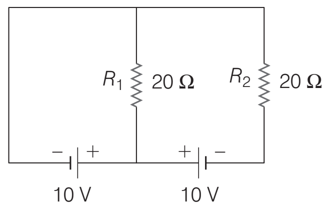
(a) 404.5 Ω (b) 401.5 Ω
(c) 402.5 Ω (d) 403.5 Ω

7. In the given circuit diagram, the currents $I_1 = -0.3$ A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2, I_3 and I_6 respectively, are **(2019 Main, 10 Jan II)**



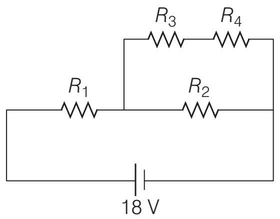
(a) 1.1 A, 0.4 A, 0.4 A (b) 1.1 A, – 0.4 A, 0.4 A
(c) 0.4 A, 1.1 A, 0.4 A (d) – 0.4 A, 0.4 A, 1.1 A

8. In the given circuit, the cells have zero internal resistance. The currents (in Ampere) passing through resistances R_1 and R_2 respectively are **(2019 Main, 10 Jan I)**



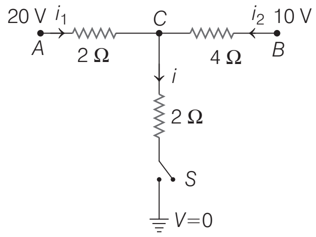
(a) 0.5, 0 (b) 1, 2
(c) 2, 2 (d) 0, 1

9. In the given circuit, the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be **(2019 Main, 9 Jan II)**



(a) 550 Ω (b) 230 Ω
(c) 300 Ω (d) 450 Ω

10. When the switch S in the circuit shown is closed, then the value of current i will be **(2019 Main, 9 Jan I)**

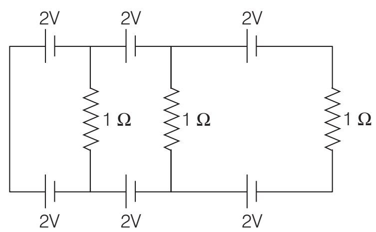


(a) 4A (b) 3A
(c) 2A (d) 5A

11. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω, respectively. The voltage across the load lies between

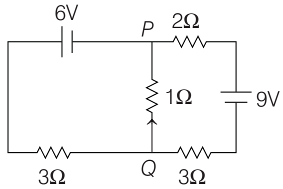
(a) 11.7 V and 11.8 V (b) 11.6 V and 11.7 V
(c) 11.5 V and 11.6 V (d) 11.4 V and 11.5 V

12. In the below circuit, the current in each resistance is **(2017 Main)**



(a) 0.25 A (b) 0.5 A
(c) 0 A (d) 1 A

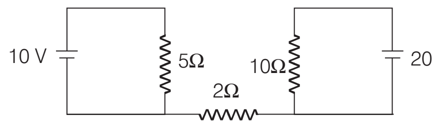
13. In the circuit shown below, the current in the 1Ω resistor is



(2015 Main)

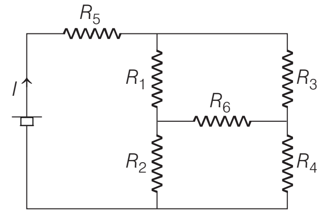
(a) 1.3 A, from P to Q (b) 0.13 A, from Q to P
(c) 0 A (d) 0.13 A, from P to Q

14. Find out the value of current through $2\,\Omega$ resistance for the given circuit. **(2005, 2M)**



(a) 5 A (b) 2 A (c) zero (d) 4 A

15. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then, the resistance values must satisfy **(2001, 2M)**



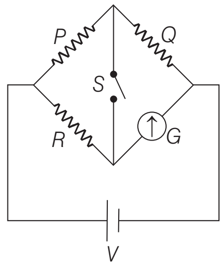
(a) $R_1 R_2 R_5 = R_3 R_4 R_6$

(b) $\dfrac{1}{R_5} + \dfrac{1}{R_6} = \dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3 + R_4}$

(c) $R_1 R_4 = R_2 R_3$

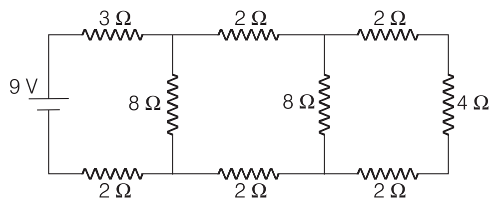
(d) $R_1 R_3 = R_2 R_4$

16. In the circuit shown $P \neq R$, the reading of galvanometer is same with switch S open or closed. Then **(1999, 2M)**



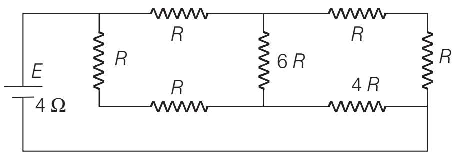
(a) $I_R = I_G$ (b) $I_P = I_G$ (c) $I_Q = I_G$ (d) $I_Q = I_R$

17. In the circuit shown in the figure, the current through **(1998, 2M)**



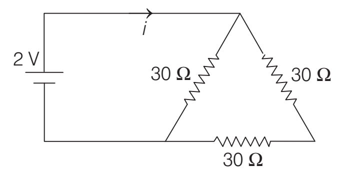
(a) the $3\,\Omega$ resistor is 0.50 A
(b) the $3\,\Omega$ resistor is 0.25 A
(c) the $4\,\Omega$ resistor is 0.50 A
(d) the $4\,\Omega$ resistor is 0.25 A

18. A battery of internal resistance $4\,\Omega$ is connected to the network of resistances as shown in figure. In order that the maximum power can be delivered to the network, the value of R in Ω should be **(1995, 2M)**



(a) $\dfrac{4}{9}$ (b) 2 (c) $\dfrac{8}{3}$ (d) 18

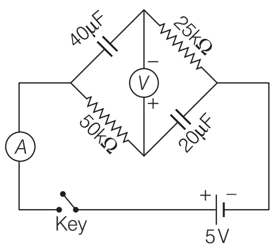
19. The current i in the circuit (see figure) is **(1983, 1M)**



(a) $\dfrac{1}{45}$ A (b) $\dfrac{1}{15}$ A (c) $\dfrac{1}{10}$ A (d) $\dfrac{1}{5}$ A

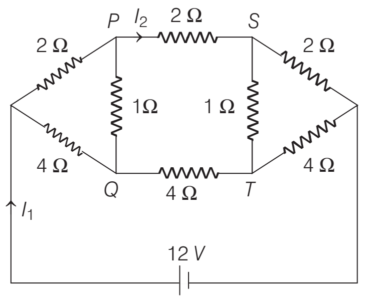
Objective Questions II (One or more correct option)

20. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is (are) true? **(2016 Adv.)**



(a) The voltmeter display -5 V as soon as the key is pressed and displays $+5$ V after a long time
(b) The voltmeter will display 0 V at time $t = \ln 2$ seconds
(c) The current in the ammeter becomes $1/e$ of the initial value after 1 second
(d) The current in the ammeter becomes zero after a long time

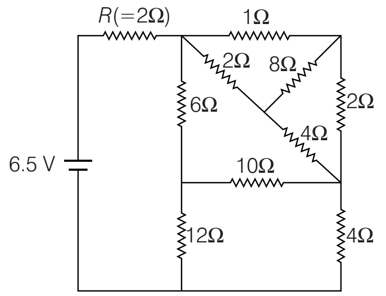
21. For the resistance network shown in the figure, choose the correct option(s). **(2012)**



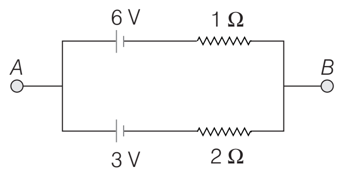
(a) The current through PQ is zero
(b) $I_1 = 3\,A$
(c) The potential at S is less than that at Q
(d) $I_2 = 2\,A$

Integer Answer Type Questions

22. In the following circuit, the current through the resistor $R(= 2\Omega)$ is I amperes. The value of I is **(2015 Adv.)**

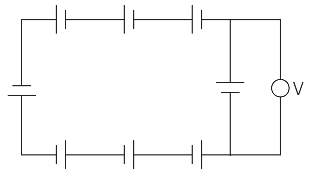


23. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volt is **(2011)**



Fill in the Blanks

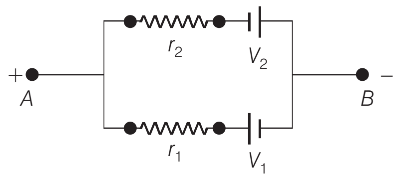
24. In the circuit shown below, each battery is 5 V and has an internal resistance of 0.2 Ω. **(1997, 2M)**



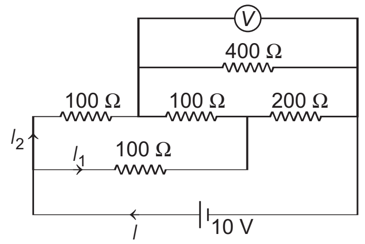
The reading in the ideal voltmeter V is ……V.

Anlytical & Descriptive Questions

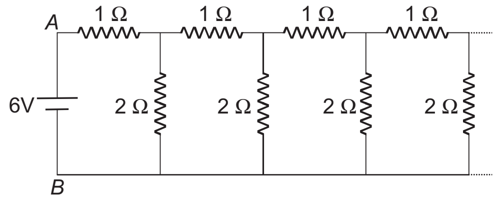
25. Find the emf (V) and internal resistance (r) of a single battery which is equivalent to a parallel combination of two batteries of emfs V_1 and V_2 and internal resistances r_1 and r_2 respectively, with polarities as shown in figure **(1997C, 5M)**



26. An electrical circuit is shown in figure. Calculate the potential difference across the resistor of 400 Ω as will be measured by the voltmeter V of resistance 400 Ω either by applying Kirchhoff's rules or otherwise. **(1996, 5M)**



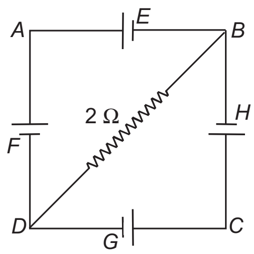
27. An infinite ladder network of resistances is constructed with 1 Ω and 2 Ω resistances, as shown in figure.



The 6 V battery between A and B has negligible internal resistance. **(1987, 7M)**

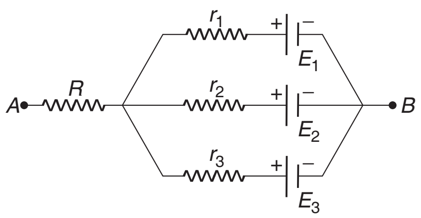
(a) Show that the effective resistance between A and B is 2Ω.
(b) What is the current that passes through the 2 Ω resistance nearest to the battery?

28. In the circuit shown in figure E, F, G, H are cells of emf 2, 1, 3 and 1 V respectively, and their internal resistances are 2, 1, 3 and 1 Ω respectively.
Calculate **(1984, 4M)**



(a) the potential difference between B and D and
(b) the potential difference across the terminals of each cells G and H.

29. In the circuit shown in figure $E_1 = 3\,V$, $E_2 = 2\,V$, $E_3 = 1\,V$ and $R = r_1 = r_2 = r_3 = 1\,\Omega$. **(1981, 6M)**



(a) Find the potential difference between the points A and B and the currents through each branch.
(b) If r_2 is short-circuited and the point A is connected to point B, find the currents through E_1, E_2, E_3 and the resistor R.

Topic 3 Heat and Power Generation

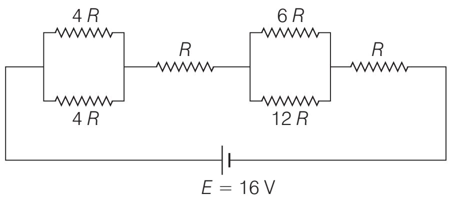
Objective Questions I (Only one correct option)

1. One kilogram of water at 20°C is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to

[Specific heat of water = 4200 J/(kg°C), Latent heat of water = 2260 kJ/kg] **(2019 Main, 12 April II)**

(a) 16 min (b) 22 min (c) 3 min (d) 10 min

2. The resistive network shown below is connected to a DC source of 16 V. The power consumed by the network is 4 W. The value of R is **(2019 Main, 12 April I)**



(a) 6 Ω (b) 8 Ω (c) 1 Ω (d) 16 Ω

3. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when **(8 April 2019, II)**

(a) $R = 2r$ (b) $R = r$ (c) $R = 0.001\,r$ (d) $R = 1000\,r$

4. Two electric bulbs rated at 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then **(2019 Main, 12 Jan I)**

(a) $P_1 = 16$ W, $P_2 = 4$ W (b) $P_1 = 4$ W, $P_2 = 16$ W
(c) $P_1 = 9$ W, $P_2 = 16$ W (d) $P_1 = 16$ W, $P_2 = 9$ W

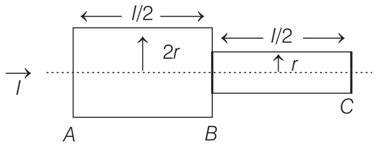
5. Two equal resistances when connected in series to a battery consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be **(2019 Main, 11 Jan I)**

(a) 60 W (b) 30 W (c) 240 W (d) 120 W

6. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is **(2019 Main, 10 Jan II)**

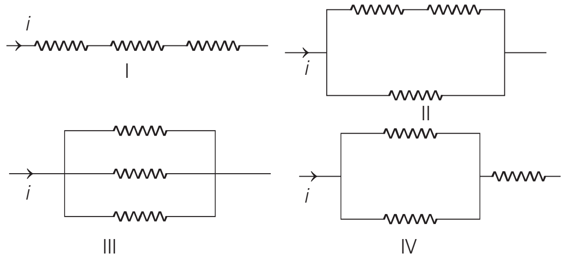
(a) 11×10^{-4} W (b) 11×10^{-5} W
(c) 11×10^{5} W (d) 11×10^{-3} W

7. Two bars of radius r and $2r$ are kept in contact as shown. An electric current I is passed through the bars. Which one of following is correct ? **(2006, 3M)**



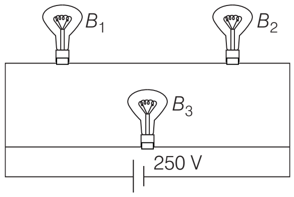
(a) Heat produced in bar BC is 4 times the heat produced in bar AB
(b) Electric field in both halves is equal
(c) Current density across AB is double that of across BC
(d) Potential difference across AB is 4 times that of across BC

8. The three resistances of equal value are arranged in the different combinations shown below. Arrange them in increasing order of power dissipation **(2003, 2M)**



(a) III < II < IV < I (b) II < III < IV < I
(c) I < IV < III < II (d) I < III < II < IV

9. A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source, as shown in the figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3 respectively. Then, **(2002, 2M)**

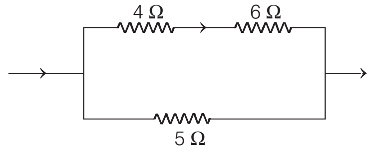


(a) $W_1 > W_2 = W_3$ (b) $W_1 > W_2 > W_3$
(c) $W_1 < W_2 = W_3$ (d) $W_1 < W_2 < W_3$

10. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time. The value of N is **(2001, 2M)**

(a) 4 (b) 6 (c) 8 (d) 9

11. In the circuit shown in figure the heat produced in the 5 Ω resistor due to the current flowing through it is 10 cal/s. **(1981, 2M)**



The heat generated in the 4 Ω resistor is

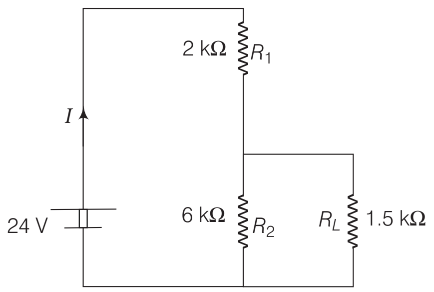
(a) 1 cal/s (b) 2 cal/s (c) 3 cal/s (d) 4 cal/s

Objective Questions II (One or more correct option)

12. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true? **(2016 Adv.)**

(a) The temperature distribution over the filament is uniform.
(b) The resistance over small sections of the filament decreases with time.
(c) The filament emits more light at higher band of frequencies before it breaks up.
(d) The filament consumes less electrical power towards the end of the life of the bulb.

13. For the circuit shown in the figure **(2009)**



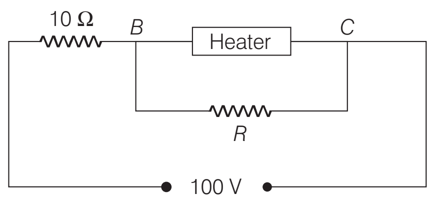
(a) the current I through the battery is 7.5 mA
(b) the potential difference across R_L is 18 V
(c) ratio of powers dissipated in R_1 and R_2 is 3
(d) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

Fill in the Blank

14. An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb delivers 500 W is ……Ω. **(1987, 2M)**

Analytical & Descriptive Question

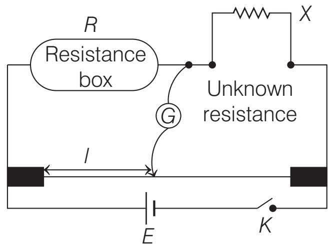
15. A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected in combination with a resistance of 10 Ω and a resistance R, to a 100 V mains as shown in the figure. What will be the value of R so that the heater operates with a power of 62.5 W ? **(1978)**



Topic 4 Electrical Instruments

Objective Questions I (Only one correct option)

1. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure **(2019 Main, 10 April I)**

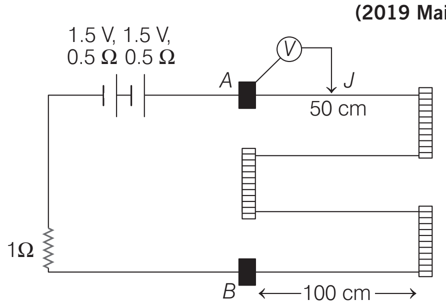


S. No.	R (Ω)	l (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent?

(a) 3 (b) 2
(c) 1 (d) 4

2. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01$ Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be **(2019 Main, 8 April II)**



(a) 0.20 V (b) 0.75 V (c) 0.25 V (d) 0.50V

3. A galvanometer whose resistance is 50 Ω, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of **(Main 2019, 12 Jan II)**

(a) 250 Ω (b) 6200 Ω (c) 200 Ω (d) 6250 Ω

4. In a meter bridge, the wire of length 1m has a non-uniform cross-section such that the variation $\frac{dR}{dl}$ of its resistance R with length l is $\frac{dR}{dl} \propto \frac{1}{\sqrt{l}}$.

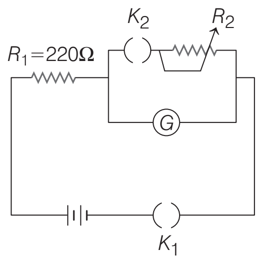
Two equal resistance are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP? **(2019 Main, 12 Jan I)**

(a) 0.3 m (b) 0.25 m (c) 0.2 m (d) 0.35 m

5. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5 Ω. The value of R to give a potential difference of 5 mV across 10 cm of potentiometer wire is **(2019 Main, 12 Jan I)**

(a) 395 Ω (b) 495 Ω (c) 490 Ω (d) 480 Ω

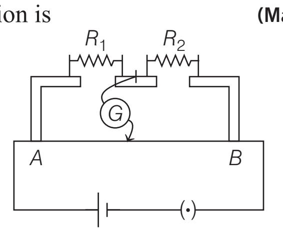
6. The galvanometer deflection, when key K_1 is closed but K_2 is open equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is given by (neglect the internal resistance of battery) : **(Main 2019, 12 Jan I)**



(a) 22Ω (b) 5Ω (c) 25Ω (d) 12Ω

7. In the experimental set up of meter bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10 Ω resistor is connected in series with R_1, the null point shifts by 10 cm.

The resistance that should be connected in parallel with $(R_1 + 10)$ Ω such that the null point shifts back to its initial position is **(Main 2019, 11 Jan II)**

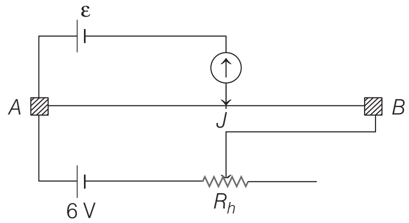


(a) 60 Ω (b) 20 Ω (c) 30 Ω (d) 40 Ω

8. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt is **(2019 Main, 11 Jan II)**

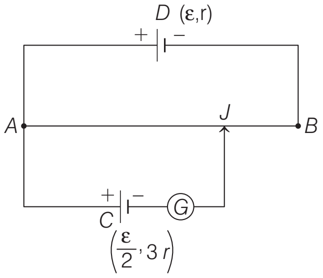
(a) 100 Ω (b) 125 Ω (c) 120 Ω (d) 80 Ω

9. The resistance of the meter bridge AB in given figure is 4 Ω. With a cell of emf $\varepsilon = 0.5$ V and rheostat resistance $R_h = 2\Omega$. The null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$, the same null point J is found for $R_h = 6\Omega$. The emf ε_2 is **(2019 Main, 11 Jan I)**



(a) 0.6 V (b) 0.3 V (c) 0.5 V (d) 0.4 V

10. A potentiometer wire AB having length L and resistance $12r$ is joined to a cell D of EMF ε and internal resistance r. A cell C having emf $\frac{\varepsilon}{2}$ and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is **(2019 Main, 10 Jan I)**



(a) $\frac{5}{12}L$ (b) $\frac{11}{12}L$

(c) $\frac{13}{24}L$ (d) $\frac{11}{24}L$

11. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances? **(2018 Main)**

(a) 990 Ω (b) 910 Ω
(c) 900 Ω (d) 550 Ω

12. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **(2018 Main)**

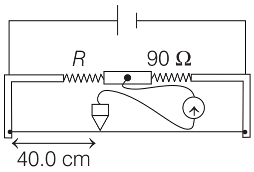
(a) 1 Ω (b) 2 Ω
(c) 1.5 Ω (d) 2.5 Ω

13. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is **(2017 Main)**

(a) 2.045×10^3 Ω (b) 2.535×10^3 Ω
(c) 4.005×10^3 Ω (d) 1.985×10^3 Ω

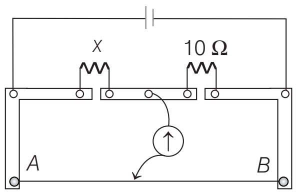
14. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is **(2016 Main)**
(a) 0.01 Ω (b) 2 Ω (c) 0.1 Ω (d) 3 Ω

15. During an experiment with a meter bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as show in the scale used in the meter bridge is 1 mm. The unknown resistance is **(2014 Adv.)**



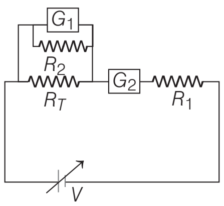
(a) $60 \pm 0.15\ \Omega$ (b) $135 \pm 0.56\ \Omega$
(c) $60 \pm 0.25\ \Omega$ (d) $135 \pm 0.23\ \Omega$

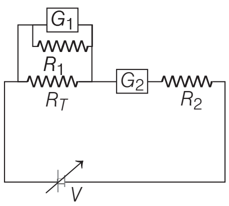
16. A meter bridge is set-up as shown in figure, to determine an unknown resistance X using a standard 10 Ω resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is **(2011)**

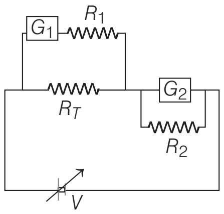


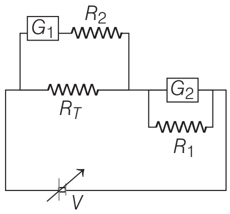
(a) 10.2 Ω (b) 10.6 Ω (c) 10.8 Ω (d) 11.1 Ω

17. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is **(2010)**

(a)


(b)


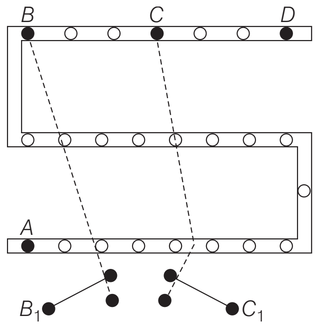
(c)


(d)


18. A resistance of 2Ω is connected across one gap of a meter-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is **(2007, 3M)**
(a) 3 Ω (b) 4 Ω (c) 5 Ω (d) 6 Ω

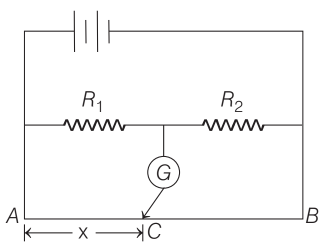
19. A moving coil galvanometer of resistance 100 Ω is used as an ammeter using a resistance 0.1 Ω. The maximum deflection current in the galvanometer is 100 μA. Find the current in the circuit, so that the ammeter shows maximum deflection. **(2005, 2M)**
(a) 100.1 mA (b) 1000.1 mA
(c) 10.01 mA (d) 1.01 mA

20. For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between **(2004, 2M)**



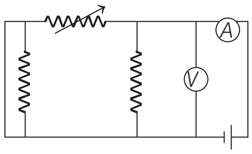
(a) B and C (b) C and D
(c) A and D (d) B_1 and C_1

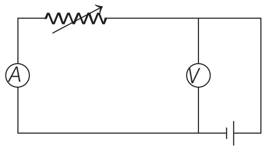
21. In the shown arrangement of the experiment of the meter bridge if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled ? **(2003, S)**

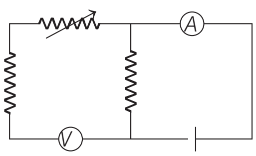


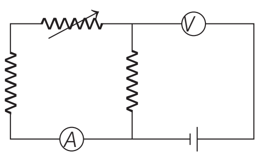
(a) x (b) $x/4$
(c) $4x$ (d) $2x$

22. Which of the following set-up can be used to verify Ohm's law ? **(2003, 2M)**

(a)


(b)


(c)


(d)


Assertion and Reason

Mark your answer as

(a) *If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I*
(b) *If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I*
(c) *If Statement I is true; Statement II is false*
(d) *If Statement I is false; Statement II is true*

23. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **(2013 Main)**

Statement I Higher the range, greater is the resistance of ammeter.

Statement II To increase the range of ammeter, additional shunt needs to be used across it.

24. **Statement I** In a meter bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

Statement II Resistance of a metal increases with increase in temperature. **(2008, 3M)**

Objective Questions II
(One or more correct option)

25. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_c < R/2$, which of the following statement(s) about anyone of the galvanometers is (are) true? **(2016 Adv.)**

(a) The maximum voltage range is obtained when all the components are connected in series
(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer
(c) The maximum current range is obtained when all the components are connected in parallel
(d) The maximum current range is obtained when the two galvanometers are connected in series, and the combination is connected in parallel with both the resistors

26. A microammeter has a resistance of 100 Ω and full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination (s) **(1991, 2M)**

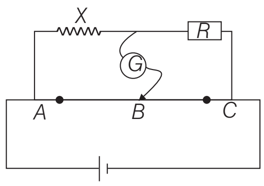
(a) 50 V range with 10 k Ω resistance in series
(b) 10 V range with 200 k Ω resistance in series
(c) 5 mA range with 1 Ω resistance in parallel
(d) 10 mA range with 1 Ω resistance in parallel

Integer Answer Type Question

27. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0-30 V. If connected to a $\frac{2n}{249}$ Ω resistance, it becomes an ammeter of range 0-1.5 A. The value of n is **(2014 Adv.)**

Analytical & Descriptive Questions

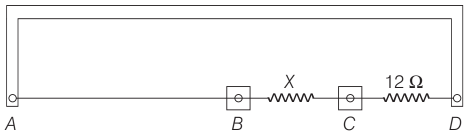
28. R_1, R_2, R_3 are different values of R. A, B, C are the null points obtained corresponding to R_1, R_2 and R_3 respectively. For which resistor, the value of X will be the most accurate and why? **(2005, 2M)**



29. Draw the circuit for experimental verification of Ohm's law using a source of variable DC voltage, a main resistance of 100 Ω, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter. **(2004, 4M)**

30. Show by diagram, how can we use a rheostat as the potential divider ? **(2003, 2M)**

31. A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in the figure. A battery and galvanometer (with a sliding jockey connected to it are also available). Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions. **(2002, 5M)**



(a) Are there positive and negative terminals on the galvanometer?
(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.
(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

32. Two resistors, 400 Ω and 800 Ω are connected in series with a 6 V battery. It is desired to measure the current in the circuit. An ammeter of 10 Ω resistance is used for this purpose. What will be the reading in the ammeter ? Similarly, if a voltmeter of 1000 Ω resistance is used to measure the potential difference across the 400 Ω resistor, what will be the reading in the voltmeter ? **(1982, 6M)**

Topic 5 Miscellaneous Problems

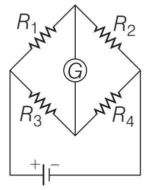
Objective Questions I (Only one correct option)

1. A 200 Ω resistor has a certain colour code. If one replaces the red colour by green in the code, the new resistance will be **(2019 Main, 8 April I)**

(a) 100 Ω (b) 400 Ω
(c) 300 Ω (d) 500 Ω

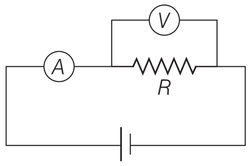
2. The Wheatstone bridge shown in figure here, gets balanced when the carbon resistor is used as R_1 has the color code (orange, red, brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω, respectively.

Assuming that the color code for the carbon resistors gives their accurate values, the color code for the carbon resistor is used as R_3 would be **(2019 Main, 10 Jan II)**



(a) brown, blue, black (b) brown, blue, brown
(c) grey, black, brown (d) red, green, brown

3. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is **(2019 Main, 10 Jan II))**

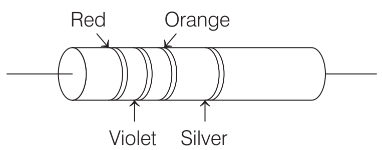


(a) 600 Ω (b) 570 Ω
(c) 350 Ω (d) 35 Ω

4. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is **(2019 Main, 10 Jan I)**

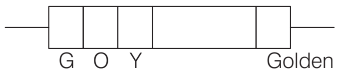
(a) 0.4 mA (b) 63 mA
(c) 20 mA (d) 100 mA

5. A resistance is shown in the figure. Its value and tolerance are given respectively by **(2019 Main, 9 April I)**



(a) 270 Ω, 5% (b) 27 k Ω, 20%
(c) 27 k Ω, 10% (d) 270 k Ω, 10%

6. A carbon resistance has a following color code. What is the value of the resistance?



(a) 5.3 MΩ ± 5% (b) 64 kΩ ± 10%
(c) 6.4 MΩ ± 5% (d) 530 kΩ ± 5%

7. Which of the following statements is false? **(2017 Main)**

(a) In a balanced Wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed
(b) A rheostat can be used as a potential divider
(c) Kirchhoff's second law represents energy conservation
(d) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

8. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be **(2014 Main)**

(a) 8 A (b) 10 A (c) 12 A (d) 14 A

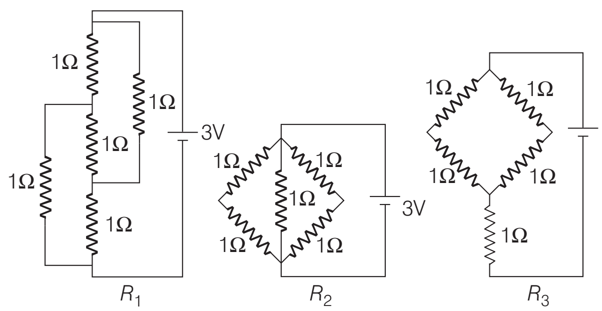
9. The supply voltage in a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb? **(2013 Main)**

(a) zero (b) 2.9 V
(c) 13.3 V (d) 10.4V

10. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is **(2010)**

(a) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (b) $R_{100} = R_{40} + R_{60}$
(c) $R_{100} > R_{60} > R_{40}$ (d) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

11. Figure shows three resistor configurations R_1, R_2 and R_3 connected to 3 V battery. If the power dissipated by the configuration R_1, R_2 and R_3 is P_1, P_2 and P_3, respectively, then **(2008, 3M)**



(a) $P_1 > P_2 > P_3$ (b) $P_1 > P_3 > P_2$
(c) $P_2 > P_1 > P_3$ (d) $P_3 > P_2 > P_1$

12. A rigid container with thermally insulated walls contains a coil of resistance 100 Ω, carrying current 1 A. Change in internal energy after 5 min will be **(2005, 2M)**
(a) zero (b) 10 kJ (c) 20 kJ (d) 30 kJ

Objective Question II (One or more correct option)

13. When a potential difference is applied across, the current passing through **(1999, 3M)**
(a) an insulator at 0 K is zero
(b) a semiconductor at 0 K is zero
(c) a metal at 0 K is finite
(d) a *p-n* diode at 300 K is finite, if it is reverse biased

Value Based Type Question

14. A moving coil galvanometer has 50 turns and each turn has an area $2 \times 10^{-4}\, m^2$. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is $10^{-4} N$ - m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs, if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range $0-1.0$ A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance in ohms, is ....

Integer Answer Type Questions

15. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25 J_2$ then the value of R in Ω is **(2010)**

Analytical & Desrcriptive Questions

16. A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor? **(1982q, 2M)**

17. A copper wire having cross-sectional area of 0.5 mm^2 and a length of 0.1 m is initially at 25°C and is thermally insulated from the surrounding. If a current of 1.0 A is set up in this wire, (a) find the time in which the wire will start melting. The change of resistance with the temperature of the wire may be neglected. (b) What will this time be, if the length of the wire is doubled ? **(1979)**

Melting point of copper $= 1075°$ C, Specific resistance of copper $= 1.6 \times 10^{-8}$ Ωm, Density of copper $= 9 \times 10^3$ kg/m^3, Specific heat of copper $= 9 \times 10^{-2}$ cal/kg°C

18. A 25 W and a 100 W bulb are joined in series and connected to the mains. Which bulb will glow brighter ? **(1979)**

Answers

Topic 1

1. (b) **2.** (c) **3.** (c) **4.** (d)
5. (d) **6.** (b) **7.** (a) **8.** (d)
9. (b) **10.** (c) **11.** (a)
12. (b) **13.** (c) **14.** (c) **15.** (a)
16. (a) **17.** (d) **18.** (c) **19.** (d)
20. $R/2$ **21.** T **22.** F **23.** 2 Ω
24. R **25.** 0.2% increase

Topic 2

1. (a) **2.** (b) **3.** (d) **4.** (d)
5. (d) **6.** (c) **7.** (a) **8.** (a)
9. (c) **10.** (d) **11.** (c)
12. (c) **13.** (b) **14.** (c) **15.** (c)
16. (a) **17.** (d) **18.** (b) **19.** (c)
20. (a,b,c,d) **21.** (a, b, c, d) **22.** 1
23. 5 **24.** zero
25. $V = \dfrac{V_1 r_2 - V_2 r_1}{r_1 + r_2}, r = \dfrac{r_1 r_2}{r_1 + r_2}$
26. $\dfrac{20}{3}$ V **27.** (b) 1.5 A
28. (a) $\dfrac{2}{13}$ V, (b) $\dfrac{21}{13}$ V, $\dfrac{19}{13}$ V
29. (a) 2 V, 1A, 0, – 1A (b) 1A, 2A, – 1A, 2A

Topic 3

1. (b) **2.** (b) **3.** (b) **4.** (a)
5. (c) **6.** (b) **7.** (a) **8.** (a)
9. (d) **10.** (b) **11.** (b) **12.** (c, d)
13. (a, d) **14.** 20 Ω **15.** 5 Ω

Topic 4

1. (d) **2.** (c) **3.** (c) **4.** (b)
5. (a) **6.** (a) **7.** (a) **8.** (d)
9. (b) **10.** (c) **11.** (d) **12.** (c)
13. (d) **14.** (a) **15.** (c) **16.** (b)
17. (c) **18.** (a) **19.** (a) **20.** (c)
21. (a) **22.** (a) **23.** (d) **24.** (d)
25. (a,c) **26.** (b, c) **27.** 5
28. B is most accurate answer
31. (a) No (c) 8 Ω
32. 4.96 mA, 1.58 V

Topic 5

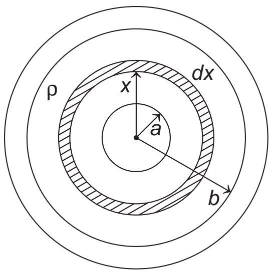
1. (d) **2.** (b) **3.** (b) **4.** (c)
5. (c) **6.** (d) **7.** (a) **8.** (c)
9. (d) **10.** (d) **11.** (c) **12.** (d)
13. (a, b, d) **14.** (5.55) **15.** (4)
16. Yes **17.** (a) 55.5 s (b) 55.5 s **18.** 25 W bulb

Hints & Solutions

Topic 1 Equivalent Resistance, Drift Velocity, Resistivity and Conductivity

1. **Key Idea** Resistance between surface of inner shell and a circumferential point of outer shell can be formed by finding resistance of a thin (differentially thin) shell in between these two shells. Then, this result can be integrated (summed up) to get resistance of the complete arrangement.

For an elemental shell of radius x and thickness dx,



Resistance, $dR = \rho \dfrac{l}{A}$

$\Rightarrow \quad dR = \rho \dfrac{dx}{4\pi x^2}$

So, resistance of complete arrangement is

$$R = \int_a^b dR = \int_a^b \rho \frac{dx}{4\pi x^2} = \frac{\rho}{4\pi}\int_a^b x^{-2} dx$$

$$\Rightarrow \quad R = \frac{\rho}{4\pi}\left(\frac{x^{-1}}{-1}\right)_a^b = \frac{\rho}{4\pi}\left(-\frac{1}{x}\right)_a^b$$

$$= \frac{\rho}{4\pi}\left(-\frac{1}{b} - \left(-\frac{1}{a}\right)\right) = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ohm}$$

2. Given, $I = 5\text{A}$,

$\rho = 1.7 \times 10^{-8}\ \Omega\text{-m}$,

$r = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$,

$v_d = 1.1 \times 10^{-3}\text{ m/s}$

Mobility of charges in a conductor is given by

$$\mu = \frac{v_d}{E} \qquad \ldots\text{(i)}$$

and resistivity is given by

$$\rho = \frac{E}{J} = \frac{E}{I/A} \qquad \left(\because J = \sigma E = \frac{1}{\rho} \times E\right)$$

$$\Rightarrow \quad \rho = \frac{EA}{I}$$

or $$E = \frac{\rho I}{A} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

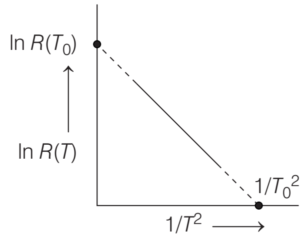
$$\mu = \frac{v_d A}{\rho I}$$

Substituting the given values, we get

$$= \frac{1.1 \times 10^{-3} \times \pi \times (5 \times 10^{-3})^2}{1.7 \times 10^{-8} \times 5}$$

$$= \frac{86.35 \times 10^{-9}}{8.5 \times 10^{-8}} = 10.1 \times 10^{-1} \Rightarrow \mu \approx 1\ \text{m}^2/\text{V-s}$$

3. From the given graph,



We can say that, $\ln R(T) \propto -\dfrac{1}{T^2}$

Negative sign implies that the slope of the graph is negative.

or $$\ln R(T) = \text{constant}\left(-\frac{1}{T^2}\right)$$

$$\Rightarrow \quad \ln R(T) = \frac{\exp(\text{const.})}{\exp\left(\dfrac{1}{T^2}\right)}$$

$$\Rightarrow \quad \ln R(T) = R_0 \exp\left(-\frac{T_0^2}{T^2}\right)$$

Alternate Solution

From graph,

$$\frac{\dfrac{1}{T^2}}{\dfrac{1}{T_0^2}} + \frac{\ln R(T)}{\ln R(T_0)} = 1$$

$$\Rightarrow \quad \ln R(T) = [\ln R(T_0)] \cdot \left[1 - \frac{T_0^2}{T^2}\right]$$

or $$R(T) = R_0 \exp\left(\frac{-T_0^2}{T^2}\right)$$

4. Initial resistance of wire is $3\ \Omega$. Let its length is l and area is A.

Then, $$R_{\text{initial}} = \rho \frac{l}{A} = 3\ \Omega \qquad \ldots\text{(i)}$$

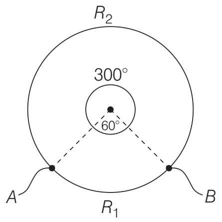
When wire is stretched twice its length, then its area becomes A', on equating volume, we have

$$Al = A'2l \Rightarrow A' = \frac{A}{2}$$

So, after stretching, resistance of wire will be

$$R' = R_{\text{final}} = \rho\frac{l'}{A'} = 4\rho\frac{l}{A} = 12\ \Omega \qquad \text{[using Eq. (i)]}$$

Now, this wire is made into a circle and connected across two points A and B (making 60º angle at centre) as



Now, above arrangement is a combination of two resistances in parallel,

$$R_1 = \frac{60 \times R'}{360} = \frac{1}{6} \times 12 = 2\ \Omega$$

and $$R_2 = \frac{300}{360} \times R' = \frac{5}{6} \times 12 = 10\ \Omega$$

Since, R_1 and R_2 are connected in parallel.

So, $$R_{AB} = \frac{R_1R_2}{R_1 + R_2} = \frac{10 \times 2}{12} = \frac{5}{3}\ \Omega$$

5. Resistivity of a conductor is

$$\rho = \frac{m_e}{ne^2\tau} \qquad \ldots \text{(i)}$$

where, m_e = mass of electron = 9.1×10^{-31} kg,

n = free charge density = $8.5 \times 10^{28}\ \text{m}^{-3}$,

τ = mean free time = 25 fs = 25×10^{-15} s

and e = charge of electron = 1.6×10^{-19} C

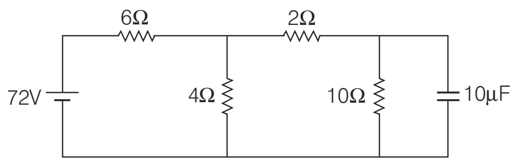
Substituting values in Eq. (i), we get

$$\rho = \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-15}}$$

$$= \frac{9.1 \times 10^{-6}}{8.5 \times 2.56 \times 25} = 0.016 \times 10^{-6}$$

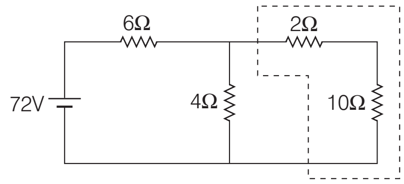
$$= 1.6 \times 10^{-8}\ \Omega\text{-m} \simeq 10^{-8}\ \Omega\text{-m}$$

6. Given circuit is



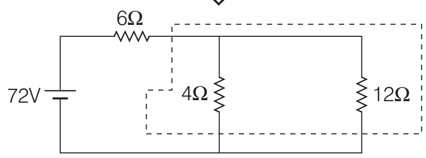
To find charge on capacitor, we need to determine voltage across it.

In steady state, capacitor will acts as open circuit and circuit can be reduced as



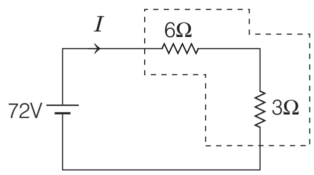
In series, $R_{\text{eq}} = 2\Omega + 10\Omega = 12\Omega$

$$\Downarrow$$

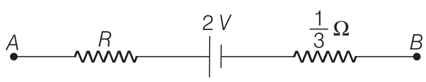


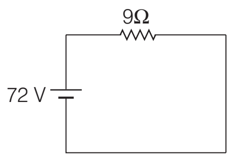
In parallel, $R_{\text{eq}} = \dfrac{4 \times 12}{4 + 12} = 3\Omega$

$$\Downarrow$$



In series, $R_{\text{eq}} = 6\Omega + 3\Omega = 9\Omega$



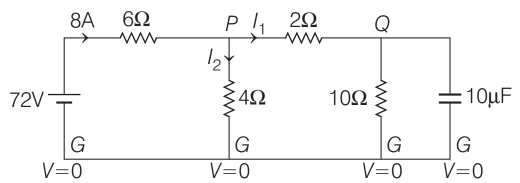


So, current in steady state, $I = \dfrac{V}{R} = \dfrac{72}{9} = 8\text{A}$

Now, by using current division, at point P, current in 6 Ω branch is

$$\frac{72 - V_P}{6\Omega} = 8\text{A}$$

$$\Rightarrow \qquad V_P = 72 - 48 = 24\ \text{V}$$



Current in 4 Ω branch is,

$$I_2 = \frac{V_P - 0}{4} = \frac{24 - 0}{4} = 6\Omega$$

So, current in 2 Ω resistance is

$$I_1 = 8 - I_2 \qquad [\because I = I_1 + I_2]$$

$$= 8 - 6 = 2\text{A}$$

$\therefore$ Potential difference across 10Ω resistor is

$$V_{QG} = 2\text{A} \times 10\,\Omega = 20\,\text{V}$$

Same potential difference will be applicable over the capacitor (parallel combination).

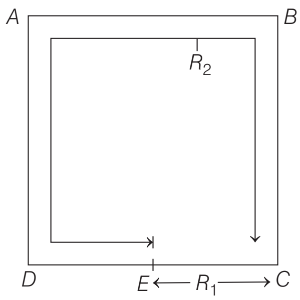
So, charge stored in the capacitor will be

$$Q = CV = 10 \times 10^{-6} \times 20$$

$$\Rightarrow \qquad Q = 2 \times 10^{-4}\,\text{C} = 200\,\mu\text{C}$$

7. Let the length of each side of square $ABCD$ is a.

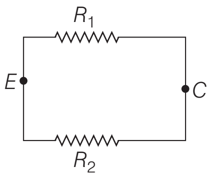
$\therefore$ Resistance per unit length of each side $= \dfrac{R}{4a}$



Now, $\qquad R_1 = \dfrac{R}{4a}[EC] = \dfrac{R}{4a} \times \dfrac{a}{2} = \dfrac{R}{8}$

Similarly, $\qquad R_2 = \dfrac{R}{4a}[EDABC] = \dfrac{R}{4a} \times \dfrac{7a}{2} = \dfrac{7R}{8}$

Now, effective resistance between E and C is the equivalent resistance of R_1 and R_2 that are connected in parallel as shown below.



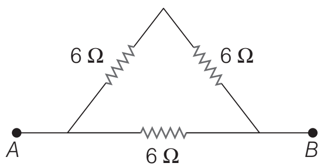
$$R_{EC} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(R/8) \times (7R/8)}{(R/8) + (7R/8)}$$

$$= \frac{7R^2}{64} \times \frac{8}{8R} = \frac{7}{64}R$$

8. Resistance of each arm of equilateral triangle will be

$$R = \frac{18}{3} = 6\,\Omega$$

So we have following combination will be



Equivalent resistance is

$$\therefore \qquad R_{AB} = \frac{12 \times 6}{12 + 6} = \frac{12 \times 6}{18} = 4\,\Omega$$

9. Electrical resistance of wire of length 'l', area of cross-section 'A' and resistivity 'ρ' is given as

$$R = \rho \frac{l}{A} \qquad \text{...(i)}$$

Since we know, volume of the wire is

$$V = A \times l \qquad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$R = \rho \frac{l^2}{V} \qquad \text{...(iii)}$$

As, the length has been increased to 0.5%.

$\therefore$ New length of the wire,

$$l' = l + 0.5\% \text{ of } l$$

$$= l + 0.005\,l = 1.005\,l$$

But V and ρ remains unchanged.

So, new resistance, $R' = \dfrac{\rho[(1.005)\,l]^2}{V} \qquad \text{...(iv)}$

Dividing Eq. (iv) and Eq. (iii), we get

$$\frac{R'}{R} = (1.005)^2$$

$\Rightarrow$ % change in the resistance

$$= \left(\frac{R'}{R} - 1\right) \times 100$$

$$= [(1.005)^2 - 1] \times 100 = 1.0025\% \simeq 1\%$$

10. Since, it is an n-type semiconductor and concentration of the holes has been ignored. So, its conductivity is given as

$$\sigma = n_e e \mu_e$$

where, n_e is the number density of electron, e is the charge on electron and μ_e is its mobility.

Substituting the given values, we get

$$\sigma = 10^{19} \times 1.6 \times 10^{-19} \times 1.6 = 2.56$$

As, resistivity, $\rho = \dfrac{1}{\sigma} = \dfrac{1}{2.56}$

or $\qquad \rho = 0.39 \simeq 0.4\,\Omega\text{-m}$

11. Relation between current (I) flowing through a conducting wire and drift velocity of electrons (v_d) is given as

$$I = neAv_d$$

where, n is the electron density and A is the area of cross-section of wire.

$$\Rightarrow \qquad v_d = \frac{I}{neA}$$

Substituting the given values, we get

$$v = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$$

$$v = \frac{1.5 \times 10^{-3}}{72}\,\text{m/s} = 0.2 \times 10^{-4}\,\text{m/s}$$

or $\qquad v = 0.02\,\text{mm/s}$

12. $\frac{1}{R}=\frac{1}{R_{Al}}+\frac{1}{R_{Fe}}=\left(\frac{A_{Al}}{\rho_{Al}}+\frac{A_{Fe}}{\rho_{Fe}}\right)\frac{1}{\ell}$

$$=\left[\frac{(7^2-2^2)}{2.7}+\frac{2^2}{10}\right]\frac{10^{-6}}{10^{-8}}\times\frac{1}{50\times10^{-3}}$$

Solving we get,

$$R=\frac{1875}{64}\times10^{-6}\ \Omega=\frac{1875}{64}\ \mu\Omega$$

13. $i=neAv_d$ or $\frac{V}{R}=neAv_d$

or $\frac{V}{\left(\frac{\rho l}{A}\right)}=neAv_d$

$\therefore$ $\rho=\frac{V}{nelv_d}$ = resistivity of wire

Substituting the given values we have

$$\rho=\frac{5}{(8\times10^{28})(1.6\times10^{-19})(0.1)(2.5\times10^{-4})}$$

$$\approx 1.6\times10^{-5}\ \Omega\text{-m}$$

14. $R=\frac{\rho\,(L)}{A}=\frac{\rho L}{tL}=\frac{\rho}{t}$

i.e. R is independent of L.

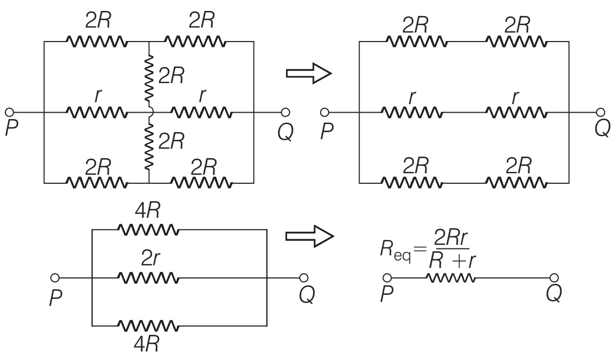
Hence, the correct option is (c).

15. $R_{PQ}=\frac{5}{11}r$, $R_{QR}=\frac{4}{11}r$ and $R_{PR}=\frac{3}{11}r$

$\therefore$ R_{PQ} is maximum.

Therefore, the correct option is (a).

16. The circuit can be redrawn as follows :



17. $i=neA\,v_d$

Drift speed, $v_d=\frac{1}{neA}\propto\frac{1}{A}$

Therefore, for non-uniform cross-section (different values of A) drift speed will be different at different sections. Only current (or rate of flow of charge) will be same.

18. Resistivity of conductors increases with increase in temperature because rate of collisions between free electrons and ions increase with increase of temperature. However, the resistivity of semiconductors decreases with increase in temperature, because more and more covalent bonds are broken at higher temperatures.

19. Copper is metal and germanium is semiconductor. Resistance of a metal decreases and that of a semiconductor increases with decrease in temperature.

$\therefore$ Correct option is (d).

20. All the three resistances are in parallel.

Therefore, $\frac{1}{R_{eq}}=\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}=\frac{2}{R}$

$\therefore$ $R_{eq}=\frac{R}{2}$

21. $\frac{I}{V}$ = slope of given graph $=\frac{1}{R}$ or $R=\frac{1}{\text{slope}}$

Resistance of a metallic wire increases with increase in temperature.

$$(\text{slope})_{T_2}<(\text{slope})_{T_1}$$

$\therefore$ $\frac{1}{(\text{slope})_{T_2}}>\frac{1}{(\text{slope})_{T_1}}$

or $R_{T_2}>R_{T_1}$ or $T_2>T_1$

22. Due to thermal energy, free electrons are always in *zig-zag* motion inside a conductor.

23. The given circuit is a simple circuit of series and parallel combinations. $R_{AB}=2\ \Omega$

24. The given circuit makes a balanced Wheatstone's bridge. The resistance between P and Q can be removed. All resistors have value R.

$\therefore$ $R_{AB}=R$

25. $R=\rho\frac{l}{A}=\frac{\rho\cdot l}{V/l}=\frac{\rho l^2}{V}$ (V = volume of wire)

$\therefore$ $R\propto l^2$ (ρ and V = constant)

For small percentage change

% change R = 2 (% change in l) = 2 (0.1%) = 0.2%

Since $R\propto l^2$, with increase in the value of l, resistance will also increase.

Topic 2 Kirchhoffs Laws and Combination of Batteries

1. Given circuit in a series combination of internal resistance of cell (r) and external resistance R.

$\therefore$ Effective resistance in the circuit,

$$R_{eff}=r+R$$

$\therefore$ Current in the circuit,

$$I=\frac{E}{R+r}\text{ or }E=IR+Ir$$

Voltage difference across resistance R is V, so

$$E=V+Ir \qquad \text{...(i)}$$

Now, from graph at $I = 0, V = 1.5\,\text{V}$

From Eq. (i) at $I = 0$,

$$E = V = 1.5\,\text{V} \qquad \ldots\text{(ii)}$$

At $I = 1000\,\text{mA}$ (or 1A), $V = 0$

From Eq. (i) at $I = 1\text{A}$ and $V = 0$

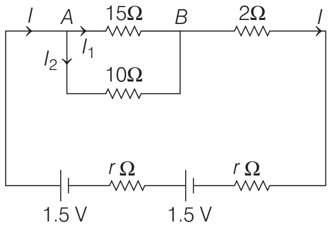
$$\Rightarrow \qquad E = I \times r = r \qquad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we can get

$$r = E = V = 1.5\text{V}$$

$$\therefore \qquad r = 1.5\,\Omega$$

2. For the given circuit



Given, $V_{AB} = 2\text{V}$

$\therefore$ Current in circuit,

$$I = I_1 + I_2 = \frac{2}{15} + \frac{2}{10} \qquad [\because V = IR \text{ or } I = V/R]$$

$$= \frac{4+6}{30} = \frac{1}{3}\text{A} \qquad \ldots\text{(i)}$$

Also, voltage drop across $(r + r)$ resistors is

= voltage of the cell − voltage drop across AB

$$= 3 - 2 = 1\,\text{V}$$

Using $V = IR$ over the entire circuit

$$\Rightarrow \qquad 1 = I\,(2 + 2r) = \frac{1}{3}(2 + 2r) \qquad \text{[using Eq. (i)]}$$

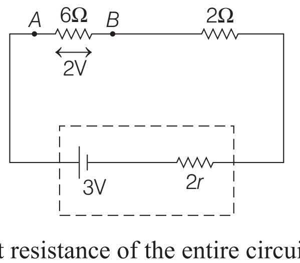
$$\Rightarrow \qquad 3 = 2 + 2r \quad \text{or} \quad 2r = 1\,\Omega$$

$$\text{or} \qquad r = \frac{1}{2}\Omega = 0.5\,\Omega$$

Alternative Solution

Equivalent resistance between A and B is

$$\frac{1}{R_{AB}} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6} \Rightarrow R_{AB} = 6\,\Omega$$



$\therefore$ Equivalent resistance of the entire circuit is, R_{eq}

$$= 6\,\Omega + 2\,\Omega + 2r = 8 + 2r$$

Now, current passing through the circuit is given as,

$$I = \frac{E_{\text{net}}}{R + r_{\text{eq}}} = \frac{E_{\text{net}}}{R_{\text{eq}}}$$

where, R is external resistance, r_{eq} is net internal resistance and E_{net} is the emf of the cells.

Here, $E_{\text{net}} = 1.5 + 1.5 = 3\,\text{V}$

$$r_{\text{eq}} = r + r = 2r$$

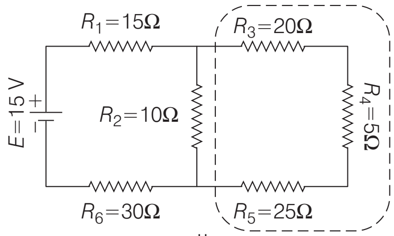
$$\Rightarrow \qquad I = \frac{3}{8 + 2r}$$

Also, reading of the voltmeter, $V = 2\text{V} = I \cdot R_{AB}$

$$2 = \left(\frac{3}{8 + 2r}\right) \times 6$$

$$\Rightarrow \qquad 8 + 2r = 9 \quad \text{or} \quad r = \frac{1}{2} = 0.5\,\Omega$$

3. Given circuit is redrawn and can be simplified as



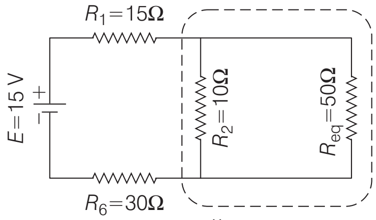
In series,

$R_{\text{eq}} = R_3 + R_4 + R_5$

$= 20 + 5 + 25$

$= 50\Omega$

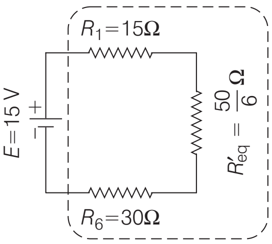
$\Downarrow$



In parallel,

$$R'_{\text{eq}} = \frac{R_{\text{eq}} \times R_2}{R_{\text{eq}} + R_2} = \frac{50 \times 10}{50 + 10} = \frac{50}{6}\,\Omega$$

$\Downarrow$



In series,

$$R''_{\text{eq}} = R_1 + R'_{\text{eq}} + R_6$$

$$= 30 + \frac{50}{6} + 15$$

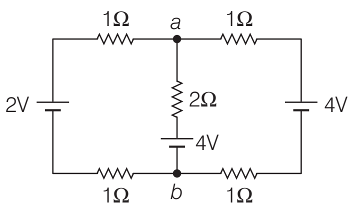
$$= \frac{320}{6} = \frac{160}{3}\Omega$$

So, current drawn through cell is

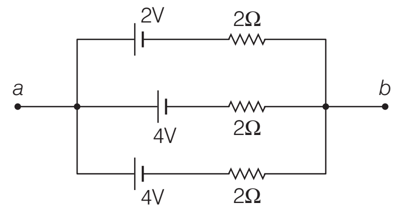
$$i = \frac{\text{Voltage}}{\text{Net resistance of the circuit}}$$

$$= \frac{V}{R''_{\text{eq}}} = \frac{15}{(160/3)} = \frac{9}{32}\text{A}$$

4. Given circuit is



Above circuit can be viewed as

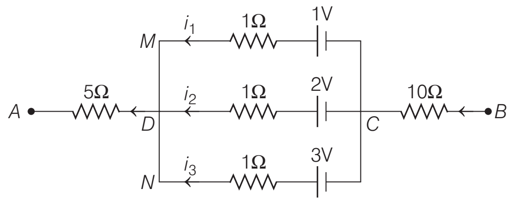


This is a parallel combination of three cells or in other words, a parallel grouping of three cells with internal resistances.

So, $V_{ab} = E_{eq} = \dfrac{I_{eq}}{r_{eq}} = \dfrac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}}$

$$= \frac{\frac{2}{2} + \frac{4}{2} + \frac{4}{2}}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = \frac{10}{3}\text{ V} \approx 3.3\text{ V}$$

5. In the given circuit, let's assume currents in the arms are i_1, i_2 and i_3, respectively.



Now, $i_1 = \dfrac{V_1}{R_1} = \dfrac{1}{1} = 1\text{ A}$

Similarly, $i_2 = \dfrac{2}{1} = 2\text{ A}$

and $i_3 = \dfrac{3}{1} = 3\text{ A}$

Total current in the arm DA is

$$i = i_1 + i_2 + i_3 = 6\text{ A}$$

As all three resistors between D and C are in parallel.

$\therefore$ Equivalent resistance between terminals D and C is

$$\frac{1}{R_{DC}} = \left(\frac{1}{1} + \frac{1}{1} + \frac{1}{1}\right)$$

$\therefore$ $R_{DC} = \dfrac{1}{3}\Omega$

So, potential difference across D and C is

$$V_{DC} = iR_{DC} = 6 \times \frac{1}{3}$$

$\Rightarrow$ $V_{DC} = 2\text{ V}$

Now, V_{AD} and $V_{CB} = 0$

(In case of open circuits, $I = 0$)

So, $V_{AB} = V_{AD} + V_{DC} + V_{CB} = V_{DC}$

So, $V_{AB} = 2\text{V}$

6. For a balanced Wheatstone bridge,

$$\frac{P}{R} = \frac{Q}{X}$$

In first case when $R = 400\ \Omega$, the balancing equation will be

$$\frac{P}{R} = \frac{Q}{X} \Rightarrow \frac{P}{400\ \Omega} = \frac{Q}{X}$$

$\Rightarrow$ $P = \dfrac{400 \times Q}{X}$ ...(i)

In second case, P and Q are interchanged and $R = 405\ \Omega$

$\therefore$ $\dfrac{Q}{R} = \dfrac{P}{X}$

$\Rightarrow$ $\dfrac{Q}{405} = \dfrac{P}{X}$ ...(ii)

Substituting the value of P from Eq. (i) in Eq. (ii), we get

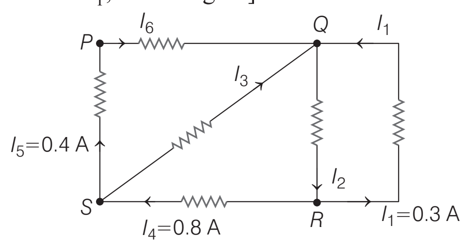
$$\frac{Q}{405} = \frac{Q \times 400}{X^2}$$

$\Rightarrow$ $X^2 = 400 \times 405$

$\Rightarrow$ $X = \sqrt{400 \times 405} = 402.5$

The value of X is close to $402.5\ \Omega$.

7. Given circuit with currents as shown in the figure below, [In the question $I_1 = 0.3$ A is given, due to it we change the direction of I_1, in this figure]



From Kirchoff's junction rule, $\Sigma I = 0$

At junction S, $I_4 = I_5 + I_3$

$\Rightarrow$ $0.8 = 0.4 + I_3$

$\Rightarrow$ $I_3 = 0.4$ A

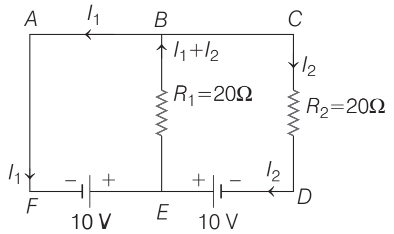
At junction P, $I_5 = I_6$

$\Rightarrow$ $I_6 = 0.4$ A

At junction Q, $I_2 = I_1 + I_3 + I_6$

$= 0.3 + 0.4 + 0.4 = 1.1$ A

8. By Kirchhoff's loop rule in the given loop $ABEFA$, we get



$$10-(I_1+I_2)R_1=0$$

$$\Rightarrow \quad 10-(I_1+I_2)\,20=0$$

$$\text{or} \quad I_1+I_2=\frac{1}{2} \quad \ldots\text{(i)}$$

and from loop $BCDEB$, we get

$$10-(I_1+I_2)R_1-I_2R_2=0$$

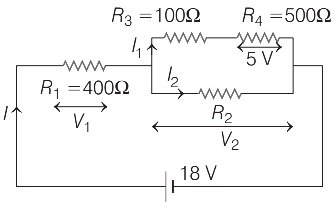
$$\Rightarrow \quad 10-(I_1+I_2+I_2)20=0$$

$$\Rightarrow \quad I_1+2I_2=\frac{1}{2} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$I_2=0 \text{ and } I_1=0.5 \text{ A.}$$

9. According to question, the voltage across R_4 is 5 volt, then the current across it



According to Ohm's law,

$$\Rightarrow \quad V=IR \quad \Rightarrow \quad 5=I_1\times R_4$$

$$\Rightarrow \quad 5=I_1\times 500$$

$$I_1=\frac{5}{500}=\frac{1}{100}\text{A}$$

The potential difference across series combination of R_3 and R_4

$$\Rightarrow \quad V_2=(R_3+R_4)I=600\times\frac{1}{100}=6\text{Volt}$$

So, potential difference (across R_1)

$$V_1=18-6=12\text{ V}$$

Current through R_1 is,

$$I=\frac{V_1}{R_1}=\frac{12}{400}=\frac{3}{100}\text{A}$$

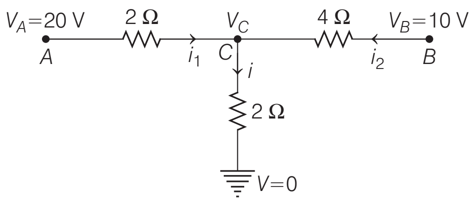
So current through R_2 is,

$$I_2=I-I_1=\frac{3}{100}-\frac{1}{100}\text{A}=\frac{2}{100}\text{A}$$

Now, from $V=IR$, we have,

$$R_2=\frac{V_2}{I_2}=\frac{6}{(2/100)}=300\ \Omega.$$

10. When the switch 'S' is closed the circuit, hence formed is given in the figure below,



Then, according to Kirchhoff's current law, which states that the sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from that point.

Since, in the above circuit, that point is 'C'

$$\therefore \quad i_1+i_2=i$$

$$\Rightarrow \quad \frac{V_A-V_C}{2}+\frac{V_B-V_C}{4}=\frac{V_C-V}{2}$$

$$(\because \text{ using Ohm's law, } V=iR)$$

$$\text{or} \quad \frac{20-V_C}{2}+\frac{10-V_C}{4}=\frac{V_C-0}{2}$$

$$\Rightarrow \quad 20-V_C+(10-V_C)2=V_C$$

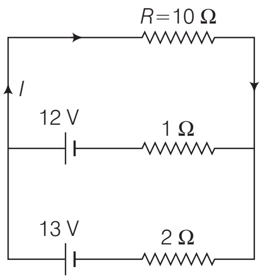
$$40=V_C+3V_C$$

$$40=4V_C$$

$$\text{or} \quad V_C=10\text{ V}$$

$$\therefore \text{ The current, } i=\frac{V_C}{2}=\frac{10}{2}=5\text{A}$$

11.



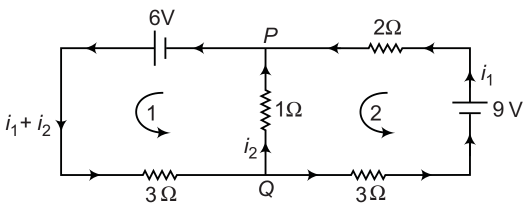
$$\boldsymbol{E}_{\text{eq}}=\frac{\boldsymbol{\Sigma E/r}}{\boldsymbol{\Sigma(1/r)}}=\frac{\frac{\mathbf{12}}{\mathbf{1}}+\frac{\mathbf{13}}{\mathbf{2}}}{\frac{\mathbf{1}}{\mathbf{1}}+\frac{\mathbf{1}}{\mathbf{2}}}=\frac{\mathbf{37}}{\mathbf{3}}\text{V}$$

$$r_{\text{eq}}=\frac{r_1r_2}{r_1+r_2}=\frac{2}{3}\Omega \quad\Rightarrow\quad I=\frac{\frac{37}{3}}{\frac{2}{3}+10}=\frac{37}{32}\text{A}$$

$$\text{Voltage across load}=IR=\left(\frac{37}{32}\right)(10)=11.56\text{ V}$$

12. A potential drop across each resistor is zero, so the current through each of resistor is zero.

13.



Applying Kirchhoff's loop law in loops 1 and 2 in the directions shown in figure we have

$$6-3(i_1+i_2)-i_2=0 \quad \ldots\text{(i)}$$

$$9-2i_1+i_2-3i_1=0 \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii) we get,

$$i_2 = 0.13 \text{ A}$$

Hence, the current in 1 Ω resister is 0.13 A from Q to P.

14. Current in the respective loop will remain confined in the loop itself.

Therefore, current through 2 Ω resistance = 0

∴ Correct answer is (c).

15. Current I can be independent of R_6 only when R_1, R_2, R_3, R_4 and R_6 form a balanced Wheatstone's bridge.

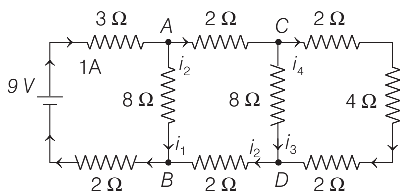
Therefore, $$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{or} \quad R_1R_4 = R_2R_3$$

16. As there is no change in the reading of galvanometer with switch S open or closed. It implies that bridge is balanced. Current through S is zero and $I_R = I_G, I_P = I_Q$.

17. Net resistance of the circuit is 9 Ω.

∴ Current drawn from the battery,

$$i = \frac{9}{9} = 1 \text{ A} = \text{current through } 3\ \Omega \text{ resistor}$$



Potential difference between A and B is

$$V_A - V_B = 9 - 1(3+2) = 4\text{V} = 8i_1$$

∴ $$i_1 = 0.5 \text{ A}$$

∴ $$i_2 = 1 - i_1 = 0.5 \text{ A}$$

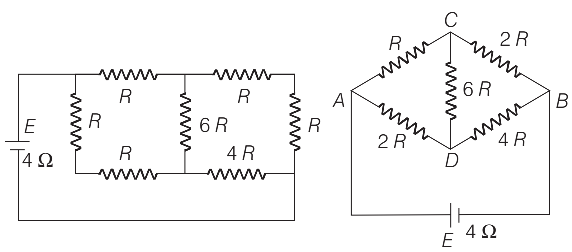
Similarly, potential difference between C and D

$$V_C - V_D = (V_A - V_B) - i_2(2+2)$$
$$= 4 - 4i_2 = 4 - 4(0.5) = 2V = 8i_3$$

∴ $$i_3 = 0.25 \text{ A}$$

Therefore, $$i_4 = i_2 - i_3 = 0.5 - 0.25$$
$$i_4 = 0.25 \text{ A}$$

18. The given circuit is a balanced Wheatstone's bridge.

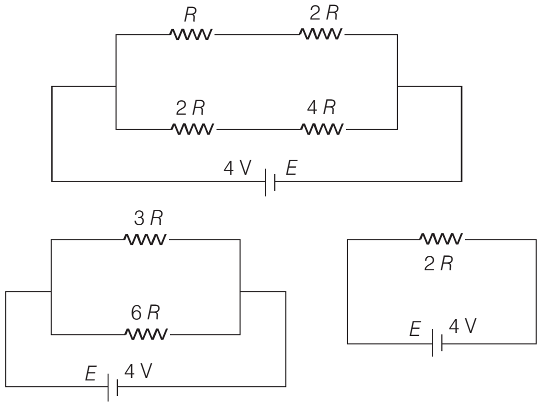


Thus, no current will flow across $6R$ of the side CD. The given circuit will now be equivalent to

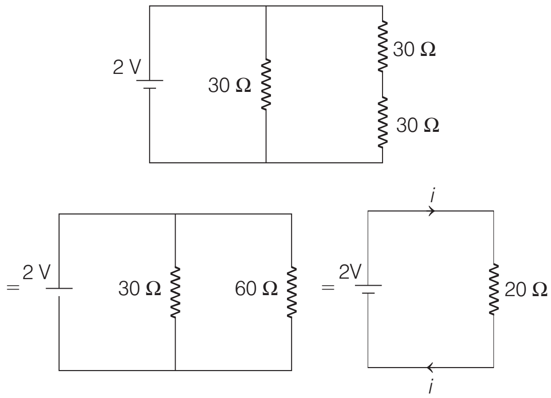
For maximum power, net external resistance

= Total internal resistance

or $$2R = 4 \quad \text{or} \quad R = 2\ \Omega$$

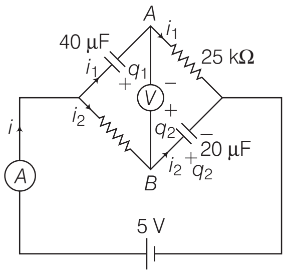


19. The simplified circuit is shown in the figure.



Therefore, current $i = \frac{2}{20} = \frac{1}{10}$ A

20.



Just after pressing key,

$$5 - 25000\, i_1 = 0$$
$$5 - 50000\, i_2 = 0$$

(As charge in both capacitors)

⇒ $i_1 = 0.2$ mA ⇒ $i_2 - 0.1$ mA

And $V_B + 25000\, i_1 = V_A$

⇒ $V_B - V_A = -5\text{V}$

After a long time, i_1 and $i_2 = 0$ (steady state)

⇒ $$5 - \frac{q_1}{40} = 0$$

⇒ $$q_1 = 200 \mu\text{C}$$

$$5-\frac{q_2}{20}=0 \Rightarrow q_2=100\mu C$$

$$V_B-\frac{q_2}{20}=V_A$$

$$\Rightarrow \quad V_B-V_A=+5V$$

$\Rightarrow$ Option (a) is correct.

For capacitor 1, $q_1=200[1-e^{-t/1}]\,\mu C$

$$i_1=\frac{1}{5}e^{-t/1}\text{mA}$$

For capacitor 2, $q_2=100[1-e^{-t/1}]\,\mu C$

$$i_2=\frac{1}{10}e^{-t/1}\text{mA}$$

$$\Rightarrow \quad V_B-\frac{q_2}{20}+i_1\times 25=V_A$$

$\Rightarrow V_B-V_A=5[1-e^{-t}]-5e^{-t}=-5[1-2e^{-t}]$

At $\quad t=\ln 2, V_B-V_A=5[1-1]=0$

$\Rightarrow$ Option (b) is correct.

At $\quad t=1, i=i_1+i_2=\frac{1}{5}e^{-1}+\frac{1}{10}e^{-1}=\frac{3}{10}\cdot\frac{1}{e}$

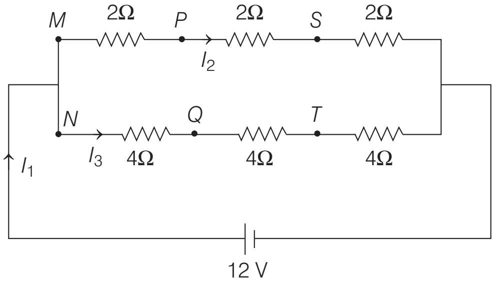
At $\quad t=0, \quad i=i_1+i_2=\frac{1}{5}+\frac{1}{10}=\frac{3}{10}$

$\Rightarrow$ (c) is correct.

After a long time, $i_1=i_2=0$

$\Rightarrow$ Option (d) is correct.

21. Due to symmetry on upper side and lower side, points P and Q are at same potentials. Similarly, points S and T are at same potentials. Therefore, the simple circuit can be drawn as shown below



$$I_2=\frac{12}{2+2+2}=2\text{ A}$$

$$I_3=\frac{12}{4+4+4}=1\text{ A}$$

$$\therefore \quad I_1=I_2+I_3=3\text{ A}$$

$$I_{PQ}=0$$

because $\quad V_P=V_Q$

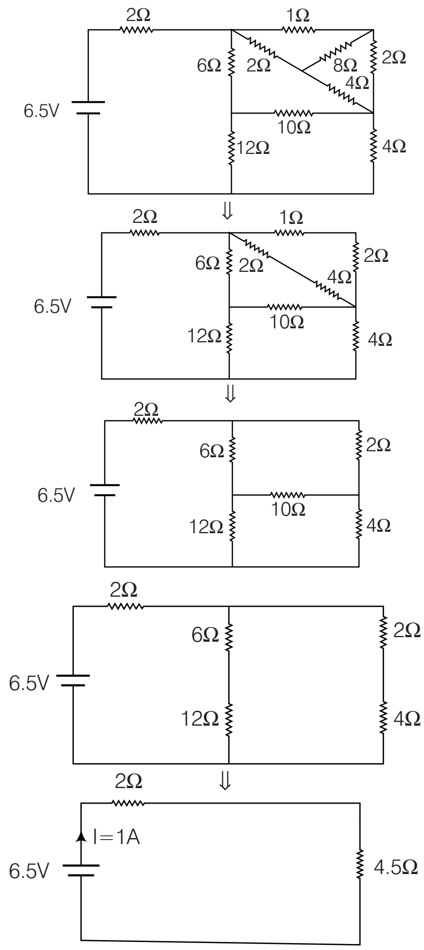
Potential drop (from left to right) across each resistance is

$$\frac{12}{3}=4\text{ V}$$

$$\therefore \quad V_{MS}=2\times 4=8\text{ V}$$

$$V_{NQ}=1\times 4=4\text{ V or } V_S<V_Q$$

22.



23. V_{AB} = Equivalent emf of two batteries in parallel.

$$=\frac{E_1/r_1+E_2/r_2}{1/r_1+1/r_2}=\frac{(6/1)+(3/2)}{(1/1)+(1/2)}=5\text{ V}$$

$\therefore$ Answer is 5.

24. Current in the circuit,

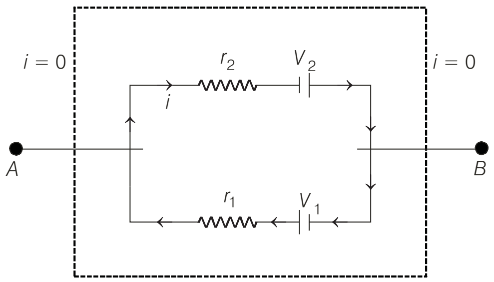
$$i=\frac{8\times 5}{8\times 0.2}=25\text{A} \qquad \text{(counter clockwise)}$$

Therefore, PD across the terminals of the battery

$$V=E-ir=5-(25)(0.2)=0$$

25. (a) **Equivalent emf (V) of the battery**

PD across the terminals of the battery is equal to its emf when current drawn from the battery is zero. In the given circuit,



Current in the internal circuit,

$$i=\frac{\text{Net emf}}{\text{Total resistance}}=\frac{V_1+V_2}{r_1+r_2}$$

Therefore, potential difference between A and B would be

$$V_A-V_B=V_1-ir_1$$

$$\therefore \quad V_A-V_B=V_1-\left(\frac{V_1+V_2}{r_1+r_2}\right)r_1=\frac{V_1r_2-V_2r_1}{r_1+r_2}$$

So, the equivalent emf of the battery is

$$V=\frac{V_1r_2-V_2r_1}{r_1+r_2}$$

Note that if $V_1r_2=V_2r_1 : V=0$.

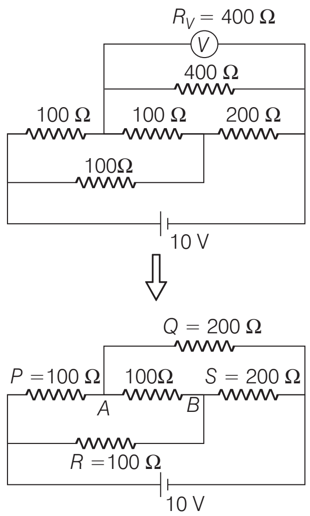
If $V_1r_2>V_2r_1 : V_A-V_B=$ Positive i.e. A side of the equivalent battery will become the positive terminal and *vice-versa.*

(b) **Internal resistance (r) of the battery**

r_1 and r_2 are in parallel. Therefore, the internal resistance r will be given by

$$1/r=1/r_1+1/r_2 \text{ or } r=\frac{r_1r_2}{r_1+r_2}$$

26. The given circuit actually forms a balanced Wheatstone's bridge (including the voltmeter) as shown below



$$R_V=400\Omega$$

Here, we see that $\dfrac{P}{Q}=\dfrac{R}{S}$

Therefore, resistance between A and B can be ignored and equivalent simple circuit can be drawn as follows

The voltmeter will read the potential difference across resistance Q.

$$\text{Currents } i_1=i_2=\frac{10}{100+200}=\frac{1}{30}\text{A}$$

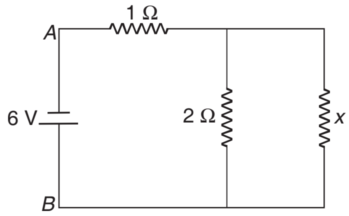
$\therefore$ Potential difference across voltmeter

$$=Qi_1=(200)\left(\frac{1}{30}\right)\text{V}=\frac{20}{3}\text{V}$$

Therefore, reading of voltmeter will be $\dfrac{20}{3}$V.

27. (a) Let $R_{AB}=x$. Then, we can break one chain and connect a resistance of magnitude x in place of it.

Thus, the circuit remains as shown in figure.



Now, $2x$ and x are in parallel. So, their combined resistance is $\dfrac{2x}{2+x}$

$$\text{or} \qquad R_{AB}=1+\frac{2x}{2+x}$$

But R_{AB} is assumed as x. Therefore,

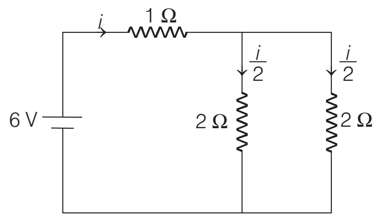
$$x=1+\frac{2x}{2+x}$$

Solving this equation, we get

$$x=2\,\Omega$$

Hence proved.

(b) Net resistance of circuit $R=1+\dfrac{2\times 2}{2+2}=2\,\Omega$



$\therefore$ Current through battery $i=\dfrac{6}{2}=3$A

This current is equally distributed in $2\,\Omega$ and $2\,\Omega$ resistances. Therefore, the desired current is $\dfrac{i}{2}$ or 1.5 A.

28. Applying Kirchhoff's second law in loop $BADB$

$$2-2i_1-i_1-1-2(i_1-i_2)=0 \qquad \ldots\text{(i)}$$

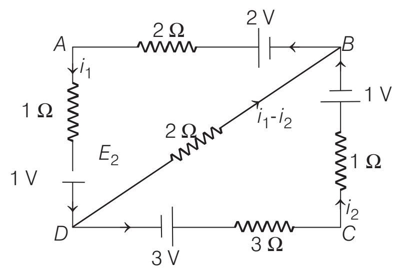
Similarly applying Kirchhoff's second law in loop $BDCB$

$$2(i_1-i_2)+3-3i_2-i_2-1=0 \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$i_1=\frac{5}{13}, i_2=\frac{6}{13}$$

$$\text{and} \qquad i_1-i_2=-\frac{1}{13}$$



(a) Potential difference between B and D.

$$V_B + 2(i_1 - i_2) = V_D$$

$$\therefore \quad V_B - V_D = -2(i_1 - i_2) = \frac{2}{13}\text{ V}$$

(b) $$V_G = E_G - i_2 r_G = 3 - \frac{6}{13} \times 3 = \frac{21}{13}\text{ V}$$

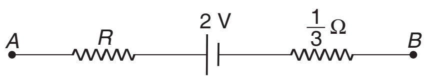
$$V_H = E_H + i_2 r_H = 1 + \frac{6}{13} \times 1 = \frac{19}{13}\text{ V}$$

29. (a) Equivalent emf of three batteries would be

$$E_{\text{eq}} = \frac{\Sigma(E/r)}{\Sigma(1/r)} = \frac{(3/1 + 2/1 + 1/1)}{(1/1 + 1/1 + 1/1)} = 2\text{ V}$$

Further r_1, r_2 and r_3 each are of 1 Ω. Therefore, internal resistance of the equivalent battery will be $\frac{1}{3}\Omega$ as all three are in parallel.

The equivalent circuit is therefore shown in the given figure.



Since, no current is taken from the battery.

$$V_{AB} = 2\text{ V (From } V = E - ir)$$

Further, $V_{AB} + V_A - V_B = E_1 - i_1 r_1$

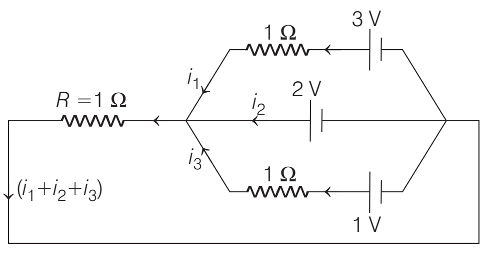
$$\therefore \quad i_1 = \frac{V_B - V_A + E_1}{r_1} = \frac{-2+3}{1} = 1\text{ A}$$

Similarly, $$i_2 = \frac{V_B - V_A + E_2}{r_2} = \frac{-2+2}{1} = 0$$

and $$i_3 = \frac{V_B - V_A + E_3}{r_3} = \frac{-2+1}{1} = -1\text{ A}$$

(b) r_2 is short circuited means resistance of this branch becomes zero. Making a closed circuit with a battery and resistance R. Applying Kirchhoff's second law in three loops so formed.

$$3 - i_1 - (i_1 + i_2 + i_3) = 0 \quad \ldots\text{(i)}$$



$$2 - (i_1 + i_2 + i_3) = 0 \quad \ldots\text{(ii)}$$

$$1 - i_3 - (i_1 + i_2 + i_3) = 0 \quad \ldots\text{(iii)}$$

From Eq. (ii) $\quad i_1 + i_2 + i_3 = 2\text{A}$

$\therefore$ Substituting in Eq. (i), we get, $i_1 = 1\text{A}$

Substituting in Eq. (iii) we get, $i_3 = -1\text{A}$

$\therefore \quad i_2 = 2\text{ A}$

Topic 3 Heat and Power Generation

1. Heat required by water for getting hot and then evaporated is

$$\Delta Q = ms\Delta T + mL$$

Here, $m = 1\text{kg}$, $\Delta T = 100° - 20° = 80°\text{ C}$,
$s = 4200\text{ J kg}^{-1}\text{ °C}^{-1}$, $L = 2260 \times 10^3\text{ J kg}^{-1}$

So, heat required is

$$\Delta Q = 1 \times 4200 \times 80 + 1 \times 2260 \times 10^3$$
$$= 336 \times 10^3 + 2260 \times 10^3$$
$$= 2596 \times 10^3\text{ J} \quad \ldots\text{(i)}$$

This heat is provided by a heating coil of resistance $R = 20\ \Omega$ connected with AC mains $V_{\text{rms}} = 200\text{ V}$

So, heat supplied by heater coil is

$$Q = Pt = \frac{V_{\text{rms}}^2}{R} \times t$$

where, P = power and t = time.

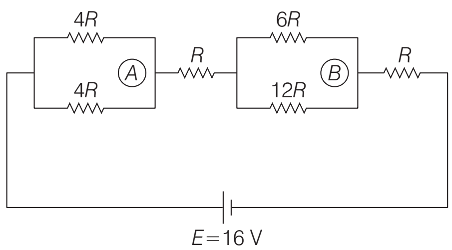
$$= \frac{(200)^2}{20} \times t$$
$$= 2 \times 10^3 \times t \quad \ldots\text{(ii)}$$

Substituting the value of heat from Eq. (ii), we get

$$t = \frac{2956 \times 10^3}{2 \times 10^3}\text{ s}$$
$$= \frac{2956}{2 \times 60}\text{ min} = 24.63\text{ min}$$

Nearest answer is 22 min.

2. Given circuit is



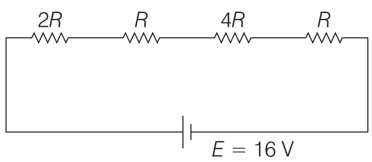
Equivalent resistance of part A,

$$R_A = \frac{4R \times 4R}{4R + 4R} = 2R$$

Equivalent resistance of part B,

$$R_B = \frac{6R \times 12R}{6R + 12R} = \frac{72}{18}R = 4R$$

$\therefore$ Equivalent circuit is

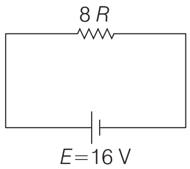


$\therefore$ Total resistance of the given network is

$$R_s = 2R + R + 4R + R = 8R$$

As we know, power of the circuit,

$$P = \frac{E^2}{R_s} = \frac{(16)^2}{8R} = \frac{16 \times 16}{8R} \quad \ldots \text{(i)}$$

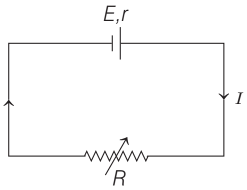


According to question, power consumed by the network, $P = 4$ W.

From Eq. (i), we get

$$\therefore \quad \frac{16 \times 16}{8R} = 4 \Rightarrow R = \frac{16 \times 16}{8 \times 4} = 8\,\Omega$$

3. Given circuit is shown in the figure below



Net current, $$I = \frac{E}{R + r} \quad \ldots \text{(i)}$$

Power across R is given as

$$P = I^2 R = \left(\frac{E}{R + r}\right)^2 \cdot R \quad \text{[using Eq. (i)]}$$

For the maximum power,

$$\frac{dP}{dR} = 0$$

$$\Rightarrow \quad \frac{dP}{dR} = \frac{d}{dR}\left(\left(\frac{E}{R + r}\right)^2 \cdot R\right)$$

$$= E^2 \frac{d}{dR}\left(\frac{R}{(R + r)^2}\right)$$

$$= E^2\left[\frac{(R + r)^2 \times 1 - 2R \times (R + r)}{(R + r)^4}\right] = 0$$

$$\Rightarrow \quad (R + r)^2 = 2R(R + r) \quad \text{or} \quad R + r = 2R \Rightarrow r = R$$

$\therefore$ The power delivered by the cell to the external resistance is maximum when $R = r$.

Alternate Solution

From maximum power theorem, power dissipated will be maximum when internal resistance of source will be equals to external load resistance, i.e.

$r = R$.

4. Resistance of a bulb of power P and with a voltage source V is given by

$$R = \frac{V^2}{P}$$

Resistance of the given two bulbs are

$$R_1 = \frac{V^2}{P_1} = \frac{(220)^2}{25}$$

and $$R_2 = \frac{V^2}{P_2} = \frac{(220)^2}{100}$$

Since, bulbs are connected in series. This means same amount of current flows through them.

$\therefore$ Current in circuit is

$$i = \frac{V}{R_{\text{total}}} = \frac{220}{\frac{(220)^2}{25} + \frac{(220)^2}{100}} = \frac{1}{11}\text{A}$$

Power drawn by bulbs are respectively,

$$P_1 = i^2 R_1 = \left(\frac{1}{11}\right)^2 \times \frac{220 \times 220}{25} = 16\text{ W}$$

and $$P_2 = i^2 R_2 = \left(\frac{1}{11}\right)^2 \times \frac{220 \times 220}{100} = 4\text{ W}$$

5. Let P_1 and P_2 be the individual electric powers of the two resistances, respectively.

In series combination, power is

$$P_0 = \frac{P_1 P_2}{P_1 + P_2} = 60\text{W}$$

Since, the resistances are equal and the current through each resistor in series combination is also same. Then,

$$P_1 = P_2 = 120\text{ W}$$

In parallel combination, power is

$$P = P_1 + P_2 = 120 + 120 = 240\text{ W}$$

Alternate method

Let R be the resistance.

$\therefore$ Net resistance in series $= R + R = 2R$

$$P = \frac{V^2}{2R} = 60\text{ W}$$

$$\Rightarrow \quad \frac{V^2}{R} = 120\text{ W}$$

New resistance in parallel $= \frac{R \times R}{R + R} = R/2$

$$P' = \frac{V^2}{R/2} = 2\left(\frac{V^2}{R}\right) = 240\text{ W}$$

6. Power dissipated by any resistor R, when I current flows through it is,

$$P = I^2 R \quad \ldots \text{(i)}$$

Given $I = 2\text{ mA} = 2 \times 10^{-3}$ A and $P = 4.4$ W

Using Eq. (i), we get

$$4.4 = (2 \times 10^{-3})^2 \times R$$

or $$R = \frac{4.4}{4 \times 10^{-6}} \cdot \Omega \quad \ldots \text{(ii)}$$

When this resistance R is connected with 11 V supply then power dissipated is

$$P = \frac{V^2}{R} \text{ or } P = \frac{(11)^2}{4.4} \times 4 \times 10^{-6} \quad [\because \text{ Using Eq. (ii)}]$$

$$\Rightarrow \quad P = \frac{11 \times 11 \times 4 \times 10^{-6}}{44 \times 10^{-1}} \text{ W}$$

$$\text{or} \quad P = 11 \times 10^{-5} \text{ W}$$

7. Current flowing through both the bars is equal. Now, the heat produced is given by

$$H = I^2 Rt \quad \text{or} \quad H \propto R \quad \text{or} \quad \frac{H_{AB}}{H_{BC}} = \frac{R_{AB}}{R_{BC}}$$

$$= \frac{(1/2r)^2}{(1/r)^2} \qquad \left(\text{as } R \propto \frac{1}{A} \propto \frac{1}{r^2}\right)$$

$$= \frac{1}{4}$$

$$\text{or} \quad H_{BC} = 4 H_{AB}$$

8. $P = i^2 R$

Current is same, so $P \propto R$.

In the first case it is $3r$, in second case it is $\frac{2}{3}r$, in third case it is $\frac{r}{3}$ and in fourth case the net resistance is $\frac{3r}{2}$.

$$R_{\text{III}} < R_{\text{II}} < R_{\text{IV}} < R_{\text{I}}$$

$$\therefore \quad P_{\text{III}} < P_{\text{II}} < P_{\text{IV}} < P_{\text{I}}$$

9. $$P = \frac{V^2}{R} \quad \text{so,} \quad R = \frac{V^2}{P}$$

$$\therefore \quad R_1 = \frac{V^2}{100} \quad \text{and} \quad R_2 = R_3 = \frac{V^2}{60}$$

$$\text{Now,} \quad W_1 = \frac{(250)^2}{(R_1 + R_2)^2} \cdot R_1$$

$$W_2 = \frac{(250)^2}{(R_1 + R_2)^2} \cdot R_2 \quad \text{and} \quad W_3 = \frac{(250)^2}{R_3}$$

$$W_1 : W_2 : W_3 = 15 : 25 : 64 \quad \text{or} \quad W_1 < W_2 < W_3$$

10. In the first case $\frac{(3E)^2}{R} t = ms\,\Delta T$ ...(i)

$$\left[H = \frac{V^2}{R} t\right]$$

When length of the wire is doubled, resistance and mass both are doubled.

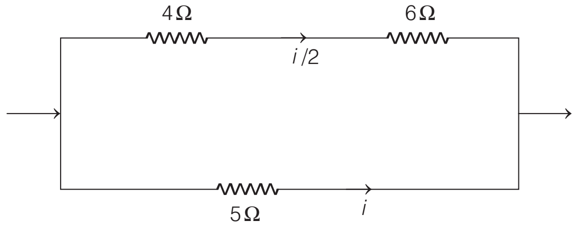
Therefore, in the second case,

$$\frac{(NE)^2}{2R} \cdot t = (2m) s\,\Delta T \qquad \text{...(ii)}$$

Dividing Eq. (ii) by (i), we get

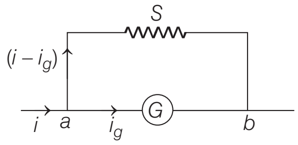
$$\frac{N^2}{18} = 2 \text{ or } N^2 = 36 \text{ or } N = 6$$

11. Since, resistance in upper branch of the circuit is twice the resistance in lower branch. Hence, current there will be half.



Now, $\quad P_4 = (i/2)^2 (4) \qquad (P = i^2 R)$

$P_5 = (i)^2 (5)$



$$\text{or} \quad \frac{P_4}{P_5} = \frac{1}{5}$$

$$\therefore \quad P_4 = \frac{P_5}{5} = \frac{10}{5} = 2 \text{ cal/s}$$

$\therefore$ Correct option is (b).

12. Because of non-uniform evaporation at different section, area of cross-section would be different at different sections.

Region of highest evaporation rate would have rapidly reduced area and would become break up cross-section.

Resistance of the wire as whole increases with time. Overall resistance increases hence power decreases.

$$\left(p = \frac{V^2}{R} \text{ or } p \propto \frac{1}{R} \text{ as } V \text{ is constant}\right).$$

At break up junction temperature would be highest, thus light of highest band frequency would be emitted at those cross-section.

13. $R_{\text{total}} = 2 + \frac{6 \times 1.5}{6 + 1.5} = 3.2 \text{ k}\Omega$

(a) $I = \frac{24 \text{V}}{3.2 \text{ k}\Omega} = 7.5 \text{mA} = I_{R_1}$

$$I_{R_2} = \left(\frac{R_L}{R_L + R_2}\right) I$$

$$I = \frac{1.5}{7.5} \times 7.5 = 1.5 \text{mA}$$

$$I_{R_L} = 6 \text{mA}$$

(b) $V_{R_L} = (I_{R_L})(R_L) = 9 \text{V}$

(c) $\frac{P_{R_1}}{P_{R_2}} = \frac{(I_{R_1}^2) R_1}{(I_{R_2}^2) R_2} = \frac{(7.5)^2 (2)}{(1.5)^2 (6)} = \frac{25}{3}$

(d) When R_1 and R_2 are inter changed, then

$$\frac{R_2 R_L}{R_2 + R_L} = \frac{2 \times 1.5}{3.5} = \frac{6}{7} \text{ k}\Omega$$

Now potential difference across R_L will be

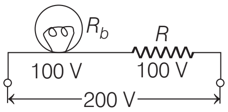
$$V_L = 24\left[\frac{6/7}{6+6/7}\right] = 3\text{ V}$$

Earlier it was 9 V

Since, $P = \frac{V^2}{R}$ or $P \propto V^2$

In new situation potential difference has been decreased three times. Therefore, power dissipated will decrease by a factor of 9.

14. Resistance of the given bulb



$$R_b = \frac{V^2}{P} = \frac{(100)^2}{500} = 20\Omega$$

To get 100 V out of 200 V across the bulb,

$$R = R_b = 20\Omega .$$

15. From $P = \frac{V^2}{R}$.

Resistance of heater,

$$R = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\ \Omega$$

From, $P = i^2R$

Current required across heater for power of 62.5 W

$$i = \sqrt{\frac{P}{R}} = \sqrt{\frac{62.5}{10}} = 2.5\text{ A}$$

Main current in the circuit,

$$I = \frac{100}{10 + \frac{10R}{10+R}} = \frac{100\,(10+R)}{100+20R} = \frac{10\,(10+R)}{10+2R}$$

This current will distribute in inverse ratio of resistance between heater and R.

$$\therefore \quad i = \left(\frac{R}{10+R}\right) I$$

or $$2.5 = \left(\frac{R}{10+R}\right)\left[\frac{10\,(10+R)}{10+2R}\right] = \frac{10R}{10+2R}$$

Solving this equation, we get

$$R = 5\ \Omega$$

Topic 4 Electrical Instruments

1. Unknown resistance 'X' in meter bridge experiment is given by

$$X = \left(\frac{100-l}{l}\right) R$$

Case (1) When $R = 1000\Omega$ and $l = 60$ cm, then

$$X = \frac{(100-60)}{60} \times 1000 = \frac{40\times 1000}{60}$$

$$\Rightarrow \quad X = \frac{2000}{3}\ \Omega \approx 667\Omega$$

Case (2) When $R = 100\ \Omega$ and $l = 13$ cm, then

$$X = \left(\frac{100-13}{13}\right)\times 100 = \frac{100\times 87}{13} = \frac{8700}{13}\ \Omega \approx 669\ \Omega$$

Case (3) When $R = 10\ \Omega$ and $l = 1.5$ cm, then

$$X = \left(\frac{100-1.5}{1.5}\right)\times 10 = \frac{98.5}{1.5}\times 10 = \frac{9850}{15}\ \Omega \approx 656\ \Omega$$

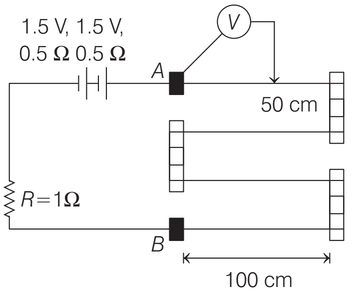
Case (4) When $R = 1\ \Omega$ and $l = 1.0$ cm, then

$$X = \left(\frac{100-1}{1}\right)\times 1$$

$$\therefore \quad X = 99\ \Omega$$

Thus, from the above cases, it can be concluded that, value calculated in case (4) is inconsistent.

2. In given potentiometer, resistance per unit length is $x = 0.01\ \Omega\text{cm}^{-1}$.



Length of potentiometer wire is $L = 400$ cm

Net resistance of the wire AB is

R_{AB} = resistance per unit length × length of AB

$= 0.01\times 400$

$$\Rightarrow \quad R_{AB} = 4\ \Omega$$

Net internal resistance of the cells connected in series,

$$r = 0.5 + 0.5 = 1\Omega$$

∴ Current in given potentiometer circuit is

$$I = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{\text{Net emf}}{r + R + R_{AB}} = \frac{3}{1+1+4} = 0.5\text{ A}$$

Reading of voltmeter when the jockey is at 50 cm (l') from one end A,

$$V = IR = I(xl') = 0.5\times 0.01\times 50 = 0.25\text{ V}$$

3. Current for deflection of pointer by

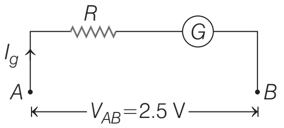
$$1\text{ division} = 4\times 10^{-4}\text{ A}$$

So, current for full-scale deflection = I_g

= Number of divisions × Current for 1 division

$\Rightarrow \quad I_g = 25 \times 4 \times 10^{-4} = 1 \times 10^{-2}$ A

Now, let a resistance of R is put in series with galvanometer to make it a voltmeter of range 2.5 V.



Then, $\quad I_g(R+G) = V_{AB}$

$$1 \times 10^{-2}\,(R+50) = 2.5$$

[∵ Given, $G = 50\,\Omega, V_{AB} = 2.5$ V]

$\therefore \quad R = 250 - 50 = 200\,\Omega$

4. As, galvanometer shows zero deflection.

This means, the meter bridge is balanced.

$$\frac{R'}{R_{AP}} = \frac{R'}{R_{PB}}$$

$\Rightarrow \quad R_{AP} = R_{PB} \quad$ ...(i)

Now, for meter bridge wire

$$\frac{dR}{dl} = \frac{k}{\sqrt{l}}$$

where, 'k' is the constant of proportionality.

$\Rightarrow \quad dR = \dfrac{k}{\sqrt{l}}\,dl$

Integrating both sides, we get

$\Rightarrow \quad R = \displaystyle\int \frac{k}{\sqrt{l}}\,dl$

So, $\quad R_{AP} = \displaystyle\int_0^l \frac{k}{\sqrt{l}}\,dl = k(2\sqrt{l})\Big|_0^l = 2k\sqrt{l}$

and $\quad R_{PB} = \displaystyle\int_l^1 \frac{k}{\sqrt{l}}\,dl = 2k(\sqrt{l})\Big|_l^1$

$$= 2k\,(\sqrt{1} - \sqrt{l}) = 2k\,(1 - \sqrt{l})$$

Substituting values of R_{AP} and R_{PB} in eq. (i), we get

$$R_{AP} = R_{PB}$$

$\Rightarrow \quad 2k\sqrt{l} = 2k(1-\sqrt{l})$

$\Rightarrow \quad \sqrt{l} = \dfrac{1}{2}$ or $l = \dfrac{1}{4} = 0.25$ m

5. Given, potential difference of 5 mV is across 10 m length of potentiometer wire. So potential drop per unit length is

$$= \frac{5 \times 10^{-3}}{10 \times 10^{-2}} = 5 \times 10^{-2}\left(\frac{\text{V}}{\text{m}}\right)$$

Hence, potential drop across 1 m length of potentiometer wire is

$$V_{AB} = 5 \times 10^{-2}\left(\frac{\text{V}}{\text{m}}\right) \times 1 = 5 \times 10^{-2}\ \text{V}$$

Now, potential drop that must occurs across resistance R is

$$V_R = 4 - 5 \times 10^{-2} = \frac{395}{100}\,V$$

Now, circuit current is

$$i = \frac{V}{R_{\text{total}}} = \frac{4}{R+5}$$

Hence, for resistance R, using $V_R = iR$, we get

$$\frac{395}{100} = \frac{4}{R+5} \times R$$

$$395\,(R+5) = 400R$$

$$395 \times 5 = (400 - 395)R$$

$\Rightarrow \quad R = 395\,\Omega$

6. For a galvanometer, $i_g \propto \theta$ or $i_g = C\theta$

where, I_g = current through coil of galvanometer,

θ = angle of deflection of coil

and C is the constant of proportionality.

Now, K_1 is closed and K_2 is opened, circuit resistance is

$$R_{\text{eq}} = R_1 + R_g$$

where, R_g = galvanometer resistance.

Hence, galvanometer current is

i.e., $\quad i_g = \dfrac{V}{R_1 + R_g}$

where, V = supply voltage.

As deflection is given θ_0, we have

$$i_g = \frac{V}{R_1 + R_g} = C\theta_0 \quad \text{...(i)}$$

When both keys K_1 and K_2 are closed, circuit resistance.

$$R_{\text{eq}} = R_1 + \left(\frac{R_2 \times R_g}{R_2 + R_g}\right)$$

Current through galvanometer will be

$$i_{g_2} = \frac{V}{\left(R_1 + \dfrac{R_2 R_g}{R_2 + R_g}\right)} \times \frac{R_2}{(R_2 + R_g)}$$

$$= \frac{VR_2}{R_1R_2 + R_1R_g + R_2R_g} = C \cdot \frac{\theta_0}{5} \quad \text{...(ii)}$$

Now, dividing Eq. (i) by Eq. (ii), we get

$$\frac{R_1R_2 + R_1R_g + R_2R_g}{R_2\,(R_1 + R_g)} = 5$$

Substituting $R_1 = 220\,\Omega$ and $R_2 = 5\,\Omega$, we get

$$\frac{1100 + 220\,R_g + 5\,R_g}{5\,(220 + R_g)} = 5$$

$$1100 + 225\,R_g = 5500 + 25R_g$$

$\Rightarrow \quad 200\,R_g = 4400$

$\Rightarrow \quad R_g = \dfrac{4400}{200} = 22\,\Omega$

$\therefore \quad R_g = 22\,\Omega$

7. For meter bridge, if balancing length is l cm, then in first case, $\dfrac{R_1}{R_2} = \dfrac{l}{(100-l)}$

It is given that, $l = 40$ cm,

So, $\dfrac{R_1}{40} = \dfrac{R_2}{100-40}$

or $\dfrac{R_1}{R_2} = \dfrac{2}{3}$ ...(i)

In second case, $R_1' = R_1 + 10$, and balancing length is now 50 cm then

$$\frac{R_1+10}{50} = \frac{R_2}{(100-50)}$$

or $R_1 + 10 = R_2$ ...(ii)

Substituting value of R_2 from (ii) to (i) we get,

or $\dfrac{R_1}{10+R_1} = \dfrac{2}{3}$

$\Rightarrow \quad 3R_1 = 20 + 2R_1$

or $R_1 = 20\,\Omega$

$\Rightarrow \quad R_2 = 30\,\Omega$

Let us assume the parallel connected resistance is x.

Then equivalent resistance is $\dfrac{x\,(R_1+10)}{x + R_1 + 100}$

So, this combination should be again equal to R_1.

$$\frac{(R_1+10)x}{R_1+10+x} = R_1$$

$\Rightarrow \quad \dfrac{30x}{30+x} = 20$

or $30x = 600 + 20x$

or $x = 60\,\Omega$

8. Resistance of galvanometer, $(G) = 20\Omega$

Number of divisions on both side = 30

Figure of merit = 0.005 ampere/division

$\therefore$ Full scale deflection current (I_g)

= Number of divisions × figure of merit

$\Rightarrow \quad I_g = 30 \times 0.005 = 0.15\,\text{A}$

Now, for measuring 15V by this galvanometer, let we use a resistance of R ohm in series with the galvanometer.

$\therefore$ Effective resistance of voltmeter, $R_{\text{eff}} = R + 20$

As,maximum potential measured by voltmeter.

$$V = I_g \cdot R_{\text{eff}} \text{ or } 15 = 0.15(R+20)$$

or $R + 20 = 100$ or $R = 80\Omega$

9. Let length of null point 'J' be 'x' and length of the potentiometer wire be 'L'.

In first case, current in the circuit

$$I_1 = \frac{6}{4+2} = 1\,\text{A}$$

$\therefore$ Potential gradient $= I \times R = \dfrac{1\times 4}{L}$

$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{1\times 4}{L} \times x = \varepsilon_1$$

Given, $\varepsilon_1 = 0.5 = \dfrac{4x}{L}$ or $\dfrac{x}{L} = \dfrac{1}{8}$ ...(i)

In second case, current in the circuit

$$I_2 = \frac{6}{4+6} = 0.6\,\text{A}$$

$\therefore$ Potential gradient $= \dfrac{0.6\times 4}{L}$

$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{0.6\times 4}{L} \times x = \varepsilon_2$$

$\Rightarrow \quad \varepsilon_2 = \dfrac{0.6\times 4}{L} \times \dfrac{L}{8}$ [using Eq. (i)]

$\Rightarrow \quad \varepsilon_2 = 0.3\,\text{V}$

10. Given, length of potentiometer wire $(AB) = L$

Resistance of potentiometer wire $(AB) = 12\,r$

EMF of cell D of potentiometer = ε

Internal resistance of cell 'D' = r

EMF of cell 'C' = $\dfrac{\varepsilon}{2}$

Internal resistance of cell 'C' = $3r$

Current in potentiometer wire

$$i = \frac{\text{EMF of cell of potentiometer}}{\text{Total resistance of potentiometer circuit}}$$

$\Rightarrow i = \dfrac{\varepsilon}{r+12r} = \dfrac{\varepsilon}{13r}$

Potential drop across the balance length AJ of potentiometer wire is

$$V_{AJ} = i \times R_{AJ}$$

$\Rightarrow \quad V_{AJ} = i$ (Resistance per unit length of potentiometer wire × length AJ)

$\Rightarrow \quad V_{AJ} = i\left(\dfrac{12r}{L} \times x\right)$

where, x = balance length AJ.

As null point occurs at J so potential drop across balance length AJ = EMF of the cell 'C'.

$\Rightarrow \quad V_{AJ} = \dfrac{\varepsilon}{2}$

$\Rightarrow \quad i\left(\dfrac{12r}{L} \times x\right) = \dfrac{\varepsilon}{2}$

$$\Rightarrow \qquad \frac{\varepsilon}{13r} \times \frac{12r}{L} \times x = \frac{\varepsilon}{2}$$

$$\Rightarrow \qquad x = \frac{13}{24}L$$

11. $$\frac{R_1}{R_2} = \frac{l}{100 - l} \qquad \ldots\text{(i)}$$

and $$\frac{R_2}{R_1} = \frac{l - 10}{100 - (l - 10)} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$\Rightarrow \qquad l = 55\text{cm}$

Substituting in Eq. (i), we get

$$\frac{R_1}{R_2} = \frac{55}{45} \qquad \ldots\text{(iii)}$$

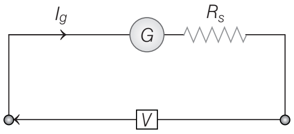
Further, $\qquad R_1 + R_2 = 100\ \Omega \qquad \ldots\text{(iv)}$

Solving Eqs. (iii) and (iv),

$$R_1 = 550\ \Omega$$

12. Using formula, $r = R\left(\frac{l_1}{l_2} - 1\right) = 5\left(\frac{52}{40} - 1\right) = 5 \times \frac{12}{40} = 1.5\ \Omega$

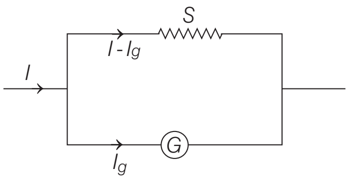
13. For a voltmeter,



$$I_g\ (G + R_s) = V \Rightarrow R = \frac{V}{I_g} - G$$

$\Rightarrow \quad R = 1985 = 1.985\ \textbf{k}\Omega \quad \text{or} \quad R = 1.985 \times 10^3\ \Omega$

14.



In parallel, current distributes in inverse ratio of resistance. Hence,

$$\frac{I - I_g}{I_g} = \frac{G}{S} \Rightarrow S = \frac{GI_g}{I - I_g}$$

As I_g is very small, hence

$$S = \frac{GI_g}{I}$$

$$b = \frac{(100)\,(1 \times 10^{-3})}{10} = 0.01\ \Omega$$

15. For balanced meter bridge

$$\frac{X}{R} = \frac{l}{(100 - l)} \qquad (\text{where, } R = 90\ \Omega)$$

$$\therefore \qquad \frac{X}{90} = \frac{40}{100 - 40}$$

$$\therefore \qquad X = 60\ \Omega$$

$$X = R\,\frac{l}{(100 - l)}$$

$$\frac{\Delta X}{X} = \frac{\Delta l}{l} + \frac{\Delta l}{100 - l} = \frac{0.1}{40} + \frac{0.1}{60}$$

$$\Delta X = 0.25$$

So, $\qquad X = (60 \pm 0.25)\ \Omega$

16. Using the concept of balanced Wheat stone bridge, we have,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\therefore \qquad \frac{X}{(52 + 1)} = \frac{10}{(48 + 2)}$$

$$\therefore \qquad X = \frac{10 \times 53}{50} = 10.6\ \Omega$$

$\therefore$ Correct option is (b).

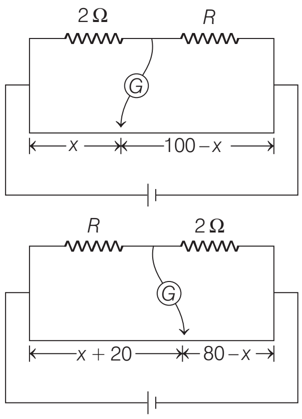
17. We will require a voltmeter, an ammeter, a test resistor and a variable battery to verify Ohm's law.

Voltmeter which is made by connecting a high resistance with a galvanometer is connected in parallel with the test resistor.

Further, an ammeter which is formed by connecting a low resistance in parallel with galvanometer is required to measure the current through test resistor.

The correct option is (c).

18. $R > 2\ \Omega \quad \Rightarrow \quad \therefore \quad 100 - x > x$



Applying, $\qquad \frac{P}{Q} = \frac{R}{S}$

We have $\qquad \frac{2}{R} = \frac{x}{100 - x} \qquad \ldots\text{(i)}$

$$\frac{R}{2} = \frac{x + 20}{80 - x} \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get, $R = 3\ \Omega$

$\therefore$ Correct option is (a).

19. $V_{ab} = i_g.G = (i - i_g)S$

$$\Rightarrow \qquad i = \left(1 + \frac{G}{S}\right)i_g$$

Substituting the values, we get

$$i = 100.1 \text{ mA}$$

$\therefore$ Correct answer is (a).

20. BC, CD and BA are known resistances.
The unknown resistance is connected between A and D.
Hence, the correct option is (c).

21. The ratio $\frac{AC}{CB}$ will remain unchanged.

22. Circuit in option (a) can be used to verify the equation $V = IR$ by varying net resistance of the circuit. No circuit is given to verify Ohm's law.

23. Statement I is false and Statement II is true.

24. With increase in temperature, the value of unknown resistance will increase.
In balanced Wheat stone bridge condition,

$$\frac{R}{X} = \frac{l_1}{l_2}$$

Here, R = value of standard resistance.
X = value of unknown resistance.

To take null point at same point

or $\frac{l_1}{l_2}$ to remain unchanged, $\frac{R}{X}$ should also remain unchanged.

Therefore, if X is increasing R, should also increase.
$\therefore$ Correct option is (d).

25. For maximum range of voltage resistance should be maximum. So, all four should be connected in series. For maximum range of current, net resistance should be least. Therefore, all four should be connected in parallel.

26. To increase the range of ammeter a parallel resistance (called shunt) is required which is given by

$$S = \left(\frac{i_g}{i - i_g}\right)G$$

For option (c)

$$S = \left(\frac{50 \times 10^{-6}}{5 \times 10^{-3} - 50 \times 10^{-6}}\right)(100) \approx 1\,\Omega$$

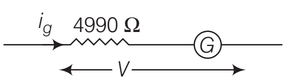
To change it in voltmeter, a high resistance R is put in series, where, R is given by $R = \frac{V}{i_g} - G$

For option (b), $R = \frac{10}{50 \times 10^{-6}} - 100$

$$\approx 200 \text{ k}\Omega$$

Therefore, options (b) and (c) are correct.

27.



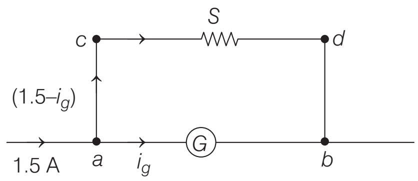
$$i_g (G + 4990) = V$$

$$\Rightarrow \quad \frac{6}{1000}(G + 4990) = 30$$

$$\Rightarrow \quad G + 4990 = \frac{30000}{6} = 5000$$

$$\Rightarrow \quad G = 10\,\Omega$$

$$V_{ab} = V_{cd}$$



$$\Rightarrow \quad i_g G = (1.5 - i_g)\,S$$

$$\Rightarrow \quad \frac{6}{1000} \times 10 = \left(1.5 - \frac{6}{1000}\right) S$$

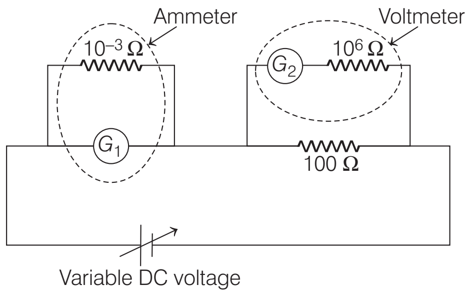
$$\Rightarrow \quad S = \frac{60}{1494} = \frac{2n}{249}$$

$$\Rightarrow \quad n = \frac{249 \times 30}{1494} = \frac{2490}{498} = 5$$

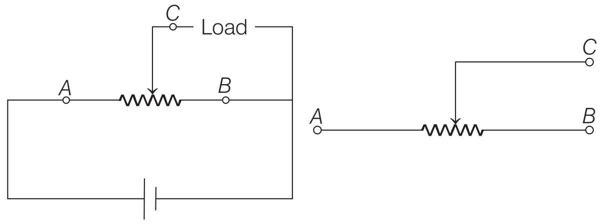
28. Slide wire bridge is most sensitive when the resistance of all the four arms of bridge is same.

Hence, B is the most accurate answer.

29.

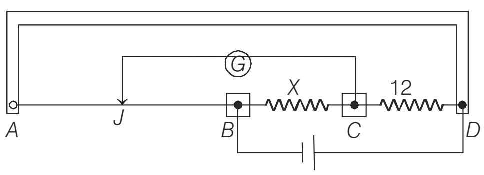


30. The rheostat is as shown in figure. Battery should be connected between A and B and the load between C and B.



31. (a) There are no positive and negative terminals on the galvanometer because only zero deflection is needed.

(b)



(c) $AJ = 60\,\text{cm}$

$\therefore \quad BJ = 40\,\text{cm}$

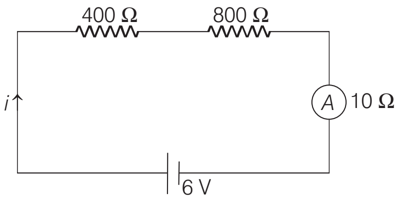
If no deflection is taking place. Then, the Wheatstone's bridge is said to be balanced.

Hence, $$\frac{X}{12} = \frac{R_{BJ}}{R_{AJ}}$$

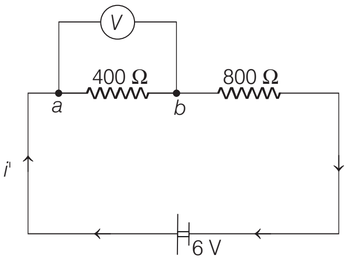
or $$\frac{X}{12} = \frac{40}{60} = \frac{2}{3}$$

or $$x = 8\,\Omega$$

32.



1000 Ω (a)



(b)

Refer figure (a) Current through ammeter,

$$i = \frac{\text{net emf}}{\text{net resistance}}$$
$$= \frac{6}{400 + 800 + 10}$$
$$= 4.96 \times 10^{-3}\,\text{A}$$
$$= 4.96\,\text{mA}$$

Refer figure (b) Combined resistance of 1000 Ω voltmeter and 400 Ω resistance is,

$$R = \frac{1000 \times 400}{1000 + 400} = 285.71\,\Omega$$

$$\therefore \quad i = \frac{6}{(285.71 + 800)} = 5.53 \times 10^{-3}\ \text{A}$$

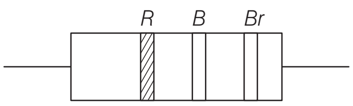
Reading of voltmeter

$$= V_{ab} = i'R = (5.53 \times 10^{-3})\,(285.71) = 1.58\,\text{V}$$

Topic 5 Miscellaneous Problems

1. Given, resistance is 200 Ω.

$$= 20 \times 10^1\,\Omega$$



So, colour scheme will be red, black and brown.

Significant figure of red band is 2 and for green is 5. When red (2) is replaced with green (5), new resistance will be

$$200\text{ ohm} \longrightarrow 500\text{ ohm}$$

2. The value of R_1 (orange, red, brown)

$$\Rightarrow \quad = 32 \times 10 = 320\,\Omega$$

Given, $R_2 = 80\,\Omega$ and $R_4 = 40\,\Omega$

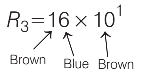
In balanced Wheatstone bridge condition,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow \quad R_3 = R_4 \times \frac{R_1}{R_2}$$

$$\Rightarrow \quad R_3 = \frac{40 \times 320}{80}$$

or $$R_3 = 160\,\Omega = 16 \times 10^1$$

Comparing the value of R_3 with the colours assigned for the carbon resistor, we get



3. Measured value of R = 5% less than actual value of R.

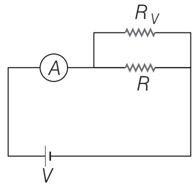
Actual values of $R = 30\,\Omega$

So, measured value of R is

$$R' = 30 - (5\%\text{ of }30) = 30 - \frac{5}{100} \times 30$$

$$\Rightarrow \quad R' = 28.5\,\Omega \quad \ldots \text{(i)}$$

Now, let us assume that internal resistance of voltmeter R_V. Replacing voltmeter with its internal resistance, we get following circuit.



It is clear that the measured value, R' should be equal to parallel combination of R and R_V. Mathematically,

$$R' = \frac{RR_V}{R + R_V} = 28.5\,\Omega$$

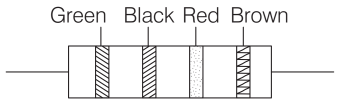
Given, $$R = 30\,\Omega \Rightarrow \frac{30R_V}{30 + R_V} = 28.5$$

$$\Rightarrow \quad 30R_V = (28.5 \times 30) + 28.5\,R_V$$

$$\Rightarrow \quad 1.5R_V = 28.5 \times 30$$

$$\Rightarrow \quad R_V = \frac{28.5 \times 30}{1.5} = 19 \times 30 \text{ or } R_V = 570\,\Omega$$

4. Colour code of carbon resistance is shown in the figure below



So, resistance value of resistor using colour code is

$$R = 502 \times 10 = 50.2 \times 10^2 \ \Omega$$

Here, we must know that for given carbon resistor first three colours give value of resistance and fourth colour gives multiplier value.

Now using power, $P = i^2 R$

we get, $$i = \sqrt{\frac{P}{R}} = \sqrt{\frac{2}{50.2 \times 10^2}}$$

$$\simeq 20 \times 10^{-3} \text{ A} = 20 \text{ mA}.$$

5. The value of a carbon resistor is given as,

$$R = AB \times C \pm D\% \quad \dots \text{(i)}$$

where, colour band A and B (first two colours) indicate the first two significant figures of resistance in ohms, C (third color) indicates the decimal multiplier and D (forth colour) indicates the tolerance in % as per the indicated value.

The table for colour code of carbon resistor is given below,

Colour Codes	**Values (A, B)**	**Multiplier (C)**	**Tolerance (D) (%)**
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold			$\pm 5\%$
Silver			$\pm 10\%$
No colour			$\pm 20\%$

$\therefore$ Comparing the colors given in the question and the table and writing in the manner of Eq. (i), we get

$$R = 27 \times 10^3 \ \Omega, \pm 10\%$$

$$= 27 \text{ k}\Omega, \pm 10\% \quad .$$

6. The value of a carbon resistor is given as

$$R = AB \times C \pm D\% \quad \dots\text{(i)}$$

where, colour band A and B (first two colour) indicates the first two significant figures of resistance in Ohms.

C (third colour) indicate the decimal multiplies and D (fourth colour) indicates the tolerance in % as per the indicated value.

The table for colour code,

Colour code	**Values (AB)**	**Multiplier (C)**	**Tolerance**
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Voilet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold	-	-	$\pm$ 5%
Silver	-	-	$\pm$ 10%
No colour	-	-	$\pm$ 20%

$\because$ Comparing the colours given in the question and table and writing in the manner of Eq. (i), we get

$$R = 53 \times 10^4 \pm 5\% \ \Omega$$

or $$R = 530 \times 10^3 \ \Omega \pm 5\% \text{ or } R = 530 \text{ k}\Omega \pm 5\%$$

7. In a balanced Wheatstone bridge, there is no effect on position of null point, if we exchange the battery and galvanometer. So, option (a) is incorrect.

8. Total power (P) consumed

$$= (15 \times 40) + (5 \times 100) + (5 \times 80) + (1 \times 1000) = 2500 \text{ W}$$

As we know,

Power, $P = VI \Rightarrow I = \dfrac{2500}{220} \text{ A} = \dfrac{125}{11} = 11.3 \text{ A}$

Minimum capacity should be 12 A.

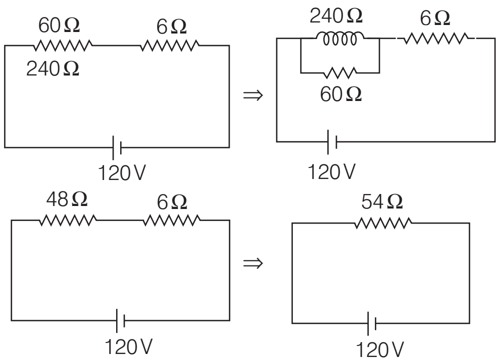
9. As, $P = \dfrac{V^2}{R}$

where, P = power dissipates in the circuit,

V = applied voltage,

R = net resistance of the circuit

$$R = \frac{120 \times 120}{60} = 240 \ \Omega \qquad \text{[resistance of bulb]}$$



$$R_{eq} = 240 + 6 = 246\ \Omega$$

$$\Rightarrow \quad i_1 = \frac{V}{R_{eq}} = \frac{120}{246} \qquad \text{[before connecting heater]}$$

$$R = \frac{V^2}{R} = \frac{120 \times 120}{240}$$

$$\Rightarrow \quad R = 60\ \Omega \qquad \text{[resistance of heater]}$$

So, from figure,

$$V_1 = \frac{240}{246} \times 120 = 117.073\ \text{V} \qquad [\because V = IR]$$

$$\Rightarrow i_2 = \frac{V}{R_2} = \frac{120}{54} \quad \Rightarrow V_2 = \frac{48}{54} \times 120 = 106.66\ \text{V}$$

$$V_1 - V_2 = 10.41\ \text{V}$$

10. $R = \dfrac{V^2}{P}$ or $R \propto \dfrac{1}{P}$

$$\therefore \quad \frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$$

Hence, the correct option is (d).

11. Applying $P = \dfrac{V^2}{R}$, $R_1 = 1\Omega, R_2 = 0.5\Omega$ and $R_3 = 2\Omega$

$$V_1 = V_2 = V_3 = 3\text{V}$$

$$\therefore \quad P_1 = \frac{(3)^2}{1} = 9\ \text{W}$$

$$P_2 = \frac{(3)^2}{0.5} = 18\ \text{W} \text{ and } P_3 = \frac{(3)^2}{2} = 4.5\ \text{W}$$

$$\therefore \quad P_2 > P_1 > P_3$$

$\therefore$ Correct option is (c).

12. $W = 0$. Therefore, from first law of thermodynamics,

$$\Delta U = \Delta Q = i^2Rt = (1)^2\ (100)\ (5 \times 60)\ \text{J} = 30\ \text{kJ}.$$

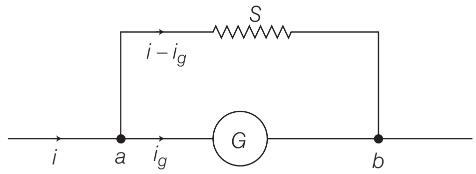
$\therefore$ Correct answer is (d).

13. At 0 K, a semiconductor becomes a perfect insulator. Therefore, at 0 K, if some potential difference is applied across an insulator or semiconductor, current is zero. But a conductor will become a super conductors at 0 K. Therefore, current will be infinite. In reverse biasing at 300 K through a *p-n* junction diode, a small finite current flows due to minority charge carriers.

14. Given, $N = 50$, $A = 2 \times 10^{-4}\ \text{m}^2, C = 10^{-4}, R = 50\Omega$, $B = 0.02\ T, \theta = 0.2$ rad $Ni_g AB = C\theta$

$$\Rightarrow \quad i_g = \frac{C\,\theta}{N\ AB} = \frac{10^{-4} \times 0.2}{50 \times 2 \times 10^{-4} \times 0.02} = 0.1\ \text{A}$$

$$\therefore \quad V_{ab} = i_g \times G = (i - i_g)S \Rightarrow 0.1 \times 50 = (1 - 0.1) \times S$$



$$5 = 0.9 \times S$$

$$\therefore \quad S = \frac{50}{9}\ \Omega = 5.55\ \Omega$$

15. In series, $i = \dfrac{2E}{2+R} \Rightarrow J_1 = i^2R = \left(\dfrac{2E}{2+R}\right)^2 \cdot R$

In parallel, $\quad i = \dfrac{E}{0.5 + R}$

$$\therefore \quad J_2 = i^2R = \left(\frac{E}{0.5+R}\right)^2 \cdot R$$

$$\frac{J_1}{J_2} = 2.25$$

$$= \frac{4\ (0.5+R)^2}{(2+R)^2} \text{ or } 1.5 = \frac{2\ (0.5+R)}{(2+R)}$$

Solving we get, $R = 4\ \Omega$

$\therefore$ The answer is 4.

16. In electrostatic condition, electric field inside a conductor is zero. But when a current flows through the conductor electric field is non-zero.

17. (a) $I^2Rt = mS\ \Delta\theta = m\ (S\ \Delta\theta)$

$$I^2\left(\rho_1 \cdot \frac{l}{A}\right) t = Al\rho_2\ (S\ \Delta\theta)$$

Here, $\quad \rho_1$ = specific resistance

and $\quad \rho_2$ = density

$$\therefore \quad t = \frac{A^2\rho_2\ (S\ \Delta\theta)}{I^2\rho_1} \qquad \ldots\text{(i)}$$

Substituting the values, we have

$$t = \frac{(0.5 \times 10^{-6})^2 (9 \times 10^3)(9 \times 10^{-2} \times 4.18 \times 1050)}{(1.0)^2 (1.6 \times 10^{-8})}$$

$$= 55.5\ \text{s}$$

(b) From Eq. (i) we can see that time is independent of length of wire.

18. In series current is same. Therefore, from the relation $P = i^2R$, a bulb having more resistance consumes more power or glows brighter. 25W bulb has more resistance $\left(R = \dfrac{V^2}{P}\right)$ compared to 100 W bulb. Hence, it will glow brighter.

Download Chapter Test
http://tinyurl.com/y56mg5vk *or*

14

Electrostatics

Topic 1 Electrostatic Force and Field Strength

Objective Questions I (Only one correct option)

1. Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance a, from the centre. If A and B do not experience any force, then **(Main 2019, 12 April II)**

(a) $a = 8^{-1/4} R$ (b) $a = \dfrac{3R}{2^{1/4}}$

(c) $a = 2^{-1/4} R$ (d) $a = R/\sqrt{3}$

2. Four point charges $-q, +q, +q$ and $-q$ are placed on Y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the X-axis at $x = D$, with $D >> d$, will behave as **(Main 2019, 9 April II)**

(a) $E \propto \dfrac{1}{D}$ (b) $E \propto \dfrac{1}{D^3}$ (c) $E \propto \dfrac{1}{D^2}$ (d) $E \propto \dfrac{1}{D^4}$

3. Two point charges q_1 $(\sqrt{10}\,\mu C)$ and q_2 $(-25\,\mu C)$ are placed on the x-axis at $x = 1\,m$ and $x = 4\,m$, respectively. The electric field (in V/m) at a point $y = 3\,m$ on Y-axis is **(Main 2019, 9 Jan II)**

$\left(\text{Take, } \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N-m}^2\text{C}^{-2}\right)$

(a) $(63\,\hat{\mathbf{i}} - 27\hat{\mathbf{j}}) \times 10^2$ (b) $(81\,\hat{\mathbf{i}} - 81\hat{\mathbf{j}}) \times 10^2$

(c) $(-81\,\hat{\mathbf{i}} + 81\hat{\mathbf{j}}) \times 10^2$ (d) $(-63\,\hat{\mathbf{i}} + 27\hat{\mathbf{j}}) \times 10^2$

4. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \dfrac{A}{r^2} e^{-\frac{2r}{a}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is **(Main 2019, 9 Jan Shift II)**

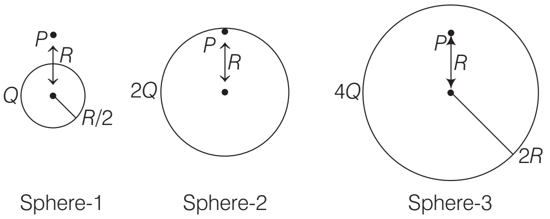
(a) $a \log\left(\dfrac{1}{1 - \dfrac{Q}{2\pi a A}}\right)$ (b) $a \log\left(1 - \dfrac{Q}{2\pi a A}\right)$

(c) $\dfrac{a}{2} \log\left(1 - \dfrac{Q}{2\pi a A}\right)$ (d) $\dfrac{a}{2} \log\left(\dfrac{1}{1 - \dfrac{Q}{2\pi a A}}\right)$

5. Three charges $+Q, q, +Q$ are placed respectively at distance $0, \dfrac{d}{2}$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x = 0$ is zero, then value of q is **(Main 2019, 9 Jan Shift I)**

(a) $\dfrac{+Q}{2}$ (b) $\dfrac{+Q}{4}$ (c) $\dfrac{-Q}{2}$ (d) $\dfrac{-Q}{4}$

6. Charges $Q, 2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2, R$ and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then **(2014 Adv.)**



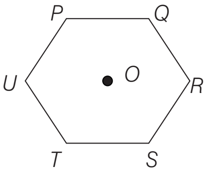
(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_1 > E_2$

(c) $E_2 > E_1 > E_3$ (d) $E_3 > E_2 > E_1$

7. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \dfrac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $y\,(y << a)$ along the y-axis, the net force acting on the particle is proportional to **(2013 Main)**

(a) y (b) $-y$ (c) $\dfrac{1}{y}$ (d) $-\dfrac{1}{y}$

8. Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charge is possible for, P, Q, R, S, T and U respectively? **(2004, 1M)**



(a) $+, -, +, -, -, +$ (b) $+, -, +, -, +, -$

(c) $+, +, -, +, -, -$ (d) $-, +, +, -, +, -$

9. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also, initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to **(1997, 1M)**
(a) 1 (b) $(m_p/m_e)^{1/2}$ (c) $(m_e/m_p)^{1/2}$ (d) 1836

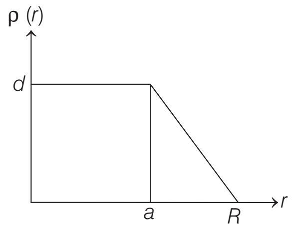
10. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to **(1987, 2M)**
(a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$ (c) $+\frac{Q}{4}$ (d) $+\frac{Q}{2}$

11. Two equal negative charges $-q$ are fixed at points $(0, -a)$ and $(0, a)$ on y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will **(1984, 2M)**
(a) execute simple harmonic motion about the origin
(b) move to the origin and remain at rest
(c) move to infinity
(d) execute oscillatory but not simple harmonic motion

Passage Based Questions

Passage

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ (charge per unit volume) is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.



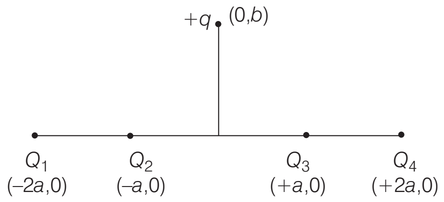
12. The electric field at $r = R$ is **(2008, 4M)**
(a) independent of a (b) directly proportional to a
(c) directly proportional to a^2 (d) inversely proportional to a

13. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is **(2008, 4M)**
(a) $\frac{3Ze}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$ (c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

14. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies **(2008, 4M)**
(a) $a = 0$ (b) $a = \frac{R}{2}$ (c) $a = R$ (d) $a = \frac{2R}{3}$

Match the Column

15. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x-axis at $x = -2a, -a, +a$ and $+2a$ respectively. A positive charge q is placed on the positive y-axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists. **(2014 Adv.)**



	List I		List II
P.	Q_1, Q_2, Q_3, Q_4 all positive	1.	$+x$
Q.	Q_1, Q_2 positive; Q_3, Q_4 negative	2.	$-x$
R.	Q_1, Q_4 positive; Q_2, Q_3 negative	3.	$+y$
S.	Q_1, Q_3 positive; Q_2, Q_4 negative	4.	$-y$

Codes
(a) P-3, Q-1, R-4, S-2 (b) P-4, Q-2, R-3, S-1
(c) P-3, Q-1, R-2, S-4 (d) P-4, Q-2, R-1, S-3

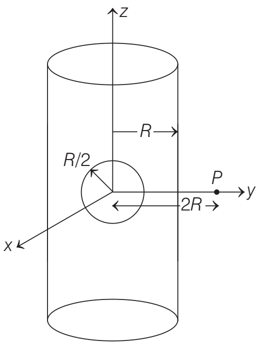
Objective Questions II (One or more correct option)

16. Let $E_1(r), E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then **(2014 Adv.)**
(a) $Q = 4\sigma\pi r_0^2$
(b) $r_0 = \frac{\lambda}{2\pi\sigma}$
(c) $E_1(r_0/2) = 2E_2(r_0/2)$
(d) $E_2(r_0/2) = 4E_3(r_0/2)$

17. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre **(1998, 2M)**
(a) increases as r increases for $r < R$
(b) decreases as r increases for $0 < r < \infty$
(c) decreases as r increases for $R < r < \infty$
(d) is discontinuous at $r = R$

18. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$.

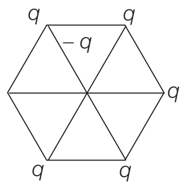
The value of k is **(2012)**



19. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a **(2009)**

Fill in the Blanks

20. Five point charges, each of value $+q$ coulomb, are placed on five vertices of a regular hexagon of side L metre. The magnitude of the force on the point charge of value $-q$ coulomb placed at the centre of the hexagon is ....newton. **(1992, 1 M)**



21. Two small balls having equal positive charges Q (coulomb) on each are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole setup is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the strings is ...... and the tension in each string is ......newton. **(1986, 2M)**

True/False

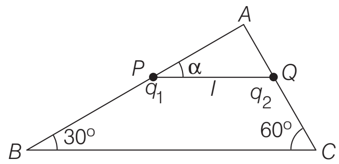
22. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring. **(1988, 2M)**

Analytical & Descriptive Questions

23. Three particles, each of mass 1 g and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. (Take $g = 10\ \text{m/s}^2$). **(1988, 5M)**

24. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} C is at rest in a horizontal uniform electric field of 20,000 V/m. Find the tension in the thread of the pendulum and the angle it makes with the vertical.
(Take $g = 9.8\ \text{ms}^{-2}$) **(1979)**

25. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane as shown in figure. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires.



Considering the case when the beads are stationary determine

(a) (i) The angle α **(1978)**

(ii) The tension in the cord

(iii) The normal reaction on the beads

(b) If the cord is now cut what are the value of the charges for which the beads continue to remain stationary?

Topic 2 Electrostatics Potential, Potential Energy, Work Done and Energy Conservation

Objective Questions I (Only one correct option)

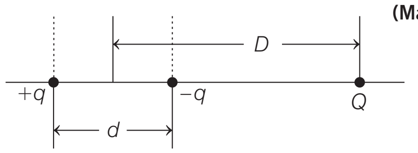
1. In free space, a particle A of charge $1\,\mu$C is held fixed at a point P. Another particle B of the same charge and mass $4\,\mu$g is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is

$$\left[\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N-m}^2\text{C}^{-2}\right]$$ **(Main 2019, 10 April II)**

(a) 1.5×10^2 m/s (b) 3.0×10^4 m/s

(c) 1.0 m/s (d) 2.0×10^3 m/s

2. A system of three charges are placed as shown in the figure **(Main 2019, 9 April I)**

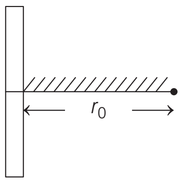


If $D >> d$, the potential energy of the system is best given by

(a) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} + \frac{2qQd}{D^2}\right]$ (b) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

(c) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{2D^2}\right]$ (d) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

3. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to **(Main 2019, 8 April II)**



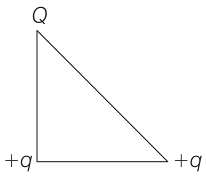
(a) $v \propto \left(\frac{r}{r_0}\right)$ (b) $v \propto e^{+\frac{r}{r_0}}$

(c) $v \propto \ln\left(\frac{r}{r_0}\right)$ (d) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

4. The electric field in a region is given by $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$ SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is **(Main 2019, 8 April II)**

(a) -48 V (b) -520 V
(c) 180 V (d) 320 V

5. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is **(Main 2019, 11 Jan I)**



(a) $-2q$ (b) $\frac{-q}{1+\sqrt{2}}$

(c) $+q$ (d) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

6. Four equal point charges Q each are placed in the xy-plane at (0, 2), (4, 2), (4, –2) and (0, – 2). The work required to put a fifth charge Q at the origin of the coordinate system will be **(Main 2019, 10 Jan II)**

(a) $\frac{Q^2}{4\pi\varepsilon_0}$ (b) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$

(c) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (d) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

7. A charge Q is distributed over three concentric spherical shells of radii $a, b, c\ (a < b < c)$ such that their surface charge densities are equal to one another.

The total potential at a point at distance r from their common centre, where $r < a$ would be **(Main 2019, 10 Jan I)**

(a) $\frac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$ (b) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

(c) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$ (d) $\frac{Q}{12\pi\varepsilon_0}\cdot\frac{ab+bc+ca}{abc}$

8. A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the Z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is **(Main 2019, 10 April I)**

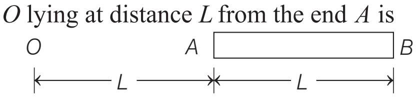
(a) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (b) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (d) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

9. Assume that an electric field $\mathbf{E} = 30x^2\,\hat{\mathbf{i}}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is **(2014 Main)**

(a) 120 J (b) -120 J (c) -80 J (d) 80 J

10. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is **(2013 Main)**

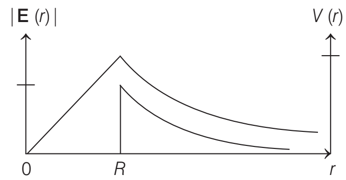


(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$

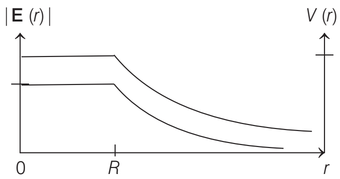
(c) $\frac{Q}{4\pi\varepsilon_0 L\ln 2}$ (d) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$

11. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\mathbf{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph? **(2012)**

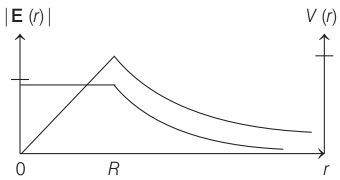
(a)



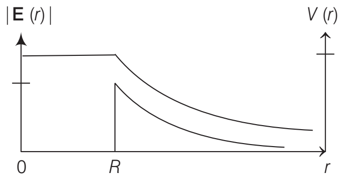
(b)



(c)



(d)



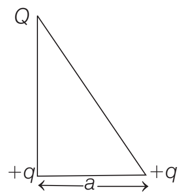
12. Positive and negative point charges of equal magnitude are kept at $(0, 0, a/2)$ and $(0, 0, -a/2)$, respectively. The work done by the electric field when another positive point charge in moved from $(-a, 0, 0)$ to $(0, a, 0)$ is **(2007, 3M)**

(a) positive
(b) negative
(c) zero
(d) depends on the path connecting the initial and final positions

13. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy **(2001, 1M)**

(a) $V_A < V_B$ (b) $V_A > V_B$ (c) $V_A < V_C$ (d) $V_A > V_C$

14. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle triangle (isosceles triangle) as shown. The net electrostatic energy of the configuration is zero, if Q is equal to **(2000, 2M)**



(a) $\dfrac{-q}{1+\sqrt{2}}$ (b) $\dfrac{-2q}{2+\sqrt{2}}$
(c) $-2q$ (d) $+q$

15. A charge $+q$ is fixed at each of the points $x = x_0, x = 3x_0, x = 5x_0 \dots \infty$ on the x-axis and a charge $-q$ is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0 \dots, \infty$. Here, x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\varepsilon_0 r$. Then the potential at the origin due to the above system of charges is **(1998, 2M)**

(a) zero (b) $\dfrac{q}{8\pi\,\varepsilon_0 x_0 \ln 2}$
(c) infinite (d) $\dfrac{q \ln(2)}{4\pi\,\varepsilon_0\, x_0}$

16. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field E everywhere in space. The value of the integral $\int_{l=\infty}^{l=0} -\mathbf{E}\cdot\mathbf{dl}$ ($l = 0$ being centre of the ring) in volt is **(1997, 2M)**

(a) +2 (b) -1
(c) -2 (d) zero

17. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of **(1981, 2M)**

(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
(c) If Statement I is true; Statement II is false.
(d) If Statement I is false; Statement II is true.

18. Statement I For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

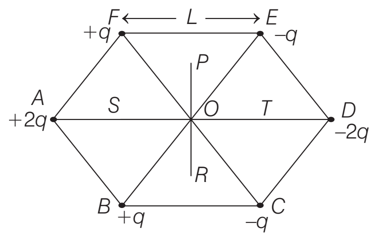
Statement II The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\dfrac{Q}{4\pi\varepsilon_0 R}$. **(2008, 3M)**

Objective Questions II (One or more correct option)

19. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\dfrac{3V_0}{2}, \dfrac{5V_0}{4}, \dfrac{3V_0}{4}$ and $\dfrac{V_0}{4}$ have radius R_1, R_2, R_3, and R_4 respectively. Then,

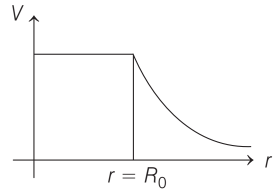
(a) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$ **(2015 Main)**
(b) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(c) $2R < R_4$
(d) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

20. Six point charges are kept at the vertices of a regular hexagon of side L and centre O as shown in the figure. Given that $K = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{L^2}$, which of the following statements(s) is(are) correct. **(2012)**



(a) The electric field at O is $6K$ along OD
(b) The potential at O is zero
(c) The potential at all points on the line PR is same
(d) The potential at all points on the line ST is same

21. For spherical symmetrical charge distribution, variation of electric potential with distance from centre is given in diagram. Given that **(2006, 5M)**



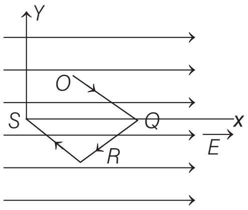
$$V = \frac{q}{4\pi\,\varepsilon_0 R_0} \text{ for } r \leq R_0 \quad \text{and} \quad V = \frac{q}{4\pi\varepsilon_0 r} \text{ for } r \geq R_0$$

Then, which option(s) are correct
(a) Total charge within $2R_0$ is q
(b) Total electrostatic energy for $r \leq R_0$ is zero
(c) At $r = R_0$ electric field is discontinuous
(d) There will be no charge anywhere except at $r = R_0$

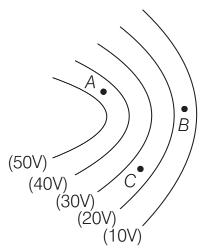
Fill in the Blanks

22. The electric potential V at any point x, y, z (all in metre) in space is given by $V = 4x^2$ volt. The electric field at the point (1m, 0, 2 m) is ……..V/m. **(1992, 1M)**

23. A point charge q moves from point P to point S along the path $PQRS$ (Fig.) in a uniform electric field E pointing parallel to the positive direction of the X-axis. The coordinates of points P, Q, R and S are $(a, b, 0)$, $(2a, 0, 0)$ $(a, -b, 0)$ $(0,0,0)$ respectively. The work done by the field in the above process is given by the expression …… **(1989, 2M)**



24. Figure shows lines of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of the electric field is greatest at the point…… . **(1984, 2M)**

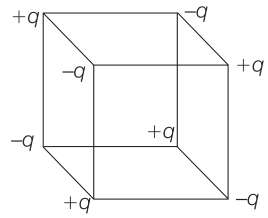


True/False

25. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. **(1981, 2M)**

Analytical & Descriptive Questions

26. Eight point charges are placed at the corners of a cube of edge a as shown in figure. Find the work done in disassembling this system of charges. **(2003, 2M)**

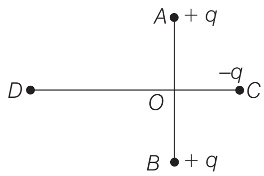


27. Four point charges $+8\,\mu C, -1\,\mu C, -1\,\mu C$ and $+8\,\mu C$ are fixed at the points $-\sqrt{27/2}$ m, $-\sqrt{3/2}$ m, $+\sqrt{3/2}$ m and $+\sqrt{27/2}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge $+0.1\,\mu C$ moves along the x-direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. $(1/4\pi\varepsilon_0 = 9 \times 10^9$ Nm^2/C$^2)$. **(2000, 10 M)**

28. Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the x-y plane. **(1991, 8M)**
(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.
(b) Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.
(c) If a particle of charge $+q$ starts form rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

29. Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges ? **(1987, 7M)**

30. Two fixed, equal, positive charges, each of magnitude $q = 5 \times 10^{-5}$C are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 J. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C. **(1985, 6M)**

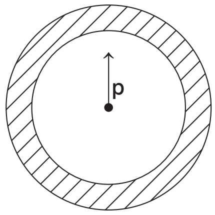


Topic 3 Gauss Theorem and Spherical Shells

Objective Questions I (Only one correct option)

1. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b and carries charge Q. At its centre is a dipole $\mathbf{p}$ as shown.

In this case, **(Main 2019, 12 April I)**

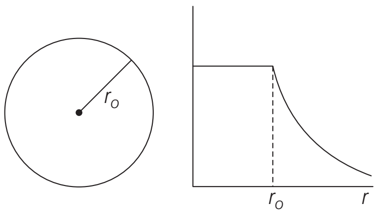


(a) surface charge density on the inner surface is uniform and equal to $\dfrac{\left(\dfrac{Q}{2}\right)}{4\pi a^2}$
(b) electric field outside the shell is the same as that of a point charge at the centre of the shell
(c) surface charge density on the outer surface depends on $|\mathbf{p}|$
(d) surface charge density on the inner surface of the shell is zero everywhere

2. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-4\,Q$, the new potential difference between the same two surfaces is **(Main 2019, 8 April I)**

(a) $-2V$ (b) $2V$ (c) $4V$ (d) V

3. The given graph shows variation (with distance r from centre) of **(Main 2019, 11 Jan I)**



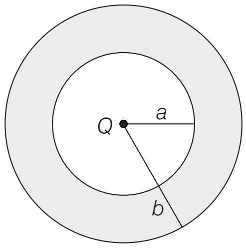
(a) electric field of a uniformly charged spherical shell
(b) potential of a uniformly charged spherical shell
(c) electric field of a uniformly charged sphere
(d) potential of a uniformly charged sphere

4. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma, -\sigma$ and $+\sigma$, respectively. The potential of shell B is **(2018 Main)**

(a) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{b^2-c^2}{c}+a\right)$ (b) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{a^2-b^2}{a}+c\right)$

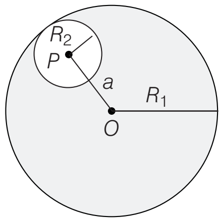
(c) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{a^2-b^2}{b}+c\right)$ (d) $\dfrac{\sigma}{\varepsilon_0}\left(\dfrac{b^2-c^2}{b}+a\right)$

5. The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \dfrac{A}{r}$, where, A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A, such that the electric field in the region between the spheres will be constant, is **(2016 Main)**



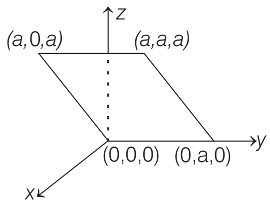
(a) $\dfrac{Q}{2\pi a^2}$ (b) $\dfrac{Q}{2\pi(b^2-a^2)}$ (c) $\dfrac{2Q}{\pi(a^2-b^2)}$ (d) $\dfrac{2Q}{\pi a^2}$

6. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\mathbf{r}$ is $\mathbf{E}(\mathbf{r})$, then the correct statements is/are



(a) $\mathbf{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\mathbf{r}$
(b) $\mathbf{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\mathbf{r}$
(c) $\mathbf{E}$ is uniform, its magnitude is independent of 'a' but its direction depends on $\mathbf{a}$
(d) $\mathbf{E}$ is uniform and both its magnitude and direction depend on $\mathbf{a}$

7. Consider an electric field $\mathbf{E} = E_0\,\hat{\mathbf{x}}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **(2011)**



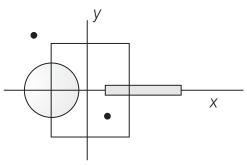
(a) $2E_0a^2$ (b) $\sqrt{2}\,E_0a^2$

(c) E_0a^2 (d) $\dfrac{E_0a^2}{\sqrt{2}}$

8. Three concentric metallic spherical shells of radii R, $2R$ and $3R$ are given charges Q_1, Q_2 and Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is **(2009)**

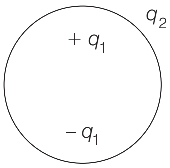
(a) 1 : 2 : 3 (b) 1 : 3 : 5 (c) 1 : 4 : 9 (d) 1 : 8 : 18

9. A disc of radius $\frac{a}{4}$ having a uniformly distributed charge 6C is placed in the x-y plane with its centre at $\left(\frac{-a}{2}, 0, 0\right)$. A rod of length a carrying a uniformly distributed charge $8C$ is placed on the x-axis from $x = \frac{a}{4}$ to $x = \frac{5a}{4}$. Two point charges −7C and 3C are placed at $\left(\frac{a}{4}, \frac{-a}{4}, 0\right)$ and $\left(\frac{-3a}{4}, \frac{3a}{4}, 0\right)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm\frac{a}{2}$, $y = \pm\frac{a}{2}$, $z = \pm\frac{a}{2}$. The electric flux through this cubical surface is **(2009)**



(a) $\frac{-2\,\text{C}}{\varepsilon_0}$ (b) $\frac{2\text{C}}{\varepsilon_0}$ (c) $\frac{10\text{C}}{\varepsilon_0}$ (d) $\frac{12\text{C}}{\varepsilon_0}$

10. Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to **(2004, 1M)**



(a) q_2 (b) only the positive charges
(c) all the charges (d) $+q_1$ and $-q_1$

11. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a change of $-3Q$, the new potential difference between the same two surfaces is **(1989, 2M)**

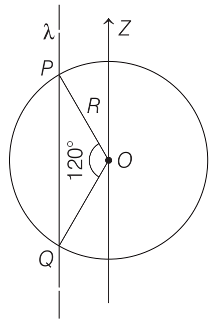
(a) V (b) 2V
(c) 4V (d) – 2V

12. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is **(1983, 1M)**

(a) zero
(b) 10 V
(c) same as at a point 5 cm away from the surface
(d) same as at a point 25 cm away from the surface

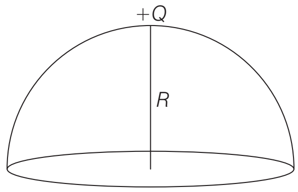
Objective Questions II (One or more correct option)

13. An infinitely long thin non-conducting wire is parallel to the Z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true? **(2018 Adv.)**



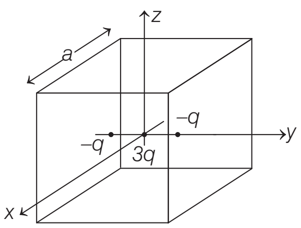
(a) The electric flux through the shell is $\sqrt{3}\, R\lambda / \varepsilon_0$
(b) The z-component of the electric field is zero at all the points on the surface of the shell
(c) The electric flux through the shell is $\sqrt{2}\, R\lambda / \varepsilon_0$
(d) The electric field is normal to the surface of the shell at all points

14. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct? **(2017 Adv.)**



(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$.
(b) The component of the electric field normal to the flat surface is constant over the surface
(c) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$
(d) The circumference of the flat surface is an equipotential

15. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$. Choose the correct option(s). **(2012)**



(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$
(c) The net electric flux crossing the entire region is q/ε_0
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$

16. Which of the following statement(s) is/are correct? **(2011)**

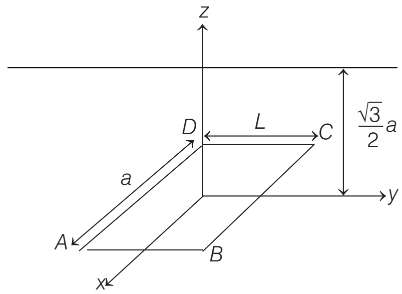
(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss's law will still be valid
(b) The Gauss's law can be used to calculate the field distribution around an electric dipole
(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same
(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

17. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then **(2011)**

(a) $E_A^{\text{inside}} = 0$ (b) $Q_A > Q_B$
(c) $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ (d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

Integer Answer Type Question

18. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \dfrac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\dfrac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is **(2015 Adv.)**



Analytical & Descriptive Questions

19. Three concentric spherical metallic shells, A, B and C of radii a, b and c $(a < b < c)$ have surface charge densities $\sigma, -\sigma$ and σ respectively. **(1990, 7M)**

(a) Find the potential of the three shells A, B and C.
(b) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c.

20. A charge Q is distributed over two concentric hollow spheres of radii r and R $(> r)$ such that the surface densities are equal. Find the potential at the common centre. **(1981, 3M)**

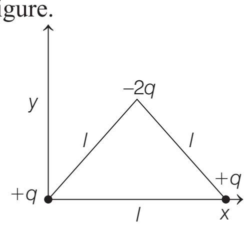
Topic 4 Electric Field Lines, Behaviour of Conductor and Electric Dipole

Objective Questions I (Only one correct option)

1. A point dipole $\mathbf{p} = -p_0\,\hat{\mathbf{x}}$ is kept at the origin. The potential and electric field due to this dipole on the Y-axis at a distance d are, respectively [Take, $V = 0$ at infinity] **(Main 2019, 12 Apr I)**

(a) $\dfrac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \dfrac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (b) $0, \dfrac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$
(c) $0, \dfrac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (d) $\dfrac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \dfrac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

2. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure. **(Main 2019, 12 Jan I)**

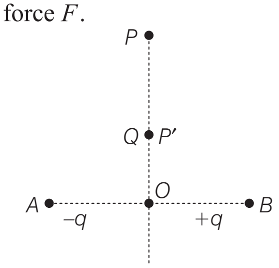


(a) $\sqrt{3}\,ql\,\dfrac{\hat{\mathbf{j}} - \hat{\mathbf{i}}}{\sqrt{2}}$ (b) $2ql\,\hat{\mathbf{j}}$
(c) $-\sqrt{3}\,ql\,\hat{\mathbf{j}}$ (d) $(ql)\dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$

3. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C-m. What is the potential energy of the electric dipole? **(Main 2019, 11 Jan II)**

(a) -9×10^{-20} J (b) -10×10^{-29} J
(c) -20×10^{-18} J (d) -7×10^{-27} J

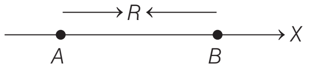
4. Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P, where $OP = y$ and $y >> 2a$. The charge Q experiences an electrostatic force F. **(Main 2019, 10 Jan II)**



If Q is now moved along the equatorial line to P' such that $OP' = \left(\dfrac{y}{3}\right)$, the force on Q will be close to $\left(\dfrac{y}{3} >> 2a\right)$

(a) $\dfrac{F}{3}$ (b) $3F$ (c) $9F$ (d) $27F$

5. Two electric dipoles, A, B with respective dipole moments $\mathbf{d}_A = -4\,qa\,\hat{\mathbf{i}}$ and $\mathbf{d}_B = -2\,qa\,\hat{\mathbf{i}}$ are placed on the X-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is **(Main 2019, 10 Jan I)**

(a) $\dfrac{\sqrt{2}\,R}{\sqrt{2}+1}$ (b) $\dfrac{\sqrt{2}\,R}{\sqrt{2}-1}$ (c) $\dfrac{R}{\sqrt{2}+1}$ (d) $\dfrac{R}{\sqrt{2}-1}$

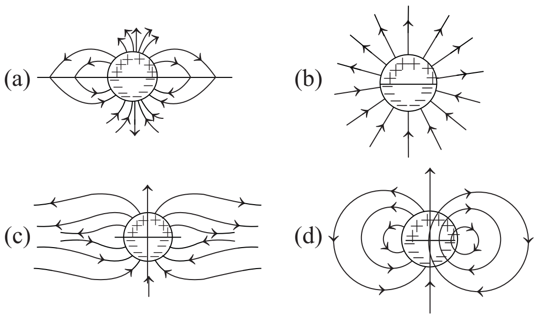
6. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then, value of h is **(Main 2019, 9 Jan I)**

(a) $\dfrac{R}{\sqrt{2}}$ (b) $R\sqrt{2}$ (c) R (d) $\dfrac{R}{\sqrt{5}}$

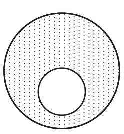
7. An electric dipole has a fixed dipole moment $\mathbf{P}$, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\,\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is **(2017 Main)**

(a) 45° (b) 60° (c) 90° (d) 30°

8. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) **(2015 Main)**

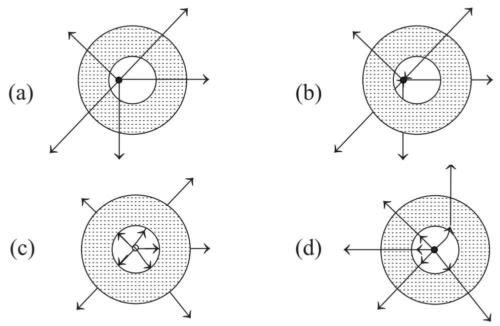


9. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then **(2007, 3M)**

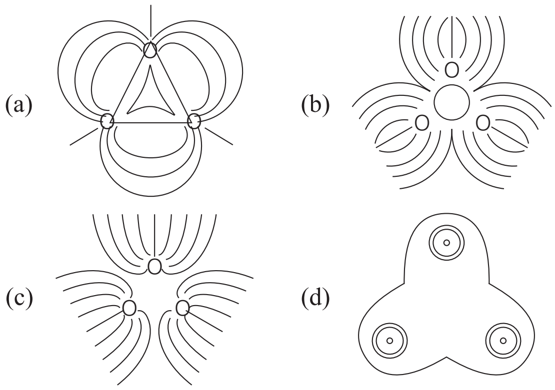


(a) negative and distributed uniformly over the surface of the sphere
(b) negative and appears only at the point on the sphere closest to the point charge
(c) negative and distributed non-uniformly over the entire surface of the sphere
(d) zero

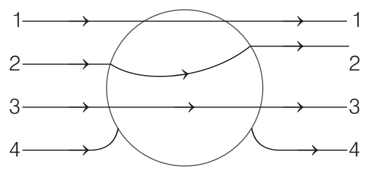
10. A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces ? **(2003, 1M)**



11. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **(2001, 1M)**



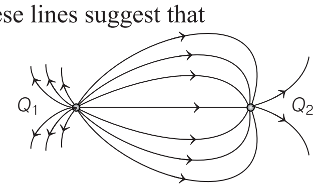
12. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as **(1996, 2M)**



(a) 1 (b) 2 (c) 3 (d) 4

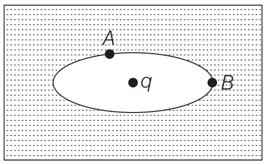
Objective Questions II (One or more correct option)

13. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that **(2010)**



(a) $|Q_1| > |Q_2|$
(b) $|Q_1| < |Q_2|$
(c) at a finite distance to the left of Q_1 the electric field is zero
(d) at a finite distance to the right of Q_2 the electric field is zero

14. An elliptical cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then **(1999, 3M)**



(a) electric field near A in the cavity = electric field near B in the cavity.
(b) charge density at A = charge density at B
(c) potential at A = potential at B
(d) total electric field flux through the surface of the cavity is q/ε_0.

True/False

15. An electric line of force in the x-y plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1$, $y = 0$ in the x-y plane, will move along the circular line of force. **(1988, 2M)**

Analytical & Descriptive Questions

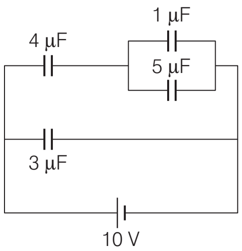
16. A positive point charge q is fixed at origin. A dipole with a dipole moment $\mathbf{p}$ is placed along the x-axis far away from the origin with $\mathbf{p}$ pointing along positive x-axis. Find : (a) the kinetic energy of the dipole when it reaches a distance d from the origin, and (b) the force experienced by the charge q at this moment. **(2003, 4M)**

17. A charged particle is free to move in an electric field. Will it always move along an electric line of force? **(1979)**

Topic 5 Capacitors

Objective Questions I (Only one correct option)

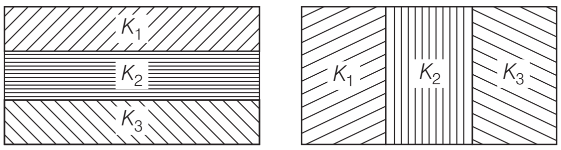
1. In the given circuit, the charge on $4\,\mu F$ capacitor will be **(Main 2019, 12 April II)**



(a) $5.4\,\mu C$ (b) $9.6\,\mu C$ (c) $13.4\,\mu C$ (d) $24\,\mu C$

2. Two identical parallel plate capacitors of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics of equal thickness and dielectric constants K_1, K_2 and K_3.

The first capacitor is filled as shown in Fig. I, and the second one is filled as shown in Fig. II. If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) : **(Main 2019, 12 April I)**



(I) (II)

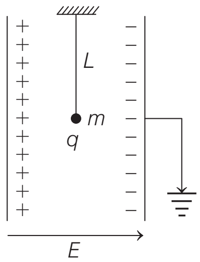
(a) $\dfrac{E_1}{E_2} = \dfrac{K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

(b) $\dfrac{E_1}{E_2} = \dfrac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{K_1K_2K_3}$

(c) $\dfrac{E_1}{E_2} = \dfrac{9\,K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

(d) $\dfrac{E_1}{E_2} = \dfrac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{9\,K_1K_2K_3}$

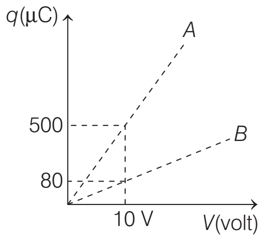
3. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by **(Main 2019, 10 April II)**



(a) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}}$ (b) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2 - \dfrac{q^2E^2}{m^2}}}}$

(c) $2\pi\sqrt{\dfrac{L}{\left(g + \dfrac{qE}{m}\right)}}$ (d) $2\pi\sqrt{\dfrac{L}{\left(g - \dfrac{qE}{m}\right)}}$

4. Figure shows charge (q) *versus* voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are **(Main 2019, 10 April I)**



(a) $60\,\mu F$ and $40\,\mu F$ (b) $50\,\mu F$ and $30\,\mu F$
(c) $20\,\mu F$ and $30\,\mu F$ (d) $40\,\mu F$ and $10\,\mu F$

5 The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is **(Main 2019, 9 April II)**

(a) $\dfrac{(n+1)V}{(K+n)}$ (b) $\dfrac{nV}{K+n}$

(c) V (d) $\dfrac{V}{K+n}$

6. A capacitor with capacitance $5\,\mu\text{F}$ is charged to $5\,\mu\text{C}$. If the plates are pulled apart to reduce the capacitance to $2\,\mu\text{F}$, how much work is done? **(Main 2019, 9 April I)**

(a) 6.25×10^{-6} J (b) 2.16×10^{-6} J

(c) 2.55×10^{-6} J (d) 3.75×10^{-6} J

7. A parallel plate capacitor has $1\,\mu\text{F}$ capacitance. One of its two plates is given $+2\mu\text{C}$ charge and the other plate $+4\mu\text{C}$ charge. The potential difference developed across the capacitor is **(Main 2019, 8 April II)**

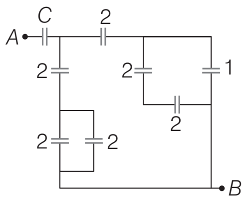
(a) 1 V (b) 5 V (c) 2 V (d) 3 V

8. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is $10^{-4}\,\text{m}^2$. What is the dielectric constant, if the capacitance is 15 pF?

(Take, $\varepsilon_0 = 8.86\times10^{-12}\,\text{C}^2/\text{N-m}^2$) **(Main 2019, 8 April I)**

(a) 3.8 (b) 8.5 (c) 4.5 (d) 6.2

9. In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5\,\mu\text{F}$. All values in the circuit are in μF. **(Main 2019, 12 Jan II)**



(a) $\dfrac{6}{5}\,\mu\text{F}$ (b) $4\,\mu\text{F}$

(c) $\dfrac{7}{10}\,\mu\text{F}$ (d) $\dfrac{7}{11}\,\mu\text{F}$

10. A parallel plate capacitor with plates of area $1\,\text{m}^2$ each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is

$\left(\text{Take, } \varepsilon_0 = 8.85\times10^{-12}\,\dfrac{\text{C}^2}{\text{N}-\text{m}^2}\right)$ **(Main 2019, 12 Jan II)**

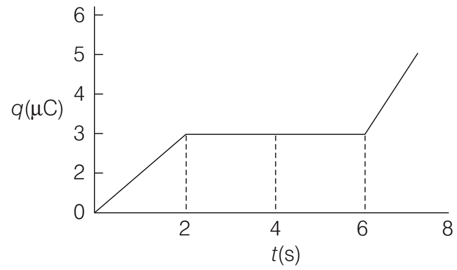
(a) 9.85×10^{-10} C

(b) 8.85×10^{-10} C

(c) 7.85×10^{-10} C

(d) 6.85×10^{-10} C

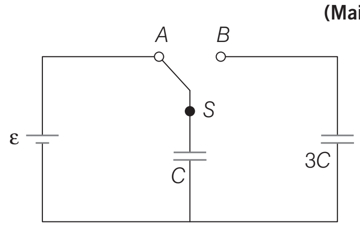
11. The charge on a capacitor plate in a circuit as a function of time is shown in the figure. **(Main 2019, 12 Jan II)**



What is the value of current at $t = 4$ s?

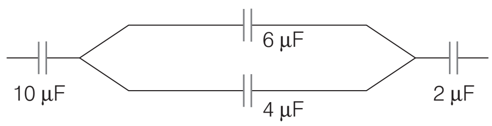
(a) $2\,\mu\text{A}$ (b) $1.5\,\mu\text{A}$ (c) Zero (d) $3\,\mu\text{A}$

12. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is **(Main 2019, 12 Jan I)**



(a) $\dfrac{3}{4}\cdot\dfrac{Q^2}{C}$ (b) $\dfrac{5}{8}\cdot\dfrac{Q^2}{C}$ (c) $\dfrac{1}{8}\cdot\dfrac{Q^2}{C}$ (d) $\dfrac{3}{8}\cdot\dfrac{Q^2}{C}$

13. In the figure shown below, the charge on the left plate of the $10\,\mu\text{F}$ capacitor is $-30\,\mu\text{C}$. The charge on the right plate of the $6\,\mu\text{F}$ capacitor is **(Main 2019, 11 Jan I)**

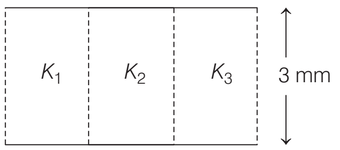


(a) $+12\,\mu\text{C}$ (b) $+18\,\mu\text{C}$ (c) $-12\,\mu\text{C}$ (d) $-18\,\mu\text{C}$

14. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is **(Main 2019, 10 Jan II)**

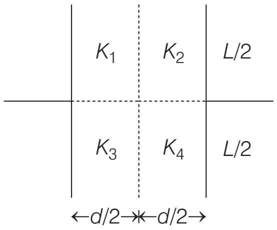
(a) 560 pJ (b) 508 pJ (c) 692 pJ (d) 600 pJ

15. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which give same capacitance when fully inserted in above capacitor, would be **(Main 2019, 10 Jan I)**



(a) 4 (b) 36 (c) 12 (d) 14

16. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be **(Main 2019, 9 Jan II)**



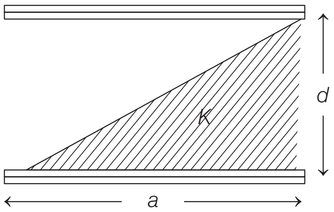
(a) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

(b) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(c) $K = \dfrac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(d) $K = \dfrac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

17. A parallel plate capacitor is made of two square plates of side 'a' separated by a distance d ($d<<a$). The lower triangular portions is filled with a dielectric of dielectric constant k, as shown in the figure. Capacitance of this capacitor is **(Main 2019, 9 Jan I)**



(a) $\dfrac{K\varepsilon_0 a^2}{d} \ln \text{K}$ (b) $\dfrac{K\varepsilon_0 a^2}{d(K-1)} \ln K$

(c) $\dfrac{K\varepsilon_0 a^2}{2d(K+1)}$ (d) $\dfrac{1}{2} \cdot \dfrac{K\varepsilon_0 a^2}{d}$

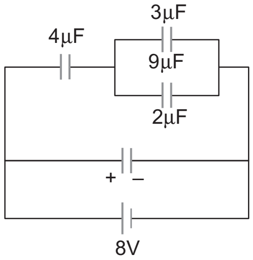
18. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = 5/3$ is inserted between the plates, the magnitude of the induced charge will be **(2018 Main)**

(a) 0.9 μC (b) 1.2 μC
(c) 0.3 μC (d) 2.4 μC

19. A capacitance of 2 μ**F** is required in an electrical circuit across a potential difference of 1kV. A large number of 1 μ**F** capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **(2017 Main)**

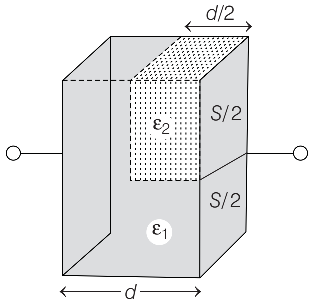
(a) 16 (b) 24
(c) 32 (d) 2

20. A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal to **(2016 Main)**



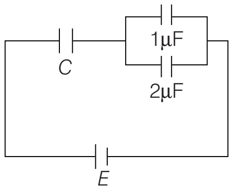
(a) 240 N/C (b) 360 N/C (c) 420 N/C (d) 480 N/C

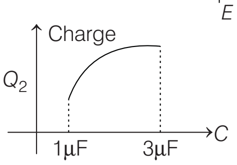
21. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\dfrac{C_2}{C_1}$ is **(2015 Adv.)**

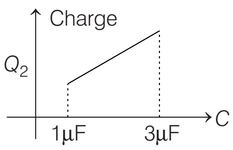


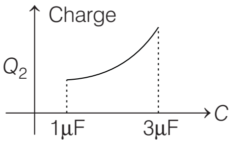
(a) $\dfrac{6}{5}$ (b) $\dfrac{5}{3}$ (c) $\dfrac{7}{5}$ (d) $\dfrac{7}{3}$

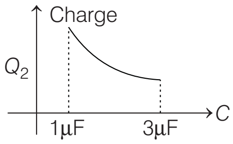
22. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of C is given properly by (figures are drawn schematically and are not to scale) **(2015 Main)**



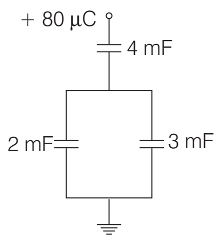
(a)


(b)


(c)


(d)


23. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is **(2012)**



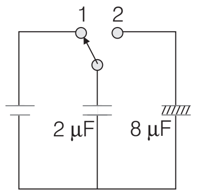
(a) + 32 μC
(b) + 40 μC
(c) + 48 μC
(d) + 80 μC

24. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to **(2014 Main)**

(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2

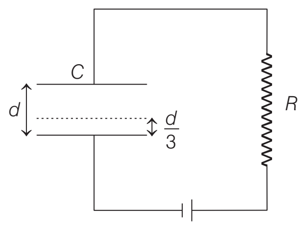
(c) 3×10^4 C/m^2 (d) 6×10^4 C/m^2

25. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **(2011)**



(a) 0% (b) 20% (c) 75% (d) 80%

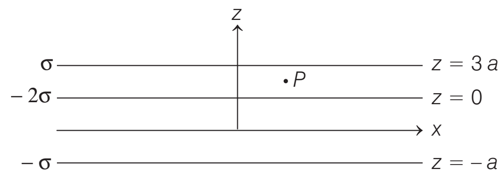
26. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time t is **(2008, 3M)**



(a) $\dfrac{6\varepsilon_0 R}{5d + 3vt}$ (b) $\dfrac{(15d + 9vt)\varepsilon_0 R}{2d^2 - 3dvt - 9v^2t^2}$

(c) $\dfrac{6\varepsilon_0 R}{5d - 3vt}$ (d) $\dfrac{(15d - 9vt)\varepsilon_0 R}{2d^2 + 3dvt - 9v^2t^2}$

27. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is **(2005, 1M)**



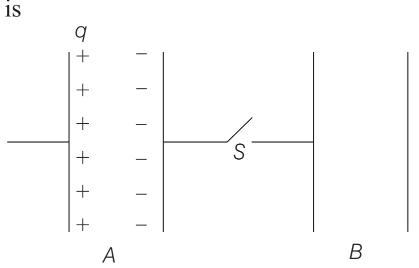
(a) $\dfrac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (b) $-\dfrac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (c) $\dfrac{4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (d) $-\dfrac{4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$

28. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. Likely charged plates are then connected. Then, the decrease in energy of the combined system is **(2002, 1M)**

(a) $\dfrac{1}{4}C(V_1^2 - V_2^2)$ (b) $\dfrac{1}{4}C(V_1^2 + V_2^2)$

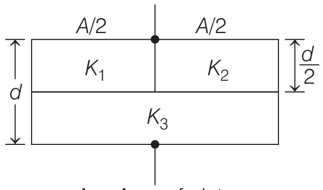
(c) $\dfrac{1}{4}C(V_1 - V_2)^2$ (d) $\dfrac{1}{4}C(V_1 + V_2)^2$

29. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is **(2001, 1M)**



(a) zero (b) $q/2$ (c) q (d) $2q$

30. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor then its dielectric constant K is given by **(2000, 2M)**

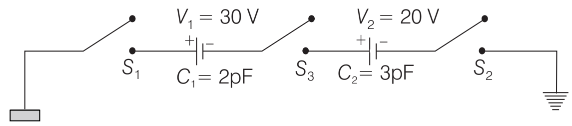


A = Area of plates

(a) $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{2K_3}$ (b) $\dfrac{1}{K} = \dfrac{1}{K_1 + K_2} + \dfrac{1}{2K_3}$

(c) $\dfrac{1}{K} = \dfrac{K_1K_2}{K_1 + K_2} + 2K_3$ (d) $K = \dfrac{K_1K_3}{K_1 + K_3} + \dfrac{K_2K_3}{K_2 + K_3}$

31. For the circuit shown, which of the following statements is true ? **(1999, 2M)**



(a) With S_1 closed, $V_1 = 15\text{V}, V_2 = 20\text{V}$

(b) With S_3 closed, $V_1 = V_2 = 25\text{V}$

(c) With S_1 and S_2 closed, $V_1 = V_2 = 0$

(d) With S_3 closed, $V_1 = 30\text{V}, V_2 = 20\text{V}$

32. Two identical metal plates are given positive charges Q_1 and Q_2 $(< Q_1)$ respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is **(1999, 2M)**

(a) $(Q_1 + Q_2)/2C$ (b) $(Q_1 + Q_2)/C$

(c) $(Q_1 - Q_2)/C$ (d) $(Q_1 - Q_2)/2C$

33. A parallel combination of 0.1 MΩ resistor and a 10 μF capacitor is connected across a 1.5 V source of negligible resistance. The time required for the capacitor to get charged upto 0.75 V is approximately (in second) **(1997, 2M)**

(a) infinite (b) $\log_e 2$

(c) $\log_{10} 2$ (d) zero

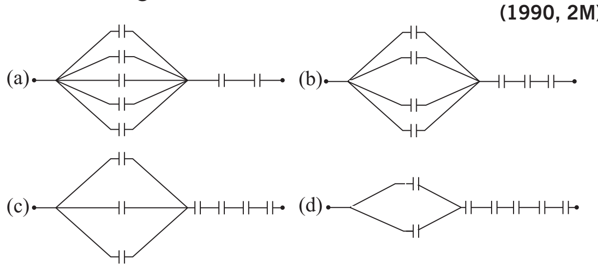
34. The magnitude of electric field **E** in the annular region of a charged cylindrical capacitor **(1996, 2M)**

(a) is same throughout.
(b) is higher near the outer cylinder than near the inner cylinder.
(c) varies as $1/r$ where r is the distance from the axis.
(d) varies as $1/r^2$ where r is the distance from the axis.

35. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is **(1995, 2M)**

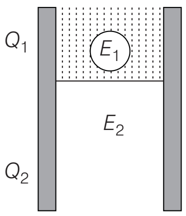
(a) zero (b) $\frac{3}{2}CV^2$ (c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$

36. Seven capacitors each of capacitance 2μF are connected in a configuration to obtain an effective capacitance $\frac{10}{11}\mu$F. Which of the following combination will achieve the desired result? **(1990, 2M)**



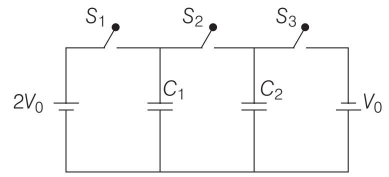
Objective Questions II (One or more correct option)

37. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. **(2014 Adv.)**



(a) $\frac{E_1}{E_2} = 1$ (b) $\frac{E_1}{E_2} = \frac{1}{K}$
(c) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (d) $\frac{C}{C_1} = \frac{2+K}{K}$

38. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time **(2013 Adv.)**



(a) the charge on the upper plate of C_1 is $2\,CV_0$
(b) the charge on the upper plate of C_1 is CV_0
(c) the charge on the upper plate of C_2 is 0
(d) the charge on the upper plate of C_2 is $-CV_0$

39. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then **(2013 Main)**

(a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$
(c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

40. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$ **(1998, 2M)**

(a) the magnitude of the electric field remains the same.
(b) the direction of the electric field remains the same.
(c) the electric potential increases continuously.
(d) the electric potential increases at first, then decreases and again increases.

41. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielecric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and work done on the system, in question, in the process of inserting the slab, then **(1991, 2M)**

(a) $Q = \frac{\varepsilon_0 AV}{d}$ (b) $Q = \frac{\varepsilon_0 KAV}{d}$
(c) $E = V/Kd$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left[1 - 1/K\right]$

42. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles **(1987, 2M)**

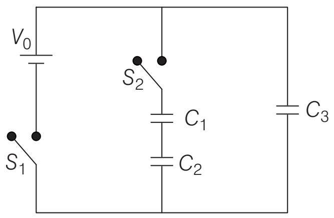
(a) the charge on the capacitor increases
(b) the voltage across the plates increases
(c) the capacitance increases
(d) the electrostatic energy stored in the capacitor increases

43. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as **(1985, 2M)**

(a) $Q > Q_0$ (b) $V > V_0$
(c) $E > E_0$ (d) $U > U_0$

Numerical Value Based Question

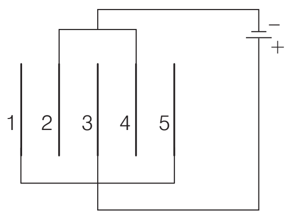
44. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8$V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of ε_r =... **(2018 Adv.)**



Fill in the Blanks

45. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected and the region between the plates of capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes…. **(1988, 2M)**

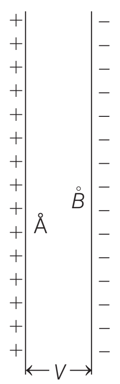
46. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at a distance d apart, the plates are connected to a source of emf V as shown in the figure. **(1984, 2M)**



The charge on plate 1 is …… and on plate 4 is …… .

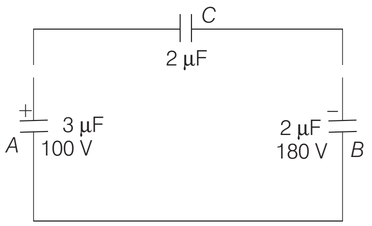
True / False

47. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical. **(1986, 3M)**



Analytical & Descriptive Questions

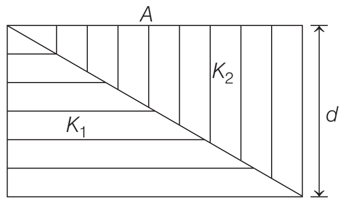
48. Two capacitors A and B with capacities 3μF and 2μF are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire of each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate **(1997, 5M)**



(a) the final charge on the three capacitors and

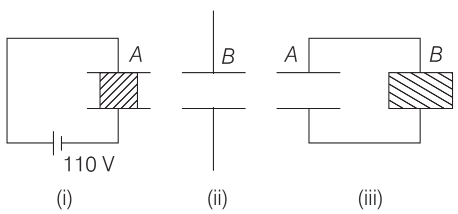
(b) the amount of electrostatic energy stored in the system before and after completion of the circuit.

49. The capacitance of a parallel plate capacitor with plate area A and separation d, is C. The space between the plates is filled with two wedges of dielectric constants K_1 and K_2 respectively (figure). Find the capacitance of the resulting capacitor. **(1996, 2M)**



50. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. Calculate the current drawn from the battery during the process. (Dielectric constant of oil = 11, $\varepsilon_0 = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2}). **(1994, 6M)**

51. Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$m between the plates. The plate areas of A and B are 0.04 m^2 and 0.02 m^2 respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B. **(1993, 7M)**

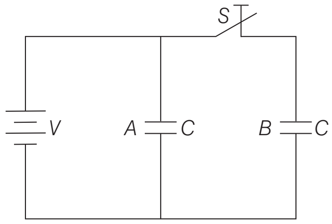


(a) The dielectric slab is placed inside A as shown in figure (i) A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

(b) The battery is disconnected and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.

(c) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (iii). Calculate the energy stored in the system.

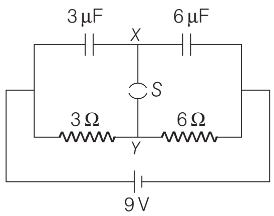
52. The figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **(1983, 6M)**



Topic 6 *C-R* Circuits

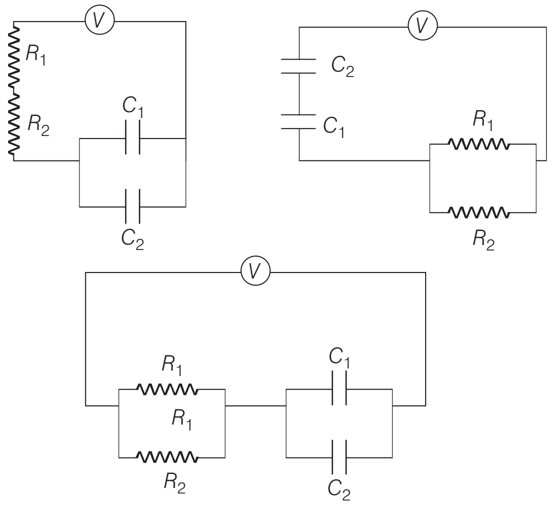
Objective Questions I (Only one correct option)

1. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is **(2007, 3M)**



(a) zero (b) 54 μC (c) 27 μC (d) 81 μC

2. Find the time constant for the given RC circuits in correct order (in μs). **(2006, 3M)**



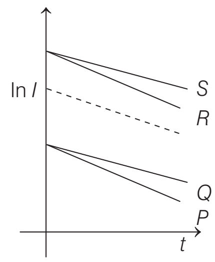
$R_1 = 1\,\Omega, R_2 = 2\,\Omega, C_1 = 4\,\mu\text{F}, C_2 = 2\mu\text{F}.$

(a) 18, 4, 8/9 (b) 18, 8/9, 4 (c) 4, 18, 8/9 (d) 4, 8/9, 18

3. A 4μF capacitor and a resistance of 2.5 MΩ are in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [Given, ln (2) = 0.693] **(2005, 2M)**

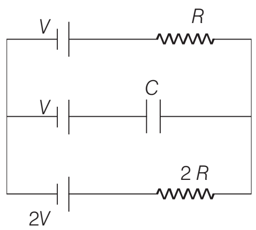
(a) 13.86 s (b) 6.93 s (c) 7 s (d) 14 s

4. A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of $\ln I$ with respect to time. If the resistance is changed to $2x$, the new graph will be **(2004, 2M)**



(a) P (b) Q (c) R (d) S

5. In the given circuit, with steady current, the potential difference across the capacitor must be **(2001, 2M)**



(a) V (b) $V/2$ (c) $V/3$ (d) $2V/3$

Objective Questions II (One or more correct option)

6. Capacitor C_1 of capacitance 1 μF and capacitor C_2 of capacitance 2 μF are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time $t = 0$. **(1989, 2M)**

(a) The current in each of the two discharging circuits is zero at $t = 0$

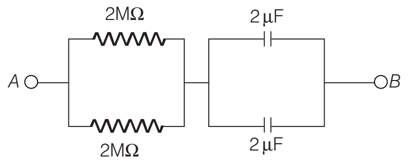
(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero

(c) The currents in the two discharging circuits at $t = 0$ are unequal

(d) Capacitor C_1, loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge

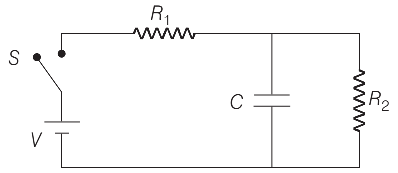
Integer Answer Type Questions

7. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in second) does the voltage across them become 4 V? [Take : $\ln\ 5 = 1.6$, $\ln\ 3 = 1.1$] **(2010)**

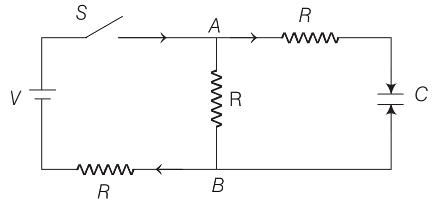


Analytical & Descriptive Questions

8. At $t = 0$, switch S is closed. The charge on the capacitor is varying with time as $Q = Q_0(1 - e^{-\alpha t})$. Obtain the value of Q_0 and α in the given circuit parameters. **(2005, 4M)**



9. In the circuit shown in figure, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$. **(1998, 8M)**

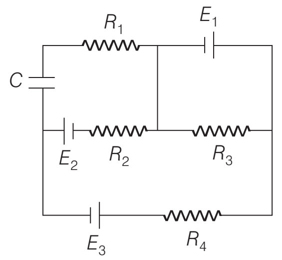


(a) Find the charge Q on the capacitor at time t.

(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$?

10. A leaky parallel plate capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\ m^{-1}$. If the charge on the capacitor at instant $t = 0$ is $q = 8.85\ \mu C$, then calculate the leakage current at the instant $t = 12$ s. **(1997 C, 5M)**

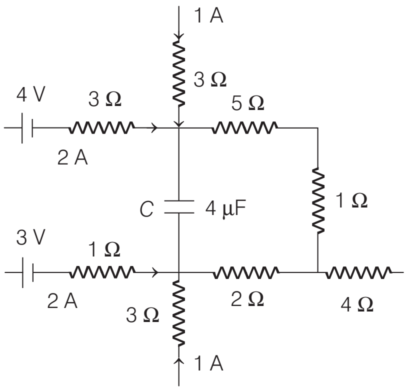
11. In the given circuit, **(1988, 5M)**



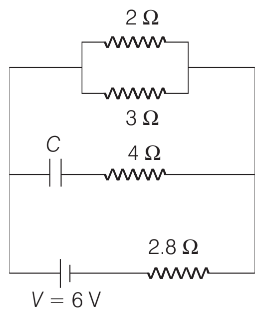
$E_1 = 3E_2 = 2E_3 = 6$ V and $R_1 = 2R_4 = 6\Omega$,
$R_3 = 2R_2 = 4\Omega,\ C = 5\mu F$.

Find the current in R_3 and the energy stored in the capacitor.

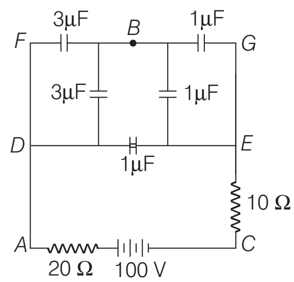
12. A part of circuit in a steady state along with the currents flowing in the branches, the values of resistances etc, is shown in the figure. Calculate the energy stored in the capacitor $C\,(4\mu F)$. **(1986, 4M)**



13. Calculate the steady state current in the 2 Ω resistor shown in the circuit (see figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 μF. **(1982, 5M)**



14. Find the potential difference between the points A and B and between the points B and C in the steady state. **(1980)**



Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. The magnetic field of a plane electromagnetic wave is given by

$\mathbf{B} = B_0[\cos(kz - \omega t)]\hat{\mathbf{i}} + B_1 \cos(kz + \omega t)\hat{\mathbf{j}}$

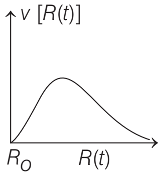
where, $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to **(Main 2019, 9 April I)**

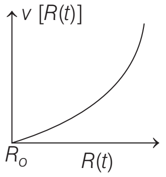
(a) 0.1 N (b) 3×10^{-2} N (c) 0.6 N (d) 0.9 N

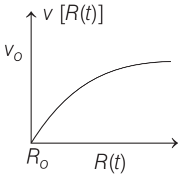
2. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is **(Main 2019, 8 April II)**

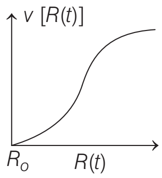
(a) $\sqrt{\dfrac{2qE}{md}}$ (b) $2\sqrt{\dfrac{qE}{md}}$ (c) $\sqrt{\dfrac{qE}{md}}$ (d) $\sqrt{\dfrac{qE}{2md}}$

3. There is uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed $v[R(t)]$ of the distribution as a function of its instantaneous radius $R(t)$ is **(Main 2019, 12 Jan I)**

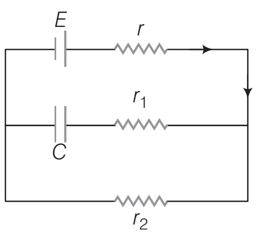
(a)


(b)


(c)


(d)


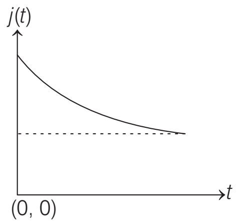
4. In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be **(2017 Main)**

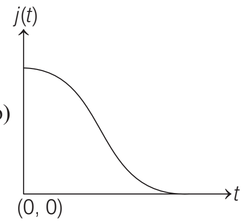


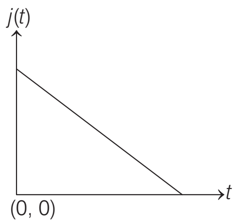
(a) $CE \dfrac{r_1}{(r_2 + r)}$ (b) $CE \dfrac{r_2}{(r + r_2)}$

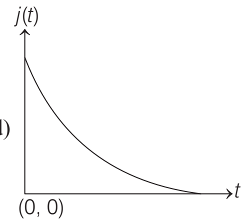
(c) $CE \dfrac{r_1}{(r_1 + r)}$ (d) CE

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material? **(2016 Adv.)**

(a)


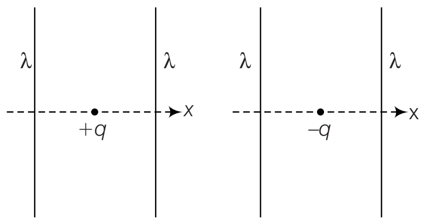
(b)


(c)


(d)


6. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other

In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statements is/are



(2015 Adv.)

(a) both charges execute simple harmonic motion.

(b) both charges will continue moving in the direction of their displacement.

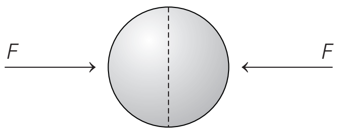
(c) charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.

(d) charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement.

7. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly **(2012)**

(a) 1×10^{-5} V (b) 1×10^{-7} V
(c) 1×10^{-9} V (d) 1×10^{-10} V

8. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to **(2010)**

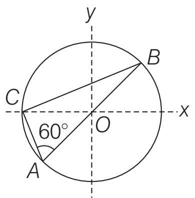


(a) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2R$
(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

9. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5\ \text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2\times10^{-3}\ \text{ms}^{-1}$. Given $g = 9.8\ \text{ms}^{-2}$, viscosity of the air $= 1.8\times10^{-5}\ \text{Ns m}^{-2}$ and the density of oil $= 900\ \text{kg m}^{-3}$, the magnitude of q is **(2010)**

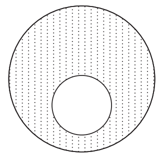
(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C
(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

10. Consider a system of three charges $q/3$, $q/3$ and $-2q/3$ placed at points A, B and C respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle $CAB = 60°$ **(2008, 3M)**



(a) The electric field at point O is $\frac{q}{8\pi\varepsilon_0R^2}$ directed along the negative x-axis.
(b) The potential energy of the system is zero
(c) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0R^2}$.
(d) The potential at point O is $\frac{q}{12\pi\varepsilon_0R}$.

11. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is **(2007, 3M)**



(a) zero everywhere (b) non-zero and uniform
(c) non-uniform (d) zero only at its centre

12. A long, hollow conducting cylinder is kept co-axially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. **(2007, 3M)**

(a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
(d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

13. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to **(2002, 1M)**

(a) x (b) x^2 (c) x^3 (d) $1/x$

14. Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$ respectively of a x-y co-ordinate system. Then **(1995, 2M)**

(a) the electric field E at all points on the x-axis has the same direction.
(b) work has to be done in bringing a test charge from ∞ to the origin.
(c) electric field at all point on y-axis is along x-axis.
(d) the dipole moment is $2qd$ along the x-axis.

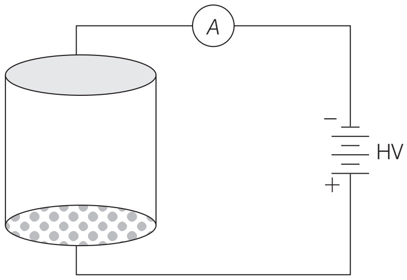
15. Two identical thin rings, each of radius R, are coaxially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is **(1992, 2M)**

(a) zero
(b) $\frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{\sqrt{2}(4\pi\varepsilon_0 R)}$
(c) $\frac{q\sqrt{2}(Q_1 + Q_2)}{(4\pi\varepsilon_0 R)}$
(d) $q(Q_1/Q_2)(\sqrt{2}+1)\sqrt{2}(4\pi\varepsilon_0 R)$

Passage Based Questions

Passage

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now, a high voltage source (HV) connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charge, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to te soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



16. Which one of the following statement is correct? **(2016 Adv.)**

(a) The balls will execute simple harmonic motion between the two plates.
(b) The balls will bounce back to the bottom plate carrying the same charge they went up with.
(c) The balls will stick to the top plate and remain there.
(d) The balls will bounce back to the bottom plate carrying the opposite charge they went up with.

17. The average current in the steady state registered by the ammeter in the circuit will be **(2016 Adv.)**

(a) proportional to V_0^2.
(b) proportional to the potential V_0.
(c) zero
(d) proportions to $V_0^{1/2}$

Match the Column

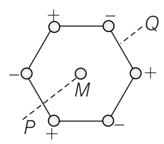
18. The electric field E is measured at a point $P\,(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

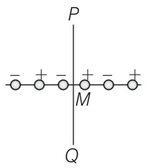
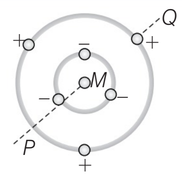
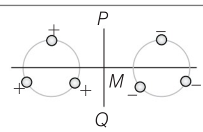
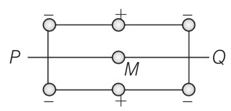
	List-I		List-II
P.	E is independent of d	1.	A point charge Q at the origin
Q.	$E \propto \dfrac{1}{d}$	2.	A small dipole with point charges Q at $(0, 0, l)$ and $-Q$ at $(0, 0, -1)$. (Take, $2l << d$)
R.	$E \propto \dfrac{1}{d^2}$	3.	An infinite line charge coincident with the X-axis, with uniform linear charge density λ.
S.	$E \propto \dfrac{1}{d^3}$	4.	Two infinite wires carrying a uniform linear charge density parallel to the X- axis. The one along $(y = 0, z = l)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -l)$ has a charge density $-\lambda$. (Take, $2l << d$).
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density.

(2018 Adv.)

(a) P → 5; Q → 3,4; R → 1; S → 2
(b) P → 5; Q → 3; R → 1, 4; S → 2
(c) P → 5; Q → 3; R → 1, 2; S → 4
(d) P → 4; Q → 2, 3; R → 1; S → 5

19. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current. **(2009)**

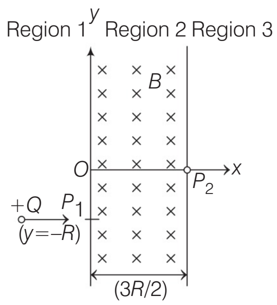
Column I		Column II
(A) $E = 0$	(p)	 Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.

Column I		Column II
(B) $V \neq 0$	(q)	 Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.
(C) $B = 0$	(r)	 Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.
(D) $\mu \neq 0$	(s)	 Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.
	(t)	 Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid points between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

Objective Questions II (One or more correct option)

20. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along X-axis enters region 2 from region 1 at point P_1 $(y = -R)$. **(2017 Main)**

Which of the following option(s) is/are correct?



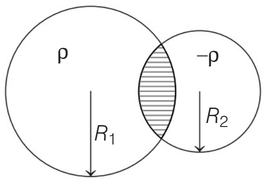
(a) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from Y-axis is $\frac{p}{\sqrt{2}}$.

(b) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on X-axis.

(c) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1.

(d) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

21. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region **(2013 Adv.)**



(a) the electrostatic field is zero.
(b) the electrostatic potential is constant.
(c) the electrostatic field is constant in magnitude.
(d) the electrostatic field has same direction.

22. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centre of the spheres, is zero. The ratio ρ_1 / ρ_2 can be **(2013 Adv.)**

(a) -4 (b) $\frac{-32}{25}$ (c) $\frac{32}{25}$ (d) 4

23. Under the influence of the coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s). **(2009)**

(a) The angular momentum of the charge $-q$ is constant
(b) The linear momentum of the charge $-q$ is constant
(c) The angular velocity of the charge $-q$ is constant
(d) The linear speed of the charge $-q$ is constant

24. A positively charged thin metal ring of radius R is fixed in the x-y plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0,0,z_0)$ where $z_0 > 0$. Then the motion of P is **(1998, 2M)**

(a) periodic for all values of z_0 satisfying $0 < z_0 < \infty$
(b) simple harmonic for all values of z_0 satisfying $0 < z_0 \le R$
(c) approximately simple harmonic provided $z_0 << R$
(d) such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$

Integer Answer Type Questions

25. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where k is a constant. Then N is **(2011)**

True/False

26. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If high energy X-ray beam falls on the ball, the ball will be deflected in the direction of the field. **(1983, 2M)**

27. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulomb and the other an equal negative charge. Their masses after charging are different. **(1983, 2M)**

Analytical & Descriptive Questions

28. A conducting bubble of radius a, thickness t $(t << a)$ has potential V. Now the bubble collapses into a droplet. Find the potential of the droplet. **(2005, 2M)**

29. There are two large parallel metallic plates S_1 and S_2 carrying surface charge densities σ_1 and σ_2 respectively $(\sigma_1 > \sigma_2)$ placed at a distance d apart in vacuum. Find the work done by the electric field in moving a point charge q a distance a $(a < d)$ from S_1 towards S_2 along a line making an angle $\pi/4$ with the normal to the plates. **(2004, 2M)**

30. A small ball of mass 2×10^{-3} kg having a charge of $1 \mu C$ is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball, so that it can make complete revolution. **(2001, 5M)**

31. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc from a height H with zero initial velocity. The particle has $q/m = 4\varepsilon_0 g/\sigma$. **(1999, 10M)**

(a) Find the value of H if the particle just reaches the disc.
(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

32. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.

S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q and it is again brought into contact with S_2 and removed. This procedure is repeated n times. **(1998, 8M)**

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.
(b) What is the limiting value of this energy as $n \to \infty$?

33. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere. **(1996, 3M)**

34. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point P $(R\sqrt{3}, 0, 0)$ on the positive x-axis directly towards O, with an initial speed v. Find the smallest (non-zero) value of the speed v such that the particle does not return to P. **(1993, 4M)**

35. Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg-m.

(a) If the same charge of Q as in part (a) above is given to a spherical conductor of the same radius R, what will be the energy of the system? **(1992, 10M)**
(b) A charge of Q is uniformly distributed over a spherical volume of radius R. Obtain an expression for the energy of the system.
(c) What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles?

Answers

Topic 1

1. (a) **2.** (d) **3.** (a)
4. (d) **5.** (d) **6.** (c) **7.** (a)
8. (d) **9.** (b) **10.** (b) **11.** (d)
12. (a) **13.** (b) **14.** (c) **15.** (a)
16. (c) **17.** (a, c) **18.** 6 **19.** 2
20. $9 \times 10^9 \frac{q^2}{L^2}$ **21.** $180°$, $\frac{Q^2}{16\pi\varepsilon_0 L^2}$ **22.** F
23. 3.17×10^{-9} C **24.** $T = 8.79 \times 10^{-4}$ N and $\theta = 27°$

25. (a) (i) $\alpha = 60°$ (ii) $T = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1q_2}{l^2} + mg$

(iii) $N_P = \sqrt{3}mg$, $N_Q = mg$ (b) $q_1q_2 = -(-4\pi\varepsilon_0)mgl^2$

Topic 2

1. (d) **2.** (d) **3.** (d) **4.** (c)
5. (d) **6.** (d) **7.** (b) **8.** (d) **9.** (c)
10. (d) **11.** (d) **12.** (c) **13.** (b)
14. (b) **15.** (d) **16.** (a) **17.** (c)
18. (b) **19.** (b, c) **20.** (a, b, c)
21. (a, b, c, d) **22.** $-8\hat{\mathbf{i}}$ **23.** $-qEa$ **24.** B
25. F **26.** $W = 5.824\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\right)$

27. $(v_0)_{\min} = 3$ m/s, $K = 3 \times 10^{-4}$ J

28. (a) Radius $= 4a$, Centre $= (5a, 0)$

(b) $V_x = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{3a - x} - \frac{2}{3a + x}\right)$ for $x \le 3a$,

$V_x = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{x - 3a} - \frac{2}{3a + x}\right)$ for $x > 3a$

(c) $v = \sqrt{\frac{Qq}{8\pi\varepsilon_0 ma}}$

29. (a) Charge q should be at a distance of 3 cm from $2q$ (b) Electric field = 0

30. Maximum distance from $O = 8.48$ m

Topic 3

1. (b) **2.** (d) **3.** (b) **4.** (c)
5. (a) **6.** (d) **7.** (c) **8.** (b)
9. (a) **10.** (c) **11.** (a) **12.** (b)
13. (a,b) **14.** (a, d) **15.** (a, c) **16.** (c, d)
17. (a, b, c, d) **18.** (6)

19. (a) $V_A = \frac{\sigma}{\varepsilon_0}(a - b + c)$, $V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b} - b + c\right)$,

$V_C = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$ (b) $a + b = c$ **20.** $\frac{Q\,(R + r)}{4\pi\varepsilon_0(R^2 + r^2)}$

Topic 4

1. (b) **2.** (c) **3.** (d) **4.** (d)
5. (a) **6.** (a) **7.** (b) **8.** (d)
9. (d) **10.** (c) **11.** (c) **12.** (d)
13. (a, d) **14.** (c, d) **15.** F
16. (a) KE $= \frac{qp}{4\pi\varepsilon_0 d^2}$ (b) $\mathbf{F} = \frac{pq}{2\pi\varepsilon_0 d^3}\hat{\mathbf{i}}$ **17.** No

Topic 5

1. (d) **2.** (d) **3.** (a) **4.** (d)
5. (a) **6.** (d) **7.** (a) **8.** (b)
9. (d) **10.** (b) **11.** (c) **12.** (d)
13. (b) **14.** (b) **15.** (c) **16.** (*)
17. (b) **18.** (b)
19. (c) **20.** (c) **21.** (d) **22.** (a)
23. (c) **24.** (a) **25.** (d) **26.** (a)
27. (b) **28.** (c) **29.** (a) **30.** (d)
31. (d) **32.** (d) **33.** (d) **34.** (c)
35. (b) **36.** (a) **37.** (a, d) **38.** (b, d)
39. (b, c) **40.** (b, c) **41.** (a, c, d) **42.** (b, d)
43. (a, d) **44.** (1.50) **45.** $\left(\frac{3}{K + 2}\right)V$

46. $\frac{\varepsilon_0 AV}{d}, -\frac{2\varepsilon_0 AV}{d}$ **47.** T

48. (a) $q_1 = 90\,\mu$C, $q_2 = 210\,\mu$C, $q_3 = 150\,\mu$C (b) (i) $U_i = 47.4$ mJ (ii) $U_f = 18$ mJ

49. $C_R = \frac{CK_1K_2}{K_2 - K_1}\ln\frac{K_2}{K_1}$ where, $C = \frac{\varepsilon_0 A}{d}$

50. $i = 4.43 \times 10^{-9}$ A

51. (a) $C_A = 2 \times 10^{-9}$ F, $U_A = 1.21 \times 10^{-5}$ J (b) $W = 4.84 \times 10^{-5}$ J (c) $U = 1.1 \times 10^{-5}$ J **52.** 3/5

Topic 6

1. (c) **2.** (b) **3.** (a) **4.** (b)
5. (c) **6.** (b, d) **7.** 2

8. $Q_0 = \frac{CVR_2}{R_1 + R_2}, \alpha = \frac{R_1 + R_2}{CR_1R_2}$ **9.** (a) $Q = \frac{CV}{2}\,(1 - e^{-2t/3RC})$

(b) $i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}, \frac{V}{2R}$ **10.** 0.198 μA

11. 1.5 A from right to left, 1.44×10^{-5} J

12. 0.288 mJ **13.** 0.9 A **14.** $V_{AB} = 25$ V, $V_{BC} = 75$ V

Topic 7

1. (c) **2.** (a) **3.** (c)
4. (b) **5.** (d) **6.** (c) **7.** (c)
8. (a) **9.** (d) **10.** (c) **11.** (b)
12. (a) **13.** (b) **14.** (c) **15.** (b)
16. (d) **17.** (a) **18.** (b)
19. A$\rightarrow$ p, r, s; B$\rightarrow$ r, s ; C$\rightarrow$ p, q, t; D$\rightarrow$ r, s **20.** (b, c)
21. (c, d) **22.** (b, d) **23.** (a) **24.** (a, c)
25. 3 **26.** T **27.** T **28.** $V' = V\left(\frac{a}{3t}\right)^{1/3}$

29. $W = \frac{(\sigma_1 - \sigma_2)qa}{\sqrt{2}\varepsilon_0}$ **30.** 5.86 m/s

31. (a) $H = \frac{4}{3}a$ (b) $H = \frac{a}{\sqrt{3}}$

32. (a) $U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$

(b) $U_\infty = \frac{Q^2R}{8\pi\varepsilon_0 r^2}$ Here, $q_n = \frac{QR}{r}\left[1 - \left(\frac{R}{R + r}\right)^n\right]$

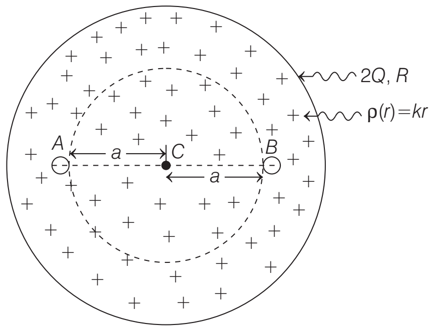
33. $\frac{5}{6}\sigma$ **34.** $v_{\min} = \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$ **35.** (a) $U = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 R}$

(b) $U = -\frac{3}{5}\frac{GM^2}{R}$, $E = 1.5 \times 10^{32}$ J (c) $U = \frac{Q^2}{8\pi\varepsilon_0 R}$

Hints & Solutions

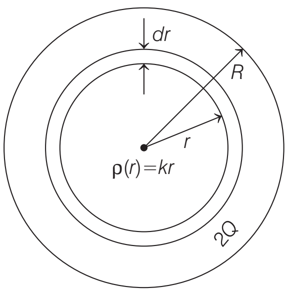
Topic 1 Electrostatics Force and Field Strength

1.



Key Idea Force on A is zero only when repulsion of A and B = attraction of positive charge distribution of radius a and charge A.

In given charge distribution, let r is radius of a shell of thickness dr.



Charge dQ present in shell of thickness dr = Volume of shell × Volumetric charge density

$$\Rightarrow \qquad dQ = (4\pi r^2 \times dr) \times (kr)$$

$$= 4\pi k r^3 \ dr$$

Total charge in sphere is

$$2Q = \int_0^R dQ = \int_0^R 4\pi k r^3 \ dr$$

$$\Rightarrow \qquad 2Q = 4\pi k \left[\frac{r^4}{4}\right]_0^R \Rightarrow k = \frac{2Q}{\pi R^4}$$

Now, using Gauss' law, electric field on the surface of sphere of radius a is

$$E \int_0^a dA = \frac{1}{\varepsilon_0} \cdot \int_0^a (kr \, 4\pi r^2 \ dr)$$

$$\Rightarrow \qquad E \cdot 4\pi a^2 = \frac{1}{\varepsilon_0} \cdot 4\pi k \left(\frac{a^4}{4}\right)$$

$$\Rightarrow \qquad E = \frac{ka^2}{4\varepsilon_0} = \frac{2Q\,a^2}{4\pi\varepsilon_0 R^4}$$

Force of attraction on charge A (or B) due to this field is

$$F_1 = QE = \frac{2Q^2 a^2}{4\pi\varepsilon_0 R^4}$$

Force of repulsion on charge A due to B is

$$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{(2a)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{4a^2}$$

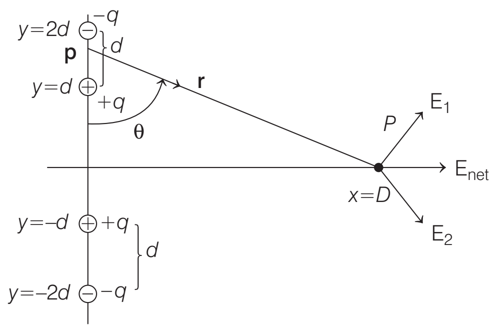
If charge A (or B) does not feel any force, then

$$F_1 = F_2$$

$$\Rightarrow \qquad \frac{2Q^2 a^2}{4\pi\varepsilon_0 R^4} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{4a^2}$$

$$\Rightarrow \qquad 8a^4 = R^4 \quad \Rightarrow a = 8^{-1/4} R$$

2. Given charge distribution is as shown below



So, we can view above point charges as combination of pair of dipoles or a quadrupole.

By symmetry, the field components parallel to quadrupole cancels and the resultant perpendicular field is

$$E = \frac{2q}{4\pi\varepsilon_0} \left(\frac{1}{D^2} - \frac{D}{(D^2 + d^2)^{3/2}}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0 D^2} \left(1 - \left(1 + \frac{d^2}{D^2}\right)^{-3/2}\right)$$

As, $\left(1 + \frac{d^2}{D^2}\right)^{-\frac{3}{2}} \approx \left(1 - \frac{3}{2}\frac{d^2}{D^2}\right)$

(using binomial expansion)

We have, $\quad E = \frac{3qd^2}{4\pi\varepsilon_0 D^4}$

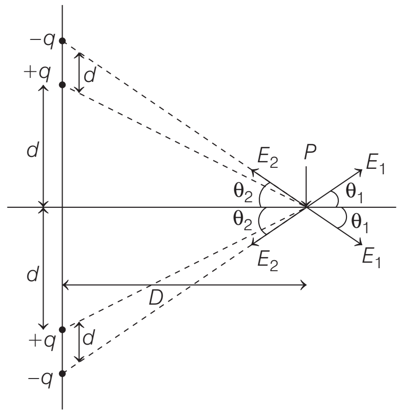
$$\therefore \qquad E \propto \frac{1}{D^4}$$

NOTE Dependence of field for a point charge is

$$E \propto \frac{1}{r^2}$$

For a dipole, it is $E \propto \frac{1}{r^3}$

For a quadrupole, it is $E \propto \frac{1}{r^4}$ ..... etc.



Alternate Solution The given distribution of charges can be shown as the figure below

Electric field at point P,

$$E = E_1\cos\theta_1 + E_1\cos\theta_1 - E_2\cos\theta_2 - E_2\cos\theta_2$$

$$= 2E_1\cos\theta_1 - 2E_2\cos\theta_2$$

$$= \frac{2kq}{(d^2 + D^2)}\cos\theta_1 - \frac{2kq}{(2d)^2 + D^2}\cos\theta_2$$

As, $\cos\theta_1 = \dfrac{D}{(d^2 + D^2)^{1/2}}$

Similarly, $\cos\theta_2 = \dfrac{D}{[(2d)^2 + D^2]^{1/2}}$

$$= 2kqD[(d^2 + D^2]^{-3/2} - (4d^2 + D^2)^{-3/2}]$$

$$= \frac{2kqD}{D^3}\left[\left(1 + \frac{d^2}{D^2}\right)^{-3/2} - \left(1 + \frac{4d^2}{D^2}\right)^{-3/2}\right]$$

As $D >> d$, then by applying binomial approximation, we get

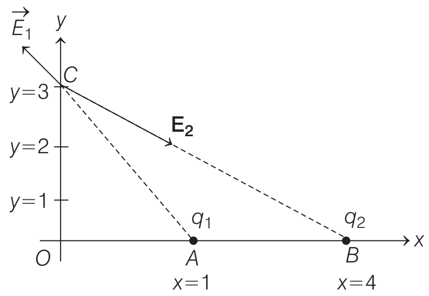
$$= \frac{2kq}{D^2}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3}{2}\left(\frac{4d^2}{D^2}\right)\right)\right]$$

$$= \frac{2kq}{D^2}\left[\frac{9d^2}{2D^2}\right] = \frac{9kqd^2}{D^4}$$

$$\Rightarrow \qquad E \propto \frac{1}{D^4}$$

3. Here, $q_1 = \sqrt{10}\,\mu\text{C} = \sqrt{10} \times 10^{-6}$ C

$q_2 = -25\,\mu\text{C} = -25 \times 10^{-6}$ C



Let $\mathbf{E}_1$ and $\mathbf{E}_2$ are the values of electric field due to q_1 and q_2 respectively.

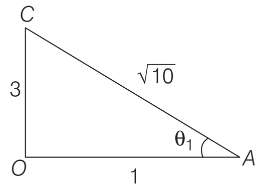
Here, $E_1 = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1}{AC^2} = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{\sqrt{10} \times 10^{-6}}{(1^2 + 3^2)}$

$$= 9 \times 10^9 \times \sqrt{10} \times 10^{-7}$$

$$= 9\sqrt{10} \times 10^2$$

$\therefore \quad \mathbf{E}_1 = 9\sqrt{10} \times 10^2[\cos\theta_1(-\hat{\mathbf{i}}) + \sin\theta_1\hat{\mathbf{j}}]$

From ΔOAC,



$\sin\theta_1 = \dfrac{3}{\sqrt{10}}$ and $\cos\theta_1 = \dfrac{1}{\sqrt{10}}$

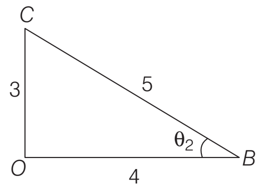
$\therefore \quad E_1 = 9\sqrt{10} \times 10^2\left[\dfrac{1}{\sqrt{10}}(-\hat{\mathbf{i}}) + \dfrac{3}{\sqrt{10}}\hat{\mathbf{j}}\right]$

$$= 9 \times 10^2[-\hat{\mathbf{i}} + 3\hat{\mathbf{j}}]$$

$$= (-9\hat{\mathbf{i}} + 27\hat{\mathbf{j}}) \times 10^2 \text{ V/m}$$

and $E_2 = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{-25 \times 10^{-6}}{(4^2 + 3^2)} = 9 \times 10^3$ V/m

From ΔOBC,



$\sin\theta_2 = \dfrac{3}{5}$

$\cos\theta_2 = \dfrac{4}{5}$

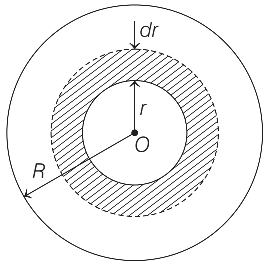
$\therefore \quad \mathbf{E}_2 = 9 \times 10^3[\cos\theta_2\hat{\mathbf{i}} - \sin\theta_2\hat{\mathbf{j}}]$

$$\mathbf{E}_2 = 9 \times 10^3\left[\frac{4}{5}\hat{\mathbf{i}} - \frac{3}{5}\hat{\mathbf{j}}\right] = (72\hat{\mathbf{i}} - 54\hat{\mathbf{j}}) \times 10^2$$

$\therefore \quad \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (63\,\hat{\mathbf{i}} - 27\hat{\mathbf{j}}) \times 10^2$ V/m

4. Here, volume charge density,

$$\rho(r) = \frac{A}{r^2}\cdot e^{-\frac{2r}{a}}$$



where, a and A are constant.

Let a spherical region of small element of radius r.
If Q is total charge distribution upto radius R, then

$$Q = \int_0^R \rho \cdot dV = \int_0^R \frac{A}{r^2} e^{-2r/a} (4\pi r^2\ dr)$$

(From figure, we observe $dV = A \cdot dr = 4\pi r^2 \cdot dr$)

$$= 4\pi A \int_0^R e^{-2r/a} dr = 4\pi A \left(\frac{e^{-2r/a}}{-2/a} \right)_0^R$$

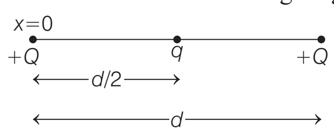
$$= 4\pi A \times \left(\frac{-a}{2} \right) (e^{-2R/a} - e^0)$$

$$= 2\pi A(-a)[e^{-2R/a} - 1]$$

or $$Q = 2\pi a\ A\ (1 - e^{-2R/a})$$

or $$R = \frac{a}{2} \log \left(\frac{1}{1 - \frac{Q}{2\pi a A}} \right)$$

5. The given condition is shown in the figure given below,



Then, according to the Coulomb's law, the electrostatic force between two charges q_1 and q_2 such that the distance between them is (r) given as,

$$F = \frac{1 \cdot q_1 q_2}{4\pi\varepsilon_0 \cdot r^2}$$

$\therefore$ Net force on charge 'Q' placed at origin i.e. at $x = 0$ in accordance with the principle of superposition can be given as

$$F_{\text{net}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2}$$

Since, it has been given that, $F_{\text{net}} = 0$.

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2} \text{ or } q = -\frac{Q}{4}$$

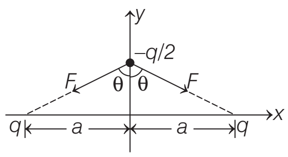
6. $E_1 = \dfrac{kQ}{R^2}$, where $k = \dfrac{1}{4\pi\varepsilon_0}$

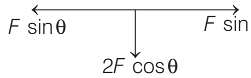
$$\Rightarrow \quad mE_2 = \frac{k(2Q)}{R^2}$$

$$\Rightarrow \quad E_2 = \frac{2kQ}{R^2} \quad \Rightarrow E_3 = \frac{k(4Q)R}{(2R)^3}$$

$$\Rightarrow \quad E_3 = \frac{kQ}{2R^2}$$

7.





$$F_{\text{net}} = 2F\cos\theta$$

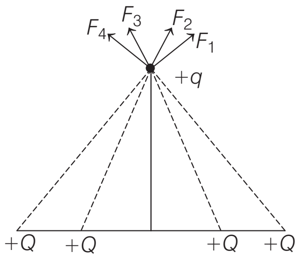
$$\Rightarrow \quad F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2 + a^2})^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

$$F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right) y}{(y^2 + a^2)^{3/2}} \Rightarrow \frac{kq^2 y}{a^3} \propto y$$

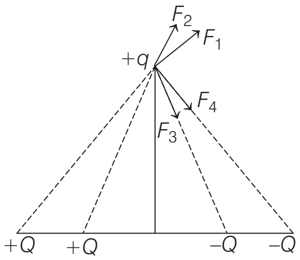
8. According to option (d) the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up. Hence, the correct option is (d).

(P) Component of forces along x-axis will vanish. Net force along positive y-axis

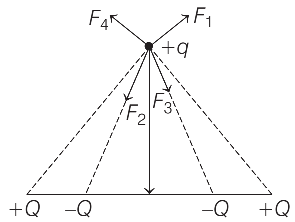
$$E_3 < E_1 < E_2$$



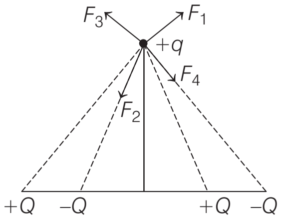
(Q) Component of forces along y-axis will vanish. Net force along positive x-axis



(R) Component of forces along x-axis will vanish. Net force along negative y-axis



(S) Component of forces along y-axis will vanish. Net force along negative x-axis



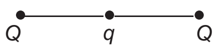
9. Electrostatic force, $F_e = eE$ (for both the particles)

But acceleration of electron, $a_e = F_e / m_e$ and

acceleration of proton, $a_p = F_e / m_p$

$$S = \frac{1}{2}a_e t_1^2 = \frac{1}{2}a_p t_2^2 \quad \Rightarrow \quad \therefore \quad \frac{t_2}{t_1} = \sqrt{\frac{a_e}{a_p}} = \sqrt{\frac{m_p}{m_e}}$$

10. Since, q is at the centre of two charges Q and Q, net force on it is zero, whatever the magnitude and sign of charge



on it. For the equilibrium of Q, q should be negative because other charge Q will repel it, so q should attract it. Simultaneously these attractions and repulsions should be equal.

$$\frac{1}{4\pi\varepsilon_0}\frac{QQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(r/2)^2}$$

or $\qquad q = \dfrac{Q}{4}$

or with sign $\qquad q = -\dfrac{Q}{4}$

11. Motion is simple harmonic only if Q is released from a point not very far from the origin on x-axis. Otherwise motion is periodic but not simple harmonic.

12. At $r = R$, from Gauss's law

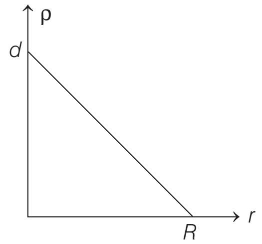
$$E\,(4\pi R^2) = \frac{q_{\text{net}}}{\varepsilon_0} = \frac{Ze}{\varepsilon_0} \text{ or } E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze}{R^2}$$

E is independent of a.

$\therefore$ Correct option is (a).

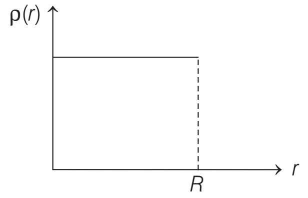
13. For $a = 0 \ \Rightarrow\ \rho(r) = \left(-\dfrac{d}{R}\cdot r + d\right)$

Now $\qquad \displaystyle\int_0^R (4\pi r^2)\left(d - \frac{d}{R}r\right)dr$ = net charge = Ze.



Solving this equation, we get $d = \dfrac{3Ze}{\pi R^3}$

14. In case of solid sphere of charge of uniform volume density



$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^3}\cdot r \qquad \text{or} \quad E \propto r$$

Thus, for E to be linearly dependent on r, volume charge density should be constant.

or $\qquad a = R$.

$\therefore$ Correct option is (c).

16. $\dfrac{Q}{4\pi\varepsilon_0 r_0^2} = \dfrac{\lambda}{2\pi\varepsilon_0 r_0} = \dfrac{\sigma}{2\varepsilon_0} \quad\Rightarrow\quad Q = 2\pi\sigma r_0^2$

(a) is incorrect, $\ r_0 = \dfrac{\lambda}{\pi\sigma}$

(b) is incorrect, $\ E_1\left(\dfrac{r_0}{2}\right) = 4E_1(r_0)$

As $\qquad E_1 \propto \dfrac{1}{r^2}$

$$E_2\left(\frac{r_0}{2}\right) = 2E_2(r_0) \text{ as } E_2 \propto \frac{1}{r}$$

(c) is correct

$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0)$$

as $E_3 \propto r^0 \Rightarrow$ (d) option is incorrect

17. Inside the sphere $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{R^3}r$

$\Rightarrow \qquad E \propto r$ for $r \le R$

i.e. E at centre $= 0$ as $r = 0$ and E at surface $= \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{R^2}$

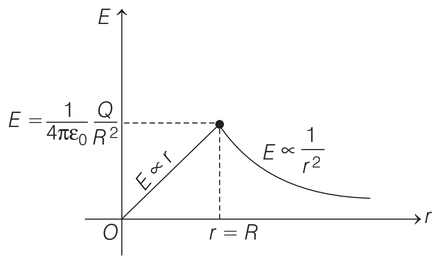
as $\qquad r = R$

Outside the sphere

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} \text{ for } r \ge R$$

or $\qquad E \propto \dfrac{1}{r^2}$

Thus, variation of electric field (E) with distance (r) from the centre will be as shown



18. Volume of cylinder per unit length ($l = 1$) is

$$V = \pi R^2 l = (\pi R^2)$$

$\therefore$ Charge per unit length,

λ = (Volume per unit length) $\times$ (Volume charge density)

$= (\pi R^2 \rho)$

Now at *P*

$$E_R = E_T - E_C$$

R = Remaining portion

T = Total portion and

C = cavity

$$\therefore \quad E_R = \frac{\lambda}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

Q = charge on sphere

$$= \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{\pi R^3 \rho}{6}$$

Substituting the values, we have

$$E_R = \frac{(\pi R^2 \rho)}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{(\pi R^3\, \rho/6)}{4R^2}$$

$$= \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{(16)\,(6)\,\varepsilon_0}$$

$$\therefore \quad k = 6$$

19. From Gauss theorem,

$$E \propto \frac{q}{r^2} \qquad (q = \text{charge enclosed})$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{q_2}{q_1} = \frac{r_1^2}{r_2^2}$$

$$\text{or} \quad 8 = \frac{\int_0^R (4\pi r^2) k r^a\, dr}{\int_0^{R/2} (4\pi r^2) k r^a\, dr} \times \frac{(R/2)^2}{(R)^2}$$

Solving this equation we get,

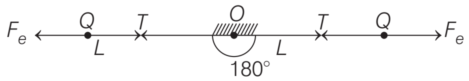
$$a = 2.$$

20. Force on $-q$ due to charges at 1 and 4 are equal and opposite. Similarly, forces on $-q$ due to charges at 2 and 5 are also equal and opposite. Therefore, net force on $-q$ due to charges at 1, 2, 4 and 5 is zero. Only unbalanced force is between $-q$ and $+q$ at 3 which is equal to

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{L^2}$$

$$\text{or} \quad 9.0 \times 10^9 \, \frac{q^2}{L^2} \text{ (attraction)}$$

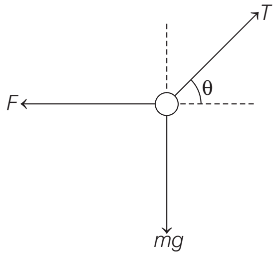
21. Due to electrostatic repulsion the charges will move as farthest as possible and the angle between the two strings will be 180° as shown in figure. Tension in each string will be equal to the electrostatic repulsion between the two charges. Thus,



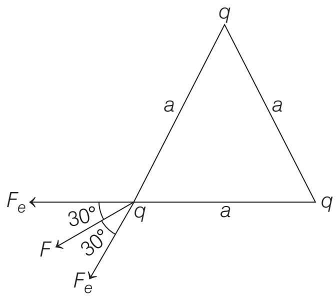
$$T = F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q \times Q}{(2L)^2} = \frac{Q^2}{16\pi\varepsilon_0 L^2}$$

22. Motion is simple harmonic only when charge $-q$ is not very far from the centre of ring on its axis. Otherwise motion is periodic but not simple harmonic in nature.

23.



F is the resultant of electrostatic forces between two charges.



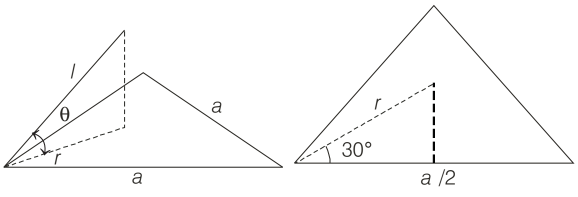
$$F = 2F_e \cos 30°$$

$$= 2\left[\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2}\right]\frac{\sqrt{3}}{2}$$

$$= \frac{2 \times 10^9 \times 9 \times q^2 \times \sqrt{3}}{(3 \times 10^{-2})^2 \times 2}$$

$$= \sqrt{3} \times 10^{13} q^2 \qquad \ldots\text{(i)}$$

θ is the angle of string with horizontal in equilibrium,



$$\cos\theta = \frac{r}{l} = \frac{(a/2)\sec 30°}{l} = \frac{a}{\sqrt{3}l} = \frac{3}{100\sqrt{3}}$$

$$\therefore \quad \theta \approx 89°$$

Now, the particle is in equilibrium under three concurrent forces, F, T and mg. Therefore, applying Lami's theorem

$$\frac{F}{\sin\,(90° + \theta)} = \frac{mg}{\sin\,(180° - \theta)}$$

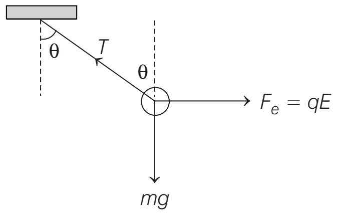
$$\text{or} \quad \sqrt{3} \times 10^{13} q^2 = (1 \times 10^{-3})(10) \cot\, 89°$$

Solving this equation, we get

$$q = 0.317 \times 10^{-8}\,\text{C} \quad \text{or} \quad q = 3.17 \times 10^{-9}\,\text{C}$$

24. For equilibrium of bob

$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = qE$$



From these two equations we find

$$T = \sqrt{(mg)^2 + (qE)^2} \text{ and } \theta = \tan^{-1}\left(\frac{qE}{mg}\right)$$

Substituting the values we have

$$T = \sqrt{(80 \times 10^{-6} \times 9.8)^2 + (2 \times 10^{-8} \times 20000)^2}$$

$$= 8.79 \times 10^{-4}\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{2 \times 10^{-8} \times 20000}{80 \times 10^{-6} \times 9.8}\right) = 27^\circ$$

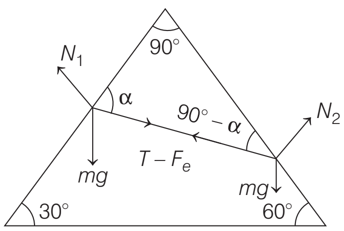
25. Tension and electrostatic force are in opposite direction and along the string. Now each bead is in equilibrium under three concurrent forces

(i) Normal reaction (N)

(ii) Weight (mg) and

(iii) $T - F_e$, where $F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{l^2}$

Applying Lami's theorem for both beads.



$$\frac{N_1}{\sin(90^\circ - \alpha)} = \frac{mg}{\cos\alpha} = \frac{T - F_e}{\cos 60^\circ} \qquad \text{...(i)}$$

$$\frac{N_2}{\sin(60^\circ + \alpha)} = \frac{mg}{\sin\alpha} = \frac{T - F_e}{\cos 30^\circ} \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we have

$$\tan\alpha = \frac{\cos 30^\circ}{\cos 60^\circ} = \sqrt{3} \Rightarrow \alpha = 60^\circ$$

$$T = F_e + mg = \left(\frac{1}{4\pi\varepsilon_0}\right) \cdot \frac{q_1 q_2}{l^2} + mg \qquad \text{...(iii)}$$

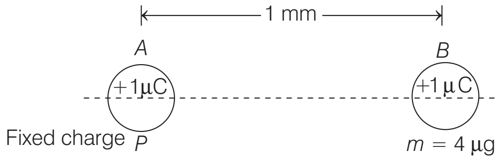
$$N_1 = \sqrt{3}\,mg \quad \text{and} \quad N_2 = mg$$

From Eq. (iii) $T = 0$ when string is cut.

or $\qquad q_1 q_2 = -(4\pi\varepsilon_0)\,mgl^2$

Topic 2 Electrostatic Potential, Potential Energy, Work Done and Energy Conservation

1. Given situation is shown in the figure below,



When charged particle B is released due to mutual repulsion, it moves away from A. In this process, potential energy of system of charges reduces and this change of potential energy appears as kinetic energy of B.

Now, potential energy of system of charges at separation of 1 mm is

$$U_1 = \frac{Kq_1q_2}{r}$$

Here, $\qquad q_1 = q_2 = 1 \times 10^{-6}\ \text{C}$

$$r = 1\,\text{mm} = 1 \times 10^{-3}\ \text{m}$$

$\therefore \qquad U_1 = \dfrac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1 \times 10^{-3}} = 9\ \text{J}$

Potential energy of given system of charges at separation of 9 mm is

$$U_2 = \frac{Kq_1q_2}{r} = \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{9 \times 10^{-3}} = 1\,\text{J}$$

By energy conservation,
Change in potential energy of system of A and B
= Kinetic energy of charged particle B

$\Rightarrow \qquad U_1 - U_2 = \dfrac{1}{2} m_B v_B^2$

where, m_B = mass of particle B = 4 μg
$= 4 \times 10^{-6} \times 10^{-3}\ \text{kg} = 4 \times 10^{-9}\ \text{kg}$

and v_B = velocity of particle B at separation of 9 mm

$\Rightarrow \quad 9 - 1 = \dfrac{1}{2} \times 4 \times 10^{-9} \times v_B^2$

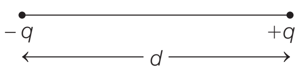
$\Rightarrow \quad v_B^2 = 4 \times 10^9 \Rightarrow v_B = 2 \times 10^3\ \text{ms}^{-1}$

2. The system of two charges, i.e. $+q$ and $-q$ that are separated by distance d can be considered as a dipole. Thus, the charge Q would be at D distance from the centre of an electric dipole on its axial line.

So, the total potential energy of the system will be due to two components.

(1) Potential energy of dipole's own system

$$(\text{PE})_1 = \frac{Kq_1q_2}{d} = -\frac{Kq^2}{d} \qquad \text{...(i)}$$



(2) Potential energy of charge Q and dipole system

$$(\text{PE})_2 = -\frac{KQq}{D^2}\cdot d \quad \text{...(ii)}$$

Hence, total potential energy of the system

$$(\text{PE})_{\text{total}} = (\text{PE})_1 + (\text{PE})_2 = -\frac{Kq^2}{d} - \frac{KQq}{D^2}\cdot d$$

$$\Rightarrow \quad (\text{PE})_{\text{total}} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{d} + \frac{Qqd}{D^2}\right]$$

3. For a positive line charge or charged wire with uniform density λ, electric field at distance x is

$$E = \frac{2k\lambda}{x} = \frac{\lambda}{2\pi\varepsilon_0 x} \quad \text{...(i)}$$

So, force on charge q which is at a distance r_0 due to this line charge is $F = qE = \frac{2kq\lambda}{x}$ ...(ii) [using Eq. (i)]

Now, work done when charge is pushed by field by a small displacement dx is

$$dW = F\cdot dx = \frac{2kq\lambda}{x}\cdot dx \quad \text{[using Eq. (ii)]}$$

$\therefore$ Total work done by field of wire in taking charge q from distance r_0 to distance r will be

$$W = \int_{r_0}^{r} dW = \int_{r_0}^{r}\frac{2kq\lambda}{x}\cdot dx$$

$$= 2kq\lambda[\log x]_{r_0}^{r} = 2kq\lambda(\log r - \log r_0)$$

$$= 2kq\lambda \log\left|\frac{r}{r_0}\right| \quad \text{...(iii)}$$

As we know, from work-kinetic energy theorem,

$$K_{\text{final}} - K_{\text{initial}} = W$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - 0 = 2kq \times \log\left|\frac{r}{r_0}\right| \quad \text{[using Eq. (iii)]}$$

$$\Rightarrow \quad v = \left(\frac{4kq\lambda}{m}\log\left|\frac{r}{r_0}\right|\right)^{1/2}$$

$$\therefore \quad v \propto \left(\log\left|\frac{r}{r_0}\right|\right)^{1/2}$$

4. Given, $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}$ N-C^{-1}

The relation between electric field and potential is given as

$$dV = -\mathbf{E}\cdot d\mathbf{x}$$

Integrating on both sides within the specified limits, we get

$$\therefore \quad \int_1^2 dV = V_2 - V_1 = -\int_{x_1}^{x_2}\mathbf{E}\cdot d\mathbf{x}$$

$$\Rightarrow \quad V_1 - V_2 = \int_{x_1}^{x_2}\mathbf{E}\cdot d\mathbf{x}$$

$$= \int_{x_1}^{x_2}(Ax+B)\hat{\mathbf{i}}\cdot(dx\hat{\mathbf{i}}) = \int_{x_1}^{x_2}(Ax+B)\cdot dx$$

Here, $A = 20$ SI unit, $B = 10$ SI unit, $x_1 = 1$ and $x_2 = -5$

$$\Rightarrow \quad V_1 - V_2 = \int_1^{-5}(20x+10)\cdot dx$$

$$= \left[\frac{20x^2}{2} + 10x\right]_1^{-5} = 10[x^2 + x]_1^{-5}$$

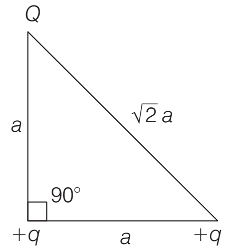
$$= 10[(-5)^2 + (-5) - (1)^2 - (1)]$$

$$= 10(25 - 5 - 2) = 180\text{ V}$$

5. Electrostatic energy between two charges q_1 and q_2 such that the distance between them r is given as

$$U = \frac{K\, q_1 q_2}{r}$$

In accordance to the principle of superposition, total energy of the charge system as shown in the figure below is



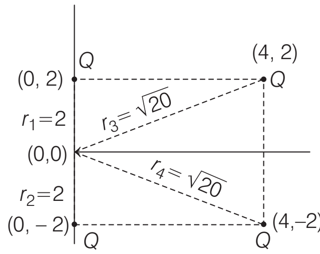
$$U = \frac{Kq^2}{a} + \frac{KQq}{a} + \frac{KQq}{\sqrt{2}a}$$

It is given that, $\quad U = 0$

$$\therefore \quad \frac{Kq}{a}\left[q + Q + \frac{Q}{\sqrt{2}}\right] = 0$$

$$\Rightarrow \quad Q = \frac{-\sqrt{2}\times q}{(\sqrt{2}+1)}$$

6. The four charges are shown in the figure below,



Electric potential at origin (0, 0) due to these charges can be found by scalar addition of electric potentials due to each charge.

$$\therefore \quad V = \frac{KQ}{r_1} + \frac{KQ}{r_2} + \frac{KQ}{r_3} + \frac{KQ}{r_4} \quad \text{... (i)}$$

$$\Rightarrow V = KQ\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{\sqrt{20}} + \frac{1}{\sqrt{20}}\right] = KQ\left[1 + \frac{1}{\sqrt{5}}\right]$$

$$\Rightarrow V = KQ\frac{(\sqrt{5}+1)}{\sqrt{5}}\text{ V} \quad \text{...(ii)}$$

Now, if another charge Q is placed at origin, then work done to get the charge at origin

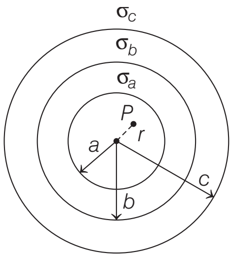
$$W = QV \quad \ldots\text{(iii)}$$

By putting the value of V from Eq. (ii) in Eq. (iii), we get

$$W = KQ^2 \frac{(\sqrt{5}+1)}{\sqrt{5}} \text{ J}$$

or $$W = \frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right) \text{ J}$$

7. Given charge distribution is shown in the figure below,



Given surface charge densities of each shell are same.

$\therefore$ $$\sigma_a = \sigma_b = \sigma_c \quad \ldots\text{(i)}$$

As, surface charge density of shell of radius 'r' and having charge 'Q' is given as $\sigma = \dfrac{Q}{4\pi r^2}$

So, relation (i) can be rewritten as

$$\frac{Q_a}{4\pi a^2} = \frac{Q_b}{4\pi b^2} = \frac{Q_c}{4\pi c^2}$$

$\Rightarrow$ $$Q_a : Q_b : Q_c = a^2 : b^2 : c^2$$

where Q_a, Q_b and Q_c are charges on shell of radius a, b and c, respectively.

Also, $$Q_a + Q_b + Q_c = Q$$

Hence, $$Q_a = \frac{a^2}{a^2+b^2+c^2}\cdot Q$$

$$Q_b = \frac{b^2}{a^2+b^2+c^2}\cdot Q$$

$$Q_c = \frac{c^2}{a^2+b^2+c^2}\cdot Q$$

As we know for charged spherical shell with charge Q of radius 'R', the potential at a point 'P' at distance r such that $r < R$ is

$$V_P = \frac{kQ}{R}.$$

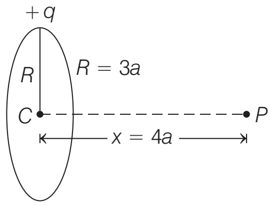
$\therefore$ potential at point P at a distance 'r' = Potential due to Q_a + Potential due to Q_b + Potential due to Q_c

$$= \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$$

Substituting the values of Q_a, Q_b and Q_c, we get

$$V = \frac{Q\,(a+b+c)}{4\pi\varepsilon_0\,(a^2+b^2+c^2)}.$$

8. Potential at any point at distance x from the centre of the ring is given by



$$V_p = \frac{Kq}{\sqrt{R^2+x^2}}$$

Given, $R = 3a$ and $x = 4a$

$\therefore$ $$V_P = \frac{Kq}{\sqrt{9a^2+16a^2}} = \frac{Kq}{5a} \quad \ldots\text{(i)}$$

At centre, $x = 0$

So, potential at centre is

$$V_C = \frac{Kq}{R} = \frac{Kq}{3a} \quad \ldots\text{(ii)}$$

Now, energy required to get this charge from $x = 4a$ to the centre is

$$\Delta U = q\,\Delta V = q\,[V_C - V_P] = q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

$$= \frac{Kq^2}{a}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$\Delta U = \frac{2}{15}\frac{Kq^2}{a} \quad \ldots\text{(iii)}$$

This energy must be equal to (or less than) the kinetic energy of the charge, i.e.

$$\frac{1}{2}mv^2 \geq \frac{2}{15}\frac{Kq^2}{a}$$

So, minimum energy required is

$$\frac{1}{2}mv^2 = \frac{2}{15}\times\frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{a} \quad (\text{put } K = 1/4\pi\varepsilon_0)$$

$\therefore$ Minimum velocity,

$$v^2 = \frac{2}{m}\times\frac{2}{4\pi\varepsilon_0}\times\frac{q^2}{15a}$$

or $$v = \sqrt{\frac{2}{m}}\times\sqrt{\frac{2q^2}{4\pi\varepsilon_0 a\times 15}}$$

9. As we know, potential difference $V_A - V_O$ is

$$dV = -E\,dx$$

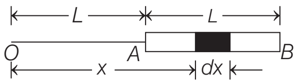
$\Rightarrow$ $$\int_{V_o}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_O = -30\times\left[\frac{x^3}{3}\right]_0^2$$

$$= -10\times[2^3 - (0)^3]$$

$$= -10\times 8 = -80 \text{ J}$$

10.



$$V = \int_L^{2L} \frac{kdQ}{x} = \int_L^{2L} \frac{k\left(\frac{Q}{L}\right) dx}{x} = \frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\left(\frac{1}{x}\right)dx$$

$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e x]_L^{2L}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e 2L - \log_e L] = \frac{Q}{4\pi\varepsilon_0 L}\ln(2)$$

11. **For inside points** $(r \le R)$

$E = 0 \Rightarrow V = \text{constant} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$

For outside points $(r \ge R)$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \quad \text{or} \quad E \propto \frac{1}{r^2}$$

and $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \text{or} \quad V \propto \frac{1}{r}$$

On the surface $(r = R)$

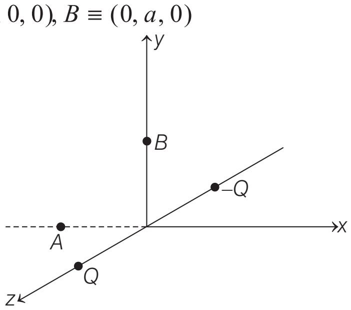
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}$$

where, $\sigma = \dfrac{q}{4\pi R^2}$ = surface charge density

corresponding to above equations the correct graphs are shown in option (d).

12. $A \equiv (-a, 0, 0)$, $B \equiv (0, a, 0)$

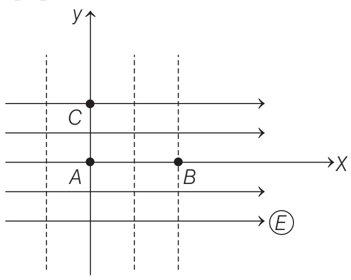


Point charge is moved from A to B

$$V_A = V_B = 0 \Rightarrow W = 0$$

or the correct option is (c).

13. Potential decreases in the direction of electric field. Dotted lines are equipotential lines.



$\therefore \quad V_A = V_C$ and $V_A > V_B$

14. Net electrostatic energy of the configuration will be

$$U = K\left[\frac{q.q}{a} + \frac{Q.q}{\sqrt{2}a} + \frac{Q.q}{a}\right].$$

Here, $K = \dfrac{1}{4\pi\varepsilon_0}$

Putting $U = 0$ we get, $Q = \dfrac{-2q}{2+\sqrt{2}}$

15. Potential at origin will be given by

$$V = \frac{q}{4\pi\,\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + ...\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right] = \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

16. $-\int_{l=\infty}^{l=0} \mathbf{E}\cdot\mathbf{dl} = \int_{l=\infty}^{l=0} dV = V\text{ (centre)} - V\text{ (infinity)}$

but V (infinity) = 0

$\therefore \; -\int_{l=\infty}^{l=0} \mathbf{E}\cdot\mathbf{dl}$ corresponds to potential at centre of ring.

and $\quad V\text{ (centre)} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{R}$

$$= \frac{(9\times 10^9)(1.11\times 10^{-10})}{0.5} \approx 2\text{V}$$

17. From conservation of mechanical energy

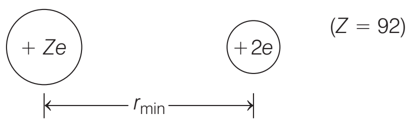
decrease in kinetic energy = increase in potential energy

or $\quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(2e)}{r_{\min}} = 5\text{ MeV}$

$$= 5\times 1.6\times 10^{-13}\text{ J}$$

$$\therefore \quad r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5\times 1.6\times 10^{-13}}$$

$$= \frac{(9\times 10^9)(2)(92)(1.6\times 10^{-19})^2}{5\times 1.6\times 10^{-13}}$$



$$= 5.3\times 10^{-14}\text{ m}$$

$$= 5.3\times 10^{-12}\text{ cm}$$

i.e. $r_{\min}$ is of the order of 10^{-12} cm.

$\therefore$ Correct option is (c).

18. Option (b) is correct.

19. V_0 = potential on the surface $= \dfrac{Kq}{R}$

where, $K = \dfrac{1}{4\pi\varepsilon_0}$ and q is total charge on sphere.

Potential at centre $= \dfrac{3}{2}\dfrac{Kq}{R} = \dfrac{3}{2}V_0$

Hence, $R_1 = 0$

From centre to surface potential varies between $\frac{3}{2}V_0$ and V_0

From surface to infinity, it varies between V_0 and 0, $\frac{5V_0}{4}$ will be potential at a point between centre and surface. At any point, at a distance $r(r \le R)$ from centre potential is given by

$$V = \frac{Kq}{R^3}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right)$$

$$= \frac{V_0}{R^2}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right) \qquad (\text{as } V_0 = \frac{Kq}{R})$$

Putting $V = \frac{5}{4}V_0$ and $r = R_2$ in this equation, we get

$$R_2 = \frac{R}{\sqrt{2}}$$

$\frac{3V_0}{4}$ and $\frac{V_0}{4}$ are the potentials lying between V_0 and zero hence these potentials lie outside the sphere. At a distance $r(\ge R)$ from centre potential is given by $V = \frac{Kq}{r} = \frac{V_0 R}{r}$

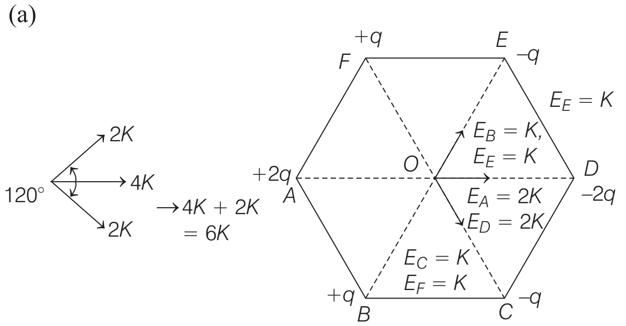
Putting $V = \frac{3}{4}V_0$ and $r = R_3$ in this equation we get, $R_3 = \frac{4}{3}R$

Further putting $V = \frac{V_0}{4}$ and $r = R_4$ in above equation,

we get $\qquad R_4 = 4R$

Thus, $R_1 = 0$, $R_2 = \frac{R}{\sqrt{2}}$, $R_3 = \frac{4R}{3}$ and $R_4 = 4R$ with these values, option (b) and (c) are correct.

20. (a)



Resultant of $2K$ and $2K$ (at 120°) is also $2K$ towards $4K$. Therefore, net electric field is $6K$.

(b) $$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{L} + \frac{q_B}{L} + \frac{q_C}{L} + \frac{q_D}{L} + \frac{q_E}{L} + \frac{q_F}{L}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 L}(q_A + \ldots + q_F) = 0$$

Because $q_A + q_B + q_C + q_D + q_E + q_F = 0$

(c) Only line PR, potential is same (= 0).

21. The given graph is of charged conducting sphere of radius R_0. The whole charge q distributes on the surface of the sphere.

22. $\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right] \Rightarrow V = 4x^2$

Therefore, $\frac{\partial V}{\partial x} = 8x$ and $\frac{\partial V}{\partial y} = 0 = \frac{\partial V}{\partial z}$

$$\mathbf{E} = -8x\hat{\mathbf{i}}$$

or $\mathbf{E}$ at (1 m, 0, 2 m) is $-8\hat{\mathbf{i}}$ V/m

23. $W_{Fe} = \mathbf{F}\cdot\mathbf{d}$ ($\mathbf{d}$ = displacement)

$= (qE\hat{\mathbf{i}})\cdot[\mathbf{r}_s - \mathbf{r}_p] = qE\hat{\mathbf{i}}\cdot[(-a\hat{\mathbf{i}} - b\hat{\mathbf{j}})] = -qEa$

24. Magnitude of electric field is greatest at a point where electric lines of force are most close to each other.

24. Electrostatic force is conservative in nature and in conservative force field work done is path independent.

26. For potential energy of the system of charges, total number of charge pairs will be ${}^8C_2$ or 28 of these 28 pairs 12 unlike charges are at a separation a, 12 like charges are at separation $\sqrt{2}\,a$ and 4 unlike charges are at separation $\sqrt{3}\,a$. Therefore, the potential energy of the system is given as

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(12)(q)(-q)}{a} + \frac{(12)(q)(q)}{\sqrt{2}a} + \frac{(4)(q)(-q)}{\sqrt{3}a}\right]$$

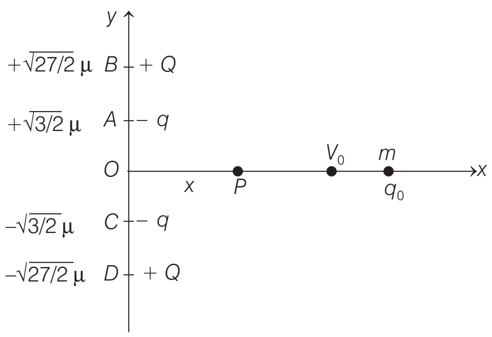
$$= -5.824\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a}\right)$$

The binding energy of this system is therefore,

$$|U| = 5.824\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\right).$$

So, work done by external forces in disassembling, this system of charges is $W = 5.824\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a}\right)$

27.



In the figure, $q = 1\,\mu\text{C} = 10^{-6}\,\text{C}$, $q_0 = +0.1\,\mu\text{C} = 10^{-7}\,\text{C}$ and $m = 6\times10^{-4}$ kg and $Q = 8\,\mu\text{C} = 8\times10^{-6}$ C

Let P be any point at a distance x from origin O. Then

$$AP = CP = \sqrt{\frac{3}{2} + x^2}$$

$$BP = DP = \sqrt{\frac{27}{2} + x^2}$$

Electric potential at point P will be, $V = \dfrac{2KQ}{BP} - \dfrac{2Kq}{AP}$

where, $K = \dfrac{1}{4\pi\varepsilon_0} = 9\times 10^9\,\text{Nm}^2/\text{C}^2$

$$\therefore \quad V = 2\times 9\times 10^9\left[\frac{8\times 10^{-6}}{\sqrt{\frac{27}{2}+x^2}} - \frac{10^{-6}}{\sqrt{\frac{3}{2}+x^2}}\right]$$

$$V = 1.8\times 10^4\left[\frac{8}{\sqrt{\frac{27}{2}+x^2}} - \frac{1}{\sqrt{\frac{3}{2}+x^2}}\right] \quad \ldots\text{(i)}$$

$\therefore$ Electric field at point P is

$$E = -\frac{dV}{dX} = 1.8\times 10^4\left[(8)\left(\frac{-1}{2}\right)\left(\frac{27}{2}+x^2\right)^{-3/2} \left(-\frac{1}{2}\right)\left(\frac{3}{2}+x^2\right)^{-3/2}\right](2x)$$

$E = 0$ on x-axis where $x = 0$,

$$\text{or} \quad \frac{8}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \quad \frac{(4)^{3/2}}{\left(\frac{27}{2}+x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2}+x^2\right)^{3/2}}$$

$$\Rightarrow \quad \left(\frac{27}{2}+x^2\right) = 4\left(\frac{3}{2}+x^2\right)$$

This equation gives, $x = \pm\sqrt{\frac{5}{2}}$ m

The least value of kinetic energy of the particle at infinity should be enough to take the particle upto $x = +\sqrt{\frac{5}{2}}$ m

because at $x = +\sqrt{\frac{5}{2}}$ m, $E = 0$.

$\Rightarrow$ Electrostatic force on charge q is zero or $F_e = 0$.

For at $x > \sqrt{\frac{5}{2}}$ m, E is repulsive (towards positive x-axis)

and for $x < \sqrt{\frac{5}{2}}$ m, E is attractive (towards negative x-axis)

Now, from Eq. (i), potential at $x = \sqrt{\frac{5}{2}}$ m

$$V = 1.8\times 10^4\left[\frac{8}{\sqrt{\frac{27}{2}+\frac{5}{2}}} - \frac{1}{\sqrt{\frac{3}{2}+\frac{5}{2}}}\right]$$

$$V = 2.7\times 10^4 \text{ V}$$

Applying energy conservation at $x = \infty$ and $x = \sqrt{\frac{5}{2}}$ m

$$\frac{1}{2}mv_0^2 = q_0V \quad \ldots\text{(ii)}$$

$$\therefore \quad v_0 = \sqrt{\frac{2q_0V}{m}}$$

Substituting these values,

$$v_0 = \sqrt{\frac{2\times 10^{-7}\times 2.7\times 10^4}{6\times 10^{-4}}}. \Rightarrow v_0 = 3\text{ m/s}$$

$\therefore$ Minimum value of v_0 is 3 m/s

From Eq. (i), potential at origin $(x = 0)$ is

$$V_0 = 1.8\times 10^4\left[\frac{8}{\sqrt{\frac{27}{2}}} - \frac{1}{\sqrt{\frac{3}{2}}}\right] \approx 2.4\times 10^4 \text{ V}$$

Let K be the kinetic energy of the particle at origin.

Applying energy conservation at $x =$`0 and at $x = \infty$

$$K + q_0V_0 = \frac{1}{2}mv_0^2$$

$$\text{But} \quad \frac{1}{2}mv_0^2 = q_0V \quad \text{[from Eq. (ii)]}$$

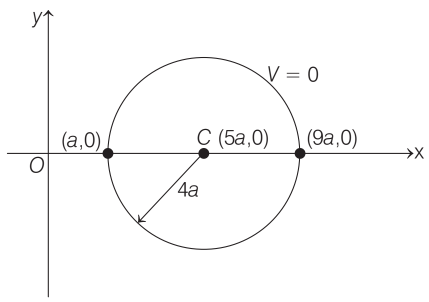
$$K = q_0(V - V_0)$$

$$K = (10^{-7})(2.7\times 10^4 - 2.4\times 10^4)$$

$$K = 3\times 10^{-4}\text{ J}$$

NOTE $E = 0$ or F_e on q_0 is zero at $x = 0$ and $x = \pm\sqrt{\frac{5}{2}}$ m of these $x = 0$ is stable equilibrium position and $x = \pm\sqrt{\frac{5}{2}}$ is unstable equilibrium position.

28. (a) Let $P(x, y)$ be a general point on x-y plane. Electric potential at point P would be,



$V =$ (potential due to Q) + (potential due to $-2Q$)

$$\text{or} \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{\sqrt{(3a-x)^2+y^2}}\right] + \frac{1}{4\pi\varepsilon_0}\left[\frac{-2Q}{\sqrt{(3a+x)^2+y^2}}\right] \quad \ldots\text{(i)}$$

Given $V = 0$

$\therefore \quad 4[(3a - x)^2 + y^2] = (3a + x)^2 + y^2$

On simplifying, we get $(x - 5a)^2 + y^2 = (4a)^2$

This is the equation of a circle of radius $4a$ and centre at $(5a, 0)$.

(b) On x-axis, potential will be undefined (or say $\pm\infty$) at $x = 3a$ and $x = -3a$, because charge Q and $-2Q$ are placed at these two points.

So, between $-3a < x < 3a$ we can find potential by putting $y = 0$ in Eq. (i). Therefore,

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right] \text{ for } -3a < x < 3a$$

$$V = 0 \text{ at } x = a$$

$$V \to -\infty \text{ at } x \to -3a$$

and $\quad V \to +\infty$ at $x \to 3a$

For $x > 3a$, there is again a point where potential will become zero so for $x > 3a$, we can write

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x - 3a} - \frac{2}{3a + x}\right] \text{ for } x > 3a$$

$$V = 0 \text{ at } x = 9a$$

For $x < -3a$, we can write

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{3a - x} - \frac{2}{3a - x}\right] \text{ for } x < -3a$$

In this region potential will be zero only at $x \to -\infty$

Thus, we can summarise it as under.

(i) At $x = 3a, V = +\infty$

(ii) At $x = -3a, V = -\infty$

(iii) For $x < -3a$, $V = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{3a - x} - \dfrac{2}{3a + x}\right]$

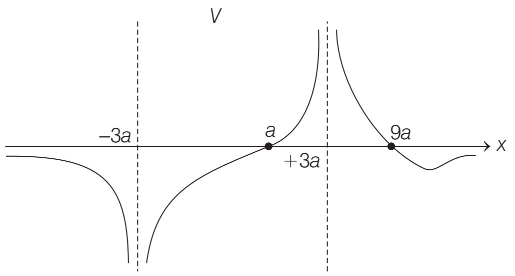
(iv) For $-3a < x < 3a$, expression of V is same i.e.

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right]$$

(v) For $x > 3a$, $V = \dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{x - 3a} - \dfrac{2}{3a + x}\right]$

Potential on x-axis is zero at two places at $x = a$ and $x = 9a$.

The V- x graph is shown below,



Exercise (i) Find potential at $x = 0$.

(ii) In the graph for $x > 9a$, find where $|V|$ will be maximum and what will be its value ?

(c) Potential at centre i.e. at $x = 5a$ will be,

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{2a} - \frac{2}{8a}\right] = \frac{Q}{16\pi\varepsilon_0 a} = \text{positive}$$

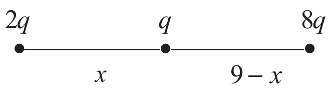
Potential on the circle will be zero.

Since, potential at centre > potential on circumference on it, the particle will cross the circle because positive charge moves from higher potential to lower potential. Speed of particle, while crossing the circle would be,

$$v = \sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{Qq}{8\pi\varepsilon_0 ma}}$$

Here, ΔV is the potential difference between the centre and circumference of the circle.

29.



For potential energy to be minimum the bigger charges should be farthest. Let x be the distance of q from $2q$. Then potential energy of the system shown in figure would be

$$U = K\left[\frac{(2q)(q)}{x} + \frac{(8q)(q)}{(9 - x)} + \frac{(2q)(8q)}{9}\right]$$

Here, $\quad K = \dfrac{1}{4\pi\varepsilon_0}$

For U to be minimum $\dfrac{2}{x} + \dfrac{8}{9 - x}$ should be minimum.

$$\frac{d}{dx}\left[\frac{2}{x} + \frac{8}{9 - x}\right] = 0$$

$$\therefore \quad \frac{-2}{x^2} + \frac{8}{(9 - x)^2} = 0$$

$$\therefore \quad \frac{x}{9 - x} = \frac{1}{2} \text{ or } x = 3 \text{ cm}$$

i.e. distance of charge q from $2q$ should be 3 cm.

Electric field at q

$$E = \frac{K(2q)}{(3 \times 10^{-2})^2} - \frac{K(8q)}{(6 \times 10^{-2})^2} = 0$$

30. Equating the energy of $(-q)$ at C and D

$$K_C + U_C = K_D + U_D$$

Here, $\quad K_C = 4\text{ J}$

$$U_C = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{AC}\right]$$

$$= \frac{-2 \times 9 \times 10^9 \times (5 \times 10^{-5})^2}{5} = -9\text{J}$$

$$K_D = 0$$

and $$U_D = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{AD}\right]$$

$$= \frac{-2\times 9\times 10^9\times(5\times 10^{-5})^2}{AD}$$

$$= -\frac{45}{AD}$$

Substituting these values in Eq. (i)

$$4 - 9 = 0 - \frac{45}{AD}$$

$\therefore$ $$AD = 9\,\text{m}$$

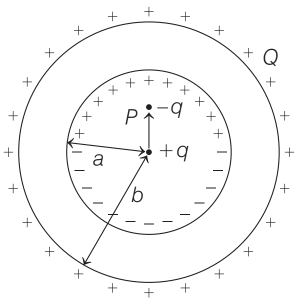
$\therefore$ $$OD = \sqrt{AD^2 - OA^2}$$

$$= \sqrt{(9)^2 - (3)^2}$$

$$= \sqrt{81 - 9} = 8.48\text{ m}$$

Topic 3 Gauss Theorem and Spherical Shells

1. Electric charge distribution at inner and outer surface of spherical shell due to the electric dipole can be shown as below



Here, we need to consider two different factors

(i) charge on the spherical shell is $+Q$ which will be distributed on its outer surface as shown in figure.

(ii) Electric dipole will create non-uniform electric field inside the shell which will distribute the charges on inner surface as shown in figure. But its net contribution to the outer side of the shell will be zero as net charge of a dipole is zero.

$\therefore$ Net charge on outer surface of shell will be $+Q$.

Hence, using (ii), option (a) is incorrect as field inside shell is not uniform. Option (b) is correct, as net charge on outer surface is $+Q$ even in the presence of dipole.

Option (c) is incorrect, as surface charge density at outer surface is uniform $\left(= \frac{Q}{A} = \frac{Q}{4\pi b^2}\right)$.

Option (d) is incorrect, as surface charge density at inner surface is non-zero.

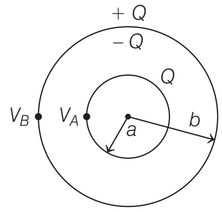
So, option (b) is correct.

Alternate Solution Using Gauss' law at outer surface, let charge on dipole is q,

$$\phi = \frac{\Sigma q}{\varepsilon_0} = E\cdot A \text{ or } E = \frac{1}{A\varepsilon_0}\Sigma q$$

$$= \frac{(+Q + q - q)}{A\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \text{constant}$$

2. Initially when uncharged shell encloses charge Q, charge distribution due to induction will be as shown,



The potential on surface of inner shell is

$$V_A = \frac{kQ}{a} + \frac{k(-Q)}{b} + \frac{kQ}{b} \qquad \ldots\text{(i)}$$

where, k = proportionality constant.

Potential on surface of outer shell is

$$V_B = \frac{kQ}{b} + \frac{k(-Q)}{b} + \frac{kQ}{b} \qquad \ldots\text{(ii)}$$

Then, potential difference is

$$\Delta V_{AB} = V_A - V_B = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

Given, $$\Delta V_{AB} = V$$

So, $$kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \qquad \ldots\text{(iii)}$$

Finally after giving charge $-4Q$ to outer shell, potential difference will be

$$\Delta V_{AB} = V_A - V_B = \left(\frac{kQ}{a} + \frac{k(-4Q)}{b}\right) - \left(\frac{kQ}{b} + \frac{k(-4Q)}{b}\right)$$

$$= kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \qquad \text{[from Eq. (iii)]}$$

Hence, we obtain that potential difference does not depend on the charge of outer sphere, hence potential difference remains same.

3. For a uniformly charged spherical shell, electric potential inside it is given by

$$V_{\text{inside}} = V_{\text{surface}} = kq / r_0 = \text{constant},$$

(where r_0 = radius of the shell).

and electric potential outside the shell at a distance r is

$$V_{\text{outside}} = \frac{kq}{r} \Rightarrow V \propto 1/r$$

$\therefore$ The given graph represents the variation of r and potential of a uniformly charged spherical shell.

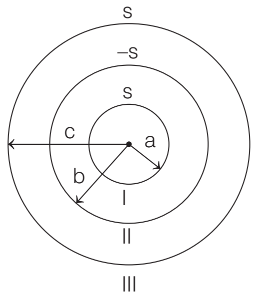
4. (c) $q_I = (4\pi a^2)(\sigma), q_{II} = 4\pi b^2(-\sigma), q_{III} = 4\pi c^2(\sigma)$

V_B = (Potential due to I) + (Potential due to II) + (Potential due to III)

$$V_B = \frac{K\,4\pi a^2\sigma}{b} + \frac{K\,4\pi b^2}{b}(-\sigma) + \frac{K\,4\pi c^2(\sigma)}{c}$$

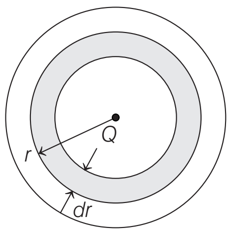
Substituting $K = \dfrac{1}{4\pi\varepsilon_0}$, we get

$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$



5. As E is constant,

Hence, $E_a = E_b$



As per Guass theorem, only Q_{in} contributes in electric field.

$$\therefore \qquad \frac{kQ}{a^2} = \frac{k\left[Q + \int_a^b 4\pi r^2 dr \cdot \frac{A}{r}\right]}{b^2}$$

Here, $\qquad k = \dfrac{1}{4\pi\varepsilon_0}$

$$\Rightarrow \qquad Q\frac{b^2}{a^2} = Q + 4\pi A\left[\frac{r^2}{2}\Bigg|_a^b\right] = Q + 4\pi A \cdot \left(\frac{b^2 - a^2}{2}\right)$$

$$\Rightarrow \qquad Q\left(\frac{b^2}{a^2}\right) = Q + 2\pi A\,(b^2 - a^2)$$

$$\Rightarrow \qquad Q\left(\frac{b^2 - a^2}{a^2}\right) = 2\pi A\,(b^2 - a^2) \;\Rightarrow\; A = \frac{Q}{2\pi a^2}$$

6. The sphere with cavity can be assumed as a complete sphere with positive charge of radius R_1 + another complete sphere with negative charge and radius R_2.

$E_+ \rightarrow$ **E** due to total positive charge

$E_- \rightarrow$ **E** due to total negative charge.

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$$

If we calculate it at P, then $\mathbf{E}_-$ comes out to be zero.

$\therefore \qquad \mathbf{E} = \mathbf{E}_+$

and $\mathbf{E}_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R_1^3}(OP)$, in the direction of OP.

Here, q is total positive charge on whole sphere.

It is in the direction of OP or **a**.

Now, inside the cavity electric field comes out to be uniform at any point. This is a standard result.

7. Electric flux, $\phi = \mathbf{E}\cdot\mathbf{S}$ or $\phi = ES\cos\theta$

Here, θ is the angle between **E** and **S**.

In this question $\theta = 45°$, because **S** is perpendicular to surface.

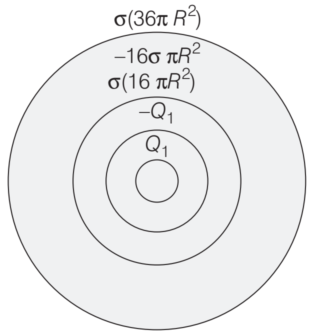
$$E = E_0$$

$$S = (\sqrt{2}a)\,(a) = \sqrt{2}a^2$$

$$\therefore \qquad \phi = (E_0)\,(\sqrt{2}a^2)\cos 45° = E_0\, a^2$$

$\therefore$ Correct option is (c).

8. $Q_1 = \sigma(4\pi R^2) = 4\pi\sigma R^2$



$$Q_2 = 16\pi\sigma R^2 - Q_1 = 12\pi\sigma R^2$$

$$Q_3 = 36\pi\sigma R^2 - 16\pi\sigma R^2 = 20\pi\sigma R^2$$

$$Q_1 : Q_2 : Q_3 = 1 : 3 : 5$$

9. Total enclosed charge as already shown is

$$q_{\text{net}} = \frac{6C}{2} + \frac{8C}{4} - 7C = -\,2C$$

From Gauss-theorem, net flux, $\phi_{\text{net}} = \dfrac{q_{\text{net}}}{\varepsilon_0} = \dfrac{-2C}{\varepsilon_0}$

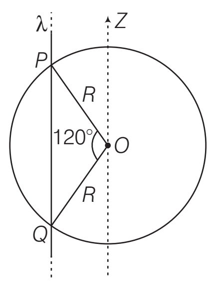
10. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ and q_2. Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

Hence, the correct option is (c).

11. In such situation potential difference depends only on the charge on inner sphere. Since, charge on inner sphere is unchanged. Therefore, potential difference V will remain unchanged.

12. Electric potential at any point inside a hollow metallic sphere is constant. Therefore, if potential at surface is 10 V, potential at centre will also be 10 V.

13. $PQ = (2)\,R\sin 60°$



$$= (2R)\frac{\sqrt{3}}{2} = (\sqrt{3}R)$$

$$q_{\text{enclosed}} = \lambda\,(\sqrt{3}R)$$

We have, $\phi = \dfrac{q_{\text{enclosed}}}{\varepsilon_0}$

$$\Rightarrow \qquad \phi = \left(\frac{\sqrt{3}\lambda R}{\varepsilon_0}\right)$$

Also, electric field is perpendicular to wire, so Z-component will be zero.

14. (a) $\Omega = 2\pi(1-\cos\theta);\ \theta = 45°$

$$\phi = -\frac{\Omega}{4\pi}\times\frac{Q}{\varepsilon_0} = -\frac{2\pi(1-\cos\theta)}{4\pi}\frac{Q}{\varepsilon_0}$$

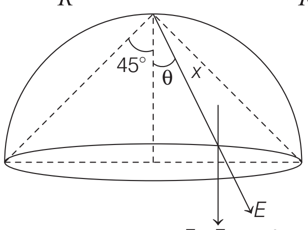
$$= -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

(b) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increases (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase. Hence, the component of the electric field normal to the flat surface is not constant.

Alternate solution

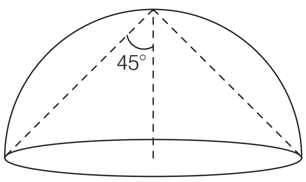
$$x = \frac{R}{\cos\theta}$$

$$E = \frac{KQ}{x^2} = \frac{KQ\cos^2\theta}{R^2} \Rightarrow E_\perp = \frac{KQ\cos^3\theta}{R^2}$$



As we move away from centre $\theta\uparrow \cos\theta\downarrow$ so $E_\perp\downarrow$

(c) Total flux ϕ due to charge Q is $\dfrac{Q}{\varepsilon_0}$.



So, ϕ through the curved and flat surface will be less than $\dfrac{Q}{\varepsilon_0}$.

(d) Since, the circumference is equidistant from Q it will be equipotential $V = \dfrac{KQ}{\sqrt{2}R}$.

15. Option (a) is correct due to symmetry.

Option (b) is wrong again due to symmetry.

Option (c) is correct because as per Gauss's theorem, net electric flux passing through any closed surface $= \dfrac{q_{\text{in}}}{\varepsilon_0}$

Here, $q_{\text{in}} = 3q - q - q = q$

$\therefore$ Net electric flux $= \dfrac{q}{\varepsilon_0}$

Option (d) is wrong because there is no symmetry in two given planes.

16. If charges are of opposite signs then the two fields are along the same direction. So, they cannot be zero. Hence, the charges should be of same sign.

Therefore, option (c) is correct.

Further, Work done by external force = change in potential energy

$\therefore\ W_{A\to B} = q\,(\Delta V) = (+1)\,(V_B - V_A)$

or $\qquad W_{A\to B} = V_B - V_A$

Therefore, option (d) is also correct.

$\therefore$ Correct options are (c) and (d).

17. Inside a conducting shell electric field is always zero. Therefore, option (a) is correct. When the two are connected, their potentials become the same.

$$\therefore \qquad V_A = V_B \quad \text{or} \quad \frac{Q_A}{R_A} = \frac{Q_B}{R_B} \qquad \left(V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\right)$$

Since, $R_A > R_B\ \therefore\ Q_A > Q_B$

$\therefore$ Option (b) is correct.

Potential is also equal to, $V = \dfrac{\sigma R}{\varepsilon_0},\ V_A = V_B$

$$\therefore \quad \sigma_A R_A = \sigma_B R_B \quad \text{or} \quad \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A} \quad \text{or} \quad \sigma_A < \sigma_B$$

$\therefore$ Option (c) is correct.

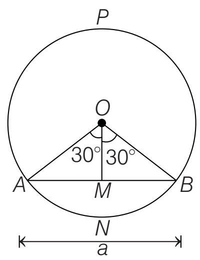
Electric field on surface, $E = \dfrac{\sigma}{\varepsilon_0}$ or $E \propto \sigma$

Since, $\sigma_A < \sigma_B\ \ \therefore\ \ E_A < E_B$

$\therefore$ Option (d) is also correct.

$\therefore$ Correct options are (a), (b), (c) and (d).

18. $ANBP$ is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.



$$AM = \frac{a}{2},\ MO = \frac{\sqrt{3}a}{2}$$

$$\therefore \quad \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Electric flux passing from the whole cylinder

$$\phi_1 = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$\therefore$ Electric flux passing through $ABCD$ plane surface (shown only AB) = Electric flux passing through cylindrical surface ANB

$$= \left(\frac{60°}{360°}\right)(\phi_1)$$

$$= \frac{\lambda L}{6\varepsilon_0}$$

$$\therefore \quad n = 6$$

19. (a) Potential at any shell will be due to all three charges.

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

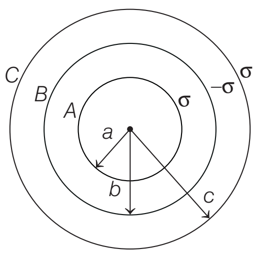
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(4\pi a^2)(\sigma)}{a} + \frac{(4\pi b^2)(-\sigma)}{b} + \frac{(4\pi c^2)(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}(a - b + c)$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{b} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(4\pi a^2)(\sigma)}{b} + \frac{(4\pi b^2)(-\sigma)}{b} + \frac{(4\pi c^2)(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$



Similarly, $$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{c} + \frac{q_B}{c} + \frac{q_C}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(4\pi a^2)(\sigma)}{c} + \frac{(4\pi b^2)(-\sigma)}{c} + \frac{(4\pi c^2)(\sigma)}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right]$$

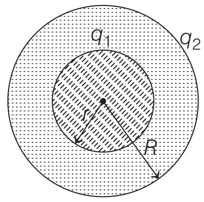
(b) Given $V_A = V_C$

$$\therefore \quad \frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

$$\therefore \quad a - b + c = \frac{a^2}{c} - \frac{b^2}{c} + c$$

or $\quad a + b = c$

20. Let q_1 and q_2 be the charges on them.



$$\sigma_1 = \sigma_2$$

$$\therefore \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \Rightarrow \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

i.e., charge on them is distributed in above ratio.

or $\quad q_1 = \frac{r^2}{r^2 + R^2}Q$ and $q_2 = \frac{R^2}{r^2 + R^2}Q$

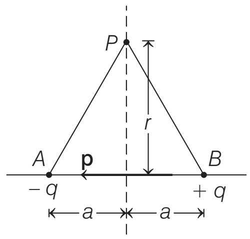
Potential at centre V = potential due to q_1 + potential due to q_2

or $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{R}$$

$$= \frac{Q(R + r)}{4\pi\varepsilon_0(r^2 + R^2)}$$

Topic 4 Electric Field Lines, Behaviour of Conductor and Electric Dipole

1. The given problem can be shown as clearly potential difference at point P due to dipole is



$$V = V_{AP} + V_{BP} \text{ (scalar addition)}$$

$$\Rightarrow \quad V = \frac{k(-q)}{AP} + \frac{k(q)}{BP} \quad \ldots\text{(i)}$$

Here, $AP = BP = \sqrt{a^2 + r^2}$

$$\therefore \quad V = -\frac{kq}{\sqrt{a^2 + r^2}} + \frac{kq}{\sqrt{a^2 + r^2}} = 0 \quad \ldots\text{(ii)}$$

Now, electric field at any point on Y-axis, i.e. equatorial line of the dipole can be given by

$$\mathbf{E} = -\frac{k\,\mathbf{p}}{r^3} \quad \text{(standard expression)}$$

$$\Rightarrow \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{r^3}$$

Given, $\quad r = d$

$$\therefore \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{d^3} \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), correct option is (b).

Alternate Solution

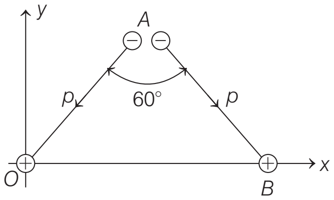
Electric field at any point at θ angle from axial line of dipole is given by

$$\mathbf{E} = -\frac{k\mathbf{p}}{r^3}\sqrt{3\cos^2\theta + 1}$$

Here, $\theta = 90^\circ \Rightarrow \cos\theta = \cos 90^\circ = 0$ and $r = d$

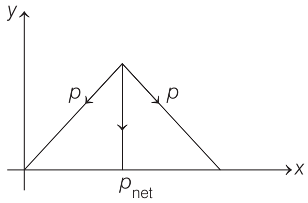
$$\therefore \qquad \mathbf{E} = -\frac{k\mathbf{p}}{d^3} = -\frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$$

2. Given system is equivalent to two dipoles inclined at 60° to each other as shown in the figure below,



Now, magnitude of resultant of these dipole moments is

$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2p\cdot p\cos 60^\circ} = \sqrt{3}p = \sqrt{3}ql$$



As, resultant is directed along negative y-direction

$$p_{\text{net}} = -\sqrt{3}p\hat{\mathbf{j}} = -\sqrt{3}ql\hat{\mathbf{j}}$$

3. Given, $\mathbf{E} = 1000$ V/m

$\theta = 45^\circ$

and $\mathbf{p} = 10^{-29}$ C-m

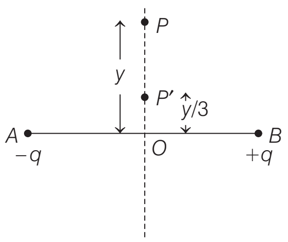
We know that, electric potential energy stored in an electric dipole kept in uniform electric field is given by the relation

$$U = -\mathbf{p}\cdot\mathbf{E} = -PE\cos\theta$$
$$= -10^{-29} \times 1000 \times \cos 45^\circ$$
$$\Rightarrow \qquad U \approx -7 \times 10^{-27}\ \text{J}$$

4. Electric field on the equatorial line of a dipole at any point, which is at distance r from the centre is given by

$$E = \frac{2kP}{(r^2 + a^2)^{3/2}} \qquad \dots\text{(i)}$$

where, P is the dipole moment of the charges.



In first case $\quad r = y$

$$\Rightarrow \qquad E_1 = \frac{2kP}{(y^2 + a^2)^{3/2}}$$

Here, $\quad y^2 >> a^2$

$$\Rightarrow \qquad y^2 + a^2 \approx y^2$$

or $$E_1 = \frac{2kP}{y^3} \qquad \dots\text{(ii)}$$

So, force on the charge in its position at P will be

$$F = QE_1 = \frac{2kPQ}{y^3} \qquad \dots\text{(iii)}$$

In second case $r = y/3$

From Eq. (i), electric field at point P' will be

$$E_2 = \frac{2kP}{\left[\left(\frac{y}{3}\right)^2 + a^2\right]^{3/2}}$$

Again, $\dfrac{y}{3} >> a \Rightarrow \left(\dfrac{y}{3}\right)^2 + a^2 \approx \left(\dfrac{y}{3}\right)^2$

$$\Rightarrow \qquad E_2 = \frac{2kP}{(y/3)^3} \Rightarrow E_2 = 27 \times \frac{2kP}{y^3}$$

Force on charge in this position,

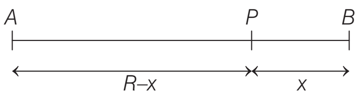
$$F' = QE_2 = 27 \times \frac{2kPQ}{y^3} \qquad \dots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$F' = 27F$$

5. **Key Idea** As, dipole moments points in same direction

So, potential of both dipoles can be same at some point between A and B. Let potentials are same at P, distant x from B as shown below



Then, $$\frac{4qa}{(R-x)^2} = \frac{2qa}{(x)^2}$$

$$2x^2 = (R-x)^2$$

$$\sqrt{2}x = R - x \quad \Rightarrow x = \frac{R}{\sqrt{2}+1}$$

Distance from A is

$$\Rightarrow \qquad R - x = R - \frac{R}{\sqrt{2}+1} = \frac{\sqrt{2}R}{\sqrt{2}+1}$$

6. Electric field at a distance 'h' from the centre of uniformly charged ring of total charge q (say) on its axis is given as,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qh}{(h^2 + R^2)^{3/2}}$$

For the magnitude to be maximum, then

$$\frac{dE}{dh} = 0$$

$$\Rightarrow \qquad \frac{dE}{dh} = \frac{q}{4\pi\varepsilon_0}$$

$$\left[\frac{(h^2 + R^2)^{3/2} - h[3/2(h^2 + R^2)^{1/2}\,2h]}{(h^2 + R^2)^3}\right] = 0$$

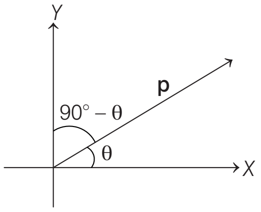
$$\Rightarrow \qquad 0 = \frac{(h^2 + R^2)^{3/2} - 3h^2(h^2 + R^2)^{1/2}}{(h^2 + R^2)^3}$$

$$\Rightarrow (h^2 + R^2)^{3/2} = 3h^2(h + R^2)^{1/2} \Rightarrow 3h^2 = (h^2 + R^2)$$

$$3h^2 - h^2 = R^2$$

$$2h^2 = R^2 \Rightarrow h = \pm\frac{R}{\sqrt{2}}$$

$\therefore$ At $\frac{R}{\sqrt{2}}$, the value of electric field associated with a charged ring on its axis has the maximum value.

7. 

Torque applied on a dipole, $\tau = pE\sin\theta$

where, θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\,\hat{\mathbf{i}}$.

it means field is directed along positive X direction, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1 = pE_1\sin\theta$$

In electric field $E_2 = \sqrt{3}\,E\hat{\mathbf{j}}$, it means field is directed along positive Y-axis, so angle between dipole and field will be $90 - \theta$

Torque in this direction $T_2 = pE\sin(90° - \theta)$.

$$= p\sqrt{3}\,E_1\cos\theta$$

According to question $\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$

$$\therefore \qquad pE_1\sin\theta = p\sqrt{3}\,E_1\cos\theta$$

$$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60°$$

$$\therefore \qquad \theta = 60°$$

8. Electric field lines originate from position charge and termination negative charge. They cannot form closed loops and they are smooth curves. Hence, the most appropriate answer is (d).

9. Charge will be induced in the conducting sphere, but net charge on it will be zero.

$\therefore$ Option (d) is correct.

10. Electric field is zero everywhere inside a metal (conductor) i.e. field lines do not enter a metal. Simultaneously these are perpendicular to a metal surface (equipotential surface).

11. Electric lines of force never form a closed loop.

12. Electric field lines never enter a metallic conductor ($E = 0$, inside a conductor) and they fall normally on the surface of a metallic conductor (because whole surface is at same potential and lines are perpendicular to equipotential surface).

13. From the behaviour of electric lines, we can say that Q_1 is positive and Q_2 is negative. Further, $|Q_1| > |Q_2|$

At some finite distance to the right of Q_2, electric field will be zero. Because electric field due to Q_1 is towards right (away from Q_1) and due to Q_2 is towards left (towards Q_2). But since magnitude of Q_1 is more, the two fields may cancel each other because distance of that point from Q_1 will also be more.

$\therefore$ The correct options are (a) and (d).

14. Under electrostatic condition, all points lying on the conductor are at same potential. Therefore, potential at A = potential at B. Hence, option (c) is correct. From Gauss theorem, total flux through the surface of the cavity will be q/ε_0.

NOTE Instead of an elliptical cavity, if it would had been a spherical cavity then options (a) and (b) were also correct.

15. Electric field lines of force does not represent the path of the charged particle but tangent to the path at any point on the line shows the direction of electric force on it and it is not always necessary that motion of the particle is in the direction of force acting on it.

16. (a) Applying energy conservation principle, increase in kinetic energy of the dipole = decrease in electrostatic potential energy of the dipole.

$\therefore$ Kinetic energy of dipole at distance d from origin

$$= U_i - U_f$$

or $$\text{KE} = 0 - (-\,\mathbf{p}\cdot\mathbf{E}) = \mathbf{p}\cdot\mathbf{E}$$

$$= (p\hat{\mathbf{i}})\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\hat{\mathbf{i}}\right) = \frac{qp}{4\pi\varepsilon_0 d^2}$$

(b) Electric field at origin due to the dipole,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{\mathbf{i}} \quad (\mathbf{E}_{\text{axis}} \uparrow\uparrow \mathbf{p})$$

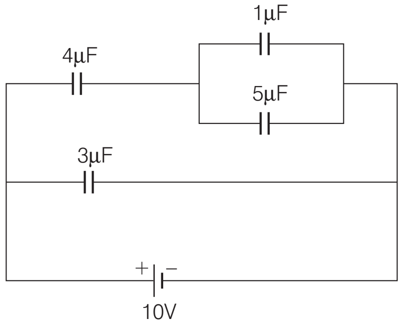
$\therefore$ Force on charge q

$$\mathbf{F} = q\mathbf{E} = \frac{pq}{2\pi\varepsilon_0 d^3}\hat{\mathbf{i}}$$

17. Force on the charged particle is along the tangent of electric line. A particle not always moves in the direction of force acting on it.

Topic 5 Capacitors

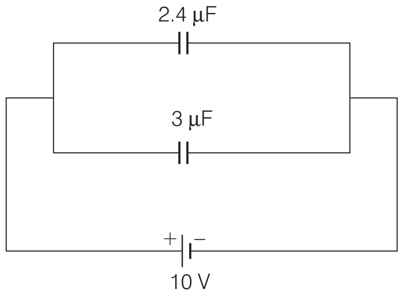
1. Given circuit is



In parallel, $C_{eq} = 5 + 1 = 6\,\mu F$

and in series, $C'_{eq} = \dfrac{6 \times 4}{6 + 4} = 2.4\,\mu F$

This is equivalent to



So, potential difference across upper branch $= 10\,V$

Using, $Q = C \times V$, charge delivered to upper branch is

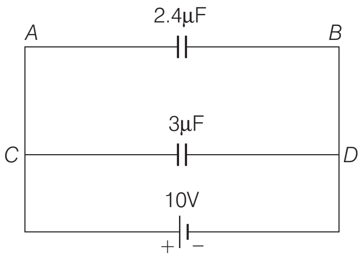
$$Q = C'_{eq} \cdot V = 2.4\,\mu F \times 10 V$$
$$= 24\,\mu C$$

As we know, in series connection, same charge is shared by capacitors, so charge on 4 μF capacitor and 6 μF capacitor would be same,

i.e., $Q'_{4\mu F} = 24 \mu C$

Alternate Solution

The circuit obtained,



This can be further simplified as, 2.4 μF and 3 μF are in parallel.

So, net capacitance, $C_{net} = 2.4 + 3 = 5.4\,\mu F$

Net charge flow through circuit,

$$Q = C_{net}V = 5.4 \times 10 = 54\,\mu C$$

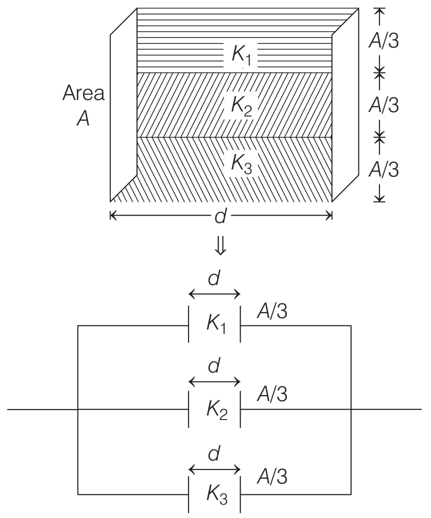
∴ This charge will be distributed in the ratio of capacitance in the two branches AB and CD as

$$\frac{Q_1}{Q_2} = \frac{2.4}{3} = \frac{4}{5} \Rightarrow 9x = 54 \mu C \text{ or } x = 6\,\mu C$$

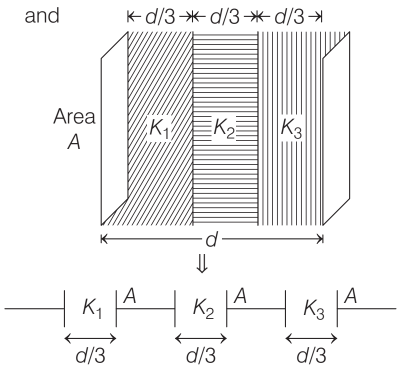
∴ Charge on 4 μF capacitor is $= 4 \times 6\,\mu C = 24\,\mu C$

2. **Key Idea** A capacitor filled with dielectrics can be treated/compared as series/parallel combinations of capacitor having individual dielectric.

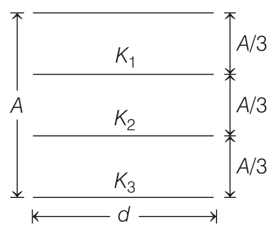
e.g.



and



Case I



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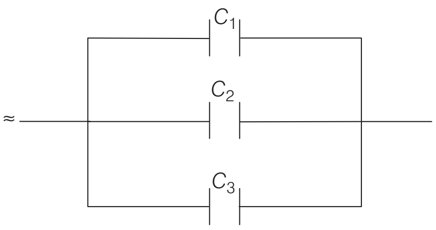


Capacitance in the equivalent circuit are

$$C_1 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_1 = \frac{\varepsilon_0 A}{3d} K_1,\ C_2 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_2 = \frac{\varepsilon_0 A}{3d} K_2$$

and $$C_3 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_3 = \frac{\varepsilon_0 A}{3d} K_3$$

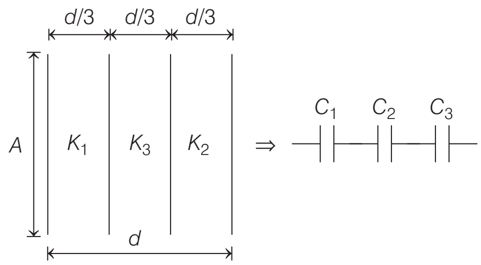
So, equivalent capacitance,

$$C_{\text{I}} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0 A}{3d} K_1 + \frac{\varepsilon_0 A}{3d} K_2 + \frac{\varepsilon_0 A}{3d} K_3$$

$$C_{\text{I}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \qquad \text{...(i)}$$

Case II



Capacitance of equivalent circuit are

$$C_1 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} \cdot K_1 = \frac{3\varepsilon_0 A}{d} K_1$$

$$C_2 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_2 = \frac{3\varepsilon_0 A}{d} K_2$$

and $$C_3 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_3 = \frac{3\varepsilon_0 A}{d} K_3$$

So, equivalent capacitance,

$$\frac{1}{C_{\text{II}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{d}{3\varepsilon_0 AK_1} + \frac{d}{3\varepsilon_0 AK_2} + \frac{d}{3\varepsilon_0 AK_3}$$

$$\Rightarrow \frac{1}{C_{\text{II}}} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right]$$

$$= \frac{d}{3\varepsilon_0 A}\left[\frac{K_2K_3 + K_1K_3 + K_1K_2}{K_1K_2K_3}\right]$$

$$C_{\text{II}} = \frac{3\varepsilon_0 A}{d}\left[\frac{K_1 K_2 K_3}{K_1 K_2 + K_2 K_3 + K_3 K_1}\right] \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{C_{\text{I}}}{C_{\text{II}}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \times \frac{d(K_1K_2 + K_2K_3 + K_3K_1)}{3\varepsilon_0 A(K_1K_2K_3)}$$

$$= \frac{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}{9K_1K_2K_3}$$

Now, energy stored in capacitor, $E = \frac{1}{2}CV^2$

$$\Rightarrow \qquad E \propto C$$

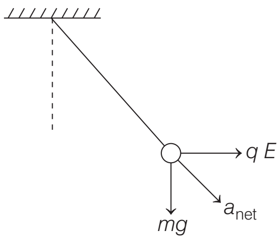
$$\therefore \qquad \frac{E_{\text{I}}}{E_{\text{II}}} = \frac{C_{\text{I}}}{C_{\text{II}}}$$

3. When pendulum is oscillating between capacitor plates, it is subjected to two forces;

(i) Weight downwards = mg

(ii) Electrostatic force acting horizontally = qE

So, net acceleration of pendulum bob is resultant of accelerations produced by these two perpendicular forces.



Net acceleration is, $a_{\text{net}} = \sqrt{a_1^2 + a_2^2} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$

So, time period of oscillations of pendulum is

$$T = 2\pi\sqrt{\frac{l}{a_{\text{net}}}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

4. In the given figure,

Slope of OA > Slope of OB

Since, we know that, net capacitance of parallel combination > net capacitance of series combination

$\therefore$ Parallel combination's capacitance,

$$C_P = C_1 + C_2 = \frac{500\,\mu\text{C}}{10\text{ V}} = 50\,\mu\text{F} \qquad \text{... (i)}$$

Series combination's capacitance,

$$C_S = \frac{C_1C_2}{C_1 + C_2} = \frac{80\,\mu\text{C}}{10\text{V}} = 8\,\mu\text{F} \qquad \text{... (ii)}$$

or $$C_1C_2 = 8 \times (C_1 + C_2) = 8 \times 50\,\mu\text{F}$$
$$= 400\,\mu\text{F} \qquad \text{[using Eq. (i)]} \qquad \text{...(iii)}$$

From Eqs. (i) and (iii), we get

$$C_1 = 50 - C_2$$

and $$C_1C_2 = 400$$

$$\Rightarrow \qquad C_2(50 - C_2) = 400$$

$$\Rightarrow \qquad 50C_2 - C_2^2 = 400$$

or $$C_2^2 - 50C_2 + 400 = 0$$

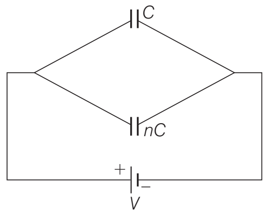
$$\Rightarrow C_2 = \frac{+50 \pm \sqrt{2500 - 1600}}{2} = \frac{+50 \pm 30}{2}$$

$\Rightarrow \quad C_2 = +40\,\mu F \text{ or } +10\,\mu F$

Also, $C_1 = 50 - C_2 \Rightarrow C_1 = +10\,\mu F \text{ or } +40\,\mu F$

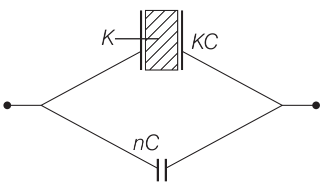
Hence, capacitance of two given capacitors is $10\,\mu F$ and $40\,\mu F$.

5. When parallel combination is fully charged, charge on the combination is



$$Q = C_{eq}V = C(1+n)V$$

When battery is removed and a dielectric slab is placed between two plates of first capacitor, then charge on the system remains same. Now, equivalent capacitance after insertion of dielectric is



$$C_{eq} = KC + nC = (n+K)C$$

If potential value after insertion of dielectric is V', then charge on system is

$$Q' = C_{eq}V' = (n+K)CV'$$

As $Q = Q'$, we have

$$C(1+n)V = (n+K)CV'$$

$$\therefore \quad V' = \frac{(1+n)V}{(n+K)}$$

6. Potential energy stored in a capacitor is

$$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

So, initial energy of the capacitor, $U_i = \frac{1}{2}Q^2 / C_1$

Final energy of the capacitor, $U_f = \frac{1}{2}Q^2 / C_2$

As we know, work done, $W = \Delta U = U_f - U_i$

$$= \frac{1}{2}Q^2\left[\frac{1}{C_2} - \frac{1}{C_1}\right]$$

Here, $Q = 5\,\mu C = 5 \times 10^{-6}$ C,

$C_1 = 5\,\mu F = 5 \times 10^{-6}$F,

$C_2 = 2\,\mu F = 2 \times 10^{-6}$ F

$$\Rightarrow \quad \Delta U = \frac{1}{2} \times (5 \times 10^{-6})^2 \left[\frac{1}{2 \times 10^{-6}} - \frac{1}{5 \times 10^{-6}}\right]$$

$$= \frac{1}{2} \times \frac{5 \times 5 \times 10^{-12}}{10^{-6}} \times \frac{3}{10}$$

$$= \frac{25 \times 3}{20} \times 10^{-6} \text{ J}$$

$$\Rightarrow \quad \Delta U = 3.75 \times 10^{-6} \text{ J}$$

$\therefore$ Work done in reducing the capacitance from $5\,\mu F$ to $2\,\mu F$ by pulling plates of capacitor apart is 3.75×10^{-6} J.

7. Net value of charge on plates of capacitor after steady state is reached is

$$q_{net} = \frac{q_2 - q_1}{2}$$

where, q_2 and q_1 are the charges given to plates.

(Note that this formula is valid for any polarity of charge.)

Here, $q_2 = 4\mu C$, $q_1 = 2\mu C$

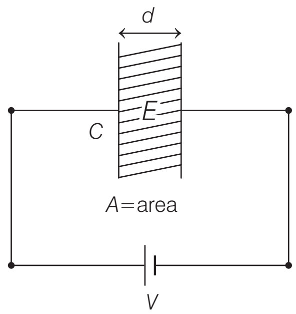
$\therefore$ Charge of capacitor is $q = \Delta q_{net} = \frac{4-2}{2} = 1\,\mu C$

Potential difference between capacitor plates is

$$V = \frac{Q}{C} = \frac{1\mu C}{1\mu F} = 1\,V$$

8. As we know, capacitance of a capacitor filled with dielectric medium,

$$C = \frac{\varepsilon_0 KA}{d} \quad \ldots(i)$$



and potential difference between plates is

$$E = \frac{V}{d} \Rightarrow d = \frac{V}{E} \quad \ldots(ii)$$

So, by combining both Eqs. (i) and (ii), we get

$$K = \frac{CV}{\varepsilon_0 AE} \quad \ldots(iii)$$

Given, $C = 15\text{pF} = 15 \times 10^{-12}$ F,

$V = 500$ V, $E = 10^6$ Vm^{-1},

$A = 10^{-4}$ m^2

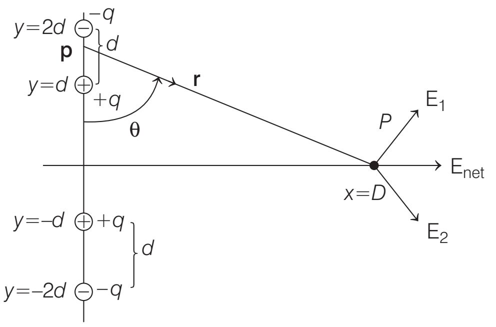
and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2}

Substituting the values in Eq. (iii), we get

$$K = \frac{15 \times 10^{-12} \times 500}{8.85 \times 10^{-12} \times 10^{-4} \times 10^6}$$

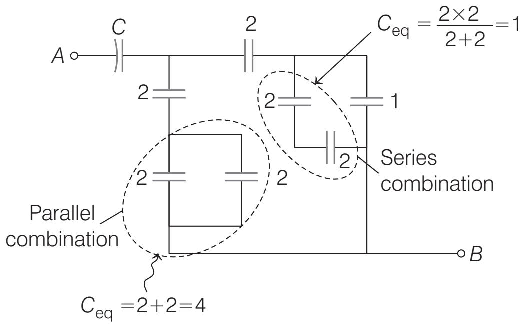
$$= 8.47 \approx 8.5$$

9. **Key Idea** Two capacitors C_1 and C_2, if connected in series, then their equivalent capacitance is

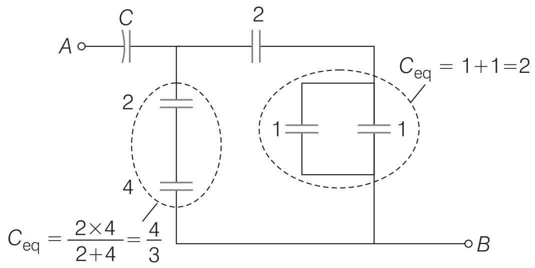


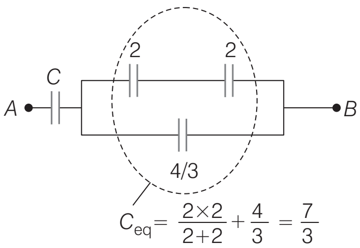
$$C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$$

If they are connected in parallel, then their equivalent capacitance is, $C_{eq} = C_1 + C_2$.



We simplify given circuit as





So, $$C_{AB} = \frac{C \times 7/3}{C + 7/3} = \frac{1}{2} \quad \text{(given)}$$

$$\Rightarrow \quad \frac{7C}{3} = \frac{C}{2} + \frac{7}{6}$$

$$\Rightarrow \quad \left(\frac{14-3}{6}\right) C = \frac{7}{6} \Rightarrow C = \frac{7}{11} \mu\text{F}$$

10. If Q = charge on each plate, then

$$Q = CV = \frac{\varepsilon_0 A}{d} \cdot Ed = \varepsilon_0 AE$$

Here, $A = 1\,\text{m}^2, E = 100\,\text{N/C}$

and $$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}-\text{m}^2}$$

So, by substituting given values, we get

$Q = 8.85 \times 10^{-12} \times 1 \times 100 = 8.85 \times 10^{-10}$ C

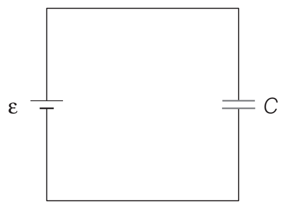
11. As we know, Current,

$$I = \frac{dq}{dt}$$

= Slope of q *versus* t graph

= Zero at t = 4s; (as graph is a line parallel to time axis at t = 4s)

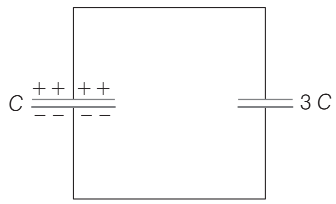
12. In position 'A' of switch, we have a capacitor joined with battery.



So, energy stored in position

$$U_1 = \frac{1}{2} C \varepsilon^2$$

When switch is turned to position B, we have a charged capacitor joined to an uncharged capacitor.



Common potential in steady state will be

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{C\varepsilon}{4C} = \frac{\varepsilon}{4}$$

Now, energy stored will be

$$U_2 = \frac{1}{2} (C_{eq})(V_{\text{common}})^2$$

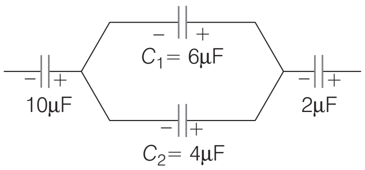
$$= \frac{1}{2} 4C \times \left(\frac{\varepsilon}{4}\right)^2 = \frac{1}{8} C\varepsilon^2$$

So, energy dissipated is

$$\Delta U = U_1 - U_2 = \frac{1}{2} C\varepsilon^2 - \frac{1}{8} C\varepsilon^2$$

$$= \frac{3}{8} C\varepsilon^2 = \frac{3}{8} C\left(\frac{Q}{C}\right)^2 = \frac{3}{8} \cdot \frac{Q^2}{C}$$

13. Applying the concept of charge conservation on isolated plates of $10\,\mu F$, $6\,\mu F$ and $4\,\mu F$. Since, $6\,\mu F$ and $4\,\mu F$ are in parallel, so total charge on this combination will be $30\,\mu C$.



$\therefore$ Charge on $6\,\mu F$, capacitor

$$= \left(\frac{C_1}{C_1 + C_2}\right) q = \frac{6}{6+4} \times 30 = 18\,\mu C$$

Since, the charge has been asked on the right plate of the capacitor. Thus, it would be $+18\,\mu C$.

Alternative method

Let charge on $6\,\mu F$ capacitor is $q\,\mu C$.

Now, V at $6\,\mu F = V$ at $4\,\mu F$

$$\therefore \quad \frac{q}{6\,\mu F} = \frac{30 - q}{4\,\mu F} \qquad (\because V = q/C)$$

$$\Rightarrow \quad 4q = -6q + 180$$

$$\Rightarrow \quad q = 18\,\mu C.$$

14. Energy stored in a charged capacitor is given by

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \cdot \frac{Q^2}{C} \qquad \ldots \text{(i)}$$

Here, $C = 12 \times 10^{-12}$ F and $V = 10$ V.

$$\Rightarrow \quad U = \frac{1}{2} \times 12 \times 10^{-12} \times 100$$

$$U = 6 \times 10^{-10} \text{ J} \qquad \ldots \text{(ii)}$$

After insertion of slab, capacitance will be $C' = KC$ and final energy,

$$U' = \frac{1}{2} \cdot \frac{Q^2}{C'} = \frac{1}{2} \frac{Q^2}{KC}$$

$$\Rightarrow U' = \frac{1}{K} U = \frac{1}{6.5} \times 6 \times 10^{-10} \text{J} \quad \ldots \text{(iv)} \quad (\because \text{given}, K = 6.5)$$

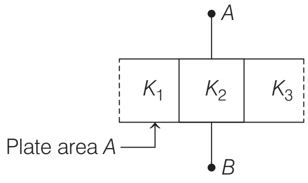
So, energy dissipated in the process will be equal to work done on the slab, i.e.

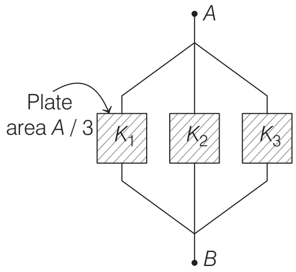
$$\Delta U = U - U' = \left(1 - \frac{1}{6.5}\right) \times 6 \times 10^{-10} \text{J}$$

$$\Rightarrow \Delta U = \frac{5.5}{6.5} \times 6 \times 10^{-10} \text{ J}$$

$$\cong 5.08 \times 10^{-10} \text{ J or } 508 \text{ pJ}$$

15. In the given arrangement, capacitor can be viewed as three-different capacitors connected in parallel as shown below,





So, equivalent capacity of the system is

$$C_{eq} = C_1 + C_2 + C_3$$

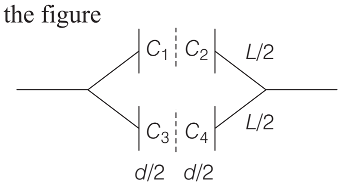
$$\Rightarrow \frac{K\varepsilon_0 A}{d} = \frac{K_1 \varepsilon_0 A/3}{d} + \frac{K_2 \varepsilon_0 A/3}{d} + \frac{K_3 \varepsilon_0 A/3}{d}$$

$$\Rightarrow \quad K = \frac{K_1}{3} + \frac{K_2}{3} + \frac{K_3}{3}$$

Here, $K_1 = 10, K_2 = 12$ and $K_3 = 14$

$$\text{So,} \quad K = \frac{10 + 12 + 14}{3} \Rightarrow K = 12$$

16 This capacitor system can be converted into two parts as shown in the figure



where C_1, C_2, C_3 and C_4 are capacitance of the capacitor having dielectric constants K_1, K_2, K_3 and K_4 respectively.

$$\text{Here,} \quad C_1 = \frac{K_1 \varepsilon_0 A/2}{d/2} = \frac{K_1 \varepsilon_0 A}{d}$$

Similarly, $C_2 = \dfrac{K_2 \varepsilon_0 A}{d}, C_3 = \dfrac{K_3 \varepsilon_0 A}{d}$ and $C_4 = \dfrac{K_4 \varepsilon_0 A}{d}$

Since, equivalent capacitance in series combination is

$$C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Here, C_1, C_2 and C_3, C_4 are in series combination.

$$\therefore \quad (C_{eq})_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{\dfrac{K_1 \varepsilon_0 A}{d} \cdot \dfrac{K_2 \varepsilon_0 A}{d}}{\dfrac{K_1 \varepsilon_0 A}{d} + \dfrac{K_2 \varepsilon_0 A}{d}}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d}$$

$$\text{Similarly,} \ (C_{eq})_{34} = \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\varepsilon_0 A}{d}$$

Now, $(C_{eq})_{12}$ and $(C_{eq})_{34}$ are in parallel combination.

$$\therefore \quad C_{net} = (C_{eq})_{12} + (C_{eq})_{34}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d} + \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C_{net} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right) \frac{\varepsilon_0 A}{d} \qquad \ldots \text{(i)}$$

If K is effective dielectric constant, then

$$C_{\text{net}} = \frac{K\,\varepsilon_0 A}{d} \qquad \text{...(ii)}$$

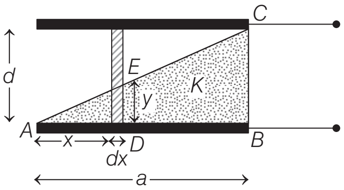
From Eqs. (i) and (ii),

$$\frac{K\,\varepsilon_0 A}{d} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 A}{d}$$

or $$K = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)$$

17. Let's consider a strip of thickness 'dx' at a distance of 'x' from the left end as shown in the figure. From the figure, ΔABC and ΔADE are similar triangles,

$$\Rightarrow \quad \frac{y}{x} = \frac{d}{a} \Rightarrow y = \left(\frac{d}{a}\right)x \qquad \text{...(i)}$$



We know that, the capacitance of parallel plate capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

$$C_1 = \frac{\varepsilon_0 (adx)}{(d - y)} \text{ and } C_2 = \frac{K\varepsilon_0 (adx)}{y}$$

Here, two capacitor are placed in series with variable thickness, therefore

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C_{\text{eq}} = \frac{K\varepsilon_0 adx}{Kd + (1 - K)y} \qquad \text{...(ii)}$$

Now, integrate it from 0 to a

$$C = \int_0^a \frac{K\varepsilon_0 adx}{Kd + (1 - K)y}$$

Using Eq. (i), $y = \left(\frac{d}{a}\right)x$, we get

$$C = \varepsilon_0 a \int_0^a \frac{dx}{d + \left(\frac{1}{K} - 1\right)\frac{d}{a}x}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a}{\left(\frac{1 - K}{K}\right)\frac{d}{a}} \ln\left[\frac{1}{K}\right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a^2 K \ln K}{(K - 1)d}$$

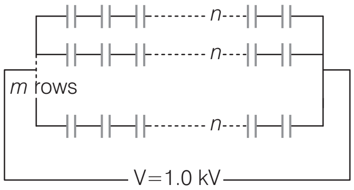
18. $$Q = KC_0 V$$

$$|Q_{\text{in}}| = \left|Q\left(1 - \frac{1}{K}\right)\right|$$

$$= (90 \times 10^{-12})(20)\left(\frac{5}{3}\right)\left(1 - \frac{3}{5}\right)\text{C}$$

$$= 1200 \times 10^{-12}\ \text{C} = 1.2\ \mu\text{C}$$

19. Let there are n capacitors in a row with m such rows in parallel.



As voltage not to exceed 300 V

$$\therefore \quad n \times 300 > 1000$$

[a voltage greater than 1 kV to be withstand]

$$\Rightarrow \quad n > \frac{10}{3} \Rightarrow n = 4 \qquad \text{(or 3.33)}$$

Also, $$C_{\textbf{Eq}} = \frac{mC}{n} = 2\mu\textbf{F}$$

$$\Rightarrow \quad \frac{m}{n} = 2 \Rightarrow m = 8 \qquad [\because C = 1\mu\textbf{F}]$$

So, total number of capacitors required

$$= m \times n = 8 \times 4 = 32$$

20. $3\,\mu\text{F}$ and $9\mu\text{F} = 12\mu\text{F}$

$$4\mu\,\text{F and } 12\mu\text{F} = \frac{4 \times 12}{4 + 12} = 3\mu\text{F}$$

$$Q = CV = 3 \times 8 = 24\mu\text{C (on } 4\mu\text{F and } 3\mu\text{F)}$$

Now, this 24μC distributes in direct ratio of capacity between 3μF and 9μF. Therefore,

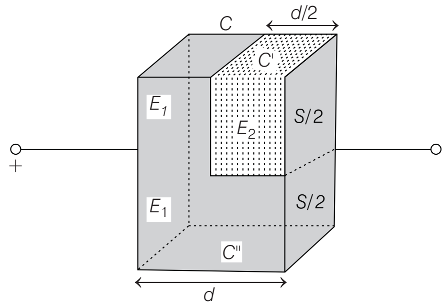
$$Q_{9\,\mu\text{F}} = 18\,\mu\text{C}$$

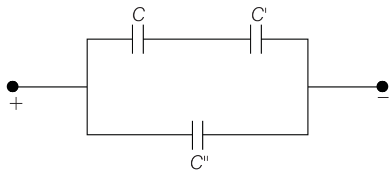
$$\therefore\ Q_{4\mu\text{F}} + Q_{9\,\mu\text{F}} = 24 + 18 = 42\mu\text{C} = Q \qquad \text{(say)}$$

$$E = \frac{kQ}{R^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{30^2}$$

$$= 420\ \text{N/C}$$

21.





$$C_1 = \frac{\varepsilon_0 s}{d}, C = \frac{2\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{2\varepsilon_0 s}{d} \Rightarrow C' = \frac{4\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{4\varepsilon_0 s}{d}$$

and $$C'' = \frac{2\varepsilon_0 \frac{s}{2}}{d} = \frac{\varepsilon_0 s}{d}$$

$$C_2 = \frac{CC'}{C + C'} + C'' = \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d} = \frac{7}{3}\frac{\varepsilon_0 s}{d} \quad \frac{C_2}{C_1} = \frac{7}{3}$$

22. Resultant of 1 μF and 2 μF is 3 μF. Now in series, potential difference distributes in inverse ratio of capacity.

$$\therefore \quad \frac{V_{3\mu F}}{V_c} = \frac{c}{3} \quad \text{or} \quad V_{3\mu F} = \left(\frac{c}{c+3}\right)E$$

This is also the potential difference across 2 μF.

$$\therefore \quad Q_2 = (2\,\mu F)(V_{2\mu F})$$

or $$Q_2 = \left(\frac{2cE}{c+3}\right) = \left(\frac{2}{1 + \frac{3}{c}}\right)E$$

From this expression of Q_2, we can see that Q_2 will increase with increase in the value of c (but not linearly). Therefore, only options (a) and (b) may be correct.

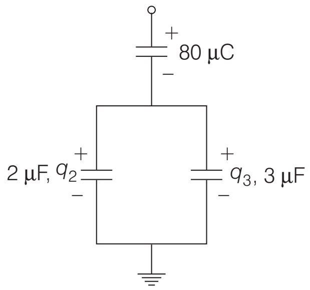
Further, $$\frac{d}{dc}(Q_2) = 2E\left[\frac{(c+3) - c}{(c+3)^2}\right] = \frac{6E}{(c+3)^2}$$

= Slope of Q_2 verus c graph.

i.e. slope of Q_2 versus c graph decreases with increase in the value of c. Hence, the correct graph is (a).

23. Between 3 μF and 2 μF (in parallel), total charge of 80 μC will distribute in direct ratio of capacity.

$$\frac{q_3}{q_2} = \frac{3}{2}$$



$$q_3 = \left(\frac{3}{3+2}\right)(80) = 48\,\mu C$$

24. When free space between parallel plates of capacitor,

$$E = \frac{\sigma}{\varepsilon_0}$$

When dielectric is introduced between parallel plates of capacitor, $E' = \dfrac{\sigma}{K\varepsilon_0}$

Electric field inside dielectric, $\dfrac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

where, K = dielectric constant of medium = 2.2

ε_0 = permitivity of free space = 8.85×10^{-12}

$$\Rightarrow \quad \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$
$$= 6.6 \times 8.85 \times 10^{-8} = 5.841 \times 10^{-7}$$
$$= 6 \times 10^{-7}\ \text{C/m}^2$$

25. $q_i = C_i V = 2V = q$ (say)

This charge will remain constant after switch is shifted from position 1 to position 2.

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2 \times 2} = \frac{q^2}{4}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2 \times 10} = \frac{q^2}{20}$$

$$\therefore \text{Energy dissipated} = U_i - U_f = \frac{q^2}{5}$$

This energy dissipated $\left(= \dfrac{q^2}{5}\right)$ is 80% of the initial stored energy $\left(= \dfrac{q^2}{4}\right)$.

∴ Correct option is (d).

26. After time t, thickeness of liquid will remain $\left(\dfrac{d}{3} - vt\right)$.

Now, time constant as function of time

$$\tau_c = CR = \frac{\varepsilon_0 (1) \cdot R}{(d - \frac{d}{3} + vt) + \frac{d/3 - vt}{2}}$$

$$\left(\text{Applying } C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}\right)$$

$$= \frac{6\varepsilon_0 R}{5d + 3vt}$$

∴ Correct option is (a).

27. All the three plates will produce electric field at P along negative z-axis. Hence,

$$\mathbf{E}_p = \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{\mathbf{k}}) = -\frac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$$

∴ Correct answer is (b).

28. ΔU = decrease in potential energy

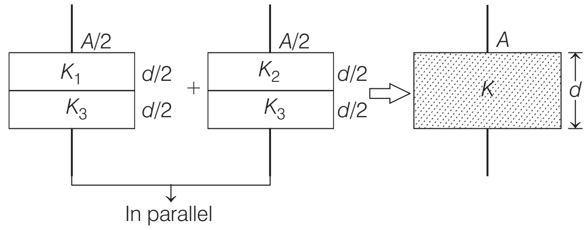
$$= U_i - U_f$$

$$= \frac{1}{2}C(V_1^2 + V_2^2) - \frac{1}{2}(2C)\left(\frac{V_1 + V_2}{2}\right)^2$$

$$= \frac{1}{4}C(V_1 - V_2)^2$$

29. Due to attraction with positive charge, the negative charge on capacitor A will not flow through the switch S.

30. Applying $C = \dfrac{\varepsilon_0 A}{d - t_1 - t_2 + \dfrac{t_1}{K_1} + \dfrac{t_2}{K_2}}$, we have



$$\frac{\varepsilon_0(A/2)}{d - d/2 - d/2 + \frac{d/2}{K_1} + \frac{d/2}{K_3}}$$

$$+ \frac{\varepsilon_0(A/2)}{d - d/2 - d/2 + \frac{d/2}{K_2} + \frac{d/2}{K_3}} = \frac{K\varepsilon_0 A}{d}$$

Solving this equation, we get

$$K = \frac{K_1K_3}{K_1 + K_3} + \frac{K_2K_3}{K_2 + K_3}$$

31. When S_3 is closed, due to attraction with opposite charge, no flow of charge takes place through S_3. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

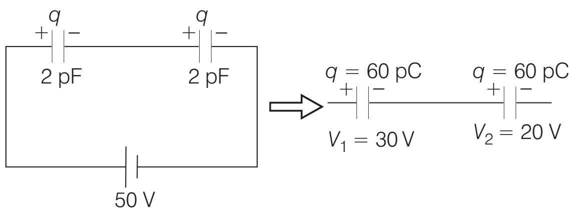
Alternate Solution

Charges on the capacitors are

$$q_1 = (30)(2) = 60 \text{ pC}$$

and $q_2 = (20)(3) = 60$ pC or $q_1 = q_2 = q$ (say)

The situation is similar as the two capacitors in series are first charged with a battery of emf 50 V and then disconnected.

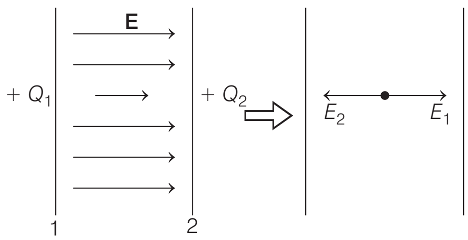


∴ When S_3 is closed,

$V_1 = 30$ V

and $V_2 = 20$ V.

32. Electric field within the plates $\mathbf{E} = \mathbf{E}_{Q_1} + \mathbf{E}_{Q_2}$

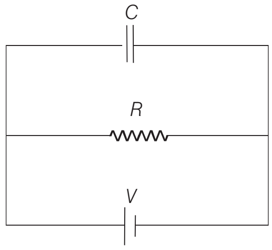


$$E = E_1 - E_2 = \frac{Q_1}{2A\varepsilon_0} - \frac{Q_2}{2A\varepsilon_0} \Rightarrow E = \frac{Q_1 - Q_2}{2A\varepsilon_0}$$

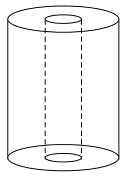
∴ Potential difference between the plates

$$V_A - V_B = Ed = \left(\frac{Q_1 - Q_2}{2A\varepsilon_0}\right) d = \frac{Q_1 - Q_2}{2\left(\frac{A\varepsilon_0}{d}\right)} = \frac{Q_1 - Q_2}{2C}$$

33. Since, the capacitor plates are directly connected to the battery, it will take no time in charging.



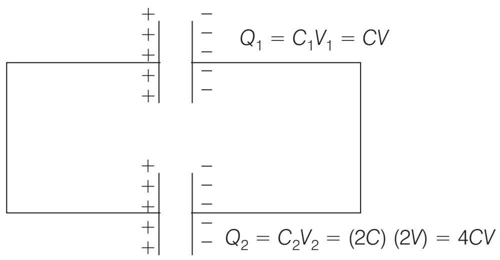
34. The magnitude of electric field at a distance r from the axis is given as :



$$E = \frac{\lambda}{2\pi\,\varepsilon_0 r} \text{ i.e. } E \propto \frac{1}{r}$$

Here, λ is the charge per unit length of the capacitor.

35. The diagramatic representation of given problem is shown in figure.



The net charge shared between the two capacitors is

$$Q' = Q_2 - Q_1 = 4CV - CV = 3CV$$

The two capacitors will have the same potential, say V'.

The net capacitance of the parallel combination of the two capacitors will be

$$C' = C_1 + C_2 = C + 2C = 3C$$

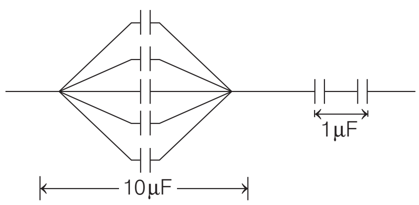
The potential difference across the capacitors will be

$$V' = \frac{Q'}{C'} = \frac{3CV}{3C} = V$$

The electrostatic energy of the capacitors will be

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$$

36. In series, $C = \dfrac{C_1C_2}{C_1 + C_2}$



$$\therefore \qquad C_{\text{net}} = \frac{(10)(1)}{10 + 1} = \frac{10}{11}\,\mu\text{F}$$

37.
$$C = C_1 + C_2$$
$$C_1 = \frac{K\varepsilon_0 A/3}{d}$$
$$C_2 = \frac{\varepsilon_0 2A/3}{d}$$
$$\Rightarrow \quad C = \frac{(K+2)\varepsilon_0 A}{3d}$$
$$\Rightarrow \quad \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = V/d$, where V is potential difference between the plates.

38. After pressing S_1 charge on upper plate of C_1 is $+2CV_0$.

After pressing S_2 this charge equally distributes in two capacitors. Therefore, charge an upper plates of both capacitors will be $+CV_0$.

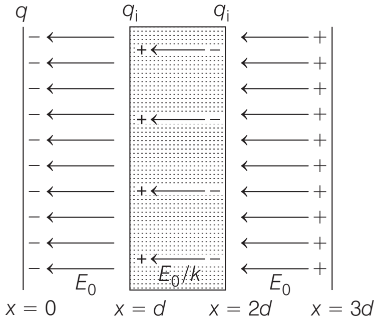
When S_2 is released and S_3 is pressed, charge on upper plate of C_1 remains unchanged $(= +CV_0)$ but charge on upper plate of C_2 is according to new battery $(= -CV_0)$.

39. Polarity should be mentioned in the question. Potential on each of them can be zero if, $q_{\text{net}} = 0$

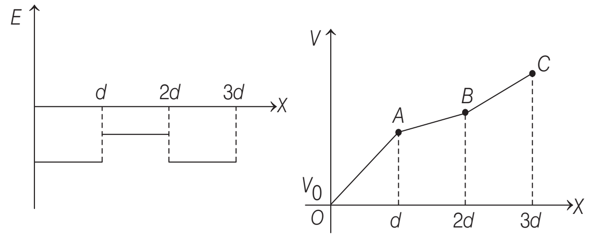
or $\quad q_1 \pm q_2 = 0$

or $\quad 120C_1 \pm 200C_2 = 0$ or $3C_1 \pm 5C_2 = 0$

40. The magnitude and direction of electric field at different points are shown in figure. The direction of the electric field remains the same. Hence, option (b) is correct. Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from $x = 0$ to $x = 3d$.



Therefore, option (c) is also correct. The variation of electric field (E) and potential (V) with x will be as follows



$OA \parallel BC$ and $(\text{Slope})_{OA} > (\text{Slope})_{AB}$

Because $\quad E_{O-d} = E_{2d-3d}$

and $\quad E_{O-d} > E_{d-2d}$

41. Battery is removed. Therefore, charge stored in the plates will remain constant.

$$Q = CV = \frac{\varepsilon_0 A}{d} V \quad \text{or } Q = \text{constant.}$$

Now, dielectric slab is inserted. Therefore, C will increase. New capacity will be,

$$C' = KC = \frac{\varepsilon_0 KA}{d} \Rightarrow V' = \frac{Q}{C'} = \frac{V}{K}$$

and new electric field, $E = \dfrac{V'}{d} = \dfrac{V}{K \cdot d}$

Potential energy stored in the capacitor,

Initially, $\quad U_i = \dfrac{1}{2}CV^2 = \dfrac{\varepsilon_0 AV^2}{2d}$

Finally, $\quad U_f = \dfrac{1}{2}C'V'^2 = \dfrac{1}{2}\left(\dfrac{K\varepsilon_0 A}{d}\right)\left(\dfrac{V}{K}\right)^2 = \dfrac{\varepsilon_0 AV^2}{2Kd}$

Work done on the system will be

$$|\Delta U| = \frac{\varepsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$$

$\therefore$ Correct options are (a), (c) and (d).

42. Charging battery is removed. Therefore, q = constant

Distance between the plates is increased. Therefore, C decreases.

Now, $V = q/C$, q is constant and C is decreasing. Therefore, V should increase.

$U = \dfrac{1}{2}\dfrac{q^2}{C}$ again q is constant and C is decreasing.

Therefore U should increase.

$\therefore$ Correct options are (b) and (d).

43. When dielectric slab is introduced capacity gets increased while potential difference remains unchanged.

$\therefore \quad V = V_0, \ C > C_0$

$Q = CV \quad \therefore \quad Q > Q_0$

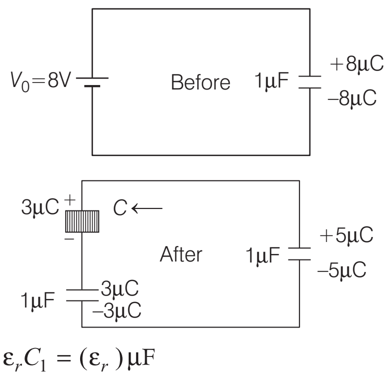
$U = \dfrac{1}{2}CV^2 \quad \therefore \quad U > U_0$

$E = \dfrac{V}{d}$ but V and d both are unchanged.

Therefore, $\quad E = E_0$

Therefore, correct options are (a) and (d).

44.



$C = \varepsilon_r C_1 = (\varepsilon_r)\,\mu F$

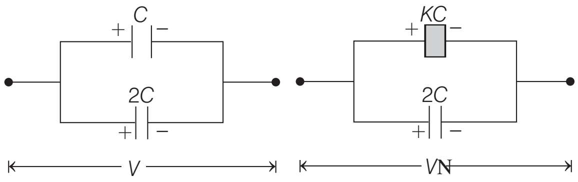
Applying loop rule,

$$\frac{5}{1}-\frac{3}{\varepsilon_r}-\frac{3}{1}=0 \quad \Rightarrow \quad \frac{3}{\varepsilon_r}=2$$

$$\varepsilon_r = 1.50$$

45. Total charge will remain unchanged.

Hence, $Q = Q'$ or $3CV = (KC + 2C)V'$

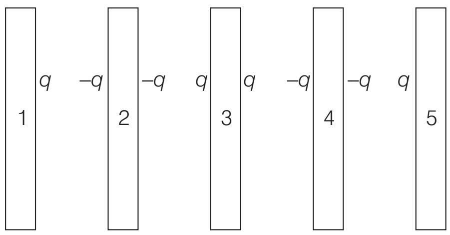


$$\therefore \qquad V' = \left(\frac{3}{K+2}\right)V$$

46. In the circuit shown in figure, there is a capacitor between plates 1 and 2 , the capacity of which is : $C_1 = \frac{\varepsilon_0 A}{d}$ and potential difference between its plates is V. Therefore, charge stored in it is, $q = C_1V = \frac{\varepsilon_0 AV}{d}$

Since, plate 1 is connected with positive terminal, hence this charge q will be positive.

Plate 4 is making two capacitors, one with 3 and other with 5. Hence, charge on it will be $-2q$ or $\frac{-2\varepsilon_0 AV}{d}$. Charge on it is negative because this is connected with negative plate. Charges on both sides of the plates are shown below.

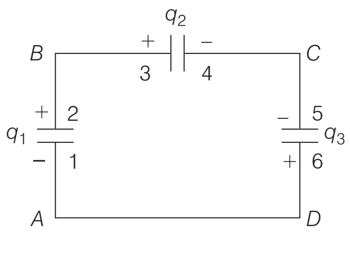


Here, $$q = \frac{\varepsilon_0 AV}{d}$$

47. Electric field between the plates of capacitor is almost uniform. Therefore, force on both the protons will be identical. It hardly matters whether they are placed near positive plate or negative plate.

48. (a) Charge on capacitor A, before joining with an uncharged capacitor

$$q_A = CV = (100)(3)\mu C = 300\,\mu C$$



Similarly, charge on capacitor B

$$q_B = (180)(2)\,\mu C = 360\,\mu C$$

Let q_1, q_2 and q_3 be the charges on the three capacitors after joining them as shown in figure.

(q_1, q_2 and q_3 are in microcoulombs)

From conservation of charge

Net charge on plates 2 and 3 before joining

= net charge after joining

$$\therefore \qquad 300 = q_1 + q_2 \qquad \ldots\text{(i)}$$

Similarly, net charge on plates 4 and 5 before joining

= net charge after joining $-360 = -q_2 - q_3$

$$\text{or} \qquad 360 = q_2 + q_3 \qquad \ldots\text{(ii)}$$

Applying Kirchhoff's second law in closed loop $ABCDA$

$$\frac{q_1}{3}-\frac{q_2}{2}+\frac{q_3}{2}=0$$

$$\text{or} \qquad 2q_1 - 3q_2 + 3q_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs. (i) , (ii) and (iii), we get

$$q_1 = 90\,\mu C,\ q_2 = 210\,\mu C$$

and $$q_3 = 150\ \mu C$$

(b) (i) Electrostatic energy stored before, completing the circuit

$$U_i = \frac{1}{2}(3\times10^{-6})(100)^2 + \frac{1}{2}(2\times10^{-6})(180)^2 \qquad \left(\because U = \frac{1}{2}CV^2\right)$$

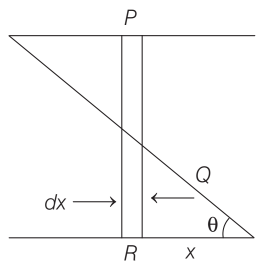
$$= 4.74\times10^{-2}\text{ J} \text{ or } U_i = 47.4\text{ mJ}$$

(ii) Electrostatic energy stored after, completing the circuit

$$U_f = \frac{1}{2}\frac{(90\times10^{-6})^2}{(3\times10^{-6})} + \frac{1}{2}\frac{(210\times10^{-6})^2}{(2\times10^{-6})} + \frac{1}{2}\frac{(150\times10^{-6})^2}{(2\times10^{-6})} \qquad \left[U = \frac{1}{2}\frac{q^2}{C}\right]$$

$$= 1.8\times10^{-2}\text{ J} \quad \text{or} \quad U_f = 18\text{ mJ}$$

49. Let length and breadth of the capacitor be l and b respectively and d be the distance between the plates as shown in figure. Then, consider a strip at a distance x of width dx.



Now, $QR = x\tan\theta$

and $PQ = d - x\tan\theta$

where, $\tan\theta = d/l$

Capacitance of PQ

$$C_1 = \frac{K_1\varepsilon_0(bdx)}{d - x\tan\theta} = \frac{K_1\varepsilon_0(bdx)}{d - \frac{xd}{l}}$$

$$C_1 = \frac{K_1\varepsilon_0 bldx}{d(l-x)} = \frac{K_1\,\varepsilon_0 A(dx)}{d(l-x)}$$

and $\quad C_2 =$ capacitance of $QR = \dfrac{K_2\varepsilon_0\, b(dx)}{x\tan\theta}$

$$C_2 = \frac{K_2\varepsilon_0 A\,(dx)}{xd} \qquad \left(\tan\theta = \frac{d}{l}\right)$$

Now, C_1 and C_2 are in series. Therefore, their resultant capacity C_0 will be given by

$$\frac{1}{C_0} = \frac{1}{C_1} + \frac{1}{C_2}$$

Then, $\dfrac{1}{C_0} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{d\,(l-x)}{K_1\,\varepsilon_0 A\,(dx)} + \dfrac{x.d}{K_2\varepsilon_0 A(dx)}$

$$\frac{1}{C_0} = \frac{d}{\varepsilon_0 A(dx)}\left(\frac{l-x}{K_1} + \frac{x}{K_2}\right)$$

$$= \frac{d\,\{K_2(l-x) + K_1 x\}}{\varepsilon_0 A K_1 K_2 (dx)}$$

$\therefore$ $$C_0 = \frac{\varepsilon_0 A K_1 K_2}{d\,\{K_2(l-x) + K_1 x\}}dx$$

$$C_0 = \frac{\varepsilon_0 A K_1 K_2}{d\{K_2 l + (K_1 - K_2)x\}}dx$$

Now, the net capacitance of the given parallel plate capacitor is obtained by adding such infinitesimal capacitors placed parallel from $x = 0$ to $x = l$

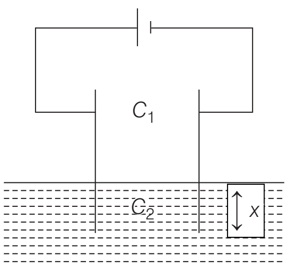
i.e. $\quad C_R = \displaystyle\int_{x=0}^{x=l} C_0 = \int_0^l \frac{\varepsilon_0 A K_1 K_2}{d\,\{K_2 l + (K_1 - K_2)x\}}\,dx$

Finally we get $C_R = \dfrac{K_1 K_2 \varepsilon_0 A}{(K_2 - K_1)d}\ln\dfrac{K_2}{K_1}$

$$= \frac{CK_1K_2}{K_2 - K_1}\ln\frac{K_2}{K_1} \qquad \text{where, } C = \frac{\varepsilon_0 A}{d}$$

50. Let a be the side of the square plate.

As shown in figure, C_1 and C_2 are in parallel. Therefore, total capacity of capacitors in the position shown is



$$C = C_1 + C_2$$

$$C = \frac{\varepsilon_0 a(a-x)}{d} + \frac{K\varepsilon_0 ax}{d}$$

$\therefore \quad q = CV = \dfrac{\varepsilon_0 aV}{d}(a - x + Kx)$

As plates are lowered in the oil, C increases or charge stored will increase.

Therefore, $\quad i = \dfrac{dq}{dt} = \dfrac{\varepsilon_0 aV}{d}(K-1)\cdot\dfrac{dx}{dt}$

Substituting the values

$$\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N-m}^2$$

$$a = 1\,\text{m}, V = 500\,\text{volt}, d = 0.01\,\text{m}, K = 11$$

and $\quad \dfrac{dx}{dt} =$ speed of plate $= 0.001\,\text{m/s}$

We get current $i = \dfrac{(8.85\times10^{-12})(1)(500)\,(11-1)\,(0.001)}{(0.01)}$

$$i = 4.43\times10^{-9}\ \text{A}$$

51. (a) Capacitor A is a combination of two capacitors C_K and C_O in parallel. Hence,

$$C_A = C_K + C_O = \frac{K\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = (K+1)\frac{\varepsilon_0 A}{d}$$

Here, $A = 0.02\ \text{m}^2$. Substituting the values, we have

$$C_A = (9+1)\frac{8.85\times10^{-12}\ (0.02)}{(8.85\times10^{-4})}$$

$$C_A = 2.0\times10^{-9}\ \text{F}$$

Energy stored in capacitor A, when connected with a 110 V battery is

$$U_A = \frac{1}{2}C_A V^2 = \frac{1}{2}(2\times10^{-9})\,(110)^2$$

$$U_A = 1.21\times10^{-5}\ \text{J}$$

(b) Charge stored in the capacitor

$$q_A = C_A V = (2.0\times10^{-9})\,(110) \Rightarrow q_A = 2.2\times10^{-7}\text{C}$$

Now, this charge remains constant even after battery is disconnected. But when the slab is removed, capacitance of A will get reduced. Let it be C'_A

$$C'_A = \frac{\varepsilon_0(2A)}{d} = \frac{(8.85\times10^{-12})\,(0.04)}{8.85\times10^{-4}}$$

$$C'_A = 0.4\times10^{-9}\ \text{F}$$

Energy stored in this case would be

$$U'_A = \frac{1}{2}\frac{(q_A)^2}{C'_A} = \frac{1}{2}\frac{(2.2\times10^{-7})^2}{(0.4\times10^{-9})}$$

$$U'_A = 6.05\times10^{-5}\ \text{J} > U_A$$

Therefore, work done to remove the slab would be

$$W = U'_A - U_A = (6.05 - 1.21)\times10^{-5}\ \text{J}$$

or $$W = 4.84\times10^{-5}\ \text{J}$$

(c) Capacity of B when filled with dielectric is

$$C_B = \frac{K\varepsilon_0 A}{d} = \frac{(9)\,(8.85\times10^{-12})\,(0.02)}{(8.85\times10^{-4})}$$

$$C_B = 1.8\times10^{-9}\ \text{F}$$

These two capacitors are in parallel. Therefore, net capacitance of the system is

$$C = C'_A + C_B = (0.4 + 1.8) \times 10^{-9}\text{ F}$$

$$C = 2.2 \times 10^{-9}\text{ F}$$

Charge stored in the system is $q = q_A = 2.2 \times 10^{-7}\text{C}$

Therefore, energy stored, $U = \frac{1}{2}\frac{q^2}{C}$

$$U = \frac{1}{2}\frac{(2.2 \times 10^{-7})^2}{(2.2 \times 10^{-9})} \quad \text{or} \quad U = 1.1 \times 10^{-5}\text{ J}$$

52. Before opening the switch potential difference across both the capacitors is V, as they are in parallel. Hence, energy stored in them is,

$$U_A = U_B = \frac{1}{2}CV^2$$

$$\therefore \quad U_{\text{Total}} = CV^2 = U_i \quad \ldots\text{(i)}$$

After opening the switch, potential difference across it is V and its capacity is $3C$

$$\therefore \quad U_A = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$$

In case of capacitor B, charge stored in it is $q = CV$ and its capacity is also $3C$.

Therefore, $U_B = \frac{q^2}{2(3C)} = \frac{CV^2}{6}$

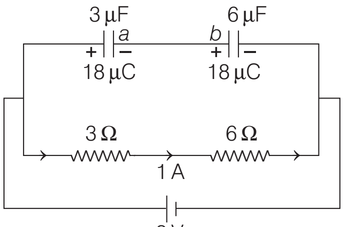
$$\therefore \quad U_{\text{Total}} = \frac{3CV^2}{2} + \frac{CV^2}{6} = \frac{10}{6}CV^2 = \frac{5CV^2}{3} = U_f \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get $\frac{U_i}{U_f} = \frac{3}{5}$

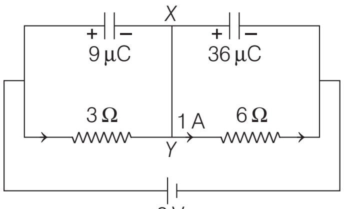
Topic 6 *C-R* Circuits

1. From Y to X charge flows to plates a and b.

$$(q_a + q_b)_i = 0, \ (q_a + q_b)_f = 27\,\mu\text{C}$$



Initial figure (when switch was open)



Final figure (when switch is closed)

$\therefore$ $27\,\mu$C charge flows from Y to X.

$\therefore$ Correct option is (c).

2. $\tau = CR$

$$\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18\,\mu\text{s}$$

$$\tau_2 = \left(\frac{C_1C_2}{C_1 + C_2}\right)\left(\frac{R_1R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9}\,\mu\text{s}$$

$$\tau_3 = (C_1 + C_2)\left(\frac{R_1R_2}{R_1 + R_2}\right) = (6)\left(\frac{2}{3}\right) = 4\,\mu\text{s}$$

3. Given : $V_C = 3V_R = 3(V - V_C)$

Here, V is the applied potential.

$$\therefore \quad V_C = \frac{3}{4}V \quad \text{or} \quad V(1 - e^{-t/\tau_c}) = \frac{3}{4}V$$

$$\therefore \quad e^{-t/\tau_c} = \frac{1}{4} \quad \ldots\text{(i)}$$

Here, $\tau_c = CR = 10\text{ s}$

Substituting this value of τ_c in Eq. (i) and solving for t, we get

$$t = 13.86\text{ s}$$

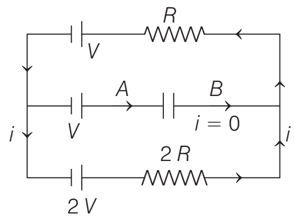
$\therefore$ Correct answer is (a).

4. Charging current, $I = \frac{E}{R}e^{-\frac{t}{RC}}$

Taking log on both sides, $\log I = \log\left(\frac{E}{R}\right) - \frac{t}{RC}$

When R is doubled, slope of curve decreases. Also at $t = 0$, the current will be less. Graph Q represents the best. Hence, the correct option is (b).

5. In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,



$$i = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

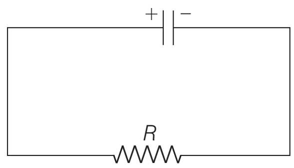
Potential difference between A and B

$$V_A - V + V + iR = V_B$$

$$\therefore \quad V_B - V_A = iR = \left(\frac{V}{3R}\right)R = \frac{V}{3}$$

NOTE In this problem charge stored in the capacitor can also be asked, which is equal to $q = C\frac{V}{3}$ with positive charge on B side and negative on A side because $V_B > V_A$.

6. The discharging current in the circuit is,



$$i = i_0 e^{-t/CR}$$

Here, $i_0 =$ initial current $= \dfrac{V}{R}$

Here, V is the potential with which capacitor was charged.

Since, V and R for both the capacitors are same, initial discharging current will be same but non-zero.

$\therefore$ Correct option is (b).

Further, $\tau_c = CR \Rightarrow C_1 < C_2$ or $\tau_{C_1} < \tau_{C_2}$

or C_1 loses its 50% of initial charge sooner than C_2.

$\therefore$ Option (d) is also correct .

7. Voltage across the capacitors will increase from 0 to 10 V exponentially. The voltage at time t will be given by

$$V = 10\,(1 - e^{-t/\tau_C})$$

Here, $\tau_c = C_{\text{net}}\, R_{\text{net}}$

$$= (1 \times 10^6)\,(4 \times 10^{-6}) = 4\text{ s}$$

$\therefore$ $V = 10\,(1 - e^{-t/4})$

Substituting $V = 4$ volt, we have

$$4 = 10\,(1 - e^{-t/4})$$

or $e^{-t/4} = 0.6 = \dfrac{3}{5}$

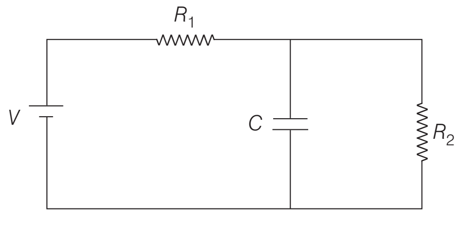
Taking log on both sides we have,

$$-\frac{t}{4} = \ln 3 - \ln 5$$

or $t = 4\,(\ln 5 - \ln 3) = 2\text{ s}$

Hence, the answer is 2 .

8.



Q_0 is the steady state charge stored in the capacitor.

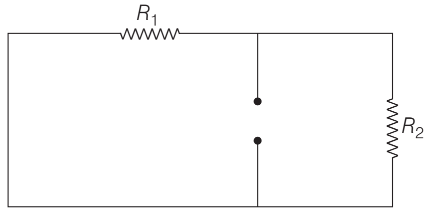
$Q_0 = C$ [PD across capacitor in steady state]

$= C$ [steady state current through R_2] (R_2)

$$= C\left(\frac{V}{R_1 + R_2}\right) \cdot R_2$$

$\therefore$ $Q_0 = \dfrac{C\,V\,R_2}{R_1 + R_2}$

α is $1/\tau_c$ or $\dfrac{1}{C\,R_{\text{net}}}$



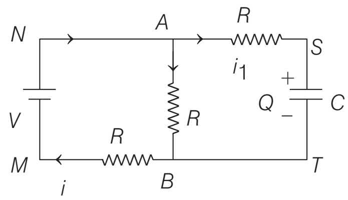
Here, R_{net} is equivalent resistance across capacitor after short circuiting the battery. Thus,

$$R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2} \qquad (\text{As } R_1 \text{ and } R_2 \text{ are in parallel})$$

$$\alpha = \frac{1}{C\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)} = \frac{R_1 + R_2}{C\,R_1 R_2}$$

9. Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t.

Therefore,



$$i_1 = \frac{dQ}{dt}$$

(a) Applying Kirchhoff 's second law in loop $MNABM$

$$V = (i - i_1)\,R + iR$$

or $V = 2iR - i_1 R$ …(i)

Similarly, applying Kirchhoff’s second law in loop $MNSTM$, we have

$$V = i_1 R + \frac{Q}{C} + iR \qquad \text{(ii)}$$

Eliminating i from Eqs. (i) and (ii), we get

$$V = 3 i_1 R + \frac{2Q}{C}$$

or $3 i_1 R = V - \dfrac{2Q}{C}$ or $i_1 = \dfrac{1}{3R}\left(V - \dfrac{2Q}{C}\right)$

or $\dfrac{dQ}{dt} = \dfrac{1}{3R}\left(V - \dfrac{2Q}{C}\right)$ or $\dfrac{dQ}{V - \dfrac{2Q}{C}} = \dfrac{dt}{3R}$

or $\displaystyle\int_0^Q \frac{dQ}{V - \dfrac{2Q}{C}} = \int_0^t \frac{dt}{3R}$

This equation gives $Q = \dfrac{CV}{2}\,(1 - e^{-2t/3RC})$

(b) $i_1 = \dfrac{dQ}{dt} = \dfrac{V}{3R}\,e^{-2t/3RC}$

From Eq. (i) $i = \dfrac{V + i_1 R}{2R} = \dfrac{V + \dfrac{V}{3}e^{-2t/3RC}}{2R}$

$\therefore$ Current through AB

$$i_2 = i - i_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

$$i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}$$

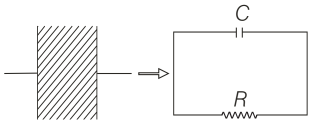
$$i_2 = \frac{V}{2R} \text{ as } t \to \infty$$

10. The problem is basically of discharging of CR circuit, because between the plates of the capacitor, there is capacitor as well as resistance.

$$R = \frac{d}{\sigma A} \qquad \left(R = \frac{l}{\sigma A}\right)$$

and $$C = \frac{K\varepsilon_0 A}{d}$$

$\therefore$ Time constant, $\tau_c = CR = \dfrac{K\varepsilon_0}{\sigma}$



Substituting the values, we have

$$\tau_c = \frac{5 \times 8.86 \times 10^{-12}}{7.4 \times 10^{-12}} = 5.98\,\text{s}$$

Charge at any time decreases exponentially as

$$q = q_0 e^{-t/\tau_c}$$

Here, $q_0 = 8.85 \times 10^{-6}$ C (Charge at time $t = 0$)

Therefore, discharging (leakage) current at time t will be given by

$$i = \left(-\frac{dq}{dt}\right) = \frac{q_0}{\tau_c}e^{-t/\tau_c}$$

or current at $t = 12$ s is

$$i = \frac{(8.85 \times 10^{-6})}{5.98}e^{-12/5.98}$$

$$= 0.198 \times 10^{-6}\ \text{A} = 0.198\,\mu\text{A}$$

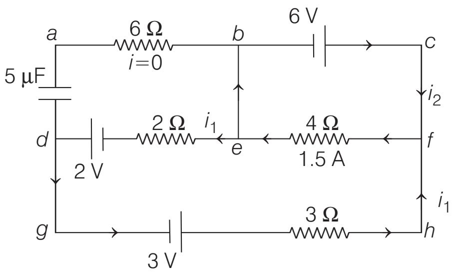
$$i = 0.198\,\mu\text{A}$$

11. In steady state no current will flow through $R_1 = 6\Omega$.

Potential difference across R_3 or 4 Ω is E_1 or 6 V

$\therefore$ Current through it will be $\dfrac{6}{4} = 1.5$A from right to left.

Because left hand side of this resistance is at higher potential.



Now, suppose this 1.5 A distributes in i_1 and i_2 as shown. Applying Kirchhoff's second law in loop $dghfed$

$$3 - 3i_1 - 4 \times 1.5 - 2i_1 + 2 = 0$$

$$\therefore \qquad i_1 = -\frac{1}{5}\text{A} = -\ 0.2\,\text{A}$$

To find energy stored in capacitor we will have to find potential difference across it. Or V_{ad}.

Now, $\qquad V_a - 2i_1 + 2 = V_d$

or $\qquad V_a - V_d = 2i_1 - 2 = -\ 2.4$ V

or $\qquad V_d - V_a = 2.4\ \text{V} = V_{da}$

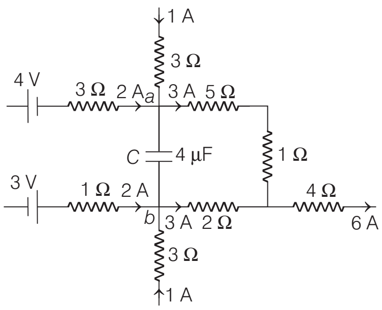
Energy stored in capacitor:

$$U = \frac{1}{2}CV_{da}^2$$

$$= \frac{1}{2}(5 \times 10^{-6})(2.4)^2$$

$$= 1.44 \times 10^{-5}\ \text{J}$$

12. Using Kirchhoff's first law at junctions a and b, we have found the current in other wires of the circuit on which currents were not shown.



Now, to calculate the energy stored in the capacitor we will have to first find the potential difference V_{ab} across it.

$$V_a - 3 \times 5 - 3 \times 1 + 3 \times 2 = V_b$$

$$\therefore \qquad V_a - V_b = V_{ab} = 12\,\text{V}$$

$$\therefore \qquad U = \frac{1}{2}CV_{ab}^2 = \frac{1}{2}(4 \times 10^{-6})(12)^2\ \text{J} = 0.288\,\text{mJ}$$

13. In steady state situation no current will flow through the capacitor. 2Ω and 3Ω are in parallel.

Therefore, their combined resistance will be

$$R = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

Net current through the battery

$$i = \frac{6}{1.2 + 2.8} = 1.5\ \text{A}$$

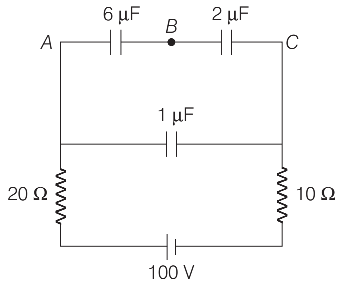
This current will distribute in inverse ratio of their resistances in 2Ω and 3Ω.

$$\therefore \qquad \frac{i_2}{i_3} = \frac{3}{2}$$

or $$i_2 = \left(\frac{3}{3 + 2}\right)(1.5) = 0.9\text{A}$$

14. Two capacitors of $3\,\mu F$ each and two capacitors of $1\,\mu F$ each are in parallel.

Therefore, simplified circuit can be drawn as below.



In steady state no current will flow in the circuit and capacitors are fully charged.

Points A, B and C in original circuit are shown in the simplified circuit.

Between points A and C, $6\,\mu F$ and $2\,\mu F$ are in series. 100 V is applied across this series combination. In series potential drops in inverse ratio of capacity.

$$\therefore \quad V_{AB} = V_{6\,\mu F} = \left(\frac{2}{6+2}\right) \times 100 = 25\text{ V}$$

$$V_{BC} = V_{2\,\mu F} = \left(\frac{6}{6+2}\right) \times 100 = 75\text{ V}$$

Topic 7 Miscellaneous Problems

1. Given, magnetic field of an electromagnetic wave is

$$\mathbf{B} = B_0[\cos(kz - \omega t)]\,\hat{\mathbf{i}} + B_1[\cos(kz + \omega t)]\,\hat{\mathbf{j}}$$

Here, $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T

Also, stationary charge, $Q = 10^{-4}$C at $z = 0$

As charge is released from the rest at $z = 0$, in this condition.

Maximum electric field, $E_0 = cB_0$ and $E_1 = cB_1$

So, $E_0 = c \times 3 \times 10^{-5}$ and $E_1 = c \times 2 \times 10^{-6}$

Now,the direction of electric field of an electromagnetic wave is perpendicular to $\mathbf{B}$ and to the direction of propagation of wave $(\mathbf{E} \times \mathbf{B})$ which is $\hat{\mathbf{k}}$.

So, for E_0, $\mathbf{E}_0 \times \mathbf{B}_0 = \hat{\mathbf{k}} \Rightarrow \mathbf{E}_0 \times \hat{\mathbf{i}} = \hat{\mathbf{k}} \Rightarrow \mathbf{E}_0 = -\hat{\mathbf{j}}$

Similarly,

for E_1 $\mathbf{E}_1 \times \mathbf{B}_1 = \mathbf{k} \Rightarrow \mathbf{E}_1 \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$

$$\Rightarrow \quad \mathbf{E}_1 = \hat{\mathbf{i}}$$

$$\therefore \quad \mathbf{E}_0 = c \times 3 \times 10^{-5}(-\hat{\mathbf{j}})\text{NC}^{-1}$$

$$\mathbf{E} = c \times 2 \times 10^{-6}(+\hat{\mathbf{i}})\text{NC}^{-1}$$

$\therefore$ Maximum force experienced by stationary charge is

$$\mathbf{F}_{\max} = Q\mathbf{E} = Q(\mathbf{E}_0 + \mathbf{E}_1)$$
$$= Q \times c\,[-3 \times 10^{-5}\,\hat{\mathbf{j}} + 2 \times 10^{-6}\,\hat{\mathbf{i}}]$$

$$\Rightarrow |\mathbf{F}_{\max}| = 10^{-4} \times 3 \times 10^{8} \times \sqrt{(3 \times 10^{-5})^2 + (2 \times 10^{-6})^2}$$
$$= 3 \times 10^{4} \times 10^{-6}\sqrt{900 + 4}$$
$$= 3 \times 10^{-2} \times \sqrt{904} \approx 0.9\text{ N}$$

$\therefore$ rms value of experienced force is

$$F_{\text{rms}} = \frac{F_{\max}}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} = 0.707 \times 0.9$$
$$= 0.6363\text{ N} \approx 0.6\text{ N}$$

2. **Key Idea** When an electric dipole is placed in an electric field $\mathbf{E}$ at some angle θ, then two forces equal in magnitude but opposite in direction acts on the +ve and –ve charges, respectively. These forces forms a couple which exert a torque, which is given as

$$\tau = \mathbf{p} \times \mathbf{E}$$

where, $\mathbf{p}$ is dipole moment.

Torque on the dipole is given as

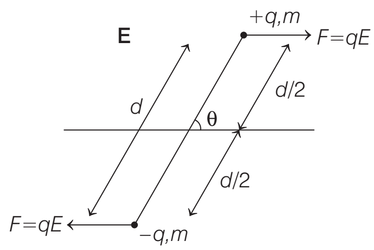
$$\tau = I\alpha = -\,pE\sin\theta$$

where, I is the moment of inertia and α is angular acceleration.

For small angles, $\sin\theta \approx \theta$

$$\therefore \quad \alpha = -\left(\frac{pE}{I}\right)\theta \qquad \text{...(i)}$$

Moment of inertia of the given system is



$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{2md^2}{4} = \frac{md^2}{2}$$

Substituting the value of I in Eq. (i), we get

$$\Rightarrow \quad \alpha = -\left(\frac{2pE}{md^2}\right)\cdot\theta \qquad \text{...(ii)}$$

The above equation is similar to the equation for a system executing angular SHM.

Comparing Eq. (ii), with the general equation of angular SHM, i.e.

$$\alpha = -\omega^2\theta$$

where, ω is the angular frequency,

we get

$$\omega^2 = \frac{2pE}{md^2} \quad \text{or} \quad \omega = \sqrt{\frac{2pE}{md^2}}$$

As, $\quad p = qd$

$$\therefore \quad \omega = \sqrt{\frac{2qdE}{md^2}} = \sqrt{\frac{2qE}{md}}$$

3. **Key Idea** As, electrostatic force is conserved in nature so, total energy of charge distribution remains constant in absence of any external interaction.

Let radius of distribution at some instant t is R. At $t = 0$, radius is given R_0

Now by conservation of energy, we have

$$0 + \frac{kQ^2}{2R_0} = \frac{1}{2}mv^2 + \frac{kQ^2}{2R}$$

($\because$ The distribution starts from rest, so, initial kinetic energy is zero.)

Differentiating this equation with respect to R, we get

$$\frac{1}{2}m2v\frac{dv}{dR} - \frac{kQ^2}{2R^2} = 0 \text{ or } \frac{dv}{dR} = \frac{kQ^2}{2mvR^2}$$

Here, $\frac{dv}{dR}$ = slope of v *versus* R graph. It decreases with increasing v and R.

Also, slope $\longrightarrow 0$ as $R \longrightarrow \infty$.

From above conclusions, we can see that the best suited graph is given in option (c).

4. In steady state no current flows through the capacitor.

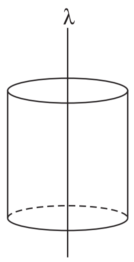
So, the current in circuit $I = \frac{E}{r + r_2}$

$\because$ Potential drop across capacitor = Potential drop across r_2

$$= Ir_2 = \frac{Er_2}{r + r_2}$$

$\therefore$ Stored charge of capacitor, $Q = CV = \frac{CEr_2}{r + r_2}$

5.



Suppose charger per unit length at any instant is λ.

Initial value of λ is suppose λ_0.

Electric field at a distance r at any instant is

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$J = \sigma E = \sigma\frac{\lambda}{2\pi\varepsilon r}$$

$$i = \frac{dq}{dt} = J(A) = -J\sigma 2\pi rl$$

$$\frac{d\lambda l}{dt} = -\frac{\lambda}{2\pi\varepsilon r} \times \sigma 2\pi rl \qquad (q = \lambda l)$$

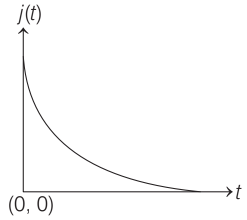
$$\int_{\lambda}^{\lambda}\frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon}\int_{a}^{t} dt$$

$$\Rightarrow \qquad \lambda = \lambda_0 e^{-\frac{\alpha}{\varepsilon}t}$$

$$J = \frac{\sigma}{2\pi\varepsilon r}\lambda = \frac{\sigma\lambda_0}{2\pi\varepsilon r}e^{-\frac{\sigma}{\varepsilon}t} = J_0 e^{-\frac{\sigma}{\varepsilon}t}$$

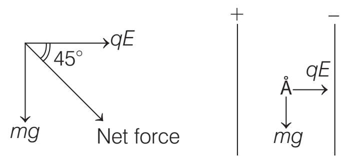
Here, $\quad J_0 = \frac{\sigma\lambda_0}{2\pi\varepsilon r}$

$\therefore$ $J(t)$ decreases exponentially as shown in figure below.



6. At the shown position, net force on both charges is zero. Hence they are in equilibrium. But equilibrium of $+q$ is stable equilibrium. So, it will start oscillations when displaced from this position. These small oscillations are simple harmonic in nature. While equilibrium of $-q$ is unstable. So, it continues to move in the direction of its displacement.

7.



Net force is at 45° from vertical.

$$\therefore \qquad qE = mg \text{ or } \frac{qX}{d} = mg \qquad \left(\because E = \frac{X}{d}\right)$$

$$\text{or} \qquad X = \frac{mgd}{q} = \frac{(1.67 \times 10^{-27})(9.8)(10^{-2})}{(1.6 \times 10^{-19})}$$

$$\approx 1 \times 10^{-9} \text{ V}$$

8. Electrical force per unit area $= \frac{1}{2}\varepsilon_0 E^2$

$$= \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{\sigma^2}{2\varepsilon_0}$$

Projected area $= \pi R^2$

$\therefore$ Net electrical force $= \left(\frac{\sigma^2}{2\varepsilon_0}\right)(\pi R^2)$

In equilibrium, this force should be equal to the applied force.

$$\therefore \qquad F = \frac{\pi\sigma^2 R^2}{2\varepsilon_0} \text{ or } F \propto \frac{\sigma^2 R^2}{\varepsilon_0}$$

$\therefore$ The correct option is (a).

9.

$$qE = mg \qquad \ldots(\text{i})$$

$$6\pi\eta rv = mg$$

$$\frac{4}{3}\pi r^3 \rho g = mg \qquad \ldots(\text{ii})$$

$$\therefore \qquad r = \left(\frac{3mg}{4\pi\rho g}\right)^{1/3} \qquad \ldots\text{(iii)}$$

Substituting the value of r in Eq. (ii) we get,

$$6\pi\eta v \left(\frac{3mg}{4\pi\,\rho g}\right)^{1/3} = mg$$

or $$(6\pi\eta v)^3\left(\frac{3mg}{4\pi\rho g}\right) = (mg)^3$$

Again substituting $mg = qE$ we get,

$$(qE)^2 = \left(\frac{3}{4\pi\,\rho g}\right)(6\pi\eta v)^3$$

or $$qE = \left(\frac{3}{4\pi\rho g}\right)^{1/2}(6\pi\eta v)^{3/2}$$

$$\therefore \qquad q = \frac{1}{E}\left(\frac{3}{4\pi\rho g}\right)^{1/2}(6\pi\eta v)^{3/2}$$

Substituting the values we get ,

$$q = \frac{7}{81\pi \times 10^5}\sqrt{\frac{3}{4\pi \times 900 \times 9.8} \times 216\,\pi^3} \times \sqrt{(1.8 \times 10^{-5} \times 2 \times 10^{-3})^3}$$

$$= 8.0 \times 10^{-19}\ \text{C}$$

10. Distance $BC = AB\sin 60° = (2R)\dfrac{\sqrt{3}}{2} = \sqrt{3}R$

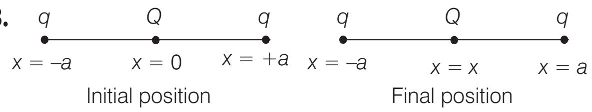
$$\therefore \qquad |Fe_{BC}| = \frac{1}{4\pi\varepsilon_0}\frac{(q/3)(2q/3)}{(\sqrt{3}R)^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}$$

11. Inside the cavity, field at any point is uniform and non-zero. Therefore, correct option is (b).

12. There will be an electric field between two cylinders (using Gauss theorem). This electric field will produce a potential difference.

$\therefore$ Correct answer is (a).

13.



$$U_i = \frac{2KQq}{a} + \frac{K.q.q.}{2a}$$

and $$U_f = KQq\left[\frac{1}{a+x} + \frac{1}{a-x}\right] + \frac{K.q.q.}{2a}$$

Here, $$K = \frac{1}{4\pi\varepsilon_0}$$

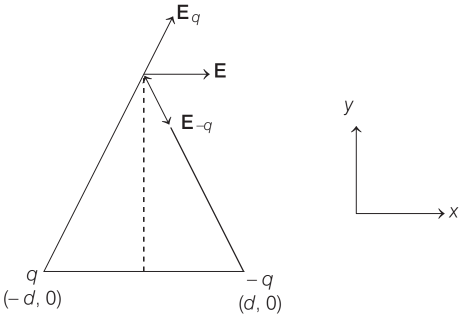
$$\Delta U = U_f - U_i$$

or $$|\Delta U| = \frac{2KQqx^2}{a^3}$$

For $\quad x << a$

$\therefore \quad \Delta U \propto x^2$

14. The diagramatic representation of the given question is shown in figure.



The electrical field **E** at all points on the x-axis will not have the same direction.

For $-d \le x \le d$, electric field is along positive x-axis while for all other points it is along negative x-axis.

The electric field **E** at all points on the y-axis will be parallel to the x-axis (i.e. $\hat{\mathbf{i}}$) [option (c)]

The electrical potential at the origin due to both the charges is zero, hence, no work is done in bringing a test charge from infinity to the origin.

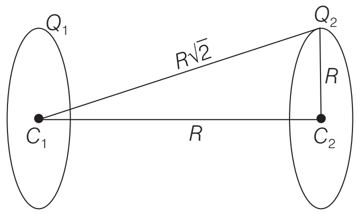
Dipole moment is directed from the $-q$ charge to the $+q$ charge (i.e. $-\hat{\mathbf{i}}$ direction).

15. $$V_{C_1} = V_{Q_1} + V_{Q_2} = \frac{1}{4\,\pi\varepsilon_0}\frac{Q_1}{R} + \frac{1}{4\pi\,\varepsilon_0}\frac{Q_2}{R\sqrt{2}}$$

$$= \frac{1}{4\pi\,\varepsilon_0 R}\left(Q_1 + \frac{Q_2}{\sqrt{2}}\right)$$

Similarly $$V_{C_2} = \frac{1}{4\pi\,\varepsilon_0 R}\left(Q_2 + \frac{Q_1}{\sqrt{2}}\right)$$

$$\therefore \quad \Delta V = V_{C_1} - V_{C_2} = \frac{1}{4\pi\varepsilon_0 R}\left[(Q_1 - Q_2) - \frac{1}{\sqrt{2}}(Q_1 - Q_2)\right]$$



$$= \frac{Q_1 - Q_2}{\sqrt{2}(4\pi\,\varepsilon_0 R)}(\sqrt{2} - 1)$$

$$W = q\Delta V = q\,(Q_1 - Q_2)\,(\sqrt{2} - 1)/\sqrt{2}(4\pi\,\varepsilon_0 R)$$

16. Balls will gain positive charge and hence move towards negative plate.

On reaching negative plate, balls will attain negative charge and come back to positive plate.

So on, balls will keep oscillating.

But oscillation is not S.H.M.,

As fore on balls is not $\propto x$.

$\Rightarrow$ (d) is correct.

17. As the balls keep on carrying charge form one plate to another, current will keep on flowing even in steady state. When at bottom plate, if all balls attain charge q,

$$\frac{kq}{r} = V_0 \qquad \left(\therefore k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\Rightarrow \qquad q = \frac{V_0 r}{k}$$

Inside cylinder, electric field $E = [V_0 - (-V_0)]h$

$$= 2V_0 h.$$

$\Rightarrow$ Acceleration of each ball,

$$a = \frac{qE}{m} = \frac{2hr}{km}\cdot V_0^2$$

$\Rightarrow$ Time taken by balls to reach other plate,

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h.km}{2hrV_0^2}} = \frac{1}{V_0}\sqrt{\frac{km}{r}}$$

If there are n balls, then

Average current, $i_{av} = \dfrac{nq}{t} = n \times \dfrac{V_0 r}{k} \times V_0\sqrt{\dfrac{r}{km}}$

$$\Rightarrow \qquad i_{av} \propto V_0^2$$

18. **List-II**

(1) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{d^2}$

$\Rightarrow E \propto \dfrac{1}{d^2}$

(2) $E_{\text{axis}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q(2l)}{d^3}$

$$\Rightarrow \qquad E \propto \frac{1}{d^3}$$

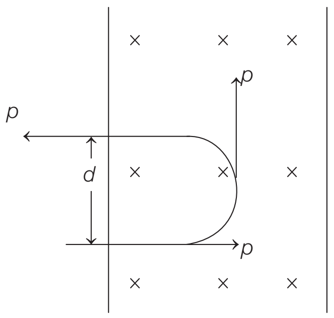
(3) $E = \dfrac{\lambda}{2\pi\varepsilon_0 d} \Rightarrow E \propto \dfrac{1}{d}$

(4) $E = \dfrac{\lambda}{2\pi\varepsilon_0(d-l)} - \dfrac{\lambda}{2\pi\varepsilon_0(d+l)} = \dfrac{\lambda(2l)}{2\pi\varepsilon_0 d^2}$

$\Rightarrow \quad E \propto \dfrac{1}{d^2}$

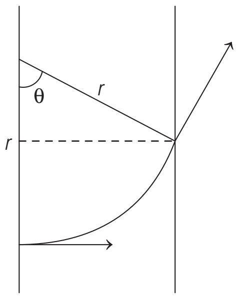
(5) $E = \dfrac{\sigma}{2\varepsilon_0} \Rightarrow$ E is independent of d

20. (a)



$|\Delta \mathbf{P}| = \sqrt{2}p$

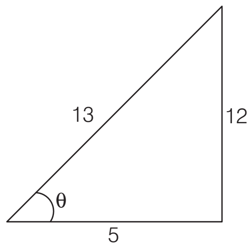
(b) $r(1-\cos\theta) = R$



$$r\sin\theta = \frac{3R}{2} \Rightarrow \frac{\sin\theta}{1-\cos\theta} = \frac{3}{2}$$

$$\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{3}{2} \Rightarrow \cot\frac{\theta}{2} = \frac{3}{2}$$

$$\Rightarrow \qquad \tan\frac{\theta}{2} = \frac{2}{3} \Rightarrow \tan\theta = \frac{2\left(\frac{2}{3}\right)}{1-\frac{4}{9}} = \frac{\frac{4}{3}}{\frac{5}{9}} = \frac{12}{5}$$



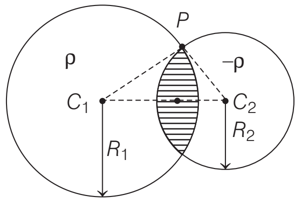
$$\sin\theta = \frac{12}{13}$$

$$r\left(\frac{12}{13}\right) = \frac{3R}{2}; \quad r = \frac{13R}{8} = \frac{P}{QB}; \quad B = \frac{8P}{13QR}$$

(c) $\dfrac{P}{QB} < \dfrac{3R}{2}, \ B > \dfrac{2P}{3QR}$

(d) $r = \dfrac{mv}{QB}, \ d = 2r = \dfrac{2mv}{QB} \Rightarrow d \propto m$

21.



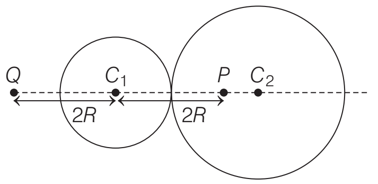
For electrostatic field,

$$\mathbf{E}_P = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\rho}{3\varepsilon_0}\mathbf{C}_1\mathbf{P} + \frac{(-\rho)}{3\varepsilon_0}\mathbf{C}_1\mathbf{P}$$

$$= \frac{\rho}{3\varepsilon_0}(\mathbf{C}_1\mathbf{P} + \mathbf{PC}_2)$$

$$\mathbf{E}_P = \frac{\rho}{3\varepsilon_0}\mathbf{C}_1\mathbf{C}_2$$

22. **At point P**



If resultant electric field is zero, then

$$\frac{KQ_1}{4R^2} = \frac{KQ_2}{8R^3}R \Rightarrow \frac{\rho_1}{\rho_2} = 4$$

At point Q

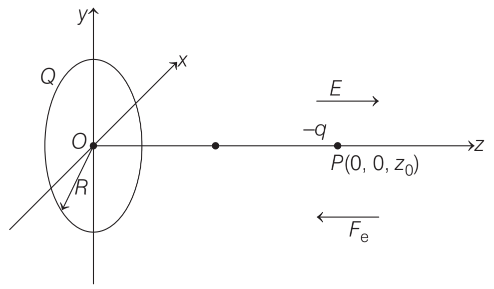
If resultant electric field is zero then

$$\frac{KQ_1}{4R^2} + \frac{KQ_2}{25R^2} = 0$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25} \qquad (\rho_1 \text{ must be negative})$$

23. Net torque on $(-q)$ about a point (say P) lying over $+Q$ is zero. Therefore, angular momentum of $(-q)$ about point P should remain constant.

24. Let Q be the charge on the ring, the negative charge $-q$ is released from point P $(0, 0, z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz_0}{(R^2+z_0^2)^{3/2}}$$

$E = 0$ at centre of the ring because $z_0 = 0$

Force on charge at P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(R^2+z_0^2)^{3/2}}\cdot z_0 \qquad \text{...(i)}$$

Similarly, when it crosses the origin, the force is again towards centre O.

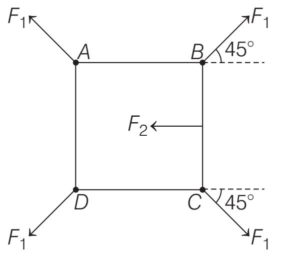
Thus, the motion of the particle is periodic for all values of z_0 lying between 0 and ∞.

Secondly, if $z_0 << R$, $(R^2 + z_0^2)^{3/2} \approx R^3$

$$F_e \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}\cdot z_0 \qquad \text{[From Eq. (i)]}$$

i.e. the restoring force $F_e \propto -z_0$. Hence, the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position.)

25.



F_1 = Net electrostatic force on any one charge due to rest of three charges

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

F_2 = Surface tension force = γa

If we see the equilibrium of line BC, then

$$2F_1 \cos 45° = F_2$$

or $$\sqrt{2}F_1 = F_2$$

or $$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(2+\frac{1}{\sqrt{2}}\right) = \gamma a$$

$\therefore$ $$a^3 = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{\gamma}$$

or $$a = \left\{\frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\right\}^{1/3}\left[\frac{q^2}{\gamma}\right]^{1/3} = k\left[\frac{q^2}{\gamma}\right]^{1/3}$$

where, $k = \left\{\frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\right\}^{1/3}$

Therefore, $N = 3$

Answer is 3.

26. When X-rays fall on the metal ball, some electrons emit from it due to photoelectric effect. The ball thus gets positively charged and on a positively charged ball an electrostatic force in the direction of electric field acts. Due to this force ball gets deflected in the direction of electric field.

27. Mass of negatively charged sphere will be slightly more than the mass of positively charged sphere because some electrons will be given to the negatively charged sphere while some electrons will be taken out from the positively charged sphere.

28. Let q be the charge on the bubble, then

$$V = \frac{Kq}{a} \qquad \left(\text{Here, } K = \frac{1}{4\pi\varepsilon_0}\right)$$

$\therefore$ $$q = \frac{Va}{K}$$

Let after collapsing, the radius of droplet becomes R, then equating the volume, we have

$$(4\pi a^2)\,t = \frac{4}{3}\pi R^3$$

$\therefore \qquad R = (3a^2t)^{1/3}$

Now, potential of droplet will be $V' = \dfrac{Kq}{R}$

Substituting the values, we have

$$V' = \frac{(K)\left(\frac{Va}{K}\right)}{(3a^2t)^{1/3}} \quad \text{or} \quad V' = V\left(\frac{a}{3t}\right)^{1/3}$$

29. Electric field near a large metallic plate is given by $E = \sigma/\varepsilon_0$. In between the plates the two fields will be in opposite directions. Hence,

$$E_{\text{net}} = \frac{\sigma_1 - \sigma_2}{\varepsilon_0} = E_0 \text{ (say)}$$

Now, $W = (q)$(potential difference)

$$= q\,(E_0 a \cos 45°) = (q)\left(\frac{\sigma_1 - \sigma_2}{\varepsilon_0}\right)\left(\frac{a}{\sqrt{2}}\right)$$

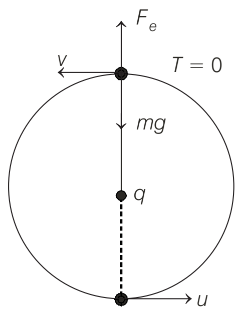
$$= \frac{(\sigma_1 - \sigma_2)\,qa}{\sqrt{2}\varepsilon_0}$$

30. Given : $q = 1\mu\text{C} = 10^{-6}\,\text{C}$

$m = 2 \times 10^{-3}$ kg

and $\quad l = 0.8$m

Let u be the speed of the particle at its lowest point and v its speed at highest point.



At highest point three forces are acting on the particle.

(a) Electrostatic repulsion

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2} \qquad \text{(outwards)}$$

(b) Weight $w = mg$ (inwards)

(c) Tension T (inwards)

$T = 0$, if the particle has just to complete the circle and the necessary centripetal force is provided by $w - F_e$ i.e.

$$\frac{mv^2}{l} = w - F_e$$

or

$$v^2 = \frac{l}{m}\left(mg - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\right)$$

$$v^2 = \frac{0.8}{2\times10^3}\left(2\times10^{-3}\times10 - \frac{9.0\times10^9\times(10^{-6})^2}{(0.8)^2}\right)\text{m}^2/\text{s}^2$$

or $\qquad v^2 = 2.4\,\text{m}^2/\text{s}^2 \qquad \ldots\text{(i)}$

Now, the electrostatic potential energy at the lowest and highest points are equal. Hence, from conservation of mechanical energy.

Increase in gravitational potential energy

= Decrease in kinetic energy.

or $\qquad mg\,(2l) = \dfrac{1}{2}m\,(u^2 - v^2)$

or $\qquad u^2 = v^2 + 4gl$

Substituting the values of v^2 from Eq. (i), we get

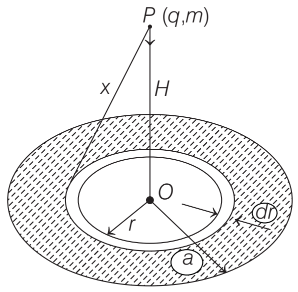
$$u^2 = 2.4 + 4\,(10)\,(0.8) = 34.4\ \text{m}^2/\text{s}^2$$

$\therefore \qquad u = 5.86$ m/s

Therefore, minimum horizontal velocity imparted to the lower ball, so that it can make complete revolution, is 5.86 m/s.

31. Potential at a height *H* on the axis of the disc *V* (*P*)

The charge dq contained in the ring shown in figure



$$dq = (2\pi r\,dr)\,\sigma$$

Potential at P due to this ring

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{x} \text{ where } x = \sqrt{H^2 + r^2}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2\pi r dr)\sigma}{\sqrt{H^2 + r^2}} = \frac{\sigma}{2\varepsilon_0}\frac{rdr}{\sqrt{H^2 + r^2}}$$

$\therefore$ Potential due to the complete disc

$$V_p = \int_{r=0}^{r=a} dV$$

$$= \frac{\sigma}{2\varepsilon_0}\int_{r=0}^{r=a}\frac{rdr}{\sqrt{H^2 + r^2}}$$

$$V_p = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2 + H^2} - H]$$

Potential at centre, (O) will be

$$V_O = \frac{\sigma a}{2\varepsilon_0} \qquad H = 0$$

(a) Particle is released from P and it just reaches point O. Therefore, from conservation of mechanical energy

Decrease in gravitational potential energy = Increase in electrostatic potential energy

($\Delta\text{KE} = 0$ because $K_i = K_f = 0$)

$\therefore\ mgH = q[V_O - V_p]$

or $gH = \left(\dfrac{q}{m}\right)\left(\dfrac{\sigma}{2\varepsilon_0}\right)[a - \sqrt{a^2 + H^2} + H]$ ...(i)

$$\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$$

$$\therefore \qquad \frac{q\sigma}{2\varepsilon_0 m} = 2g$$

Substituting in Eq. (i), we get

$$gH = 2g\,[a + H - \sqrt{a^2 + H^2}\,]$$

or $\qquad \dfrac{H}{2} = (a + H) - \sqrt{a^2 + H^2}$

or $\qquad \sqrt{a^2 + H^2} = a + \dfrac{H}{2}$

or $\qquad a^2 + H^2 = a^2 + \dfrac{H^2}{4} + aH$

or $\qquad \dfrac{3}{4}H^2 = aH$

or $\qquad H = \dfrac{4}{3}a$

and $\qquad H = 0$

$\therefore \qquad H = (4/3)a$

(b) Potential energy of the particle at height H = Electrostatic potential energy + gravitational potential energy

$\therefore \qquad U = qV + mgH$

Here V = Potential at height H

$$U = \frac{\sigma q}{2\varepsilon_0}[\sqrt{a^2 + H^2} - H] + mgH \qquad \text{...(ii)}$$

At equilibrium position $F = \dfrac{-\,dU}{dH} = 0$

Differentiating Eq. (ii) w.r.t. H

or $\ mg + \sigma\dfrac{q}{2\,\varepsilon_0}\left[\left(\dfrac{1}{2}\right)(2H)\dfrac{1}{\sqrt{a^2 + H^2}} - 1\right] = 0$

$$\left(\because \frac{\sigma q}{2\varepsilon_0} = 2mg\right)$$

$\therefore \qquad mg + 2mg\left[\dfrac{H}{\sqrt{a^2 + H^2}} - 1\right] = 0$

or $\qquad 1 + \dfrac{2H}{\sqrt{a^2 + H^2}} - 2 = 0$

or $\qquad \dfrac{2H}{\sqrt{a^2 + H^2}} = 1$ or $\dfrac{H^2}{a^2 + H^2} = \dfrac{1}{4}$

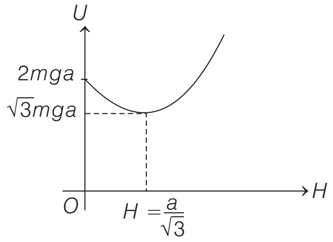
or $\qquad 3H^2 = a^2$ or $H = \dfrac{a}{\sqrt{3}}$

From Eq. (ii), we can write

$U - H$ equation as

$U = mg(2\sqrt{a^2 + H^2} - H)$ (Parabolic variation)

$U = 2mga$ at $H = 0$



and $U = U_{\min} = \sqrt{3}mga$ at $H = \dfrac{a}{\sqrt{3}}$

Therefore, $U - H$ graph will be as shown

Note that at $H = \dfrac{a}{\sqrt{3}}$, U is minimum.

Therefore, $\quad H = \dfrac{a}{\sqrt{3}}$ is stable equilibrium position.

32. Capacities of conducting spheres are in the ratio of their radii. Let C_1 and C_2 be the capacities of S_1 and S_2, then

$$\frac{C_2}{C_1} = \frac{R}{r}$$

(a) Charges are distributed in the ratio of their capacities. Let in the first contact, charge acquired by S_2 is q_1. Therefore, charge on S_1 will be $Q - q_1$. Say it is q_1'

$$\therefore \qquad \frac{q_1}{q_1'} = \frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r}$$

$$\therefore \qquad q_1 = Q\left(\frac{R}{R + r}\right) \qquad \text{...(i)}$$

In the second contact, S_1 again acquires the same charge Q.

Therefore, total charge in S_1 and S_2 will be

$$Q + q_1 = Q\left(1 + \frac{R}{R + r}\right)$$

This charge is again distributed in the same ratio. Therefore, charge on S_2 in second contact,

$$q_2 = Q\left(1 + \frac{R}{R + r}\right)\left(\frac{R}{R + r}\right)$$

$$= Q\left[\frac{R}{R + r} + \left(\frac{R}{R + r}\right)^2\right]$$

Similarly, $q_3 = Q\left[\dfrac{R}{R + r} + \left(\dfrac{R}{R + r}\right)^2 + \left(\dfrac{R}{R + r}\right)^3\right]$

and $\qquad q_n = Q\left[\dfrac{R}{R + r} + \left(\dfrac{R}{R + r}\right)^2 + \ldots + \left(\dfrac{R}{R + r}\right)^n\right]$

or $$q_n = Q\frac{R}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\left[S_n=\frac{a\,(1-r^n)}{(1-r)}\right] \quad \text{...(ii)}$$

Therefore, electrostatic energy of S_2 after n such contacts

$$= \frac{q_n^2}{2\,(4\pi\varepsilon_0 R)} \quad \text{or } U_n = \frac{q_n^2}{8\pi\varepsilon_0 R}$$

where, q_n can be written from Eq. (ii).

(b) As $n \to \infty \quad q_\infty = Q\frac{R}{r}$

$$\therefore\ U_\infty = \frac{q_\infty^2}{2C} = \frac{Q^2R^2/r^2}{8\pi\varepsilon_0 R}$$

or $$U_\infty = \frac{Q^2R}{8\,\pi\varepsilon_0 r^2}$$

33. Let q_1 and q_2 be the charges on the two spheres before connecting them.

Then, $q_1 = \sigma(4\pi R^2)$, and $q_2 = \sigma(4\pi)\,(2R)^2 = 16\sigma\pi R^2$

Therefore, total charge (q) on both the spheres is

$$q = q_1 + q_2 = 20\,\sigma\pi R^2$$

Now, after connecting, the charge is distributed in the ratio of their capacities, which in turn depends on the ratio of their radii ($C = 4\pi\varepsilon_0 R$).

$$\therefore \quad \frac{q_1'}{q_2'} = \frac{R}{2R} = \frac{1}{2}$$

$$\therefore \quad q_1' = \frac{q}{3} = \frac{20}{3}\sigma\pi R^2$$

and $$q_2' = \frac{2q}{3} = \frac{40}{3}\sigma\pi R^2$$

Therefore, surface charge densities on the spheres are

$$\sigma_1 = \frac{q_1'}{4\pi R^2} = \frac{(20/3)\,\sigma\pi R^2}{4\pi R^2} = \frac{5}{3}\sigma$$

and $$\sigma_2 = \frac{q_2'}{4\pi(2R)^2} = \frac{(40/3)\sigma\pi R^2}{16\pi R^2} = \frac{5}{6}\sigma$$

Hence, surface charge density on the bigger sphere is σ_2 i.e. $(5/6)\sigma$.

34. Total charge in the ring is $Q = (2\pi R)\,\lambda$

Potential due to a ring at a distance of x from its centre on its axis is given by $V(x) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{\sqrt{R^2+x^2}}$

and at the centre is $V_{\text{centre}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

Using the above formula

$$V_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi R\lambda}{\sqrt{R^2+3R^2}} = \frac{\lambda}{4\varepsilon_0}$$

$$V_o = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\,\pi R\lambda}{R} = \frac{\lambda}{2\varepsilon_0}$$

$$V_o > V_p$$

PD between points O and P is

$$V = V_o - V_p = \frac{\lambda}{2\varepsilon_0} - \frac{\lambda}{4\varepsilon_0} = \frac{\lambda}{4\varepsilon_0}$$

$$\therefore \quad \frac{1}{2}mv^2 \ge qV \text{ or } v \ge \sqrt{\frac{2qV}{m}}$$

or $$v \ge \sqrt{\frac{2q\lambda}{4\varepsilon_0 m}}$$

or $$v \ge \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$$

Therefore, minimum value of speed v should be

$$v_{\min} = \sqrt{\frac{q\lambda}{2\varepsilon_0 m}}$$

35. (a) In this case the electric field exists from centre of the sphere to infinity. Potential energy is stored in electric field, with energy density

$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad (\because \text{Energy/Volume})$$

(i) Energy stored within the sphere (U_1)

Electric field at a distance r is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}\cdot r$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}r\right\}^2$$

Volume of element,

$$dV = (4\pi r^2)\,dr$$

Energy stored in this volume, $dU = u\,(dV)$

$$dU = (4\pi r^2 dr)\frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^3}r\right\}^2$$

$$dU = \frac{1}{8\pi\varepsilon_0}\cdot\frac{Q^2}{R^6}\cdot r^4 dr$$

$$\therefore \quad U_1 = \int_0^R dU = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{R^6}\int_0^R r^4 dr$$

$$= \frac{Q^2}{40\pi\varepsilon_0 R^6}[r^5]_0^R$$

$$U_1 = \frac{1}{40\pi\varepsilon_0}\cdot\frac{Q^2}{R} \quad \text{...(i)}$$

(ii) Energy stored outside the sphere (U_2)

Electric field at a distance r is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$$

$$\therefore \quad u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\right\}^2$$

$$dU = (4\pi r^2 dr)$$

$$\therefore \quad dU = u \cdot dV$$

$$= (4\pi r^2 dr)\left[\frac{\varepsilon_0}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\right)^2\right]$$

$$dU = \frac{Q^2}{8\pi\varepsilon_0}\frac{dr}{r^2}$$

$$\therefore \quad U_2 = \int_R^\infty dU = \frac{Q^2}{8\pi\varepsilon_0}\cdot\int_R^\infty\frac{dr}{r^2}$$

$$U_2 = \frac{Q^2}{8\pi\varepsilon_0 R} \qquad \ldots\text{(ii)}$$

Therefore, total energy of the system is

$$U = U_1 + U_2 = \frac{Q^2}{40\pi\varepsilon_0 R} + \frac{Q^2}{8\pi\varepsilon_0 R}$$

or $$U = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 R}$$

(b) Comparing this with gravitational forces, the gravitational potential energy of earth will be

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

by replacing Q^2 by M^2 and $\frac{1}{4\pi\varepsilon_0}$ by G

$$g = \frac{GM}{R^2}$$

$$\therefore \quad G = \frac{gR^2}{M}$$

$$U = \frac{-3}{5}MgR$$

Therefore, energy needed to completely disassemble the earth against gravitational pull amongst its constituent particles will be given by

$$E = |U| = \frac{3}{5}MgR$$

Substituting the values, we get

$$E = \frac{3}{5}(10\text{m/s}^2)(2.5\times 10^{31}\text{ kg -m})$$

$$E = 1.5\times 10^{32}\text{ J}$$

(c) This is the case of a charged spherical conductor of radius R, energy of which is given by $= \frac{1}{2}\frac{Q^2}{C}$

or $$U = \frac{1}{2}.\frac{Q^2}{4\pi\varepsilon_0 R} \text{ or } U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

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http://tinyurl.com/y4er64a8 *or*

15

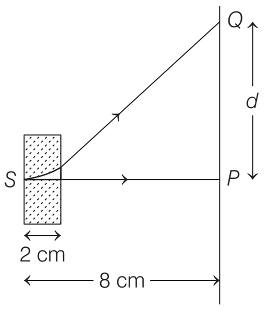
Magnetics

Topic 1 Magnetic Force and Path of Charged Particle in Uniform Fields

Objective Questions I (Only one correct option)

1. An electron moving along the X-axis with an initial energy of 100 eV, enters a region of magnetic field $\mathbf{B} = (1.5 \times 10^{-3}\,\text{T})\,\hat{\mathbf{k}}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is

(Take, electron's charge $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg) **(2019 Main, 12 April II)**



(a) 11.65 cm (b) 12.87 cm
(c) 1.22 cm (d) 2.25 cm

2. A proton, an electron and a helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane.

Let r_p, r_e and r_{He} be their respective radii, then **(2019 Main, 10 April I)**

(a) $r_e < r_p = r_{He}$ (b) $r_e > r_p = r_{He}$
(c) $r_e < r_p < r_{He}$ (d) $r_e > r_p > r_{He}$

3. A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field B is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be **(2019 Main, 12 Jan I)**

(a) $1 : \sqrt{2}$ (b) $1 : \sqrt{3}$ (c) $1 : 3$ (d) $1 : 2$

4. A particle of mass m and charge q is in an electric and magnetic field is given by

$$\mathbf{E} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}},\ \mathbf{B} = 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}.$$

The charged particle is shifted from the origin to the point $P(x = 1;\ y = 1)$ along a straight path. The magnitude of the total work done is **(Main 2019, 11 Jan II)**

(a) $(0.35)\,q$ (b) $(0.15)\,q$ (c) $(2.5)\,q$ (d) $5\,q$

5. In an experiment, electrons are accelerated, from rest by applying a voltage of 500 V. Calculate the radius of the path, if a magnetic field 100 mT is then applied.
(Take, charge of the electron $= 1.6 \times 10^{-19}$ C and mass of the electron $= 9.1 \times 10^{-31}$ kg) **(2019 Main, 11 Jan I)**

(a) 7.5×10^{-2} m (b) 7.5×10^{-4} m
(c) 7.5×10^{-3} m (d) 7.5 m

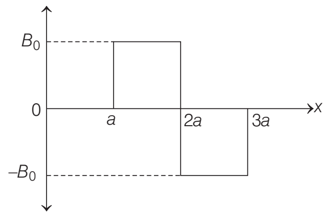
6. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Take, charge of electron $= 1.6 \times 10^{-19}$ C) **(2019 Main, 9 Jan II)**

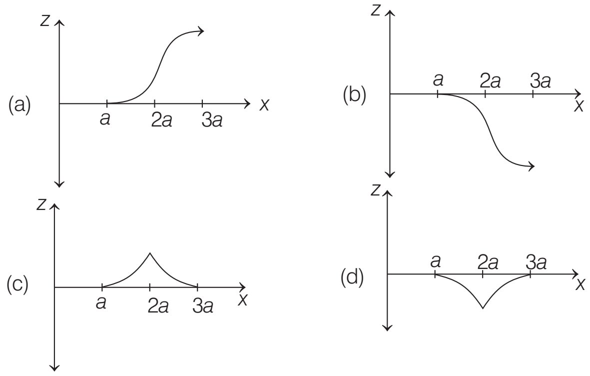
(a) 1.6×10^{-19} kg (b) 1.6×10^{-27} kg
(c) 9.1×10^{-31} kg (d) 2.0×10^{-24} kg

7. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively, in a uniform magnetic field B. The relation between r_e, r_p, r_α is **(2018 Main)**

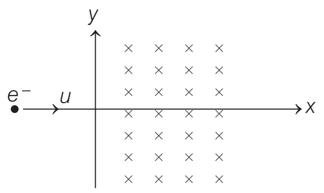
(a) $r_e < r_\alpha < r_p$ (b) $r_\alpha > r_e = r_p$
(c) $r_e < r_p = r_\alpha$ (d) $r_e < r_p < r_\alpha$

8. A magnetic field $\mathbf{B} = B_0\hat{\mathbf{j}}$ exists in the region $a < x < 2a$ and $\mathbf{B} = -B_0\hat{\mathbf{j}}$, in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$, where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like **(2007, 3M)**



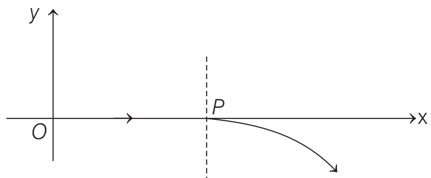


9. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field $\mathbf{B} = -B_0\hat{\mathbf{k}}$ which exists to the right of y-axis. The electron exits from the region after sometime with the speed v at coordinate y, then **(2004, 2M)**



(a) $v > u, y < 0$ (b) $v = u, y > 0$
(c) $v > u, y > 0$ (d) $v = u, y < 0$

10. For a positively charged particle moving in a x-y plane initially along the x-axis, there is a sudden change in its path due to the presence of electric and/or magnetic fields beyond P. The curved path is shown in the x - y plane and is found to be non-circular. **(2003, 2M)**



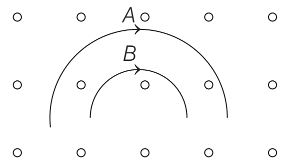
Which one of the following combinations is possible?

(a) $\mathbf{E} = 0; \mathbf{B} = b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ (b) $\mathbf{E} = a\hat{\mathbf{i}}; \mathbf{B} = c\hat{\mathbf{k}} + a\hat{\mathbf{i}}$
(c) $\mathbf{E} = 0; \mathbf{B} = c\hat{\mathbf{j}} + b\hat{\mathbf{k}}$ (d) $\mathbf{E} = a\hat{\mathbf{i}}; \mathbf{B} = c\hat{\mathbf{k}} + b\hat{\mathbf{j}}$

11. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is **(2002, 2M)**

(a) $\dfrac{qbB}{m}$ (b) $\dfrac{q(b-a)B}{m}$ (c) $\dfrac{qaB}{m}$ (d) $\dfrac{q(b+a)B}{2m}$

12. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B, respectively and the trajectories are as shown in the figure. Then **(2001, 2M)**



(a) $m_A v_A < m_B v_B$ (b) $m_A v_A > m_B v_B$
(c) $m_A < m_B$ and $v_A < v_B$ (d) $m_A = m_B$ and $v_A = v_B$

13. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$-direction and a magnetic field along the $+z$-direction, then **(2000, 2M)**

(a) positive ions deflect towards $+y$-direction and negative ions towards $-y$-direction
(b) all ions deflect towards $+y$-direction
(c) all ions deflect towards $-y$-direction
(d) positive ions deflect towards $-y$-direction and negative ions towards $-y$-direction

14. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a **(1999, 2M)**

(a) straight line (b) circle
(c) helix (d) cycloid

15. A proton, a deutron and an α-particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote, respectively the radii of the trajectories of these particles, then **(1997, 1M)**

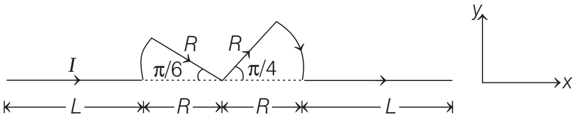
(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

16. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is **(1988, 2M)**

(a) $(R_1/R_2)^{1/2}$ (b) R_2/R_1
(c) $(R_1/R_2)^2$ (d) R_1/R_2

Objective Questions II (One or more correct option)

17. A conductor (shown in the figure) carrying constant current I is kept in the x- y plane in a uniform magnetic field $\mathbf{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statements is/are **(2015 Adv.)**



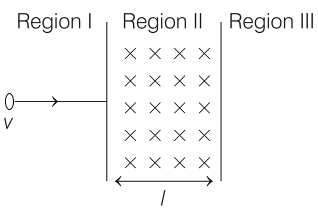
(a) if **B** is along $\hat{\mathbf{z}}$, $F \propto (L+R)$
(b) if **B** is along $\hat{\mathbf{x}}$, $F = 0$
(c) if **B** is along $\hat{\mathbf{y}}$, $F \propto (L+R)$
(d) if **B** is along $\hat{\mathbf{z}}$, $F = 0$

18. A particle of mass M and positive charge Q, moving with a constant velocity $\mathbf{u}_1 = 4\hat{\mathbf{i}}\ \text{ms}^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from $x=0$ to $x=L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\mathbf{u}_2 = 2(\sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-1}$. The correct statement(s) is (are)
(a) the direction of the magnetic field is $-z$ direction.
(b) the direction of the magnetic field is $+z$ direction
(c) the magnitude of the magnetic field is $\dfrac{50\pi M}{3Q}$ units.
(d) the magnitude of the magnetic field is $\dfrac{100\pi M}{3Q}$ units.
(2013 Adv.)

19. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\mathbf{E} = E_0\hat{\mathbf{j}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$. At time $t=0$, this charge has velocity $\mathbf{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t>0$? **(2012)**
(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.
(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis
(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.
(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

20. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? **(2011)**
(a) They will never come out of the magnetic field region
(b) They will come out travelling along parallel paths
(c) They will come out at the same time
(d) They will come out at different times

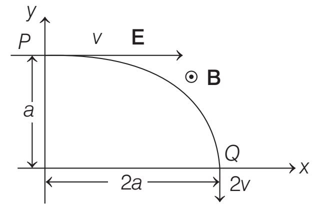
21. A particle of mass m and charge q, moving with velocity v enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is l. Choose the correct choice (s). **(2008, 4M)**



(a) The particle enters Region III only if its velocity $v > \dfrac{qlB}{m}$
(b) The particle enters Region III only if its velocity $v < \dfrac{qlB}{m}$
(c) Path length of the particle in Region II is maximum when velocity $v = qlB/m$
(d) Time spent in Region II is same for any velocity v as long as the particle returns to Region I.

22. H^+, He^+ and O^{2+} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1 amu, 4 amu and 16 amu respectively. Then **(1994, 2M)**
(a) H^+ will be deflected most
(b) O^{2+} will be deflected most
(c) He^+ and O^{2+} will be deflected equally
(d) all will be deflected equally

23. A particle of charge $+q$ and mass m moving under the influence of a uniform electric field $E\hat{\mathbf{i}}$ and uniform magnetic field $B\hat{\mathbf{k}}$ follows a trajectory from P to Q as shown in figure. The velocities at P and Q are $v\hat{\mathbf{i}}$ and $-2\hat{\mathbf{j}}$. Which of the following statement(s) is/are correct ? **(1991, 2M)**



(a) $E = \dfrac{3}{4}\left[\dfrac{mv^2}{qa}\right]$
(b) Rate of work done by the electric field at P is $\dfrac{3}{4}\left[\dfrac{mv^3}{a}\right]$
(c) Rate of work done by the electric field at P is zero
(d) Rate of work done by both the fields at Q is zero

24. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively. Then, this region of space may have **(1985, 2M)**
(a) $E = 0, B = 0$
(b) $E = 0, B \neq 0$
(c) $E \neq 0, B = 0$
(d) $E \neq 0, B \neq 0$

Match the Columns

Directions (Q.Nos. 25-27) Matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin ($x=0$, $y=0$, $z=0$) with a given initial velocity $\mathbf{v}$. A uniform electric field $\mathbf{E}$ and a uniform magnetic field $\mathbf{B}$ exist everywhere. The velocity $\mathbf{v}$, electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1	Column 2	Column 3
(I) Electron with $v = 2\frac{E_0}{B_0}\hat{x}$	(i) $E = E_0\hat{z}$	(P) $B = -B_0\hat{x}$
(II) Election with $v = \frac{E_0}{B_0}\hat{y}$	(ii) $E = -E_0\hat{y}$	(Q) $B = B_0\hat{x}$
(III) Proton with $v = 0$	(iii) $E = -E_0\hat{x}$	(R) $B = B_0\hat{y}$
(IV) Proton with $v = 2\frac{E_0}{B_0}\hat{x}$	(iv) $E = E_0\hat{x}$	(S) $B = B_0\hat{z}$

(2017 Adv.)

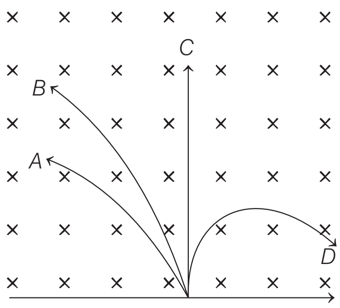
25. In which case would the particle move in a straight line along the negative direction of Y-axis (i.e. move along $-\hat{y}$)?
(a) (IV) (ii) (S) (b) (II) (iii) (Q)
(c) (III), (ii) (R (d) (III) (ii) (P)

26. In which case will the particle move in a straight line with constant velocity?
(a) (II) (iii) (S) (b) (III) (iii) (P)
(c) (IV) (i) (S) (d) (III) (ii) (R)

27. In which case will the particle describe a helical path with axis along the positive z-direction?
(a) (II) (ii) (R) (b) (III) (iii) (P)
(c) (IV) (i) (S) (d) (IV) (ii) (R)

Fill in the Blanks

28. A neutron, a proton, an electron and an alpha particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inward normal to the plane of the paper. The tracks of the particles are labelled in figure. The electron follows track...... and the alpha particle follows track...... **(1984, 2M)**



True/False

29. An electron and a proton are moving with the same kinetic energy along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular path of the same radius. **(1985, 3M)**

30. A charged particle enters a region of uniform magnetic field at an angle of 85° to the magnetic line of force. The path of the particle is a circle. **(1983, 2M)**

31. There is no change in the energy of a charged particle moving in magnetic field although a magnetic force is acting on it. **(1983, 2M)**

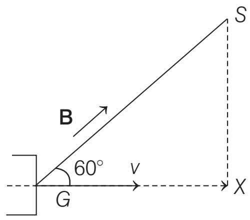
Analytical & Descriptive Questions

32. A proton and an alpha particle, after being accelerated through same potential difference, enter uniform magnetic field, the direction of which is perpendicular to their velocities. Find the ratio of radii of the circular paths of the two particles. **(2004, 2M)**

33. The region between $x = 0$ and $x = L$ is filled with uniform steady magnetic field $-B_0\hat{\mathbf{k}}$. A particle of mass m, positive charge q and velocity $v_0\hat{\mathbf{i}}$ travels along x-axis and enters the region of the magnetic field. **(1999, 10M)**
Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.
(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now expands upto 2.1 L.

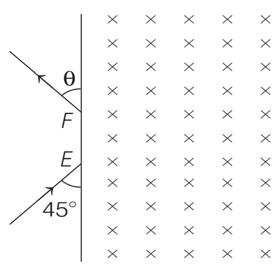
34. An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S where $GS = 0.1$ m, and the line GS make an angle of 60° with the x-axis as shown in figure. A uniform magnetic field **B** parallel to GS exists in the region outside the electron gun.



Find the minimum value of B needed to make the electrons hit S. **(1993, 7M)**

35. A beam of protons with a velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the protons beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation). **(1986, 6M)**

36. A particle of mass $m = 1.6 \times 10^{-27}$ kg and charge $q = 1.6 \times 10^{-19}$C enters a region of uniform magnetic field of strength 1 T along the direction shown in figure. The speed of the particle is 10^7m/s. **(1984, 8M)**

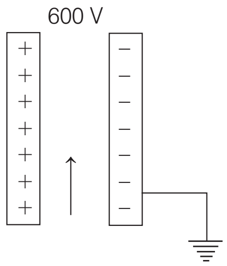


(a) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.

(b) If the direction of the field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.

37. A particle of mass 1×10^{-26} kg and charge $+1.6 \times 10^{-19}$ C travelling with a velocity 1.28×10^{6} m/s in the $+x$ direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0, E_z = -102.4$ kV/m and $B_x = B_z = 0$, $B_y = 8 \times 10^{-2}$ T. The particle enters this region at the origin at time $t = 0$. Determine the location (x, y and z coordinates) of the particle at $t = 5 \times 10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t = 7.45 \times 10^{-6}$ s? **(1982, 7M)**

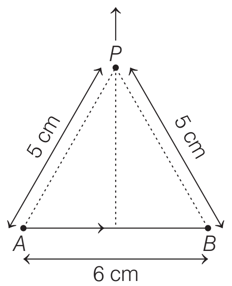
38. A potential difference of 600 V is applied across the plates of a parallel plate condenser. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^{6} m/s moves undeflected between the plates. Find the magnitude and direction of the magnetic field in the region between the condenser plates. (Neglect the edge effects).
(Charge of the electron $= 1.6 \times 10^{-19}$ C) **(1981, 3M)**



Topic 2 Biot-Savart Law and Ampere's Circuital Law

Objective Questions I (Only one correct option)

1. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A (See figure). (Take, $\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2}) **(2019 Main, 12 April II)**



(a) 2.0×10^{-5} T (b) 1.5×10^{-5} T
(c) 3.0×10^{-5} T (d) 2.5×10^{-5} T

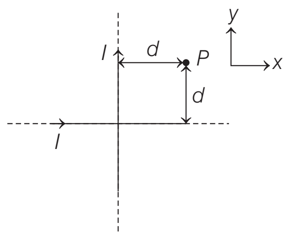
2. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2). **(2019 Main, 12 April I)**

(a) 2×10^{-6} C (b) 3×10^{-5} C
(c) 4×10^{-5} C (d) 7×10^{-6} C

3. The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is
[Take, $\mu_0 = 4\pi \times 10^{-7}$NA^{-2}] **(2019 Main, 10 April II)**

(a) 9 μT (b) 1 μT
(c) 3 μT (d) 18 μT

4. Two very long, straight and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure

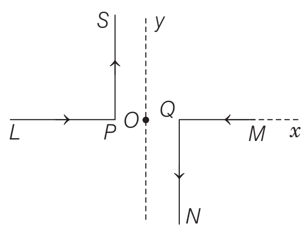


These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be **(2019 Main, 8 April II)**

(a) zero (b) $\frac{+\mu_0 I}{\pi d}(\hat{\mathbf{z}})$

(c) $-\frac{\mu_0 I}{2\pi d}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$ (d) $\frac{\mu_0 I}{2\pi d}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$

5. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the X-axis, while segments PS and QN are parallel to the Y-axis. If $OP = OQ = 4$ cm and the magnitude of the magnetic field at O is 10^{-4} T and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be (Take, $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}) **(2019 Main, 12 Jan I)**

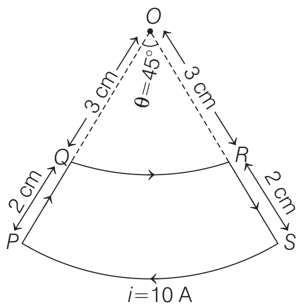


(a) 40 A, perpendicular out of the page
(b) 20 A, perpendicular into the page
(c) 20 A, perpendicular out of the page
(d) 40 A, perpendicular into the page

6. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the centre of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be **(2019 Main, 9 Jan II)**

(a) $\frac{1}{N}$ (b) N (c) $\frac{1}{N^2}$ (d) N^2

7. A current loop, having two circular arcs joined by two radial lines as shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to **(2019 Main, 9 Jan I)**

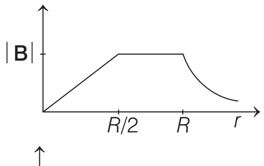


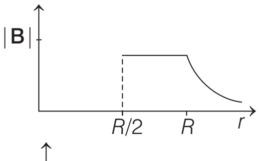
(a) 1.0×10^{-5} T (b) 1.0×10^{-7} T
(c) 1.5×10^{-7} T (d) 1.5×10^{-5} T

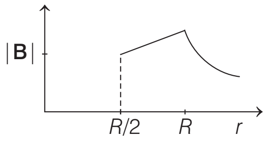
8. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is **(2016 Main)**

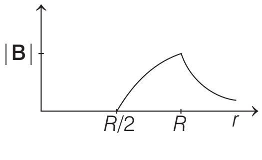
(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

9. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\mathbf{B}|$ as a function of the radial distance r from the axis is best represented by **(2012)**

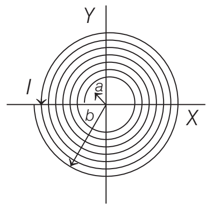
(a)


(b)


(c)


(d)


10. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X-Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the centre of the spiral is **(2011)**



(a) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$ (b) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$
(c) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b+a}{b-a}\right)$

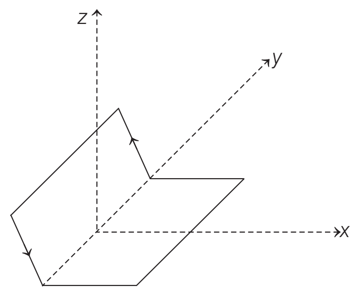
11. A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic vector field $\mathbf{B}$ at a point having coordinate (x, y) on the $z = 0$ plane is **(2002, 2M)**

(a) $\frac{\mu_0 I(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})}{2\pi(x^2+y^2)}$ (b) $\frac{\mu_0 I(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})}{2\pi(x^2+y^2)}$
(c) $\frac{\mu_0 I(x\hat{\mathbf{j}} - y\hat{\mathbf{i}})}{2\pi(x^2+y^2)}$ (d) $\frac{\mu_0 I(x\hat{\mathbf{i}} - y\hat{\mathbf{j}})}{2\pi(x^2+y^2)}$

12. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is **(2001, 2M)**

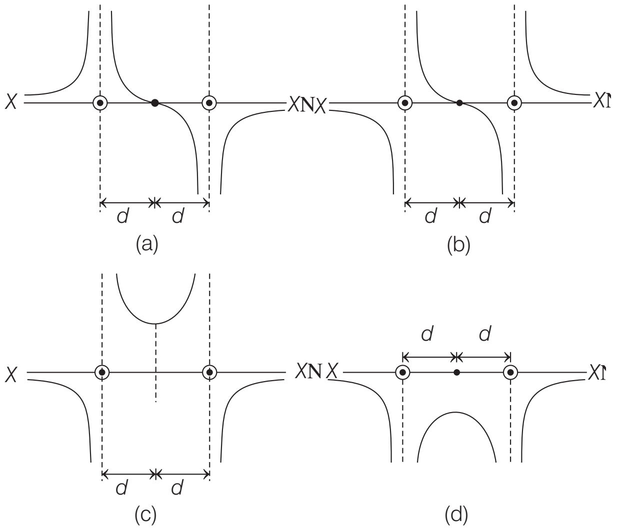
(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)} \ln\frac{b}{a}$ (d) $\frac{\mu_0 I^N}{2(b-a)} \ln\frac{b}{a}$

13. A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point $P\ (a, 0, a)$ points in the direction **(2001, 2M)**

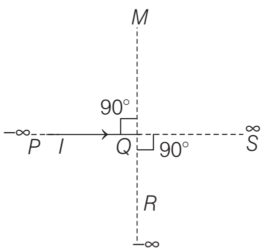


(a) $\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$ (b) $\frac{1}{\sqrt{3}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}} + \hat{\mathbf{i}})$
(c) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ (d) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$

14. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by **(2000, 2M)**



15. An infinitely long conductor PQR is bent to form a right angle as shown in figure. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q, so that current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by **(2000, 2M)**



(a) 1/2 (b) 1 (c) 2/3 (d) 2

16. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is **(1995, 2M)**

(a) proportional to $(180° - \theta)$ (b) inversely proportional to r

(c) zero, only if $(\theta = 180°)$ (d) zero for all values of θ

17. A current I flows along the length of an infinitely long, straight, thin-walled pipe. Then **(1993, 2M)**

(a) the magnetic field at all points inside the pipe is the same, but not zero

(b) the magnetic field at any point inside the pipe is zero

(c) the magnetic field is zero only on the axis of the pipe

(d) the magnetic field is different at different points inside the pipe

Objective Questions II (One or more correct option)

18. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are) **(2013 Adv.)**

(a) In the region $0 < r < R$, the magnetic field is non-zero

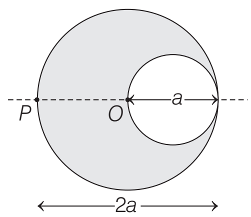
(b) In the region $R < r < 2R$, the magnetic field is along the common axis

(c) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis

(d) In the region $r > 2R$, the magnetic field is non-zero

Integer Answer Type Questions

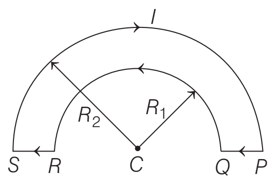
19. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is **(2012)**



20. Two parallel wires in the plane of the paper are distance x_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance x_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{x_0}{x_1} = 3$, and value of $\frac{R_1}{R_2}$ is **(2014 Adv.)**

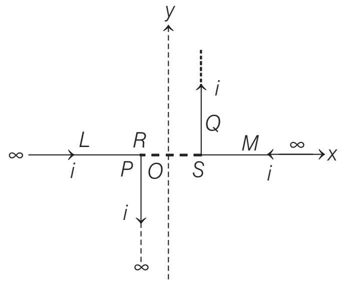
Fill in the Blank

21. The wire loop $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown. The magnitude of the magnetic induction at the centre C is …… **(1988, 2M)**



Analytical & Descriptive Questions

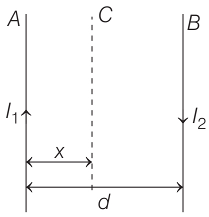
22. A pair of stationary and infinitely long bent wires are placed in the xy plane as shown in figure. The wires carry current of $i = 10$ A each as shown. The segments L and M are along the x-axis. The segments P and Q are parallel to the y-axis such that $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic induction at the origin O. **(1989, 6M)**



Topic 3 Magnetic Force on Current Carrying Wires

Objective Questions I (Only one correct option)

1. Two wires A and B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are **(2019 Main, 10 April I)**



(a) $x = \left(\dfrac{I_2}{I_1 + I_2}\right) d$ and $x = \left(\dfrac{I_2}{I_1 - I_2}\right) d$

(b) $x = \left(\dfrac{I_1}{I_1 - I_2}\right) d$ and $x = \left(\dfrac{I_2}{I_1 + I_2}\right) d$

(c) $x = \left(\dfrac{I_1}{I_1 + I_2}\right) d$ and $x = \left(\dfrac{I_2}{I_1 - I_2}\right) d$

(d) $x = \pm \dfrac{I_1 d}{(I_1 - I_2)}$

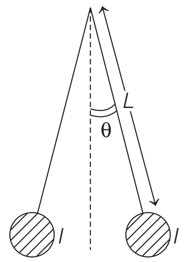
2. A circular coil having N turns and radius r carries a current I. It is held in the XZ-plane in a magnetic field $B\hat{\mathbf{i}}$. The torque on the coil due to the magnetic field (in N-m) is **(2019 Main, 8 April I)**

(a) $\dfrac{Br^2 I}{\pi N}$ (b) $B\pi r^2 IN$ (c) $\dfrac{B\pi r^2 I}{N}$ (d) Zero

3. Two coaxial solenoids of different radii carry current I in the same direction. Let $\mathbf{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then, **(2015 Main)**

(a) $\mathbf{F}_1$ is radially outwards and $\mathbf{F}_2 = 0$
(b) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2$ is radially outwards
(c) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2 = 0$
(d) $\mathbf{F}_1 = \mathbf{F}_2 = 0$

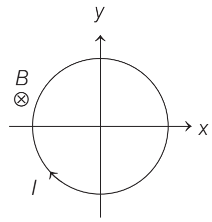
4. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then, the value of I is (g = gravitational acceleration) **(2015 Main)**



(a) $2\sin\theta \sqrt{\dfrac{\pi\lambda g L}{\mu_0 \cos\theta}}$ (b) $\sin\theta \sqrt{\dfrac{\pi\lambda g L}{\mu_0 \cos\theta}}$

(c) $2\sqrt{\dfrac{\pi g L}{\mu_0}\tan\theta}$ (d) $\sqrt{\dfrac{\pi\lambda g L}{\mu_0}\tan\theta}$

5. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to **(2003, 2M)**



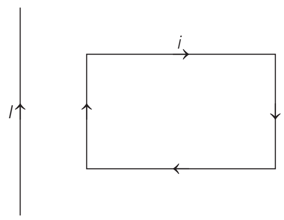
(a) contract
(b) expand
(c) move towards + ve x-axis
(d) move towards –ve x-axis

6. Two thin long parallel wires separated by a distance b are carrying a current i ampere each. The magnitude of the force per unit length exerted by one wire on the other is **(1986, 2M)**

(a) $\dfrac{\mu_0 i^2}{b^2}$ (b) $\dfrac{\mu_0 i^2}{2\pi b}$

(c) $\dfrac{\mu_0 i}{2\pi b}$ (d) $\dfrac{\mu_0 i}{2\pi b^2}$

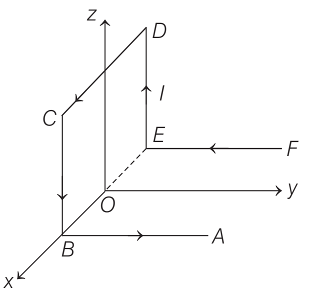
7. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If steady current I is established in the wire as shown in the figure, the loop will **(1985, 2M)**



(a) rotate about an axis parallel to the wire
(b) move away from the wire
(c) move towards the wire
(d) remain stationary

Fill in the Blank

8. A wire $ABCDEF$ (with each side of length L) bent as shown in figure and carrying a current I is placed in a uniform magnetic induction B parallel to the positive y-direction. The force experienced by the wire is ……in the …… direction. **(1990, 2M)**



True/False

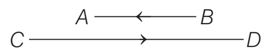
9. No net force acts on a rectangular coil carrying a steady current when suspended freely in a uniform magnetic field. **(1981, 2M)**

Analytical & Descriptive Questions

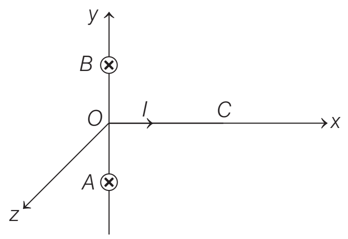
10. Three infinitely long thin wires, each carrying current i in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$. **(1997, 5M)**

(a) Find the locus of the points for which the magnetic field B is zero.
(b) If the central wire is displaced along the z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation.

11. A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations. **(1994, 6M)**



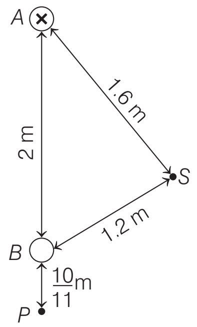
12. A straight segment OC (of length L) of a circuit carrying a current I is placed along the x-axis . Two infinitely long straight wires A and B, each extending from $z = -\infty$ to $+\infty$, are fixed at $y = -a$ and $y = +a$ respectively, as shown in the figure. If the wires A and B each carry a current I into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed? **(1992, 10M)**



13. Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper.

The wire A carries a current of 9.6 A, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P, at a distance of $10/11$ m from the wire B, is zero. **(1987, 7M)**

Find

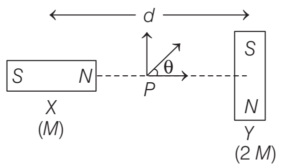


(a) the magnitude and direction of the current in B.
(b) the magnitude of the magnetic field of induction at the point S.
(c) the force per unit length on the wire B.

Topic 4 Magnetic Dipole

Objective Questions I (Only one correct option)

1. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole) **(2019 Main, 8 April II)**



(a) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3} \times qv$ (b) 0

(c) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3} \times qv$ (d) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(\frac{d}{2}\right)^3} \times qv$

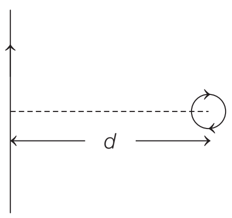
2. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop (in A-m) will be **(2019 Main, 10 April II)**

(a) $\frac{4m}{\pi}$ (b) $\frac{3m}{\pi}$ (c) $\frac{2m}{\pi}$ (d) $\frac{m}{\pi}$

3. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin ($x = 0$) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is **(2019 Main, 10 Jan I)**

(a) $n\rho l^3$ (b) $\pi n \rho l^3$ (c) $\frac{\pi}{3} n \rho l^3$ (d) $\frac{\pi}{4} n \rho l^3$

4. An infinitely long current-carrying wire and a small current-carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d\,(d >> a)$. If the loop applies a force F on the wire, then **(2019 Main, 9 Jan I)**

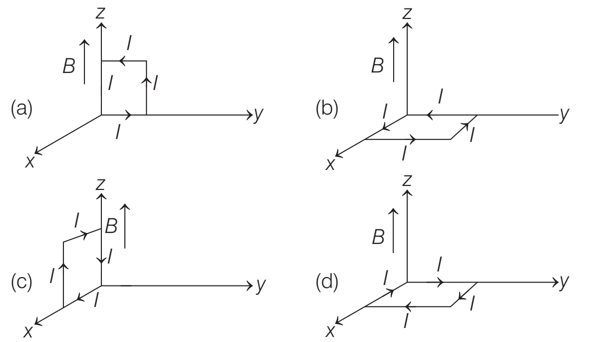


(a) $F \propto \left(\frac{a^2}{d^3}\right)$ (b) $F = 0$ (c) $F \propto \left(\frac{a}{d}\right)$ (d) $F \propto \left(\frac{a}{d}\right)^2$

5. The dipole moment of a circular loop carrying a current I is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is **(2018 Main)**

(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{2}$

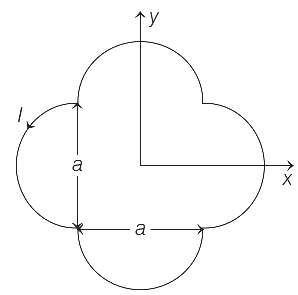
6. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below. **(2015 Main)**



If there is a uniform magnetic field of 0.3 T in the positive z-direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(a) (a) and (b) respectively (b) (b) and (d) respectively
(c) (a) and (c) respectively (d) (b) and (c) respectively

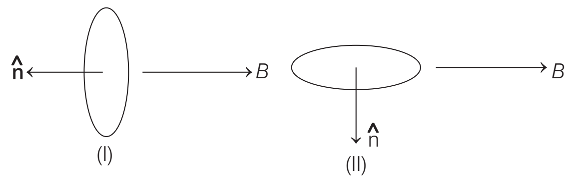
7. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{\mathbf{k}}$ is coming out of the plane of the paper. The magnetic moment of the current loop is **(2012)**

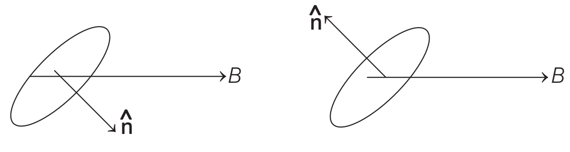


(a) $a^2 I\,\hat{\mathbf{k}}$ (b) $\left(\frac{\pi}{2}+1\right)a^2\,I\,\hat{\mathbf{k}}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2\,I\,\hat{\mathbf{k}}$ (d) $(2\pi+1)\,a^2\,I\,\hat{\mathbf{k}}$

8. A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III and IV. Arrange them in the decreasing order of potential energy. **(2003, 2M)**





(III) (IV)

(a) I > III > II > IV (b) I > II > III > IV
(c) I > IV > II > III (d) III > IV > I > II

9. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on **(2000, 2M)**

(a) ω and q (b) ω, q and m
(c) q and m (d) ω and m

10. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length $2R$. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is **(1998, 2M)**

(a) $q/2m$ (b) q/m
(c) $2q/m$ (d) $q/\pi m$

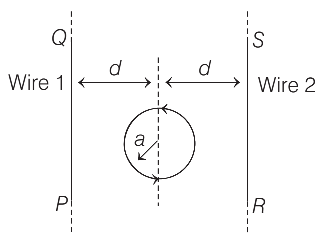
11. A conducting circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field $\mathbf{B}_0$ such that $\mathbf{B}_0$ is perpendicular to the plane of the loop. The magnetic force acting on the loop is **(1983, 1M)**

(a) $ir\mathbf{B}_0$ (b) $2\pi ir\mathbf{B}_0$ (c) zero (d) $\pi ir\mathbf{B}_0$

Passage Based Questions

Passage

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counter-clockwise direction if seen from above. **(2014 Adv.)**



12. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

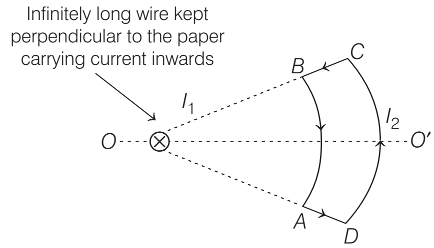
(a) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$
(b) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$
(c) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$
(d) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

13. Consider $d >> a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(a) $\dfrac{\mu_0 I^2 a^2}{d}$ (b) $\dfrac{\mu_0 I^2 a^2}{2d}$
(c) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{d}$ (d) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{2d}$

Questions II (One or more correct option)

14. Which of the following statement is (are) correct in the given figure ? **(2006, 5M)**



(a) Net force on the loop is zero
(b) Net torque on the loop is zero
(c) Loop will rotate clockwise about axis OO' when seen from O
(d) Loop will rotate anticlockwise about OO' when seen from O

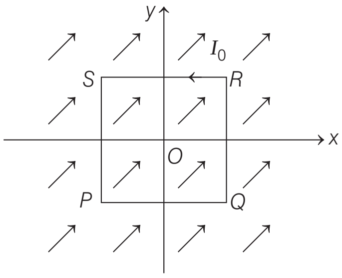
Fill in the Blanks

15. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10^{16} revolutions per second. The magnetic moment associated with the orbital motion of the electron is ...... **(1988, 2M)**

16. A wire of length L metre carrying a current i ampere is bent in the form of circle. The magnitude of its magnetic moment is ...... in MKS units. **(1987, 2M)**

Analytical & Descriptive Questions

17. A uniform constant magnetic field $\mathbf{B}$ is directed at an angle of 45° to the x-axis in x-y plane. $PQRS$ is rigid square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position shown in the figure with its sides parallel to x and y-axes. Each side of the frame is of mass M and length L **(1998, 8M)**

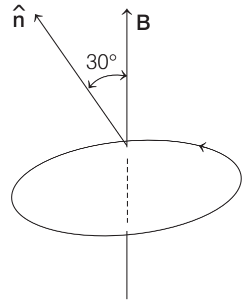


(a) What is the magnitude of torque τ acting on the frame due to the magnetic field ?

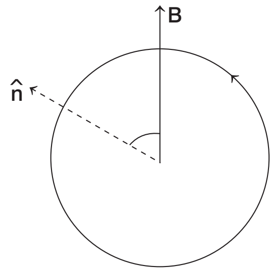
(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt and the axis about which this rotation occurs (Δt is so short that any variation in the torque during this interval may be neglected). Given : the moment of inertia of the frame about an axis through its centre perpendicular to its plane is $\frac{4}{3}ML^2$.

18. An electron in the ground state of hydrogen atom is revolving in anti-clockwise direction in a circular orbit of radius R . **(1996, 5M)**

(a) Obtain an expression for the orbital magnetic moment of the electron.



(b) The atom is placed in a uniform magnetic induction **B** such that the normal to the plane of electron's orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.



Topic 5 Magnetism

Objective Questions I (Only one correct option)

1. A magnetic compass needle oscillates 30 times per minute at a place, where the dip is 45° and 40 times per minute, where the dip is 30°. If B_1 and B_2 are respectively, the total magnetic field due to the earth at the two places, then the ratio $\frac{B_1}{B_2}$ is best given by **(2019 Main, 12 April I)**

(a) 1.8 (b) 0.7 (c) 3.6 (d) 2.2

2. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses
a torsion band of torsion constant
10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately **(2019 Main, 9 April II)**

(a) 10^{-3} (b) 10^{-4} (c) 10^{-1} (d) 10^{-2}

3. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is **(2019 Main, 12 Jan II)**

(a) 3.726×10^{-4} (b) 3.672×10^{-4}
(c) 2.672×10^{-4} (d) 3.267×10^{-4}

4. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is **(2019 Main, 11 Jan II)**

(a) 3.3×10^{-4} (b) 3.3×10^{-2}
(c) 4.3×10^{-2} (d) 2.3×10^{-2}

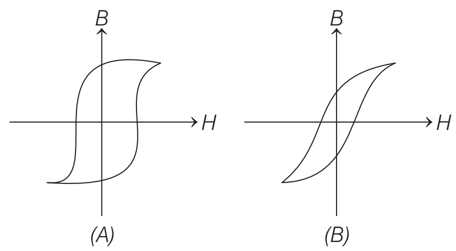
5. At some location on earth, the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A-m is suspended from its mid point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is **(2019 Main, 10 Jan II)**

(a) 6.5×10^{-5} N (b) 3.6×10^{-5} N
(c) 1.3×10^{-5} N (d) 1.8×10^{-5} N

6. A bar magnet is demagnetised by inserting it inside a solenoid of length 0.2 m, 100 turns and carrying a current of 5.2 A. The coercivity of the bar magnet is **(Main 2019, 9 Jan I)**

(a) 1200 A/m (b) 285 A/m (c) 2600A/m (d) 520A/m

7. Hysteresis loops for two magnetic materials A and B are as given below: **(2016 Main)**

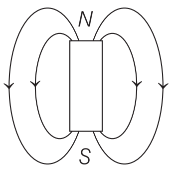


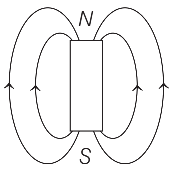
These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use
(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

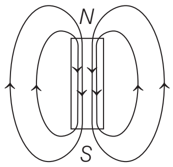
8. The coercivity of a small magnet where the ferromagnet gets demagnetised is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is **(2014 Main)**
(a) 30 mA (b) 60 mA (c) 3 A (d) 6 A

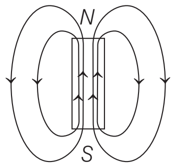
9. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is 3.6×10^{-5} Wb/m^2) **(2013 Main)**
(a) 3.6×10^{-5} Wb / m^2
(b) 2.56×10^{-4} Wb / m^2
(c) 3.50×10^{-4} Wb / m^2
(d) 5.80×10^{-4} Wb / m^2

10. The magnetic field lines due to a bar magnet are correctly shown in **(2002, 2M)**

(a)


(b)


(c)


(d)


11. A magnetic needle is kept in a non-uniform magnetic field. It experiences **(1982, 3M)**
(a) a force and a torque (b) a force but not a torque
(c) a torque but not a force (d) neither a force nor a torque

Assertion and Reason

Mark your answer as
(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) If Statement I is true; Statement II is false
(d) If Statement I is false; Statement II is true

12. **Statement I** The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil. **(2008, 3M)**

Statement II Soft iron has a high magnetic permeability and cannot be easily magnetised or demagnetised.

Analytical & Descriptive Questions

13. A moving coil galvanometer experiences torque = ki, where i is current. If N coils of area A each and moment of inertia I is kept in magnetic field B. **(2005, 6M)**
(a) Find k in terms of given parameters.
(b) If for current i deflection is $\frac{\pi}{2}$, find out torsional constant of spring.
(c) If a charge Q is passed suddenly through the galvanometer, find out maximum angle of deflection.

Topic 6 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V $(V = GI_0)$ by connecting a series resistance R_V to it. Then, **(2019 Main, 12 April II)**

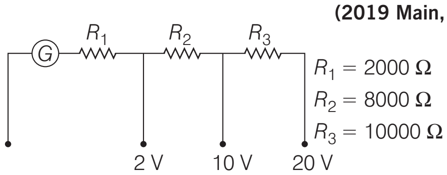
(a) $R_A R_V = G^2 \left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

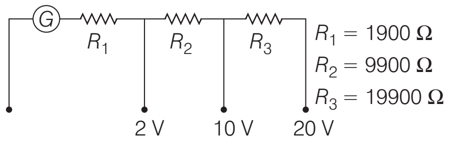
(b) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

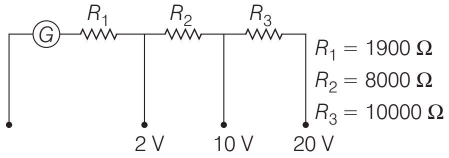
(c) $R_A R_V = G^2 \left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

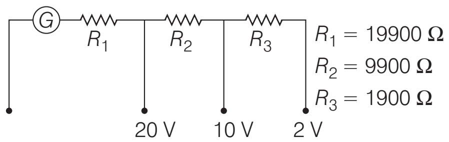
(d) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

2. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is **(2019 Main, 12 April I)**

(a)


(b) 

(c) 

(d) 

3. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore, the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0.10 mA is **(2019 Main, 10 April I)**

(a) 100 Ω (b) 500 Ω (c) 200 Ω (d) 10 Ω

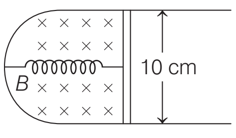
4. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0-0.5 A? **(2019 Main, 9 April II)**

(a) 0.2 ohm (b) 0.5 ohm
(c) 0.002 ohm (d) 0.02 ohm

5. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to **(2019 Main, 9 April I)**

(a) 40 V (b) 10 V (c) 15 V (d) 20 V

6. A thin strip 10 cm long is on an U-shaped wire of negligible resistance and it is connected to a spring of spring constant $0.5\ \text{Nm}^{-1}$ (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10 Ω and air drag negligible, N will be close to **(2019 Main, 8 April I)**



(a) 1000 (b) 50000 (c) 5000 (d) 10000

7. The mean intensity of radiation on the surface of the sun is about $10^8\ \text{W/m}^2$. The rms value of the corresponding magnetic field is closest to **(Main 2019, 12 Jan II)**

(a) 1 T (b) 10^2 T (c) 10^{-4} T (d) 10^{-2} T

8. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then **(2019 Main, 10 Jan II)**

(a) $T_h = 0.5\,T_c$ (b) $T_h = T_c$ (c) $T_h = 2\,T_c$ (d) $T_h = 1.5\,T_c$

9. A magnet of total magnetic moment $10^{-2}\,\hat{\mathbf{i}}$ A-m^2 is placed in a time varying magnetic field, $B\hat{\mathbf{i}}$ $(\cos\omega t)$, where $B = 1$ T and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at $t = 1$ s is **(2019 Main, 10 Jan I)**

(a) 0.01 J (b) 0.007 J (c) 0.014 J (d) 0.028 J

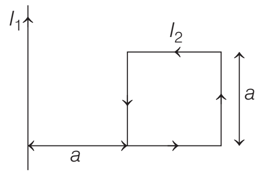
10. A conducting circular loop is made of a thin wire has area $3.5\times10^{-3}\ \text{m}^2$ and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4\text{T})\sin(0.5\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to **(2019 Main, 9 Jan I)**

(a) 6 mC (b) 21 mC (c) 7 mC (d) 14 mC

11. A rectangular coil (dimension 5 cm $\times$ 2.5 cm) with 100 turns, carrying a current of 3A in the clockwise direction, is kept centred at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is **(2019 Main, 9 April I)**

(a) 0.27 N-m (b) 0.38 N-m (c) 0.42 N-m (d) 0.55 N-m

12. A rigid square loop of side a and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be **(2019 Main, 9 April I)**



(a) repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{2\pi}$

(b) attractive and equal to $\dfrac{\mu_0 I_1 I_2}{3\pi}$

(c) zero

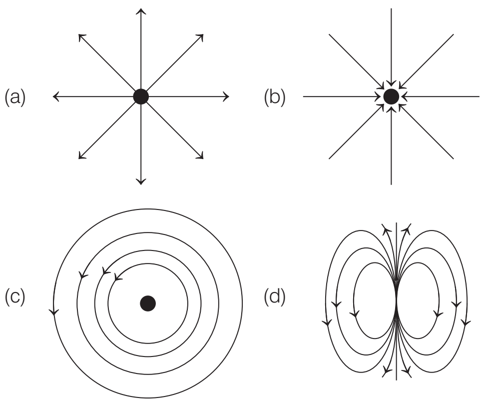
(d) repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{4\pi}$

13. The region between $y = 0$ and $y = d$ contains a magnetic field $\mathbf{B} = B\hat{\mathbf{k}}$. A particle of mass m and charge q enters the region with a velocity $\mathbf{v} = v\hat{\mathbf{i}}$. If $d = \dfrac{mv}{2qB}$ then the acceleration of the charged particle at the point of its emergence at the other side is **(2019 Main, 11 Jan II)**

(a) $\dfrac{qvB}{m}\left(\dfrac{\sqrt{3}}{2}\hat{\mathbf{i}}+\dfrac{1}{2}\hat{\mathbf{j}}\right)$ (b) $\dfrac{qvB}{m}\left(\dfrac{1}{2}\hat{\mathbf{i}}-\dfrac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$

(c) $\dfrac{qvB}{m}\left(\dfrac{-\hat{\mathbf{j}}+\hat{\mathbf{i}}}{\sqrt{2}}\right)$ (d) $\dfrac{qvB}{m}\left(\dfrac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)$

14. Which of the field patterns given in the figure is valid for electric field as well as for magnetic field? **(2011)**



15. Two very long straight parallel wires carry steady currents I and $-I$ respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\mathbf{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is **(1998, 2M)**

(a) $\dfrac{\mu_0 Iqv}{2\pi d}$ (b) $\dfrac{\mu_0 Iqv}{\pi d}$ (c) $\dfrac{2\mu_0 Iqv}{\pi d}$ (d) zero

Objective Questions II (One or more correct option)

16. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive, if it is in the $+\hat{\mathrm{j}}$-direction. Which of the following statements regarding the magnetic field B is (are) true? **(2018 Adv.)**

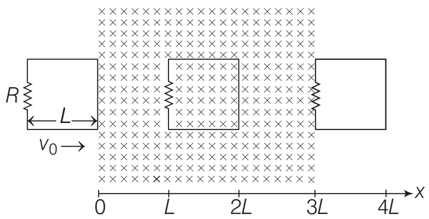
(a) If $I_1 = I_2$, then B cannot be equal to zero at the origin (0,0,0)

(b) If $I_1 > 0$ and $I_2 < 0$, then B can be equal to zero at the origin (0,0,0)

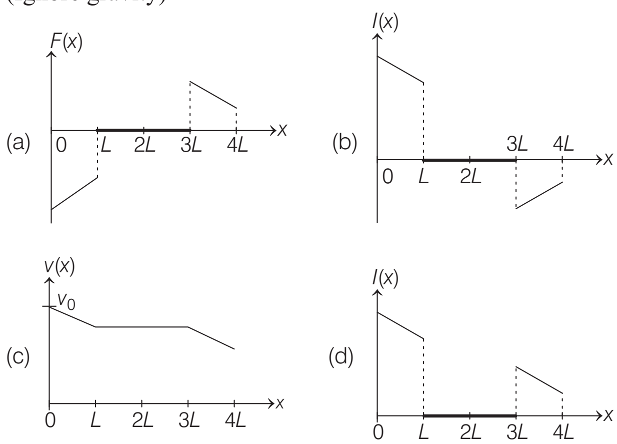
(c) If $I_1 < 0$ and $I_2 > 0$, then B can be equal to zero at the origin (0,0,0)

(d) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\dfrac{\mu_o I}{2R}\right)$

17. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$, where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. **(2016 Adv.)**

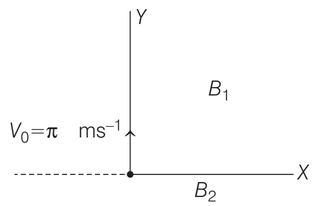


Which of the following schematic plot(s) is (are) correct? (Ignore gravity)



Numerical Value

18. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{\mathrm{k}}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{\mathrm{k}}$. A positively charged particle is projected from the origin along the positive Y-axis with speed $v_0 = \pi\ \mathrm{ms}^{-1}$ at $t = 0$, as shown in figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the X-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the X-axis in the time interval T is........... . **(2018 Adv.)**

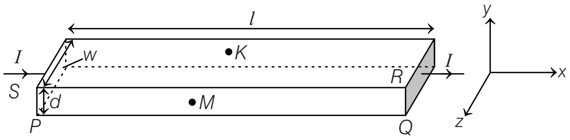


Passage Based Questions

Passage 1

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively. A uniform magnetic field $\mathbf{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction.

This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



19. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statements is/are **(2015 Adv.)**

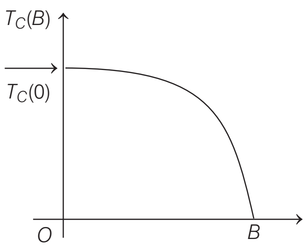
(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$
(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$
(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$
(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

20. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct options is/are **(2015 Adv.)**

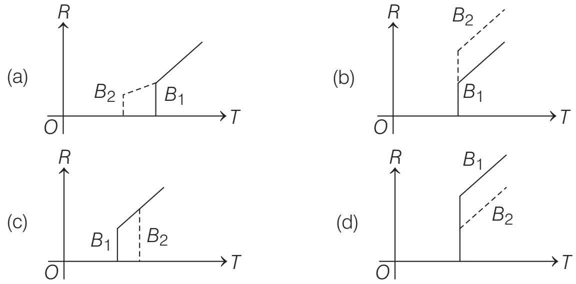
(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Passage 2

Electrical resistance of certain materials, known as superconductors, changes abruptly from a non-zero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field i.e. the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure. **(2010)**



21. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (*solid line*) and B_2 (*dashed line*). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

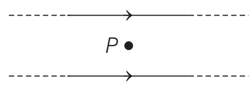
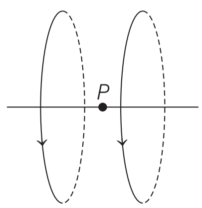
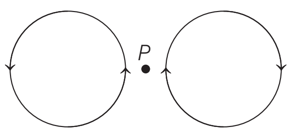


22. A superconductor has $T_C(0) = 100\,\text{K}$. When a magnetic field of 7.5 T is applied, its T_C decreases to 75 K. For this material one can definitely say that when **(Note** T = Tesla**)**

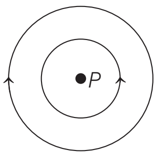
(a) $B = 5\,\text{T}, T_C(B) = 80\,\text{K}$
(b) $B = 5\,\text{T}, 75\,\text{K} < T_C(B) < 100\,\text{K}$
(c) $B = 10\,\text{T}, 75\,\text{K} < T_C(B) < 100\,\text{K}$
(d) $B = 10\,\text{T}, T_C(B) = 70\,\text{K}$

Match the Columns

23. Two wires each carrying a steady current I are shown in four configurations in Column I. Some of the resulting effects are described in Column II. Match the satements in Column I with the statements in Column II. **(2007, 6M)**

	Column I		Column II
(A)	Point P is situated midway between the wires. 	(p)	The magnetic fields (B) at P due to the currents in the wires are in the same direction.
(B)	Point P is situated at the mid-point of the line joining the centres of the circular wires, which have same radii. 	(q)	The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(C)	Point P is situated at the mid-point of the line joining the centres of the circular wires, which have same radii. 	(r)	There is no magnetic field at P.

	Column I		Column II
(D)	Point P is situated at the common centre of the wires.	(s)	The wires repel each other.



24. **Column I** gives certain situations in which a straight metallic wire of resistance R is used and **Column II** gives some resulting effects. Match the statements in **Column I** with the statements in **Column II.** **(2007, 6M)**

	Column I		Column II
(A)	A charged capacitor is connected to the ends of the wire	(p)	A constant current flows through the wire
(B)	The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	(q)	Thermal energy is generated in the wire
(C)	The wire is placed in a constant electric field that has a direction along the length of the wire	(r)	A constant potential difference develops between the ends of the wire
(D)	A battery of constant emf is connected to the ends of the wire	(s)	Charges of constant magnitude appear at the ends of the wire

25. Some laws/processes are given in **Column I.** Match these with the physical phenomena given in **Column II.** **(2006, 4M)**

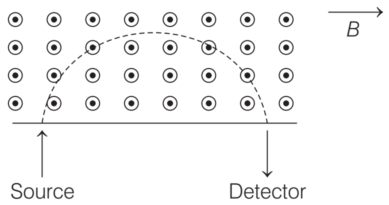
	Column I		Column II
(A)	Dielectric ring uniformly charged	(p)	Time independent electrostatic field out of system
(B)	Dielectric ring uniformly charged rotating with angular velocity ω	(q)	Magnetic field
(C)	Constant current in ring i_o	(r)	Induced electric field
(D)	$i = i_0 \cos \omega t$	(s)	Magnetic moment

Integer Answer Type Question

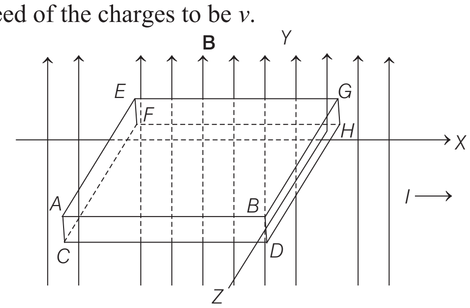
26. A steady current I goes through a wire loop PQR having shape of a right angle triangle with $PQ = 3x$, $PR = 4x$ and $QR = 5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\dfrac{\mu_0 I}{48\pi x}\right)$, find the value of k. **(2009)**

Fill in the Blanks

27. A uniform magnetic field with a slit system as shown in figure is to be used as momentum filter for high-energy charged particles. With a field B Tesla, it is found that the filter transmits α-particles each of energy 5.3 MeV. The magnetic field is increased to 2.3 B Tesla and deuterons are passed into the filter. The energy of each deuteron transmitted by the filter is ……MeV. **(1997C, 1M)**

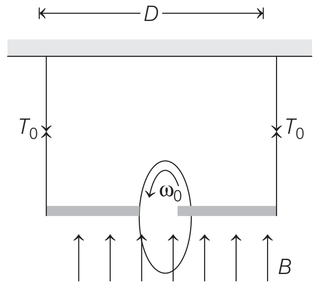


28. A metallic block carrying current I is subjected to a uniform magnetic induction **B** as shown in figure. The moving charges experience a force **F** given by ……which results in the lowering of the potential of the face……. Assume the speed of the charges to be v. **(1996, 2M)**

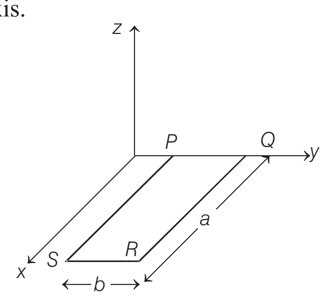


Analytical & Descriptive Questions

29. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now, a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_0/2$. **(2003, 4M)**



30. A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis.



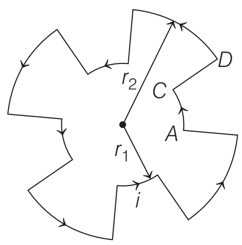
A uniform magnetic field $\mathbf{B} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **(2002, 5M)**

(a) What is the direction of the current I in PQ ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

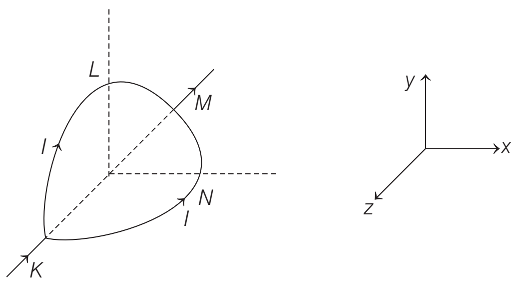
31. A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii $r_1 = 0.08$ m and $r_2 = 0.12$ m. Each subtends the same angle at the centre. **(2001, 10M)**



(a) Find the magnetic field produced by this circuit at the centre.

(b) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre ?

32. A circular loop of radius R is bent along a diameter and given a shape as shown in figure. One of the semicircles (KNM) lies in the x-z plane and the other one (KLM) in the y-z plane with their centres at origin. Current I is flowing through each of the semicircles as shown in figure. **(2000, 10M)**

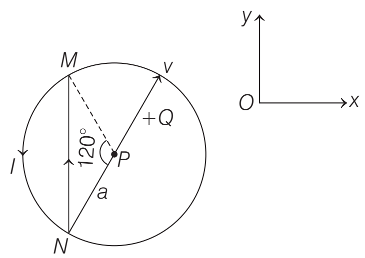


(a) A particle of charge q is released at the origin with a velocity $\mathbf{v} = -v_0\hat{\mathbf{i}}$. Find the instantaneous force $\mathbf{F}$ on the particle. Assume that space is gravity free.

(b) If an external uniform magnetic field $B_0\hat{\mathbf{j}}$ is applied determine the force $\mathbf{F}_1$ and $\mathbf{F}_2$ on the semicircles KLM and KNM due to the field and the net force $\mathbf{F}$ on the loop.

33. A particle of mass m and charge q is moving in a region where uniform, constant electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are present. $\mathbf{E}$ and $\mathbf{B}$ are parallel to each other. At time $t = 0$, the velocity $\mathbf{v}_0$ of the particle is perpendicular to $\mathbf{E}$ (Assume that its speed is always $<< c$, the speed of light in vacuum). Find the velocity $\mathbf{v}$ of the particle at time t. You must express your answer in terms of t, q, m, the vector $\mathbf{v}_0$, $\mathbf{E}$ and $\mathbf{B}$ and their magnitudes v_0, E and B. **(1998, 8M)**

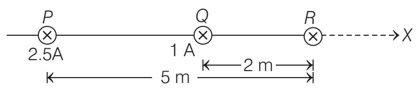
34. A wire loop carrying a current I is placed in the x-y plane as shown in figure. **(1991, 4+4M)**



(a) If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity $\mathbf{v}$ along NP (see figure), find its instantaneous acceleration.

(b) If an external uniform magnetic induction field $\mathbf{B} = B\hat{\mathbf{i}}$ is applied, find the force and the torque acting on the loop due to this field.

35. Two long parallel wires carrying currents 2.5 A and I (ampere) in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 m and 2 m respectively from a collinear point R (see figure). **(1990, 8M)**



(a) An electron moving with a velocity of 4×10^5 m/s along the positive x-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of I.

(b) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed, so that the magnetic induction at R is zero.

Answers

Topic 1

1. (*) **2.** (c) **3.** (a) **4.** (d)
5. (b) **6.** (d) **7.** (c)
8. (a) **9.** (d) **10.** (b) **11.** (b)
12. (b) **13.** (c) **14.** (a) **15.** (a)
16. (c) **17.** (a,b.c) **18.** (a, c) **19.** (c, d)
20. (b, d) **21.** (a, c, d) **22.** (a, c) **23.** (a, b, d)
24. (a, b, d) **25.** (c) **26.** (a) **27.** (c)
28. D, B **29.** F **30.** F **31.** T
32. $\frac{1}{\sqrt{2}}$ **33.** (a) $L = \frac{mv_0}{2B_0q}$ (b) $\mathbf{v}_f = -v_0\hat{\mathbf{i}}$, $t = \frac{\pi m}{B_0 q}$
34. 4.73×10^{-3} T **35.** 1.2×10^{-2}m, 4.37×10^{-2}m
36. (a) 0.14 m, 45° (b) 4.712×10^{-8} s
37. (6.4 m, 0, 0), (6.4 m, 0, 2 m)
38. 0.1 T (perpendicular to paper inwards)

Topic 2

1. (b) **2.** (b) **3.** (d) **4.** (a)
5. (b) **6.** (c) **7.** (a) **8.** (d)
9. (d) **10.** (a) **11.** (a) **12.** (c)
13. (d) **14.** (b) **15.** (c) **16.** (d)
17. (b) **18.** (a, d) **19.** 5 **20.** 3
21. $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (perpendicular to paper outwards)
22. 10^{-4} T, perpendicular to paper outwards

Topic 3

1. (d) **2.** (b) **3.** (d) **4.** (a)
5. (b) **6.** (b) **7.** (c) **8.** ILB, positive-z
9. T
10. (a) $x = 0 = z$ and $z = 0, x = \pm\frac{d}{\sqrt{3}}$ (b) $f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$
11. 0.2 s **12.** $\mathbf{F} = \frac{-\mu_0 I^2}{2\pi}\ln\left(\frac{L^2 + a^2}{a^2}\right)\hat{\mathbf{k}}$, zero
13. (a) 3 A, perpendicular to paper outwards
(b) 13×10^{-7} T
(c) 2.88×10^{-6} N / m

Topic 4

1. (b) **2.** (a) **3.** (d) **4.** (d)
5. (d) **6.** (b) **7.** (b) **8.** (c)
9. (c) **10.** (a) **11.** (c) **12.** (c)
13. (b) **14.** (a, c) **15.** 1.26×10^{-23} A-m^2
16. $\frac{L^2 i}{4\pi}$ **17.** (a) $|\tau| = I_0L^2B$ (b) $\theta = \frac{3}{4}\frac{I_0B}{M}(\Delta t)^2$
18. (a) $M = \frac{eh}{4\pi m}$ (b) $\tau = \frac{ehB}{8\pi m}$, perpendicular to both **M** and **B**.

Topic 5

1. (b) **2.** (a) **3.** (d) **4.** (a)
5. (a) **6.** (c) **7.** (d) **8.** (c)
9. (b) **10.** (d) **11.** (a) **12.** (c)
13. (a) $k = BNA$ (b) $k = \frac{2BiNA}{\pi}$ (c) $Q\sqrt{\frac{BNA\pi}{2I}}$

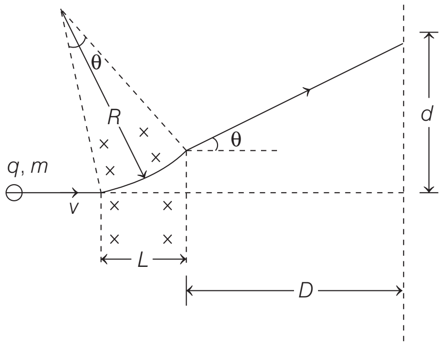
Topic 6

1. (b) **2.** (c) **3.** (*) **4.** (a)
5. (d) **6.** (c) **7.** (c) **8.** (b)
9. (c) **10.** (d) **11.** (a) **12.** (d)
13. (*) **14.** (c) **15.** (d) **16.** (a,b,d)
17. (b,c) **18.** (2) **19.** (a,d)
20. (a,c) **21.** (a) **22.** (b)
23. (A) $\to$ q, r (B) $\to$ p (C) $\to$ q, r (D) $\to$ q, s or q
24. (A) $\to$ q (B) $\to$ r, s (C) $\to$ s (D) $\to$ p, q, r
25. (A) $\to$ p (B) $\to$ p, q, s (C) $\to$ q, s (D) $\to$ q, r
26. 7 **27.** 14.0185 **28.** $evB\hat{\mathbf{k}}$, ABCD **29.** $\omega_{max} = \frac{DT_0}{BQR^2}$
30. (a) P to Q (b) $IbB_0\,(3\hat{\mathbf{k}} - 4\hat{\mathbf{i}})$ (c) $\frac{mg}{6bB_0}$
31. (a) 6.54×10^{-5} T (vertically upward or outward normal to the paper) (b) Zero, Zero, 8.1×10^{-6} N (inwards)
32. (a) $\mathbf{F} = -\frac{\mu_0 q v_0 I}{4R}\hat{\mathbf{k}}$ (b) $\mathbf{F}_1 = \mathbf{F}_2 = 2BIR\hat{\mathbf{i}}$, $\mathbf{F} = 4BIR\hat{\mathbf{i}}$
33. $\mathbf{v} = \cos\left(\frac{qB}{m}t\right)(\mathbf{v}_0) + \left(\frac{q}{m}t\right)(\mathbf{E}) + \sin\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0\times\mathbf{B}}{B}\right)$
34. (a) $\frac{0.11\mu_0 IQv}{2am}(\hat{\mathbf{j}} - \sqrt{3}\,\hat{\mathbf{i}})$ (b) zero, $(0.61Ia^2B)\hat{\mathbf{j}}$
35. (a) 4A (b) At distance 1m from R to the left or right of it, current is outwards if placed to the left and inwards if placed to the right of 6R.

Hints & Solutions

Topic 1 Magnetic Force and Path of Charged Particle in Uniform Fields

1.



When electron enters the region of magnetic field, it experiences a Lorentz force which rotates electron in a circular path of radius R.

So, Lorentz force acts like a centripetal force and we have

$$\frac{mv^2}{R} = Bqv$$

where, m = mass of electron,

q = charge of electron, v = speed of electron,

R = radius of path,

and B = magnetic field intensity.

Radius of path of electron,

$$R = \frac{mv}{Bq}$$

Now, from geometry of given arrangement, comparing values of $\tan\theta$, we have

$$\tan\theta = \frac{L}{R} = \frac{d}{D} \Rightarrow d = \frac{LD}{R} = \frac{Bq\,LD}{mv}$$

$$\Rightarrow \quad d = \frac{BqLD}{\sqrt{2mk}} \quad [\because mv = \sqrt{2mk}]$$

where, k = kinetic energy of electron

Here, $B = 1.5 \times 10^{-3}$ T,

$q = 1.6 \times 10^{-19}$ C, $L = 2 \times 10^{-2}$ m, $D = 6 \times 10^{-2}$ m,

$m = 9.1 \times 10^{-31}$ kg, $k = 100 \times 1.6 \times 10^{-19}$ J

$$\text{So, } d = \frac{(1.5 \times 10^{-3} \times 1.6 \times 10^{-19} \times 2 \times 10^{-2} \times 6 \times 10^{-2})}{\sqrt{(2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19})}}$$

$$= \frac{28.8 \times 10^{-26}}{\sqrt{29.12 \times 10^{-48}}} = \frac{28.8 \times 10^{-26}}{5.39 \times 10^{-24}} = 5.34 \times 10^{-2}\text{ m}$$

= 5.34 cm

No option is matching.

2. When a moving charged particle is placed in a magnetic field $\mathbf{B}$.

Then, the net magnetic force acting on it is

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$$

$$\mathbf{F}_m = q\,vB\sin\theta$$

Here, $\theta = 90°$

$$\mathbf{F}_m = q\,vB$$

Also, due to this net force, the particle transverses a circular path, whose necessary centripetal force is being provided by F_m, i.e.

$$\frac{mv^2}{r} = q\,vB$$

$$\Rightarrow \quad r = \frac{mv^2}{qvB} = \frac{mv}{qB}$$

$$\Rightarrow \quad r \propto m$$

So, for electron,

$$r_e = \frac{m_e v}{eB}$$

or $\quad r_e \propto m_e$

For proton,

$$r_p = \frac{m_p v}{eB}$$

or $\quad r_p \propto m_p$

For He-particle,

$$r_{\text{He}} = \frac{4m_p v}{2eB} = \frac{2m_p v}{eB}$$

Clearly, $\quad r_{\text{He}} > r_p \quad (\because r_{\text{He}} = 2r_p)$

and we know that, $m_p > m_e$ $\quad \left[\begin{array}{l}\because m_p \approx 10^{-27}\text{ kg,} \\ m_e \approx 10^{-31}\text{ kg}\end{array}\right]$

$\therefore \quad r_p > r_e$

$\Rightarrow \quad r_{\text{He}} > r_p > r_e$

3. Radius of path of charged particle q in a uniform magnetic field B of mass 'm' moving with velocity v is

$$r = \frac{mv}{Bq} = \frac{m\sqrt{(2qV/m)}}{Bq}$$

$$\Rightarrow \quad r \propto \frac{\sqrt{m}}{\sqrt{q}} \text{ so, required ratio is}$$

$$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha}} \times \sqrt{\frac{q_\alpha}{q_p}}$$

$$= \sqrt{\frac{1}{4}} \times \sqrt{\frac{2}{1}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

4. Here, $\mathrm{E} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}}$, $\mathrm{B} = 4\hat{\mathrm{j}} + 6\hat{\mathrm{k}}$,

q = charge on a particle.

Initial position, $r_1 = (0,0)$

Final position, $r_2 = (1,1)$

Net force experienced by charge particle inside electromagnetic field is

$$\mathbf{F}_{net} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$
$$= q(2\hat{i} + 3\hat{j}) \qquad [\text{Here, } \mathbf{v} \times \mathbf{B} = 0]$$
$$= (2q\,\hat{i} + 3q\,\hat{j})$$

$\therefore \qquad dW = \mathbf{F}_{net}.\mathbf{dr}$

$$\Rightarrow \qquad \int dW = \int_{r_1}^{r_2} (2q\,\hat{i} + 3q\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$$

$$[\text{Here, } dr = dx\,\hat{i} + dy\,\hat{j}]$$

$$\Rightarrow \qquad W = 2q\int_0^1 dx + 3q\int_0^1 dy$$

or $\qquad W = 2q + 3q$ or $W = 5q$

5. During the circular motion of accelerated electron in the presence of magnetic field, its radius is given by

$$r = \frac{mv}{Be} = \frac{\sqrt{2meV}}{eB}$$

where, v is velocity and V is voltage.

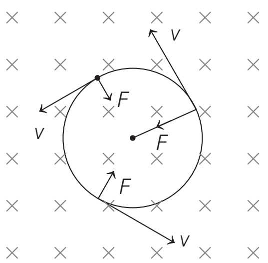
After substituting the given values, we get

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}}{1.6 \times 10^{-19} \times 100 \times 10^{-3}}$$

$$= 10\left[\frac{2 \times 9.1 \times 500}{1.6} \times 10^{-12}\right]^{1/2}$$

$$r = 7.5 \times 10^{-4} \text{ m}$$

6. According to given condition, when a particle having charge same as electron move in a magnetic field on circular path, then the force always acts towards the centre and perpendicular to the velocity.



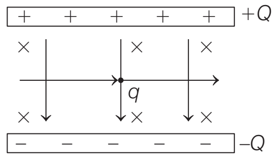
Here, $\qquad B = 0.5\,\text{T}$

$\qquad R = $ radius of circular path $= 5\,\text{cm}$

Now, the magnetic force is

$$F_m = q(v \times B) = q\,vB \sin 90°$$
$$F_m = qvB \qquad \ldots\text{(i)}$$

When the electric field applied, then the particle moves in a straight path, then this is the case of velocity selector.



Here, the electric force on charge,

$$F_e = qE \qquad \ldots\text{(ii)}$$

In velocity selector, $\quad F_m = F_e$

$$\Rightarrow \qquad qvB = qE \qquad \ldots\text{(iii)}$$

Initially particle moves under the magnetic field, So the radius of circular path taken by the particle is

$$R = \frac{mv}{qB} \qquad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv),

$$m = \frac{qB^2R}{E}$$

$$m = \frac{1.6 \times 10^{-19} \times 0.25 \times 0.5 \times 10^{-2}}{10^2}$$

$$m = 2 \times 10^{-24} \text{ kg}$$

7. **For circular path in magnetic field,**

$$r = \frac{\sqrt{2mK}}{qB}$$

where, $K =$ kinetic energy.

$$\Rightarrow \qquad r \propto \frac{\sqrt{m}}{q}$$

Further,

	e	p	α
m	$1/1836$	1	14
q	$-e$	$+e$	$2e$

$\therefore \qquad r_p = r_\alpha > r_e$

8. $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

9. Magnetic force does not change the speed of charged particle. Hence, $v = u$. Further magnetic field on the electron in the given condition is along negative y-axis in the starting. Or it describes a circular path in clockwise direction. Hence, when it exits from the field, $y < 0$.

10. Electric field can deviate the path of the particle in the shown direction only when it is along negative y-direction. In the given options $\mathbf{E}$ is either zero or along x-direction. Hence, it is the magnetic field which is really responsible for its curved path. Options (a) and (c) cannot be accepted as the path will be circular in that case. Option (d) is wrong because in that case component of net force on the particle also comes in $\hat{\mathbf{k}}$ direction which is not acceptable as the particle is moving in x-y plane. Only in option (b), the particle can move in x-y plane.

In option (d)

$$\mathbf{F}_{net} = q\,\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$

Initial velocity is along x-direction. So, let $\mathbf{v} = v\hat{\mathbf{i}}$

$\therefore \quad \mathbf{F}_{net} = qa\hat{\mathbf{i}} + q[(v\hat{\mathbf{i}}) \times (c\hat{\mathbf{k}} + b\hat{\mathbf{j}})]$

$$= qa\hat{\mathbf{i}} - qvc\hat{\mathbf{j}} + qvb\hat{\mathbf{k}}$$

In option (b)

$$\mathbf{F}_{net} = q(a\hat{\mathbf{i}}) + q[(v\hat{\mathbf{i}}) \times (c\hat{\mathbf{k}} + a\,\hat{\mathbf{i}})] = qa\hat{\mathbf{i}} - qvc\hat{\mathbf{j}}$$

11. If $(b - a) \geq r$

(r = radius of circular path of particle)

The particle cannot enter the region $x > b$.

So, to enter in the region $x > b$

$$r > (b - a) \text{ or } \frac{mv}{Bq} > (b - a)$$

or $v > \dfrac{q(b-a)B}{m}$

12. Radius of the circle $= \dfrac{mv}{Bq}$

or radius $\propto mv$ if B and q are same.

$$(\text{Radius})_A > (\text{Radius})_B$$

$\therefore \quad m_A v_A > m_B v_B$

13. We can write $\mathbf{E} = E \cdot \hat{\mathbf{i}}$ and $\mathbf{B} = B\hat{\mathbf{k}}$

Velocity of the particle will be along q. $\mathbf{E}$ direction. Therefore, we can write

$$\mathbf{v} = Aq\,E\hat{\mathbf{i}}$$

In $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{v}$, A, E and B are positive constants while q can be positive or negative.

Now, magnetic force on the particle will be

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = q\,\{AqE\hat{\mathbf{i}}\} \times \{B\hat{\mathbf{k}}\}$$

$$= q^2 AEB\,(\hat{\mathbf{i}} \times \hat{\mathbf{k}})$$

$$\mathbf{F}_m = q^2 AEB\,(-\hat{\mathbf{j}})$$

Since, $\mathbf{F}_m$ is along negative y-axis, no matter what is the sign of charge q. Therefore, all ions will deflect towards negative y-direction.

14. The charged particle will be accelerated parallel (if it is a positive charge) or antiparallel (if it is a negative charge) to the electric field, i.e. the charged particle will move parallel or antiparallel to electric and magnetic field. Therefore, net magnetic force on it will be zero and its path will be a straight line.

15. Radius of the circular path is given by

$$r = \frac{mv}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Here, K is the kinetic energy to the particle.

Therefore, $r \propto \dfrac{\sqrt{m}}{q}$ if K and B are same.

$$\therefore \quad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$$

Hence, $\quad r_\alpha = r_p < r_d$

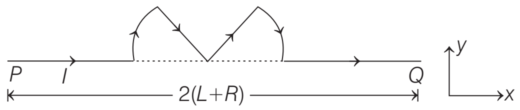
16. $R = \dfrac{\sqrt{2qVm}}{Bq}$

or $\quad R \propto \sqrt{m}$

or $\quad \dfrac{R_1}{R_2} = \sqrt{\dfrac{m_X}{m_Y}}$

or $\quad \dfrac{m_X}{m_Y} = \left(\dfrac{R_1}{R_2}\right)^2$

17.



Force on the complete wire = force on straight wire PQ carrying a current I.

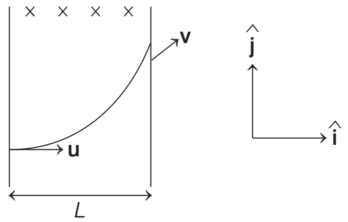
$$\mathbf{F} = I(\mathbf{PQ} \times \mathbf{B}) = I[\{2(L + R)\hat{\mathbf{i}}\} \times \mathbf{B}]$$

This force is zero if $\mathbf{B}$ is along $\hat{\mathbf{i}}$ direction or x-direction. If magnetic field is along $\hat{\mathbf{j}}$ direction or $\hat{\mathbf{k}}$ direction,

$$|\mathbf{F}| = F = (I)(2)(L + R)B\sin 90^\circ$$

or $\quad F = 2I(L + R)B \quad$ or $\quad F \propto (L + R)$

18. $\mathbf{u} = 4\hat{\mathbf{i}}$; $\mathbf{v} = 2(\sqrt{3}\,\hat{\mathbf{i}} + \hat{\mathbf{j}})$



According to the figure, magnetic field should be in ⊗ direction, or along $-z$ direction.

Further, $\quad \tan\theta = \dfrac{v_y}{v_x} = \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\therefore \quad \theta = 30^\circ$

or $\quad \dfrac{\pi}{6}$ = angle of $\mathbf{v}$ with x-axis

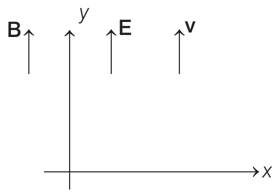
= angle rotated by the particle $= Wt = \left(\dfrac{BQ}{M}\right)t$

$\therefore \quad B = \dfrac{\pi M}{6Qt} = \dfrac{50\pi M}{3Q}$ units $\quad$ (as $t = 10^{-3}$ s)

19. and Magnetic field will rotate the particle in a circular path (in x-z plane or perpendicular to B). Electric field will exert a constant force on the particle in positive y-direction. Therefore, resultant path is neither purely circular nor helical or the options (a) and (b) both are wrong.

$v_\perp$ and $\mathbf{B}$ will rotate the particle in a circular path in x- z plane (or perpendicular to $\mathbf{B}$). Further $v_\parallel$ and $\mathbf{E}$ will move the particle (with increasing speed) along positive y-axis (or along the axis of above circular path). Therefore, the resultant path is helical with increasing pitch, along the y-axis (or along **B** and **E**). Therefore option (c) is correct.

(d)



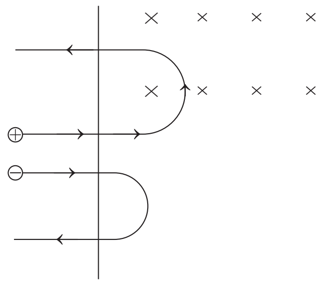
Magnetic force is zero, as θ between **B** and **v** is zero. But electric force will act in y-direction. Therefore, motion is 1-D and uniformly accelerated (towards positive y-direction).

20. $r = \dfrac{mv}{Bq}$ or $r \propto m$

$\therefore$ $r_e < r_p$ as $m_e < m_p$

Further, $T = \dfrac{2\pi m}{Bq}$

or $T \propto m$



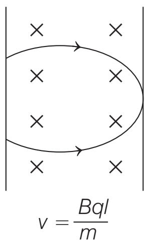
$\therefore$ $T_e < T_p$, $t_e = \dfrac{T_e}{2}$ and $t_p = \dfrac{T_p}{2}$ or $t_e < t_p$

21. $\mathbf{v} \perp \mathbf{B}$ in region II. Therefore, path of particle is circle in region II.

Particle enters in region III if,

radius of circular path, $r > l$

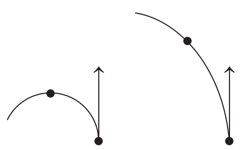
or $\dfrac{mv}{Bq} > l$ or $v > \dfrac{Bql}{m}$



If $v = \dfrac{Bql}{m}$, $r = \dfrac{mv}{Bq} = l$, particle will turn back and path length will be maximum. If particle returns to region I, time spent in region II will be

$$t = \frac{T}{2} = \frac{\pi m}{Bq}, \text{ which is independent of } v.$$

22. $$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq} \text{ i.e. } r \propto \frac{\sqrt{m}}{q}$$



If K and B are same.

i.e., $r_{H^+} : r_{He^+} : r_{O^{2+}} = \dfrac{\sqrt{1}}{1} : \dfrac{\sqrt{4}}{1} : \dfrac{\sqrt{16}}{2} = 1 : 2 : 2$

Therefore, He^+ and O^{2+} will be deflected equally but H^+ having the least radius will be deflected most.

23. Magnetic force does not do work. From work-energy theorem :

$$W_{F_e} = \Delta \text{KE} \quad \text{or} \quad (qE)(2a) = \frac{1}{2}m[4v^2 - v^2]$$

or $$E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$$

At P, rate of work done by electric field

$$= \mathbf{F}_e \cdot \mathbf{v} = (qE)(v)\cos 0°$$
$$= q\left(\frac{3}{4}\frac{mv^2}{qa}\right)v = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

Therefore, option (b) is also correct.

Rate of work done at Q :

of electric field $= \mathbf{F}_e \cdot \mathbf{v} = (qE)(2v)\cos 90° = 0$

and of magnetic field is always zero. Therefore, option (d) is also correct.

Note that $\mathbf{F}_e = qE\hat{\mathbf{i}}$.

24. If both E and B are zero, then $\mathbf{F}_e$ and $\mathbf{F}_m$ both are zero. Hence, velocity may remain constant. Therefore, option (a) is correct.

If $E = 0$, $B \neq 0$ but velocity is parallel or antiparallel to magnetic field, then also $\mathbf{F}_e$ and $\mathbf{F}_m$ both are zero. Hence, option (b) is also correct.

If $E \neq 0$, $B \neq 0$ but $\mathbf{F}_e + \mathbf{F}_m = 0$, then also velocity may remain constant or option (d) is also correct.

25. For particle to move in negative y-direction, either its velocity must be in negative y-direction (if initial velocity $\neq 0$) and force should be parallel to velocity or it must experience a net force in negative y-direction only (if initial velocity = 0)

26. $\mathbf{F}_{\text{net}} = \mathbf{F}_e + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

For particle to move in straight line with constant velocity, $\mathbf{F}_{\text{net}} = 0$

$\therefore$ $q\mathbf{E} + q\mathbf{v} \times \mathbf{B} = 0$

27. For path to be helix with axis along positive z-direction, particle should experience a centripetal acceleration in xy-plane.

For the given set of options, only option (c) satisfy the condition. Path is helical with increasing pitch.

28. ZPath C is undeviated. Therefore, it is of neutron's path. From Fleming's left hand rule magnetic force on positive charge will be leftwards and on negative charge is rightwards. Therefore, track D is of electron. Among A and B one is of proton and other of α-particle.

Further, $r = \dfrac{mv}{Bq}$ or $r \propto \dfrac{m}{q}$

Since, $\left(\dfrac{m}{q}\right)_\alpha > \left(\dfrac{m}{q}\right)_P$

$\therefore$ $r_\alpha > r_P$

or track B is of α-particle.

29. $r = \dfrac{\sqrt{2km}}{Bq}$ or $r \propto \sqrt{m}$ (k, q and B are same)

$m_p > m_e \Rightarrow \therefore r_p > r_e$

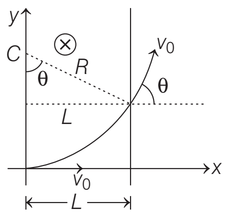
30. The path will be a helix. Path is circle when it enters normal to the magnetic field.

31. Magnetic force acting on a charged particle is always perpendicular to its velocity or work done by a magnetic force is always zero. Hence, a magnetic force cannot change the energy of charged particle.

32. $r = \dfrac{\sqrt{2qVm}}{Bq}$ or $r \propto \sqrt{\dfrac{m}{q}}$

$$\frac{r_P}{r_\alpha} = \sqrt{\frac{m_P}{m_\alpha}}\sqrt{\frac{q_\alpha}{q_P}} = \sqrt{\frac{1}{4}}\sqrt{\frac{2}{1}} = \frac{1}{\sqrt{2}}$$

33. (a) $\theta = 30°$



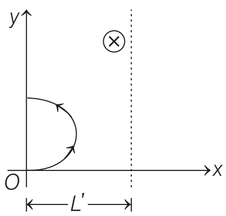
$\sin\theta = \dfrac{L}{R}$

Here, $R = \dfrac{mv_0}{B_0 q}$

$\therefore$ $\sin 30° = \dfrac{L}{\dfrac{mv_0}{B_0 q}}$

or $\dfrac{1}{2} = \dfrac{B_0 qL}{mv_0} \Rightarrow L = \dfrac{mv_0}{2B_0 q}$

(b) In part (a)



$\sin 30° = \dfrac{L}{R}$ or $\dfrac{1}{2} = \dfrac{L}{R}$

or $L = R/2$

Now when $L' = 2.1\,L$

or $\dfrac{2.1}{2}R \Rightarrow L' > R$

Therefore, deviation of the particle is $\theta = 180°$ as shown.

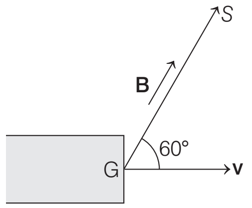
$\therefore$ $\mathbf{v}_f = -v_0\hat{\mathbf{i}}$

and $t_{AB} = T/2 = \dfrac{\pi m}{B_0 q}$

34. Kinetic energy of electron, $K = \dfrac{1}{2}mv^2 = 2$ keV

$\therefore$ Speed of electron, $v = \sqrt{\dfrac{2K}{m}}$

$$v = \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-16}}{9.1 \times 10^{-31}}}\ \text{m/s} = 2.65 \times 10^7\ \text{m/s}$$



Since, the velocity ($\mathbf{v}$) of the electron makes an angle of $\theta = 60°$ with the magnetic field $\mathbf{B}$, the path will be a helix. So, the particle will hit S if GS = np

Here, $n = 1, 2, 3$ ....

p = pitch of helix $= \dfrac{2\pi m}{qB} v\cos\theta$

But for B to be minimum, $n = 1$

Hence, $GS = p = \dfrac{2\pi m}{qB} v\cos\theta$

$$B = B_{\min} = \frac{2\pi mv\cos\theta}{q(GS)}$$

Substituting the values, we have

$$B_{\min} = \frac{(2\pi)(9.1 \times 10^{-31})(2.65 \times 10^7)\left(\frac{1}{2}\right)}{(1.6 \times 10^{-19})(0.1)}$$

or $B_{\min} = 4.73 \times 10^{-3}$ T

35. (i) $r = \dfrac{mv\sin\theta}{Bq} = \dfrac{(1.67 \times 10^{-27})(4 \times 10^5)(\sin 60°)}{(0.3)(1.6 \times 10^{-19})}$

$= 1.2 \times 10^{-2}$ m

(ii) $p = \left(\dfrac{2\pi m}{Bq}\right)(v\cos\theta)$

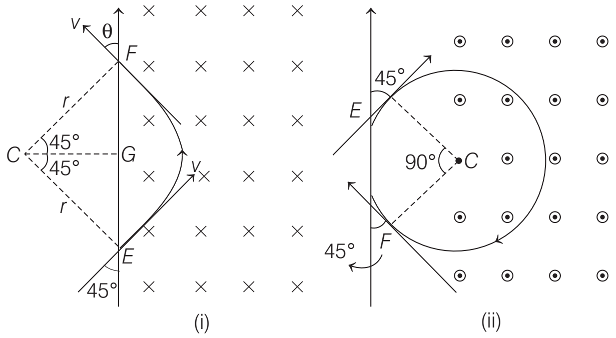
$$= \frac{(2\pi)(1.67 \times 10^{-27})(4 \times 10^5)(\cos 60°)}{(0.3)(1.6 \times 10^{-19})}$$

$= 4.37 \times 10^{-2}$ m

36. Inside a magnetic field speed of charged particle does not change. Further, velocity is perpendicular to magnetic field in both the cases hence path of the particle in the magnetic field will be circular. Centre of circle can be obtained by drawing perpendiculars to velocity (or tangent to the circular path) at E and F. Radius and angular speed of circular path would be

$$r = \frac{mv}{Bq}$$

and $\omega = \dfrac{Bq}{m}$



(i) (ii)

(a) Refer figure (i)

$\angle CFG = 90° - \theta$ and $\angle CEG = 90° - 45° = 45°$

Since, $CF = CE$

$\therefore \angle CFG = \angle CEG$

or $90° - \theta = 45°$ or $\theta = 45°$

Further, $FG = GE = r\cos 45°$

$$\therefore EF = 2FG = 2r\cos 45° = \frac{2mv\cos 45°}{Bq}$$

$$= \frac{2\,(1.6\times 10^{-27})\,(10^7)\left(\frac{1}{\sqrt{2}}\right)}{(1)\,(1.6\times 10^{-19})} = 0.14 \text{ m}$$

NOTE That in this case particle completes 1/4th of circle in the magnetic field.

(b) Refer figure (ii) In this case particle will complete $\frac{3}{4}$th of circle in the magnetic field. Hence, the time spent in the magnetic field

$$t = \frac{3}{4} \quad \text{(time period of circular motion)}$$

$$= \frac{3}{4}\left(\frac{2\pi m}{Bq}\right) = \frac{3\pi m}{2Bq}$$

$$= \frac{(3\pi)\,(1.6\times 10^{-27})}{(2)\,(1)(1.6\times 10^{-19})}$$

$$= 4.712\times 10^{-8} \text{ s}$$

37. $\mathbf{F}_e = q\,\mathbf{E} = (1.6\times 10^{-19})(-\,102.4\times 10^3)\hat{\mathbf{k}}$

$$= -\,(1.6384\times 10^{-16})\hat{\mathbf{k}} \text{ N}$$

$$\mathbf{F}_m = q\,(\mathbf{v}\times\mathbf{B}) = 1.6\times 10^{-19}\,(1.28\times 10^6\,\hat{\mathbf{i}}\times 8\times 10^{-2}\,\hat{\mathbf{j}})$$

$$= 1.6384\times 10^{-16}\,\hat{\mathbf{k}} \text{ N}$$

Since, $\mathbf{F}_e + \mathbf{F}_m = 0$

$\therefore$ Net force on the charged particle is zero. Particle will move undeviated.

In time $t = 5\times 10^{-6}$ s, the x-coordinate of particle will become,

$$x = v_x t = (1.28\times 10^6)\,(5\times 10^{-6}) = 6.4 \text{ m}$$

while y and z-coordinates will be zero.

At $x = 5\times 10^{-6}$ s, electric field is switched-off. Only magnetic field is left which is perpendicular to its velocity. Hence, path of the particle will now become circular.

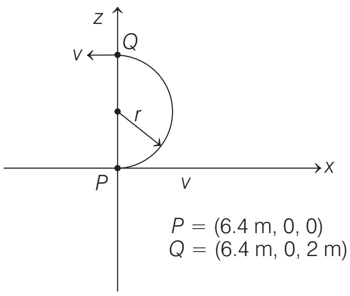
Plane of circle will be perpendicular to magnetic field i.e. x-z. Radius and angular velocity of circular path will become

$$r = \frac{mv}{Bq} = \frac{(10^{-26})\,(1.28\times 10^6)}{(8\times 10^{-2})\,(1.6\times 10^{-19})} = 1\text{m}$$

$$\omega = \frac{Bq}{m} = \frac{(8\times 10^{-2})\,(1.6\times 10^{-19})}{(10^{-26})}$$

$$= 1.28\times 10^6 \text{ rad/s}$$

In the remaining time i.e. $(7.45 - 5)\times 10^{-6} = 2.45\times 10^{-6}$s



Angle rotated by particle,

$$\theta = \omega t = (1.28\times 10^6)\,(2.45\times 10^{-6}) = 3.14 \text{ rad} \approx 180°$$

$\therefore$ z-coordinate of particle will become

$$z = 2r = 2m$$

while y-coordinate will be zero.

$\therefore$ Position of particle at $t = 5\times 10^{-6}$ s is $P \equiv (6.4 \text{ m}, 0, 0)$

and at $t = 7.45\times 10^{-6}$ s is $Q \equiv (6.4\text{m}, 0, 2\text{m})$

38. Electron pass undeviated. Therefore,

$$|\mathbf{F}_e| = |\mathbf{F}_m| \quad \text{or} \quad eE = eBv$$

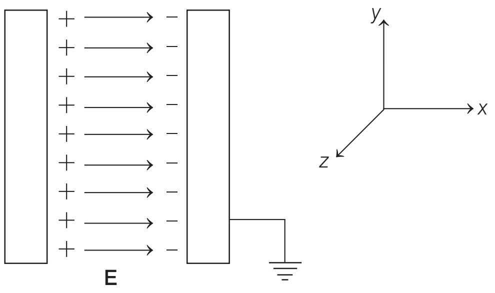
or $B = \dfrac{E}{v} = \dfrac{V/d}{v}$

(V = potential difference between the plates)

or $B = \dfrac{V}{dv}$

Substituting the values, we have

$$B = \frac{600}{3\times 10^{-3}\times 2\times 10^6} = 0.1\text{T}$$



Further, direction of $\mathbf{F}_e$ should be opposite of $\mathbf{F}_m$.

or $\quad e\mathbf{E} \uparrow\downarrow e(\mathbf{v} \times \mathbf{B})$

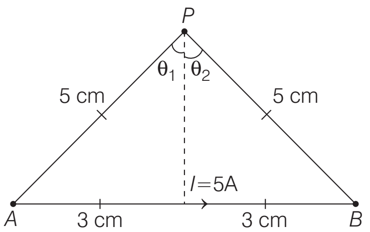
$\therefore \quad \mathbf{E} \uparrow\downarrow \mathbf{v} \times \mathbf{B}$

Here, **E** is in positive x-direction.

Therefore, $\mathbf{v} \times \mathbf{B}$ should be in negative x-direction or **B** should be in negative z-direction or perpendicular to paper inwards, because velocity of electron is in positive y-direction.

Topic 2 Biot-Savart Law and Ampere's Circuital Law

1. The given figure can be drawn as shown below



By Biot-Savart's law, magnetic field due to a wire segment at point P is

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

Here, $\quad \theta_1 = \theta_2 = \theta \quad$ (say)

Then, $$B = \frac{\mu_0 I}{4\pi d} \times 2\sin\theta \quad \text{...(i)}$$

From given data,

$$I = 5\text{A},$$
$$\mu = 4\pi \times 10^{-7}\ \text{NA}^{-2}$$
$$d = \sqrt{5^2 - 3^2} = \sqrt{16} = 4\ \text{cm}$$
$$\sin\theta = \frac{3}{5}$$

On substituting these values in Eq. (i), we get

$$B = \frac{\mu_0 I}{4\pi d} \times 2\sin\theta = \frac{4\pi \times 10^{-7} \times 5}{4\pi \times 4 \times 10^{-2}} \times 2 \times \left(\frac{3}{5}\right)$$
$$= \frac{5 \times 2 \times 3}{4 \times 5} \times 10^{-5} = 1.5 \times 10^{-5}\ \text{T}$$

2. Given, $\quad \mu_0 = 4\pi \times 10^{-7}\ \text{N/A}^2$,

$$\omega = 40\pi\ \text{rad/s}$$
$$B_{\text{at centre}} = 3.8 \times 10^{-9}\ \text{T}$$

and $\quad R = 10\,\text{cm} = 0.1\,\text{m}$

Now, we know that, magnetic field at the centre of a current carrying ring is given by

$$B = \frac{\mu_0 I}{2r} \quad \text{...(i)}$$

Here, I can be determined by flow of charge per rotation, i.e.

$$I = \frac{Q}{T} \quad \text{...(ii)}$$

Here, $$T = \frac{2\pi}{\omega}$$

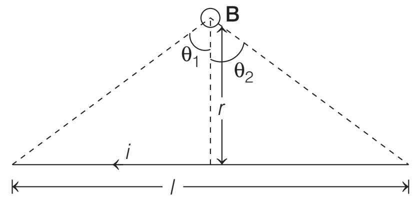
$$\Rightarrow \quad I = \frac{Q\omega}{2\pi} \quad \text{...(iii)}$$

By putting value of I from Eq. (iii) to Eq. (i), we get

$$B = \frac{\mu_0 Q\omega}{2r \times 2\pi} \text{ or } Q = \frac{2Br \times 2\pi}{\mu_0 \omega}$$
$$= \frac{2 \times 3.8 \times 10^{-9} \times 0.1 \times 2\pi}{4\pi \times 10^{-7} \times 40\pi}$$
$$= \frac{2 \times 3.8 \times 0.1}{2 \times 40\pi} \times 10^{-2}$$
$$= 0.003022 \times 10^{-2}\ \text{C}$$
$$= 3.022 \times 10^{-5}\ \text{C}$$

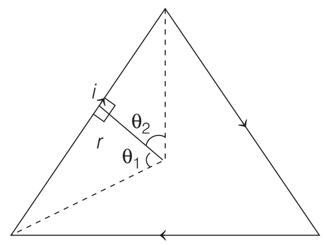
or $\quad Q = 3 \times 10^{-5}\ \text{C}$

3. For a current carrying wire, from result obtained by Biot-Savart's law, magnetic field at a distance r is given by



$$B = \frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

Now, in given case,



Due to symmetry of arrangement, net field at centre of triangle is

$$B_{\text{net}} = \text{Sum of fields of all wires (sides)}$$
$$= 3 \times \frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = \theta_2 = 60°$

$\therefore \quad \sin\theta_1 = \sin\theta_2 = \dfrac{\sqrt{3}}{2}, i = 10\text{A}, \dfrac{\mu_0}{4\pi} = 10^{-7}\text{NA}^{-2}$

and $\quad r = \dfrac{1}{3} \times \text{altitude}$

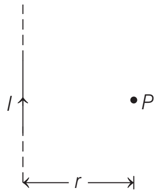
$$= \frac{1}{3} \times \frac{\sqrt{3}}{2} \times \text{sides length} = \frac{1}{2\sqrt{3}} \times 1\,\text{m} = \frac{1}{2\sqrt{3}}\,\text{m}$$

So,

$$B_{\text{net}} = \frac{3 \times 10^{-7} \times 10 \times 2\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2\sqrt{3}}\right)} = 18 \times 10^{-6}\ \text{T}$$

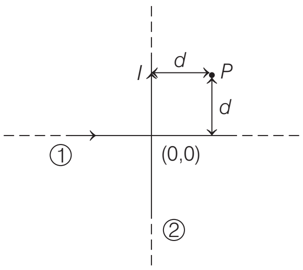
$\Rightarrow B_{\text{net}} = 18\,\mu\text{T}$

4. Magnetic field due to an infinitely long straight wire at point P is given as



$$|\mathbf{B}| = \frac{\mu_0 I}{2\pi r}$$

Thus, in the given situation, magnetic field due to wire 1 at point P is



$$\mathbf{B}_1 = \frac{\mu_0}{2\pi}\frac{I}{d}, \otimes$$

Similarly, magnetic field due to wire 2 at point P is

$$\mathbf{B}_2 = \frac{\mu_0}{2\pi}\frac{I}{d},$$

Resultant field at point P is

$$\mathbf{B}_{\text{net}} = \mathbf{B}_1 + \mathbf{B}_2$$

Since, $|\mathbf{B}_1| = |\mathbf{B}_2|$, but they are opposite in direction.

Thus, $\mathbf{B}_{\text{net}} = 0$

$\therefore$ Net magnetic field at point P will be zero.

5. There is no magnetic field along axis of a current-carrying wire.

Also, magnetic field near one of end of an infinitely long wire is $\frac{\mu_0 I}{4\pi r}$ tesla.

Hence, magnetic field due to segments LP and MQ at 'O' is zero.

Using right hand rule, we can check that magnetic field due to segments PS and QN at 'O' is in same direction perpendicularly into the plane of paper.

Hence, $B_O = B_{PS} + B_{QN}$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} = \frac{\mu_0 i}{2\pi r}$$

So, $\quad i = \frac{2\pi r B_0}{\mu_0}$

Here, $r = OP = OQ = 4$ cm

and $B_O = 10^{-4}$ T.

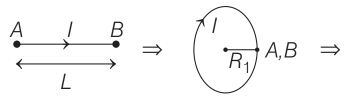
Substituting values, we get

$$\Rightarrow i = \frac{2\pi \times 4 \times 10^{-2} \times 10^{-4}}{4\pi \times 10^{-7}}$$

$\Rightarrow i = 20\text{A}$, Also, magnetic field points perpendicular into the plane of paper.

6. Let consider the length of first wire is L, then according to question, if radius of loop formed is R_1, then,

For wire 1,

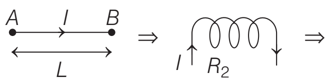


$$L = 2\pi R_1 \Rightarrow R_1 = \frac{L}{2\pi}$$

The magnetic field due to this loop at its centre is

$$B_L = \frac{\mu_0 I}{2R_1} = \frac{\mu_0 I}{2L} \times 2\pi \qquad \ldots\text{(i)}$$

Now, several wire is made into a coil of N- turns.



(R_2=radius of coil having N loops)

Thin, $\quad L = N(2\pi R_2) \Rightarrow R_2 = \frac{L}{2\pi N}$

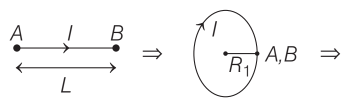
The magnetic field due to this circular coil of N-turns is

$$B_C = \left(\frac{\mu_0 I}{2R_2}\right) N = N \cdot \frac{\mu_0 I \cdot (2\pi N)}{2L} \qquad \ldots\text{(ii)}$$

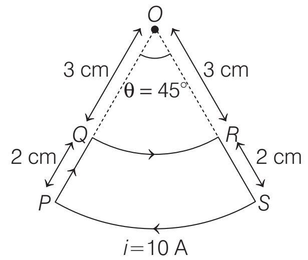
Using Eqs. (i) and (ii), the ratio of $\frac{B_L}{B_C}$ is :

$$\Rightarrow \quad \frac{B_L}{B_C} = \frac{\frac{\mu_0 I}{2R_1}}{N\frac{\mu_0 I}{2R_2}} = \frac{\frac{\mu_0 I}{2L}\cdot(2\pi)}{\frac{\mu_0 I}{2L}\cdot(2\pi)N^2} = \frac{1}{N^2}$$

7. **Key Idea** When a point 'P' lies on the axial position of current-carrying conductor, then magnetic field at P is always zero.



From the given figure as shown below,



The magnetic field at point 'O' due to wires PQ and RS will be zero.

Magnetic field due to arc QR at point 'O' will be

$$B_1 = \frac{\theta}{2\pi}\left(\frac{\mu_0 i}{2a}\right)$$

Here, $\theta = 45° = \frac{\pi}{4}$ rad, $i = 10$ A

and $\quad a = 3 \text{ cm} = 3 \times 10^{-2}$ m

$$\Rightarrow \quad B_1 = \frac{\pi}{2\pi \times 4}\left(\frac{\mu_0 \times 10}{2 \times 3 \times 10^{-2}}\right)$$

$$= \frac{\mu_0 \times 5}{2 \times 12 \times 10^{-2}} = \frac{5 \times \mu_0 \times 10^2}{24}$$

Direction of field B_1 will be coming out of the plane of figure.

Similarly, field at point 'O' due to arc SP will be

$$B_2 = \frac{\pi}{4}\left(\frac{1}{2\pi}\right)\left[\frac{\mu_0 \times 10}{2 \times (2+3) \times 10^{-2}}\right] = \frac{\mu_0 \times 5}{2 \times 20 \times 10^{-2}}$$

$$= \frac{\mu_0}{2 \times 4 \times 10^{-2}} = \frac{\mu_0 \times 10^2}{8}$$

Direction of B_2 is going into the plane of the figure.

$\therefore$ The resultant field at O is

$$B = B_1 - B_2 = \frac{1}{2}\left(\frac{5 \times \mu_0}{12 \times 10^{-2}} - \frac{\mu_0}{4 \times 10^{-2}}\right)$$

$$= \frac{1}{2}\left(\frac{5\mu_0 - 3\mu_0}{12 \times 10^{-2}}\right) = \frac{1}{2}\left(\frac{2\mu_0}{12 \times 10^{-2}}\right)$$

$$= \frac{4\pi \times 10^{-7}}{12 \times 10^{-2}} \cong 1 \times 10^{-5} \text{ T}$$

8. B at centre of a circle $= \frac{\mu_0 I}{2R}$

B at centre of a square

$$= 4 \times \frac{\mu I}{4\pi \cdot \frac{l}{2}}[\sin 45° + \sin 45°] = 4\sqrt{2}\,\frac{\mu_0 I}{2\pi l}$$

Now, $\quad R = \frac{L}{2\pi}$ and $l = \frac{L}{4}$ $\qquad$ (as $L = 2\pi R = 4l$)

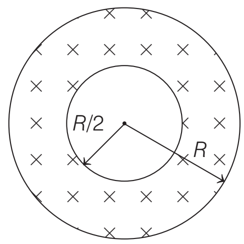
where, $\quad L$ = length of wire.

$$\therefore \quad B_A = \frac{\mu_0 I}{2 \cdot \frac{L}{2\pi}} = \frac{\pi \mu_0 I}{L} = \pi\left[\frac{\mu_0 I}{L}\right]$$

$$B_B = 4\sqrt{2}\,\frac{\mu_0 I}{2\pi\left(\frac{L}{4}\right)} = \frac{8\sqrt{2}\mu_0 I}{\pi L} = \frac{8\sqrt{2}}{\pi}\left[\frac{\mu_0 I}{L}\right]$$

$$\therefore \quad \frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

9.



r = distance of a point from centre.

For $r \le R/2$ Using Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} \quad \text{or} \quad Bl = \mu_0 (I_{in})$$

or $\qquad B\,(2\pi r) = \mu_0\,(I_{in})$ or $\quad B = \frac{\mu_0}{2\pi}\frac{I_{in}}{r}$ $\qquad$ ...(i)

Since, $\quad I_{in} = 0 \Rightarrow \therefore \quad B = 0$

For $\frac{R}{2} \le r \le R$ $\quad I_{in} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right]\sigma$

Here, σ = current per unit area

Substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi}\frac{\left[\pi r^2 - \pi\frac{R^2}{4}\right]\sigma}{r} = \frac{\mu_0 \sigma}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $\quad r = \frac{R}{2}$, $B = 0$

At $\quad r = R$, $B = \frac{3\mu_0 \sigma R}{8}$

For $r \ge R$ $\qquad I_{in} = I_{Total} = I$ (say)

Therefore, substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \quad \text{or} \quad B \propto \frac{1}{r}$$

10. If we take a small strip of dr at distance r from centre, then number of turns in this strip would be,

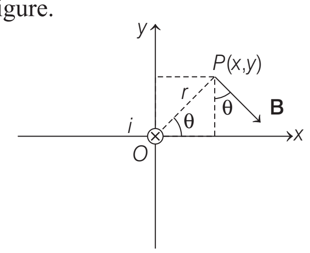
$$dN = \left(\frac{N}{b-a}\right)dr$$

Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0\,(dN)I}{2r} = \frac{\mu_0 NI}{(b-a)}\frac{dr}{r}$$

$$\therefore \qquad B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$$

11. Magnetic field at P is **B**, perpendicular to OP in the direction shown in figure.



So, $\mathbf{B} = B\sin\theta\hat{\mathbf{i}} - B\cos\theta\hat{\mathbf{j}}$

Here, $B = \dfrac{\mu_0 I}{2\pi r}$

$$\sin\theta = \frac{y}{r} \quad \text{and} \quad \cos\theta = \frac{x}{r}$$

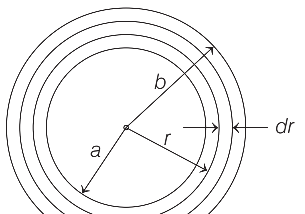
$$\therefore \quad \mathbf{B} = \frac{\mu_0 I}{2\pi}.\frac{1}{r^2}(y\hat{\mathbf{i}} - x\hat{\mathbf{i}})$$

$$= \frac{\mu_0 I(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})}{2\pi(x^2 + y^2)} \qquad (\text{as } r^2 = x^2 + y^2)$$

12. Consider an element of thickness dr at a distance r from the centre. The number of turns in this element,

$$dN = \left(\frac{N}{b-a}\right)dr$$

Magnetic field due to this element at the centre of the coil will be

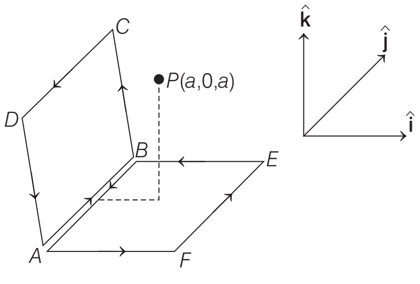


$$dB = \frac{\mu_0(dN)I}{2r} = \frac{\mu_0 I}{2}\frac{N}{b-a}.\frac{dr}{r}$$

$$\therefore \quad B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

NOTE The idea of this question is taken from question number 3.245 of IE Irodov.

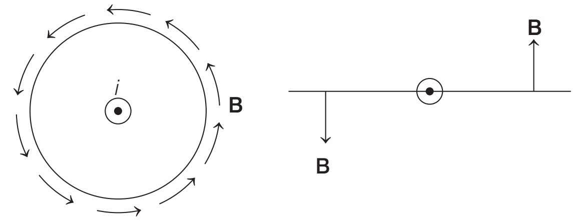
13. The magnetic field at $P(a, 0, a)$ due to the loop is equal to the vector sum of the magnetic fields produced by loops $ABCDA$ and $AFEBA$ as shown in the figure.



Magnetic field due to loop $ABCDA$ will be along $\hat{\mathbf{i}}$ and due to loop $AFEBA$, along $\hat{\mathbf{k}}$. Magnitude of magnetic field due to both the loops will be equal. Therefore, direction of resultant magnetic field at P will be $\dfrac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$.

NOTE This is a common practice, when by assuming equal currents in opposite directions in an imaginary wire (here AB) loops are completed and solution becomes easy.

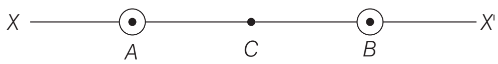
14. If the current flows out of the paper, the magnetic field at points to the right of the wire will be upwards and to the left will be downwards as shown in figure.



Now, let us come to the problem.

Magnetic field at $C = 0$

Magnetic field in region BX' will be upwards (+ve) because all points lying in this region are to the right of both the wires.



Magnetic field in region AC will be upwards (+ve), because points are closer to A, compared to B. Similarly, magnetic field in region BC will be downwards (–ve).

Graph (b) satisfies all these conditions. Therefore, correct answer is (b).

15. H_1 = Magnetic field at M due to PQ + Magnetic field at M due to QR. But magnetic field at M due to $QR = 0$

$\therefore$ Magnetic field at M due to PQ (or due to current I in PQ) $= H_1$

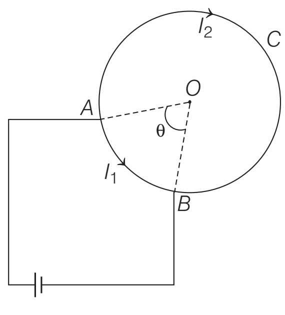
Now H_2 = Magnetic field at M due to PQ (current I)
+ magnetic field at M due to QS (current $I/2$)
+ magnetic field at M due to QR

$$= H_1 + \frac{H_1}{2} + 0 = \frac{3}{2}H_1$$

$$\frac{H_1}{H_2} = \frac{2}{3}$$

NOTE Magnetic field at any point lying on the current carrying straight conductor is zero.

16. For a current flowing into a circular arc, the magnetic induction at the centre is



$$B = \left(\frac{\mu_0 i}{4\pi r}\right)\theta \text{ or } B \propto i\theta$$

In the given problem, the total current is divided into two arcs

$$i \propto \frac{1}{\text{resistance of arc}} \propto \frac{1}{\text{length of arc}}$$

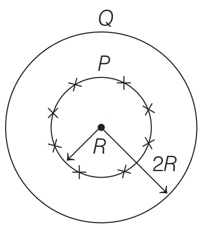
$$\propto \frac{1}{\text{angle subtended at centre }(\theta)}$$

or $\quad i\theta = \text{constant}$

i.e. magnetic field at centre due to arc AB is equal and opposite to the magnetic field at centre due to arc ACB. Or the net magnetic field at centre is zero.

17. Using Ampere's circuital law over a circular loop of any radius less than the radius of the pipe, we can see that net current inside the loop is zero. Hence, magnetic field at every point inside the loop will be zero.

18.



$P \rightarrow$ Hollow cylindrical conductor
$Q \rightarrow$ Solenoid

In the region, $0 < r < R$

$$B_P = 0,$$

$B_Q \neq 0$, along the axis

$\therefore \quad B_{\text{net}} \neq 0$

In the region, $\quad R < r < 2R$

$B_P \neq 0$, tangential to the circle of radius r, centred on the axis.

$B_Q \neq 0$, along the axis.

$\therefore\ B_{\text{net}} \neq 0$ neither in the directions mentioned in options (b) or (c). In region, $r > 2R$

$$B_P \neq 0$$

$$B_Q \neq 0 \Rightarrow B_{\text{net}} \neq 0$$

19. $B_R = B_T - B_C$

R = Remaining portion

T = Total portion and

C = Cavity

$$B_R = \frac{\mu_0 I_T}{2a\pi} - \frac{\mu_0 I_C}{2\,(3a/2)\pi} \qquad \ldots\text{(i)}$$

$$I_T = J\,(\pi a^2)$$

$$I_C = J\left(\frac{\pi a^2}{4}\right)$$

Substituting the values in Eq. (i), we have

$$B_R = \frac{\mu_0}{a\pi}\left[\frac{I_T}{2} - \frac{I_C}{3}\right]$$

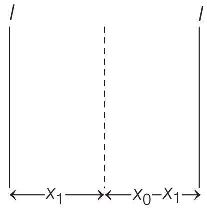
$$= \frac{\mu_0}{a\pi}\left[\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right] = \frac{5\mu_0 aJ}{12}$$

$\therefore \quad N = 5$

20. $B_2 = \dfrac{\mu_0 I}{2\pi x_1} + \dfrac{\mu_0 I}{2\pi\,(x_0 - x_1)}$

(when currents are in opposite directions)

$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi\,(x_0 - x_1)}$$



(when currents are in same direction)

Substituting $x_1 = \dfrac{x_0}{3} \qquad \left(\text{as } \dfrac{x_0}{x_1} = 3\right)$

$$B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$$

$$R_1 = \frac{mv}{qB_1} \text{ and } B_2 = \frac{9\mu_0 I}{4\pi x_0}$$

$$R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \qquad \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$$

21. At C magnetic field due to wires PQ and RS will be zero. Due to wire QR,

$$B_1 = \frac{1}{2}\left(\frac{\mu_0 I}{2R_1}\right) = \frac{\mu_0 I}{4R_1} \quad \text{(perpendicular to paper outwards)}$$

And due to wire SP,

$$B_2 = \frac{1}{2}\left(\frac{\mu_0 I}{2R_2}\right) = \frac{\mu_0 I}{4R_2} \quad \text{(perpendicular to paper inwards)}$$

$\therefore$ Net magnetic field would be,

$$B = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(perpendicular to paper outwards)}$$

22. Magnetic field at O due to L and M is zero. Due to P magnetic field at O is

$$B_1 = \frac{1}{2}\left(\frac{\mu_0}{2\pi}\frac{i}{OR}\right) = \frac{(10^{-7})(10)}{0.02} = 5.0 \times 10^{-5}\,\text{T}$$

(perpendicular to paper outwards)

Similarly, field at O due to Q would be,

$$B_2 = \frac{1}{2}\left(\frac{\mu_0}{2\pi}\frac{i}{OS}\right)$$

$$= \frac{(10^{-7})(10)}{0.02} = 5.0 \times 10^{-5}\,\text{T}$$

(perpendicular to paper outwards)

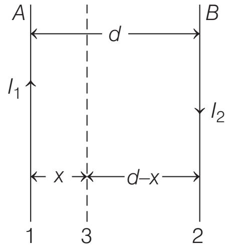
Since, both the fields are in same direction, net field will be sum of these two.

$\therefore \qquad B_{\text{net}} = B_1 + B_2 = 10^{-4}\,\text{T}$

Direction of field is perpendicular to the paper outwards.

Topic 3 Magnetic Force on Current Carrying Wires

1. Net force on the third wire, carrying current I in the following first case is



$$F_{13} + F_{23} = 0$$

Using thumb rule, direction of **B** at inside region of wires A and B will be same.

$$\therefore \quad \frac{\mu_0 I_1 I}{2\pi x} + \frac{\mu_0 I_2 I}{2\pi (d-x)} = 0$$

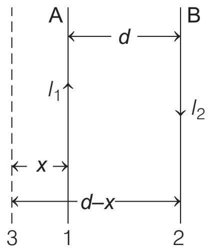
$$\Rightarrow \quad \frac{I_1}{x} + \frac{I_2}{d-x} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{x-d} \text{ or } (x-d) I_1 = x I_2$$

$$\Rightarrow \quad x(I_1 - I_2) = dI_1$$

$$\Rightarrow \quad x = \frac{I_1}{(I_1 - I_2)} \cdot d \qquad \ldots \text{(i)}$$

Second case of balanced force can be as shown



Using thumb rule, directions of **B** at any point on wires A and B will be opposite, so net force,

$$\frac{\mu_0 I_1 I}{2\pi x} - \frac{\mu_0 I_2 I}{2\pi (d+x)} = 0 \text{ or } \frac{I_1}{x} - \frac{I_2}{(d+x)} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{d+x}$$

$$\Rightarrow \quad (d+x) I_1 = x I_2$$

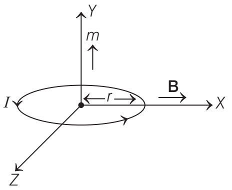
$$\Rightarrow \quad (I_2 - I_1) x = dI_1$$

$$\Rightarrow \quad x = -\frac{I_1}{(I_1 - I_2)} \cdot d \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), it is clear that

$$x = \pm \frac{I_1}{(I_1 - I_2)} d$$

2. According to the question, the situation can be drawn as



Let the current I is flowing in anti-clockwise direction, then the magnetic moment of the coil is

$$m = NIA$$

where, $\quad N$ = number of turns in coil

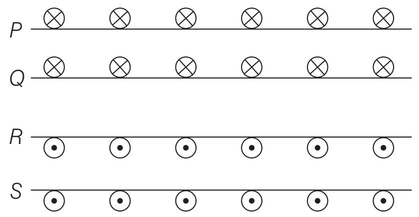
and A = area of each coil = πr^2.

Its direction is perpendicular to the area of coil and is along Y-axis.

Then, torque on the current coil is

$$\tau = \mathbf{m} \times \mathbf{B} = mB \sin 90^\circ = NIAB = NI\pi r^2 B \text{(N-m)}$$

3.

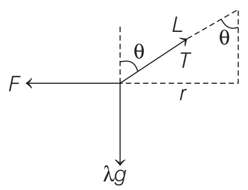


If we calculate the force on inner solenoid. Force on Q due to P is outwards (attraction between currents in same direction. Similarly, force on R due to S is also outwards. Hence, net force $\mathbf{F}_1$ is zero)

Force on P due to Q and force on S due to R is inwards. Hence, net force $\mathbf{F}_2$ is also zero.

Alternate Thought Field of one solenoid is uniform and other solenoid may be assumed a combination of circular closed loops. In uniform magnetic field, net force on a closed current carrying loop is zero.

4.



$$r = L \sin\theta$$

F = Magnetic force (repulsion) per unit length

$$= \frac{\mu_0}{2\pi} \frac{I^2}{2r} = \frac{\mu_0}{4\pi} \frac{I^2}{L \sin\theta}$$

λg = weight per unit length

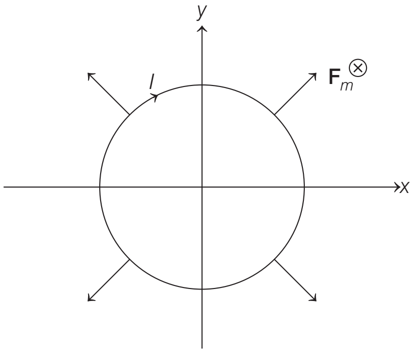
Each wire is in equilibrium under three concurrent forces as shown in figure. Therefore applying Lami's theorem.

$$\frac{F}{\sin(180^\circ - \theta)} = \frac{\lambda g}{\sin(90^\circ + \theta)}$$

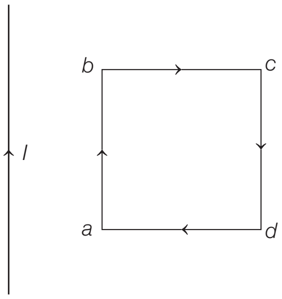
or $$\frac{\frac{\mu_0}{4\pi}\frac{I^2}{L\sin\theta}}{\sin\theta}=\frac{\lambda g}{\cos\theta}$$

$\therefore$ $$I=2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$$

5. Net force on a current carrying loop in uniform magnetic field is zero. Hence, the loop cannot translate. So, options (c) and (d) are wrong. From Fleming's left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\mathbf{F}_m$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



6. Force per unit length between two wires carrying currents i_1 and i_2 at distance r is given by



$$\frac{F}{l}=\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$$

Here, $i_1=i_2=i$

and $r=b$

$\therefore$ $$\frac{F}{l}=\frac{\mu_0 i^2}{2\pi b}$$

7. Straight wire will produce a non-uniform field to the right of it. $\mathbf{F}_{bc}$ and $\mathbf{F}_{da}$ will be calculated by integration but these two forces will cancel each other. Further force on wire ab will be towards the long wire and on wire cd will be away from the long wire. But since the wire ab is nearer to the long wire, force of attraction towards the long wire will be more. Hence, the loop will move towards the wire.

8. Net force is on an imaginary wire FA having current I from F to A.

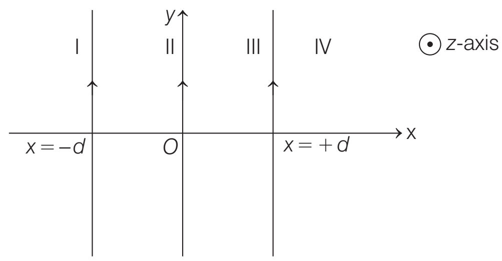
$$\mathbf{F}=I[(L\hat{\mathbf{i}})\times(B\hat{\mathbf{j}})]=ILB\hat{\mathbf{k}}$$

$\because$ Magnitude of force is ILB and direction of force is positive z.

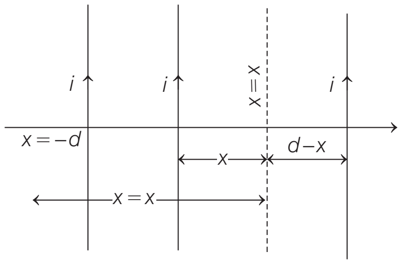
9. A current carrying coil is a magnetic dipole. Net magnetic force on a magnetic dipole in uniform field is zero.

10. Magnetic field will be zero on the y-axis i.e.

$$x=0=z$$



Magnetic field cannot be zero in region I and region IV because in region I magnetic field will be along positive z-direction due to all the three wires, while in region IV magnetic field will be along negative z-axis due to all the three wires. It can zero only in region II and III.



Let magnetic field is zero on line ($z=0$) and $x=x$. Then magnetic field on this line due to wires 1 and 2 will be along negative z-axis and due to wire 3 along positive z-axis. Thus

$$B_1+B_2=B_3$$

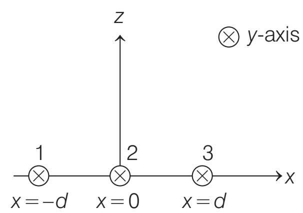
or $$\frac{\mu_0}{2\pi}\frac{i}{d+x}+\frac{\mu_0}{2\pi}\frac{i}{x}=\frac{\mu_0}{2\pi}\frac{i}{d-x}$$

or $$\frac{1}{d+x}+\frac{1}{x}=\frac{1}{d-x}$$

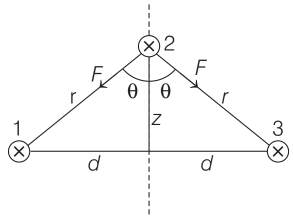
This equation givesz $x=\pm\dfrac{d}{\sqrt{3}}$

where magnetic field is zero.

(b) In this part, we change our coordinate axes system, just for better understanding.



There are three wires 1, 2 and 3 as shown in figure. If we displace the wire 2 towards the z-axis, then force of attraction per unit length between wires (1 and 2) and (2 and 3) will be given as



$$F = \frac{\mu_0}{2\pi}\frac{i^2}{r}$$

The components of F along x-axis will be cancelled out. Net resultant force will be towards negative z-axis (or mean position) and will be given by

$$F_{\text{net}} = \frac{\mu_0}{2\pi}\frac{i^2}{r}(2\cos\theta) = 2\left\{\frac{\mu_0}{2\pi}\frac{i^2}{r}\right\}\frac{z}{r}$$

$$F_{\text{net}} = \frac{\mu_0}{\pi}\frac{i^2}{(z^2+d^2)}z$$

If $z << d$, then

$$z^2 + d^2 = d^2 \text{ and } F_{\text{net}} = -\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right).z$$

Negative sign implies that F_{net} is restoring in nature

Therefore, $F_{\text{net}} \propto -z$

i.e. the wire will oscillate simple harmonically.

Let a be the acceleration of wire in this position and λ is the mass per unit length of this wire then

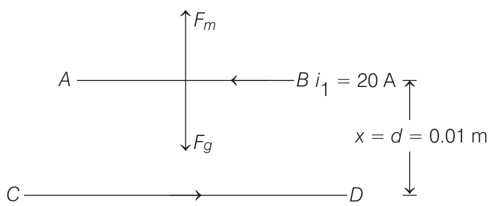
$$F_{\text{net}} = \lambda\,.\,a = -\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right)z \quad \text{or} \quad a = -\left(\frac{\mu_0 i^2}{\pi\lambda d^2}\right).z$$

$\therefore$ Frequency of oscillation

$$f = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}} = \frac{1}{2\pi}\sqrt{\frac{a}{z}}$$

$$= \frac{1}{2\pi}\frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}} \quad \text{or} \quad f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

11. Let m be the mass per unit length of wire AB. At a height x above the wire CD, magnetic force per unit length on wire AB will be given by



$$F_m = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{x} \quad \text{(upwards)} \qquad \ldots\text{(i)}$$

Weight per unit length of wire AB is

$$F_g = mg \qquad \text{(downwards)}$$

Here, $m =$ mass per unit length of wire AB

At $x = d$, wire is in equilibrium i.e.,

$$F_m = F_g$$

or $$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$$

or $$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d^2} = \frac{mg}{d} \qquad \ldots\text{(ii)}$$

When AB is depressed, x decreases therefore, F_m will increase, while F_g remains the same. Let AB is displaced by dx downwards.

Differentiating Eq. (i) w.r.t. x, we get

$$dF_m = -\frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2}.dx \qquad \ldots\text{(iii)}$$

i.e. restoring force, $F = dF_m \propto -dx$

Hence, the motion of wire is simple harmonic.

From Eqs. (ii) and (iii), we can write

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad (\because x = d)$$

$\therefore$ Acceleration of wire $a = -\left(\dfrac{g}{d}\right).dx$

Hence, period of oscillation

$$T = 2\pi\sqrt{\left|\frac{dx}{a}\right|}$$

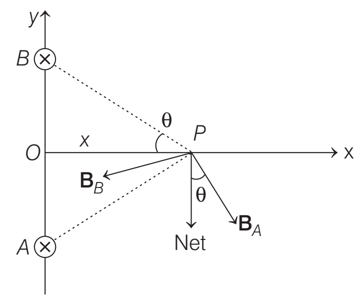
$$= 2\pi\sqrt{\frac{|\text{displacement}|}{|\text{acceleration}|}}$$

or $$T = 2\pi\sqrt{\frac{d}{g}}$$

$$= 2\pi\sqrt{\frac{0.01}{9.8}}$$

or $$T = 0.2 \text{ s}$$

12. (a) Let us assume a segment of wire OC at a point P, a distance x from the centre of length dx as shown in figure.



Magnetic field at P due to current in wires A and B will be in the directions perpendicular to AP and BP respectively as shown.

$$|\mathbf{B}| = \frac{\mu_0}{2\pi}\frac{I}{AP}$$

Therefore, net magnetic force at P will be along negative y-axis as shown

$$B_{\text{net}} = 2|\mathbf{B}|\cos\theta = 2\left(\frac{\mu_0}{2\pi}\right)\frac{I}{AP}\left(\frac{x}{AP}\right)$$

$$B_{\text{net}} = \left(\frac{\mu_0}{\pi}\right)\frac{I.x}{(AP)^2}$$

$$B_{\text{net}} = \frac{\mu_0}{\pi}\cdot\frac{Ix}{(a^2+x^2)}$$

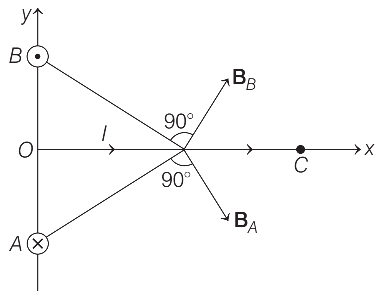
Therefore, force on this element will be

$$dF = I\left\{\frac{\mu_0}{\pi}\frac{Ix}{a^2+x^2}\right\}dx \qquad \text{(in negative } z\text{-direction)}$$

$\therefore$ Total force on the wire will be

$$F = \int_{x=0}^{x=L} dF = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{xdx}{x^2+a^2}$$

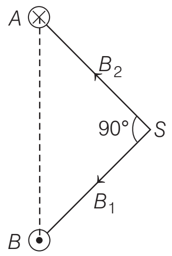
$$= \frac{\mu_0 I^2}{2\pi}\ln\left(\frac{L^2+a^2}{a^2}\right) \qquad \text{(in negative } z\text{-axis)}$$



Hence, $\mathbf{F} = -\dfrac{\mu_0 I^2}{2\pi}\ln\left(\dfrac{L^2+a^2}{a^2}\right)\hat{\mathbf{k}}$

(b) When direction of current in B is reversed, net magnetic field is along the current. Hence, force is zero.

13. (a) Direction of current at B should be perpendicular to paper outwards. Let current in this wire be i_B. Then,



$$\frac{\mu_0}{2\pi}\frac{i_A}{\left(2+\frac{10}{11}\right)} = \frac{\mu_0}{2\pi}\frac{i_B}{(10/11)}$$

or $\qquad \dfrac{i_B}{i_A} = \dfrac{10}{32}$

or $\qquad i_B = \dfrac{10}{32}\times i_A = \dfrac{10}{32}\times 9.6 = 3\text{A}$

(b) Since, $AS^2 + BS^2 = AB^2$

$\therefore \quad \angle ASB = 90°$

At S : B_1 = Magnetic field due to i_A

$$= \frac{\mu_0}{2\pi}\frac{i_A}{1.6}$$

$$= \frac{(2\times10^{-7})(9.6)}{1.6} = 12\times10^{-7}\ \text{T}$$

B_2 = Magnetic field due to i_B

$$= \frac{\mu_0}{2\pi}\frac{i_B}{1.2} = \frac{(2\times10^{-7})(3)}{1.2} = 5\times10^{-7}\ \text{T}$$

Since, B_1 and B_2 are mutually perpendicular. Net magnetic field at S would be

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{(12\times10^{-7})^2 + (5\times10^{-7})^2}$$

$$= 13\times10^{-7}\ \text{T}$$

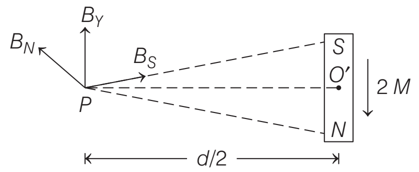
(c) Force per unit length on wire B

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_A i_B}{r} \qquad (r = AB = 2\text{ m})$$

$$= \frac{(2\times10^{-7})(9.6\times3)}{2} = 2.88\times10^{-6}\ \text{N/m}$$

Topic 4 Magnetic Dipole

1. Let $2l_1$ and $2l_2$ be the length of dipole X and Y, respectively. For dipole X, point P lies on its axial line. So, magnetic field strength at P due to X is



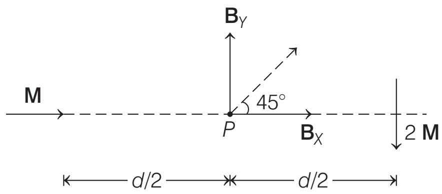
$$\mathbf{B}_X = \frac{\mu_0}{4\pi}\cdot\frac{2Mr}{(r^2-l_1^2)^2}, \text{ along } OP$$

Here, $\qquad r = \dfrac{d}{2}$

Also, $\qquad d >> l_1$

$\Rightarrow \quad |\mathbf{B}_X| = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2M(d/2)}{(d/2)^4} = \dfrac{\mu_0}{4\pi}\dfrac{2M}{(d/2)^3}$

Similarly, for dipole Y, point P lies on its equatorial line. So, magnetic field strength at P due to Y is



$$\mathbf{B}_Y = \frac{\mu_0}{4\pi}\cdot\frac{2M}{(r^2+l_2^2)^{3/2}}, \text{ (along a line perpendicular to } O'P)$$

Here, $\qquad r = \dfrac{d}{2}$

Also, $\qquad d >> l_2$

$\Rightarrow \qquad |\mathbf{B}_Y| = \dfrac{\mu_0}{4\pi}\dfrac{2M}{(d/2)^3}$

Thus, the resultant magnetic field due to X and Y at P is

$$\mathbf{B}_{\text{net}} = \mathbf{B}_X + \mathbf{B}_Y$$

Since, $\qquad |\mathbf{B}_Y| = |\mathbf{B}_X|$

Thus, the resultant magnetic field ($\mathbf{B}_{net}$) at P will be at 45° with the horizontal.

This means, direction of $\mathbf{B}_{net}$ and velocity of the charged particle is same.

$\therefore$ Force on the charged particle moving with velocity v in the presence of magnetic field which is

$$\mathbf{B} = q(\mathbf{v} \times \mathbf{B}) = q\,|\mathbf{v}||\mathbf{B}|\sin\theta$$

where, θ is the angle between $\mathbf{B}$ and $\mathbf{v}$.

According to the above analysis, we get

$$\theta = 0$$

$$\therefore \qquad \mathbf{F} = 0$$

Thus, magnitude of force on the particle at that instant is zero.

2. **Key Idea** Magnetic dipole moment of a current carrying loop is

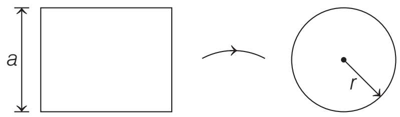
$$m = IA (\text{A - m}^2)$$

where, I = current in loop and A = area of loop.

Let the given square loop has side a, then its magnetic dipole moment will be

$$m = Ia^2$$

When square is converted into a circular loop of radius r,



Then, wire length will be same in both areas,

$$\Rightarrow \qquad 4a = 2\pi r \quad \Rightarrow \quad r = \frac{4a}{2\pi} = \frac{2a}{\pi}$$

Hence, area of circular loop formed is, $A' = \pi r^2$

$$= \pi\left(\frac{2a}{\pi}\right)^2 = \frac{4a^2}{\pi}$$

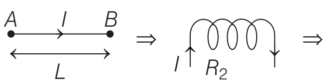
Magnitude of magnetic dipole moment of circular loop will be

$$m' = IA' = I\frac{4a^2}{\pi}$$

Ratio of magnetic dipole moments of both shapes is,

$$\frac{m'}{m} = \frac{I \cdot \frac{4a^2}{\pi}}{Ia^2} = \frac{4}{\pi} \quad \Rightarrow \quad m' = \frac{4m}{\pi}(\text{A-m})$$

3.

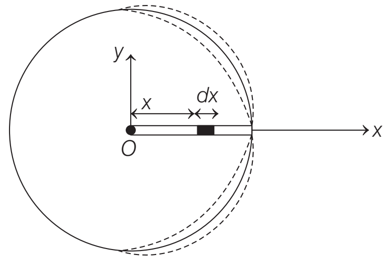


(R_2=radius of coil having N loops)

Key Idea A rotating charge constitutes a current. Hence, a rotating charged rod behaves like a current carrying coil.
If charge q rotates with a frequency n, then equivalent current is $I = qn$ and magnetic moment associated with this current is $M = IA$

where, A = area of coil or area swept by rotating rod.

Let dq be the charge on dx length of rod at a distance x from origin as shown in the figure below.



The magnetic moment dm of this portion dx is given as

$$dm = (dI)\,A$$

$$dm = ndqA \qquad [\because I = qn, \; \therefore dI = n.dq]$$

$$= n\,\rho\,dx\,A$$

where, ρ = charge density of rod $= \rho_0 \dfrac{x}{l}$.

So, $$dm = \frac{n\rho_0 x\,dx\,\pi x^2}{l} = \frac{\pi\,n\,\rho_0}{l} \cdot x^3 \cdot dx$$

Total magnetic moment associated with rotating rod is sum of all the magnetic moments of such differentiable elements of rod.

So, magnetic moment associated with complete rod is

$$M = \int_{x=0}^{x=l} dm$$

$$= \int_0^l \frac{\pi\,n\,\rho_0}{l} \cdot x^3\,dx = \frac{\pi\,n\,\rho_0}{l} \cdot \int_0^l x^3\,dx$$

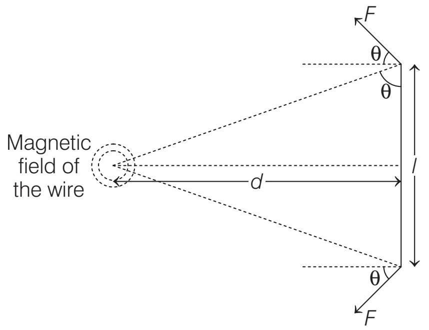
$$= \frac{\pi\,n\,\rho_0}{l}\left[\frac{x^4}{4}\right]_0^l = \frac{\pi\,n\,\rho_0\,l^3}{4}$$

at $\qquad x = l$

$$\rho = \rho_0$$

$$\therefore \qquad M = \frac{\pi}{4} n\rho l^3$$

4. In the given condition, the current-carrying loop is at a very large distance from the long current-carrying conducting wire. Thus, it can be considered as a dipole (a magnet with north pole facing in upward direction and south in the downward direction). Suppose the effective length of this dipole be 'l'. Thus, the top view of the condition can be shown in the figure given below.



Now, the net force on the loop (i.e. at the two poles) due to the wire is given as,

$$F_{\text{net}} = 2F\cos\theta = 2mB\cos\theta$$

where, m is the pole strength.

From the figure, we have

$$\cos\theta = \frac{\frac{l}{2}}{\sqrt{d^2 + \frac{l^2}{4}}}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{2mB\,l}{2\sqrt{d^2 + \frac{l^2}{4}}} \quad \text{...(i)}$$

Since, the magnetic moment of a loop of radius r is

$$M = IA = I\pi r^2 = ml \quad \text{...(ii)}$$

and magnetic field due to a straight infinitely long current-carrying conductor at a distance x is

$$B = \frac{\mu_0 I'}{2\pi x} \quad \text{...(iii)}$$

$\therefore$ Using Eqs. (ii) and (iii), rewriting Eq. (i), we get

$$F_{\text{net}} = \frac{I\pi a^2 \mu_0 I'}{2\pi\left(\sqrt{d^2 + \frac{l^2}{4}}\right)^2}$$

$$\Rightarrow \quad F_{\text{net}} \propto \frac{a^2}{\left(d^2 + \frac{l^2}{4}\right)}$$

l can be neglected as the loop is kept at very large distance.

$$\text{or} \quad F_{\text{net}} \propto \left(\frac{a}{d}\right)^2$$

5. $m = I \times \pi R^2$

$$2m = I \times \pi (R')^2 \quad \Rightarrow \quad R' = \sqrt{2}R$$

$$B = \frac{\mu_0 I}{2\pi R} \Rightarrow B \propto \frac{1}{R} \Rightarrow \frac{B_1}{B_2} = \frac{R'}{R} = \sqrt{2}$$

6. Direction of magnetic dipole moment **M** is given by screw law and this is perpendicular to plane of loop.

In stable equilibrium position, angle between **M** and **B** is 0° and in unstable equilibrium this angle is 180°.

7. Area of the given loop is

A = (area of two circles of radius $\frac{a}{2}$ and area of a square of side a)

$$= 2\pi\left(\frac{a}{2}\right)^2 + a^2 = \left(\frac{\pi}{2} + 1\right)a^2$$

$$|\mathbf{M}| = IA = \left(\frac{\pi}{2} + 1\right)a^2 I$$

From screw law direction of **M** is outwards or in positive z-direction.

$$\therefore \quad \mathbf{M} = \left(\frac{\pi}{2} + 1\right)a^2\, I\,\hat{\mathbf{k}}$$

8. $U = -\mathbf{M}\mathbf{B} = -MB\cos\theta$

Here, **M** = magnetic moment of the loop

θ = angle between **M** and **B**

U is maximum when $\theta = 180°$ and minimum when $\theta = 0°$. So, as θ decreases from 180° to 0°, its PE also decreases.

9. Ratio of magnetic moment and angular momentum is given by

$$\frac{M}{L} = \frac{q}{2m}$$

which is a function of q and m only. This can be derived as follows,

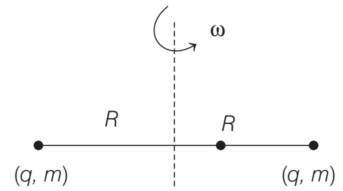
$$M = i\,A = (q\,f).(\pi r^2)$$

$$= (q)\left(\frac{\omega}{2\pi}\right)(\pi r^2) = \frac{q\omega r^2}{2}$$

$$\text{and} \quad L = I\omega = (mr^2\omega)$$

$$\therefore \quad \frac{M}{L} = \frac{q\frac{\omega r^2}{2}}{mr^2\omega} = \frac{q}{2m}$$

10. Current, i = (frequency) (charge) $= \left(\frac{\omega}{2\pi}\right)(2q) = \frac{q\omega}{\pi}$



Magnetic moment, $M = (i)(A) = \left(\frac{q\omega}{\pi}\right)(\pi R^2) = (q\omega R^2)$

Angular momentum, $L = 2I\omega = 2(mR^2)\,\omega$

$$\therefore \quad \frac{M}{L} = \frac{q\omega R^2}{2(mR^2)\omega} = \frac{q}{2m}$$

11. Magnetic force on a current carrying loop in uniform magnetic field is zero.

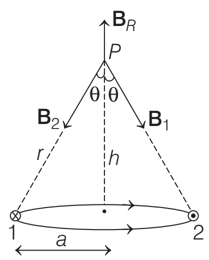
12. $\mathbf{B}_R$ = **B** due to ring

$\mathbf{B}_1$ = **B** due to wire-1

$\Rightarrow$ $\mathbf{B}_2$ = **B** due to wire-2

In magnitudes,

$$\mathbf{B}_1 = \mathbf{B}_2 = \frac{\mu_0 I}{2\pi r}$$



Resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$

$$= 2\mathbf{B}_1\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r}\right)\left(\frac{h}{r}\right) = \frac{\mu_0 I h}{\pi r^2}$$

$$\mathbf{B}_R = \frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}} = \frac{2\mu_0 I\pi a^2}{4\pi r^3}$$

As, $R = a, x = h$ and $a^2 + h^2 = r^2$

For zero magnetic field at P, $\frac{\mu_0 Ih}{\pi r^2} = \frac{2\mu_0 I\pi a^2}{4\pi r^3}$

$\Rightarrow \quad \pi a^2 = 2rh \quad \Rightarrow h \approx 1.2a$

13. Magnetic field at mid-point of two wires

$= 2$ (magnetic field due to one wire)

$$= 2\left[\frac{\mu_0}{2\pi}\frac{I}{d}\right] = \frac{\mu_0 I}{\pi d} \otimes$$

Magnetic moment of loop $M = IA = I\pi a^2$

Torque on loop $= MB \sin 150°$

$$= \frac{\mu_0 I^2 a^2}{2d}$$

14. $\mathbf{F}_{BA} = 0$, because magnetic lines are parallel to this wire.

$\mathbf{F}_{CD} = 0$, because magnetic lines are anti-parallel to this wire.

$\mathbf{F}_{CB}$ is perpendicular to paper outwards and $\mathbf{F}_{AD}$ is perpendicular to paper inwards. These two forces (although calculated by integration) cancel each other but produce a torque which tend to rotate the loop in clockwise direction about an axis OO'.

15. Equivalent current $i = qf$

and magnetic moment $m = (i\pi r^2) = \pi q f r^2$

Substituting the values, we have

$$M = (\pi)(0.5 \times 10^{-10})^2 (10^{16})(1.6 \times 10^{-19})$$

$$= 1.26 \times 10^{-23} \text{ A-m}^2$$

16. Let R be the radius of circle. Then,

$$2\pi R = L \quad \text{or} \quad R = \frac{L}{2\pi}$$

$$M = iA = i\pi R^2 = \frac{L^2 i}{4\pi}$$

17. Magnetic moment of the loop, $\mathbf{M} = (iA)\hat{\mathbf{k}} = (I_0 L^2)\hat{\mathbf{k}}$

Magnetic field, $\mathbf{B} = (B\cos 45°)\hat{\mathbf{i}} + (B \sin 45°)\hat{\mathbf{j}}$

$$= \frac{B}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

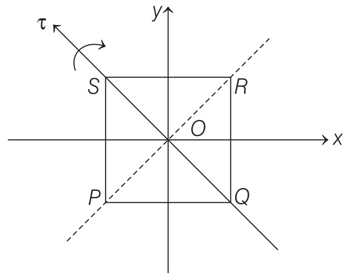
(a) Torque acting on the loop, $\tau = \mathbf{M} \times \mathbf{B}$

$$= (I_0 L^2 \hat{\mathbf{k}}) \times \left[\frac{B}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})\right]$$

$$\therefore \quad \tau = \frac{I_0 L^2 B}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

or $\quad |\tau| = I_0 L^2 B$

(b) Axis of rotation coincides with the torque and since torque is in $\hat{\mathbf{j}} - \hat{\mathbf{i}}$ direction or parallel to QS. Therefore, the loop will rotate about an axis passing through Q and S as shown in the figure.



Angular acceleration, $\alpha = \frac{|\tau|}{I}$

where, I = moment of inertia of loop about QS.

$$I_{QS} + I_{PR} = I_{ZZ}$$

(From theorem of perpendicular axis)

But $\quad I_{QS} = I_{PR}$

$$\therefore \quad 2I_{QS} = I_{ZZ} = \frac{4}{3}ML^2$$

$$I_{QS} = \frac{2}{3}ML^2$$

$$\therefore \quad \alpha = \frac{|\tau|}{I} = \frac{I_0 L^2 B}{2/3\, ML^2} = \frac{3}{2}\frac{I_0 B}{M}$$

$\therefore$ Angle by which the frame rotates in time Δt is

$$\theta = \frac{1}{2}\alpha(\Delta t)^2 \text{ or } \theta = \frac{3}{4}\frac{I_0 B}{M}.(\Delta t)^2$$

18. In ground state ($n = 1$) according to Bohr's theory

$$mvR = \frac{h}{2\pi} \text{ or } v = \frac{h}{2\pi mR}$$

Now, time period,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{h/2\pi mR} = \frac{4\pi^2 mR^2}{h}$$

Magnetic moment, $M = iA$

where, $\quad i = \frac{\text{charge}}{\text{time period}} = \frac{e}{\frac{4\pi^2 mR^2}{h}}$

$$= \frac{eh}{4\pi^2 mR^2} \quad \text{and} \quad A = \pi R^2$$

$$\therefore \quad M = (\pi R^2)\left(\frac{eh}{4\pi^2 mR^2}\right) \text{ or } M = \frac{eh}{4\pi m}$$

Direction of magnetic moment $\mathbf{M}$ is perpendicular to the plane of orbit.

(b) $\quad \tau = \mathbf{M} \times \mathbf{B} \Rightarrow |\tau| = MB \sin\theta$

where, θ is the angle between $\mathbf{M}$ and $\mathbf{B}$

$$\theta = 30°$$

$$\therefore \quad \tau = \left(\frac{eh}{4\pi m}\right)(B) \sin 30°$$

$$\therefore \quad \tau = \frac{ehB}{8\pi m}$$

The direction of τ is perpendicular to both $\mathbf{M}$ and $\mathbf{B}$.

Topic 5 Magnetism

1. Given, at first place, angle of dip, $\theta_1 = 45^\circ$

Time period, $T_1 = \dfrac{60}{30} = 2\text{s}$

At second place, angle of dip, $\theta_2 = 30^\circ$

Time period, $T_2 = \dfrac{60}{40} = \dfrac{3}{2}\text{ s}$

Now, at first place,

$$B_{H_1} = B_1 \cos\theta_1 = B_1 \cos 45^\circ = \frac{B_1}{\sqrt{2}} \quad \ldots\text{(i)}$$

and at second place,

$$B_{H_2} = B_2 \cos\theta_2 = B_2 \cos 30^\circ = \frac{\sqrt{3}}{2} B_2 \quad \ldots\text{(ii)}$$

Also, time period of a magnetic needle is given by

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \quad \ldots\text{(iii)}$$

$$\therefore \quad T \propto \sqrt{\frac{1}{B_H}} \text{ or } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \quad \ldots\text{(iv)}$$

By putting the values from Eqs. (i) and (ii) into Eq. (iv), we get

$$\frac{2}{\frac{3}{2}} = \sqrt{\frac{\sqrt{3}\frac{B_2}{2}}{\frac{B_1}{\sqrt{2}}}} \text{ or } \left(\frac{4}{3}\right)^2 = \frac{\sqrt{3}\times\sqrt{2}\, B_2}{2B_1}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{\sqrt{3}\times\sqrt{2}}{2} \times \frac{9}{16}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{9\sqrt{3}}{16\sqrt{2}}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{9\times 1.732}{16\times 1.414} = \frac{15.588}{22.624}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = 0.689 \approx 0.7\text{ T}$$

2. In a moving coil galvanometer in equilibrium, torque on coil due to current is balanced by torque of torsion band.

As, torque on coil,

$$\tau = \mathbf{M}\times\mathbf{B} = NIAB\sin\alpha$$

where, B = magnetic field strength,
I = current,
N = number of turns of coil

Since, plane of the coil is parallel to the field.

$$\therefore \quad \alpha = 90^\circ \Rightarrow \tau = NIBA$$

Torque of torsion band, $T = k\theta$

where, k = torsion constant of torsion band

and θ = deflection of coil in radians or angle of twist of restoring torque.

$$\therefore \quad BINA = k\theta \text{ or } B = \frac{k\theta}{INA} \quad \ldots\text{(i)}$$

Here, $k = 10^{-6}$ N-m/rad,
$I = 1\times 10^{-3}$ A,
$N = 175$,
$A = 1\text{ cm}^2 = 1\times 10^{-4}\text{ m}^2$

$$\theta = 1^\circ = \frac{\pi}{180}\text{ rad}$$

Substituting values in Eq. (i), we get

$$B = \frac{10^{-6}\times 22}{1\times 10^{-3}\times 175\times 7\times 180\times 10^{-4}}$$

$$= 0.998\times 10^{-3} \simeq 10^{-3}\text{ T}$$

3. From Curie's law for paramagnetic substance, we have

Magnetic susceptiblity $\chi \propto \dfrac{1}{T}$

$$\therefore \quad \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2} \Rightarrow \chi_2 = \frac{\chi_1 \cdot T_1}{T_2}$$

$$\Rightarrow \quad \chi_2 = \frac{2.8\times 10^{-4}\times 350}{300} = 3.267\times 10^{-4}$$

4. Given, side of cube = 1cm = 10^{-2} m

$\therefore$ Volume, $V = 10^{-6}\text{m}^3$

Dipole moment, $M = 20\times 10^{-6}$ J/T

Applied magnetic intensity, $H = 60\times 10^3$ A/m

Intensity of magnetisation

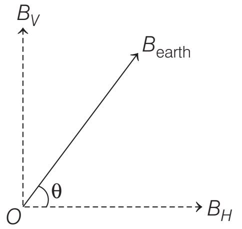
$$I = \frac{M}{V} = \frac{20\times 10^{-6}}{10^{-6}} = 20\text{ A/m}$$

Now, magnetic susceptibility χ is

$$\chi = \frac{\text{Intensity of magnetisation}}{\text{Applied magnetic intensity}} = \frac{I}{H} = \frac{20}{60\times 10^3}$$

$$\Rightarrow \quad \chi = \frac{1}{3}\times 10^{-3} = 3.33\times 10^{-4}$$

5. Without applied forces, (in equilibrium position) the needle will stay in the resultant magnetic field of earth. Hence, the dip 'θ' at this place is 45° (given).



We know that, horizontal and vertical components of earth's magnetic field (B_H and B_V) are related as

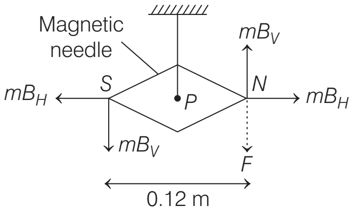
$$\frac{B_V}{B_H} = \tan\theta$$

Here, $\theta = 45^\circ$ and $B_H = 18\times 10^{-6}$T

$$\Rightarrow \quad B_V = B_H \tan 45^\circ$$

$$\Rightarrow \quad B_V = B_H = 18\times 10^{-6}\text{T} \quad (\because \tan 45^\circ = 1)$$

Now, when the external force F is applied, so as to keep the needle stays in horizontal position is shown below,



Taking torque at point P, we get

$$mB_V \times 2l = Fl$$

$$\therefore \quad F = 2 \times mB_V$$

Substituting the given values, we get

$$= 2 \times 1.8 \times 18 \times 10^{-6}$$

$$= 6.48 \times 10^{-5} = 6.5 \times 10^{-5} \text{ N}$$

6. Coercivity of a bar magnet is the value of magnetic field intensity (H) that is needed to reduce magnetisation to zero.

Since, for a solenoid magnetic induction is given as,

$$B = \mu_0 nI \qquad \text{...(i)}$$

where, n is the number of turns (N) per unit, length (l) and I is the current.

Also, $$B = \mu_0 H \qquad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\mu_0 nI = \mu_0 H \text{ or } H = nI = \frac{N}{l} I$$

Substituting the given values, we get

$$H = \frac{100}{0.2} \times 5.2 = 2600 \text{ A/m}$$

Thus, the value of coercivity of the bar magnet is 2600 A/m.

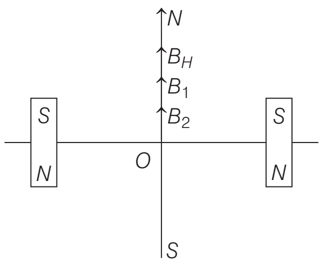
7. We need high retentivity and high coercivity for electromagnets and small area of hysteresis loop for transformers.

8. For solenoid, the magnetic field needed to be magnetised the magnet. $B = \mu_0 nI$

$$\text{where, } n = 100, l = 10\text{cm} = \frac{10}{100}\text{m} = 0.1 \text{ m}$$

$$\Rightarrow \quad 3 \times 10^3 = \frac{100}{0.1} \times I \Rightarrow I = 3\text{A}$$

9. $B_{\text{net}} = B_1 + B_2 + B_H$



$$B_{\text{net}} = \frac{\mu_0}{4\pi} \frac{(M_1 + M_2)}{r^3} + B_H$$

$$= \frac{10^{-7}(1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4} \text{ Wb/m}^2$$

10. Magnetic lines form closed loop. Inside magnet, these are directed from South to North-pole.

11. In non-uniform magnetic field, the needle will experience both a force and a torque.

12. $c\theta = BINA \Rightarrow \theta = \left(\frac{BNA}{c}\right) I$

Using iron core, value of magnetic field increases. So, deflection increases for same current. Hence, sensitivity increases.

Soft iron can be easily magnetised or demagnetised.

13. $\tau = MB = ki \Rightarrow k = \frac{MB}{i} = \frac{(NiA)B}{i} = NBA$

(b) $\tau = k \cdot \theta = BiNA$

$$\therefore \quad k = \frac{2\,BiNA}{\pi} \qquad (\text{as } \theta = \pi/2)$$

(c) $\tau = BiNA$

$$\text{or} \quad \int_0^t \tau \, dt = BNA \int_0^t i \, dt$$

$$I\omega = BNAQ$$

$$\text{or} \quad \omega = \frac{BNAQ}{I} \qquad \text{...(i)}$$

At maximum deflection, whole kinetic energy (rotational) will be converted into potential energy of spring.

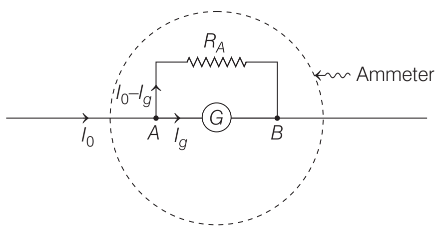
$$\text{Hence,} \quad \frac{1}{2} I\omega^2 = \frac{1}{2} k\theta_{\text{max}}^2$$

Substituting the values, we get

$$\theta_{\text{max}} = Q\sqrt{\frac{BN\pi A}{2I}}$$

Topic 6 Miscellaneous Problems

1. To use galvanometer as an ammeter, a low resistance in parallel is used.



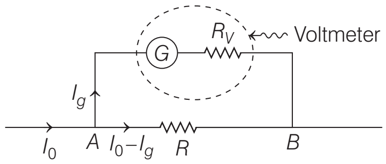
In ammeter, if I_g = full scale deflection current, then equating potential drops across points marked AB, we have

$$V_{AB} = I_g G = (I_0 - I_g) R_A$$

$$\Rightarrow \qquad R_A = \frac{I_g G}{I_0 - I_g} \qquad \ldots\text{(i)}$$

Here, G = resistance of galvanometer coil.

When a galvanometer is used as a voltmeter, a high resistance (R_V) in series is used.



Equating potential across point AB,

$$V_{AB} = (G + R_V)\, I_g$$

But $\qquad V_{AB} = I_0 G \qquad$ (given)

So, $\qquad I_0 G = (G + R_V)\, I_g$

$$\Rightarrow \qquad R_V = \frac{(I_0 - I_g)\, G}{I_g} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{R_A}{R_V} = \frac{\left(\dfrac{I_g G}{I_0 - I_g}\right)}{\dfrac{(I_0 - I_g) G}{I_g}} = \frac{I_g^2}{(I_0 - I_g)^2}$$

and $\qquad R_A \times R_V = \dfrac{I_g G}{(I_0 - I_g)} \times \dfrac{(I_0 - I_g)\, G}{I_g} = G^2$

2. Given, divisions in scale of galvanometer, $n = 50$

Sensitivity of galvanometer,

$$\frac{I_g}{n} = 20\,\mu\text{A} / \text{division}$$

$\therefore$ Current in galvanometer,

$$I_g = \frac{I_g}{n} \times n = 20\mu\text{A} \times 50$$

$\Rightarrow \qquad I_g = 1000\,\mu\text{A} = 1\text{ mA}$

Now, for R, it should be converted into 2V voltmeter.

$\therefore \qquad V_1 = I_g\ (R_1 + G)$

$\qquad\qquad 2 = 10^{-3}(R_1 + 100)$

$\Rightarrow \qquad 2000 = R_1 + 100$

$\therefore \qquad R_1 = 1900\,\Omega \qquad \ldots\text{(i)}$

For R_2, it should be converted into 10V voltmeter.

$\therefore \qquad V_2 = I_g\ [(R_1 + R_2) + G]$

$\Rightarrow \qquad 10 = 10^{-3}\ [(R_1 + R_2) + 100]$

$\Rightarrow \qquad 10000 = R_1 + R_2 + 100 = 2000 + R_2$

$\therefore \qquad R_2 = 8000\,\Omega \qquad \ldots\text{(ii)}$

For R_3, it should be converted into 20V voltmeter.

$\therefore \qquad V_3 = I_g[(R_1 + R_2 + R_3 + G]$

$\Rightarrow \qquad 20 = 10^{-3}[1900 + 8000 + R_3 + 100]$

$\Rightarrow \qquad 20000 = R_3 + 10000$

$\therefore \qquad R_3 = 10000\,\Omega \qquad \ldots\text{(iii)}$

From Eqs. (i), (ii) and (iii), it is clear that option (c) is correct.

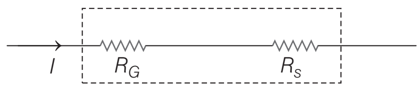
3. Given data,

$$I = 10^{-4}\text{ A},$$

$$R_S = 2\text{ m}\Omega = 2 \times 10^6\,\Omega,$$

$$V_{\max} = 5\text{ V}$$

Let internal resistance of galvanometer is R_G.



Voltmeter

Then, $\qquad I \times R_S + I \times R_G = V_{\max}$

$\Rightarrow \qquad 2 \times 10^6 \times 10^{-4} + 10^{-4} \times R_G = 5$

$\Rightarrow \qquad 10^{-4}\, R_G = 5 - 200 = -195$

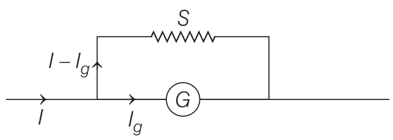
or $\qquad R_G = -195 \times 10^4\,\Omega$

Resistance cannot be negative.

$\therefore$ No option is correct

4. **Key Idea** An ammeter is a type of galvanometer with a shunt connected in parallel to the galvanometer.

Ammeter circuit is shown in the figure below,



So, $\qquad I_g G = (I - I_g)\, S$

Here, $\qquad I_g = 0.002\text{ A},$

$\qquad\qquad I = 0.5\text{A},$

$\qquad\qquad G = 50\,\Omega$

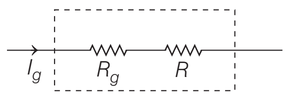
So, shunt resistance required is

$$S = \frac{I_g G}{I - I_g} = \frac{0.002 \times 50}{(0.5 - 0.002)} \approx 0.2\,\Omega$$

5. Given, resistance of galvanometer, $R_g = 50\Omega$

Current, $I_g = 4\text{ mA} = 4 \times 10^{-3}\text{ A}$

Resistance used in converting a galvanometer in voltmeter, $R = 5\text{ k}\Omega = 5 \times 10^3\,\Omega$



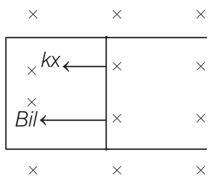
Voltmeter

$\therefore$ Maximum current in galvanometer is

$$I_g = \frac{E}{R + R_g}$$

$$\therefore \quad E = I_g(R + R_g)$$
$$= 4 \times 10^{-3} \times (5 \times 10^3 + 50)$$
$$= 5050 \times 4 \times 10^{-3}$$
$$= 20.2\,\text{V} \simeq 20\,\text{V}$$

6. There are two forces on slider



Spring force $= kx$

where, k = spring constant.

As the slider is kept in a uniform magnetic field $B = 0.1\,\text{T}$, hence it will experience a force, i.e.

Magnetic force $= Bil$

where, l = length of the strip.

Now, using $F_{\text{net}} = ma$

We have, $(-kx) + (-Bil) = ma$

$$\Rightarrow \quad -kx - Bil - ma = 0$$

$$\Rightarrow \quad -kx - \frac{B^2l^2}{R} \cdot v - ma = 0 \quad \left[\because i = \frac{Blv}{R}\right]$$

and acceleration, $a = \dfrac{d^2x}{dt^2}$

Hence, the modified equation becomes

$$\Rightarrow \quad \frac{md^2x}{dt^2} + \frac{B^2l^2}{R}\left(\frac{dx}{dt}\right) + kx = 0$$

This is the equation of damped simple harmonic motion.

So, amplitude of oscillation varies with time as

$$A = A_0 e^{-\frac{B^2l^2}{2Rm} \cdot t}$$

Now, when amplitude is $\dfrac{A_0}{e}$, then

$$\frac{A_0}{e} = \frac{A_0}{e^{\frac{B^2l^2}{2Rm} \cdot t}} \quad \text{(as given)}$$

$$\Rightarrow \quad \left(\frac{B^2l^2}{2Rm}\right)t = 1 \quad \text{or} \quad t = \frac{2Rm}{B^2l^2}$$

According to the question, magnetic field $B = 0.1\,\text{T}$, mass of strip $m = 50 \times 10^{-3}$ kg,

resistance $R = 10\,\Omega$, $l = 10\,\text{cm} = 10 \times 10^{-2}$ m

$$\therefore \quad t = \frac{2Rm}{B^2l^2} = \frac{2 \times 10 \times 50 \times 10^{-3}}{(0.1)^2 \times (10 \times 10^{-2})^2}$$
$$= \frac{1}{10^{-4}} = 10000\,\text{s}$$

Given, spring constant, $k = 0.5\,\text{Nm}^{-1}$

Also, time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{50 \times 10^{-3}}{0.5}} = \frac{2\pi}{\sqrt{10}} \approx 2\,\text{s}$$

So, number of oscillations is $N = \dfrac{t}{T} = \dfrac{10000}{2} = 5000$

7. Mean radiation intensity is

$$I = \varepsilon_0 c E_{\text{rms}}^2$$
$$= \varepsilon_0 c (cB_{\text{rms}})^2 \quad \left[\because \frac{E_{\text{rms}}}{B_{\text{rms}}} = c\right]$$
$$= \varepsilon_0 c^3 B_{\text{rms}}^2$$

$$\Rightarrow \quad B_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0 c^3}}$$

Substituting the given values, we get

$$= \sqrt{\left(\frac{10^8}{8.85 \times 10^{-12} \times (3 \times 10^8)^3}\right)}$$
$$= \sqrt{\left(\frac{10^8}{8.85 \times 27 \times 10^{12}}\right)} \approx \sqrt{(10^{-8})} \approx 10^{-4}\,\text{T}$$

8. The time-period of oscillations made by a magnet of magnetic moment M, moment of inertia I, placed in a magnetic field is given by

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad \ldots \text{(i)}$$

For the hoop, let us assume its moment of inertia I_h and magnetic moment M_h, then its time period will be

$$T_h = 2\pi\sqrt{\frac{I_h}{M_hB}} \quad \ldots \text{(ii)}$$

Similarly, for solid cylinder, time period is,

$$T_c = 2\pi\sqrt{\frac{I_c}{M_cB}} \quad \ldots \text{(iii)}$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{I_hM_c}{M_hI_c}} \quad \ldots \text{(iv)}$$

Now, it is given that,

$$M_h = 2M_c$$

and we know that, moment of inertia of hoop $I_h = mR^2$ and moment of inertia of solid

cylinder $I_c = \dfrac{1}{2}mR^2$

Substituting these values in Eq. (iv), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{mR^2 \times M_c}{\frac{1}{2}mR^2 \times 2M_c}} = 1$$

$$\Rightarrow \quad T_h = T_c$$

9. Work done in reversing dipole is

$$W = 2\,MB$$

where, M = magnetic dipole moment

$$= 10^{-2}\ \text{A-m}^2$$

and B = external field

$$= B\cos\omega t = 1\times\cos(0.125\times 1)$$

$$= \cos(7^\circ) = 0.992$$

Substituting these values, we get,

$$W = 2\times 10^{-2}\times 0.992$$

$$= 0.0198\ \text{J}$$

which is nearest to 0.014 J

10. Since, the magnetic field is dependent on time, so the net charge flowing through the loop will be given as

$$Q = \frac{\text{change in magnetic flux, } \Delta\phi_B}{\text{resistance, } R}$$

As, $\Delta\phi_B = \mathbf{B}\ \mathbf{A} = BA\cos\theta$

where, A is the surface area of the loop and 'θ' is an angle between B and A.

Here, $\theta = 0 \Rightarrow \Delta\phi_B = BA$

$\therefore$ For the time interval, $t = 0$ ms to $t = 10$ ms,

$$Q = \frac{\Delta\phi_B}{R}$$

$$= \frac{A}{R}(B_{f\ \text{at}\ 0.01\ \text{s}} - B_{i\ \text{at}\ 0\ \text{s}})$$

Substituting the given values, we get

$$= \frac{3.5\times 10^{-3}}{10}[0.4\sin(0.5\pi) - 0.4\ \sin 0]$$

$$= 3.5\times 10^{-4}\,(0.4\sin\pi/2)$$

$$= 1.4\times 10^{-4}\,\text{C} = 14\ \text{mC}$$

11. Given,

Area of the rectangular coil, $A = 5\,\text{cm}\times 2.5\,\text{cm}$

$\Rightarrow \quad A = 12.5\ \text{cm}^2 = 12.5\times 10^{-4}\ \text{m}^2$

Number of turns, $N = 100$ turns

Current through the coil, $I = 3$ A

Magnetic field applied, $B = 1$ T

Angle between the magnetic field and area vector of the coil, $\theta = 45^\circ$

As we know that, when a coil is tilted by an angle θ in the presence of some external magnetic field, then the net torque experienced by the coil is,

$$\tau = \mathbf{M}\times\mathbf{B} = NI(\mathbf{A}\times\mathbf{B}) = NIAB\sin\theta$$

Substituting the given values, we get

$$\tau = 100\times 3\times 12.5\times 10^{-4}\times 1\times\sin 45^\circ$$

$$\tau = 0.707\times 100\times 3\times 12.5\times 10^{-4}\ \text{N-m}$$

$$= 2.651\times 10^{-1}\ \text{N-m}$$

$$\approx 0.27\ \text{N-m}$$

12. **Key Idea** Net force experienced by two wires separated by same distance is attractive, if current flow in them in same direction. However, this force is repulsive in nature, if current in them flows in opposite direction.

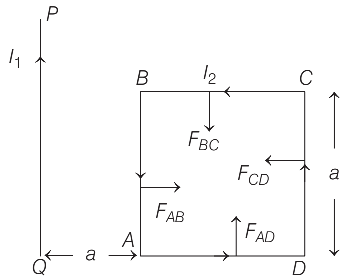
Force on a wire 1 in which current I_1 is flowing due to another wire 2 which are separated by a distance r is given as

$$\mathbf{F} = I_1(l\times\mathbf{B}_2)$$

or

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}\cdot l\sin\theta \qquad \left[\because \mathbf{B}_2 = \frac{\mu_0 I_2}{2\pi r}\right]$$

Thus, the given square loop can be drawn as shown below



$$F_{AB} = \frac{\mu_0 I_1}{2\pi a}\cdot I_2 a \text{ (away from wire } PQ)$$

$$F_{BC} = F_{AD} = 0 \qquad [\because \theta = 0^\circ]$$

$$F_{CD} = \frac{\mu_0 I_1}{2\pi(2a)}\cdot I_2 a$$

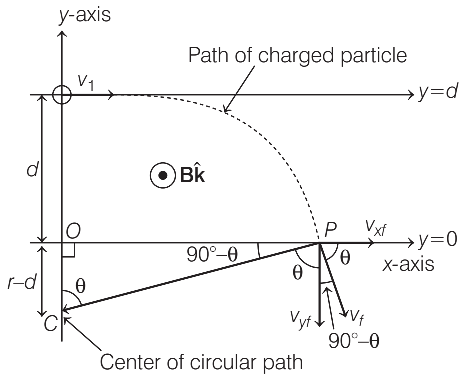
$$= \frac{\mu_0 I_1}{4\pi a}\cdot I_2 a \text{(towards the wire } PQ)$$

$$\therefore \quad F_{\text{net}} = F_{AB} - F_{CD}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi} \qquad \text{(away from wire)}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi} \qquad \text{(repulsive in nature)}$$

13. Situation given in question is shown below;



Path taken by particle of charge 'q' and mass 'm' is a circle of radius r where,

$$r = \frac{mv}{Bq}$$

Here final velocity

$$\mathbf{v}_f = \mathbf{v}_x\hat{\mathbf{i}} + \mathbf{v}_{yf}(-\hat{\mathbf{j}}) = v\cos 60\,\hat{\mathbf{i}} - v\sin 60^\circ\,\hat{\mathbf{j}}$$

$$= v\left(\frac{1}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

So, change of velocity of charged particle is

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i = v\left(\frac{1}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right) - v\hat{\mathbf{i}}$$

$$= -v\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

It t = time taken by charged particle to cross region of magnetic field then,

$$t = \frac{\text{distance } OP}{\text{speed in direction } OP}$$

$$= \frac{r \times \frac{\sqrt{3}}{2}}{v} = \frac{\frac{mv}{Bq} \times \frac{\sqrt{3}}{2}}{v} = \frac{\sqrt{3}m}{2Bq}$$

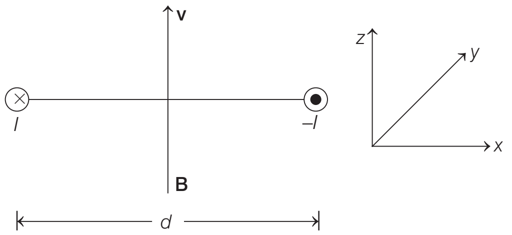
So, acceleration of charged particle at the point its emergence is;

$$\text{Acceleration, } \mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{-v\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)}{\frac{\sqrt{3}}{2}\frac{m}{Bq}}$$

$$= \frac{-Bqv}{m}\left(\frac{\hat{\mathbf{i}}}{\sqrt{3}} + \hat{\mathbf{j}}\right)\text{ms}^{-2}$$

14. Correct answer is (c), because induced electric field lines (produced by change in magnetic field) and magnetic field lines form closed loops.

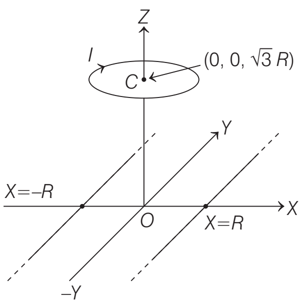
15. Net magnetic field due to both the wires will be downward as shown in the figure.



Since, angle between $\mathbf{v}$ and $\mathbf{B}$ is 180°.
Therefore, magnetic force

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = 0$$

16.

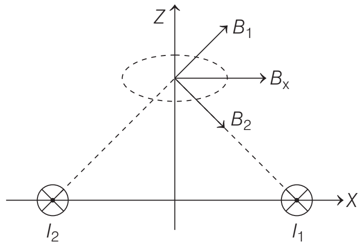


(a) At origin, $\mathbf{B} = 0$ due to two wires if $I_1 = I_2$, hence ($\mathbf{B}_{\text{net}}$) at origin is equal to $\mathbf{B}$ due to ring. which is non-zero.

(b) If $I_1 > 0$ and $I_2 < 0$, B at origin due to wires will be along $+\hat{k}$. Direction of B due to ring is along $-\hat{k}$ direction and hence B can be zero at origin.

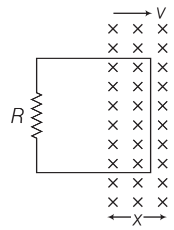
(c) If $I_1 < 0$ and $I_2 > 0$, B at origin due to wires is along $-\hat{k}$ and also along $-\hat{k}$ due to ring, hence B cannot be zero.

(d)



At centre of ring, B due to wires is along x-axis.
Hence, z-component is only because of ring which $B = \frac{\mu_0 i}{2R}(-\hat{k})$.

17.



When loop was entering (x < L)

$$\phi = BLx$$

$$e = -\frac{d\phi}{dt} = -BL\frac{dx}{dt}$$

$$|e| = BLv$$

$$i = \frac{e}{R} = \frac{BLv}{R} \qquad \text{(anti-clockwise)}$$

$$F = ilB \text{ (Left direction)} = \frac{B^2L^2v}{R} \text{ (in left direction)}$$

$$\Rightarrow \qquad a = \frac{F}{m} = -\frac{B^2L^2v}{mR}$$

$$a = v\frac{dv}{dx}$$

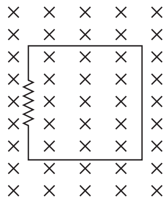
$$v\frac{dv}{dx} = -\frac{B^2L^2v}{mR}$$

$$\Rightarrow \qquad \int_{v_0}^{v} dv = -\frac{B^2L^2}{mR}\int_0^x dx$$

$$\Rightarrow v = v_0 - \frac{B^2L^2v}{mR}x \text{ (straight line of negative slope for } x < L)$$

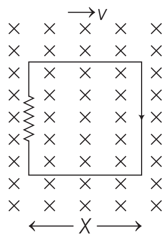
$I = \frac{BL}{R}v \Rightarrow$ (I *vs* x will also be straight line of negative slope for $x < L$)

$L \le x \le 3L$



$\frac{d\phi}{dt} = 0,\ e = 0, i = 0$

$F = 0, \quad x > 4L$



$e = Blv$

Force also will be in left direction.

$$i = \frac{BLv}{R} \text{ (clockwise)}$$

$$a = \frac{B^2L^2v}{mR} = v\frac{dv}{dx}$$

$$F = \frac{B^2L^2v}{R}$$

$$\int_L^x -\frac{B^2L^2}{mR}dx = \int_{v_i}^{v_f} dv$$

$$\Rightarrow -\frac{B^2L^2}{mR}(x - L) = v_f - v_i$$

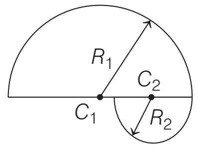
$$v_f - v_i - \frac{B^2L^2}{mR}(x - L) \text{ (straight line of negative slope)}$$

$$I = \frac{BLv}{R} \rightarrow \text{(Clockwise) (straight line of negative slope)}$$

18. If average speed is considered along x-axis,

$$R_1 = \frac{mv_0}{qB_1}, R_2 = \frac{mv_0}{qB_2} = \frac{mv_0}{4qB_1}$$

$$R_1 > R_2$$



Distance travelled along x-axis, $\Delta x = 2(R_1 + R_2) = \frac{5mv_0}{2qB_1}$

$$\text{Total time} = \frac{T_1}{2} + \frac{T_2}{2} = \frac{\pi m}{qB_1} + \frac{\pi m}{qB_2}$$

$$= \frac{\pi m}{qB_1} + \frac{\pi m}{4qB_1} = \frac{5\pi m}{4qB_1}$$

$$\text{Magnitude of average speed} = \frac{\frac{5mv_0}{2qB_1}}{\frac{5\pi m}{4qB_1}} = 2$$

19. $F_B = Bev = Be\frac{I}{nAe} = \frac{BI}{nA}$

$F_e = eE$

$F_e = F_B$

$eE = \frac{BI}{nA} \Rightarrow \quad E = \frac{B}{nAe}$

$V = Ed = \frac{BI}{nAe} \cdot w = \frac{BIw}{n(wd)e} = \frac{BI}{ned}$

$\frac{V_1}{V_2} = \frac{d_2}{d_1}$

$\Rightarrow$ if $w_1 = 2w_2$

and $d_1 = d_2$

$V_1 = V_2$

20. $V = \frac{BI}{ned} \Rightarrow \frac{V_1}{V_2} = \frac{B_1}{B_2} \times \frac{n_2}{n_1}$

If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

21. If $B_2 > B_1$, critical temperature, (at which resistance of semiconductors abruptly becomes zero) in case 2 will be less than compared to case 1.

22. With increase in temperature, T_C is decreasing.

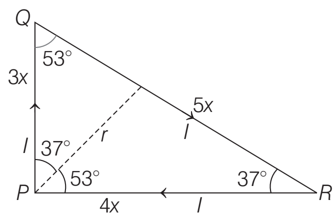
$$T_C(0) = 100 \text{ K}$$

$$T_C = 75 \text{ K at } B = 7.5 \text{ T}$$

Hence, at $B = 5$ T, T_C should lie between 75K and 100K. Hence, the correct option should be (b).

26. Magnetic field at point P due to wires RP and RQ is zero. Only wire QR will produce magnetic field at P.

$$r = 3x\cos 37° = (3x)\left(\frac{4}{5}\right) = \frac{12x}{5}$$



Now, $\quad B = \frac{\mu_0}{4\pi}\frac{I}{12x/5}$ [sin 37° + sin 53°]

$$= 7\left(\frac{\mu_0 I}{48\pi x}\right)$$

27. Radius of circular path is given by

$$R = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Radius in both the cases are equal. Therefore,

$$\frac{\sqrt{2K_\alpha m_\alpha}}{Bq_\alpha} = \frac{\sqrt{2K_d m_d}}{2.3Bq_d},$$

$$\frac{q_d}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

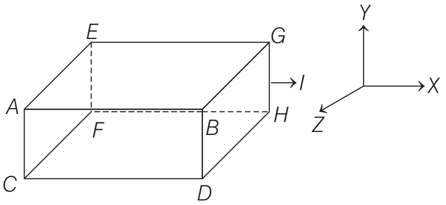
and $$\frac{m_\alpha}{m_d} = \frac{4}{2} = 2$$

$$\therefore \quad K_d = \left(\frac{2.3q_d}{q_\alpha}\right)^2 \left(\frac{m_\alpha}{m_d}\right) \cdot K_\alpha$$

$$K_d = \left(2.3 \times \frac{1}{2}\right)^2 (2)(5.3)\text{MeV},$$

$$K_d = 14.0185 \text{ MeV}$$

28.



In metals, charge carries are free electrons. Current is in positive direction of x-axis. Therefore, charge carriers will be moving in negative direction of x-axis.

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = (-e)(-v\,\hat{\mathbf{i}}) \times (B\hat{\mathbf{j}})$$

$$\mathbf{F}_m = evB\hat{\mathbf{k}}$$

Due to this magnetic force, electrons will be collecting at face $ABCD$, therefore lowering its potential.

29. In equilibrium, $2T_0 = mg$ or $T_0 = \frac{mg}{2}$ ...(i)

Magnetic moment, $M = iA = \left(\frac{\omega}{2\pi}Q\right)(\pi R^2)$

$$\tau = MB \sin 90° = \frac{\omega BQR^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on ($T_1 > T_2$).
For translational equilibrium,

$$T_1 + T_2 = mg \quad \text{...(ii)}$$

For rotational equilibrium,

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2} \text{ or } T_1 - T_2 = \frac{\omega BQR^2}{2} \quad \text{... (iii)}$$

Solving Eqs. (ii) and (iii), we have

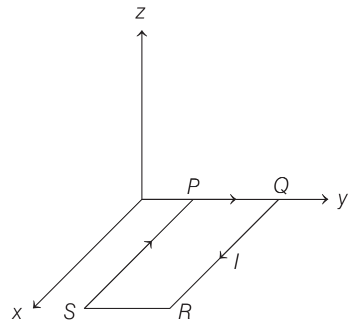
$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

As $T_1 > T_2$ and maximum values of T_1 can be $\frac{3T_0}{2}$, we have

$$\frac{3T_0}{2} = T_0 + \frac{\omega_{\max} BQR^2}{2D} \qquad \left(\frac{mg}{2} = T_0\right)$$

$$\therefore \quad \omega_{\max} = \frac{DT_0}{BQR^2}$$

30. Let the direction of current in wire PQ is from P to Q and its magnitude be I.



The magnetic moment of the given loop is

$$\mathbf{M} = -Iab\hat{\mathbf{k}}$$

Torque on the loop due to magnetic forces is

$$\tau_1 = \mathbf{M} \times \mathbf{B} = (-Iab\hat{\mathbf{k}}) \times (3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0\hat{\mathbf{i}} = -3IabB_0\hat{\mathbf{j}}$$

Torque of weight of the loop about axis PQ is

$$\tau_2 = \mathbf{r} \times \mathbf{F} = \left(\frac{a}{2}\hat{\mathbf{i}}\right) \times (-mg\hat{\mathbf{k}}) = \frac{mga}{2}\hat{\mathbf{j}}$$

We see that when the current in the wire PQ is from P to Q, τ_1 and τ_2 are in opposite directions, so they can cancel each other and the loop may remain in equilibrium. So, the direction of current I in wire PQ is from P to Q. Further for equilibrium of the loop

$$|\tau_1| = |\tau_2|$$

or $$3IabB_0 = \frac{mga}{2}$$

$$I = \frac{mg}{6bB_0}$$

(b) Magnetic force on wire RS is

$$\mathbf{F} = I(\mathbf{I} \times \mathbf{B}) = I[(-b\hat{\mathbf{j}}) \times \{(3\hat{\mathbf{i}} + 4\hat{\mathbf{k}})B_0\}]$$

$$\mathbf{F} = IbB_0(3\hat{\mathbf{k}} - 4\hat{\mathbf{i}})$$

31. Given, $i = 10$ A, $r_1 = 0.08$ m and $r_2 = 0.12$ m. Straight portions i.e. CD etc, will produce zero magnetic field at the centre. Rest eight arcs will produce the magnetic field at the centre in the same direction i.e. perpendicular to the paper outwards or vertically upwards and its magnitude is

$$B = B_{\text{inner arcs}} + B_{\text{outer arcs}}$$

$$= \frac{1}{2}\left\{\frac{\mu_0 i}{2r_1}\right\} + \frac{1}{2}\left\{\frac{\mu_0 i}{2r_2}\right\} = \left(\frac{\mu_0}{4\pi}\right)(\pi i)\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

Substituting the values, we have

$$B = \frac{(10^{-7})(3.14)(10)(0.08 + 0.12)}{(0.08 \times 0.12)} \Rightarrow B = 6.54 \times 10^{-5}\ \text{T}$$

(vertically upward or outward normal to the paper)

(b) Force on AC

Force on circular portions of the circuit i.e. AC etc, due to the wire at the centre will be zero because magnetic field due to the central wire at these arcs will be tangential ($\theta = 180°$).

Force on CD

Current in central wire is also $i = 10$ A. Magnetic field at distance x due to central wire

$$B = \frac{\mu_0}{2\pi} \cdot \frac{i}{x}$$

$\therefore$ Magnetic force on element dx due to this magnetic field

$$dF = (i)\left(\frac{\mu_0}{2\pi} \cdot \frac{i}{x}\right) \cdot dx = \left(\frac{\mu_0}{2\pi}\right) i^2 \frac{dx}{x}$$

$$(F = ilB \sin 90°)$$

Therefore, net force on CD is

$$F = \int_{x=r_1}^{x=r_2} dF = \frac{\mu_0 i^2}{2\pi} \int_{0.08}^{0.12} \frac{dx}{x} = \frac{\mu_0}{2\pi} i^2 \ln\left(\frac{3}{2}\right)$$

Substituting the values

$$F = (2 \times 10^{-7})(10)^2 \ln (1.5)$$

or $\quad F = 8.1 \times 10^{-6}$ N (inwards)

Force on wire at the centre

Net magnetic field at the centre due to the circuit is in vertical direction and current in the wire in centre is also in vertical direction. Therefore, net force on the wire at the centre will be zero. ($\theta = 180°$). Hence,

(i) force acting on the wire at the centre is zero.

(ii) force on arc $AC = 0$.

(iii) force on segment CD is 8.1×10^{-6} N (inwards).

32. Magnetic field ($\mathbf{B}$) at the origin = magnetic field due to semicircle KLM + Magnetic field due to other semicircle KNM

$$\therefore \quad \mathbf{B} = -\frac{\mu_0 I}{4R}(-\hat{\mathbf{i}}) + \frac{\mu_0 I}{4R}(\hat{\mathbf{j}})$$

$$\mathbf{B} = -\frac{\mu_0 I}{4R}\hat{\mathbf{i}} + \frac{\mu_0 I}{4R}\hat{\mathbf{j}} = \frac{\mu_0 I}{4R}(-\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

$\therefore$ Magnetic force acting on the particle

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q\{(-v_0 \hat{\mathbf{i}}) \times (-\hat{\mathbf{i}} + \hat{\mathbf{j}})\}\frac{\mu_0 I}{4R}$$

$$\mathbf{F} = -\frac{\mu_0 q v_0 I}{4R}\hat{\mathbf{k}}$$

(b) $\mathbf{F}_{KLM} = \mathbf{F}_{KMN} = \mathbf{F}_{KM}$ and $\mathbf{F}_{KM} = BI(2R)\hat{\mathbf{i}} = 2BIR\hat{\mathbf{i}}$

$$\mathbf{F}_1 = \mathbf{F}_2 = 2BIR\,\hat{\mathbf{i}}$$

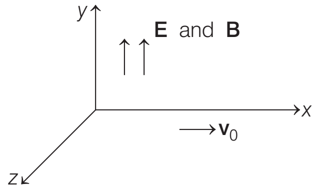
Total force on the loop, $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ or $\mathbf{F} = 4BIR\hat{\mathbf{i}}$

NOTE If a current carrying wire ADC (of any shape) is placed in a uniform magnetic field **B**.

Then, $\quad F_{ADC} = F_{AC}$ or $|F_{ADC}| = \hat{i}(AC)B$

From this we can conclude that net force on a current carrying loop in uniform magnetic field is zero. In the question, segments KLM and KNM also form a loop and they are also placed in a uniform magnetic field but in this case net force on the loop will not be zero. It would had been zero if the current in any of the segments was in opposite direction.

33. $\hat{\mathbf{j}} = \dfrac{\mathbf{E}}{E}$ or $\dfrac{\mathbf{B}}{B}$; $\hat{\mathbf{i}} = \dfrac{\mathbf{v}_0}{v_0} \Rightarrow \hat{\mathbf{k}} = \dfrac{\mathbf{v}_0 \times \mathbf{B}_0}{v_0 B}$



Force due to electric field will be along y-axis. Magnetic force will not affect the motion of charged particle in the direction of electric field (or y-axis). So,

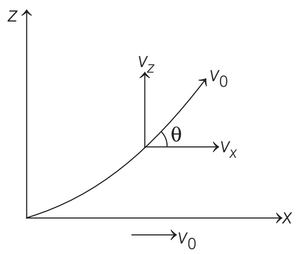
$$a_y = \frac{F_e}{m} = \frac{qE}{m} = \text{constant.}$$

Therefore, $\quad v_y = a_y t = \dfrac{qE}{m} \cdot t \quad$ ...(i)

The charged particle under the action of magnetic field describes a circle in x-z plane (perpendicular to **B**) with

$$T = \frac{2\pi m}{Bq} \text{ or } \omega = \frac{2\pi}{T} = \frac{qB}{m}$$

Initially ($t = 0$), velocity was along x-axis. Therefore, magnetic force ($\mathbf{F}_m$) will be along positive z-axis [$\mathbf{F}_m = q(\mathbf{v}_0 \times \mathbf{B})$]. Let it makes an angle θ with x-axis at time t, then



$$\theta = \omega t$$

$\therefore \quad v_x = v_0 \cos \omega t = v_0 \cos\left(\dfrac{qB}{m}t\right) \quad$ ...(ii)

$\quad v_z = v_0 \sin \omega t = v_0 \sin\left(\dfrac{qB}{m}t\right) \quad$ ...(iii)

From Eqs. (i), (ii) and (iii),

$$\mathbf{v} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

$$\therefore \quad \mathbf{v} = v_0 \cos\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0}{v_0}\right) + \frac{qE}{m}t\left(\frac{\mathbf{E}}{E}\right) + v_0 \sin\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0 \times \mathbf{B}}{v_0 B}\right)$$

or $\mathbf{v} = \cos\left(\frac{qB}{m}t\right)(\mathbf{v}_0) + \left(\frac{q}{m}t\right)(\mathbf{E}) + \sin\left(\frac{qB}{m}t\right)\left(\frac{\mathbf{v}_0 \times \mathbf{B}}{B}\right)$

NOTE The path of the particle will be a helix of increasing pitch. The axis of the helix will be along y-axis.

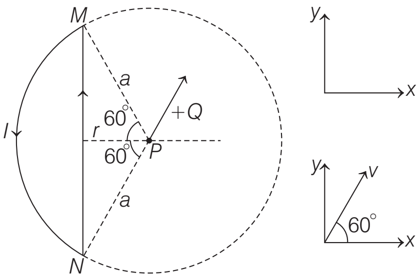
34. Magnetic field at P due to arc of circle,

Subtending an angle of 120° at centre would be

$$B_1 = \frac{1}{3}(\text{field due to circle}) = \frac{1}{3}\left(\frac{\mu_0 I}{2a}\right)$$

$$= \frac{\mu_0 I}{6a} \qquad \text{(outwards)}$$

$$= \frac{0.16\mu_0 I}{a} \qquad \text{(outwards)}$$



or $\quad \mathbf{B}_1 = \frac{0.16\mu_0 I}{a}\hat{\mathbf{k}}$

Magnetic field due to straight wire NM at P

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin 60° + \sin 60°)$$

Here, $\quad r = a\cos 60°$

$\therefore \quad B_2 = \frac{\mu_0}{4\pi}\frac{I}{a\cos 60°}(2\sin 60°)$

or $\quad B_2 = \frac{\mu_0}{2\pi}\frac{I}{a}\tan 60° = \frac{0.27\mu_0 I}{a} \qquad$ (inwards)

or $\quad \mathbf{B}_2 = -\frac{0.27\mu_0 I}{a}\hat{\mathbf{k}}$

$\therefore \quad \mathbf{B}_{\text{net}} = \mathbf{B}_1 + \mathbf{B}_2 = -\frac{0.11\mu_0 I}{a}\hat{\mathbf{k}}$

Now, velocity of particle can be written as,

$$\mathbf{v} = v\cos 60°\,\hat{\mathbf{i}} + v\sin 60°\,\hat{\mathbf{j}} = \frac{v}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}v}{2}\hat{\mathbf{j}}$$

Magnetic force

$$\mathbf{F}_m = Q(\mathbf{v}\times\mathbf{B}) = \frac{0.11\mu_0 IQv}{2a}\hat{\mathbf{j}} - \frac{0.11\sqrt{3}\,\mu_0 IQv}{2a}\hat{\mathbf{i}}$$

$\therefore$ Instantaneous acceleration

$$\mathbf{a} = \frac{\mathbf{F}_m}{m} = \frac{0.11\mu_0 IQv}{2am}(\hat{\mathbf{j}} - \sqrt{3}\hat{\mathbf{i}})$$

(b) In uniform magnetic field, force on a current loop is zero. Further, magnetic dipole moment of the loop will be,

$$\mathbf{M} = (IA)\,\hat{\mathbf{k}}$$

Here, A is the area of the loop.

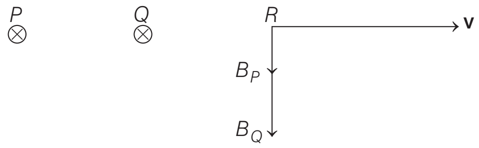
$$A = \frac{1}{3}(\pi a^2) - \frac{1}{2}[2\times a\sin 60°][a\cos 60°]$$

$$= \frac{\pi a^2}{3} - \frac{a^2}{2}\sin 120° = 0.61\,a^2$$

$\therefore \quad \mathbf{M} = (0.61\,Ia^2)\hat{\mathbf{k}}$

Given, $\quad \mathbf{B} = B\hat{\mathbf{i}} \Rightarrow \tau = \mathbf{M}\times\mathbf{B} = (0.61\,Ia^2B)\,\hat{\mathbf{j}}$

35. Magnetic field at R due to both the wires P and Q will be downwards as shown in figure.



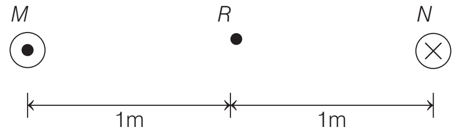
Therefore, net field at R will be sum of these two

$$B = B_P + B_Q = \frac{\mu_0}{2\pi}\frac{I_P}{5} + \frac{\mu_0}{2\pi}\frac{I_Q}{2}$$

$$= \frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right) = \frac{\mu_0}{4\pi}(I+1)$$

$$= 10^{-7}(I+1)$$

(a) Net force on the electron will be,



$$F_m = Bqv\sin 90°$$

or $(3.2\times 10^{-20}) = (10^{-7})(I+1)(1.6\times 10^{-19})(4\times 10^5)$

or $\quad I + 1 = 5 \Rightarrow I = 4\text{A}$

(b) Net field at R due to wires P and Q is

$$B = 10^{-7}(I+1)\,\text{T} = 5\times 10^{-7}\ \text{T}$$

Magnetic field due to third wire carrying a current of 2.5 A should be 5×10^{-7} T in upward direction so, that net field at R becomes zero. Let distance of this wire from R be r. Then,

$$\frac{\mu_0}{2\pi}\frac{2.5}{r} = 5\times 10^{-7} \text{ or } \frac{(2\times 10^{-7})(2.5)}{r} = 5\times 10^{-7}\,\text{m}$$

or $\quad r = 1\,\text{m}$

So, the third wire can be put at M or N as shown in figure. If it is placed at M, then current in it should be outwards and if placed at N, then current be inwards.

Download Chapter Test
http://tinyurl.com/y3qdelhb *or*

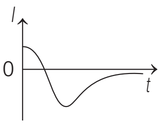
16

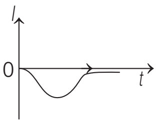
Electromagnetic Induction and Alternating Current

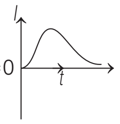
Topic 1 Magnetic Flux and Induced EMF by Change in Flux

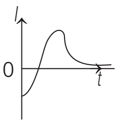
Objective Questions I (Only one correct option)

1. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}\ (k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by **(Main 2019, 9 April II)**

(a) $t=0$ 

(b) $t=0$ 

(c) $t=0$ 

(d) $t=0$ 

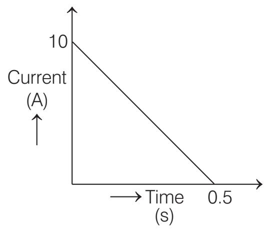
2. A 10 m long horizontal wire extends from North-East to South-West. It is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field of 0.3×10^{-4} Wb/ m^2. The value of the induced emf in wire is **(Main 2019, 12 Jan II)**

(a) 1.5×10^{-3} V (b) 1.1×10^{-3} V
(c) 0.3×10^{-3} V (d) 2.5×10^{-3} V

3. The self-induced emf of a coil is 25 V. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is **(Main 2019, 10 Jan II)**

(a) 437.5 J (b) 740 J
(c) 637.5 J (d) 540 J

4. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is **(2017 Main)**

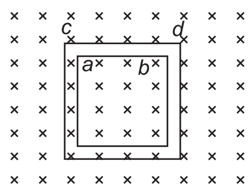


(a) 225 Wb (b) 250 Wb (c) 275 Wb (d) 200 Wb

5. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the smaller loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the bigger loop, then the flux linked with smaller loop is **(2013 Main)**

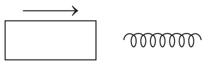
(a) 9.1×10^{-11} Wb (b) 6×10^{-11} Wb
(c) 3.3×10^{-11} Wb (d) 6.6×10^{-9} Wb

6. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then, **(2009)**

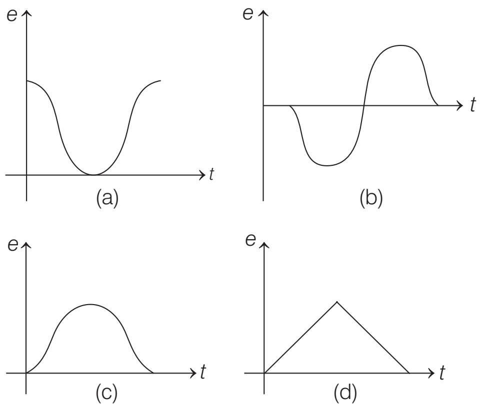


(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction ba and I_2 is in the direction cd.
(d) I_1 is in the direction ab and I_2 is in the direction dc.

7. The variation of induced emf (e) with time (t) in a coil if a



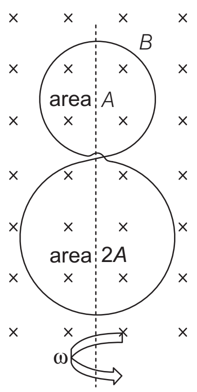
short bar magnet is moved along its axis with a constant velocity is best represented as **(2004, 2M)**



8. A circular loop of radius R, carrying current I, lies in x-y plane with its centre at origin. The total magnetic flux through x-y plane is **(1999, 2M)**
(a) directly proportional to I.
(b) directly proportional to R.
(c) inversely proportional to R.
(d) zero.

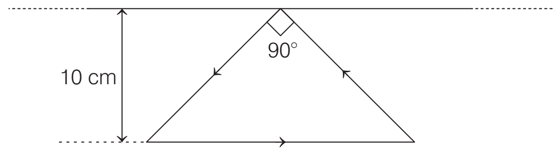
Objective Questions II (One or more correct option)

9. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field **B** points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct? **(2017 Adv.)**



(a) The emf induced in the loop is proportional to the sum of the areas of the two loops.
(b) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper.
(c) The net emf induced due to both the loops is proportional to $\cos \omega t$.
(d) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone.

10. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of $10\,\text{As}^{-1}$. Which of the following statement(s) is (are) true? **(2016 Adv.)**



(a) There is a repulsive force between the wire and the loop.
(b) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.
(c) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.
(d) The induced current in the wire is in opposite direction to the current along the hypotenuse.

11. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are) **(2012)**
(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is finite if the current decreases at a steady rate.

Fill in the Blank

12. In a straight conducting wire, a constant current is flowing from left to right due to a source of emf. When the source is switched off, the direction of the induced current in the wire will be…… **(1993, 1M)**

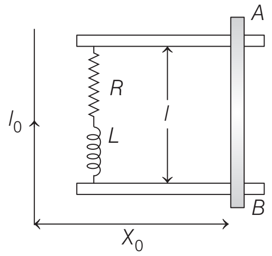
True/False

13. A coil of metal wire is kept stationary in a non-uniform magnetic field. An emf is induced in the coil. **(1986, 3M)**

Analytical & Descriptive Questions

14. A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight

wire, carrying a constant current I_0 is placed in the plane of the rails as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it made to slide on the rails away from the wire. Answer the following questions.
(2002, 5M)

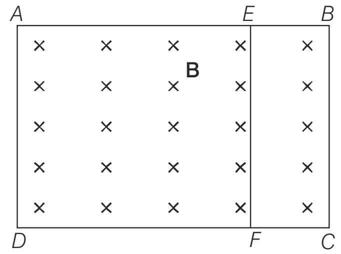


(a) Find a relation among i, $\dfrac{di}{dt}$ and $\dfrac{d\phi}{dt}$, where i is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.

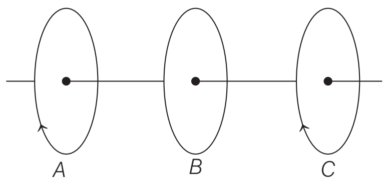
(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $i_1/4$ at time $2T$. Find the value of L/R in terms of the other given quantities.

15. A rectangular frame $ABCD$, made of a uniform metal wire, has a straight connection between E and F made of the same wire, as shown in figure. $AEFD$ is a square of side 1 m and $EB = FC = 0.5$ m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the plane of the paper and normal to it.

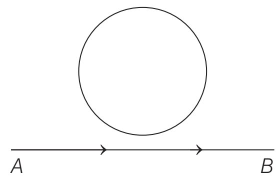


The rate of change of the magnetic field is 1 T/s. The resistance per unit length of the wire is 1 Ω/m. Find the magnitudes and directions of the currents in the segments AE, BE and EF. **(1993, 5M)**

16. Three identical closed coils A, B and C are placed with their planes parallel to one another. Coils A and C carry equal currents as shown in figure. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is there any induced current in B ? If no, give reasons. If yes, mark the direction of the induced current in the diagram. **(1982, 2M)**



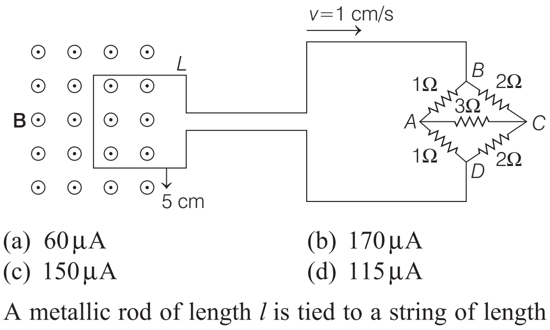
17. A current from A to B is increasing in magnitude. What is the direction of induced current, in the loop as shown in the figure? **(1979, 6M)**



Topic 2 Motional and Rotational EMF

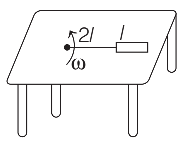
Objective Questions I (Only one correct option)

1 The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to **(Main 2019, 12 April I)**



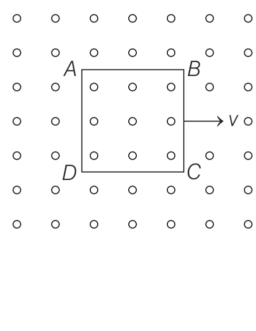
(a) 60 μA (b) 170 μA
(c) 150 μA (d) 115 μA

2. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is **(2013 Main)**



(a) $\dfrac{2B\omega l^3}{2}$ (b) $\dfrac{3B\omega l^3}{2}$ (c) $\dfrac{4B\omega l^2}{2}$ (d) $\dfrac{5B\omega l^2}{2}$

3. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. Electric field is induced **(2001, 2M)**

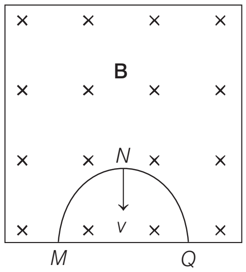


(a) in AD, but not in BC
(b) in BC, but not in AD
(c) neither in AD nor in BC
(d) in both AD and BC

4. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following. **(1998, 2M)**

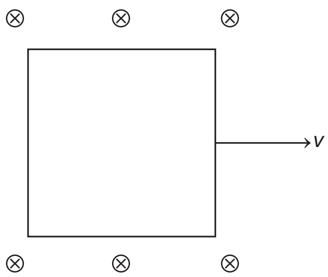
(a) The entire rod is at the same electric potential.
(b) There is an electric field in the rod.
(c) The electric potential is highest at the centre of the rod and decrease towards its ends.
(d) The electric potential is lowest at the centre of the rod and increases towards its ends.

5. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\mathbf{B}$. At the position MNQ the speed of the ring is v and the potential difference developed across the ring is **(1996, 2M)**



(a) zero
(b) $Bv\pi R^2/2$ and M is at higher potential
(c) πBRv and Q is at higher potential
(d) $2RBv$ and Q is at higher potential

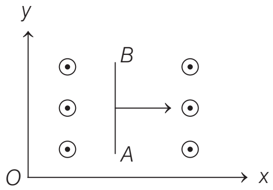
6. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular to and into the plane of the loop exists everywhere. The current induced in the loop is **(1989, 2M)**



(a) BLv/R clockwise
(b) BLv/R anti-clockwise
(c) $2BLv/R$ anti-clockwise
(d) zero

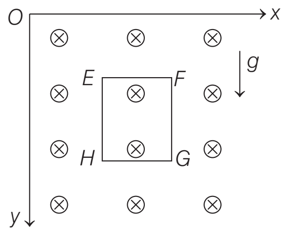
True/ False

7. A conducting rod AB moves parallel to the X-axis in a uniform magnetic field pointing in the positive z-direction. The end A of the rod gets positively charged. **(1987, 2M)**



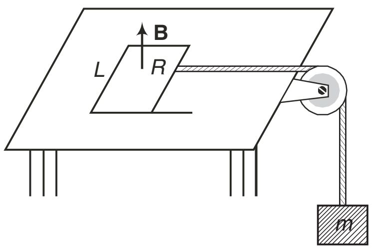
Analytical & Descriptive Questions

8. A magnetic field $B = (B_0\, y/a)\hat{\mathbf{k}}$ is acting into the paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m and resistance R in x-y plane starts falling under the influence of gravity. Note the directions of x and y in the figure. Find **(1999, 10M)**



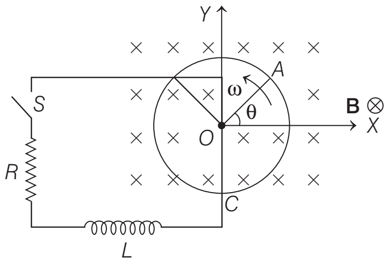
(a) the induced current in the loop and indicate its direction.
(b) the total Lorentz force acting on the loop and indicate its direction.
(c) an expression for the speed of the loop $v(t)$ and its terminal velocity.

9. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest. Calculate : **(1997, 5M)**



(a) the terminal velocity achieved by the rod, and
(b) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

10. A metal rod OA and mass m and length r kept rotating with a constant angular speed ω in a vertical plane about horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction **B** is applied perpendicular and into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open. **(1995, 10M)**



(a) What is the induced emf across the terminals of the switch?

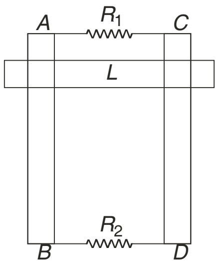
(b) The switch S is closed at time $t = 0$.

(i) Obtain an expression for the current as a function of time.

(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed. Given that the rod OA was along the positive x-axis at $t = 0$.

11. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in figure. A horizontal metallic bar of mass 0.2 kg slides without friction vertically down the rails under the action of gravity.

There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. **(1994, 6M)**



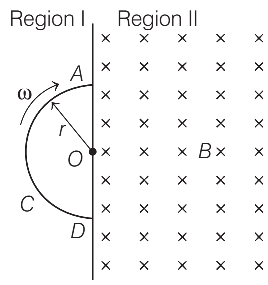
12. Space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper.

The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The effective resistance of the loop is R. **(1985, 6M)**

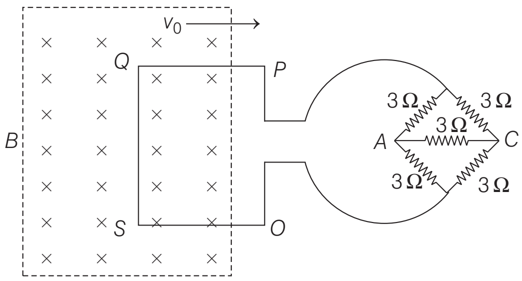
(a) Obtain an expression for the magnitude of the induced current in the loop.

(b) Show the direction of the current when the loop is entering into the region II.

(c) Plot a graph between the induced current and the time of rotation for two periods of rotation.



13. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2\ \text{Wb/m}^2$ as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 Ω.



What should be the speed of the loop so as to have a steady current of 1mA in the loop? Give the direction of current in the loop. **(1983, 6M)**

14. The two rails of a railway track, insulated from each other and the ground, are connected to a millivoltmeter. What is the reading of the millivoltmeter when a train travels at a speed of 180 km/h along the track given that the vertical components of earth's magnetic field is $0.2 \times 10^{-4}\ \text{Wb/m}^2$ and the rails are separated by 1 m? Track is South to North. **(1981, 4M)**

Topic 3 Self and Mutual Inductance

Objective Questions I (Only one correct option)

1. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are **(Main 2019, 10 April I)**

(a) 440 V and 5 A (b) 220 V and 20 A
(c) 220 V and 10 A (d) 440 V and 20 A

2. Two coils P and Q are separated by some distance. When a current of 3 A flows through coil P, a magnetic flux of 10^{-3} Wb passes through Q. No current is passed through Q. When no current passes through P and a current of 2 A passes through Q, the flux through P is **(Main 2019, 9 April II)**

(a) 6.67×10^{-3} Wb (b) 6.67×10^{-4} Wb
(c) 3.67×10^{-3} Wb (d) 3.67×10^{-4} Wb

3. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to **(Main 2019, 9 April I)**

(a) $1/L$ (b) L^2 (c) L (d) $1/L^2$

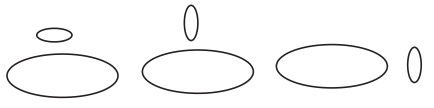
4. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self-inductance of the coil **(Main 2019, 11 Jan II)**

(a) increases by a factor of 3.
(b) decreases by a factor of $9\sqrt{3}$.
(c) increases by a factor of 27.
(d) decreases by a factor of 9.

5 There are two long coaxial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self-inductance of the inner coil is **(Main 2019, 11 Jan I)**

(a) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$ (b) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ (c) $\frac{n_2}{n_1}$ (d) $\frac{n_1}{n_2}$

6. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be **(2001, S)**



(A) (B) (C)

(a) maximum in situation (A).
(b) maximum in situation (B).
(c) maximum in situation (C).
(d) the same in all situations.

7. A small square loop of wire of side l is placed inside a large square loop of wire of side $L (L >> l)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to **(1998, 2M)**

(a) l/L (b) l^2/L (c) L/l (d) L^2/l

Objective Questions II (One or more correct option)

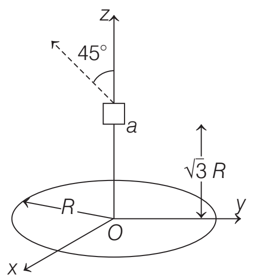
8. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then **(1994, 2M)**

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_1}{V_2} = 4$

Integer Answer Type Question

9. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side a $(a << R)$ having two turns is placed with its centre at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the Z-axis.

If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is **(2012)**



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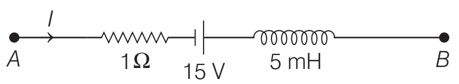
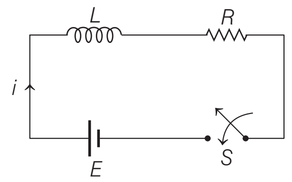
10. The network shown in figure is part of a complete circuit. If at a certain instant the current (I) is 5A and is decreasing at a rate of 10^3 A/s, then $V_B - V_A = ...$V **(1997, 1M)**



Topic 4 *L-R* Circuits and *L-C* Oscillations

Objective Questions I (Only one correct option)

1. Consider the L-R circuit shown in the figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is **(Main 2019, 12 April II)**

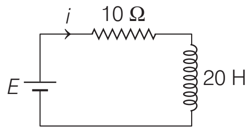


(a) $\frac{2.7EL}{R^2}$ (b) $\frac{EL}{2.7R^2}$ (c) $\frac{7.3EL}{R^2}$ (d) $\frac{EL}{7.3R^2}$

2. A coil of self inductance 10 mH and resistance $0.1\,\Omega$ is connected through a switch to a battery of internal resistance $0.9\,\Omega$. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [Take, $\ln 5 = 1.6$] **(Main 2019, 10 April II)**

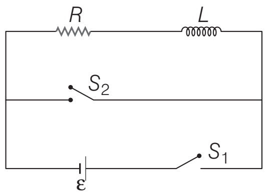
(a) 0.002 s (b) 0.324 s
(c) 0.103 s (d) 0.016 s

3. A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is **(Main 2019, 10 Jan I)**

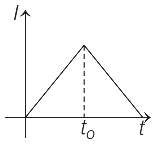


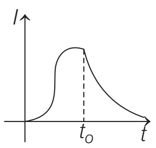
(a) $\frac{2}{\ln 2}$ (b) $\frac{1}{2}\ln 2$ (c) $2\ln 2$ (d) $\ln 2$

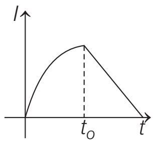
4. In the circuit shown,

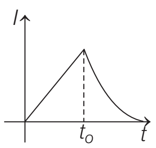


The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by **(Main 2019, 11 Jan I)**

(a) 

(b) 

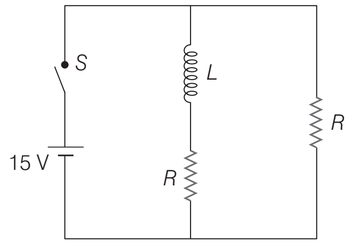
(c) 

(d) 

5. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is **(Main 2019, 9 Jan II)**

(a) 3.39×10^3 J (b) 5.65×10^2 J
(c) 2.26×10^3 J (d) 5.17×10^2 J

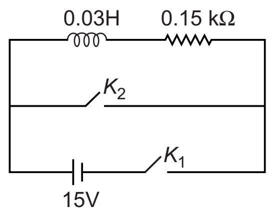
6. In the figure shown, a circuit contains two identical resistors with resistance $R = 5\Omega$ and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit.



What will be the current through the battery long after the switch is closed? **(2019 Main, 12 Jan I)**

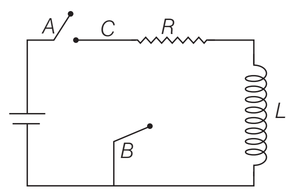
(a) 6 A (b) 3 A (c) 5.5 A (d) 7.5 A

7. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15\,\text{k}\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ms, the current in the circuit will be ($e^5 \cong 150$) **(2015 Main)**



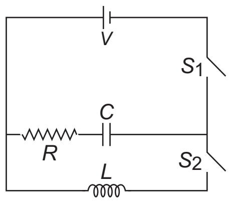
(a) 100 mA (b) 67 mA (c) 0.67mA (d) 6.7 mA

8. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to **(2014 Main)**



(a) $\frac{e}{1-e}$ (b) 1 (c) -1 (d) $\frac{1-e}{e}$

9. In a L-C-R circuit as shown below, both switches are open initially. Now, switch S_1 and S_2, are closed. (q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **(2013 Main)**



(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At $t = \tau$, $q = CV/2$

(c) At $t = 2\tau$, $q = CV\ (1 - e^{-2})$

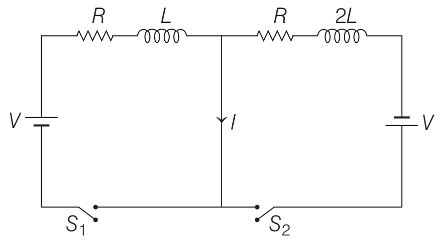
(d) At $t = \tau/2$, $q = CV\ (1 - e^{-1})$

10. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1A at approximately the time **(1999, 2M)**

(a) 500 s (b) 20 s (c) 35 ms (d) 1 ms

Objective Question II (One or More than One)

11. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statements is (are) true ? **(2018 Adv.)**



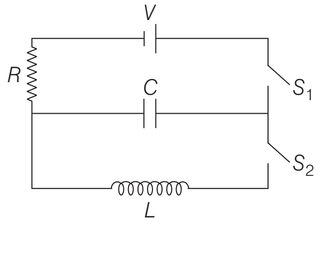
(a) $I_{max} = \dfrac{V}{2R}$ (b) $I_{max} = \dfrac{V}{4R}$

(c) $\tau = \dfrac{L}{R}\ln 2$ (d) $\tau = \dfrac{2L}{R}\ln 2$

Passage Based Questions

Passage

The capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing switch S_1 while keeping switch S_2 open. The capacitor can be connected in series with an inductor L by closing switch S_2 and opening S_1.



12. Initially, the capacitor was uncharged. Now, switch S_1 is closed and S_2 is kept open. If time constant of this circuit is τ, then **(2006, 6M)**

(a) after time interval τ, charge on the capacitor is $CV/2$

(b) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-2})$

(c) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

(d) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-1})$

13. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then, **(2006, 6M)**

(a) at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy.

(b) at any time $t > 0$, current in the circuit is in the same direction.

(c) at $t > 0$, there is no exchange of energy between the inductor and capacitor.

(d) at any time $t > 0$, maximum instantaneous current in the circuit may be $V\sqrt{\dfrac{C}{L}}$.

14. If the total charge stored in the LC circuit is Q_0, then for $t \geq 0$, **(2006, 6M)**

(a) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} + \dfrac{t}{\sqrt{LC}}\right)$

(b) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} - \dfrac{t}{\sqrt{LC}}\right)$

(c) the charge on the capacitor is $Q = -LC\dfrac{d^2Q}{dt^2}$

(d) the charge on the capacitor is $Q = -\dfrac{1}{\sqrt{LC}}\dfrac{d^2Q}{dt^2}$

Fill in the Blank

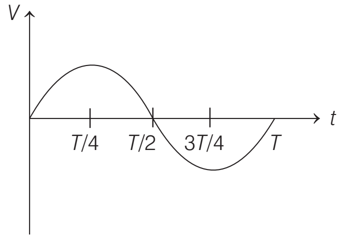
15. A uniformly wound solenoidal coil of self-inductance 1.8×10^{-4} H and resistance $6\,\Omega$ is broken up into two identical coils. These identical coils are then connected in parallel across a 15 V battery of negligible resistance. The time constant for the current in the circuit is …… s and the steady state current through the battery is … A. **(1989, 2M)**

Integer Answer Type Question

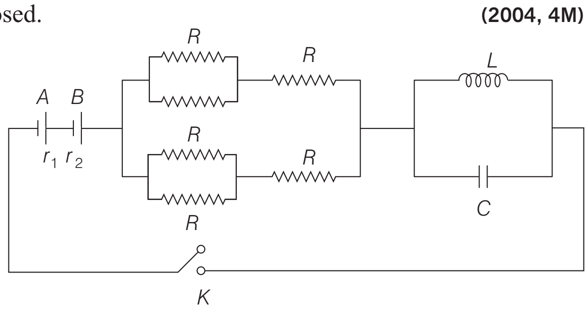
16. Two inductors L_1 (inductance 1mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is **(2016 Adv.)**

Analytical & Descriptive Questions

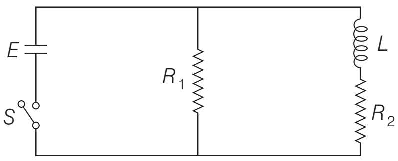
17. In an L-R series circuit, a sinusoidal voltage $V = V_0 \sin \omega t$ is applied. It is given that $L = 35$ mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\omega / 2\pi = 50$ Hz and $\pi = 22/7$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph. **(2004, 4M)**



18. In the circuit shown A and B are two cells of same emf E but different internal resistances r_1 and $r_2 (r_1 > r_2)$ respectively. Find the value of R such that the potential difference across the terminals of cell A is zero, a long time after the key K is closed. **(2004, 4M)**



19. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf $E = 12$ V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$.



What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time? **(2001, 5M)**

20. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting L-C circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I, the current in the circuit. It is found that the maximum value of Q is 200 μC. **(1998, 8M)**

(a) When $Q = 100\ \mu$C, what is the value of $|dI/dt|$?

(b) When $Q = 200\ \mu$C, what is the value of I?

(c) Find the maximum value of I.

(d) When I is equal to one-half its maximum value, what is the value of $|Q|$?

21. A solenoid has an inductance of 10 H and a resistance of 2 Ω. It is connected to a 10 V battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value? **(1996, 3M)**

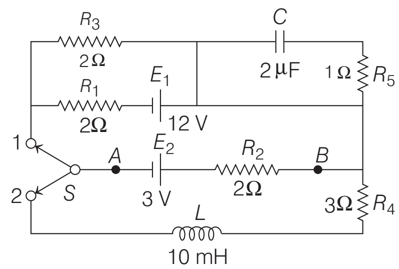
22. A circuit containing a two position switch S is shown in figure. **(1991, 4+4M)**

(a) The switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.

(b) If now the switch S is put in position 2 at $t = 0$. Find

(i) steady current in R_4 and

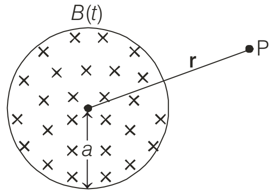
(ii) the time when current in R_4 is half the steady value. Also, calculate the energy stored in the inductor L at that time.



Topic 5 Induced Electric Field

Objective Question I (Only one correct option)

1. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region **(2000, 2M)**



(a) is zero (b) decreases as $1/r$
(c) increases as r (d) decreases as $1/r^2$

Topic 6 Alternating Current

Objective Questions I (Only one correct option)

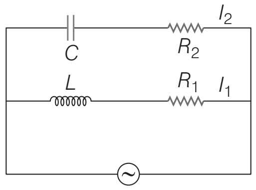
1. A circuit connected to an AC source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this? **(Main 2019, 8 Apr II)**

(a) RC circuit with $R = 1\,\text{k}\Omega$ and $C = 1\,\mu\text{F}$
(b) RL circuit with $R = 1\,\text{k}\Omega$ and $L = 1\,\text{mH}$
(c) RC circuit with $R = 1\,\text{k}\Omega$ and $C = 10\,\mu\text{F}$
(d) RL circuit with $R = 1\,\text{k}\Omega$ and $L = 10\,\text{mH}$

2. An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of $50\,\Omega$. The time taken for the current to rise from half of the peak value to the peak value is **(Main 2019, 8 April I)**

(a) 5 ms (b) 2.2 ms (c) 7.2 ms (d) 3.3 ms

3.



In the above circuit, $C = \frac{\sqrt{3}}{2}\,\mu\text{F}$, $R_2 = 20\,\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\,\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path is I_2. The voltage of AC source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is **(Main 2019, 12 Jan II)**

(a) 30º (b) 60º (c) 0º (d) 90º

4. For an R-L-C circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor, Q is given by **(2018 Main)**

(a) $\frac{CR}{\omega_0}$ (b) $\frac{\omega_0 L}{R}$ (c) $\frac{\omega_0 R}{L}$ (d) $\frac{R}{\omega_0 C}$

5. In an AC circuit, the instantaneous emf and current are given by $e = 100 \sin 30\,t$, $i = 20 \sin\left(30\,t - \frac{\pi}{4}\right)$

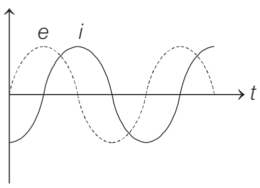
In one cycle of AC, the average power consumed by the circuit and the wattless current are, respectively **(2018 Main)**

(a) 50 , 0 (b) 50 , 10 (c) $\frac{1000}{\sqrt{2}}$, 10 (d) $\frac{50}{\sqrt{2}}$, 0

6. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased, **(2010)**

(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) total impedance of the circuit is unchanged
(d) total impedance of the circuit increases

7. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\frac{\pi}{4}$ ahead, as shown in the diagram.

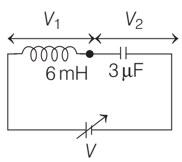


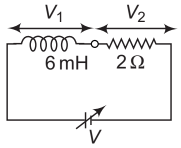
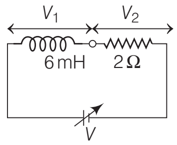
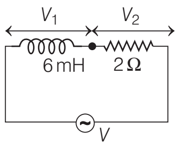
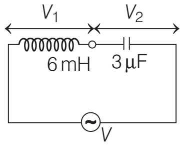
If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements. **(2003, 2M)**

(a) $R = 1\,\text{k}\Omega$, $C = 10\,\mu\text{F}$ (b) $R = 1\,\text{k}\Omega$, $C = 1\,\mu\text{F}$
(c) $R = 1\,\text{k}\Omega$, $L = 10$ H (d) $R = 1\,\text{k}\Omega$, $L = 1$H

Match the Column

8. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (*the first two circuits*) or an AC voltage source of 50 Hz frequency (*the next three circuits*) in different ways as shown in **Column II**. When a current I (*steady state for DC or rms for AC*) flows through the circuit, the corresponding voltage V_1 and V_2 (*indicated in circuits*) are related as shown in **Column I**. **(2010)**

	Column I		Column II
(A)	$I \neq 0, V_1$ is proportional to I	(p)	

	Column I		Column II
(B)	$I \neq 0, V_2 > V_1$	(q)	
(C)	$V_1 = 0, V_2 = V$	(r)	
(D)	$I \neq 0, V_2$ is proportional to I	(s)	
		(t)	

Objective Questions II (One or more correct option)

9. The instantaneous voltages at three terminals marked X, Y and Z are given by $V_X = V_0 \sin \omega t$,

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right).$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be **(2017 Adv.)**

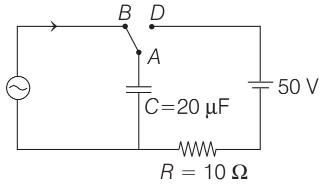
(a) $(V_{YZ})_{\text{rms}} = V_0\sqrt{\frac{1}{2}}$

(b) $(V_{XY})_{\text{rms}} = V_0\sqrt{\frac{3}{2}}$

(c) independent of the choice of the two terminals

(d) $(V_{XY})_{\text{rms}} = V_0$

10. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1\,\text{A}$ and $\omega = 500\,\text{rad s}^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = 7\pi/6\omega$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\,\mu\text{F}$, $R = 10\,\Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s). **(2014 Adv.)**



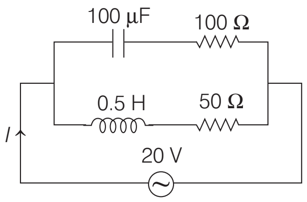
(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(c) Immediately after A is connected to D, the current in R is 10 A

(d) $Q = 2 \times 10^{-3}$ C

11. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is(are) **(2012)**



(a) the current through the circuit, I is 0.3 A

(b) the current through the circuit, I is $0.3\sqrt{2}$ A

(c) the voltage across 100 Ω resistor = $10\sqrt{2}$ V

(d) the voltage across 50 Ω resistor = 10 V

12. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true? **(2011)**

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$

(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

Integer Answer Type Question

13. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ rad/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **(2011)**

Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A solid metal cube of edge length 2 cm is moving in a positive Y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive Z-direction. The potential difference between the two faces of the cube perpendicular to the X-axis is **(Main 2019, 10 Jan I)**

(a) 2 mV (b) 12 mV
(c) 6 mV (d) 1 mV

2. A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be **(Main 2019, 9 Jan II)**

(a) 45 A (b) 50 A (c) 25 A (d) 35 A

3. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to **(2016 Main)**

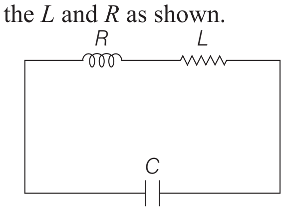
(a) 80 H (b) 0.08 H (c) 0.044 H (d) 0.065 H

4. Arrange the following electromagnetic radiations in the order of increasing energy. **(2016 Main)**

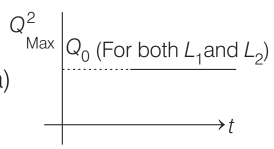
A. Blue light B. Yellow light
C. X-ray D. Radio wave

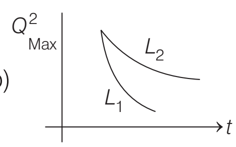
(a) D, B, A, C (b) A, B, D, C
(c) C, A, B, D (d) B, A, D, C

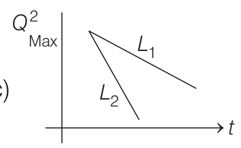
5. An L-C-R circuit is equivalent to a damped pendulum. In an L-C-R circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown.

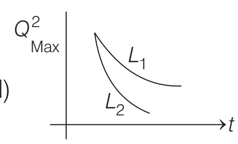


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L, then which of the following represents this graph correctly? (plots are schematic and not drawn to scale) **(2015 Main)**

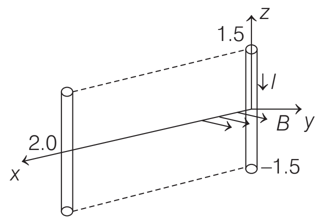
(a) 

(b) 

(c) 

(d) 

6. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\mathbf{B} = 3.0 \times 10^{-4} e^{-0.2x} a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ in 5×10^{-3} s.
Assume parallel motion along the x-axis. **(2014 Main)**

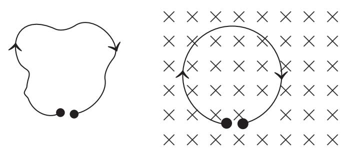


(a) 1.57 W (b) 2.97 W
(c) 14.85 W (d) 29.7 W

7. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5 s. In another 10 s, it will decrease to α times its original magnitude, where α equals **(2013 Main)**

(a) 0.7 (b) 0.81
(c) 0.729 (d) 0.6

8. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is **(2010)**



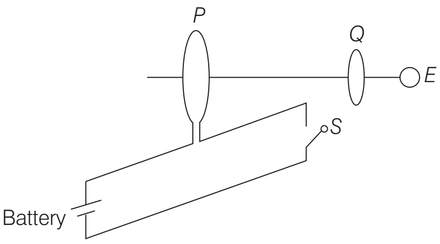
(a) IBL (b) $\dfrac{IBL}{\pi}$
(c) $\dfrac{IBL}{2\pi}$ (d) $\dfrac{IBL}{4\pi}$

9. An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive Z-axis. The direction of induced current as seen from the Z-axis will be
(a) clockwise of the +ve Z-axis **(2005, 2M)**
(b) anti-clockwise of the +ve Z-axis
(c) zero
(d) along the magnetic field

10. A short-circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled (four times) and the wire radius halved, the electrical power dissipated would be **(2002, 2M)**

(a) halved (b) the same
(c) doubled (d) quadrupled

11. As shown in the figure, P and Q are two coaxil conducting loops separated by some distance. When the switch S is closed, a clockwise current I_p flows in P (as seen by E) and an induced current I_{Q_1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q_2} flows in Q. Then the direction I_{Q_1} and I_{Q_2} (as seen by E) are **(2002, 2M)**



(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

12. A coil of wire having finite inductance and resistance has a conducting ring placed co-axially within it. The coil is connected to a battery at time $t = 0$, so that a time dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time ($t > 0$), the product $I_2(t)\,B(t)$ **(2000, 2M)**
(a) increases with time
(b) decreases with time
(c) does not vary with time
(d) passes through a maximum

13. Two identical circular loops of metal wire are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B **(1999, 2M)**
(a) remains stationary
(b) is attracted by the loop A
(c) is repelled by the loop A
(d) rotates about its CM, with CM fixed

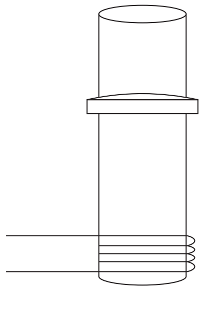
Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) If Statement I is true; Statement II is false
(d) If Statement I is false; Statement II is true

14. **Statement I** A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. There is a conducting ring round the rod as shown in the figure. The ring can float at a certain height above the coil. **(2007, 3M)**

Statement II In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.



Passage Based Questions

Passage 1

Consider a simple RC circuit as shown in Figure 1.

Process 1 In the circuit, the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e. charging continues for time $T >> RC$). In the process, some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_c.

Process 2 In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then, the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in process 1.

These two processes are depicted in Figure 2. **(2017 Adv.)**

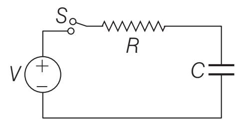


Figure 1

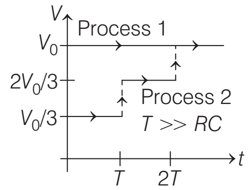


Figure 2

15. In process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by
(a) $E_C = E_D \ln 2$
(b) $E_C = E_D$
(c) $E_C = 2E_D$
(d) $E_C = \frac{1}{2}E_D$

16. In process 2, total energy dissipated across the resistance E_D is
(a) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$
(b) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$
(c) $E_D = 3CV_0^2$
(d) $E_D = \frac{1}{2}CV_0^2$

Passage 2

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value.

At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the current and voltages mentioned are rms values. **(2013 Adv.)**

17. If the direct transmission method with a cable of resistance $0.4\ \Omega\ \text{km}^{-1}$ is used, the power dissipation (in %) during transmission is
(a) 20 (b) 30
(c) 40 (d) 50

18. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is $1:10$. If the power to the consumers has to be supplied at 200V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is
(a) $200:1$ (b) $150:1$ (c) $100:1$ (d) $50:1$

Passage 3

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The applications of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ. **(2013 Adv.)**

19. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is
(a) $\frac{BR}{4}$ (b) $\frac{-BR}{2}$ (c) BR (d) $2BR$

20. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is
(a) γBQR^2 (b) $-\gamma\frac{BQR^2}{2}$ (c) $\gamma\frac{BQR^2}{2}$ (d) γBQR^2

Passage 4

Modern trains are based on Maglev technology in which trains are magnetically leviated, which runs its EDS Maglev system. There are coils on both sides of wheels. Due to motion of train, current induces in the coil of track which levitate it. This is in accordance with Lenz's law. If trains lower down then due to Lenz's law a repulsive force increases due to which train gets uplifted and if it goes much high, then there is a net downward force due to gravity. The advantage of Maglev train is that there is no friction between the train and the track, thereby reducing power consumption and enabling the train to attain very high speeds.

Disadvantage of Maglev train is that as it slows down the electromagnetic forces decreases and it becomes difficult to keep it leviated and as it moves forward according to Lenz's law, there is an electromagnetic drag force. **(2006, 6M)**

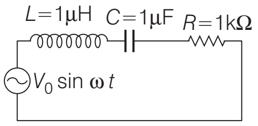
21. What is the advantage of this system ?
(a) No friction hence no power consumption
(b) No electric power is used
(c) Gravitation force is zero
(d) Electrostatic force draws the train

22. What is the disadvantage of this system ?
(a) Train experiences upward force according to Lenz's law
(b) Friction froce create a drag on the train
(c) Retardation
(d) By Lenz's law train experience a drag

23. Which force causes the train to elevate up ?
(a) Electrostatic force
(b) Time varying electric field
(c) Magnetic force
(d) Induced electric field

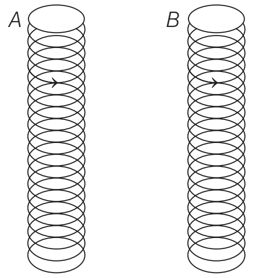
Objective Questions II (One or more correct option)

24. In the circuit shown, $L = 1\mu\text{H}$, $C = 1\mu\text{F}$ and $R = 1\,\text{k}\Omega$. They are connected in series with an AC source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct? **(2017 Adv.)**



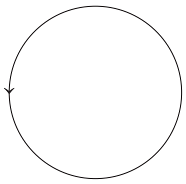
(a) At $\omega \sim 0$, the current flowing through the circuit becomes nearly zero
(b) The frequency at which the current will be in phase with the voltage is independent of R
(c) The current will be in phase with the voltage if $\omega = 10^4\ \text{rads}^{-1}$
(d) At $\omega >> 10^6\ \text{rads}^{-1}$, the circuit behaves like a capacitor

25. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is (are) **(2009)**



(a) $\rho_A > \rho_B$ and $m_A = m_B$ (b) $\rho_A < \rho_B$ and $m_A = m_B$
(c) $\rho_A > \rho_B$ and $m_A > m_B$ (d) $\rho_A < \rho_B$ and $m_A < m_B$

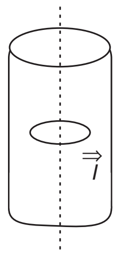
26. A field line is shown in the figure. This field cannot represent **(2006, 5M)**



(a) magnetic field (b) electrostatic field
(c) induced electric field (d) gravitational field

Integer Answer Type Question

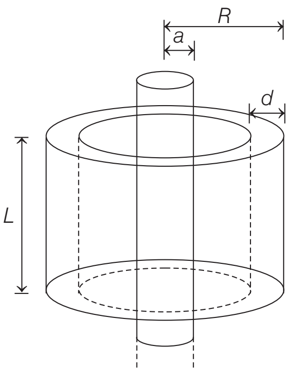
27. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube.



The current varies as $I = I_0 \cos 300\, t$ where I_0 is constant. If the magnetic moment of the loop is $N \mu_0 I_0 \sin(300t)$, then N is **(2011)**

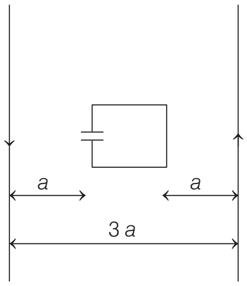
Analytical & Descriptive Questions

28. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R, thickness d ($d << R$) and length L.



A variable current $i = i_0 \sin \omega t$ flows through the solenoid. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell. **(2005, 4M)**

29. Two infinitely long parallel wires carrying currents $I = I_0 \sin \omega t$ in opposite directions are placed a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown. Find the maximum current in the square loop. Also, sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anti-clockwise direction for the current in the loop as positive. **(2003, 4M)**

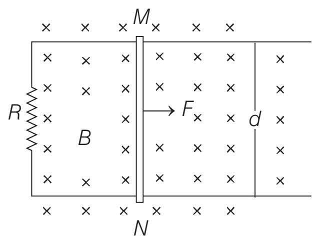


30. A thermocol vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area $5 \times 10^{-3}\, m^2$, number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform time varying magnetic field is setup to pass vertically through the coil at time $t = 0$. The field is first increased from 0 to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s.

The cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as a function of time for the first two cycles. Clearly indicate the magnitudes of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of metal = 500 J/kg-K and the specific heat of water = 4200 J/kg-K. Neglect the inductance of coil. **(2000, 10M)**

31. An infinitesimally small bar magnet of dipole moment **M** is pointing and moving with the speed v in the positive x-direction. A small closed circular conducting loop of radius a and negligible self-inductance lies in the y-z plane with its centre at $x = 0$, and its axis coinciding with the X-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a. **(1997C, 5M)**

32. Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current i flows through R.



Find the velocity of the rod and the applied force F as functions of the distance x of the rod from R.v **(1988, 6M)**

Answers

Topic 1

1. (d) **2.** (a) **3.** (a) **4.** (b)
5. (a) **6.** (d) **7.** (b) **8.** (d)
9. (b,d) **10.** (a,c) **11.** (a, c)
12. Left to right or zero **13.** F and T
14. (a) $\frac{d\phi}{dt} = iR + L\frac{di}{dt}$ (b) $\frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}(\ln(2) - Li_1)\right]$ (c) $\frac{T}{\ln(4)}$
15. $\frac{7}{22}$ A (E to A), $\frac{6}{22}$ A (B to E), $\frac{1}{22}$ A (F to E)
16. Yes, in the direction opposite to A.
17. Clockwise

Topic 2

1. (b) **2.** (d) **3.** (d) **4.** (b)
5. (d) **6.** (d) **7.** T
8. (a) $i = \frac{B_0 av}{R}$, anti-clockwise (b) $\mathbf{F} = -\frac{B_0^2 a^2 v}{R}\,\hat{\mathbf{j}}$
(c) $v = \frac{g}{K}(1 - e^{-kT})$, where $K = \frac{B_0^2 a^2}{mR}$, $v_t = \frac{g}{K} = \frac{gmR}{B_0^2 a^2}$
9. (a) $v_T = \frac{mgR}{B^2 L^2}$ (b) $a = \frac{g}{2}$
10. (a) $e = \frac{B\omega r^2}{2}$
(b) (i) $i = \frac{B\omega r^2}{2R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$ (ii) $\tau_{\text{net}} = \frac{B^2\omega r^4}{4R} + \frac{mgr}{2}\cos\omega t$
11. $v = 1\,\text{m/s}$, $R_1 = 0.47\ \Omega$, $R_2 = 0.3\ \Omega$
12. (a) $\frac{1}{2}\frac{Br^2\omega}{R}$ (b) anti-clockwise (c) see the solution
13. 0.02 m/s, direction of induced current is clockwise
14. 1 mV

Topic 3

1. (a) **2.** (b) **3.** (a) **4.** (c)
5. (c) **6.** (a) **7.** (b) **8.** (a, c, d)
9. 7 **10.** 15

Topic 4

1. (b) **2.** (d) **3.** (c) **4.** (b)
5. (d) **6.** (a) **7.** (c) **8.** (c)
9. (c)
10. (d) **11.** (b, d) **12.** (b) **13.** (d)
14. (c) **15.** 3×10^{-5}, 10 **16.** (8)
17. 20 A, $\frac{\pi}{4}$ **18.** $R = \frac{4}{3}(r_1 - r_2)$
19. $12\,e^{-5t}$ V, $6e^{-10t}$ A (clockwise)
20. (a) 10^4 A/s (b) zero (c) 2.0 A (d) 1.732×10^{-4} C
21. 3.465 s
22. (a) –5V, 24.5 W (b) (i) 0.6 A (ii) 1.386×10^{-3}s, 4.5×10^{-4}J

Topic 5

1. (b)

Topic 6

1. (c) **2.** (d) **3.** (a) **4.** (b)
5. (c) **6.** (b) **7.** (a)
8. A → r, s, t; B → q, r, s, t; C → q, p; D → q, r, s, t
9. (b, c) **10.** (c, d) **11.** (a, c)
12. (b, c) **13.** 4

Topic 7

1. (b) **2.** (a) **3.** (d) **4.** (a)
5. (d) **6.** (b) **7.** (c) **8.** (c)
9. (c) **10.** (d) **11.** (d) **12.** (d)
13. (c) **14.** (a) **15.** (b) **16.** (a)
17. (b) **18.** (a) **19.** (b) **20.** (b)
21. (a) **22.** (d) **23.** (c) **24.** (a, b)
25. (b, d) **26.** (b, d) **27.** 6
28. $i = \frac{\mu_0 Ldna^2 I_0\omega\cos\omega t}{2\rho R}$ **29.** $i_{\max} = \frac{\mu_0 a\, CI_0\omega^2\ln(2)}{\pi}$
30. 35.6°C **31.** $F = \frac{21}{4}\frac{\mu_0^2 M^2 a^4 v}{Rx^8}$ (repulsion)
32. $v = \frac{(R + 2\lambda x)\,i}{Bd}$, $F = \frac{2\lambda i^2 m}{B^2 d^2}(R + 2\lambda x)^2 + idB$

Hints & Solutions

Topic 1 Magnetic Flux and Induced EMF by change in Flux

1. Magnetic flux associated with the outer coil is

$$\phi_{\text{outer}} = \mu_0 \pi NR \cdot I = \mu_0 N\pi R(kte^{-\alpha t})$$
$$= Cte^{-\alpha t}$$

where,

$$C = \mu_0 N\pi Rk = \text{constant}$$

Induced emf,

$$e = \frac{-d\phi_{\text{outer}}}{dt} = Ce^{-\alpha t} + (-\alpha\, C\, t\, e^{-\alpha t})$$
$$= Ce^{-\alpha t}(1 - \alpha t)$$

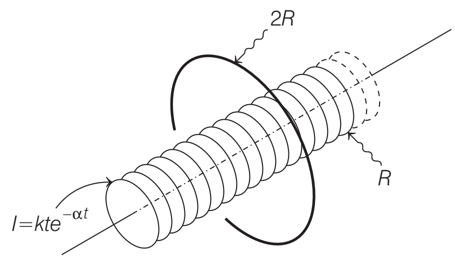
$\therefore$ Induced current, $I = \dfrac{e}{\text{Resistance}}$

$\Rightarrow$ At $t = 0$, $I = -$ ve

$\therefore$ The correct graph representing this condition is given in option (d).

Alternate Solution

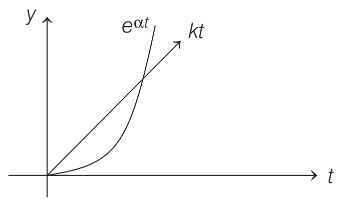
Given solenoid is shown below as,



At $t = 0$, current in solenoid

$$= I(t = 0) = k(0)\, e^{-\alpha \cdot 0} = 0$$

Graph of $e^{\alpha t}$ and kt *versus* time can be shown as,



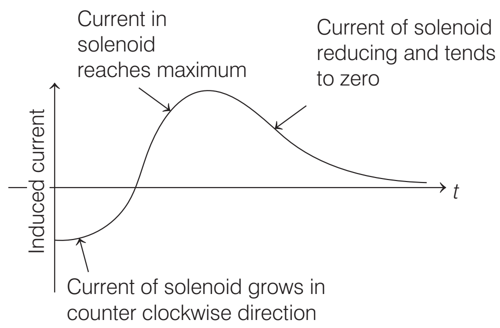
As, $\quad I = \dfrac{kt}{e^{\alpha t}}$;

Initially, $\quad kt > e^{\alpha t}$

So, current is increasing in magnitude.

Finally, after a short time $kt < e^{\alpha t}$. So, current is decreasing in magnitude.

But in both cases, it remains positive or counter clockwise. So, current induced is at first anti-clockwise (following Lenz's law) and then it becomes clockwise and finally reduces to zero as $t \longrightarrow \infty$.



So, correct graph of induced current is

2. Wire falls perpendicularly to horizontal component of earth's magnetic field, so induced electromotive force $(\varepsilon) = Blv$

Substituting the given values, we get

$$\varepsilon = 0.3 \times 10^{-4} \times 10 \times 5 = 1.5 \times 10^{-3}\ \text{V}$$

3. Energy stored in an inductor of inductance L and current I is given by

$$E = \frac{1}{2}LI^2$$

When current is being changed from I_1 to I_2, change in energy will be

$$\Delta E = E_2 - E_1 = \frac{1}{2}LI_2^2 - \frac{1}{2}LI_1^2 \qquad \ldots\text{(i)}$$

As I_1 and I_2 are given, we need to find value of L.

Now, induced emf in a coil is

$$\varepsilon = L\frac{dI}{dt}$$

Here, $\quad \varepsilon = 25\,\text{V}$,

$$dI = I_2 - I_1 = (25 - 10) = 15\ \text{A and } dt = 1\ \text{s}$$

$$\Rightarrow \quad 25 = L \times \frac{15}{1} \text{ or } L = \frac{25}{15} = 5/3\,\text{H}$$

Putting values of L, I_1 and I_2 in Eq. (i), we get

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times [25^2 - 10^2] = \frac{1}{2} \times \frac{5}{3} \times 525 \quad \Delta E = 437.5\ \text{J}$$

4. Induced constant, $I = \dfrac{e}{R}$

Here, e = induced emf $= \dfrac{d\phi}{dt}$

$$I = \frac{1}{R} = \left(\frac{d\phi}{dt}\right) \cdot \frac{1}{R}$$
$$d\phi = IRdt$$
$$\phi = \int IRdt$$

$\therefore$ Here, R is constant

$\therefore$ $$\phi = R\int Idt$$

$\int I \cdot dt =$ Area under I-t graph

$$= \frac{1}{2} \times 10 \times 0.5 = 2.5$$

$\therefore \quad \phi = R \times 2.5 = 100 \times 2.5 = 250$ Wb.

5. Magnetic field at the centre of smaller loop

$$B = \frac{\mu_0 i R_2^2}{2(R_2^2 + x^2)^{3/2}}$$

Area of smaller loop $S = \pi R_1^2$

$\therefore$ Flux through smaller loop $\phi = BS$

Substituting the values, we get, $\phi \approx 9.1 \times 10^{-11}$ Wb

6. Cross $\otimes$ magnetic field passing from the closed loop is increasing. Therefore, from Lenz's law induced current will produce dot $\odot$ magnetic field. Hence, induced current is anti-clockwise.

7. Polarity of emf will be opposite in the two cases while entering and while leaving the coil. Only in option (b) polarity is changing.

8. Total magnetic flux passing through whole of the x-y plane will be zero, because magnetic lines form a closed loop. So, as many lines will move in $-z$ direction same will return to $+z$ direction from the x-y plane.

9. The net magnetic flux through the loops at time t is

$$\phi = B(2A - A)\cos\omega t = BA\cos\omega t$$

So, $\left|\frac{d\phi}{dt}\right| = B\omega A \sin\omega t$

$\therefore \left|\frac{d\phi}{dt}\right|$ is maximum when $\phi = \omega t = \pi/2$

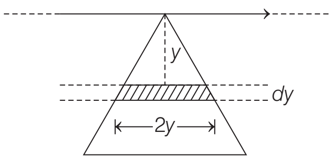
The emf induced in the smaller loop,

$$\varepsilon_{\text{smaller}} = -\frac{d}{dt}(BA\cos\omega t) = B\omega A \sin\omega t$$

$\therefore$ Amplitude of maximum net emf induced in both the loops

= Amplitude of maximum emf induced in the smaller loop alone.

10. By reciprocity theorem of mutual induction, it can be assumed that current in infinite wire is varying at 10A/s and EMF is induced in triangular loop.



Flux of magnetic field through triangle loop, if current in infinite wire is ϕ, can be calculated as follows

$$d\phi = \frac{\mu_0 i}{2\pi y} \cdot 2y\,dy$$

$$d\phi = \frac{\mu_0 i}{\pi} dy$$

$$\phi = \frac{\mu_0 i}{\pi}\left(\frac{l}{\sqrt{2}}\right)$$

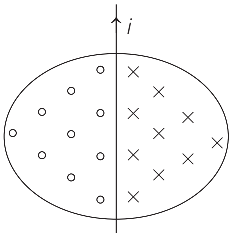
$$\Rightarrow \quad \text{EMF} = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0}{\pi}\left(\frac{l}{\sqrt{2}}\right) \cdot \frac{di}{dt}$$

$$= \frac{\mu_0}{\pi}(10\text{cm})\left(10\frac{\text{A}}{\text{s}}\right) = \frac{\mu_0}{\pi}\text{ volt}$$

If we assume the current in the wire towards right then as the flux in the loop increases we know that the induced current in the wire is counter clockwise. Hence, the current in the wire is towards right.

Field due to triangular loop at the location of infinite wire is into the paper. Hence, force on infinite wire is away from the loop. By cylindrical symmetry about infinite wire, rotation of triangular loop will not cause any additional EMF.

11.



Due to the current in the straight wire, net magnetic flux from the circular loop is zero. Because in half of the circle, magnetic field is inwards and in other half, magnetic field is outwards. Therefore, change in current will not cause any change in magnetic flux from the loop. Therefore, induced emf under all conditions through the circular loop is zero.

12. When source is switched-off, left to right current decreases to zero. Therefore, from Lenz's law, induced current will oppose the change i.e. it will be from left to right if there is some inductance in the circuit, otherwise it will be zero.

13. If field is non-uniform from position point of view, no emf will be induced. If it is non-uniform from time point of view, emf will be induced.

14. (a) Applying Kirchhoff's second law, we get

$$\frac{d\phi}{dt} - iR - L\frac{di}{dt} = 0$$

or $$\frac{d\phi}{dt} = iR + L\frac{di}{dt} \quad \ldots\text{(i)}$$

This is the desired relation between i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$.

(b) Eq. (i) can be written as

$$d\phi = iRdt + Ldi$$

Integrating, we get

$$\Delta\phi = R\,\Delta q + Li_1$$

$$\Delta q = \frac{\Delta\phi}{R} - \frac{Li_1}{R} \quad \ldots\text{(ii)}$$

Here, $\Delta\phi = \phi_f - \phi_i = \int_{x=2x_0}^{x=x_0} \frac{\mu_0}{2\pi}\frac{I_0}{x}\,l\,dx = \frac{\mu_0 I_0 l}{2\pi}\ln(2)$

So, from Eq. (ii) charge flown through the resistance upto time $t = T$, when current i_1, is

$$\Delta q = \frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}\ln(2) - L i_1\right]$$

(c) This is the case of current decay in an L-R circuit. Thus,

$$i = i_0 e^{-t/\tau_L} \qquad \ldots\text{(iii)}$$

Here, $i = \frac{i_1}{4}$, $i_0 = i_1$, $t = (2T - T) = T$ and $\tau_L = \frac{L}{R}$

Substituting these values in Eq. (iii), we get

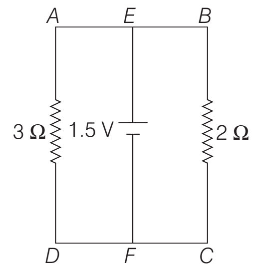
$$\tau_L = \frac{L}{R} = \frac{T}{\ln 4}$$

15. Induced emf in two loops $AEFD$ and $EBCF$ would be

$$e_1 = \left|\frac{d\phi_1}{dt}\right| = S_1\left(\frac{dB}{dt}\right) = (1\times 1)\,(1)\text{ V} = 1\text{ V}$$

Similarly, $$e_2 = \left|\frac{d\phi_2}{dt}\right| = S_2\left(\frac{dB}{dt}\right) = (0.5\times 1)\,(1)\text{ V} = 0.5\text{ V}$$

Now, since the magnetic field is increasing, the induced current will produce the magnetic field in u direction. Hence, e_1 and e_2 will be applied as shown in the figure.



Kirchhoff's first law at junction F gives

$$i_1 = i + i_2 \qquad \ldots\text{(i)}$$

Kirchhoff's second law in loop $FEADF$ gives

$$3i_1 + i = 1 \qquad \ldots\text{(ii)}$$

Kirchhoff's second law in loop $FEBCF$ gives

$$2i_2 - i = 0.5 \qquad \ldots\text{(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = \left(\frac{7}{22}\right)\text{A} \quad \text{and} \quad i_2 = \left(\frac{6}{22}\right)\text{A}$$

and $\quad i = (1/22)$ A

Therefore, current in segment AE is (7/22) A from E to A, current in segment BE is 6/22 A from B to E and current in segment EF is (1/22) A from F to E.

16. Due to the current in A, a magnetic field is from right to left. When A is moved towards B, magnetic lines passing through B (from right to left) will increase, i.e. magnetic flux passing through B will increase. Therefore, current will be induced in B. The induced current will have such a direction that it gives a magnetic field opposite to that, passing through B due to current in A. Therefore, induced current in B will be in opposite direction of current in A.

17. Magnetic field due to straight wire passing through the wire loop will be perpendicular to paper outwards. With increase in current in straight wire, outwards magnetic field through the loop will increase. Therefore, from Lenz's law, inward magnetic field will be produced by the induced current. Hence, induced current is clockwise.

Topic 2 Motional and Rotational EMF

1. Induced emf in the conductor of length L moving with velocity of 1 cm/s in the magnetic field of 1T is given by

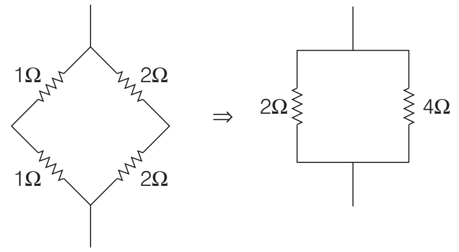
$$V = BLv \qquad \ldots\text{(i)}$$

If equivalent resistance of the circuit is R_{eq}, then current in the loop will be

$$i = \frac{V}{R_{eq}} = \frac{BLv}{R_{eq}} \qquad \ldots\text{(ii)}$$

Now, given network is a balanced Wheatstone bridge $\left(\frac{P}{Q} = \frac{R}{S}\right)$.

So, equivalent resistance of the Wheatstone bridge is



$$R_W = \frac{2\times 4}{2+4} = \frac{8}{6} = \frac{4}{3}\,\Omega$$

Again, resistance of conductor is 1.7 Ω.

So, effective resistance will be

$$R_{eq} = \frac{4}{3} + 1.7 = \frac{4}{3} + \frac{17}{10}$$

$$R_{eq} = \frac{40+51}{30} = \frac{91}{30} \simeq 3\Omega$$

By putting given values of R_{eq}, B and v in Eq. (ii), we have

$$i = \frac{(1)\,(5\times 10^{-2})\times 10^{-2}}{3}$$

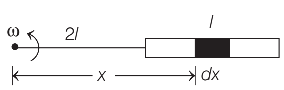
[here, $L = 5\times 10^{-2}$ m, $v = 1$ cm/s $= 10^{-2}$ m/s]

$$i = \frac{5\times 10^{-4}}{3} = 1.67\times 10^{-4}\text{ A}$$

$$i = 167\,\mu\text{A} \approx 170\,\mu\text{A}$$

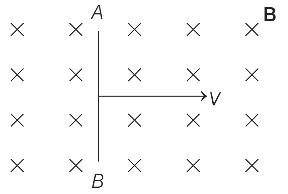
2. $$e = \int_{2l}^{3l} (\omega x)\,B\,dx = B\omega\,\frac{[(3l)^2 - (2l)^2]}{2}$$

$$= \frac{5Bl^2\omega}{2}$$



3. Electric field will be induced in both AD and BC.

4. A motional emf, $e = Blv$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



5. Induced motional emf in MNQ is equivalent to the motional emf in an imaginary wire MQ i.e.

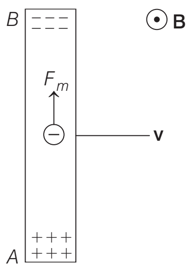
$$e_{MNQ} = e_{MQ} = Bvl = Bv(2R) \quad [l = MQ = 2R]$$

Therefore, the potential difference developed across the ring is $2RBv$ with Q at higher potential.

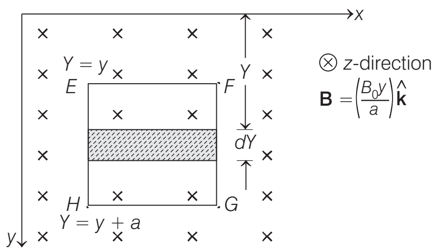
6. Net change in magnetic flux passing through the coil is zero.

$\therefore$ Current (or emf) induced in the loop is zero.

7. Magnetic force on free electrons will be towards B. Therefore, at B, there is excess of electrons (means negative charge) and at A, there is defficiency of electrons (means positive charge).



8. When the side EF is at a distance y from the X-axis, magnetic flux passing through the loop is



$$\phi = \int d\phi = \int_{Y=y}^{Y=y+a} \frac{B_0 Y}{a}(a\,dY)$$

$$\phi = \frac{B_0}{2}[(y+a)^2 - y^2]$$

(a) Induced emf is

$$e = \left|\frac{-d\phi}{dt}\right| = \left|-\frac{B_0}{2}[2(y+a) - 2y]\frac{dy}{dt}\right|$$

$$e = B_0 a\frac{dy}{dt} \Rightarrow e = B_0 va$$

where, $v = \frac{dy}{dt}$ = speed of loop

$\therefore$ Induced current, $i = \frac{e}{R} = \frac{B_0 av}{R}$

Direction $|\mathbf{B}| \propto y$ i.e. as the loop comes down ⊗ magnetic field passing through the loop increases, therefore the induced current will producer u, magnetic field or the induced current in the loop will be **counter-clockwise.**

Alternate solution (of part a)

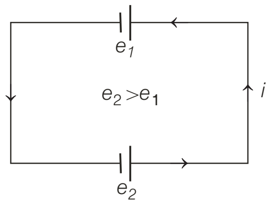
Motional emf in EH and $FG = 0$ as $\mathbf{v}||\mathbf{I}$

Motional emf in EF is $e_1 = \left(\frac{B_0 y}{a}\right)(a)v = B_0 yv \quad (\because e = Blv)$

Similarly, motional emf in GH will be

$$e_2 = \left\{\frac{B_0(y+a)}{a}\right\}(a)(v) = B_0(a+y)v$$

Polarities of e_1 and e_2 are shown in adjoining figures.



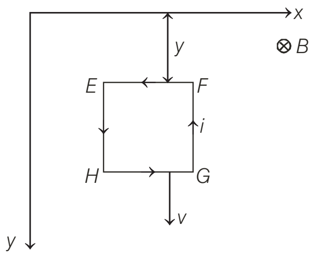
Net emf, $\quad e = e_2 - e_1$

$$e = B_0 av$$

$$\therefore \quad i = \frac{e}{R} = \frac{B_0 av}{R}$$

and direction of current will be counter-clockwise.

(b) Total Lorentz force on the loop :**3**



We have seen in part (a) that induced current passing through the loop (when its speed is v) is

$$i = \frac{B_0 av}{R}$$

Now, magnetic force on EH and FG are equal in magnitude and in opposite directions, hence they cancel each other and produce no force on the loop.

$$F_{EF} = \left(\frac{B_0 av}{R}\right)(a)\left(\frac{B_0 y}{a}\right) \quad \text{(downwards)}$$

$$(F = ilB) = \frac{B_0{}^2 avy}{R}$$

and $$F_{GH} = \left(\frac{B_0 av}{R}\right)(a)\left(\frac{B_0 (y+a)}{a}\right) \quad \text{(upwards)}$$

$$= \left(\frac{B_0{}^2 av}{R}\right)(y+a)$$

$$F_{GH} > F_{EF}$$

$\therefore$ Net Lorentz force on the loop

$$\mathbf{F} = F_{GH} - F_{EF} = \frac{B_0^2 a^2 v}{R} \quad \text{(upwards)}$$

$$\mathbf{F} = -\frac{B_0^2 a^2 v}{R}\hat{\mathbf{j}}$$

(c) Net force on the loop will be

F = weight − Lorentz force (downwards)

or $$F = mg - \frac{B_0^2 a^2 v}{R}$$

or $$m\left(\frac{dv}{dt}\right) = mg - \left(\frac{B_0^2 a^2}{R}\right)v$$

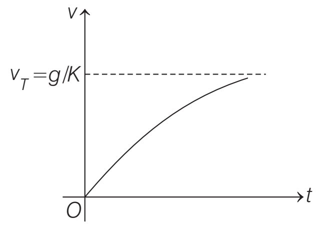
$\therefore$ $$\frac{dv}{dt} = g - \left(\frac{B_0^2 a^2}{mR}\right)v = g - Kv$$

where, $K = \dfrac{B_0^2 a^2}{mR}$ = constant

or $$\frac{dv}{g - Kv} = dt$$

or $$\int_0^v \frac{dv}{g - Kv} = \int_0^t dt$$

This equation gives $v = \dfrac{g}{K}(1 - e^{-Kt})$



Here, $$K = \left(\frac{B_0^2 a^2}{mR}\right)$$

i.e. speed of the loop is increasing exponentially with time t. Its terminal velocity will be

$$v_T = \frac{g}{K} = \left(\frac{mgR}{B_0^2 a^2}\right)$$

at $t \to \infty$

9. (a) Let v be the velocity of the wire (as well as block) at any instant of time t.

Motional emf, $e = BvL$

Motional current, $i = \dfrac{e}{r} = \dfrac{BvL}{R}$

and magnetic force on the wire

$$F_m = iLB = \frac{vB^2L^2}{R}$$

Net force in the system at this moment will be

$$F_{\text{net}} = mg - F_m = mg - \frac{vB^2L^2}{R}$$

or $$ma = mg - \frac{vB^2L^2}{R}$$

$$a = g - \frac{vB^2L^2}{mR} \quad \text{...(i)}$$

Velocity will acquire its terminal value i.e. $v = v_T$ when F_{net} or acceleration (a) of the particle becomes zero.

Thus, $$0 = g - \frac{v_T B^2 L^2}{mR}$$

or $$v_T = \frac{mgR}{B^2L^2}$$

(b) When $$v = \frac{v_T}{2} = \frac{mgR}{2B^2L^2}$$

Then from Eq. (i), acceleration of the block,

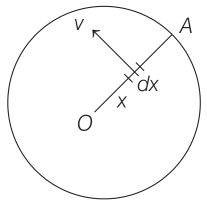
$$a = g - \left(\frac{mgR}{2B^2L^2}\right)\left(\frac{B^2L^2}{mR}\right) = g - \frac{g}{2}$$

or $$a = \frac{g}{2}$$

10. (a) Consider a small element of length dx of the rod OA situated at a distance x from O.

Speed of this element, $v = x\omega$

Therefore, induced emf developed across this element in uniform magnetic field B



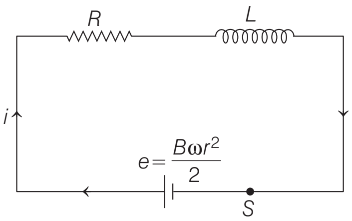
$$de = (B)(x\omega)dx \quad (\because e = Bvl)$$

Hence, total induced emf across OA,

$$e = \int_{x=0}^{x=r} de = \int_0^r B\omega x dx = \frac{B\omega r^2}{2} \Rightarrow e = \frac{B\omega r^2}{2}$$

(b) (i) A constant emf or PD, $e = \dfrac{B\omega r^2}{2}$ is induced across O and A.

The equivalent circuit can be drawn as shown in the figure.



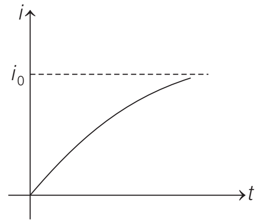
Switch S is closed at time $t = 0$. Therefore, it is case of growth of current in an L-R circuit. Current at any time t is given by

$$i = i_0(1 - e^{-t/\tau_L}),\ i_0 = \frac{e}{R} = \frac{B\omega r^2}{2R}$$

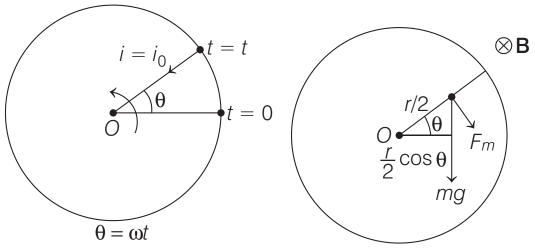
$$\tau_L = L/R$$

$$i = \frac{B\omega r^2}{2R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

The i-t graph will be as follows



(ii) At constant angular speed, net torque $= 0$



The steady state current will be $i = i_0 = \dfrac{B\omega r^2}{2R}$

From right hand rule, we can see that this current would be inwards (from circumference to centre) and corresponding magnetic force F_m will be in the direction shown in figure and its magnitude is given by

$$F_m = (i)(r)(B) = \frac{B^2\omega r^3}{2R} \qquad (\because F_m = ilB)$$

Torque of this force about centre O is

$$\tau_{F_m} = F_m \cdot \frac{r}{2} = \frac{B^2\omega r^4}{4R} \qquad \text{(clockwise)}$$

Similarly, torque of weight (mg) about centre O is

$$\tau_{mg} = (mg)\frac{r}{2}\cos\theta = \frac{mgr}{2}\cos\omega t \qquad \text{(clockwise)}$$

Therefore, net torque at any time t (after steady state condition is achieved) about centre O will be

$$\tau_{\text{net}} = \tau_{F_m} + \tau_{mg}$$

$$= \frac{B^2\omega r^4}{4R} + \frac{mgr}{2}\cos\omega t \qquad \text{(clockwise)}$$

Hence, the external torque applied to maintain a constant angular speed is $\tau_{\text{ext}} = \dfrac{B^2\omega r^4}{4R} + \dfrac{mgr}{2}\cos\omega t$ (but in anti-clockwise direction).

Note that for $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$, torque of weight will be anti-clockwise, the sign of which is automatically adjusted because $\cos\theta$ = negative for $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$.

11. Let the magnetic field be perpendicular to the plane of rails and inwards ⊗ . If v be the terminal velocity of the rails, then potential difference across E and F would be BvL with E at lower potential and F at higher potential. The equivalent circuit is shown in figure (2). In figure (2)

$$i_1 = \frac{e}{R_1} \qquad \text{... (i)}$$

$$i_2 = \frac{e}{R_2} \qquad \text{... (ii)}$$

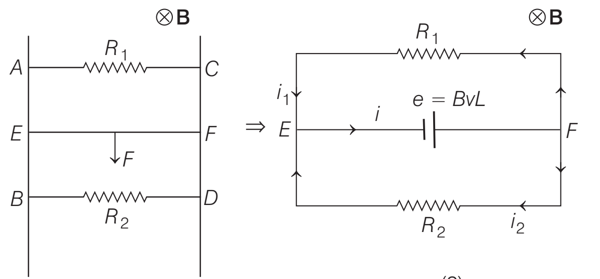
Power dissipated in R_1 is 0.76 W

Therefore $\qquad ei_1 = 0.76\ \text{W} \qquad$ ... (iii)

Similarly, $\qquad ei_2 = 1.2\ \text{W} \qquad$ ... (iv)

Now, the total current in bar EF is

$$i = i_1 + i_2 \qquad \text{(from } E \text{ to } F\text{) ...(v)}$$

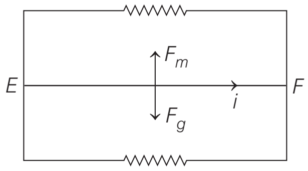


(1) (2)

Under equilibrium condition, magnetic force (F_m) on bar EF = weight (F_g) of bar EF

i.e. $\qquad F_m = F_g \quad$ or $\quad iLB = mg \qquad$ ...(vi)

From Eq. (vi) $\qquad i = \dfrac{mg}{LB} = \dfrac{(0.2)(9.8)}{(1.0)(0.6)}$



or $\qquad i = 3.27$ A

Multiplying Eq. (v) by e, we get

$$ei = ei_1 + ei_2$$

$$= (0.76 + 1.2) \quad \text{[From Eqs. (iii) and (iv)]}$$

$$= 1.96 \text{ W}$$

$$e = \frac{1.96}{i} \text{ V} = \frac{1.96}{3.27}$$

or $\quad e = 0.6 \text{ V}$

But since $\quad e = BvL$

$$v = \frac{e}{BL} = \frac{(0.6)}{(0.6)(1.0)} = 1.0 \text{ m/s}$$

Hence, terminal velocity of bar is 1.0 m/s.

Power in R_1 is 0.76 W

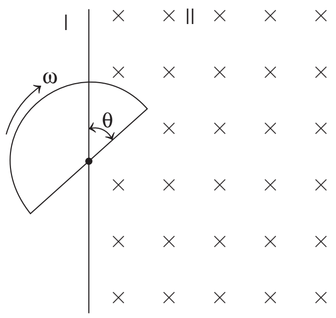
$$\therefore \quad 0.76 = \frac{e^2}{R_1} \Rightarrow \therefore \; R_1 = \frac{e^2}{0.76} = \frac{(0.6)^2}{0.76}$$

$$= 0.47\,\Omega \Rightarrow R_1 = 0.47\,\Omega$$

Similarly, $\quad R_2 = \dfrac{e^2}{1.2} = \dfrac{(0.6)^2}{1.2} = 0.3\,\Omega$

$$R_2 = 0.3\,\Omega$$

12. (a) At time $t : \theta = \omega t$



$\therefore$ Flux passing through coil $\phi = BS \cos 0°$

or $\quad \phi = B\left(\dfrac{\theta}{2\pi}\right)(\pi r^2)$

or $\quad \phi = \left(\dfrac{Br^2}{2}\right)\theta = \left(\dfrac{Br^2}{2}\right)\omega t$

Magnitude of induced emf

$$e = \frac{d\phi}{dt} = \frac{B\omega r^2}{2}$$

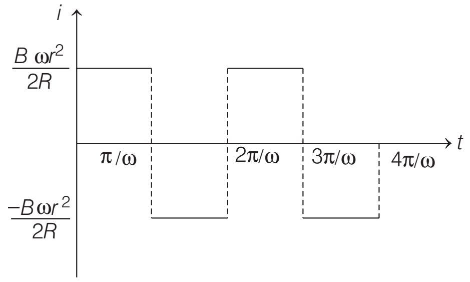
$\therefore$ Magnitude of induced current

$$i = \frac{e}{R} = \frac{B\omega r^2}{2R}$$

(b) When the loop enters in region II, magnetic field in cross direction passing through the loop is increasing. Hence, from the Lenz's law, induced current will produce magnetic field in dot direction or the current will be anti-clockwise.

(c) For half rotation $\left(t = \dfrac{T}{2} = \dfrac{\pi}{\omega}\right)$, current in the loop will be of constant magnitude $i = \dfrac{B\omega r^2}{2R}$ and anti-clockwise.

In the next half rotation when loop comes out of region II current will be clockwise, but again magnitude is constant. So, taking anti-clockwise current as the positive, i-t graph for two rotations will be as under.



13. Given network forms a balanced Wheatstone's bridge. The net resistance of the circuit is therefore $3\Omega + 1\Omega = 4\Omega$. Emf of the circuit is $Bv_0 l$. Therefore, current in the circuit would be

$$i = \frac{Bv_0 l}{R} \quad \text{or} \quad v_0 = \frac{iR}{Bl}$$

$$= \frac{(1 \times 10^{-3})(4)}{2 \times 0.1} = 0.02 \text{ m/s}$$

Cross magnetic field passing through the loop is decreasing. Therefore, induced current will produce magnetic field in cross direction. Or direction of induced current is clockwise.

14. Potential difference between the two rails :
$V = Bvl$ (When **B**, **v** and **I** all are mutually perpendicular to each other)

$$= (0.2 \times 10^{-4})\left(180 \times \frac{5}{18}\right)(1)$$

$$= 10^{-3} \text{ V} = 1 \text{ mV}$$

Topic 3 Self and Mutual Inductance

1. Given, Number of turns in primary, $N_1 = 300$

Number of turns in secondary, $N_2 = 150$

Output power, $P_2 = 2.2 \text{ kW} = 2.2 \times 10^3 \text{ W}$

Current in secondary coil, $I_2 = 10 \text{ A}$

Output power, $P_2 = I_2 V_2$

$$\Rightarrow \quad V_2 = \frac{P_2}{I_2} = \frac{2.2 \times 10^3}{10} = 220 \text{V} \quad \ldots \text{(i)}$$

We know that,

$$\frac{N_1}{N_2} = \frac{\text{Input voltage}}{\text{Output voltage}} = \frac{V_1}{V_2} \Rightarrow V_1 = \left(\frac{N_1}{N_2}\right) V_2$$

$$\Rightarrow \quad V_1 = \left(\frac{300}{150}\right) \times (220 \text{ V}) \quad \text{[using Eq. (i)]}$$

$$V_1 = 440 \text{ V} \quad \ldots \text{(ii)}$$

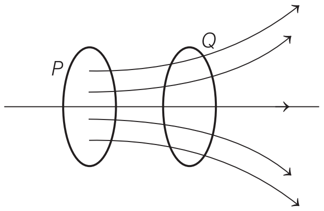
Again, $\quad \dfrac{V_1}{V_2} = \dfrac{I_2}{I_1}$

$$\Rightarrow \quad I_1 = \left(\frac{V_2}{V_1}\right) I_2 = \frac{220}{440} \times 10$$

[using Eqs. (i) and (ii)]

$$\Rightarrow \quad I_1 = 5 \text{A}$$

2. As, coefficient of mutual induction is same for both coils



$$\therefore \quad M_{PQ} = M_{QP}$$

$$\Rightarrow \quad \frac{N_P \phi_{PQ}}{I_Q} = \frac{N_Q \phi_{QP}}{I_P} \quad \ldots \text{(i)}$$

Here, $N_P = N_Q = 1$,

$\phi_{PQ} = ?, \phi_{QP} = 10^{-3}$ Wb

$I_Q = 2\text{A}, I_P = 3\text{A}$

Substituting values in Eq (i), we get

$$\phi_{PQ} = \frac{I_Q \cdot \phi_{QP}}{I_P} = \frac{2}{3} \times 10^{-3}$$

$$= 0.667 \times 10^{-3} = 6.67 \times 10^{-4} \text{ Wb}$$

3. *(a)* Self inductance L_{sol} of a solenoid is given by

$$L_{sol} = \mu_0 n^2 \pi r^2 L$$

(Here, $n = N / L$ and L = length of solenoid)

$$\text{or} \quad L_{sol} = \frac{\mu_0 N^2 \pi r^2}{L}$$

$$\text{Clearly,} \quad L_{sol} \propto \frac{1}{L}$$

($\because$ All other parameters are fixed)

NOTE We can determine expression of L as follows

$$\phi = NBA = L_{sol} I$$

But for a solenoid, $B = \mu_0 nI$, $A = \pi r^2$

$$\therefore \quad L_{sol} I = \mu_0 n I \pi r^2 N$$

$$\text{or} \quad L_{sol} = \mu_0 n^2 \pi r^2 L = \mu_0 \frac{N^2}{L} \pi r^2$$

4. Self-inductance of a coil is given by the relation

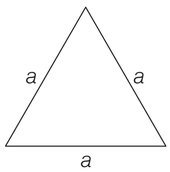
$$L = \mu_0 n^2 A \cdot l$$

where, n is number of turns per unit length. Shape of the wooden frame is equilateral triangle.

$\therefore$ Area of equilateral triangle,

$$A = \frac{\sqrt{3}}{4} a^2$$

(where, a is side of equilateral triangle)



$\therefore$ Self-inductance, $L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2\right] . l$

Here, $l = 3a \times N$ (where, N is total number of turns)

$$\therefore \quad L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2\right] \times 3aN \text{ or } L \propto a^3$$

When each side of frame is increased by a factor 3 keeping the number of turns per unit length of the frame constant.

Then, $a' = 3a$

$$\therefore \quad L' \propto (a')^3 \text{ or } L' \propto (3a)^3$$

$$\text{or} \quad L' \propto 27a^3 \text{ or } L' = 27L$$

5. Mutual inductance for a coaxial solenoid of radius r_1 and r_2 and number of turns n_1 and n_2, respectively is given as,

$M = \mu_0 n_1 n_2 \pi r_1^2 l$ (for internal coil of radius r_1)

Self inductance for the internal coil,

$$L = \mu_0 n_1^2 \pi r_1^2 l$$

$$\therefore \quad \frac{M}{L} = \frac{n_1 n_2}{n_1^2} = \frac{n_2}{n_1}$$

6. When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (A).

7. Magnetic field produced by a current i in a large square loop at its centre,

$$B \propto \frac{i}{L} \quad \text{say} \quad B = K \frac{i}{L}$$

$\therefore$ Magnetic flux linked with smaller loop,

$$\phi = B \cdot S$$

$$\phi = \left(K \frac{i}{L}\right)(l^2)$$

Therefore, the mutual inductance

$$M = \frac{\phi}{i} = K \frac{l^2}{L} \quad \text{or} \quad M \propto \frac{l^2}{L}$$

NOTE Dimensions of self inductance (L) or mutual inductance (M) are [Mutual inductance] = [Self inductance]

$$= [\mu_0] \text{ [length]}$$

Similarly, dimensions of capacitance are

$$\text{[capacitance]} = [\varepsilon_0] \text{ [length]}$$

From this point of view, options (b) and (d) may be correct.

8. From Faraday's law, the induced voltage

$V \propto L$, if rate of change of current is constant $\left(V = -L\frac{di}{dt}\right)$

$$\therefore \quad \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4} \quad \text{or} \quad \frac{V_1}{V_2} = 4$$

Power given to the two coils is same, i.e.

$$V_1 i_1 = V_2 i_2 \quad \text{or} \quad \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

Energy stored, $W = \frac{1}{2} L i^2$

$$\therefore \quad \frac{W_2}{W_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{i_2}{i_1}\right)^2 = \left(\frac{1}{4}\right)(4)^2$$

$$\text{or} \quad \frac{W_1}{W_2} = \frac{1}{4}$$

9. If I current flows through the circular loop, then magnetic flux at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2(R^2 + Z^2)^{3/2}}$$

Substituting the value of $Z\ (= \sqrt{3}R)$,

we have $$B = \frac{\mu_0 I}{16R}$$

Now, total flux through the square loop is

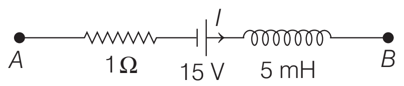
$$\phi_T = NBS \cos\theta = (2)\left(\frac{\mu_0 T}{16R}\right) a^2 \cos 45^\circ$$

Mutual inductance,

$$M = \frac{\phi_T}{I} = \frac{\mu_0 a^2}{2^{7/2} R}$$

$\therefore$ $$p = 7$$

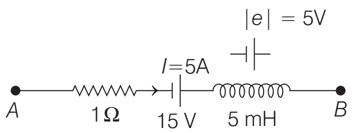
10. $\dfrac{di}{dt} = 10^3$ A/s



$\therefore$ Induced emf across inductance, $|e| = L\dfrac{di}{dt}$

$$|e| = (5 \times 10^{-3})(10^3)\ \text{V} = 5\ \text{V}$$

Since, the current is decreasing, the polarity of this emf would be so as to increase the existing current. The circuit can be redrawn as



Now, $$V_A - 5 + 15 + 5 = V_B$$

$\therefore$ $$V_A - V_B = -15\ \text{V}$$

or $$V_B - V_A = 15\ \text{V}$$

Topic 4 *L-R* Circuits and *L-C* Oscillations

1. In an L-R circuit, current during charging is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

where, $I_0 = \dfrac{E}{R}$ = saturation current

So, we have $\dfrac{dq}{dt} = I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$$\Rightarrow \quad dq = I_0\left(1 - e^{-\frac{R}{L}t}\right) dt$$

So, charge q that passes through battery from time $t = 0$ to $t = \dfrac{L}{R}$ is obtained by integrating the above equation within the specified limits, i.e.

$$q = \int_0^Q dq = \int_{t=0}^{t=\frac{L}{R}} I_0\left(1 - e^{-\frac{R}{L}t}\right) dt$$

$$= I_0\left[\left(t - \frac{1}{\left(-\frac{R}{L}\right)} \cdot e^{-\frac{R}{L}t}\right)\right]_0^{\frac{L}{R}}$$

$$= \frac{E}{R}\left[\left\{\frac{L}{R} + \frac{L}{Re'}\right\} - \left\{0 + \frac{L}{R}\right\}\right] = \frac{E}{R} \times \frac{L}{Re} = \frac{EL}{R^2 e}$$

$$\Rightarrow \quad q = \frac{EL}{2.7\,R^2} \qquad [\because e \approx 2.72]$$

2. **Key Idea** In an L-R circuit, current during charging of inductor is given by

$$i = i_0(1 - e^{-\frac{R}{L}\cdot t})$$

where, i_0 = saturation current.

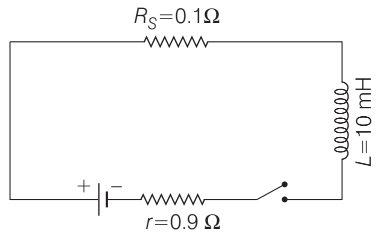
In given circuit,

Inductance of circuit is

$$L = 10\,\text{mH} = 10 \times 10^{-3}\ \text{H}$$

Resistance of circuit is

$$R = (R_s + r) = 0.1 + 0.9 = 1\Omega$$



Now, from

$$i = i_0(1 - e^{-\frac{R}{L}\cdot t}) \qquad \text{...(i)}$$

Given, $$i = 80\%\ \text{of}\ i_0$$

$$\Rightarrow \quad i = \frac{80\,i_0}{100} = 0.8\,i_0$$

Substituting the value of i in Eq. (i), we get

$$0.8 = 1 - e^{-\frac{R}{L}\cdot t} \Rightarrow e^{-\frac{R}{L}t} = 0.2 \Rightarrow e^{\frac{R}{L}t} = 5$$

$$\Rightarrow \ln(e)^{\frac{R}{L}t} = \ln 5 \quad \Rightarrow \frac{R}{L}t = \ln 5$$

$$\Rightarrow \quad t = \frac{L}{R}\cdot\ln(5) = \frac{10 \times 10^{-3}}{1} \times \ln(5)$$

$$= 10 \times 10^{-3} \times 1.6$$

$$= 1.6 \times 10^{-2}\ \text{s} = 0.016\ \text{s}$$

3. Given circuit is a series L-R circuit

In an L-R circuit, current increases as

$$i = \frac{E}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$

Now, energy stored in inductor is

$$U_L = \frac{1}{2}Li^2$$

where, L = self inductance of the coil
and energy dissipated by resistor is

$$U_R = i^2R$$

Given, rate of energy stored in inductor is equal to the rate of energy dissipation in resistor. So, after differentiating, we get

$$iL\frac{di}{dt} = i^2R \quad \Rightarrow \frac{di}{dt} = \frac{R}{L}i$$

$$\Rightarrow \qquad \frac{E}{R}\cdot\frac{R}{L}e^{-\frac{R}{L}t} = \frac{R}{L}\cdot\frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow \qquad 2e^{-\frac{R}{L}t} = 1$$

$$\Rightarrow \qquad e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log on both sides, we have

$$\Rightarrow \qquad \frac{-R}{L}t = \ln\left(\frac{1}{2}\right) \Rightarrow \frac{R}{L}t = \ln 2$$

$$\Rightarrow \qquad t = \frac{L}{R}\ln 2 = \frac{20}{10}\ln 2 \Rightarrow t = 2\ln 2$$

4. Initially in the given RL circuit with a source, when S_1 is closed and S_2 is open at $t \le t_0$.

$$I_1 = \frac{V}{R}\left[1 - \exp\left(\frac{-R}{L}t\right)\right]$$

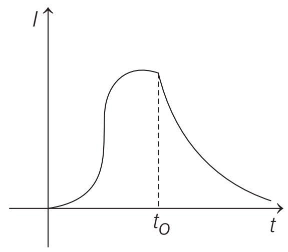
In this case, inductor L is charging.

When switch S_2 is closed and S_1 is open (after $t > t_0$), the inductor will be discharged through resistor.

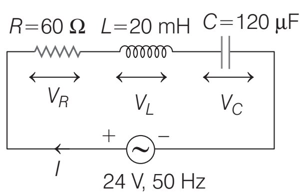
In this case ($t > t_0$),

$$I_2 = \frac{V}{R}\exp\left[-\frac{R}{L}(t - t_0)\right]$$

Thus, the variation of I with t approximately is shown below



5. The given series R-L-C circuit is shown in the figure below.



Here,
V_R = potential across resistance (R)
V_L = potential across inductor (L) and
V_C = potential across capacitor (C)
Impedance of this series circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \ldots(\text{i})$$

$$\because \qquad X_L = \omega L = (2\pi f)(L)$$

$$= 2\pi \times 50 \times 20 \times 10^{-3}\ \Omega$$

$$X_L = 6.28\Omega \qquad \ldots(\text{ii})$$

$$\text{and} \qquad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = \frac{250}{3\pi}\Omega \qquad \ldots(\text{iii})$$

$$\text{and} \qquad X_L - X_C = \left(6.28 - \frac{250}{3\pi}\right) = -20.23\ \Omega \qquad \ldots(\text{iv})$$

RMS value of current in circuit is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{24}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{\text{rms}} = \frac{24}{\sqrt{60^2 + (-20.23)^2}} = \frac{24}{63.18}$$

$$I_{\text{rms}} = 0.379\text{ A}$$

Therefore, energy dissipated is

$$= I_{\text{rms}}^2 \times R \times t$$

$$E = (0.379)^2 \times 60 \times 60$$

$$\text{or} \qquad = 517.10 = 5.17 \times 10^2\text{ J}$$

6. After a sufficiently long time, in steady state, resistance offered by inductor is zero. So, circuit is reduced to

$\therefore$ Current in circuit is

$$I = \frac{E}{R_{\text{eq}}} = \frac{15}{\left(\frac{5 \times 5}{5 + 5}\right)}$$

$$= \frac{15 \times 2}{5} = 6\text{ A}$$

7. Steady state current i_0 was already flowing in the L-R circuit when K_1 was closed for a long time. Here,

$$i_0 = \frac{V}{R} = \frac{15\text{ V}}{150\ \Omega} = 0.1\text{ A}$$

Now, K_1 is opened and K_2 is closed. Therefore, this i_0 will decrease exponentially in the $L - R$ circuit. Current i at time t will be given by $i = i_0 e^{\frac{-t}{\tau_L}}$

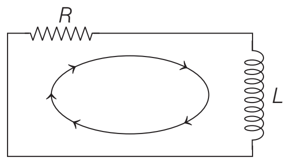
$$\text{where,} \qquad \tau_L = \frac{L}{R} \quad \Rightarrow \quad \therefore \quad i = i_0 e^{\frac{-Rt}{L}}$$

Substituting the values, we have

$$i = (0.1)e^{\frac{-(0.15 \times 10^3)(10^{-3})}{(0.03)}}$$

$$= (0.1)(e^{-5}) = \frac{0.1}{150} = 6.67 \times 10^{-4}\text{A}$$
$$= 0.67\,\text{mA}$$

8. After connecting C to B hanging the switch, the circuit will act like an L-R discharging circuit.



Applying Kirchhoff's loop equation,

$$V_R + V_L = 0 \quad \Rightarrow \quad V_R = -V_L \quad \Rightarrow \quad \frac{V_R}{V_L} = -1$$

9. For charging of capacitor $q = CV\,(1 - e^{t/\tau})$

At $t = 2\tau$

$$q = CV\,(1 - e^{-2})$$

10. The current-time (i-t) equation in L-R circuit is given by [Growth of current in L-R circuit]

$$i = i_0\,(1 - e^{-t/\tau_L}) \qquad \text{...(i)}$$

where, $\quad i_0 = \frac{V}{R} = \frac{12}{6} = 2\,\text{A}$

and $\quad \tau_L = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3}\ \text{s}$

and $\quad i = 1\,\text{A} \qquad$ (given)

$t = ?$

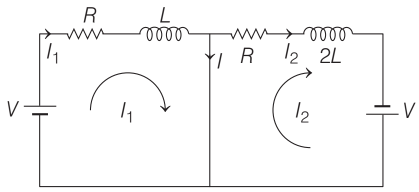
Substituting these values in Eq. (i), we get

$$t = 0.97 \times 10^{-3}\ \text{s}$$

or $\quad t = 0.97$ ms

$$t \approx 1\,\text{ms}$$

11. $$I_1 = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$



$$I_2 = \frac{V}{R}\left(1 - e^{-\frac{tR}{2L}}\right)$$

From principle of superposition,

$$I = I_1 - I_2 \;\Rightarrow\; I = \frac{V}{R} e^{-\frac{tR}{2L}}\left(1 - e^{-\frac{tR}{2L}}\right) \qquad \text{...(i)}$$

I is maximum when $\frac{dI}{dt} = 0$, which gives

$$e^{-\frac{tR}{2L}} = \frac{1}{2} \;\text{ or }\; t = \frac{2L}{R}\ln 2$$

Substituting this time in Eq. (i), we get $I_{\max} = \frac{V}{4R}$

12. Charge on capacitor at time t is

$$q = q_0\,(1 - e^{-t/\tau})$$

Here, $\quad q_0 = CV$ and $t = 2\tau$

$\therefore \quad q = CV\,(1 - e^{-2\tau/\tau}) = CV\,(1 - e^{-2})$

13. From conservation of energy,

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV^2$$

$$\therefore \quad I_{\max} = V\sqrt{\frac{C}{L}}$$

14. Comparing the L-C oscillations with normal SHM, we get

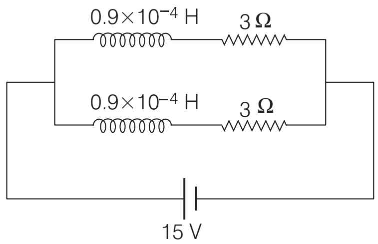
$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

Here, $\quad \omega^2 = \frac{1}{LC}$

$$\therefore \quad Q = -LC\,\frac{d^2Q}{dt^2}$$

15. Inductance of the circuit $L = \frac{0.9 \times 10^{-4}}{2} = 0.45 \times 10^{-4}$ H

(in parallel)



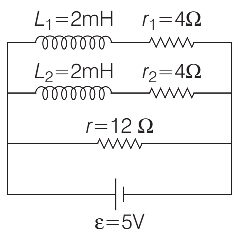
Resistance of the circuit $R = 3/2 = 1.5\ \Omega$ (in parallel)

$$\therefore \quad \tau_L\ (\text{time constant}) = \frac{L}{R} = 3.0 \times 10^{-5}\text{s}$$

Steady state current in the circuit through the battery

$$i_0 = \frac{V}{R} = \frac{15}{1.5} = 10\text{A}$$

16.



$$I_{\max} = \frac{\varepsilon}{R} = \frac{5}{12}\text{A (Initially at } t = 0)$$

$$I_{\min} = \frac{\varepsilon}{R_{eq}} = \varepsilon\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right) \text{ (finally in steady state)}$$

$$= 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right) = \frac{10}{3}\text{A}$$

$$\frac{I_{\max}}{I_{\min}} = 8$$

17. Inductive reactance,

$$X_L = \omega L = (50)(2\pi)(35 \times 10^{-3}) \approx 11\Omega$$

Impedence, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\ \Omega$

Given, $V_{\text{rms}} = 220\ \text{V}$

Hence, amplitude of voltage,

$$V_0 = \sqrt{2} V_{\text{rms}} = 220\sqrt{2}\ \text{V}$$

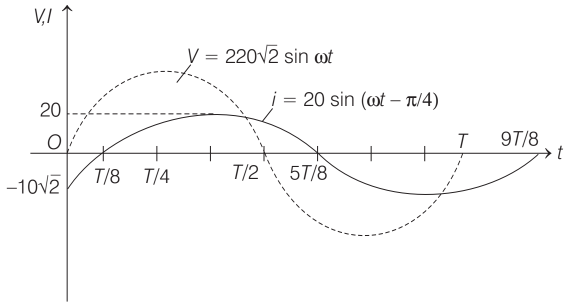
$\therefore$ Amplitude of current, $i_0 = \dfrac{V_0}{Z} = \dfrac{220\sqrt{2}}{11\sqrt{2}}$ or $i_0 = 20\ \text{A}$

Phase difference, $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{11}{11}\right) = \dfrac{\pi}{4}$

In L-R circuit, voltage leads the current. Hence, instantaneous current in the circuit is

$$i = (20\text{A}) \sin(\omega t - \pi/4)$$

Corresponding i-t graph is shown in figure.



18. After a long time, resistance across an inductor becomes zero while resistance across capacitor becomes infinite. Hence, net external resistance,

$$R_{\text{net}} = \frac{\frac{R}{2} + R}{2} = \frac{3R}{4}$$

Current through the batteries, $i = \dfrac{2E}{\frac{3R}{4} + r_1 + r_2}$

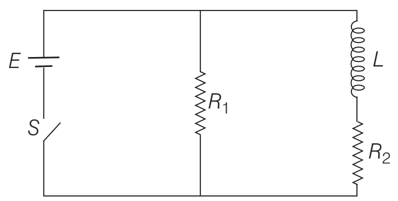
Given that potential across the terminals of cell A is zero.

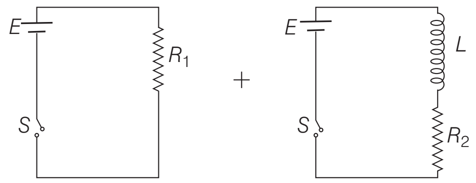
$\therefore$ $$E - ir_1 = 0$$

or $$E - \left(\frac{2E}{3R/4 + r_1 + r_2}\right) r_1 = 0$$

Solving this equation, we get, $R = \dfrac{4}{3}(r_1 - r_2)$

19. (a) Given, $R_1 = R_2 = 2\ \Omega$, $E = 12\ \text{V}$ and $L = 400\ \text{mH} = 0.4\ \text{H}$. Two parts of the circuit are in parallel with the applied battery. So, the upper circuit can be broken as





(a) (b)

Now, refer Fig. (b)

This is a simple L-R circuit, whose time constant

$$\tau_L = L/R_2 = \frac{0.4}{2} = 0.2\ \text{s}$$

and steady state current

$$i_0 = \frac{E}{R_2} = \frac{12}{2} = 6\ \text{A}$$

Therefore, if switch S is closed at time $t = 0$, then current in the circuit at any time t will be given by

$$i(t) = i_0 (1 - e^{-t/\tau_L})$$

$$i(t) = 6(1 - e^{-t/0.2})$$

$$= 6(1 - e^{-5t}) = i \qquad \text{(say)}$$

Therefore, potential drop across L at any time t is

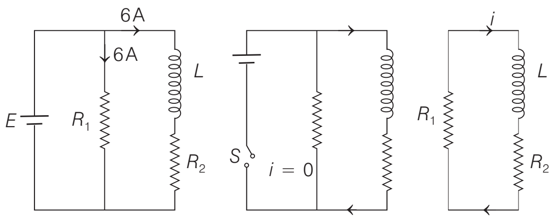
$$V = \left| L\frac{di}{dt} \right| = L(30e^{-5t}) = (0.4)(30)e^{-5t}$$

or $$V = 12e^{-5t}\ \text{V}$$

(b) The steady state current in L or R_2 is

$$i_0 = 6\ \text{A}$$

Now, as soon as the switch is opened, current in R_1 is reduced to zero immediately. But in L and R_2 it decreases exponentially. The situation is as follows



Steady state condition (c) | $t = 0$ S is open (d) | $t = t$ (e)

Refer figure (e) :

Time constant of this circuit would be

$$\tau_L' = \frac{L}{R_1 + R_2} = \frac{0.4}{(2+2)} = 0.1\ \text{s}$$

$\therefore$ Current through R_1 at any time t is

$$i = i_0 e^{-t/\tau_L'} = 6e^{-t/0.1} \text{ or } i = 6e^{-10t}\ \text{A}$$

Direction of current in R_1 is as shown in figure or clockwise.

20. This is a problem of L-C oscillations. Charge stored in the capacitor oscillates simple harmonically as

$$Q = Q_0 \sin(\omega t \pm \phi)$$

Here, Q_0= maximum value of $Q = 200\mu C = 2\times 10^{-4}$ C

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times 10^{-3})(5.0\times 10^{-6})}} = 10^4 s^{-1}$$

Let at $t = 0, Q = Q_0$, then

$$Q(t) = Q_0 \cos\omega t \quad \text{...(i)}$$

$$I(t) = \frac{dQ}{dt} = -Q_0\omega \sin\omega t \text{ and} \quad \text{...(ii)}$$

$$\frac{dI(t)}{dt} = -Q_0\omega^2 \cos(\omega t) \quad \text{...(iii)}$$

(a) $Q = 100\mu C$ or $\frac{Q_0}{2}$ at $\cos\omega t = \frac{1}{2}$ or $\omega t = \frac{\pi}{3}$

At $\cos(\omega t) = \frac{1}{2}$, from Eq. (iii) :

$$\left|\frac{dI}{dt}\right| = (2.0\times 10^{-4} C)(10^4 s^{-1})^2 \left(\frac{1}{2}\right)$$

$$\left|\frac{dI}{dt}\right| = 10^4 \text{ A/s}$$

(b) $Q = 200\,\mu C$ or Q_0 when $\cos(\omega t) = 1$ i.e. $\omega t = 0, 2\pi$ ...

At this time $I(t) = -Q_0\omega \sin\omega t$

or $I(t) = 0$ $(\sin 0° = \sin 2\pi = 0)$

(c) $I(t) = -Q_0\omega \sin\omega t$

$\therefore$ Maximum value of I is $Q_0\omega$.

$$I_{max} = Q_0\omega = (2.0\times 10^{-4})(10^4)$$

$$I_{max} = 2.0 \text{ A}$$

(d) From energy conservation,

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$$

or $$Q = \sqrt{LC(I_{max}^2 - I^2)}$$

$$I = \frac{I_{max}}{2} = 1.0 \text{ A}$$

$\therefore$ $$Q = \sqrt{(2.0\times 10^{-3})(5.0\times 10^{-6})(2^2 - 1^2)}$$

$$Q = \sqrt{3}\times 10^{-4} C \text{ or } Q = 1.732\times 10^{-4} \text{ C}$$

21. $U = \frac{1}{2}Li^2$ i.e. $U \propto i^2$

U will reach $\frac{1}{4}$th of its maximum value when current is reached half of its maximum value. In L-R circuit, equation of current growth is written as

$$i = i_0(1 - e^{-t/\tau_L})$$

Here, i_0= Maximum value of current

τ_L = Time constant = L/R

$$\tau_L = \frac{10 \text{ H}}{2\,\Omega} = 5 \text{ s}$$

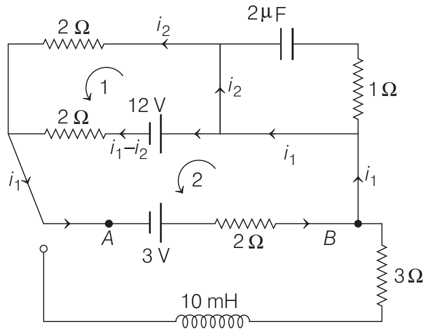
Therefore, $i = i_0/2 = i_0(1 - e^{-t/5})$

or $\frac{1}{2} = 1 - e^{-t/5}$ or $e^{-t/5} = \frac{1}{2}$

or $-t/5 = \ln\left(\frac{1}{2}\right)$ or $t/5 = \ln(2) = 0.693$

$\therefore$ $t = (5)(0.693)$ or $t = 3.465$ s

22. (a) In steady state, no current will flow through capacitor. Applying Kirchhoff's second law in loop 1



$$-2i_2 + 2(i_1 - i_2) + 12 = 0$$

$\therefore$ $2i_1 - 4i_2 = -12$

or $i_1 - 2i_2 = -6$ ...(i)

Applying Kirchhoff's second law in loop 2

$$-12 - 2(i_1 - i_2) + 3 - 2i_1 = 0$$

$\therefore$ $4i_1 - 2i_2 = -9$ ...(ii)

Solving Eqs. (i) and (ii), we get

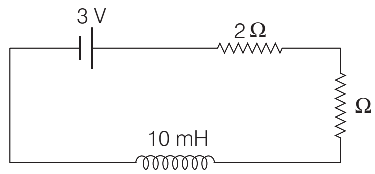
$$i_2 = 2.5 \text{ A} \quad \text{and} \quad i_1 = -1 \text{ A}$$

Now, $V_A + 3 - 2i_1 = V_B$

or $V_A - V_B = 2i_1 - 3 = 2(-1) - 3 = -5$ V

$$P_{R_1} = (i_1 - i_2)^2 R_1 = (-1 - 2.5)^2\,(2) = 24.5 \text{ W}$$

(b) **In position 2 :** Circuit is as under



(i) Steady current in R_4 :

$$i_0 = \frac{3}{3+2} = 0.6 \text{ A}$$

(ii) Time when current in R_4 is half the steady value

$$t_{1/2} = \tau_L\,(\ln 2) = \frac{L}{R}\ln(2)$$

$$= \frac{(10\times 10^{-3})}{5}\ln(2)$$

$$= 1.386\times 10^{-3} \text{ s}$$

$$U = \frac{1}{2}Li^2 = \frac{1}{2}(10\times 10^{-3})(0.3)^2$$

$$= 4.5\times 10^{-4} \text{ J}$$

Topic 5 Induced Electric Field

1. $$\int \mathbf{E} \cdot d\mathbf{l} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$$

or $$E(2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$$

For $r \geq a$,

$$\therefore \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$\therefore$ Induced electric field $\propto 1/r$

For $r \leq a$,

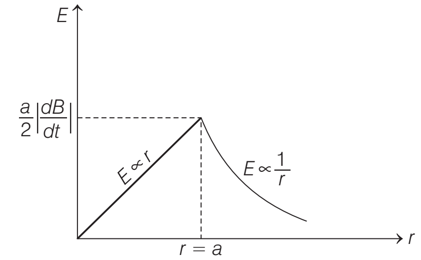
$$E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$$

or $$E = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

or $$E \propto r$$

At $r = a$, $$E = \frac{a}{2}\left|\frac{dB}{dt}\right|$$

Therefore, variation of E with r (distance from centre) will be as follows



Topic 6 Alternating Currents

1. Given, phase difference, $\phi = \frac{\pi}{4}$

As we know, for R-L or R-C circuit,

$$\tan\phi = \frac{\text{Capacitive reactance } (X_C) \text{ or inductive reactance } (X_L)}{\text{Resistance (R)}}$$

$$\Rightarrow \quad \tan\frac{\pi}{4} = \frac{X_C \text{ or } X_L}{R}$$

$$1 = \frac{X_C \text{ or } X_L}{R} \Rightarrow R = X_C \text{ or } X_L$$

Also, given $e = e_0 \sin(100t)$

Comparing the above equation with general equation of emf, i.e. $e = e_0 \sin\omega t$, we get

$$\omega = 100 \text{ rad/s} = 10^2 \text{ rad/s}$$

Now, checking option wise,

For R-C circuit, with

$$R = 1\text{k}\Omega = 10^3\ \Omega \text{ and } C = 1\mu\text{F} = 10^{-6}\text{F}$$

So, $$X_C = \frac{1}{\omega C} = \frac{1}{10^2 \times 10^{-6}} = 10^4\ \Omega \Rightarrow R \neq X_C$$

For R - L circuit, with

$$R = 1\text{k}\Omega = 10^3\ \Omega$$

and $$L = 1\text{mH} = 10^{-3}\text{ H}$$

So, $X_L = \omega L = 10^2 \times 10^{-3} = 10^{-1}\ \Omega \Rightarrow R \neq X_L$

For R - C circuit, with

$$R = 1\text{k}\Omega = 10^3\ \Omega$$

and $$C = 10\ \mu\text{F} = 10 \times 10^{-6}\text{ F} = 10^{-5}\text{ F}$$

So, $$X_C = \frac{1}{10^2 \times 10^{-5}} = 10^3\ \Omega \Rightarrow R = C$$

For R - L circuit, with

$$R = 1\text{k}\Omega = 10^3\ \Omega$$

and $$L = 10\text{ mH} = 10 \times 10^{-3}\text{H} = 10^{-2}\text{H}$$

$$X_L = 10^2 \times 10^{-2} = 1\ \Omega \Rightarrow R \neq X_L$$

Alternate Solution

Since, $\tan\frac{\pi}{4} = 1 = \frac{X_C \text{ or } X_L}{R}$

$\therefore$ For R-C circuit, we have

$$1 = \frac{1}{C\omega R} \text{ or } \omega = \frac{1}{CR} \qquad \text{...(i)}$$

Similarly, for R-L circuit, we have

$$1 = \frac{\omega L}{R} \Rightarrow \omega = \frac{R}{L} \qquad \text{...(ii)}$$

It is given in the question that, $\omega = 100$ rad/s

Thus, again by substituting the given values of R, C or L option wise in the respective Eqs. (i) and (ii), we get that only for option (c),

$$\omega = \frac{1}{CR} = \frac{1}{10 \times 10^{-6} \times 10^3} \text{ or } \omega = 100 \text{ rad/s}$$

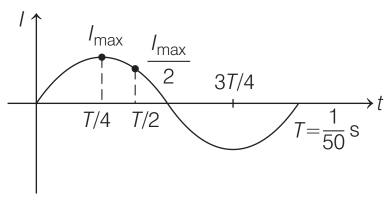
2. In an AC resistive circuit, current and voltage are in phase.

So, $$I = \frac{V}{R}$$

$$\Rightarrow \quad I = \frac{220}{50}\sin(100\pi t) \qquad \text{...(i)}$$

$\therefore$ Time period of one complete cycle of current is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50}\text{ s}$$



So, current reaches its maximum value at

$$t_1 = \frac{T}{4} = \frac{1}{200}\text{ s}$$

When current is half of its maximum value, then from Eq. (i), we have

$$I = \frac{I_{\max}}{2} = I_{\max}\sin(100\pi t_2)$$

$$\Rightarrow \sin(100\pi t_2) = \frac{1}{2} \Rightarrow 100\pi t_2 = \frac{5\pi}{6}$$

So, instantaneous time at which current is half of maximum value is $t_2 = \frac{1}{120}$ s

Hence, time duration in which current reaches half of its maximum value after reaching maximum value is

$$\Delta t = t_2 - t_1 = \frac{1}{120} - \frac{1}{200} = \frac{1}{300} \text{ s} = 3.3 \text{ ms}$$

3. Phase difference between I_2 and V, i.e. $C-R_2$ circuit is given by

$$\tan\phi = \frac{X_C}{R_2} \quad \Rightarrow \quad \tan\phi = \frac{1}{C\omega R_2}$$

Substituting the given values, we get

$$\tan\phi = \frac{1}{\frac{\sqrt{3}}{2} \times 10^{-6} \times 100 \times 20} = \frac{10^3}{\sqrt{3}}$$

$\therefore \phi_1$, is nearly 90°.

Phase difference between I_1 and V, i.e. in $L-R_1$ circuit is given by

$$\tan\phi_2 = -\frac{X_L}{R_1} = -\frac{L\omega}{R}$$

Substituting the given values, we get

$$\tan\phi_2 = -\frac{\frac{\sqrt{3}}{10} \times 100}{10} = -\sqrt{3}$$

As, $\quad \tan\phi_2 = -\sqrt{3} \Rightarrow \phi_2 = 120°$

Now, phase difference between I_1 and I_2 is

$$\Delta\phi = \phi_2 - \phi_1 = 120° - 90° = 30°$$

4. $$Q = \frac{\omega_1 - \omega_2}{\omega_0}$$

Here, $\omega_1 - \omega_2 = \text{bandwidth} = \frac{R}{L}$

Substituting the values, we get

$$Q = \frac{\omega_0 L}{R}$$

Alternative solution $\frac{\omega_0 L}{R}$ is the only dimensionless quantity, hence it must be the quality factor.

5. $$\langle P \rangle = e_{\text{rms}}\, i_{\text{rms}} \cos\phi = \left(\frac{e_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)\cos\phi$$

$$= \frac{e_0 i_0}{2}\cos\phi = \frac{(100)(20)\cos\frac{\pi}{4}}{2} = \frac{1000}{\sqrt{2}} \text{ W}$$

Wattless current $= I_{\text{rms}} \sin\phi = \frac{20}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 10\text{A}$

6. Impedance, $Z = \sqrt{R^2 + X_c^2}$, $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$, $P = I^2{}_{\text{rms}} R$

where, $X_C = \frac{1}{\omega C}$

As ω is increased, X_C will be decrease or Z will be decrease. Hence, I_{rms} or P will increase. Therefore, bulb glows brighter.

7. As the current i leads the emf e by $\pi/4$, it is an R-C circuit.

$$\tan\phi = \frac{X_C}{R}$$

or $$\tan\frac{\pi}{4} = \frac{\frac{1}{\omega C}}{R}$$

$\therefore \quad \omega CR = 1$

As $\quad \omega = 100$ rad/s

The product of C-R should be $\frac{1}{100}\text{s}^{-1}$

Option (a) satisfy this condition.

8. **In circuit (*p*)** I can't be non zero in steady state.

In circuit (*q*) $V_1 = 0$ and $V_2 = 2I = V$ (also)

In circuit (*r*) $V_1 = X_L\, I = (2\pi f L) I$

$$= (2\pi \times 50 \times 6 \times 10^{-3}) I = 1.88\, I$$

$$V_2 = 2I$$

In circuit (*s*) $V_1 = X_L\, I = 1.88 I$

$$V_2 = X_C I = \left(\frac{1}{2\pi fC}\right) I$$

$$= \left(\frac{1}{2\pi \times 50 \times 3 \times 10^{-6}}\right) I = (1061)\, I$$

In circuit (*t*)

$$V_1 = IR = (1000)\, I$$

$$V_2 = X_C\, I = (1061)\, I$$

Therefore, the correct options are as under

(A) $\rightarrow$ r, s, t $\qquad$ (B) $\rightarrow$ q, r, s, t

(C) $\rightarrow$ q or p, q $\qquad$ (D) $\rightarrow$ q, r, s, t

9. $$V_{XY} = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\omega t$$

$$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) + V_0 \sin(\omega t + \pi)$$

$$\Rightarrow \quad \phi = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\Rightarrow \quad V_0' = 2V_0 \cos\left(\frac{\pi}{6}\right) = \sqrt{3}\, V_0$$

$$\Rightarrow \quad V_{XY} = \sqrt{3} V_0 \sin(\omega t + \phi)$$

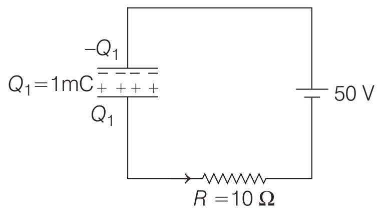
$$\Rightarrow \quad (V_{XY})_{\text{rms}} = (V_{YZ})_{\text{rms}} = \sqrt{\frac{3}{2}}\, V_0$$

10. $$\frac{dQ}{dt} = I$$

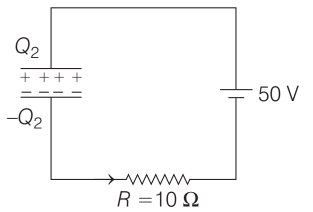
$$\Rightarrow \quad Q = \int I\, dt = \int (I_0 \cos\omega t)\, dt$$

$$\therefore \quad Q_{\max} = \frac{I_0}{\omega} = \frac{1}{500} = 2 \times 10^{-3} \text{ C}$$

Just after switching



In steady state



At $t = \dfrac{7\pi}{6\omega}$ or $\omega t = \dfrac{7\pi}{6}$

Current comes out to be negative from the given expression. So, current is anti-clockwise. Charge supplied by source from $t = 0$ to $t = \dfrac{7\pi}{6\omega}$

$$Q = \int_0^{\frac{7\pi}{6\omega}} \cos(500t)\, dt = \left[\frac{\sin 500t}{500}\right]_0^{\frac{7\pi}{6\omega}} = \frac{\sin \dfrac{7\pi}{6}}{500} = -1\ \text{mC}$$

Apply Kirchhoff's loop law, just after changing the switch to position D

$$50 + \frac{Q_1}{C} - IR = 0$$

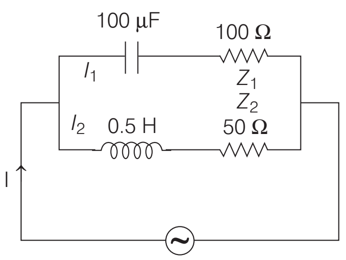
Substituting the values of Q_1, C and R, we get

$$I = 10\ \text{A}$$

In steady state $Q_2 = CV = 1\ \text{mC}$

$\therefore$ Net charge flown from battery = 2 mC

11.



***Circuit* 1**

$$X_C = \frac{1}{\omega C} = 100\ \Omega$$

$$\therefore \quad Z_1 = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ \Omega$$

$$\phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right) = 45°$$

In this circuit, current leads the voltage.

$$I_1 = \frac{V}{Z_1} = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}\ \text{A}$$

$$V_{100\ \Omega} = (100)\, I_1 = (100)\frac{1}{5\sqrt{2}}\ \text{V} = 10\sqrt{2}\ \text{V}$$

***Circuit* 2**

$$X_L = \omega L = (100)(0.5) = 50\ \Omega$$

$$Z_2 = \sqrt{(50)^2 + (50)^2} = 50\sqrt{2}\ \Omega$$

$$\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right) = 45°$$

In this circuit, voltage leads the current.

$$I_2 = \frac{V}{Z_2} = \frac{20}{50\sqrt{2}} = \frac{\sqrt{2}}{5}\ \text{A}$$

$$V_{50\ \Omega} = (50)\, I_2 = 50\left(\frac{\sqrt{2}}{5}\right) = 10\sqrt{2}\ \text{V}$$

Further, I_1 and I_2 have a mutual phase difference of 90°.

$$\therefore \quad I = \sqrt{I_1^2 + I_2^2} = 0.34$$

12. $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

In case (b), capacitance C will be more. Therefore, impedence Z will be less. Hence, current will be more.

Further, $V_C = \sqrt{V^2 - V_R^2} = \sqrt{V^2 - (IR)^2}$

In case (b), since current I is more.

Therefore, V_C will be less.

13. $Z = \sqrt{R^2 + X_C^2} = R\sqrt{1.25}$

$\therefore$ $R^2 + X_C^2 = 1.25\, R^2$ or $X_C = \dfrac{R}{2}$ or $\dfrac{1}{\omega C} = \dfrac{R}{2}$

$\therefore$ Time constant $= CR = \dfrac{2}{\omega} = \dfrac{2}{500}\ \text{s} = 4\ \text{ms}$

Topic 7 Miscellaneous Problems

1. Potential difference between opposite faces of cube is

$$V = \text{induced emf} = B\, l\, v$$

where,

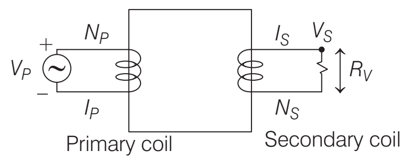
B = magnetic field = 0.1 T,

l = distance between opposite faces of cube

$= 2\ \text{cm} = 2 \times 10^{-2}$ m and v = speed of cube = 6 ms^{-1}.

Hence, $V = 0.1 \times 2 \times 10^{-2} \times 6 = 12\ \text{mV}$

2. For a transformer, there are two circuits which have N_p and N_s (number of coil turns), I_p and I_S (currents) respectively as shown below.



Here, input voltage, $V_p = 2300\ \text{V}$

Number of turns in primary coil, $N_P = 4000$

Output voltage, $V_S = 230$ volt

Output power, $P_S = V_S \cdot I_S$

Input power, $P_P = V_P I_P$

$\therefore$ The efficiency of the transformer is

$$\eta = \frac{\text{Output (secondary) power}}{\text{Input (primary) power}}$$

$$\Rightarrow \quad \eta = \frac{V_S \cdot I_S}{V_P \cdot I_P} \times 100$$

$$\Rightarrow \quad \eta = \frac{(230)(I_S)}{(2300)(5)} \times 100$$

$$90 = \frac{230\, I_S}{(2300) \times 5} \times 100$$

$$\Rightarrow \quad I_S = 45\,\text{A}$$

3. Resultant voltage, $V^2 = V_R^2 + V_L^2 \Rightarrow 220^2 = 80^2 + V_L^2$

Solving, we get

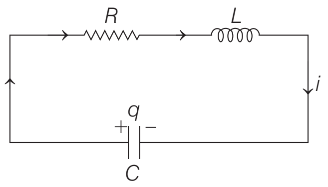
$$V_L \approx 205 \text{ V}$$

$$X_L = \frac{V_L}{I} = \frac{205}{10} = 20.5\,\Omega = \omega L$$

$$\therefore \quad L = \frac{20.5}{2\pi \times 50} = 0.065\,\text{H}$$

4. Remembering based question. Therefore, no solution is required.

5.



At a general time t, suppose charge on capacitor is q and current in the circuit is i, then applying Kirchhoff's loop law, we have

$$\frac{q}{c} - iR - L\frac{di}{dt} = 0$$

Putting $\quad i = -\dfrac{dq}{dt}$ and $\dfrac{di}{dt} = -\dfrac{d^2q}{dt^2}$

In the above equation, we have

$$\frac{d^2q}{dt^2} + \frac{R}{L}\left(\frac{dq}{dt}\right) + \frac{q}{LC} = 0 \qquad \text{...(i)}$$

Comparing this equation with the standard differential equation of damped oscillation,

$$\frac{d^2X}{dt^2} + \frac{b}{m}\frac{dX}{dt} + \frac{K}{m}X = 0$$

Which has a general solution of amplitude, $A = A_0 e^{\frac{-bt}{2m}}$

The general solution of Eq. (i) will be

$$Q_{\max} = Q_0 e^{\frac{-Rt}{2L}} \text{ or } Q_{\max}^2 = Q_0^2 e^{\frac{-Rt}{L}}$$

Hence, $Q_{\max}^2$ *versus* time graph is exponentially decreasing graph. Lesser the value of self inductance, faster will be the damping.

6. When force exerted on a current carrying conductor

$$F_{\text{ext}} = BIL$$

Average power $= \dfrac{\text{Work done}}{\text{Time taken}}$

$$P = \frac{1}{t}\int_0^2 F_{\text{ext.}} \cdot dx = \frac{1}{t}\int_0^2 B(x) IL\, dx$$

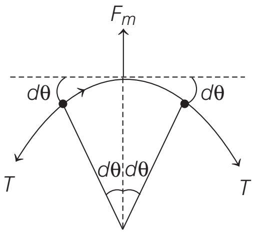
$$= \frac{1}{5 \times 10^{-3}}\int_0^2 3 \times 10^{-4} e^{-0.2x} \times 10 \times 3\, dx$$

$$= 9[1 - e^{-0.4}] = 9\left[1 - \frac{1}{e^{0.4}}\right]$$

$$= 2.967 \approx 2.97\,\text{W}$$

7. Amplitude decreases exponentially. In 5 s, it remains 0.9 times. Therefore, in total 15 s it will remains (0.9) (0.9) (0.9) = 0.729 times its original value.

8.



$$L = 2\pi R$$

$$\therefore \quad R = L/2\pi$$

$$2T \sin(d\theta) = F_m$$

For small angles, $\sin(d\theta) \approx d\theta$

$$\therefore \quad 2T\,(d\theta) = I\,(dL)\,B \sin 90^\circ$$

$$= I\,(2R \cdot d\theta) \cdot B$$

$$\therefore \quad T = IRB = \frac{ILB}{2\pi}$$

9. In uniform magnetic field, change in magnetic flux is zero. Therefore, induced current will be zero.

10. Power, $P = e^2/R$

Here, $\quad e = $ induced emf $= -\left(\dfrac{d\phi}{dt}\right)$ where, $\phi = NBA$

$$e = -NA\left(\frac{dB}{dt}\right)$$

Also, $\quad R \propto \dfrac{l}{r^2}$

where, R = resistance, r = radius, l = length.

$$\therefore \quad P \propto N^2 r^2 \Rightarrow \therefore \frac{P_2}{P_1} = 4$$

11. When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So, according to Lenz's law, induced current in Q i.e. I_{Q_1} will flow in such a direction, so that the magnetic field lines due to I_{Q_1} passes from left to right through Q.

This is possible when I_{Q_1} flows in anti-clockwise direction as seen by E. Opposite is the case when switch S is opened i.e. I_{Q_2} will be clockwise as seen by E.

12. The equations of $I_1(t)$, $I_2(t)$ and $B(t)$ will take the following forms :

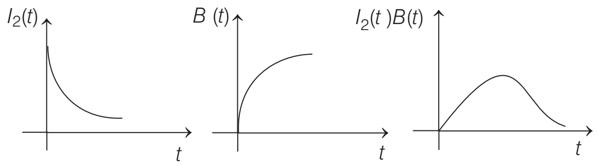
$I_1(t) = K_1(1 - e^{-k_2 t}) \rightarrow$ current growth in L-R circuit

$B(t) = K_3(1 - e^{-k_2 t}) \rightarrow B(t) \propto I_1\ (t)$

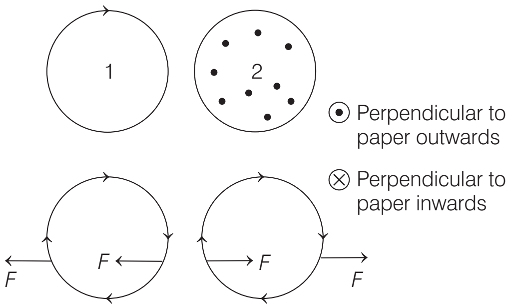
$I_2(t) = K_4 e^{-k_2 t}$

$$\left[I_2(t) = \frac{e_2}{R} \text{ and } e_2 \propto \frac{dI_1}{dt} : e_2 = -M\,\frac{dI_1}{dt}\right]$$

Therefore, the product $I_2(t)B(t) = K_5\ e^{-k_2 t}(1 - e^{-k_2 t})$. The value of this product is zero at $t = 0$ and $t = \infty$. Therefore, the product will pass through a maximum value. The corresponding graphs will be as follows :



13. For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce u magnetic field in loop 2. Therefore, increase in current in loop 1 will produce an induced current in loop 2 which produces ⊗ magnetic field passing through it i.e. induced current in loop 2 will also be clockwise as shown in the figure.



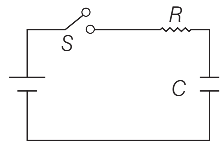
The loops will now repel each other as the currents at the nearest and farthest points of the two loops flow in opposite directions.

14. The induced current in the ring will interact with horizontal component of magnetic field and both will repel each other. This repulsion will balance the weight of ring.

Hence, option (a) is correct.

15. When switch is closed for a very long time capacitor will get fully charged and charge on capacitor will be $q = CV$

Energy stored in capacitor, $E_C = \frac{1}{2}CV^2$ …(i)



Work done by a battery, $W = Vq = VCV = CV^2$

Energy dissipated across resistance

E_D = (work done by battery) − (energy stored)

$E_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2$ …(ii)

From Eqs. (i) and (ii)

$$E_D = E_C$$

16. For process (1)

Charge on capacitor $= \frac{CV_0}{3}$

Energy stored in capacitor $= \frac{1}{2}C\frac{V_0^2}{9} = \frac{CV_0^2}{18}$

Work done by battery $= \frac{CV_0}{3} \times \frac{V}{3} = \frac{CV_0^2}{9}$

$\therefore$ Heat loss $= \frac{CV_0^2}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$

For process (2)

Charge on capacitor $= \frac{2\,CV_0}{3}$

Extra charge flow through battery $= \frac{CV_0}{3}$

Work done by battery $= \frac{CV_0}{3} \cdot \frac{2V_0}{3} = \frac{2CV_0^2}{9}$

Final energy stored in capacitor $= \frac{1}{2}C\left(\frac{2V_0}{3}\right)^2 = \frac{4CV_0^2}{18}$

Energy stored in process 2 $= \frac{4CV_0^2}{18} - \frac{CV_0^2}{18} = \frac{3CV_0^2}{18}$

Heat loss in process (2) = work done by battery in process (2) − energy stored in capacitor process (2)

$$= \frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$$

For process (3) Charge on capacitor $= CV_0$

Extra charge flown through battery $= CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$

Work done by battery in this process $= \left(\frac{CV_0}{3}\right)(V_0) = \frac{CV_0^2}{3}$

Final energy stored in capacitor $= \frac{1}{2}CV_0^2$

Energy stored in this process $= \frac{1}{2}CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$

Heat loss in process (3) $= \frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$

Now, total heat loss $(E_D) = \dfrac{CV_0^2}{18} + \dfrac{CV_0^2}{18} + \dfrac{CV_0^2}{18} = \dfrac{CV_0^2}{6}$

Final energy stored in capacitor $= \dfrac{1}{2}CV_0^2$

So, we can say that $E_D = \dfrac{1}{3}\left(\dfrac{1}{2}CV_0^2\right)$

17. $$P = Vi$$

$$\therefore \quad i = \frac{P}{V} = \frac{600 \times 10^3}{4000} = 150\ \text{A}$$

Total resistance of cables, $R = 0.4 \times 20 = 8\ \Omega$

$\therefore$ Power loss in cables $= i^2R = (150)^2\,(8)$

$$= 180000\ \text{W} = 180\ \text{kW}$$

This loss is 30% of 600 kW.

18. During step-up, $\dfrac{N_p}{N_s} = \dfrac{V_p}{V_s}$ or $\dfrac{1}{10} = \dfrac{4000}{V_s}$

or $\quad V_s = 40000\ \text{V}$

In step down transformer,

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40000}{200} = \frac{200}{1}$$

19. The induced electric field is given by,

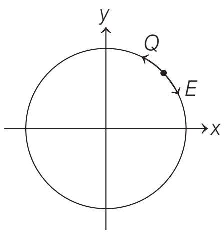
$$\oint \mathbf{E} \cdot \mathbf{dl} = -\frac{d\phi}{dt} \text{ or } El = -s\left(\frac{dB}{dt}\right)$$

$$\therefore \quad E(2\pi R) = -(\pi R^2)(B) \text{ or } E = -\frac{BR}{2}$$

20. $$\frac{M}{L} = \frac{Q}{2m}$$

$$\therefore \quad M = \left(\frac{Q}{2m}\right)L \quad \Rightarrow \quad M \propto L, \text{ where } \gamma = \frac{Q}{2m}$$

$$= \left(\frac{Q}{2m}\right)(I\omega) = \left(\frac{Q}{2m}\right)(mR^2\omega) = \frac{Q\omega R^2}{2}$$



Induced electric field is opposite direction. Therefore,

$$\omega' = \omega - \alpha t$$

$$\alpha = \frac{\tau}{I} = \frac{(QE)R}{mR^2}$$

$$= \frac{(Q)\left(\frac{BR}{2}\right)R}{mR^2} = \frac{QB}{2m}$$

$$\therefore \quad \omega' = \omega - \frac{QB}{2m} \cdot 1 = \omega - \frac{QB}{2m}$$

$$M_f = \frac{Q\omega' R^2}{2} = Q\left(\omega - \frac{QB}{2m}\right)\frac{R^2}{2}$$

$$\therefore \quad \Delta M = M_f - M_i = -\frac{Q^2BR^2}{4m}$$

$$M = -\gamma \frac{QBR^2}{2} \qquad \left(\because \gamma = \frac{Q}{2m}\right)$$

24. At $\omega \approx 0$, $X_C = \dfrac{1}{\omega C} = \infty$. Therefore, current is nearly zero.

Further at resonance frequency, current and voltage are in phase. This resonance frequency is given by

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6} \times 10^{-6}}} = 10^6 \text{ rad / s}$$

We can see that this frequency is independent of R.

Further, $X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$

At, $\omega = \omega_r = 10^6$ rad/s, $X_L = X_C$.

For $\omega > \omega_r$, $X_L > X_C$. So, circuit is inductive.

25. Induced emf $e = -\dfrac{d\phi}{dt}$. For identical rings induced emf will be same. But currents will be different. Given $h_A > h_B$. Hence, $v_A > v_B$ as $\left(h = \dfrac{v^2}{2g}\right)$.

If $\rho_A > \rho_B$, then, $I_A < I_B$. In this case given condition can be fulfilled if $m_A < m_B$.

If $\rho_A < \rho_B$, then $I_A > I_B$. In this case given condition can be fulfilled if $m_A \le m_B$.

26. Electrostatic and gravitational field do not make closed loops.

27. Take the circular tube as a long solenoid. The wires are closely wound. Magnetic field inside the solenoid is

$$B = \mu_0 ni$$

Here, n = number of turns per unit length

$\therefore$ ni = current per unit length

In the given problem $ni = \dfrac{I}{L}$

$$\therefore \quad B = \frac{\mu_0 I}{L}$$

Flux passing through the circular coil is

$$\phi = BS = \left(\frac{\mu_0 I}{L}\right)(\pi r^2)$$

Induced emf $\quad e = -\dfrac{d\phi}{dt} = -\left(\dfrac{\mu_0 \pi r^2}{L}\right) \cdot \dfrac{dI}{dt}$

Induced current, $i = \dfrac{e}{R} = -\left(\dfrac{\mu_0 \pi r^2}{LR}\right) \cdot \dfrac{dI}{dt}$

Magnetic moment, $M = iA = i\pi r^2$

or $$M = -\left(\frac{\mu_0 \pi^2 r^4}{LR}\right) \cdot \frac{dI}{dt} \quad \ldots\text{(i)}$$

Given, $\quad I = I_0 \cos\,(300t)$

$\therefore \quad \dfrac{dI}{dt} = -\,300 I_0 \sin\,(300t)$

Substituting in Eq. (i), we get

$$M = \left(\frac{300\,\pi^2 r^4}{LR}\right)\mu_0 I_0 \sin\,(300\,t)$$

$$\therefore \quad N = \frac{300\,\pi^2 r^4}{LR}$$

Substituting the values, we get

$$N = \frac{300\,(22/7)^2\,(0.1)^4}{(10)\,(0.005)} = 5.926 \text{ or } N \simeq 6$$

28. Outside the solenoid, net magnetic field is zero. It can be assumed only inside the solenoid and equal to $\mu_0 nI$.

Induced $e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(\mu_0 nI\pi a^2)$

or $\quad |e| = (\mu_0 n\pi a^2)\,(I_0\omega \cos \omega t)$

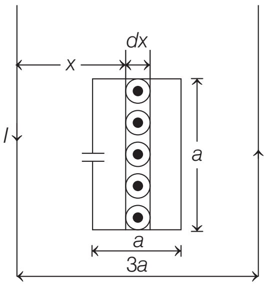
Resistance of the cylindrical vessel, $R = \dfrac{\rho l}{s} = \dfrac{\rho(2\pi R)}{Ld}$

$\therefore$ Induced current $i = \dfrac{|e|}{R} = \dfrac{\mu_0 Ldna^2 I_0\omega \cos \omega t}{2\rho R}$

29. (a) For an elemental strip of thickness dx at a distance x from left wire, net magnetic field (due to both wires)

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{3a - x} \qquad \text{(outwards)}$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a - x}\right)$$



Magnetic flux in this strip,

$$d\phi = BdS = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{3a - x}\right) a\,dx$$

$$\therefore \text{ Total flux, } \phi = \int_a^{2a} d\phi = \frac{\mu_0 Ia}{2\pi}\int_a^{2a}\left(\frac{1}{x} + \frac{1}{3a - x}\right)dx$$

or $$\phi = \frac{\mu_0 Ia}{\pi}\ln(2)$$

$$\phi = \frac{\mu_0 a \ln(2)}{\pi}(I_0 \sin \omega t) \qquad \text{...(i)}$$

Magnitude of induced emf,

$$e = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a I_0 \omega \ln(2)}{\pi}\cos\omega t = e_0 \cos\omega t$$

where, $\quad e_0 = \dfrac{\mu_0 a I_0 \omega \ln(2)}{\pi}$

Charge stored in the capacitor,

$$q = Ce = Ce_0 \cos\omega t \qquad \text{...(ii)}$$

and current in the loop

$$i = \frac{dq}{dt} = C\omega\, e_0 \sin\omega t \qquad \text{...(iii)}$$

$$i_{\max} = C\omega e_0 = \frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$$

(b) Magnetic flux passing through the square loop

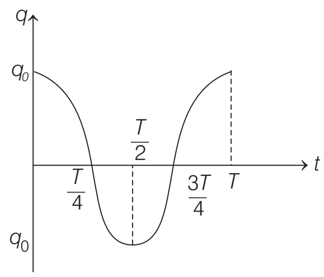
$$\phi \propto \sin\omega t \qquad \text{[From Eq. (i)]}$$

i.e. u magnetic field passing through the loop is increasing at $t = 0$. Hence, the induced current will produce ⊗ magnetic field (from Lenz's law). Or the current in the circuit at $t = 0$ will be clockwise (or negative as per the given convention). Therefore, charge on upper plate could be written as,

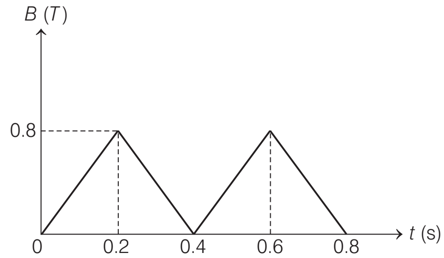
$$q = +\,q_0 \cos\omega t \qquad \text{[From Eq. (ii)]}$$

Here, $\quad q_0 = Ce_0 = \dfrac{\mu_0 a C I_0 \omega \ln(2)}{\pi}$

The corresponding q-t graph is shown in figures,



30. Magnetic field (B) varies with time (t) as shown in figure.



$$\left|\frac{dB}{dt}\right| = \frac{0.8}{0.2} = 4 \text{ T/s}$$

Induced emf in the coil due to change in magnetic flux passing through it, $e = \left|\dfrac{d\phi}{dt}\right| = NA\left|\dfrac{dB}{dt}\right|$

Here, A = Area of coil = 5×10^{-3} m^2

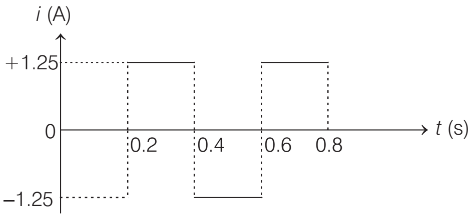
N = Number of turns = 100

Substituting the values, we get $e = (100)\,(5 \times 10^{-3})(4) = 2$ V

Therefore, current passing through the coil

$$i = \frac{e}{R} \text{ or } i = \frac{2}{1.6} = 1.25 \text{ A}$$

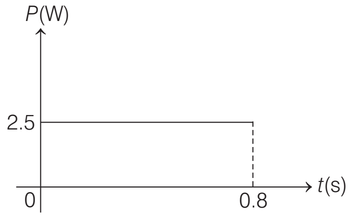
NOTE That from 0 to 0.2 s and from 0.4 s to 0.6 s, magnetic field passing through the coil increases, while during the time 0.2s to 0.4s and from 0.6s to 0.8s magnetic field passing through the coil decreases. Therefore, direction of current through the coil in these two time intervals will be opposite to each other. The variation of current (i) with time (t) will be as follows :



Power dissipated in the coil is

$$P = i^2R = (1.25)^2\ (1.6) = 2.5 \text{ W}$$

Power is independent of the direction of current through the coil. Therefore, power (P) *versus* time (t) graph for first two cycles will be as under :



Total heat obtained in 12,000 cycles will be

$$H = P.t\ = (2.5)(12000)(0.4) = 12000 \text{ J}$$

This heat is used in raising the temperature of the coil and the water. Let θ be the final temperature. Then

$$H = m_w s_w (\theta - 30) + m_c s_c (\theta - 30)$$

Here, m_w = mass of water = 0.5 kg

s_w = specific heat of water = 4200 J/kg-K

m_c = mass of coil = 0.06 kg

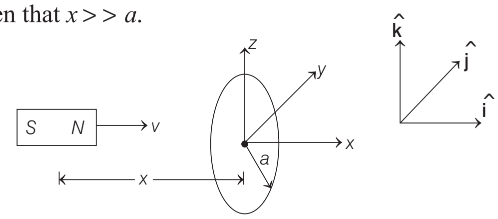
and s_c = specific heat of coil = 500 J/kg-K

Substituting the values, we get

$$12000 = (0.5)\,(4200)\,(\theta - 30) + (0.06)\,(500)\,(\theta - 30)$$

or $\theta = 35.6°$ C

31. Given that $x >> a$.



Magnetic field at the centre of the coil due to the bar magnet is

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3} = \frac{\mu_0}{2\pi}\frac{M}{x^3}$$

Due to this, magnetic flux linked with the coil will be,

$$\phi = BS = \frac{\mu_0}{2\pi}\frac{M}{x^3}(\pi a^2) = \frac{\mu_0 Ma^2}{2x^3}$$

$\therefore$ Induced emf in the coil, due to motion of the magnet is

$$e = \frac{-d\phi}{dt} = -\left(\frac{\mu_0 Ma^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right)$$

$$= \frac{\mu_0 Ma^2}{2}\left(\frac{3}{x^4}\right)\frac{dx}{dt} = \frac{3}{2}\frac{\mu_0 Ma^2}{x^4}v \quad \left(\because \frac{dx}{dt} = v\right)$$

Therefore, induced current in the coil is

$$i = \frac{e}{R} = \frac{3}{2}\frac{\mu_0 Ma^2}{Rx^4}v$$

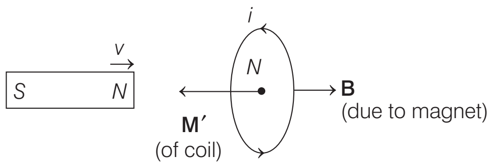
Magnetic moment of the coil due to this induced current will be,

$$M' = iS = \frac{3}{2}\frac{\mu_0 Ma^2}{Rx^4}v\,(\pi a^2) \Rightarrow M' = \frac{3}{2}\frac{\mu_0 \pi Ma^4 v}{Rx^4}$$

Potential energy of $\mathbf{M'}$ in $\mathbf{B}$ will be

$$U = -M'B\cos 180°$$

$$U = M'B = \frac{3}{2}\frac{\mu_0 \pi Ma^4 v}{Rx^4}\left(\frac{\mu_0}{2\pi}\cdot\frac{M}{x^3}\right)$$



$$U = \frac{3}{4}\frac{\mu_0^2 M^2 a^4 v}{R}\frac{1}{x^7} \Rightarrow F = -\frac{dU}{dx} = \frac{21}{4}\frac{\mu_0^2 M^2 a^4 v}{Rx^8}$$

Positive sign of F implies that there will be a repulsion between the magnet and the coil.

32. Total resistance of the circuit as function of distance x from resistance R is $R_{\text{net}} = R + 2\lambda x$

Let v be velocity of rod at this instant, then motional emf induced across the rod, $e = Bvd$

$$\therefore \quad \text{Current } i = \frac{e}{R_{\text{net}}} = \frac{Bvd}{R + 2\lambda x} \Rightarrow v = \frac{(R + 2\lambda x)\,i}{Bd}$$

Net force on the rod, $F_{\text{net}} = m\dfrac{dv}{dt} = \dfrac{2\lambda im}{Bd}(R + 2\lambda x)\cdot\dfrac{dx}{dt}$

but $$\frac{dx}{dt} = v = \frac{(R + 2\lambda x)\,i}{Bd}$$

$$F_{\text{net}} = \frac{2\lambda i^2 m}{B^2 d^2}(R + 2\lambda x)^2$$

This net force is equal to $F - F_m$ where $F_m = idB$

$$\therefore \quad F = F_{\text{net}} + F_m = \frac{2\lambda i^2 m}{B^2 d^2}(R + 2\lambda x)^2 + idB$$

17

Modern Physics

Topic 1 Bohr's Atomic Model

Objective Questions I (Only one correct option)

1. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths λ_1 / λ_2 of the photons emitted in this process is **(Main 2019, 12 April II)**

(a) 20/7 (b) 27/5
(c) 7/5 (d) 9/7

2. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n corresponding to its initial excited state is [for photon of wavelength λ, energy $E = \dfrac{1240\,\text{eV}}{\lambda\ (\text{in nm})}$] **(Main 2019, 12 April I)**

(a) $n = 4$ (b) $n = 5$
(c) $n = 7$ (d) $n = 6$

3. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ? **(Main 2019, 10 April II)**

[Take, $h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1}]

(a) 9.4 nm (b) 12.3 nm (c) 10.8 nm (d) 11.4 nm

4. A He^+ ion is in its first excited state. Its ionisation energy is **(Main 2019, 9 April II)**

(a) 54.40 eV (b) 13.60 eV (c) 48.36 eV (d) 6.04 eV

5. Taking the wavelength of first Balmer line in hydrogen spectrum ($n = 3$ to $n = 2$) as 660 nm, the wavelength of the 2$^{\text{nd}}$ Balmer line ($n = 4$ to $n = 2$) will be **(Main 2019, 9 April I)**

(a) 889.2 nm (b) 388.9 nm (c) 642.7 nm (d) 488.9 nm

6. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms fall on He^+ ions in $n = 1$ and $n = 2$ states. The possible transition of helium ions as they absorb energy from the radiation is **(Main 2019, 8 April I)**

(a) $n = 2 \to n = 3$ (b) $n = 1 \to n = 4$
(c) $n = 2 \to n = 5$ (d) $n = 2 \to n = 4$

7. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as **(Main 2019, 12 Jan I)**

(a) $r_n \propto n, E_n \propto n$ (b) $r_n \propto n^2, E_n \propto \dfrac{1}{n^2}$
(c) $r_n \propto \sqrt{n}, E_n \propto n$ (d) $r_n \propto \sqrt{n}, E_n \propto \dfrac{1}{n}$

8. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be **(Main 2019, 11 Jan II)**

(a) $\dfrac{27}{20}\lambda$ (b) $\dfrac{25}{16}\lambda$
(c) $\dfrac{20}{27}\lambda$ (d) $\dfrac{16}{25}\lambda$

9. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state in terms of Bohr radius a_0 will be (Take $hc = 12500$ eV-Å) **(Main 2019, 11 Jan I)**

(a) $4a_0$ (b) $9a_0$
(c) $16a_0$ (d) $25a_0$

10. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is **(2018 Main)**

(a) $\nu_L / 25$ (b) $16\,\nu_L$
(c) $\dfrac{\nu_L}{16}$ (d) $\dfrac{\nu_L}{20}$

11. As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion **(2015 Main)**

(a) kinetic energy, potential energy and total energy decrease
(b) kinetic energy decreases, potential energy increases but total energy remains same
(c) kinetic energy and total energy decrease but potential energy increases
(d) its kinetic energy increases but potential energy and total energy decrease

12. Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^8)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4, respectively for four elements, then approximately which one of the following is correct?

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ **(2014 Main)**
(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

13. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to **(2013 Main)**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$
(c) $\frac{1}{n^4}$ (d) $\frac{1}{n^3}$

14. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is **(2011)**

(a) 1215 Å (b) 1640 Å
(c) 2430 Å (d) 4687 Å

15. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **(2007, 3M)**

(a) 802 nm (b) 823 nm
(c) 1882 nm (d) 1648 nm

16. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of micro second another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector? **(2005, 2M)**

(a) 2 photons of energy 10.2 eV
(b) 2 photons of energy 1.4 eV
(c) One photon of energy 10.2 eV and an electron of energy 1.4 eV
(d) One photon of energy 10.2 eV and another photon of energy 1.4 eV

17. If the atom ${}_{100}Fm^{257}$ follows the Bohr's model and the radius of last orbit of ${}_{100}Fm^{257}$ is n times the Bohr radius, then find n **(2003, 2M)**

(a) 100 (b) 200 (c) 4 (d) 1/4

18. The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number. **(2003, 2M)**

(a) $r_n \propto n$ (b) $r_n \propto \frac{1}{n}$ (c) $r_n \propto n^2$ (d) $r_n \propto \frac{1}{n^2}$

19. A hydrogen atom and a Li^{2+} ion are both in the second excited state. If l_H and l_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then **(2002, 2M)**

(a) $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$
(b) $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$
(c) $l_H = l_{Li}$ and $|E_H| > |E_{Li}|$
(d) $l_H < l_{Li}$ and $|E_H| < |E_{Li}|$

20. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition **(2001, 2M)**

(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $5 \to 4$

21. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statement is true ? **(2000, 2M)**

(a) Its kinetic energy increases and its potential and total energy decreases
(b) Its kinetic energy decreases, potential energy increases and its total energy remains the same
(c) Its kinetic and total energy decreases and its potential energy increases
(d) Its kinetic, potential and total energy decreases

22. Imagine an atom made up of proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (given in terms of the Rydberg constant R for the hydrogen atom) equal to **(2000, 2M)**

(a) $9/5R$ (b) $36/5R$
(c) $18/5R$ (d) $4/R$

23. In hydrogen spectrum, the wavelength of H_α line is 656 nm; whereas in the spectrum of a distant galaxy H_α line wavelength is 706 nm. Estimated speed of galaxy with respect to earth is **(1999, 2M)**

(a) 2×10^8 m/s
(b) 2×10^7 m/s
(c) 2×10^6 m/s
(d) 2×10^5 m/s

24. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom $(Z = 3)$ is **(1997, 1M)**

(a) 1.51 (b) 13.6
(c) 40.8 (d) 122.4

25. Consider the spectral line resulting from the transition $n = 2 \to n = 1$ in the atoms and ions given below. The shortest wavelength is produced by **(1983, 1M)**

(a) hydrogen atom
(b) deuterium atom
(c) singly ionised helium
(d) doubly ionised lithium

Passage Based Questions

Passage 1

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantisation of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantised rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantisation condition.

26. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the nth level ($n = 0$ is not allowed) is **(2010)**

(a) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (b) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

27. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi \times 10^{-34}$ J-s) **(2010)**

(a) 2.76×10^{-46} kg - m^2 (b) 1.87×10^{-46} kg - m^2

(c) 4.67×10^{-47} kg - m^2 (d) 1.17×10^{-47} kg - m^2

28. In a CO molecule, the distance between C (mass = 12 amu) and O (mass = 16 amu), where 1 amu = $\frac{5}{3} \times 10^{-27}$ kg, is close to **(2010)**

(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

Passage 2

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de-Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number n taking values 1, 2, 3, ...($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. [Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C]

29. The allowed energy for the particle for a particular value of n is proportional to **(2009)**

(a) a^{-2} (b) $a^{-3/2}$ (c) a^{-1} (d) a^2

30. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to **(2009)**

(a) 0.8 meV (b) 8 meV

(c) 80 meV (d) 800 meV

31. The speed of the particle that can take discrete values is proportional to **(2009)**

(a) $n^{-3/2}$ (b) n^{-1}

(c) $n^{1/2}$ (d) n

Passage 3

In a mixture of H - He$^+$ gas (He$^+$ is singly ionized He atom), H atoms and He$^+$ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He$^+$ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

32. The quantum number n of the state finally populated in He$^+$ ions is **(2008, 4M)**

(a) 2 (b) 3 (c) 4 (d) 5

33. The wavelength of light emitted in the visible region by He$^+$ ions after collisions with H atoms is **(2008, 4M)**

(a) 6.5×10^{-7} m (b) 5.6×10^{-7} m

(c) 4.8×10^{-7} m (d) 4.0×10^{-7} m

34. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He$^+$ ion is **(2008, 4M)**

(a) 1/4 (b) 1/2

(c) 1 (d) 2

Objective Questions II (One or more correct option)

35. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principle quantum number n, where $n >> 1$. Which of the following statement(s) is (are) true? **(2016 Adv.)**

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as $1/n$

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(d) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

36. The radius of the orbit of an electron in a Hydrogen-like atom is $4.5\,a_0$ where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are) **(2013 Adv.)**

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$ (c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

37. The electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr's model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are **(1998, 2M)**

(a) $n_1 = 4, n_2 = 2$
(b) $n_1 = 8, n_2 = 2$
(c) $n_1 = 8, n_2 = 1$
(d) $n_1 = 6, n_2 = 3$

38. In the Bohr model of the hydrogen atoms **(1984, 2M)**

(a) the radius of the nth orbit is proportional n^2.
(b) the total energy of the electron in the nth orbit is inversely proportional to n.
(c) the angular momentum of the electron in an orbit is an integral multiple of h/π.
(d) the magnitude of the potential energy of the electron in any orbit is greater than its kinetic energy.

Numerical Value

39. Consider a hydrogen-like ionised atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionisation energy of the hydrogen atom is 13.6 eV. The value of Z is .... **(2018 Adv.)**

Fill in the Blanks

40. The recoil speed of a hydrogen atom after it emits a photon is going from $n = 5$ state to $n = 1$ state is .... m/s. **(1997, 1M)**

41. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is .... **(1992, 1M)**

Integer Answer Type Questions

42. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. v_i and v_f are respectively the initial and final potential energies of the electron. If $\frac{v_i}{v_f} = 6.25$, then the smallest possible n_f is **(2017 Adv.)**

43. A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}$ eVm and the ground state energy of hydrogen atom as -13.6 eV, the number of lines present in the emission spectrum is **(2016 Adv.)**

44. Consider a hydrogen atom with its electro in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is ($hc = 1242$ eV nm) **(2015 Adv.)**

Analytical & Descriptive Questions

45. Wavelengths belonging to Balmer series lying in the range of 450 nm to 750 nm were used to eject photoelectrons from a metal surface whose work function is 2.0 eV. Find (in eV) the maximum kinetic energy of the emitted photoelectrons. (Take $hc = 1242$ eV nm.) **(2004, 4M)**

46. A hydrogen-like atom (described by the Bohrs model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between – 0.85 eV and –0.544 eV (including both these values). **(2002, 5M)**

(a) Find the atomic number of the atom.
(b) Calculate the smallest wavelength emitted in these transitions.

(Take $hc = 1240$ eV-nm, ground state energy of hydrogen atom = -13.6 eV)

47. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) of this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is –13.6 eV. **(2000, 6M)**

48. An electron in a hydrogen like atom is in an excited state. It has a total energy of -3.4 eV. Calculate **(1996, 3M)**

(a) the kinetic energy,
(b) the de-Broglie wavelength of the electron.

49. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 eV and 17.0 eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. **(1994, 6M)**

Determine the values of n and Z.
(Ionization energy of H-atom = 13.6 eV)

50. A particle of charge equal to that of an electron – e, and mass 208 times of the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge + $3e$. (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system, **(1988, 6M)**

(a) derive an expression for the radius of the n^{th} Bohr orbit.
(b) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.
(c) find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit to the first orbit.

(Rydberg's constant = 1.097×10^7 m^{-1})

51. A doubly ionised lithium atom is hydrogen-like with atomic number 3. **(1985, 6M)**

(a) Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 eV.)
(b) How many spectral lines are observed in the emission spectrum of the above excited system?

52. The ionization energy of a hydrogen like Bohr atom is 4 rydberg. **(1984, 4M)**
(a) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state?
(b) What is the radius of the first orbit for this atom ?

53. Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of Planck's constant. **(1983, 4M)**

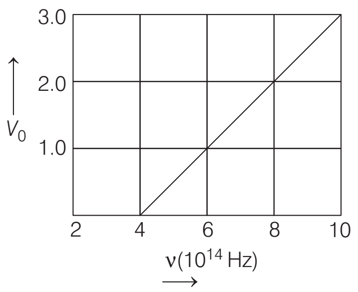
54. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975° Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV. **(1982, 5M)**

55. A single electron orbits around a stationary nucleus of charge $+Ze$. Where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. **(1981, 10M)**
Find
(a) the value of Z.
(b) the energy required to excite the electron from the third to the fourth Bohr orbit.
(c) the wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.
(d) the kinetic energy, potential energy and the angular momentum of the electron in the first Bohr orbit.
(e) the radius of the first Bohr orbit.
(The ionization energy of hydrogen atom $=13.6$ eV, Bohr radius $=5.3\times10^{-11}$m, velocity of light $=3\times10^{8}$m/s. Planck's constant $=6.6\times10^{-34}$ J-s).

Topic 2 Photo Electric Effect

Objective Questions I (Only one correct option)

1. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be
(Take, Planck's constant $(h)=6.63\times10^{-34}$ J-s, electron charge, $e=1.6\times10^{-19}$ C] **(Main 2019, 12 April I)**



(a) 1.82 eV (b) 1.66 eV
(c) 1.95 eV (d) 2.12 eV

2. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is
[Given, Planck's constant $h=6.6\times10^{-34}$ Js, speed of light $c=3.0\times10^{8}$ m/s] **(Main 2019, 10 April II)**
(a) 1×10^{16} (b) 5×10^{15}
(c) 1.5×10^{16} (d) 2×10^{16}

3. In a photoelectric effect experiment, the threshold wavelength of light is 380 nm. If the wavelength of incident light is
260 nm, the maximum kinetic energy of emitted electrons will be
Given, E (in eV) $=\dfrac{1237}{\lambda\text{ (in nm)}}$ **(Main 2019, 10 April I)**
(a) 15.1 eV (b) 3.0 eV
(c) 1.5 eV (d) 4.5 eV

4. The electric field of light wave is given as $\mathbf{E}=10^{-3}\cos\left(\dfrac{2\pi x}{5\times10^{-7}}-2\pi\times6\times10^{14}t\right)\hat{\mathbf{x}}$ NC^{-1}. This light falls on a metal plate of work function 2eV. The stopping potential of the photoelectrons is
Given, E (in eV) $=\dfrac{12375}{\lambda\text{(in Å)}}$ **(Main 2019, 9 April I)**
(a) 0.48 V (b) 0.72 V
(c) 2.0 V (d) 2.48 V

5. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the photocurrent is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is **(Main 2019, 12 Jan II)**
(a) $\dfrac{4}{3}\nu$ (b) 2ν
(c) $\dfrac{3\nu}{2}$ (d) $\dfrac{5\nu}{3}$

6. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to $\left(\dfrac{hc}{e}=1240\text{ nmV}\right)$ **(Main 2019, 11 Jan II)**
(a) 0.5 V (b) 2.0 V
(c) 1.5 V (d) 1.0 V

7. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 m W/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively will be (Take, 1 eV $= 1.6 \times 10^{-19}$ J)

(a) 10^{11} and 5 eV (b) 10^{12} and 5 eV
(c) 10^{10} and 5 eV (d) 10^{14} and 10 eV

8. The magnetic field associated with a light wave is given at the origin, by

$B = B_0 [\sin (3.14 \times 10^7) ct + \sin (6.28 \times 10^7) ct]$.

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons? **(Main 2019, 10 Jan II)**

(Take, $c = 3 \times 10^8$ ms^{-1} and $h = 6.6 \times 10^{-34}$ J-s)

(a) 7.72 eV (b) 6.82 eV (c) 8.52 eV (d) 12.5 eV

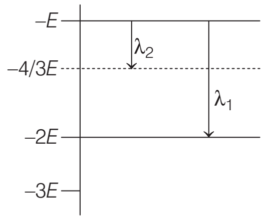
9. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then by light of wavelength $\lambda_2 = 540$ n-m. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to

(energy of photon $= \dfrac{1240}{\lambda \text{ (in n - m)}}$ eV) **(Main 2019, 9 Jan I)**

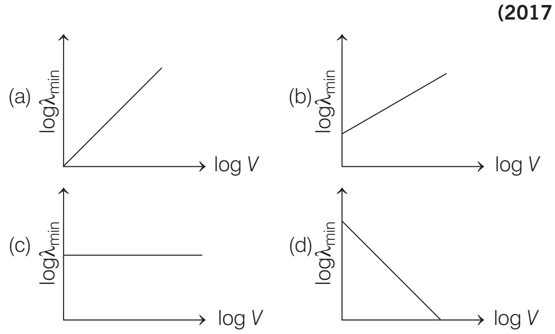
(a) 5.6 (b) 2.5 (c) 1.8 (d) 1.4

10. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$ is given by **(2017 Main)**

(a) $r = \dfrac{2}{3}$ (b) $r = \dfrac{3}{4}$ (c) $r = \dfrac{1}{3}$ (d) $r = \dfrac{4}{3}$



11. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-rays in the spectrum, the variation of $\log \lambda_{min}$ with $\log V$ is correctly represented in **(2017 Main)**



12. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\dfrac{3\lambda}{4}$, the speed of the fastest emitted electron will be **(2016 Main)**

(a) $> v\left(\dfrac{4}{3}\right)^{1/2}$ (b) $< v\left(\dfrac{4}{3}\right)^{1/2}$

(c) $= v\left(\dfrac{4}{3}\right)^{1/2}$ (d) $= v\left(\dfrac{3}{4}\right)^{1/2}$

13. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below: **(2016 Adv.)**

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ ms^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J-s) found from such an experiment is)

(a) 6.0×10^{-34} (b) 6.4×10^{-34}
(c) 6.6×10^{-34} (d) 6.8×10^{-34}

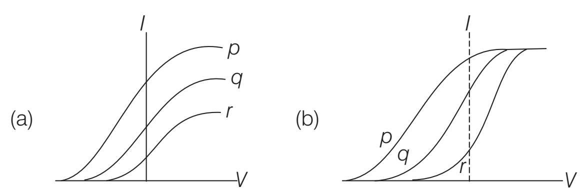
14. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240$ e V nm, the work function of the metal is nearly **(2014 Adv.)**

(a) 3.7 eV (b) 3.2 eV (c) 2.8 eV (d) 2.5 eV

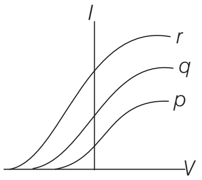
15. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to **(2014 Main)**

(a) 1.8 eV (b) 1.1 eV (c) 0.8 eV (d) 1.6 eV

16. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is **(2009)**



(c)



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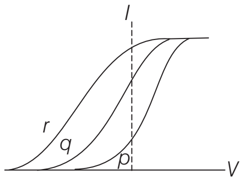
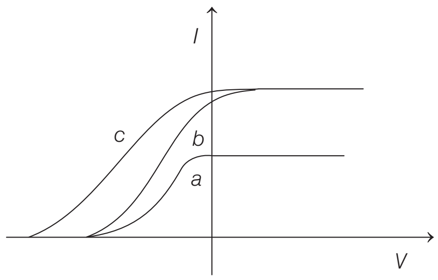


17. The figure shows the variation of photocurrent with anode potential for a photosensitive surface for three different radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves *a, b* and *c* respectively **(2004, 2M)**



(a) $f_a = f_b$ and $I_a \neq I_b$ (b) $f_a = f_c$ and $I_a = I_c$
(c) $f_a = f_b$ and $I_a = I_b$ (d) $f_b = f_c$ and $I_b = I_c$

18. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **(1998, 2M)**

(a) 540 nm (b) 400 nm (c) 310 nm (d) 220 nm

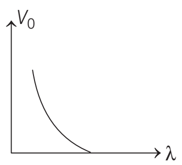
19. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential in volt is **(1997, 1M)**

(a) 2 (b) 4 (c) 6 (d) 10

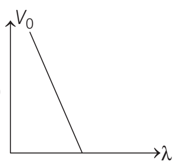
Objective Questions II (One or more correct option)

20. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $\frac{1}{\lambda}$. **(2015 Adv.)**

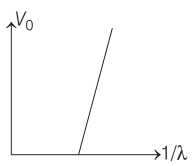
(a)



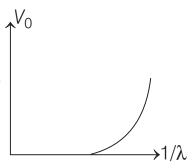
(b)



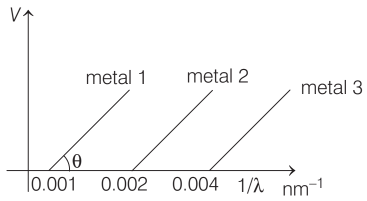
(c)



(d)



21. The graph between $1/\lambda$ and stopping potential (V) of three metals having work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statement(s) is/are correct ? (Here, λ is the wavelength of the incident ray). **(2006, 5M)**



(a) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
(b) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
(c) $\tan\theta$ is directly proportional to hc/e, where h is Planck's constant and c is the speed of light
(d) The violet colour light can eject photoelectrons from metals 2 and 3

22. When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A expressed in eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50\,\text{eV})$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then **(1994, 2M)**

(a) the work function of A is 2.25 eV
(b) the work function of B is 4.20 eV
(c) $T_A = 2.00$ eV
(d) $T_B = 2.75$ eV

23. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are respectively 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then **(1992, 2M)**

(a) the stopping potential will be 0.2 V
(b) the stopping potential will be 0.6 V
(c) the saturation current will be 6.0 mA
(d) the saturation current will be 2.0 mA

24. Photoelectric effect supports quantum nature of light because **(1987, 2M)**

(a) there is a minimum frequency of light below which no photoelectrons are emitted
(b) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
(c) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
(d) electric charge of the photoelectrons is quantized

25. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a **(1982, 3M)**

(a) 50 W infrared lamp (b) 1 W infrared lamp
(c) 50 W ultraviolet lamp (d) 1 W ultraviolet lamp

Integer Answer Type Questions

26. The work functions of silver and sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential *versus* frequency plot for silver to that of sodium is **(2013 Adv.)**

27. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^{Z}$ (where $1 < A < 10$). The value of Z is **(2011)**

Fill in the Blank

28. The maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the .... of the incident radiation. **(1984, 2M)**

True/False

29. In a photoelectric emission process, the maximum energy of the photoelectrons increases with increasing intensity of the incident light. **(1986, 3M)**

30. The kinetic energy of photoelectrons emitted by a photosensitive surface depends on the intensity of the incident radiation. **(1981, 2M)**

Analytical & Descriptive Questions

31. In a photoelectric experiment set-up, photons of energy 5 eV falls on the cathode having work function 3 eV. (a) If the saturation current is $i_A = 4\mu\text{A}$ for intensity $10^{-5}\,\text{W/m}^2$, then plot a graph between anode potential and current. (b) Also, draw a graph for intensity of incident radiation $2 \times 10^{-5}\,\text{W/m}^2$. **(2003, 2M)**

32. Two metallic plates A and B each of area $5 \times 10^{-4}\,\text{m}^2$, are placed parallel to each other at separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at $t = 0$ so that 10^{16} photons fall on it per square metre per second. Assume that one photoelectron is emitted for every 10^{6} incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV.

Determine **(2002, 5M)**

(a) the number of photoelectrons emitted up to $t = 10\,\text{s}$,

(b) the magnitude of the electric field between the plates A and B at $t = 10\text{s}$ and

(c) the kinetic energy of the most energetic photoelectrons emitted at $t = 10$ s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B. (Take $\varepsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2/\text{N-m}^2$).

33. Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α-particles. A maximum energy electron combines with an α-particle to form a He^{+} ion, emitting a single photon in this process. He^{+} ions thus formed are in their fourth excited state. Find the energies in eV of the photons lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination. **(1999, 5M)**

[Take $h = 4.14 \times 10^{-15}$ eV-s]

34. In a photoelectric effect set-up a point of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius $8.0 \times 10^{-3}\,\text{m}$. The efficiency of photoelectrons emission is one for every 10^{6} incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission. **(1995, 10M)**

(a) Calculate the number of photoelectrons emitted per second.

(b) Find the ratio of the wavelength of incident light to the de-Broglie wavelength of the fastest photoelectrons emitted.

(c) It is observed that the photoelectrons emission stops at a certain time t after the light source is switched on why ?

(d) Evaluate the time t.

35. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. **(1992, 10M)**

Find

(a) the energy of the photons causing the photoelectrons emission.

(b) the quantum numbers of the two levels involved in the emission of these photons.

(c) the change in the angular momentum of the electron in the hydrogen atom, in the above transition, and

(d) the recoil speed of the emitting atom assuming it to be at rest before the transition. (Ionization potential of hydrogen is 13.6 eV.)

36. A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of $3.6 \times 10^{-3}\,\text{Wm}^{-2}$ equally distributed amongst the three wavelengths. The beam falls normally on an area $1.0\,\text{cm}^2$ of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds. **(1989, 8M)**

Topic 3 Radioactivity

Objective Questions I (Only one correct option)

1. In a radioactive decay chain, the initial nucleus is $^{232}_{90}\text{Th}$. At the end, there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $^{A}_{Z}X$, A and Z are given by **(Main 2019, 12 Jan II)**

(a) $A = 202; Z = 80$ (b) $A = 208; Z = 82$
(c) $A = 200; Z = 81$ (d) $A = 208; Z = 80$

2. Using a nuclear counter, the count rate of emitted particles from a radioactive source is measured. At $t = 0$, it was 1600 counts per second and $t = 8$ s, it was 100 counts per second. The count rate observed as counts per second at $t = 6$ s is close to **(Main 2019, 10 Jan I)**

(a) 400 (b) 200 (c) 150 (d) 360

3. In given time $t = 0$, Activity of two radioactive substances A and B are equal. After time t, the ratio of their activities $\frac{R_B}{R_A}$ decreases according to e^{-3t}. If the half life of A is In 2, the half-life of B will be **(Main 2019, 9 Jan II)**

(a) $4 \ln 2$ (b) $\frac{\ln 2}{4}$ (c) $\frac{\ln 2}{2}$ (d) $2 \ln 2$

4. A sample of radioactive material A, that has an activity of 10 mCi ($1 \text{ Ci} = 3.7 \times 10^{10}$ decays/s) has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would, then be respectively **(Main 2019, 9 Jan I)**

(a) 20 days and 10 days (b) 5 days and 10 days
(c) 10 days and 40 days (d) 20 days and 5 days

5. A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. After sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by **(2017 Main)**

(a) $t = T \frac{\log 1.3}{\log 2}$ (b) $t = T \log 1.3$
(c) $t = \frac{T}{\log 1.3}$ (d) $t = \frac{T \log 2}{2 \log 1.3}$

6. Half-lives of two radioactive elements A and B are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of A and B nuclei will be **(2016 Main)**

(a) 1 : 16 (b) 4 : 1 (c) 1 : 4 (d) 5 : 4

7. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? **(2016 Adv.)**

(a) 64 (b) 90 (c) 108 (d) 120

8. A radioactive sample S_1 having an activity of 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 can be **(2008, 3M)**

(a) 20 yr and 5 yr, respectively
(b) 20 yr and 10 yr, respectively
(c) 10 yr each
(d) 5 yr each

9. Half-life of a radioactive substance A is 4 days. The probability that a nucleus will decay in two half-lives is **(2006, 3M)**

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 1

10. After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. The initial activity of the sample in dps is **(2004, 2M)**

(a) 6000 (b) 9000
(c) 3000 (d) 24000

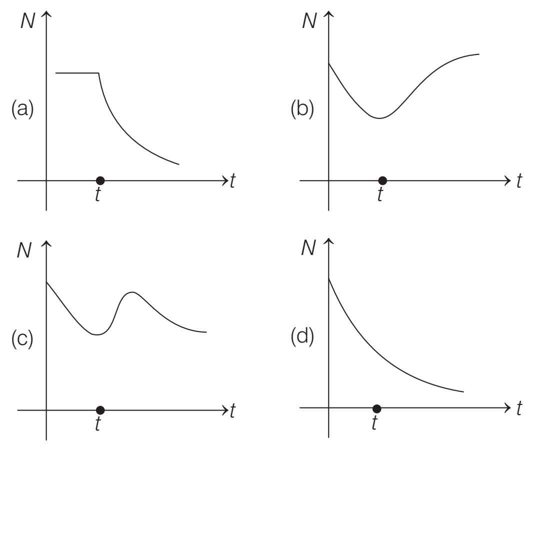
11. Which of the following processes represent a γ-decay ? **(2002, 2M)**

(a) $^{A}X_Z + \gamma \rightarrow {}^{A}X_{Z-1} + a + b$
(b) $^{A}X_Z + {}^{1}n_0 \rightarrow {}^{A-3}X_{Z-2} + c$
(c) $^{A}X_Z \rightarrow {}^{A}X_Z + f$
(d) $^{A}X_Z + e_{-1} \rightarrow {}^{A}X_{A-1} + g$

12. The half-life of ^{215}At is 100 μs. The time taken for the activity of a sample of ^{215}At to decay to $\frac{1}{16}$th of its initial value is **(2002, 2M)**

(a) 400 μs (b) 63 μs
(c) 40 μs (d) 300 μs

13. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figure best represents the form of this plot ? **(2001, 2M)**



14. The electron emitted in beta radiation originates from
(a) inner orbits of atom **(2001, 2M)**
(b) free electrons existing in nuclei
(c) decay of a neutron in a nucleus
(d) photon escaping from the nucleus

15. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time **(2000, 2M)**
(a) $1/10\lambda$ (b) $1/11\lambda$ (c) $11/10\lambda$ (d) $1/9\lambda$

16. The half-life period of a radioactive element x is same as the mean life time of another radioactive element y. Initially both of them have the same number of atoms. Then **(1999, 3M)**
(a) x and y have the same decay rate initially
(b) x and y decay at the same rate always
(c) y will decay at a faster rate than x
(d) x will decay at a faster rate than y

17. Which of the following is a correct statement ? **(1999, 2M)**
(a) Beta rays are same as cathode rays
(b) Gamma rays are high energy neutrons
(c) Alpha particles are singly ionized helium atoms
(d) Protons and neutrons have exactly the same mass

18. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that **(1998, 2M)**
(a) no nucleus will decay before $t = 4$ days
(b) no nucleus will decay before $t = 8$ days
(c) all nuclei will decay before $t = 16$ days
(d) a given nucleus may decay at any time after $t = 0$

19. Masses of two isobars $_{29}Cu^{64}$ and $_{30}Zn^{64}$ are 63.9298 u and 63.9292 u respectively. It can be concluded from these data that **(1997C, 1M)**
(a) both the isobars are stable
(b) Zn^{64} is radioactive, decaying to Cu^{64} through β-decay
(c) Cu^{64} is radioactive, decaying to Zn^{64} through γ-decay
(d) Cu^{64} is radioactive, decaying to Zn^{64} through β-decay

20. Consider α-particles, β-particles and γ-rays each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are **(1994, 1M)**
(a) α, β, γ (b) α, γ, β (c) β, γ, α (d) γ, β, α

21. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are respectively given by **(1989, 2M)**
(a) $1/\lambda$ and $(\ln 2)/\lambda$ (b) $(\ln 2)/\lambda$ and $1/\lambda$
(c) $\lambda (\ln 2)$ and $1/\lambda$ (d) $\lambda/(\ln 2)$ and $1/\lambda$

22. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is **(1988, 2M)**
(a) 6 h (b) 12 h
(c) 24 h (d) 128 h

23. During a negative beta decay, **(1987, 2M)**
(a) an atomic electron is ejected
(b) an electron which is already present within the nucleus is ejected
(c) a neutron in the nucleus decays emitting an electron
(d) a part of the binding energy of the nucleus is converted into an electron

24. Beta rays emitted by a radioactive material are **(1983, 1M)**
(a) electromagnetic radiations
(b) the electrons orbiting around the nucleus
(c) charged particles emitted by the nucleus
(d) neutral particles

25. The half-life of the radioactive radon is 3.8 days. The time, at the end of which 1/20th of the radon sample will remain undecayed, is (given $\log_{10} e = 0.4343$) **(1981, 2M)**
(a) 3.8 days (b) 16.5 days (c) 33 days (d) 76 days

Objective Questions II (One or more than one)

26. In a radioactive decay chain, $^{232}_{90}Th$ nucleus decays to $^{212}_{82}Pb$ nucleus. Let N_α and N_β be the number of α and β- particles respectively, emitted in this decay process. Which of the following statements is (are) true? **(2018 Adv.)**
(a) $N_\alpha = 5$ (b) $N_\alpha = 6$ (c) $N_\beta = 2$ (d) $N_\beta = 4$

Integer Answer Type Questions

27. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 h, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in litres is approximately (you may use $e^2 \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$). **(2017 Adv.)**

28. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive source P(mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rate of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is **(2015 Adv.)**

29. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $\ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is **(2013 Adv.)**

30. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^{-9} s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is **(2011)**

Fill in the Blanks

31. In the nuclear process, ${}_6C^{11} \rightarrow {}_5B^{11} + \beta^+ + X$, X stands for .... **(1992, 1M)**

32. When boron nucleus $\left({}^{10}_{5}B\right)$ is bombarded by neutrons, α-particles are emitted. The resulting nucleus is of the element .... and has the mass number...... . **(1986, 2M)**

33. In the uranium radioactive series, the initial nucleus is ${}^{238}_{92}U$ and the final nucleus is ${}^{206}_{82}Pb$. When the uranium nucleus decays to lead, the number of α-particles emitted is ... and the number of β-particles emitted is ..... . **(1985, 2M)**

34. The radioactive decay rate of a radioactive element is found to be 10^3 disintegration/second at a certain time. If the half-life of the element is one second, the decay rate after one second is .... and after three seconds is........ . **(1983, 2M)**

Analytical & Descriptive Questions

35. A rock is 1.5×10^9 yr old. The rock contains ${}^{238}U$ which disintegrates to form ${}^{206}Pb$. Assume that there was no ${}^{206}Pb$ in the rock initially and it is the only stable product formed by the decay. Calculate the ratio of number of nuclei of ${}^{238}U$ to that of ${}^{206}Pb$ in the rock. Half-life of ${}^{238}U$ is 4.5×10^9 yr. $(2^{1/3} = 1.259)$ **(2004, 2M)**

36. A radioactive element decays by β-emission. A detector records n beta particles in 2 s and in next 2 s it records $0.75\,n$ beta particles. Find mean life correct to nearest whole number. Given $\ln|2| = 0.6931$, $\ln|3| = 1.0986$. **(2003, 2M)**

37. A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_x = 0.1\,s^{-1}$, Y further decays to a stable nucleus Z with a decay constant $\lambda_y = 1/30\,s^{-1}$. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$. Set-up the rate equations for the populations of X, Y and Z. The population of Y nucleus as a function of time is given by $N_y(t) = \{N_0 \lambda_x / (\lambda_x - \lambda_y)\}[\exp(-\lambda_y t) - \exp(-\lambda_x t)]$. Find the time at which N_Y is maximum and determine the populations X and Z at that instant. **(2001, 5M)**

38. Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element. **(1998, 8M)**
(a) Calculate the number N of nuclei of A at time t.
(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N as $t \rightarrow \infty$.

39. At a given instant there are 25% undecayed radioactive nuclei in a sample. After 10 s the number of undecayed nuclei reduces to 12.5%. Calculate **(1996, 3M)**
(a) mean life of the nuclei,
(b) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number.

40. A small quantity of solution containing Na^{24} radio nuclide (half-life = 15 h) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5h shows an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person. **(1994, 6M)**
(1 curie = 3.7×10^{10} disintegrations per second)

41. There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? **(1986, 6M)**

42. A uranium nucleus (atomic number 92, mass number 238) emits an alpha particle and the resulting nucleus emits β-particle. What are the atomic number and mass number of the final nucleus? **(1982, 2M)**

Topic 4 X-Rays and de-Broglie Wavelength

Objective Questions I (Only one correct option)

1. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is **(Main 2019, 12 April II)**
(a) 3.5 Å (b) 6.6 Å
(c) 12.9 Å (d) 9.7 Å

2. A particle P is formed due to a completely inelastic collision of particles x and y having de-Broglie wavelengths λ_x and λ_y, respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of P is **(Main 2019, 9 April II)**
(a) $\lambda_x - \lambda_y$ (b) $\frac{\lambda_x \lambda_y}{\lambda_x - \lambda_y}$ (c) $\frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$ (d) $\lambda_x + \lambda_y$

3. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2, respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ of the final particle, is given by **(Main 2019, 8 April I)**
(a) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$
(b) $\lambda = \sqrt{\lambda_1 \lambda_2}$
(c) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$
(d) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

4. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of
50 V. Another particle B of mass '$4m$' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to **(Main 2019, 12 Jan I)**

(a) 4.47 (b) 10.00
(c) 0.07 (d) 14.14

5. If the de-Broglie wavelength of an electron is equal to 10^{-3} times, the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to
(Take, speed of light $= 3 \times 10^8$ m/s,
Planck's constant $= 6.63 \times 10^{-34}$ J-s and
mass of electron $= 9.1 \times 10^{-31}$ kg) **(Main 2019, 11 Jan I)**

(a) 1.45×10^6 m/s (b) 1.8×10^6 m/s
(c) 1.1×10^6 m/s (d) 1.7×10^6 m/s

6. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de-Broglie wavelength of the electron in the nth state and the ground state, respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants) **(2018 main)**

(a) $\Lambda_n^2 \approx \lambda$ (b) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$
(c) $\Lambda_n \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx A + B\lambda_n^2$

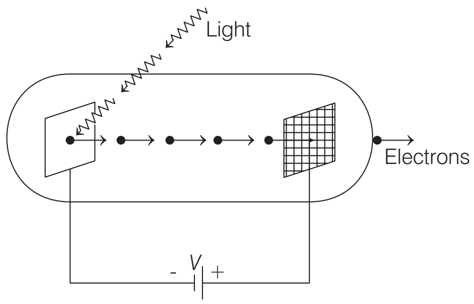
7. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is held on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is **(2017 Main)**

(a) $\frac{\lambda_A}{\lambda_B} = 2$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

8. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda \left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de-Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then, the ratio $\frac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to **(2017 Adv.)**

(a) $\frac{\lambda_d^2}{\lambda^2}$ (b) $\frac{\lambda_d}{\lambda}$ (c) $\frac{\lambda_d^3}{\lambda}$ (d) $\frac{\lambda_d^3}{\lambda^2}$

9. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statements(s) is (are) true? **(2016 Adv.)**



(a) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$
(b) λ_e is approximately halved, if d is doubled
(c) λ_e decreases with increase in ϕ and λ_{ph}
(d) For large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times.

10. If λ_{Cu} is the wavelength of K_α, X-ray line of copper (atomic number 29) and λ_{MO} is the wavelength of the K_α, X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu} / \lambda_{Mo}$ is close to **(2014 Adv.)**

(a) 1.99 (b) 2.14
(c) 0.50 (d) 0.48

11. Which one of the following statements is wrong in the context of X-rays generated from an X-ray tube ? **(2008, 3M)**

(a) Wavelength of characteristic X-rays decreases when the atomic number of the target increases
(b) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(c) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(d) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

12. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is **(2007, 3M)**

(a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$
(c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

13. K_α wavelength emitted by an atom of atomic number $Z = 11$ is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ **(2005, 2M)**

(a) $Z = 6$ (b) $Z = 4$
(c) $Z = 11$ (d) $Z = 44$

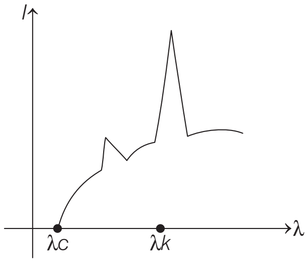
14. The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio $\frac{\lambda_1}{\lambda_2}$ is proportional to **(2004, 2M)**

(a) E^0 (b) $E^{1/2}$ (c) E^{-1} (d) E^{-2}

15. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Then the number of electrons striking the target per second is **(2002, 2M)**

(a) 2×10^{16} (b) 5×10^{6} (c) 1×10^{17} (d) 4×10^{15}

16. The intensity of X-rays from a coolidge tube is plotted against wavelength λ as shown in the figure. The minimum wavelength found is λ_c and the wavelength of the K_α line is λ_k. As the accelerating voltage is increased **(2001, 2M)**



(a) $\lambda_k - \lambda_c$ increases (b) $\lambda_k - \lambda_c$ decreases
(c) λ_k increases (d) λ_k decreases

17. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K-shell electrons of tungsten have 72.5 keV energy. X-rays emitted by the tube contain only **(2000, 2M)**

(a) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of ≈ 0.155 Å
(b) a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths
(c) the characteristic X-ray spectrum of tungsten
(d) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of ≈ 0.155Å and the characteristic X-ray spectrum of tungsten

18. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles λ_1/λ_2 is **(1999, 2M)**

(a) m_1/m_2 (b) m_2/m_1 (c) 1 (d) $\sqrt{m_2}/\sqrt{m_1}$

19. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from **(1998, 2M)**

(a) 0 to ∞ (b) $\lambda_{\min}$ to ∞ where $\lambda_{\min} > 0$
(c) 0 to $\lambda_{\max}$ where $\lambda_{\max} < \infty$
(d) $\lambda_{\min}$ to $\lambda_{\max}$ where $0 < \lambda_{\min} < \lambda_{\max} < \infty$

20. The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. The energy difference between K and L levels in this atoms is about **(1997C, 1M)**

(a) 0.51 MeV (b) 1.2 MeV (c) 59 keV (d) 13.6 eV

21. The X-ray beam coming from an X-ray tube will be

(a) monochromatic **(1985, 2M)**
(b) having all wavelengths smaller than a certain maximum wavelength
(c) having all wavelengths larger than a certain minimum wavelength
(d) having all wavelengths lying between a minimum and a maximum wavelength

22. The shortest wavelength of X-rays emitted from an X-ray tube depends on **(1982, 3M)**

(a) the current in the tube
(b) the voltage applied to the tube
(c) the nature of the gas in tube
(d) the atomic number of the target material

Assertion and Reason

Mark your answer as

(a) If Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.
(b) If Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
(c) If Statement I is true; Statement II is false.
(d) If Statement I is false; Statement II is true.

23. **Statement I** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change **(2007, 3M)**

Statement II When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

Objective Question II (One or more correct option)

24. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation **(1988, 2M)**

(a) the intensity increases
(b) the minimum wavelength increases
(c) the intensity remains unchanged
(d) the minimum wavelength decreases

Integer Answer Type Questions

25. An electron in an excited state of Li^{2+} ion has angular momentum $\frac{3h}{2\pi}$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is **(2015 Adv.)**

26. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de-Broglie wavelength (in units of fm) of the proton at its start is [Take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J-s/C;

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m] **(2013 Adv.)**

27. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is **(2010)**

Fill in the Blanks

28. The wavelength of K_α, X-rays produced by an X-ray tube is 0.76 Å. The atomic number of the anode material of the tube is .... **(1996, 2M)**

29. A potential difference of 20 kV is applied across an X-ray tube. The minimum wavelength of X-rays generated is .... Å. **(1996, 2M)**

30. In an X-ray tube, electrons accelerated through a potential difference of 15, 000 V strike a copper target. The speed of the emitted X-ray inside the tube is .... m/s. **(1992, 1M)**

31. The wavelength of the characteristic X-ray K_α line emitted by a hydrogen like element is 0.32D. The wavelength of the K_β line emitted by the same element will be .... **(1990, 2M)**

32. When the number of electrons striking the anode of an X-ray tube is increased the .... of the emitted X-rays increases, while when the speeds of the electrons striking the anode are increased the cut-off wavelength of the emitted X-rays .... **(1986, 2M)**

33. To produce characteristic X-rays using a tungsten target in an X-ray generator, the accelerating voltage should be greater than ...... V and the energy of the characteristic radiation is ...... eV. (The binding energy of the innermost electron in tungsten is 40 keV). **(1983, 2M)**

Analytical & Descriptive Questions

34. If the wavelength of the n^{th} line of Lyman series is equal to the de-Broglie wavelength of electron in initial orbit of a hydrogen like element ($Z = 11$). Find the value of n. **(2005)**

35. X-rays are incident on a target metal atom having 30 neutrons. The ratio of atomic radius of the target atom and ${}^4_2\text{He}$ is $(14)^{1/3}$. **(2005, 4M)**

(a) Find the mass number of target atom.

(b) Find the frequency of K_α line emitted by this metal.

($R = 1.1 \times 10^7 \text{ m}^{-1}$, $c = 3 \times 10^8$ m/s)

36. The potential energy of a particle varies as

$$U(x) = E_0 \quad \text{for } 0 \le x \le 1$$
$$= 0 \quad \text{for } x > 1$$

For $0 \le x \le 1$, de-Broglie wavelength is λ_1 and for $x > 1$ the de-Broglie wavelength is λ_2. Total energy of the particle is $2E_0$. Find $\dfrac{\lambda_1}{\lambda_2}$. **(2005, 2M)**

37. Characteristic X-rays of frequency 4.2×10^{18} Hz are produced when transitions from L-shell to K-shell take place in a certain target material. Use Mosley's law to determine the atomic number of the target material. Given Rydberg's constant $R = 1.1 \times 10^7 \text{ m}^{-1}$. **(2003, 2M)**

38. Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electron in eV and the least value of d for which the standing wave of the type described above can form. **(1997, 5M)**

Topic 5 Nuclear Physics

Objective Questions I (Only one correct option)

1. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If initially a sample has equal number of nuclei, then after
60 minutes, the ratio of decayed numbers of nuclei A and B will be **(Main 2019, 12 April II)**

(a) 3 : 8 (b) 1 : 8 (c) 8 : 1 (d) 9 : 8

2. Two radioactive substances A and B have decay constants 5λ and λ, respectively. At $t = 0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\dfrac{1}{e}\right)^2$ will be **(Main 2019, 10 April II)**

(a) $2/\lambda$ (b) $1/2\lambda$ (c) $1/4\lambda$ (d) $1/\lambda$

3. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be $1/e$ after a time **(Main 2019, 10 April I)**

(a) $\dfrac{1}{11\lambda}$ (b) $\dfrac{11}{10\lambda}$ (c) $\dfrac{1}{9\lambda}$ (d) $\dfrac{1}{10\lambda}$

4. The ratio of mass densities of nuclei of ${}^{40}\text{Ca}$ and ${}^{16}\text{O}$ is close to **(Main 2019, 8 April II)**

(a) 5 (b) 2 (c) 0.1 (d) 1

5. Consider the nuclear fission

$$\text{Ne}^{20} \longrightarrow 2\text{He}^4 + \text{C}^{12}$$

Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement. **(Main 2019, 10 Jan II)**

(a) Energy of 3.6 MeV will be released.

(b) Energy of 12.4 MeV will be supplied.

(c) 8.3 MeV energy will be released.

(d) Energy of 11.9 MeV has to be supplied.

6. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron, ${}^1_1\text{H}$, ${}^{15}_7\text{N}$ and ${}^{15}_8\text{O}$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ nuclei are same, $1\ \text{u} = 931.5\ \text{MeV}/c^2$ (c is the speed of light) and $e^2/(4\pi\varepsilon_0) = 1.44$ MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ is purely due to the electrostatic energy, the radius of either of the nuclei is $(1\,\text{fm} = 10^{-15}\ \text{m})$ **(2016 Adv.)**

(a) 2.85 fm (b) 3.03 fm (c) 3.42 fm (d) 3.80 fm

7. A fission reaction is given by ${}^{236}_{92}\text{U} \to {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y$, where x and y are two particles. Considering ${}^{236}_{92}\text{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2 MeV) and K_y(2 MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}\text{U}$, ${}^{140}_{54}\text{Xe}$ and ${}^{94}_{38}\text{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct options is/are **(2015 Adv.)**

(a) $x = n,\ y = n,\ K_{\text{Sr}} = 129\ \text{MeV},\ K_{\text{Xe}} = 86\ \text{MeV}$
(b) $x = p,\ y = e^-,\ K_{\text{Sr}} = 129\ \text{MeV},\ K_{\text{Xe}} = 86\ \text{MeV}$
(c) $x = p,\ y = n,\ K_{\text{Sr}} = 129\ \text{MeV},\ K_{\text{Xe}} = 86\ \text{MeV}$
(d) $x = n,\ y = n,\ K_{\text{Sr}} = 86\ \text{MeV},\ K_{\text{Xe}} = 129\ \text{MeV}$

8. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is **(2007, 3M)**

(a) $E({}^{236}_{92}\text{U}) > E({}^{137}_{53}\text{I}) + E({}^{97}_{39}\text{Y}) + 2E(n)$
(b) $E({}^{236}_{92}\text{U}) < E({}^{137}_{53}\text{I}) + E({}^{97}_{39}\text{Y}) + 2E(n)$
(c) $E({}^{236}_{92}\text{U}) < E({}^{140}_{56}\text{Ba}) + E({}^{94}_{36}\text{Kr}) + 2E(n)$
(d) $E({}^{235}_{92}\text{U}) < E({}^{140}_{56}\text{Ba}) + E({}^{94}_{36}\text{Kr}) + E(n)$

9. If a star can convert all the He nuclei completely into oxygen nuclei. The energy released per oxygen nuclei is :
(Mass of the helium nucleus is 4.0026 amu and mass of oxygen nucleus is 15.9994 amu) **(2005, 2M)**

(a) 7.6 MeV
(b) 56.12 MeV
(c) 10.24 MeV
(d) 23.4 MeV

10. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle **(2003, 2M)**

(a) 4.4 MeV (b) 5.4 MeV
(c) 5.6 MeV (d) 6.5 MeV

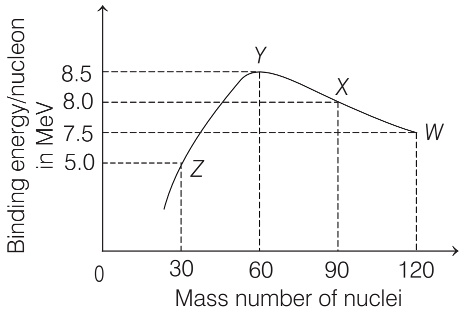
11. For uranium nucleus how does its mass vary with volume ? **(2003, 2M)**

(a) $m \propto V$ (b) $m \propto 1/V$
(c) $m \propto \sqrt{V}$ (d) $m \propto V^2$

12. Order of magnitude of density of uranium nucleus is $(m_p = 1.67 \times 10^{-27}\ \text{kg})$ **(1999, 2M)**

(a) $10^{20}\ \text{kg/m}^3$
(b) $10^{17}\ \text{kg/m}^3$
(c) $10^{14}\ \text{kg/m}^3$
(d) $10^{11}\ \text{kg/m}^3$

13. Binding energy per nucleon *versus* mass number curve for nuclei is shown in figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is **(1999, 2M)**



(a) $Y \to 2Z$ (b) $W \to X + Z$
(c) $W \to 2Y$ (d) $X \to Y + Z$

14. Fast neutrons can easily be slowed down by **(1994, 1M)**

(a) the use of lead shielding
(b) passing them through heavy water
(c) elastic collisions with heavy nuclei
(d) applying a strong electric field

15. A star initially has 10^{40} deuterons. It produces energy *via* the processes ${}_1\text{H}^2 + {}_1\text{H}^2 \to {}_1\text{H}^3 + p$ and ${}_1\text{H}^2 + {}_1\text{H}^3 \to {}_2\text{He}^4 + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of **(1993, 2M)**

(a) 10^6 s (b) 10^8 s
(c) 10^{12} s (d) 10^{16} s

The mass of the nuclei are as follows
$\text{M}(\text{H}^2) = 2.014$ amu; $\text{M}(n) = 1.008$ amu;
$\text{M}(p) = 1.007$ amu; $\text{M}(\text{He}^4) = 4.001$ amu.

16. During a nuclear fusion reaction **(1987, 2M)**

(a) a heavy nucleus breaks into two fragments by itself
(b) a light nucleus bombarded by thermal neutrons breaks up
(c) a heavy nucleus bombarded by thermal neutrons breaks up
(d) two light nuclei combine to give a heavier nucleus and possibly other products

17. The equation;
$4\,{}^1_1\text{H} \longrightarrow {}^4_2\text{He}^{2+} + 2e^- + 26\ \text{MeV}$ represents **(1983, 1M)**

(a) β-decay (b) γ-decay (c) fusion (d) fission

Passage Based Questions

Passage 1

The mass of a nucleus ${}^A_Z X$ is less that the sum of the masses of $(A - Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below

(2013 Adv.)

${}_1^1H$	1.007825u	${}_1^2H$	2.014102u
${}_3^6Li$	6.01513u	${}_3^7Li$	7.016004u
${}_{64}^{152}Gd$	151.919803u	${}_{82}^{206}Pb$	205.974455u
${}_1^3H$	3.016050u	${}_2^4He$	4.002603u
${}_{30}^{70}Zn$	69.925325u	${}_{34}^{82}Se$	81.916709u

18. The correct statement is

(a) The nucleus ${}_3^6Li$ can emit an alpha particle.

(b) The nucleus ${}_{84}^{210}Po$ can emit a proton.

(c) Deuteron and alpha particle can undergo complete fusion.

(d) The nuclei ${}_{30}^{70}Zn$ and ${}_{34}^{82}Se$ can undergo complete fusion.

19. The kinetic energy (in keV) of the alpha particle, when the nucleus ${}_{84}^{210}Po$ at rest undergoes alpha decay, is

(a) 5319 (b) 5422 (c) 5707 (d) 5818

Passage 2

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_1^2H$ known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D-D reaction is ${}_1^2H + {}_1^2H \rightarrow {}_2^3He + n$ + energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_1^4H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place.

Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s cm^{-3}.

It may be helpful to use the following : Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm.

20. In the core of nuclear fusion reactor, the gas becomes plasma because of **(2009)**

(a) strong nuclear force acting between the deuterons.
(b) Coulomb force acting between the deuterons.
(c) Coulomb force acting between deuteron-electron pairs.
(d) the high temperature maintained inside the reactor core.

21. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy $1.5\,kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range **(2009)**

(a) 1.0×10^9 K $< T < 2.0 \times 10^9$ K
(b) 2.0×10^9 K $< T < 3.0 \times 10^9$ K
(c) 3.0×10^9 K $< T < 4.0 \times 10^9$ K
(d) 4.0×10^9 K $< T < 5.0 \times 10^9$ K

22. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion ? **(2009)**

(a) Deuteron density $= 2.0 \times 10^{12}$ cm^{-3}, confinement time $= 5.0 \times 10^{-3}$ s

(b) Deuteron density $= 8.0 \times 10^{14}$ cm^{-3}, confinement time $= 9.0 \times 10^{-1}$ s

(c) Deuteron density $= 4.0 \times 10^{23}$ cm^{-3}, confinement time $= 1.0 \times 10^{-11}$ s

(d) Deuteron density $= 1.0 \times 10^{24}$ cm^{-3}, confinement time $= 4.0 \times 10^{-12}$ s

Match the Column

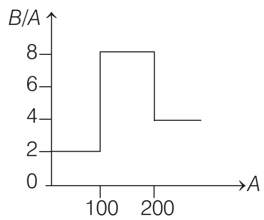
23. Some laws/processes are given in **Column I.** Match these with the physical phenomena given in **Column II.** **(2006,4M)**

	Column I		Column II
(A)	Nuclear fusion	(p)	Converts some matter into energy
(B)	Nuclear fission	(q)	Generally possible for nuclei with low atomic number
(C)	β-decay	(r)	Generally possible for nuclei with higher atomic number
(D)	Exothermic nuclear reaction	(s)	Generally possible for weak nuclear forces

Objective Questions II (One or more correct option)

24. Assume that the nuclear binding energy per nucleon (B/A) *versus* mass number (A) is as shown in the figure.

Use this plot to choose the correct choice(s) given below. **(2008, 4M)**



(a) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.

(b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.

(c) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments.

(d) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

25. Let m_p be the mass of proton, m_n the mass of neutron. M_1 the mass of ${}^{20}_{10}\text{Ne}$ nucleus and M_2 the mass of ${}^{40}_{20}\text{Ca}$ nucleus. Then **(1998, 2M)**

(a) $M_2 = 2M_1$
(b) $M_2 > 2M_1$
(c) $M_2 < 2M_1$
(d) $M_1 < 10\,(m_n + m_p)$

26. Which of the following statements(s) is (are) correct ? **(1994, 2M)**

(a) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons
(b) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons
(c) In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 amu)
(d) In nuclear fission, energy is released by fragmentation of a very heavy nucleus

27. The mass number of a nucleus is **(1986, 2M)**

(a) always less than its atomic number.
(b) always more than its atomic number.
(c) sometimes equal to its atomic number.
(d) sometimes more than and sometimes equal to its atomic number.

28. From the following equations pick out the possible nuclear fusion reactions **(1984, 3M)**

(a) ${}_6\text{C}^{13} + {}_1\text{H}^1 \longrightarrow {}_6\text{C}^{14} + 4.3\text{ MeV}$
(b) ${}_6\text{C}^{12} + {}_1\text{H}^1 \longrightarrow {}_7\text{N}^{13} + 2\text{ MeV}$
(c) ${}_7\text{N}^{14} + {}_1\text{H}^1 \longrightarrow {}_8\text{O}^{15} + 7.3\text{ MeV}$
(d) ${}_{92}\text{U}^{235} + {}_0n^1 \longrightarrow {}_{54}\text{Xe}^{140} + 36\text{Sr}^{94} + {}_0n^1 + {}_0n^1 + \gamma + 200\text{MeV}$

Fill in the Blanks

29. Consider the reaction : ${}^2_1\text{H} + {}^2_1\text{H} = {}^4_2\text{He} + Q$. Mass of the deuterium atom $= 2.0141$u. Mass of helium atom $= 4.0024$ u. This is a nuclear .... reaction in which the energy Q released is ...... MeV. **(1996, 2M)**

30. The binding energies per nucleon for deuteron (${}_1\text{H}^2$) and helium (${}_2\text{He}^4$) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus (${}_2\text{He}^4$) is .... **(1988, 2M)**

True/False

31. The order of magnitude of the density of nuclear matter is $10^4\ \text{kg/m}^3$. **(1989, 2M)**

Integer Answers Type Question

32. The isotopes ${}^{12}_5\text{B}$ having a mass 12.014 u undergoes β-decay to ${}^{12}_6\text{C}$. ${}^{12}_6\text{C}$ has an excited state of the nucleus (${}^{12}_6\text{C}^*$) at 4.041 MeV above its ground state. If ${}^{12}_5\text{B}$ decays to ${}^{12}_5\text{C}^*$, the maximum kinetic energy of the β-particle in units of MeV is ($1\text{u} = 931.5\ \text{MeV}/c^2$, where c is the speed of light in vacuum) **(2016 Adv.)**

33. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is **(2015 Adv.)**

Analytical & Descriptive Questions

34. In a nuclear reactor ${}^{235}\text{U}$ undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 yr, find the total mass of uranium required. **(2001, 5M)**

35. The element curium ${}^{248}_{96}\text{Cm}$ has a mean life of 10^{13}s. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%, each fission releases 200 MeV of energy. The masses involved in decay are as follows : **(1997, 5M)**

${}^{248}_{96}\text{Cm} = 248.072220\ \text{u}$,
${}^{244}_{94}\text{Pu} = 244.064100\ \text{u}$ and ${}^4_2\text{He} = 4.002603\ \text{u}$. Calculate the power output from a sample of 10^{20} Cm atoms.

$(1\ \text{u} = 931\ \text{MeV}/c^2)$

36. A nucleus X, initially at rest, undergoes alpha-decay according to the equation. **(1991, 2+4+2M)**

$${}^{A}_{92}X \to {}^{228}_{Z}Y + \alpha$$

(a) Find the values of A and Z in the above process.
(b) The alpha particle produced in the above process is found to move in a circular track of radius 0.11m in a uniform magnetic field of 3 T. Find the energy (in MeV) released during the process and the binding energy of the parent nucleus X. Given that m (Y) = 228.03u ; $m({}^1_0 n) = 1.009$u

$m({}^4_2\text{He}) = 4.003\ \text{u}$; $m({}^1_1\text{H}) = 1.008\ \text{u}$.

37. It is proposed to use the nuclear fusion reaction;

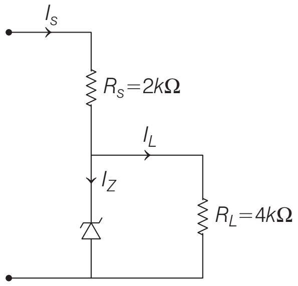
$${}^2_1\text{H} + {}^2_1\text{H} \to {}^4_2\text{He}$$

in a nuclear reactor 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day? (The masses of ${}^2_1\text{H}$ and ${}^4_2\text{He}$ are 2.0141 atomic mass units and 4.0026 atomic mass units respectively. **(1990, 8M)**

Topic 6 Semiconductor Devices, Diodes and Triodes

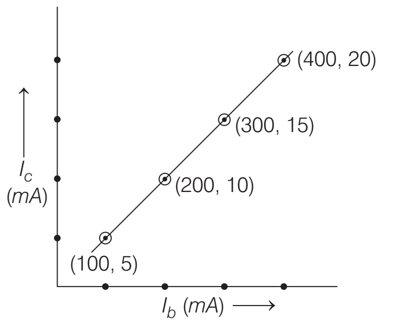
Objective Questions I (Only one correct option)

1. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current? **(Main 2019, 12 April II)**



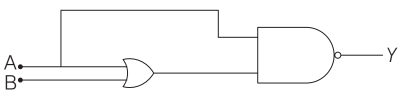
(a) 2.5 mA (b) 1.5 mA (c) 7.5 mA (d) 3.5 mA

2. The transfer characteristic curve of a transistor, having input and output resistance 100Ω and 100 kΩ respectively, is shown in the figure. The voltage and power gain, are respectively, **(Main 2019, 12 April I)**



(a) $2.5 \times 10^4, 2.5 \times 10^6$ (b) $5 \times 10^4, 5 \times 10^6$
(c) $5 \times 10^4, 5 \times 10^5$ (d) $5 \times 10^4, 2.5 \times 10^6$

3. The truth table for the circuit given in the figure is **(Main 2019, 12 April I)**



(a)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(b)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

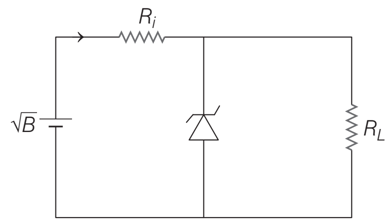
(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

4. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is $R_L = 4$ kΩ. The series resistance of the circuit is $R_i = 1$ kΩ. If the battery voltage V_B varies from 8V to 16V, what are the minimum and maximum values of the current through Zener diode? **(Main 2019, 10 April II)**

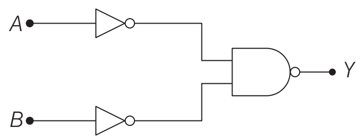


(a) 1.5 mA, 8.5 mA (b) 1 mA, 8.5 mA
(c) 0.5 mA, 8.5 mA (d) 0.5 mA, 6 mA

5. An *n-p-n* transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is **(Main 2019, 10 April I)**

(a) 10^2 (b) 6×10^2 (c) 10^4 (d) 60

6. The logic gate equivalent to the given logic circuit is **(Main 2019, 9 April II)**

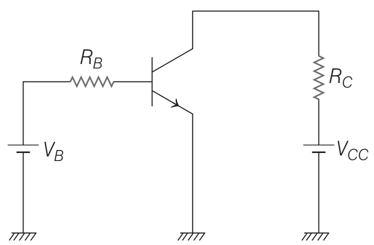


(a) NOR (b) NAND (c) OR (d) AND

7. An *n-p-n* transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are **(Main 2019, 9 April I)**

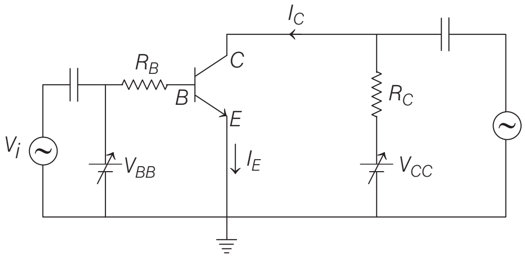
(a) 0.67 kΩ, 200 (b) 0.33 kΩ, 1.5
(c) 0.67 kΩ, 300 (d) 0.33 kΩ, 300

8. A common emitter amplifier circuit, built using an *n-p-n* transistor, is shown in the figure. Its DC current gain is 250, $R_C = 1\text{k}\Omega$ and $V_{CC} = 10\text{V}$. What is the minimum base current for V_{CE} to reach saturation? **(Main 2019, 8 April II)**



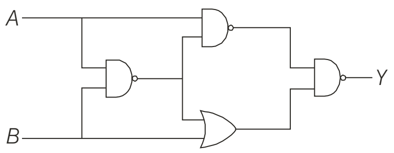
(a) 40 μA (b) 10 μA (c) 100 μA (d) 7 μA

9. In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5$ V, $\beta_{DC} = 200$, $R_B = 100$ k Ω, $R_C = 1$ k Ω and $V_{BE} = 1.0$ V. The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively **(Main 2019, 12 Jan II)**



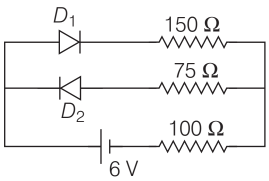
(a) 25 μA and 2.8 V (b) 25 μA and 3.5 V
(c) 20 μA and 3.5 V (d) 20 μA and 2.8 V

10. The output of the given logic circuit is **(Main 2019, 12 Jan I)**



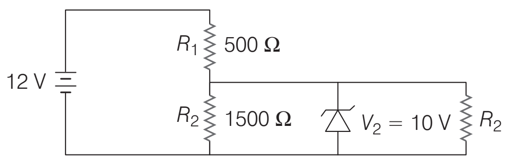
(a) $A\overline{B}$ (b) $\overline{A}B$ (c) $AB + \overline{AB}$ (d) $A\overline{B} + \overline{A}B$

11. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in ampere) is **(Main 2019, 11 Jan II)**



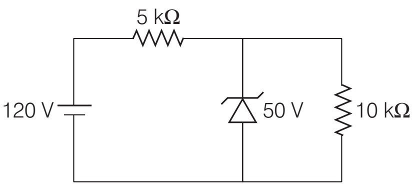
(a) 0.027 (b) 0.020 (c) 0.030 (d) 0.036

12. In the given circuit, the current through zener diode is closed to **(Main 2019, 11 Jan I)**



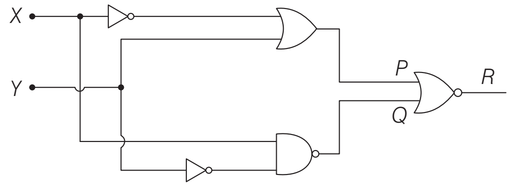
(a) 6.0 mA (b) 6.7 mA (c) 0 (d) 4.0 mA

13. For the circuit shown below, the current through the Zener diode is **(Main 2019, 10 Jan II)**



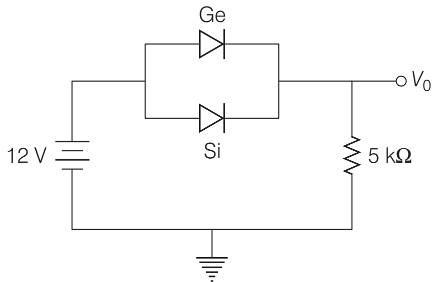
(a) 14 mA (b) zero (c) 5 mA (d) 9 mA

14. To get output '1' at R, for the given logic gate circuit, the input values must be **(Main 2019, 10 Jan I)**



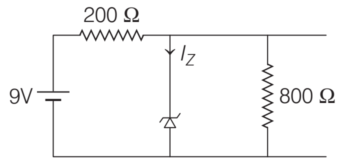
(a) $X = 0, Y = 0$ (b) $X = 1, Y = 0$
(c) $X = 1, Y = 1$ (d) $X = 0, Y = 1$

15. At 0.3V and 0.7 V, the diodes Ge and Si become conductor respectively. In given figure, if ends of diode Ge overturned, the change in potential V_0 will be **(Main 2019, 9 Jan II)**



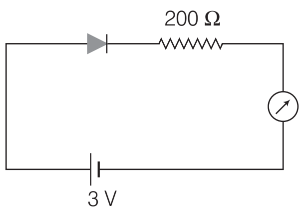
(a) 0.2 V (b) 0.6V (c) 0.4 V (d) 0.8V

16 The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



The current I_z through the Zener is **(2019 Main 8 April I)**
(a) 10 mA (b) 17 mA (c) 15 mA (d) 7 mA

17. The reading of the ammeter for a silicon diode in the given circuit is **(2018 Main)**

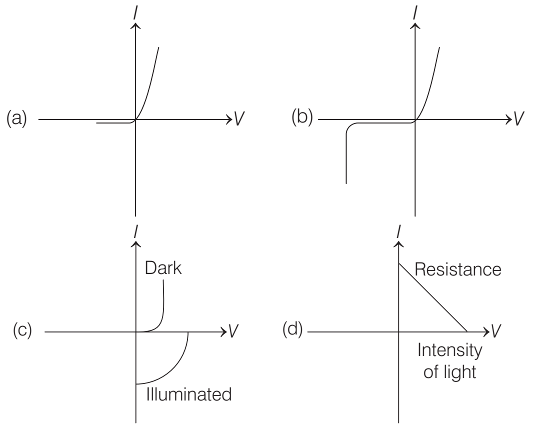


(a) 13.5 mA (b) 0 (c) 15 mA (d) 11.5 mA

18. In a common emitter amplifier circuit using an *n-p-n* transistor, the phase difference between the input and the output voltages will be **(2017 Main)**
(a) 90° (b) 135° (c) 180° (d) 45°

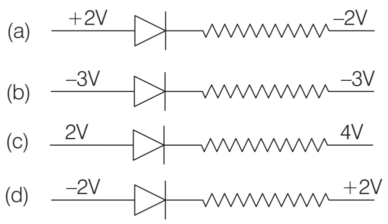
19. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by **(2016 Main)**
(a) linear increase for Cu, linear increase for Si
(b) linear increase for Cu, exponential increase for Si
(c) linear increase for Cu, exponential decrease for Si
(d) linear decrease for Cu, linear decrease for Si

20. Identify the semiconductor devices whose characteristics are as given below, in the order (a),(b),(c),(d). **(2016 Main)**

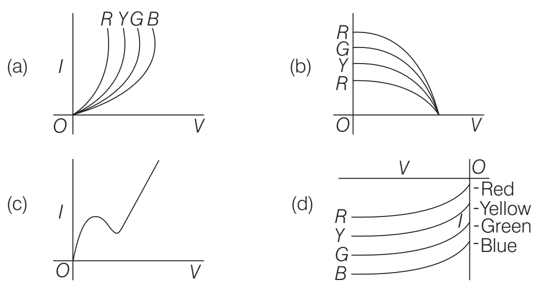


(a) Simple diode, Zener diode, Solar cell, Light dependent resistance
(b) Zener diode, Simple diode, Light dependent resistance, Solar cell
(c) Solar cell, Light dependent resistance, Zener diode, Simple diode
(d) Zener diode, Solar cell, Simple diode, Light dependent resistance

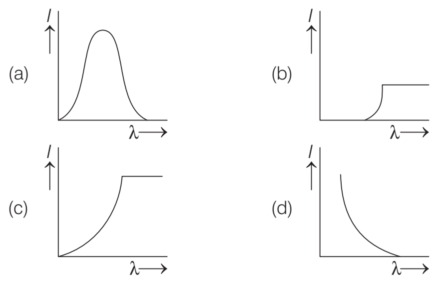
21. The forward biased diode connection is **(2014 Main)**



22. The *I-V* characteristic of an LED is **(2013 Main)**



23. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of photocell varies as follows **(2013 Main)**



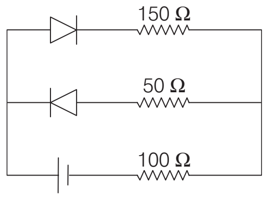
24. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pF in parallel with a load resistance 100 kΩ. Find the maximum modulated frequency which could be detected by it. **(2013 Main)**

(a) 10.62 MHz (b) 10.62 kHz
(c) 5.31 MHz (d) 5.31 kHz

25. In a p-n junction diode not connected to any circuit
(a) the potential is the same everywhere. **(1998, 2M)**
(b) the p-type side is at a higher potential than the n-type side.
(c) there is an electric field at the junction directed from the n-side to the p-type side.
(d) there is an electric field at the junction directed from the p-type side to the n-type side.

26. Which of the following statements is not true ? **(1997, 1M)**
(a) The resistance of intrinsic semiconductors decreases with increase of temperature.
(b) Doping pure Si with trivalent impurities give p-type semiconductors.
(c) The majority carriers in n-type semiconductors are holes.
(d) A p-n junction can act as a semiconductor diode.

27. The circuit shown in the figure contains two diodes each with a forward resistance of 50 Ω and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 Ω resistance (in ampere) is



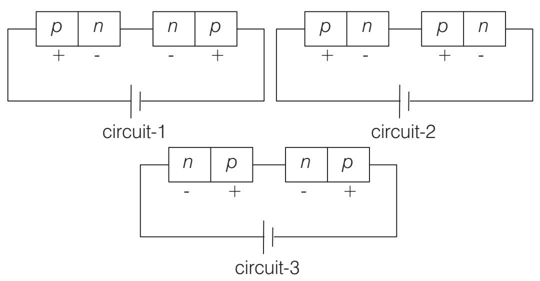
(1997, 1M)

(a) zero (b) 0.02 (c) 0.03 (d) 0.036

28. The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon p-n junctions are **(1997C, 1M)**
(a) drift in forward bias, diffusion in reverse bias.
(b) diffusion in forward bias, drift in reverse bias.
(c) diffusion in both forward and reverse bias.
(d) drift in both forward and reverse bias.

29. The electrical conductivity of a semiconductor increases when electro magnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap (in eV) for the semiconductor is **(1997C, 1M)**
(a) 0.9 (b) 0.7 (c) 0.5 (d) 1.1

30. Two identical p-n junctions may be connected in series with a battery in three ways . The potential drops across the two p-n junctions are equal in **(1989, 2M)**



(a) circuit-1 and circuit-2
(b) circuit-2 and circuit-3
(c) circuit-3 and circuit-1
(d) circuit-1 only

31. For a given plate voltage, the plate current in a triode valve is maximum when the potential of **(1985, 2M)**
(a) the grid is positive and plate is negative
(b) the grid is zero and plate is positive
(c) the grid is negative and plate is positive
(d) the grid is positive and plate is positive

32. Select the correct statement from the following **(1984, 2M)**
(a) a diode can be used as a rectifier
(b) a triode cannot be used as a rectifier
(c) the current in a diode is always proportional to the applied voltage
(d) the linear portion of the I-V characteristic of a triode is used for amplification without distortion

Objective Questions II (One or more correct option)

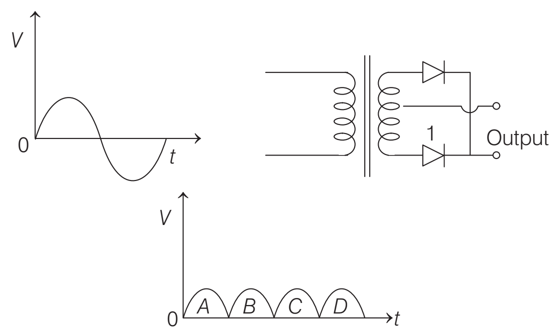
33. For a common-emitter configuration, if α and β have their usual meanings, the **incorrect** relationship between α and β is **(2016 Main)**

(a) $\frac{1}{\alpha}=\frac{1}{\beta}+1$ (b) $\alpha=\frac{\beta}{1-\beta}$ (c) $\alpha=\frac{\beta}{1+\beta}$ (d) $\alpha=\frac{\beta^2}{1+\beta^2}$

34. A transistor is used in common emitter mode as an amplifier, then **(1998, 2M)**
(a) the base emitter junction is forward biased.
(b) the base emitter junction is reverse biased.
(c) the input signal is connected in series with the voltage applied to bias the base emitter junction.
(d) the input signal is connected in series with the voltage applied to bias the base collector junction.

35. Holes are charge carriers in **(1997)**
(a) intrinsic semiconductors
(b) ionic solids
(c) p-type semiconductors
(d) metals

36. A full wave rectifier circuit along with the output is shown in figure. The contribution (s) from the diode 1 is (are) **(1996, 2M)**



(a) C (b) A, C
(c) B, D (d) A, B, C, D

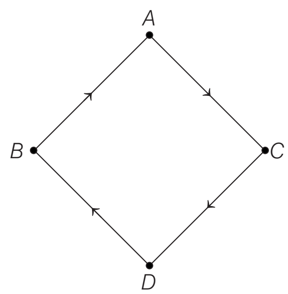
37. In an n-p-n transistor circuit, the collector current is 10 mA. If 90% of the electrons emitted reach the collector **(1992, 2M)**
(a) the emitter current will be 9 mA
(b) the emitter current will be 11mA
(c) the base current will be 1mA
(d) the base current will be -1 mA

38. The impurity atoms with which pure silicon should be doped to make a p-type semiconductor are those of **(1988, 2M)**
(a) phosphorus (b) boron
(c) antimony (d) aluminium

Fill in the Blanks

39. In a .... biased p-n junction, the net flow of holes is from the n region to the p region. **(1993, 1M)**

40. For the given circuit shown in figure to act as full wave rectifier, the AC input should be connected across .... and the DC out put would appear across .... **(1991, 1M)**



41. ....biasing of p-n junction offers high resistance to current flow across the junction. The biasing is obtained by connecting the p-side to the .... terminal of the battery. **(1990, 2M)**

42. In the forward bias arrangement of a p-n junction rectifier, the p end is connected to the .... terminal of the battery and the direction of the current is from .... to ...... in the rectifier. **(1988, 2M)**

True/False

43. For a diode the variation of its anode current I_a with the anode voltage V_a at two different cathode temperatures T_1 and T_2 is shown in the figure. The temperature T_2 is greater than T_1. **(1986, 3M)**

Analytical & Descriptive Questions

44. A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of -1 V the anode current I (in mA) is given in terms of plate voltage V by the algebraic relation :

$$I = 0.125\,\text{V} - 7.5 \qquad \textbf{(1987, 7M)}$$

For grid voltage of -3 V, the current at anode voltage of 300 V is 5 mA. Determine the plate resistance (r_p) transconductance (g_m) and the amplification factor (μ) for the triode.

Topic 7 Miscellaneous Problems

Objective Questions I (Only one correct option)

1. A plane electromagnetic wave having a frequency $\nu = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave? **(Main 2019, 12 April II)**

(a) $\mathbf{B} = 2 \times 10^7 \sin(0.5 \times 10^3 z + 1.5 \times 10^{11} t)\,\hat{\mathbf{i}}$

(b) $\mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\,\hat{\mathbf{i}}$

(c) $\mathbf{B} = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\,\hat{\mathbf{k}}$

(d) $\mathbf{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 x + 0.5 \times 10^{11} t)\,\hat{\mathbf{j}}$

2. In an amplitude modulator circuit, the carrier wave is given by $C(t) = 4\sin(20000\,\pi t)$ while modulating signal is given by, $m(t) = 2\sin(2000\,\pi t)$. The values of modulation index and lower side band frequency are **(Main 2019, 12 April II)**

(a) 0.5 and 10 kHz (b) 0.4 and 10 kHz
(c) 0.3 and 9 kHz (d) 0.5 and 9 kHz

3. An electromagnetic wave is represented by the electric field $\mathbf{E} = E_0\,\hat{\mathbf{n}} \sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z- directions to be $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, the direction of propagation $\hat{\mathbf{s}}$, is **(Main 2019, 12 April I)**

(a) $\hat{\mathbf{s}} = \dfrac{3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}}{5}$ (b) $\hat{\mathbf{s}} = \dfrac{-4\hat{\mathbf{k}} + 3\hat{\mathbf{j}}}{5}$

(c) $\hat{\mathbf{s}} = \left(\dfrac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5}\right)$ (d) $\hat{\mathbf{s}} = \dfrac{4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{5}$

4. Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be **(Main 2019, 10 April II)**

(a) 3.5×10^{-6} N-s (b) 6.3×10^{-4} N-s
(c) 1.4×10^{-6} N-s (d) 5.0×10^{-3} N-s

5. Given below in the left column are different modes of communication using the kinds of waves given in the right column.

A.	Optical fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared light
C.	Sonar	R.	Microwaves
D.	Mobile phones	S.	Radio waves

From the options given below, find the most appropriate match between entries in the left and the right column. **(Main 2019, 10 April I)**

(a) A-Q, B-S, C-R, D-P (b) A-S, B-Q, C-R, D-P
(c) A-Q, B-S, C-P, D-R (d) A-R, B-P, C-S, D-Q

6. The electric field of a plane electromagnetic wave is given by

$$\mathbf{E} = E_0 \hat{\mathbf{i}} \cos(kz) \cos(\omega t)$$

The corresponding magnetic field $\mathbf{B}$ is then given by **(Main 2019, 10 April I)**

(a) $\mathbf{B} = \dfrac{E_0}{c}\,\hat{\mathbf{j}} \sin(kz) \sin(\omega t)$

(b) $\mathbf{B} = \dfrac{E_0}{c}\,\hat{\mathbf{j}} \sin(kz) \cos(\omega t)$

(c) $\mathbf{B} = \dfrac{E_0}{c}\,\hat{\mathbf{k}} \sin(kz) \cos(\omega t)$

(d) $\mathbf{B} = \dfrac{E_0}{c}\,\hat{\mathbf{j}} \cos(kz) \sin(\omega t)$

7. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are **(Main 2019, 10 April I)**

(a) 0.25; 1×10^8 Hz (b) 4; 1×10^8 Hz
(c) 0.25; 2×10^8 Hz (d) 4; 2×10^8 Hz

8. The physical sizes of the transmitter and receiver antenna in a communication system are **(Main 2019, 9 April II)**

(a) proportional to carrier frequency
(b) inversely proportional to modulation frequency
(c) independent of both carrier and modulation frequency
(d) inversely proportional to carrier frequency

9. 50 Q/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to ($c = 3 \times 10^8$ m/s) **(Main 2019, 9 April II)**

(a) 20×10^{-8} N (b) 35×10^{-8} N
(c) 15×10^{-8} N (d) 10×10^{-8} N

10. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is **(Main 2019, 9 April I)**

(a) $(v_0 \sin\omega_0 t + A\cos\omega t$
(b) $(v_0 + A)\cos\omega t \sin\omega_0 t$
(c) $v_0 \sin[\omega_0(1 + 0.01 A \sin\omega t)t]$
(d) $v_0 \sin\omega_0 t + \dfrac{A}{2}\sin(\omega_0 - \omega)t + \dfrac{A}{2}\sin(\omega_0 + \omega)t$

11. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be

(Radius of the earth = 6.4×10^6 m) **(Main 2019, 8 April II)**

(a) 20 m (b) 32 m
(c) 40 m (d) 51 m

12. The magnetic field of an electromagnetic wave is given by

$\mathbf{B} = 1.6 \times 10^{-6} \cos(2 \times 10^7 z + 6 \times 10^{15} t)(2\hat{\mathbf{i}} + \hat{\mathbf{j}})\,\text{Wbm}^{-2}$

The associated electric field will be (Main 2019, 8 April II)

(a) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t)(-2\hat{\mathbf{j}} + \hat{\mathbf{i}})\,\mathrm{Vm}^{-1}$

(b) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t)(2\hat{\mathbf{j}} + \hat{\mathbf{i}})\,\mathrm{Vm}^{-1}$

(c) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t)(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\,\mathrm{Vm}^{-1}$

(d) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t)(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\mathrm{Vm}^{-1}$

13. A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same directions as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C respectively (Main 2019, 8 April II)

(a) $2\lambda_A, \lambda_A$ (b) $\frac{\lambda_A}{2}, \lambda_A$

(c) $\lambda_A, 2\lambda_A$ (d) $\lambda_A, \frac{\lambda_A}{2}$

14. The wavelength of the carrier waves in a modern optical fibre communication network is close to (Main 2019, 8 April I)

(a) 2400 nm (b) 1500 nm (c) 600 nm (d) 900 nm

15. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\mathrm{Vm}^{-1}$ along y-direction. Its corresponding magnetic field component, B would be (Main 2019, 8 April I)

(a) 2×10^{-8} T along z - direction

(b) 6×10^{-8} T along x - direction

(c) 6×10^{-8} T along z - direction

(d) 2×10^{-8} T along y - direction

16. To double the covering range of a TV transmission tower, its height should be multiplied by (Main 2019, 12 Jan II)

(a) $\sqrt{2}$ (b) 4 (c) 2 (d) $\frac{1}{\sqrt{2}}$

17. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to (Main 2019, 12 Jan II)

(a) 250 nm (b) 2020 nm

(c) 1700 nm (d) 220 nm

18. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index? (Main 2019, 12 Jan I)

(a) 0.4 (b) 0.5

(c) 0.6 (d) 0.3

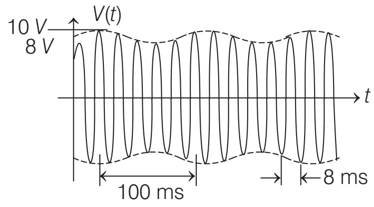
19. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by

[Take, permittivity of space, $\varepsilon_0 = 9 \times 10^{-12}$ SI units and speed of light, $c = 3 \times 10^8$ m/s] (Main 2019, 11 Jan II)

(a) 1 kV/m (b) 0.7 kV/m

(c) 2 kV/m (d) 1.4 kV/m

20. An amplitude modulated signal is plotted below (Main 2019, 11 Jan II)



Which one of the following best describes the above signal?

(a) $[1 + 9\sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)$V

(b) $[9 + \sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)$V

(c) $[9 + \sin(4\pi \times 10^4 t)]\sin(5\pi \times 10^5 t)$V

(d) $[9 + \sin(2.5\pi \times 10^5 t)]\sin(2\pi \times 10^4 t)$V

21. An amplitude modulates signal is given by

$v(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]\ \sin(5.5 \times 10^5 t)$.

Here, t is in seconds. The sideband frequencies (in kHz) are $\left(\text{Take, } \pi = \frac{22}{7}\right)$ (Main 2019, 11 Jan I)

(a) 892.5 and 857.5 (b) 89.25 and 85.75

(c) 178.5 and 171.5 (d) 1785 and 1715

22. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by (Main 2019, 11 Jan I)

(a) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$ (b) $(\sqrt{n}, \sqrt{n})$

(c) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

23. The electric field of a plane polarised electromagnetic wave in free space at time $t = 0$ is given by an expression.

$$\mathbf{E}(x, y) = 10\hat{\mathbf{j}}\cos[(6x + 8z)]$$

The magnetic field $\mathbf{B}(x, z, t)$ is given by (where, c is the velocity of light) (Main 2019, 10 Jan II)

(a) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z + 10ct)]$

(b) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$

(c) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x - 8z + 10ct)]$

(d) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$

24. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license, what broadcast frequency will you allot ? (Main 2019, 10 Jan II)

(a) 2000 kHz (b) 2250 kHz

(c) 2900 kHz (d) 2750 kHz

25. If the magnetic field of a plane electromagnetic wave is given by

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right],$$

then the maximum electric field associated with it is (Take, the speed of light = 3×10^8 m/s) **(Main 2019, 10 Jan I)**

(a) 6×10^4 N/C (b) 4×10^4 N/C
(c) 3×10^4 N/C (d) 4.5×10^4 N/C

26. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Take, radius of earth $= 6.4 \times 10^6$ m). **(Main 2019, 10 Jan I)**

(a) 65 km (b) 80 km (c) 40 km (d) 48 km

27. In an electron microscope, the resolution that can be achieved is of the order of wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to **(Main 2019, 10 Jan I)**

(a) 500 keV (b) 1 keV
(c) 100 keV (d) 25 keV

28. In free space, the energy of electromagnetic wave in electric field is U_E and in magnetic field is U_B. Then **(Main 2019, 9 Jan II)**

(a) $U_E = U_B$ (b) $U_E > U_B$
(c) $U_E < U_B$ (d) $U_E = \frac{U_B}{2}$

29. In communication system, only one percent frequency of signal of wavelength 800 nm can be used as bandwidth. How many channal of 6MHz bandwidth can be broadcast this?
$(c = 3 \times 10^8 \text{ m/s}, h = 6.6 \times 10^{-34} \text{ J-s})$ **(Main 2019, 9 Jan II)**

(a) 3.75×10^6 (b) 3.86×10^6 (c) 6.25×10^5 (d) 4.87×10^5

30. An EM wave from air enters a medium. The electric fields are

$E_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and

$E_2 = E_{02}\hat{x}\cos[k(2z - ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic.

If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(a) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ (b) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$
(c) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (d) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

31. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilised for transmission. How many telephonic channels can be transmitted simultaneously, if each channel requires a bandwidth of 5 kHz? **(2018 Main)**

(a) 2×10^6 (b) 2×10^3 (c) 2×10^4 (d) 2×10^5

32. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2MHz. The frequencies of the resultant signal is/are **(2015 Main)**

(a) 2 MHz only
(b) 2005 kHz 2000 kHz and 1995 kHz
(c) 2005 kHz and 1995 kHz
(d) 2000 kHz and 1995 kHz

33. A red LED emits light at 0.1 W uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is **(2015 Main)**

(a) 2.45 V/m (b) 1.73 V/m
(c) 5.48 V/m (d) 7.75 V/m

34. During the propagation of electromagnetic waves in a medium, **(2014 Main)**

(a) electric energy density is double of the magnetic energy density.
(b) electric energy density is half of the magnetic energy density.
(c) electric energy density is equal to the magnetic energy density.
(d) Both electric and magnetic energy densities are zero.

35. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is **(2013 Main)**

(a) 3V/m (b) 6 V/m
(c) 9 V/m (d) 12 V/m

36. A beam of electron is used in an YDSE experiment. The slit width is d. When the velocity of electron is increased, then **(2005, 2M)**

(a) no interference is observed
(b) fringe width increases
(c) fringe width decreases
(d) fringe width remains same

37. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is **(1999, 2M)**

(a) nitrogen (b) carbon (c) boron (d) oxygen

38. Four physical quantities are listed in **Column I**. Their values are listed in **Column II** in a random order **(1987, 2M)**

	Column I	Column II
A	Thermal energy of air molecules at room temperature.	(i) 0.02 eV
B	Binding energy of heavy nuclei per nucleon.	(ii) 2 eV
C	X-ray photon energy.	(iii) 10 keV
D	Photon energy of visible light.	(iv) 7 MeV

The correct matching of **Columns I** and **II** is given by :

	A	B	C	D		A	B	C	D
(a)	i	iv	iii	ii	(b)	i	iii	ii	iv
(c)	ii	i	iii	iv	(d)	ii	iv	i	iii

39. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be **(1983, 1M)**

(a) 60 (b) 32 (c) 4 (d) 64

40. The plate resistance of a triode is $3 \times 10^3 \Omega$ and its mutual conductance is 1.5×10^{-3} A/V. The amplification factor of the triode is **(1981, 2M)**

(a) 5×10^{-5} (b) 4.5 (c) 45 (d) 2×10^5

Numerical Value

41. In a photoelectric experiment, a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency, so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$N due to the impact of the electrons. The value of n is .... (Take mass of the electron, $m_e = 9 \times 10^{-31}$kg and eV $= 1.6 \times 10^{-19}$ J) **(2018 Adv.)**

Passage Based Questions

Passage

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, *i.e.*, $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum.

Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy. **(2012)**

42. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy K, of the electron?

(a) $0 \le K \le 0.8 \times 10^6$ eV (b) 3.0 eV$\le K \le 0.8 \times 10^6$ eV

(c) 3.0 eV $\le K < 0.8 \times 10^6$ eV (d) $0 \le K < 0.8 \times 10^6$ eV

43. What is the maximum energy of the anti-neutrino?

(a) Zero

(b) Much less than 0.8×10^6 eV

(c) Nearly 0.8×10^6 eV

(d) Much larger than 0.8×10^6 eV

Match the Columns

44. Match the nuclear processes given in Column I with the appropriate option(s) in Column II. **(2015 Adv.)**

Column I		Column II	
A.	Nuclear fusion	P.	absorption of thermal neutrons by $^{235}_{92}$U
B.	Fission in a nuclear reactor	Q.	$^{60}_{27}$Co nucleus
C.	β-decay	R.	Energy production in stars via hydrogen conversion to helium
D.	γ-ray emission	S.	Heavy water
		T.	Neutrino emission

45. Match Column I (fundamental experiment) with Column II (its conclusion) and select the correct option from the choices given below the list. **(2015 Main)**

Column I		Column II	
A	Franck-Hertz experiment	1	Particle nature of light
B	Photo-electric experiment	2	Discrete energy levels of atom
C	Davisson-Germer experiment	3	Wave nature of electron
		4	Structure of atom

	A	B	C
(a)	1	4	3
(b)	2	1	3
(c)	2	4	3
(d)	4	3	2

46. Match List I (Electromagnetic wave type) with List II (Its association/application) and select the correct option from the choices given below the lists. **(2014 Main)**

List I		List II	
A.	Infrared waves	1.	To treat muscular strain
B.	Radio waves	2.	For broadcasting
C.	X-rays	3.	To detect fracture of bones
D.	Ultraviolet	4.	Absorbed by the ozone layer of the atmosphere

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	1	2	4	3
(c)	3	2	1	4
(d)	1	2	3	4

47. Match List-I of the nuclear process with List-II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists. **(2013 Adv.)**

	List I		List II
P.	Alpha decay	1.	${}^{15}_{8}O \longrightarrow {}^{17}_{7}N + \ldots$
Q.	β^+ decay	2.	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + \ldots$
R.	Fission	3.	${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + \ldots$
S.	Proton emission	4.	${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + \ldots$

Codes

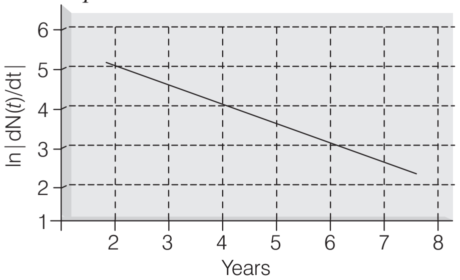
	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

48. Some laws/processes are given in Column-I. Match these with the physical phenomena given in Column-II. **(2007, 6M)**

	Column I		Column II
(A)	Transition between two atomic energy levels	(p)	Characteristic X-rays
(B)	Electron emission from a material	(q)	Photoelectric effect
(C)	Moseley's law	(r)	Hydrogen spectrum

Integer Answer Type Question

49. To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\dfrac{dN(t)}{dt}\right|$ *versus* t. Here $\dfrac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 yr, the value of p is **(2010)**



Fill in the Blank

50. Atoms having the same ...... but different ...... are called isotopes. **(1986, 2M)**

Analytical & Descriptive Questions

51. A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength, $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α-particle is 4.002 amu. Determine the total kinetic energy in the final state. Hence obtain the mass of the parent nucleus in amu. (1 amu = 931.470 MeV/c^2) **(2001, 5M)**

52. When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area 1.0×10^{-4} m^2 and work function 5.6 eV. 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take 1 eV $= 1.6 \times 10^{-19}$ J. **(2000, 4M)**

53. In the following, **Column** I lists some physical quantities and the **Column** II gives approximate energy values associated with some of them. Choose the appropriate value of energy from **Column** II for each of the physical quantities in **Column** I and write the corresponding letters A, B, C etc., against the number (i), (ii) and (iii) etc., of the physical quantity. In the answer books in your answer the sequence of column I should be maintained : **(1997, 4M)**

	Column I		Column II
(i)	Energy of thermal neutrons	(A)	0.025 eV
(ii)	Energy of X-ray	(B)	0.5eV
(iii)	Binding energy per nucleon	(C)	3eV
(iv)	Photoelectric threshold of a metal	(D)	20 eV
		(E)	8 MeV
		(F)	10 keV

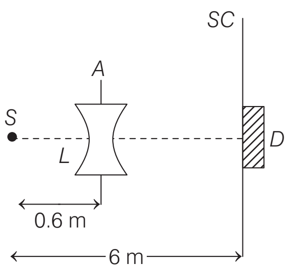
54. A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of 90° with respect to its original direction. **(1993, 9+1M)**

(a) Find the allowed values of the energy of the neutron and that of the atom after the collision.

(b) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation.

[Given : Mass of He atom = 4 × (mass of neutrons)
Ionization energy of H atom = 13.6 eV]

55. A monochromatic point source S radiating wavelength 6000 Å, with power 2 W, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in figure. A photoemissive detector D of surface area 0.5 cm^2 is placed at the centre of the



screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9. **(1991, 2+4+2M)**

(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

(b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

(c) If the work function of the photoemissive surface is 1 eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

56. Electrons in hydrogen-like atom ($Z = 3$) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 V. Calculate the work function of the metal, and the stopping potential for the photoelectrons ejected by the longer wavelength (Rydberg's constant $= 1.094 \times 10^7 \text{ m}^{-1}$) **(1990, 7M)**

57. A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make the transition to a higher energy level by absorbing monochromatic light of photon energy 2.7eV. Subsequently, the atoms emit radiation of only six different energy photons. Some of the emitted photons have an energy of 2.7 eV, some have more energy and some less than 2.7 eV. **(1989, 8M)**

(a) Find the principal quantum number of the initially excited level B.

(b) Find the ionization energy for the gas atoms.

(c) Find the maximum and the minimum energies of the emitted photons.

58. How many electrons, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24?
Find, **(1982, 2M)**

(a) Number of electrons.
(b) Number of protons.
(c) Number of neutrons.

Answers

Topic 1

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)
5. (d)	**6.** (d)	**7.** (c)	**8.** (c)
9. (c)	**10.** (a)		
11. (d)	**12.** (c)	**13.** (d)	**14.** (a)
15. (b)	**16.** (c)	**17.** (d)	**18.** (a)
19. (b)	**20.** (d)	**21.** (a)	**22.** (c)
23. (b)	**24.** (d)	**25.** (d)	**26.** (d)
27. (b)	**28.** (c)	**29.** (a)	**30.** (b)
31. (d)	**32.** (c)	**33.** (c)	**34.** (a)
35. (a, b, d)	**36.** (a, c)	**37.** (a, d)	**38.** (a, d)
39. 3			
40. 4.17	**41.** –1	**42.** (5)	**43.** 6
44. (2)	**45.** 0.55 eV	**46.** (a) $z = 3$ (b) 4052.3 nm	

47. $n = 2$, $Z = 4$, -217.6 eV, 10.58 eV

48. (a) 3.4 eV (b) 6.63 Å **49.** 6, 3

50. (a) $r_n = \dfrac{n^2h^2\varepsilon_0}{624\ \pi m_e e^2}$ (b) $n \approx 25$ (c) 0.546 Å

51. (a) 113.74 Å (b) 3 **52.** (a) 300 Å (b) 0.2645 Å

53. 6.6×10^{-34} J-s

54. Six, 1.875 μm

55. (a) 5 (b) 16.53 eV(c) 36.4 Å (d) 340 eV, – 680 eV, – 340 eV, $1.05 \times 10^{-34} \dfrac{\text{kg-m}^2}{\text{s}}$ (e) 1.06×10^{-11}m

Topic 2

1. (b)	**2.** (b)	**3.** (c)	**4.** (a)
5. (c)	**6.** (d)	**7.** (a)	**8.** (a)
9. (c)	**10.** (c)	**11.** (b)	**12.** (a)
13. (b)	**14.** (a)	**15.** (b)	**16.** (a)
17. (a)	**18.** (c)	**19.** (b)	**20.** (a, c)
21. (a, c)	**22.** (a, b, c)	**23.** (b, d)	**24.** (a, b, c)
25. (c, d)	**26.** 1	**27.** 7	**28.** Frequency
29. Question is incomplete		**30.** F	

32. (a) 5×10^7 (b) 2×10^3 N/C (c) 23 eV

33. During combination 3.4 eV. After combination 3.84 eV, 2.64 eV

34. (a) 10^5/s (b) 285.1 (d) 111 s

35. (a) 2.55 eV (b) $4 \to 2$ (c) $-\dfrac{h}{\pi}$ (d) 0.814 m/s

36. 1.1×10^{12}

Topic 3

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)
5. (a)	**6.** (d)	**7.** (c)	**8.** (a)
9. (b)	**10.** (d)	**11.** (c)	**12.** (a)
13. (d)	**14.** (c)	**15.** (d)	**16.** (c)
17. (a)	**18.** (d)	**19.** (d)	**20.** (a)
21. (b)	**22.** (b)	**23.** (c)	**24.** (c)
25. (b)	**26.** (a,c)		
27. 5	**28.** 2	**29.** 4	**30.** 1
31. Neutrino	**32.** Lithium, 7	**33.** 8, 6	
34. 500 dPs, 125 dPs		**35.** 3.861	
36. 6.947 s			

37. (a) $\dfrac{dN_X}{dt} = -\lambda_X N_X, \dfrac{dN_Y}{dt} = \lambda_X N_X - \lambda_Y N_Y, \dfrac{dN_Z}{dt} = \lambda_Y N_Y$

(b) 16.48 s (c) $N_X = 1.92 \times 10^{19}$, $N_Z = 2.32 \times 10^{19}$

38. (a) $\frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)\, e^{-\lambda t}]$ (b) (i) $\frac{3}{2} N_0$ (ii) $2N_0$

39. (a) 14.43 s (b) 40 s **40.** 5.95 L

41. 3.96×10^{-6} **42.** (a) 91 (b) 234

Topic 4

1. (d) **2.** (b) **3.** (a) **4.** (d)
5. (a) **6.** (b)
7. (a) **8.** (d) **9.** (d) **10.** (b)
11. (b) **12.** (a) **13.** (a) **14.** (b)
15. (a) **16.** (a) **17.** (d) **18.** (c)
19. (b) **20.** (c) **21.** (c) **22.** (b)
23. (b) **24.** (a, d) **25.** 2 **26.** 7
27. 3 **28.** 39 **29.** 0.62 **30.** 3×10^8
31. 0.27 Å **32.** Intensity, decrease
33. 30×10^3, 30×10^3 **34.** $n = 24$
35. (a) 56 (b) 1.55×10^{18} Hz **36.** $\sqrt{2}$ **37.** 42
38. (a) 150.8 eV (b) 0.5 Å

Topic 5

1. (d) **2.** (b) **3.** (c) **4.** (d)
5. (*)
6. (c) **7.** (a) **8.** (a) **9.** (c)
10. (b) **11.** (a) **12.** (b) **13.** (c)
14. (b) **15.** (c) **16.** (d) **17.** (c)
18. (c) **19.** (a) **20.** (d) **21.** (a)
22. (b)
23. (A) $\rightarrow$ p, q (B) $\rightarrow$ p, r (C) $\rightarrow$ p, s (D) $\rightarrow$ p, q, r
24. (b, d) **25.** (c, d) **26.** (a, d) **27.** (c, d)
28. (b, c) **29.** Fusion, 24 **30.** 23.6 MeV **31.** F
32. 9 **33.** 3 **34.** 3.847×10^4kg
35. 3.32×10^{-5}W
36. (a) 232, 90 (b) 5.3 MeV, 1823.2 MeV
37. 120.26 g

Topic 6

1. (d) **2.** (d) **3.** (c) **4.** (c)
5. (a) **6.** (c) **7.** (c) **8.** (a)
9. (b) **10.** (a) **11.** (b) **12.** (c)
13. (d) **14.** (b) **15.** (c) **16.** (a)
17. (c)
18. (c) **19.** (c) **20.** (a) **21.** (a)
22. (a) **23.** (d) **24.** (b) **25.** (c)
26. (c) **27.** (b) **28.** (b) **29.** (c)
30. (b) **31.** (d) **32.** (a) **33.** (b, d)
34. (a, d) **35.** (a, c) **36.** (b, c) **37.** (b, c)
38. (b, d) **39.** Reverse **40.** B and D, A and C
41. Reverse, negative **42.** Positive, p-side, n-side
43. T **44.** 8kΩ, 12.5×10^{-3} A/V, 100

Topic 7

1. (b) **2.** (d) **3.** (c) **4.** (d)
5. (c) **6.** (a) **7.** (c) **8.** (d)
9. (a) **10.** (d) **11.** (b) **12.** (c)
13. (b) **14.** (b) **15.** (a) **16.** (b)
17. (a) **18.** (c) **19.** (d) **20.** (b)
21. (b) **22.** (d) **23.** (b) **24.** (a)
25. (c) **26.** (a) **27.** (d) **28.** (a)
29. (c) **30.** (d) **31.** (d)
32. (b) **33.** (a) **34.** (c) **35.** (b)
36. (c) **37.** (b) **38.** (a) **39.** (a)
40. (b) **41.** (24) **42.** (d) **43.** (c)
44. A $\rightarrow$ (R or RT), B $\rightarrow$ (PS) C $\rightarrow$ (Q, T), D $\rightarrow$ (R)
45. (b) **46.** (d) **47.** (c)
48. (A) $\rightarrow$ p, r (B) $\rightarrow$ p, q, s (C) $\rightarrow$ p (D) $\rightarrow$ q
49. 8 **50.** Atomic number, mass number
51. (a) 6.25 MeV, (b) 227.62 amu
52. 6.25×10^{11}, zero, 5.0 eV
53. (i) A(ii) F (iii) E (iv) C
54. (a) 6.36 eV, 0.312 eV (of neutron), 17.84 eV, 16.328 eV (of atom) (b) 1.82×10^{15}Hz, 11.67×10^{15} Hz, 9.84×10^{15}Hz
55. (a) 2.87×10^{13} s^{-1}m^{-2}, 2.07×10^{-10}A (b) 2.06×10^{13} s^{-1}m^{-2}, 1.483×10^{-10}A (c) 1.06 V in both cases
56. 2eV, 0.74 V
57. (a) 2 (b) 14.4 eV (c) 13.5 eV, 0.7 eV
58. Zero, 11, 13

Hints & Solutions

Topic 1 Bohr's Atomic Model

1. Wavelength λ of emitted photon as an electron transits from an initial energy level n_i to some final energy level n_f is given by Balmer's formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where, R = Rydberg constant.

In transition from $n = 4$ to $n = 3$, we have

$$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= R\left(\frac{7}{9 \times 16}\right) \qquad \ldots\text{(i)}$$

In transition from $n = 3$ to $n = 2$, we have

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{5}{9 \times 4}\right) \qquad \ldots\text{(ii)}$$

So, from Eqs. (i) and (ii), the ratio of $\frac{\lambda_1}{\lambda_2}$ is

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{9 \times 16}{7R}\right)}{\left(\frac{9 \times 4}{5R}\right)} = \frac{20}{7}$$

2. Change in energy in transition from n to m stage is given by $(n > m)$,

$$E_n = -\frac{E_0 Z^2}{n^2}$$

Here, $Z = 2$

$$\Delta E_n = +13.6 \times 4\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = \frac{hc}{\lambda} \qquad \text{...(i)}$$

Let it start from n to m and then m to ground.

So, in first case,

$$13.6 \times 4 \times \left(\frac{1}{m^2} - \frac{1}{n^2}\right) = \frac{hc}{108.5 \text{ nm}} \qquad \text{...(ii)}$$

and in second case,

$$13.6 \times 4 \times \left(\frac{1}{1^2} - \frac{1}{m^2}\right) = \frac{hc}{30.4 \text{ nm}}$$

$$\Rightarrow \quad \left(1 - \frac{1}{m^2}\right) = \frac{1240 \text{ eV}}{30.4 \times 13.6 \times 4} \qquad \left(\text{Given, } E = \frac{1240 \text{ eV}}{\lambda \text{ (in nm)}}\right)$$

$$\Rightarrow \quad \left(1 - \frac{1}{m^2}\right) = 0.74980 \approx 0.75$$

or

$$\frac{1}{m^2} = 1 - 0.75 = 0.25$$

$$\Rightarrow \quad m^2 = \frac{1}{0.25} = 4$$

Hence, $m = 2$

So, by putting the value of m in Eq. (ii), we get

$$13.6 \times 4 \times \left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{1240}{108.5} \text{ eV}$$

$$\Rightarrow \quad \left(\frac{1}{4} - \frac{1}{n^2}\right) = \frac{1240}{108.5 \times 13.6 \times 4}$$

$$\Rightarrow \quad \frac{1}{4} - \frac{1}{n^2} = 0.21$$

or

$$\frac{1}{n^2} = 0.25 - 0.21 = 0.04$$

$$n^2 = \frac{1}{0.04} = 25$$

$$\Rightarrow \quad n^2 = 25$$

$$\Rightarrow \quad n = 5$$

3. Number of spectral lines produced as an excited electron falls to ground state $(n = 1)$ is,

$$N = \frac{n(n-1)}{2}$$

In given case, $N = 6$

$$\therefore \quad 6 = \frac{n(n-1)}{2}$$

$$\Rightarrow \quad n = 4$$

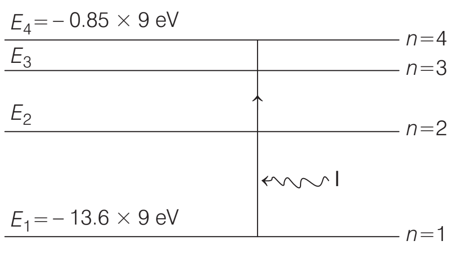
So, L^{++} electron is in it's 3rd excited state.

Now, using the expression of energy of an electron in nth energy level,

$$E_n = -\frac{13.6 Z^2}{n^2} \text{ eV}$$

where, Z is the atomic number.

$\therefore$ Energy levels of L^{++} electron are as shown



So, energy absorbed by electron from incident photon of wavelength λ is

$$\Delta E = \frac{hc}{\lambda} \Rightarrow (13.6 \times 9 - 0.85 \times 9) = \frac{hc}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{hc}{9(13.6 - 0.85)}$$

$$\Rightarrow \quad \lambda = \frac{1240 \text{ eV-nm}}{9 \times 12.75 \text{ eV}} = 10.8 \text{ nm}$$

4. Energy of a hydrogen atom like ion by Bohr's model is

$$E_n = -13.6\frac{Z^2}{n^2}$$

where, Z = atomic number

and $\quad n$ = principal quantum number.

For a He^+ ion in first excited state,

$$n = 2, \quad Z = 2$$

$$\therefore \quad E_2 = -13.6 \times \frac{4}{4} = -13.6 \text{ eV}$$

So, it's ionisation energy $= -(E_2) = 13.6$ eV.

5. Expression for the energy of the hydrogenic electron states for atoms of atomic number Z is given by

$$E = h\nu = \frac{Z^2 me^4}{8h^2 E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \qquad \text{Here, } m < n)$$

$$\text{or} \quad \frac{hc}{\lambda} = \frac{Z^2 me^4}{8h^2E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \Rightarrow \frac{1}{\lambda} \propto \left(\frac{1}{m^2} - \frac{1}{n^2}\right)Z^2$$

For first case,

$$\lambda = 660 \text{ nm}, m = 2 \text{ and } n = 3$$

$$\therefore \quad \frac{1}{660 \text{ nm}} \propto \left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right]Z^2$$

$$\Rightarrow \quad \frac{1}{660 \text{ nm}} \propto \left(\frac{1}{4} - \frac{1}{9}\right)Z^2 \text{ or } \frac{5}{36}Z^2 \qquad \text{...(i)}$$

For second case, transition is from $n = 4$ to $n = 2$, i.e. $m = 2$ and $n = 4$

$$\therefore \quad \frac{1}{\lambda} \propto \left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right)Z^2 \Rightarrow \frac{1}{\lambda} \propto \left(\frac{1}{4} - \frac{1}{16}\right)Z^2$$

$$\text{or} \quad \frac{1}{\lambda} \propto \frac{3}{16}Z^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda}{660 \text{ nm}} = \frac{5}{36} \times \frac{16}{3}$$

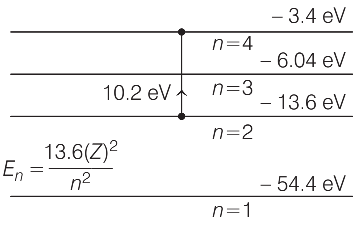
$$\Rightarrow \quad \lambda = \frac{80}{108} \times 660 \text{ nm} = 488.9 \text{ nm}$$

6. De-excitation energy of hydrogen electron in transition $n = 2$ to $n = 1$ is

$$E = 13.6 \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \text{eV} = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 10.2 \text{ eV}$$

Now, energy levels of helium ion's (He^+) electron are

(For helium, $Z = 2$)



So, a photon of energy 10.2 eV can cause a transition $n = 2$ to $n = 4$ in a He^+ ion.

Alternate Solution

For He^+ ion, when in $n = 1$ state,

$$10.2 = 13.6 \times 2^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow n = 1$$

Thus, no transition takes place.

Similarly, when in $n = 2$ state,

$$10.2 = 13.6 \times 2^2\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \Rightarrow n = 4$$

7. As, for conservative fields $\mathbf{F} = -\left(\frac{d\mathbf{U}}{d\mathbf{r}}\right)$

$\therefore$ Magnitude of force on particle is

$$\Rightarrow \quad F = \frac{dU}{dr} = \frac{d}{dr}\left(\frac{1}{2}kr^2\right)$$

$$\Rightarrow \quad F = kr$$

This force is acting like centripetal force.

$$\therefore \quad \frac{mv^2}{r} = kr \qquad \text{...(i)}$$

So, for n^{th} orbit,

$$\Rightarrow \quad m^2v_n^2 = mkr_n^2$$

$$\Rightarrow \quad \frac{n^2h^2}{4\pi^2r^2} = mkr_n^2 \qquad \left[\because v_n = \frac{nh}{2\pi mr}\right]$$

$$\text{Therefore,} \quad r_n^4 \propto n^2$$

$$\Rightarrow \quad r_n^2 \propto n$$

$$\text{So,} \quad r_n \propto \sqrt{n}$$

Energy of particle is

$$E_n = \text{PE} + \text{KE} = \frac{1}{2}kr_n^2 + \frac{1}{2}mv_n^2$$

$$= \frac{1}{2}kr_n^2 + \frac{1}{2}kr_n^2 \qquad \text{[using Eq. (i)]}$$

$$= kr_n^2$$

So, energy, $E_n \propto r_n^2$

$$\Rightarrow \quad E_n \propto n$$

8. For hydrogen or hydrogen like atoms, we know that

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \qquad \text{...(i)}$$

where, R is Rydberg constant and Z is atomic number.

When electron jumps from M - shell to the L - shell, then

$$n_1 = 2 \qquad \text{(for } L \text{ - shell)}$$

$$n_2 = 3 \qquad \text{(for } M \text{ - shell)}$$

$\therefore$ Eq (i) becomes

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}RZ^2] \qquad \text{...(ii)}$$

Now, electron jumps from N-shell to the L - shell, for this

$$n_1 = 2 \qquad \text{(for } L \text{ - shell)}$$
$$n_2 = 4 \qquad \text{(for } N \text{ - shell)}$$

$\therefore$ Eq. (i) becomes

$$\frac{1}{\lambda'} = RZ^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3}{16}RZ^2 \qquad \text{...(iii)}$$

Now, we divide Eq (ii) by Eq (iii)

$$\frac{\lambda'}{\lambda} = \left(\frac{5}{36}RZ^2\right) \div \left(\frac{3}{16}RZ^2\right) = \frac{20}{27}$$

or $$\lambda' = \frac{20}{27}\lambda$$

9. We know that net change in energy of a photon in a transition with wavelength λ is $\Delta E = hc/\lambda$.

Here, $hc = 12500\,\text{eV Å}$ and $\lambda = 980\,\text{Å}$

$$\Delta E = 12500/980 = 12.76\,\text{eV}$$

$$\Rightarrow \quad E_n - E_1 = 12.76\,\text{eV}$$

Since, the energy associated with an electron in n^{th} Bohr's orbit is given as,

$$E_n = \frac{-13.6}{n^2}\,\text{eV} \qquad \text{...(i)}$$

$$\Rightarrow \quad E_n = E_1 + 12.76\,\text{eV} \qquad \text{...(ii)}$$

$$= \frac{-13.6}{(1)^2} + 12.76 = -0.84$$

Putting this value in Eq. (i)

$$\Rightarrow \quad n^2 = \frac{-13.6}{-0.84} = 16 \Rightarrow n = 4$$

and radius of n^{th} orbit, $r_n = n^2 a_0 \Rightarrow r_n = 16\,a_0$

10. Lyman series ends at $n=1$

$\therefore$ Series limit frequency of the Lyman series is given by,

$$\nu_L = RcZ^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \Rightarrow \nu_L = RcZ^2$$

Pfund series ends at $n=5$.

$\therefore$ Series limit frequency of the Pfund series,

$$\nu_P = RcZ^2\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \frac{RcZ^2}{25} \text{ or } \nu_p = \frac{\nu_L}{25}$$

11. The expressions of kinetic energy, potential energy and total energy are

$$K_n = \frac{me^4}{8\varepsilon_0^2 n^2 h^2} \Rightarrow K_n \propto \frac{1}{n^2}$$

$$U_n = \frac{-me^4}{4\varepsilon_0^2 n^2 h^2} \Rightarrow U_n \propto -\frac{1}{n^2}$$

and $$E_n = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2} \Rightarrow E_n \propto -\frac{1}{n^2}$$

In the transition from some excited state to ground state value of n decreases, therefore kinetic energy increases, but potential and total energy decreases.

12. For hydrogen atom, we get

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \qquad \Rightarrow \frac{1}{\lambda_1} = R(1)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \frac{1}{\lambda_2} = R(1)^2\left(\frac{3}{4}\right) \qquad \Rightarrow \frac{1}{\lambda_3} = R(2)^2\left(\frac{3}{4}\right)$$

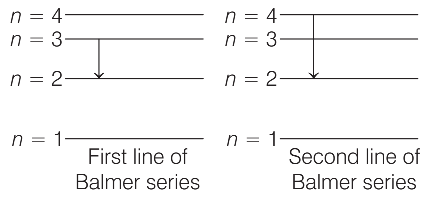
$$\Rightarrow \frac{1}{\lambda_4} = R(3)^2\left(\frac{3}{4}\right) \qquad \Rightarrow \frac{1}{\lambda_1} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} = \frac{1}{\lambda_2}$$

13. $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{k2n}{n^2(n-1)^2} \approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

14.



For hydrogen or hydrogen type atoms,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

In the transition from $n_i \longrightarrow n_f$

$$\therefore \quad \lambda \propto \frac{1}{Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

$$\lambda_2 = \frac{\lambda_1 Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

Substituting the values, we have

$$= \frac{(6561\,\text{Å})(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{(2)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)} = 1215\,\text{Å}$$

15. The series in U-V region is Lymen series. Longest wavelength corresponds to minimum energy which occurs in transition from $n = 2$ to $n = 1$.

$$\therefore \qquad 122 = \frac{\frac{1}{R}}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} \qquad \ldots(\text{i})$$

The smallest wavelength in the infrared region corresponds to maximum energy of Paschen series.

$$\therefore \qquad \lambda = \frac{\frac{1}{R}}{\left(\frac{1}{3^2} - \frac{1}{\infty}\right)} \qquad \ldots(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$$\lambda = 823.5 \text{ nm}$$

16. The first photon will excite the hydrogen atom (in ground state) to first excited state (as $E_2 - E_1 = 10.2$ eV). Hence, during de-excitation a photon of 10.2 eV will be released. The second photon of energy 15 eV can ionise the atom. Hence, the balance energy i.e. $(15 - 13.6)$ eV $= 1.4$ eV is retained by the electron. Therefore, by the second photon an electron of energy 1.4 eV will be released.

17. $$(r_m) = \left(\frac{m^2}{z}\right)(0.53 \text{ Å}) = (n \times 0.53) \text{ Å}$$

$$\therefore \quad \frac{m^2}{z} = n$$

$m = 5$ for ${}_{100}\text{Fm}^{257}$ (the outermost shell) and $z = 100$

$$\therefore \qquad n = \frac{(5)^2}{100} = \frac{1}{4}$$

18. $$U = eV = eV_0 \ln\left(\frac{r}{r_0}\right)$$

$$|F| = \left|-\frac{dU}{dr}\right| = \frac{eV_0}{r}$$

This force will provide the necessary centripetal force.

Hence, $$\frac{mv^2}{r} = \frac{eV_0}{r}$$

or $$v = \sqrt{\frac{eV_0}{m}} \qquad \ldots(\text{i})$$

Moreover, $$mvr = \frac{nh}{2\pi} \qquad \ldots(\text{ii})$$

Dividing Eq. (ii) by (i), we have

$$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$$

or $$r_n \propto n$$

19. In second excited state $n = 3$,

So, $$l_{\text{H}} = l_{\text{Li}} = 3\left(\frac{h}{2\pi}\right)$$

while $$E \propto Z^2$$

and $$Z_{\text{H}} = 1, Z_{\text{Li}} = 3$$

So, $$|E_{\text{Li}}| = 9|E_{\text{H}}|$$

or $$|E_{\text{H}}| < |E_{\text{Li}}|$$

20. Energy of infrared radiation is less than the energy of ultraviolet radiation. In options (a), (b) and (c), energy released will be more, while in option (d) only, energy released will be less.

21. $v_n \propto \frac{1}{n}$ $\therefore$ KE $\propto \frac{1}{n^2}$ (with positive sign)

Potential energy U is negative and $U_n \propto \frac{1}{r_n}$

$$\left[U_n = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r_n}\right]$$

$$E_n \propto \frac{1}{n^2} \qquad (\text{because } r_n \propto n^2)$$

Similarly, total energy $E_n \propto \frac{1}{n^2}$. (with negative sign)

Therefore, when an electron jumps from some excited state to the ground state, value of n will decrease. Therefore, kinetic energy will increase (with positive sign), potential energy and total energy will also increase but with negative sign.

Thus, finally kinetic energy will increase, while potential and total energies will decrease.

NOTE

- For hydrogen and hydrogen-like atoms

$$E_n = -13.6\,\frac{Z^2}{n^2} \text{ eV}$$

$$U_n = 2E_n = -27.2\,\frac{Z^2}{n^2} \text{ eV} \quad \textit{and} \quad K_n = |E_n| = 13.6\,\frac{Z^2}{n^2} \text{ eV}$$

From these three relations we can see that as n decreases, K_n will increase but E_n and U_n will decrease.

- As an electron comes closer to the nucleus, the electrostatic force (which provides the necessary centripetal force) increases or speed (or KE) of the electron increases.

22. In hydrogen atom, $E_n = -\frac{Rhc}{n^2}$. Also, $E_n \propto m$

where, m is the mass of the electron.

Here, the electron has been replaced by a particle whose mass is double of an electron. Therefore, for this hypothetical atom energy in n^{th} orbit will be given by

$$E_n = -\frac{2Rhc}{n^2}$$

The longest wavelength $\lambda_{\max}$ (or minimum energy) photon will correspond to the transition of particle from $n = 3$ to $n = 2$.

$$\therefore \qquad \frac{hc}{\lambda_{\max}} = E_3 - E_2 = 2Rhc\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

This gives, $\lambda_{\max} = 18/5R$

23. Since, the wavelength (λ) is increasing, we can say that the galaxy is receding. Doppler effect can be given by

$$\lambda' = \lambda\sqrt{\frac{c+v}{c-v}} \qquad \ldots(\text{i})$$

or $$706 = 656\sqrt{\frac{c+v}{c-v}}$$

or $$\frac{c+v}{c-v} = \left(\frac{706}{656}\right)^2 = 1.16$$

$\therefore$ $$c + v = 1.16\,c - 1.16v$$

$\therefore$ $$v = \frac{0.16c}{2.16} = \frac{0.16 \times 3.0 \times 10^8}{2.16}$$
$$= 0.22 \times 10^8 \text{ m/s}$$
$$v \approx 2.2 \times 10^7 \text{ m/s}$$

If we take the approximation then Eq. (i) can be written as

$$\Delta\lambda = \lambda\left(\frac{v}{c}\right) \quad \ldots\text{(ii)}$$

From here $v = \left(\frac{\Delta\lambda}{\lambda}\right)\cdot c = \left(\frac{706-656}{656}\right)(3 \times 10^8)$

$$v = 0.23 \times 10^8 \text{ m/s}$$

which is almost equal to the previous answer. So, we may use Eq. (ii) also.

24. For hydrogen and hydrogen like atoms

$$E_n = -13.6\frac{(Z^2)}{(n^2)}\text{ eV}$$

Therefore, ground state energy of doubly ionised lithium atom ($Z = 3$, $n = 1$) will be

$$E_1 = (-13.6)\frac{(3)^2}{(1)^2} = -122.4 \text{ eV}$$

$\therefore$ Ionisation energy of an electron in ground state of doubly ionised lithium atom will be 122.4 eV.

25. Shortest wavelength will correspond to maximum energy. As value of atomic number (Z) increases, the magnitude of energy in different energy states gets increased. Value of Z is maximum for doubly ionised lithium atom ($Z = 3$) among the given elements. Hence, wavelength corresponding to this will be least.

26. $L = I\omega = \frac{nh}{2\pi}$

$\therefore$ $$\omega = \frac{nh}{2\pi I}$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{nh}{2\pi I}\right)^2 = \frac{n^2h^2}{8\pi^2 I}$$

27. $h\nu = K_2 - K_1 = \frac{3h^2}{8\pi^2 I}$

$\therefore$ $$I = \frac{3h}{8\pi^2 f} = \frac{3 \times 2\pi \times 10^{-34} \times \pi}{8 \times \pi^2 \times 4 \times 10^{11}}$$
$$= 1.87 \times 10^{-46} \text{ kgm}^2$$

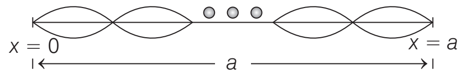
28. $I = \mu r^2$ (where, μ = reduced mass)

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{48}{7}\text{ amu} = 11.43 \times 10^{-27}\text{ kg}$$

Substituting in $I = \mu r^2$ we get,

$$r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{1.87 \times 10^{-46}}{11.43 \times 10^{-27}}}$$
$$= 1.28 \times 10^{-10}\text{ m}$$

29.



$$a = \frac{n\lambda}{2}$$

$\therefore$ $$\lambda = \frac{2a}{n} = \frac{h}{p} = \frac{h}{\sqrt{2Em}} \quad \ldots\text{(i)}$$

or $\sqrt{E} \propto \frac{1}{a} \Rightarrow \therefore E \propto \frac{1}{a^2}$

30. From Eq. (i) $E = \frac{n^2h^2}{8a^2m}$

In ground state $n = 1$

$\therefore$ $$E_1 = \frac{h^2}{8ma^2}$$

Substituting the values, we get

$$E_1 = 8\text{ meV}$$

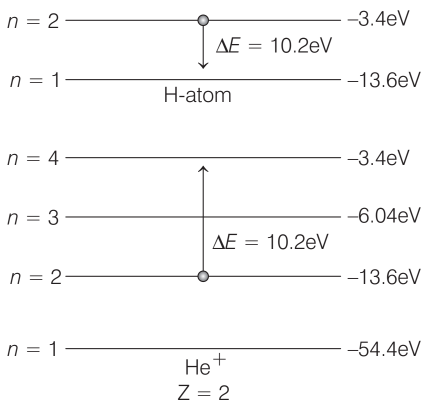
31. From Eq. (i)

$$p \propto n$$

$\therefore$ $$mv \propto n$$

or $$v \propto n$$

32.



Energy given by H-atom in transition from $n = 2$ to $n = 1$ is equal to energy taken by He^+ atom in transition from $n = 2$ to $n = 4$.

33. Visible light lies in the range, $\lambda_1 = 4000$Å to $\lambda_2 = 7000$Å. Energy of photons corresponding to these wavelengths (in eV) would be :

$$E_1 = \frac{12375}{4000} = 3.09\text{ eV}$$

$$E_2 = \frac{12375}{7000} = 1.77\text{ eV}$$

From energy level diagram of He^+ atom, we can see that in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04)$$
$$= 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{2.64} \text{Å} = 4687.5 \text{ Å}$$
$$= 4.68 \times 10^{-7} \text{m}$$

34. Kinetic energy $K \propto Z^2$

$$\therefore \quad \frac{K_H}{K_{He^+}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

35. As radius $\quad r \propto \dfrac{n^2}{z}$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{\left(\frac{n+1}{z}\right)^2 - \left(\frac{n}{z}\right)^2}{\left(\frac{n}{z}\right)^2} = \frac{2n+1}{n^2} \approx \frac{2}{n} \propto \frac{1}{n}$$

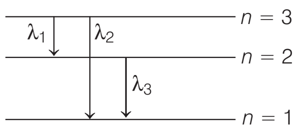
Energy, $\quad E \propto \dfrac{z^2}{n^2}$

$$\Rightarrow \quad \frac{\Delta E}{E} = \frac{\frac{z^2}{n^2} - \frac{z^2}{(n-1)^2}}{\frac{z^2}{(n+1)^2}}$$
$$= \frac{(n+1)^2 - n^2}{n^2 \cdot (n+1)^2} \cdot (n+1)^2$$
$$\Rightarrow \quad \frac{\Delta E}{E} = \frac{2n+1}{n^2} \simeq \frac{2n}{n^2} \propto \frac{1}{n}$$

Angular momentum, $L = \dfrac{nh}{2\pi}$

$$\Rightarrow \quad \frac{\Delta L}{L} = \frac{\frac{(n+1)}{2\pi} - \frac{nh}{2\pi}}{\frac{nh}{2\pi}} = \frac{1}{n} \propto \frac{1}{n}$$

36. $L = 3\left(\dfrac{h}{2\pi}\right)$



$$\therefore \quad n = 3, \text{as } L = n\left(\frac{h}{2\pi}\right)$$
$$r_n \propto \frac{n^2}{z}$$
$$r_3 = 4.5 a_0$$
$$\therefore \quad z = 2$$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 4R\left(\frac{1}{4} - \frac{1}{9}\right)$$
$$\therefore \quad \lambda_1 = \frac{9}{5R}$$
$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 4R\left(1 - \frac{1}{9}\right)$$
$$\Rightarrow \quad \lambda_2 = \frac{9}{32R}$$
$$\frac{1}{\lambda_3} = Rz^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 4R\left(1 - \frac{1}{4}\right)$$
$$\Rightarrow \quad \lambda_3 = \frac{1}{3R}$$

37. Time period, $T_n = \dfrac{2\pi r_n}{v_n}$ $\quad$ (in nth state)

i.e. $\quad T_n \propto \dfrac{r_n}{v_n}$

But $\quad r_n \propto n^2$

and $\quad v_n \propto \dfrac{1}{n}$

Therefore, $\quad T_n \propto n^3$

Given $\quad T_{n_1} = 8 T_{n_2}$

Hence, $\quad n_1 = 2n_2$

38. $r_n \propto \dfrac{n^2}{Z}$ and $|PE| = 2$ (KE)

39.
$$\Delta E_{2-1} = 13.6 \times Z^2\left[1 - \frac{1}{4}\right] = 13.6 \times Z^2\left[\frac{3}{4}\right]$$
$$\Delta E_{3-2} = 13.6 \times Z^2\left[\frac{1}{4} - \frac{1}{9}\right] = 13.6 \times Z^2\left[\frac{5}{36}\right]$$
$$\therefore \quad \Delta E_2 = \Delta E_{3-2} + 74.8$$
$$13.6 \times Z^2\left[\frac{3}{4}\right] = 13.6 \times Z^2\left[\frac{5}{36}\right] + 74.8$$
$$13.6 \times Z^2\left[\frac{3}{4} - \frac{5}{36}\right] = 74.8$$
$$Z^2 = 9$$
$$\therefore \quad Z = 3$$

40. From conservation of linear momentum,

|Momentum of recoil hydrogen atom| = |Momentum of emitted photon|

or $\quad mv = \dfrac{\Delta E}{c}$

Here, $\Delta E = E_5 - E_1 = -13.6\left[\dfrac{1}{5^2} - \dfrac{1}{1^2}\right]$ eV

$$= (13.6)(24/25) \text{ eV} = 13.056 \text{ eV}$$
$$= 13.056 \times 1.6 \times 10^{-19} \text{ J} = 2.09 \times 10^{-18} \text{ J}$$

and m = mass of hydrogen atom $= 1.67 \times 10^{-27}$ kg

$\therefore \quad v = \dfrac{\Delta E}{mc} = \dfrac{2.09 \times 10^{-18}}{(1.67 \times 10^{-27})(3 \times 10^{8})},$

$v \approx 4.17$ m/s

41. Kinetic energy of an electron in nth orbit of hydrogen atom is $K = \dfrac{me^4}{8\varepsilon_0^2 h^2 n^2}$ and total energy of electron in nth orbit is

$$E = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}, \quad \frac{K}{E} = -1$$

or $\quad K = -E$

42. Potential energy of hydrogen atom ($Z = 1$) in nth orbit (in eV)

$$\text{PE} = -\frac{27.2}{n^2}$$

$$\frac{v_f}{v_i} = \frac{-\dfrac{27.2}{n_f^2}}{-\dfrac{27.2}{n_i^2}} = \frac{1}{6.25}$$

$$6.25 = \frac{n_f^2}{n_i^2}$$

$$\frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

Hence the answer is 5.

43. Energy of incident light (in eV)

$$E = \frac{12375}{970} = 12.7\text{eV}$$

After excitation, let the electron jumps to nth state, then

$$\frac{-13.6}{n^2} = -13.6 + 12.7$$

Solving this equation, we get

$$n = 4$$

$\therefore$ Total number of lines in emission spectrum,

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

44. Kinetic energy of ejected electron

= Energy of incident photon – energy required to ionize the electron from nth orbit (all in eV)

$\therefore \quad 10.4 = \dfrac{1242}{90} - |E_n|$

$= \dfrac{1242}{90} - \dfrac{13.6}{n^2} \qquad \left(\text{as } E_n \propto \dfrac{1}{n^2} \text{ and } E_1 = -13.6\,\text{eV}\right)$

Solving this equation, we get

$$n = 2$$

45. Wavelengths corresponding to minimum wavelength ($\lambda_{\min}$) or maximum energy will emit photoelectrons having maximum kinetic energy. ($\lambda_{\min}$) belonging to Balmer series and lying in the given range (450 nm to 750 nm) corresponds to transition from ($n = 4$ to $n = 2$). Here,

$$E_4 = -\frac{13.6}{(4)^2} = -0.85\,\text{eV}$$

and $\quad E_2 = -\dfrac{13.6}{(2)^2} = -3.4$ eV

$\therefore \quad \Delta E = E_4 - E_2 = 2.55\,\text{eV}$

$K_{\max}$ = Energy of photon – work function

$= 2.55 - 2.0 = 0.55\,\text{eV}$

46. (a) Total 6 lines are emitted. Therefore,

$$\frac{n(n-1)}{2} = 6 \quad \text{or} \quad n = 4$$

So, transition is taking place between m^{th} energy state and $(m+3)^{\text{th}}$ energy state.

$$E_m = -0.85\,\text{eV}$$

or $\quad -13.6\left(\dfrac{z^2}{m^2}\right) = -0.85$

or $\quad \dfrac{z}{m} = 0.25 \qquad \ldots\text{(i)}$

Similarly, $\quad E_{m+3} = -0.544$ eV

or $\quad -13.6\dfrac{z^2}{(m+3)^2} = -0.544$

or $\quad \dfrac{z}{(m+3)} = 0.2 \qquad \ldots\text{(ii)}$

Solving Eqs. (i) and (ii) for z and m, we get

$$m = 12 \text{ and } z = 3$$

(b) Smallest wavelength corresponds to maximum difference of energies which is obviously $E_{m+3} - E_m$

$\therefore \quad \Delta E_{\max} = -0.544 - (-0.85) = 0.306$ eV

$\therefore \quad \lambda_{\min} = \dfrac{hc}{\Delta E_{\max}}$

$= \dfrac{1240}{0.306} = 4052.3$ nm.

47. Let ground state energy (in eV) be E_1.

Then, from the given condition

$$E_{2n} - E_1 = 204\text{ eV}$$

or $\quad \dfrac{E_1}{4n^2} - E_1 = 204$ eV

or $\quad E_1\left(\dfrac{1}{4n^2} - 1\right) = 204$ eV $\qquad \ldots\text{(i)}$

and $\quad E_{2n} - E_n = 40.8\,\text{eV}$

or $\quad \dfrac{E_1}{4n^2} - \dfrac{E_1}{n^2} = 40.8\,\text{eV}$

or $\quad E_1\left(\dfrac{-3}{4n^2}\right) = 40.8\,\text{eV} \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii),

$$\frac{1-\frac{1}{4n^2}}{\frac{3}{4n^2}} = 5$$

or $$1 = \frac{1}{4n^2} + \frac{15}{4n^2} \quad \text{or} \quad \frac{4}{n^2} = 1$$

or $$n = 2$$

From Eq. (ii),

$$E_1 = -\frac{4}{3} n^2 \,(40.8)\,\text{eV}$$

$$= -\frac{4}{3}(2)^2 \,(40.8)\,\text{eV}$$

or $$E_1 = -\,217.6\,\text{eV}$$

$$E_1 = -\,(13.6)\,Z^2$$

$\therefore$ $$Z^2 = \frac{E_1}{-13.6} = \frac{-217.6}{-13.6} = 16$$

$\therefore$ $$Z = 4$$

$$E_{\min} = E_{2n} - E_{2n-1}$$

$$= \frac{E_1}{4n^2} - \frac{E_1}{(2n-1)^2}$$

$$= \frac{E_1}{16} - \frac{E_1}{9} = -\frac{7}{144} E_1$$

$$= -\left(\frac{7}{144}\right)(-\,217.6)\,\text{eV}$$

$\therefore$ $$E_{\min} = 10.58\,\text{eV}$$

48. (a) Kinetic energy of electron in the orbits of hydrogen and hydrogen like atoms $= |\text{Total energy}|$

$\therefore$ Kinetic energy = 3.4 eV

(b) The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}$$

Here, K = kinetic energy of electron

Substituting the values, we have

$$\lambda = \frac{(6.6 \times 10^{-34}\,\text{J-s})}{\sqrt{2(3.4 \times 1.6 \times 10^{-19}\,\text{J})\,(9.1 \times 10^{-31}\,\text{kg})}}$$

$$\lambda = 6.63 \times 10^{-10}\,\text{m}$$

or $$\lambda = 6.63\,\text{Å}$$

49. From the given conditions

$$E_n - E_2 = (10.2 + 17)\,\text{eV} = 27.2\,\text{eV} \qquad \text{...(i)}$$

and $$E_n - E_3 = (4.25 + 5.95)\,\text{eV} = 10.2\,\text{eV} \qquad \text{...(ii)}$$

Eq. (i) – Eq. (ii) gives

$$E_3 - E_2 = 17.0\,\text{eV}$$

or $$Z^2\,(13.6)\left(\frac{1}{4} - \frac{1}{9}\right) = 17.0$$

$\Rightarrow$ $$Z^2\,(13.6)\,(5/36) = 17.0$$

$\Rightarrow$ $$Z^2 = 9 \quad \text{or} \quad Z = 3$$

From Eq. (i) $Z^2\,(13.6)\left(\frac{1}{4} - \frac{1}{n^2}\right) = 27.2$

or $$(3)^2\,(13.6)\left(\frac{1}{4} - \frac{1}{n^2}\right) = 27.2$$

or $$\frac{1}{4} - \frac{1}{n^2} = 0.222$$

or $$1/n^2 = 0.0278 \;\text{ or }\; n^2 = 36$$

$\therefore$ $$n = 6$$

50. If we assume that mass of nucleus >> mass of mu-meson, then nucleus will be assumed to be at rest, only mu-meson is revolving round it.

(a) In nth orbit the necessary centripetal force to the mu-meson will be provided by the electrostatic force between the nucleus and the mu-meson.

Hence,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)\,(e)}{r^2} \qquad \text{...(i)}$$

Further, it is given that Bohr model is applicable to this system also. Hence

Angular momentum in n^{th} orbit $= \frac{nh}{2\pi}$

or $$mvr = n\frac{h}{2\pi} \qquad \text{...(ii)}$$

We have two unknowns v and r (in n^{th} orbit). After solving these two equations, we get

$$r = \frac{n^2 h^2 \varepsilon_0}{Z\pi m e^2}$$

Substituting $Z = 3$ and $m = 208\,m_e$, we get

$$r_n = \frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2}$$

(b) The radius of the first Bohrs orbit for the hydrogen atom is $\frac{h^2 \varepsilon_0}{\pi m_e e^2}$.

Equating this with the radius calculated in part (a), we get

$$n^2 \approx 624 \quad \text{or} \quad n \approx 25$$

(c) Kinetic energy of atom $= \frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$

and potential energy $= -\frac{Ze^2}{4\pi\varepsilon_0 r}$

$\therefore$ Total energy $E_n = \frac{-\,Ze^2}{8\pi\varepsilon_0 r}$

Substituting value of r, calculated in part (a),

$$E_n = \frac{1872}{n^2}\left[-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right]$$

But $\left[-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right]$ is the ground state energy of hydrogen atom and hence is equal to -13.6 eV.

$$\therefore \quad E_n = \frac{-1872}{n^2}(13.6)\,\text{eV} = -\frac{25459.2}{n^2}\text{eV}$$

$$\therefore \quad E_3 - E_1 = -25459.2\left[\frac{1}{9}-\frac{1}{1}\right] = 22630.4 \text{ eV}$$

$\therefore$ The corresponding wavelength,

$$\lambda\,(\text{in Å}) = \frac{12375}{22630.4} = 0.546\,\text{Å}$$

51. Given $Z = 3$, $E_n \propto \frac{Z^2}{n^2}$

(a) To excite the atom from $n = 1$ to $n = 3$, energy of photon required is

$$E_{1-3} = E_3 - E_1 = \frac{(-13.6)(3)^2}{(3)^2} - \left[\frac{(-13.6)(3)^2}{(1)^2}\right]$$

$$= 108.8\,\text{eV}$$

Corresponding wavelength will be,

$$\lambda\,(\text{in Å}) = \frac{12375}{E(\text{in eV})} = \frac{12375}{108.8} = 113.74 \text{ Å}$$

(b) From n^{th} orbit total number of emission lines can be $\frac{n(n-1)}{2}$.

$$\therefore \text{ Number of emission lines} = \frac{3(3-1)}{2} = 3$$

52. (a) 1 rydberg $= 2.2\times 10^{-18}$ J $= Rhc$

Ionisation energy is given as 4 rydberg

$$= 8.8\times 10^{-18}\text{J} = \frac{8.8\times 10^{-18}}{1.6\times 10^{-19}} = 55\,\text{eV}$$

$\therefore$ Energy in first orbit $E_1 = -55$ eV

Energy of radiation emitted when electron jumps from first excited state $(n = 2)$ to ground state $(n=1)$:

$$E_{21} = \frac{E_1}{(2)^2} - E_1 = -\frac{3}{4}E_1 = 41.25\,\text{eV}$$

$\therefore$ Wavelength of photon emitted in this transition would be

$$\lambda = \frac{12375}{41.25} = 300 \text{ Å}$$

(b) Let Z be the atomic number of given element. Then

$$E_1 = (-13.6)(Z^2)$$

or $\quad -55 = (-13.6)(Z^2)$

or $\quad Z \approx 2$

Now, as $\quad r \propto \frac{1}{Z}$

Radius of first orbit of this atom,

$$r_1 = \frac{r_{H_1}}{Z} = \frac{0.529}{2} = 0.2645 \text{ Å}$$

53. When 800 Å wavelength falls on hydrogen atom (in ground state) 13.6 eV energy is used in liberating the electron. The rest goes to kinetic energy of electron.

Hence, $\quad K = E - 13.6$ (in eV) or

$$(1.8\times 1.6\times 10^{-19}) = \frac{hc}{800\times 10^{-10}} - 13.6\times 1.6\times 10^{-19} \quad \ldots\text{(i)}$$

Similarly, for the second wavelength :

$$(4.0\times 1.6\times 10^{-19}) = \frac{hc}{700\times 10^{-10}} - 13.6\times 1.6\times 10^{-19} \quad \ldots\text{(ii)}$$

Solving these two equations, we get

$$h \approx 6.6\times 10^{-34} \text{ J-s}$$

54. Energy corresponding to given wavelength

$$E\,(\text{in eV}) = \frac{12375}{\lambda\,(\text{in Å})} = \frac{12375}{975} = 12.69 \text{ eV}$$

Now, let the electron excites to n^{th} energy state. Then,

$$E_n - E_1 = 12.69 \text{ or } \frac{(-13.6)}{(n^2)} - (-13.6) = 12.69$$

$$\therefore \quad n \approx 4$$

i.e. electron excites to 4th energy state. Total number of lines in emission spectrum would be

$$\frac{n(n-1)}{2} = \frac{4\times 3}{2} = 6$$

Longest wavelength will correspond to the minimum energy and minimum energy is released in transition from $n = 4$ to $n = 3$.

$$E_{4-3} = E_4 - E_3 = \frac{-13.6}{(4^2)} - \left[\frac{-13.6}{(3)^2}\right] = 0.66\,\text{eV}$$

$\therefore$ Longest wavelength will be,

$$\lambda_{\max} = \frac{12375}{E\,(\text{in eV})}$$

$$= \frac{12375}{0.66}\text{Å} = 1.875\times 10^{-6}\text{m} = 1.875\,\mu\text{m}$$

55. (a) Given, $E_3 - E_2 = 47.2$ eV

Since $\quad E_n \propto \frac{Z^2}{n^2}$ (for hydrogen like atoms)

$$\text{or} \quad (-13.6)\left(\frac{Z^2}{9}\right) - \left[(-13.6)\left(\frac{Z^2}{4}\right)\right] = 47.2$$

Solving this equation, we get

$$Z = 5$$

(b) Energy required to excite the electron from 3rd to 4th orbit :

$$E_{3-4} = E_4 - E_3$$

$$= (-13.6)\left(\frac{25}{16}\right) - \left[(-13.6)\left(\frac{25}{9}\right)\right] = 16.53\,\text{eV}$$

(c) Energy required to remove the electron from first orbit to infinity (or the ionisation energy) will be

$$E = (13.6)(5)^2 = 340 \text{ eV}$$

The corresponding wavelength would be,

$$\lambda = \frac{hc}{E} = \frac{6.6\times10^{-34}\times3\times10^{8}}{340\times1.6\times10^{-19}}$$

$$= 0.0364\times10^{-7} \text{ m} = 36.4 \text{ Å}$$

(d) In first orbit, total energy $= -340$ eV

kinetic energy $= +340$ eV

Potential energy $= -2\times340$ eV $= -680$ eV and

$$\text{angular momentum} = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{2\pi} = 1.05\times10^{-34} \text{ kg-m}^2\text{/s}$$

(e) $r_n \propto \frac{n^2}{Z}$

Radius of first Bohrs orbit

$$r_1 = \frac{r_1^H}{Z} = \frac{5.3\times10^{-11}}{5} = 1.06\times10^{-11} \text{ m}$$

Topic 2 Photo Electric Effect

1. Given,

Planck's constant,

$$h = 6.63\times10^{-34} \text{ J-s}$$

$$e = 1.6\times10^{-19} \text{ C}$$

and there is a graph between stopping potential and frequency.

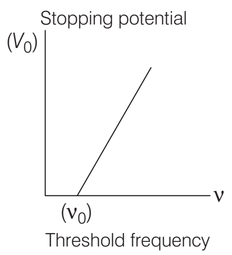
We need to determine work function W.

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{\text{max}} = eV_0 = h\nu - h\nu_0 = h\nu - W \quad [\because W = h\nu_0]$$

or $$V_0 = \frac{h}{e}\nu - \frac{W}{e}$$

From graph at $V_0 = 0$ and $\nu = 4\times10^{14}$ Hz



$\therefore$ $$0 = \frac{6.63\times10^{-34}}{e}\times4\times10^{14} - \frac{W}{e}$$

$\Rightarrow$ $$\frac{W}{e} = \frac{6.63\times10^{-34}\times4\times10^{14}}{e} \text{ J}$$

or $$W = 6.63\times4\times10^{-20} \text{ J}$$

or $$W = \frac{6.63\times4\times10^{-20}}{1.6\times10^{-19}} \text{ eV} = 1.657 \text{ eV}$$

$\therefore$ $$W = 1.66 \text{ eV}$$

Alternate Solution

From graph, threshold frequency,

$$\nu_0 = 4\times10^{14} \text{ Hz (where, } V_0 = 0)$$

$\therefore$ Work function, $W = h\nu_0$

$\Rightarrow$ $$W = 6.63\times10^{-34}\times4\times10^{14} \text{ J}$$

$\Rightarrow$ $$W = \frac{6.63\times4\times10^{-20}}{1.6\times10^{-19}} \text{ eV} = 1.657 \text{ eV} \approx 1.66 \text{ eV}$$

2. Power of laser is given as

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Number of photons emitted}\times\text{Energy of one photon}}{\text{Time}}$$

$\Rightarrow$ $$P = \frac{NE}{t} = \left(\frac{N}{t}\right)\cdot E$$

So, number of photons emitted per second

$$= \frac{N}{t} = \frac{P}{E}$$

$$= \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \qquad \left[\because E = h\nu = \frac{hc}{\lambda}\right]$$

Here, $h = 6.6\times10^{-34}$ J-s, $\lambda = 500$ nm $= 500\times10^{-9}$ m

$c = 3\times10^{8}$ ms^{-1}

$P = 2$ mW $= 2\times10^{-3}$ W

$\therefore$ $$\frac{N}{t} = \frac{2\times10^{-3}\times500\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}}$$

$$= 5.56\times10^{15}$$

$$\approx 5\times10^{15} \text{ photons per second}$$

3. Given, threshold wavelength, $\lambda_0 = 380$ nm

Wavelength of incident light, $\lambda = 260$ nm

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{\text{max}} = eV_0 = h\nu - h\nu_0 \qquad \ldots\text{(i)}$$

But $$h\nu = E = \frac{1237}{\lambda(\text{nm})} \text{ eV}$$

$\therefore$ $$E_0 = \frac{1237}{\lambda_0(\text{nm})} \text{ eV} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$(\text{KE})_{\text{max}} = E - E_0\left(\frac{1237}{\lambda} - \frac{1237}{\lambda_0}\right) \text{ eV}$$

$$= 1237\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \text{ eV } (\lambda \text{ in nm}) \qquad \ldots\text{(iii)}$$

By putting values of λ and λ_0 in Eq. (iii), we get

$$(\text{KE})_{\max} = 1237\left(\frac{1}{260} - \frac{1}{380}\right)\text{eV}$$

$$= 1237 \times \left[\frac{380-260}{380 \times 260}\right]\text{eV}$$

$$\Rightarrow (\text{KE})_{\max} = 1.5 \text{ eV}$$

4. Given, $\mathbf{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi \times 6 \times 10^{14} t\right)\hat{\mathbf{x}}\text{NC}^{-1}$

By comparing it with the general equation of electric field of light, i.e.

$$E = E_0 \cos(kx - \omega t)\,\hat{\mathbf{x}}, \text{ we get}$$

$$k = \frac{2\pi}{5 \times 10^{-7}} = 2\pi/\lambda$$

(from definition, $k = 2\pi/\lambda$)

$$\Rightarrow \quad \lambda = 5 \times 10^{-7} \text{ m} = 5000\,\text{Å} \quad \ldots\text{(i)}$$

Or

The value of λ can also be calculated as, after comparing the given equation of $\mathbf{E}$ with standard equation, we get

$$\omega = 6 \times 10^{14} \times 2\pi$$

$$\Rightarrow \quad \nu = 6 \times 10^{14} \quad [\because 2\pi\nu = \omega]$$

As, $\quad c = \nu\lambda$

$$\Rightarrow \quad \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7} \text{ m} = 5000\,\text{Å}$$

According to Einstein's equation for photoelectric effect, i.e.,

$$\frac{hc}{\lambda} - \phi = (\text{KE})_{\max} = eV_0 \quad \ldots\text{(ii)}$$

For photon, substituting the given values,

$$E = \frac{hc}{\lambda} = \frac{12375 \text{ eV}}{\lambda} \quad \text{[given]}$$

$$\text{or} \quad \frac{hc}{\lambda} = \frac{12375}{5000}\text{ eV} \quad \text{[using Eq. (i)]} \ldots\text{(iii)}$$

Now, substituting the values from Eq. (iii) in Eq. (ii), we get

$$\frac{12375}{5000}\text{ eV} - 2\text{eV} = eV_0$$

$$\Rightarrow \quad 2.475\,\text{eV} - 2\text{eV} = eV_0$$

$$\text{or} \quad V_0 = 2.475\text{V} - 2\text{V}$$

$$= 0.475\,\text{V} \Rightarrow V_0 \approx 0.48\,\text{V}$$

5. Relation between stopping potential and incident light's frequency is $eV_0 = hf - \phi_0$.

where, V_0 is the stopping potential and ϕ_0 is the the work function of the photosensitive surface.

So, from given data, we have,

$$-e\frac{V_0}{2} = h\nu - \phi_0 \quad \ldots\text{(i)}$$

$$\text{and} \quad -eV_0 = \frac{h\nu}{2} - \phi_0 \quad \ldots\text{(ii)}$$

Subtracting Eqs. (i) from (ii), we have

$$-eV_0 - \left(-\frac{eV_0}{2}\right) = \frac{h\nu}{2} - h\nu \Rightarrow -\frac{eV_0}{2} = -\frac{h\nu}{2}$$

$$\Rightarrow \quad eV_0 = h\nu$$

Substituting this in Eq. (i), we get

$$-\frac{eV_0}{2} = eV_0 - \phi_0$$

$$\Rightarrow \quad -\left(\frac{3}{2}eV_0\right) = -\phi_0 \quad \text{or} \quad \frac{3}{2}h\nu = \phi_0$$

If threshold frequency is ν_0 then

$$h\nu_0 = \frac{3}{2}h\nu \Rightarrow \nu_0 = \frac{3}{2}\nu$$

6. Given, $\lambda_1 = 300$ nm

$$\lambda_2 = 400\,\text{nm}$$

$$\frac{hc}{e} = 1240\,\text{nm}$$

Using Einstein equation for photoelectric effect,

$$E = h\nu = \phi + \text{eV}_0 \quad \ldots\text{(i)}$$

(here, ϕ is work function of the metal and V_0 is stopping potential)

For λ_1 wavelength's wave,

$$E_1 = h\nu_1 = \phi + \text{eV}_{01}$$

$$\text{or} \quad \frac{hc}{\lambda_1} = \phi + \text{eV}_{01} \quad \ldots\text{(ii)}$$

$$\text{Similarly,} \quad \frac{hc}{\lambda_2} = \phi + \text{eV}_{02} \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = e(V_{01} - V_{02}) \text{ or } \frac{hc}{e}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = \Delta V$$

By using given values,

$$\Delta V = 1240\left[\frac{1}{300} - \frac{1}{400}\right]\frac{\text{nmV}}{\text{nm}}$$

$$= 1240 \times \frac{1}{1200}\text{ V}$$

$$\Rightarrow \quad \Delta V = 1.03.\text{ V} \approx 1\text{V}$$

7. We know that, intensity of a radiation I with energy 'E' incident on a plate per second per unit area is given as

$$\Rightarrow \quad I = \frac{dE}{dA \times dt} \Rightarrow \frac{dE}{dt} = IdA \text{ or } IA$$

i.e., energy incident per unit time = IA

Substituting the given values, we get

$$\frac{dE}{dt} = 16 \times 10^{-3} \times 1 \times 10^{-4}$$

$$\frac{dE}{dt} = 16 \times 10^{-7} \text{ W} \quad \ldots\text{(i)}$$

Using Einstein's photoelectric equation, we can find kinetic energy of the incident radiation as

$$E = \frac{1}{2}mv^2 + \phi$$

(Here, ϕ is work function of metal)

or $\quad E = \text{KE} + \phi$

$$\text{KE} = E - \phi = 10\text{ eV} - 5\text{ eV}$$

$\Rightarrow \quad \text{KE} = 5\text{ eV} \quad \ldots\text{(ii)}$

Now, energy per unit time for incident photons will be

$\because \quad E = Nh\nu$

$\therefore \quad \frac{dE}{dt} = h\nu\frac{dN}{dt}$ or $h\nu\dot{N} \quad \ldots\text{(iii)}$

From Eqs. (i) and (iii), we get

$$h\nu\dot{N} = 16\times10^{-7} \text{ or } E\dot{N} = 16\times10^{-7}$$

But $\quad E = 10$ eV, so

$$\dot{N}(10\times1.6\times10^{-19}) = 16\times10^{-7} \Rightarrow \dot{N} = 10^{12}$$

$\because$ Only 10% of incident photons emit electrons.

So, emitted electrons per second are

$$\frac{10}{100}\times10^{12} = 10^{11}$$

8. According to question, the wave equation of the magnetic field which produce photoelectric effect

$$B = B_0[(\sin(3.14\times10^7 ct) + \sin(6.28\times10^7 ct)]$$

Here, the photoelectric effect produced by the angular frequency $(\omega) = 6.28\times10^7$ c

$\Rightarrow \quad \omega = 6.28\times10^7\times3\times10^8$

$\omega = 2\pi\times10^7\times3\times10^8$ rad/s $\quad \ldots\text{(i)}$

Using Eqs. (i)

$$h\nu = \frac{h\omega}{2\pi} = \frac{h\times2\pi\times10^7\times3\times10^8}{2\pi}$$

$$h\nu = 12.4\text{ eV}$$

Therefore, according to Einstein equation for photoelectric effect

$$E = h\nu = \phi + \text{KE}_{\max}$$

$\Rightarrow \quad KE_{\max} = E - \phi$

(where, ϕ = work-function = 4.7 eV)

$$\text{KE}_{\max} = 12.4 - 4.7 = 7.7\text{ eV}$$

or $\quad \text{KE}_{\max} = 7.7$ eV

9. Let maximum speed of photo electrons in first case is v_1 and maximum speed of photo electrons in second case is v_2

Assumption I if we assume difference in maximum speed in two cases is 2 then $v_1 = v$ and $v_2 = 3v$

According to Einstein's photo electron equation

Energy of incident photon = work function + KE

i.e. $\quad \frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2$

Where $hc = 1240$ eV, λ is wavelength of light incident, ϕ_0 is work function and v is speed of photo electrons.

When $\quad \lambda_1 = 350$ nm

$\therefore \quad \frac{hc}{350} = \phi_0 + \frac{1}{2}mv^2$

or $\quad \frac{hc}{350} - \phi_0 = \frac{1}{2}mv^2 \quad \ldots\text{(i)}$

when $\quad \lambda_2 = 540$ nm

$\therefore \quad \frac{hc}{540} = \phi_0 + \frac{1}{2}m(3v^2)$

$\therefore \quad \frac{hc}{540} - \phi_0 = (\frac{1}{2}mv^2)\times9 \quad \ldots\text{(ii)}$

Now, we divide Eq. (i) by Eq. (ii), we get

$$\frac{\frac{hc}{350} - \phi_0}{\frac{hc}{540} - \phi_0} = \frac{\frac{1}{2}mv^2}{(\frac{1}{2}mv^2)\times9} = \frac{1}{9}$$

or $\quad 9\left(\frac{hc}{350} - \phi_0\right) = \frac{hc}{540} - \phi_0$

or $\quad 8\phi_0 = hc\left[\frac{9}{350} - \frac{1}{540}\right]$

or $\quad \phi_0 = \frac{1}{8}\times1240\left[\frac{9\times540 - 350}{350\times540}\right] = 3.7$ eV.

No option given is correct.

Alternate Method

Assumption II If we assume velocity of one is twice in factor with second, then.

Let $v_1 = 2v$ and $v_2 = v$

We know that from Einstein's photoelectric equation, energy of incident radiation = work function + KE

or $\quad \frac{hc}{\lambda} = \phi + \frac{1}{2}mv^2$

Let when $\lambda_1 = 350$ nm then $v_1 = 2v$

and when $\lambda_1 = 540$ nm then $v_2 = v$

$\therefore$ Above Eq. becomes

$$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v^2)$$

or $\quad \frac{hc}{\lambda_1} - \phi = \frac{1}{2}m\times4v^2 \quad \ldots\text{(i)}$

and $\quad \frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$

or $\quad \frac{hc}{\lambda_2} - \phi = \frac{1}{2}mv^2 \quad \ldots\text{(ii)}$

Now, we divide Eq. (i) by (ii) Eq.

$$\frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \frac{\frac{1}{2}m\times4v^2}{\frac{1}{2}mv^2} = 4$$

or $\quad \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$

or $\phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right) = \frac{1}{3}\times 1240\left(\frac{4\times 350 - 540}{350\times 540}\right)$

or $\qquad \phi = 1.8\,\text{eV}$

According the assumption II, correct option is (c).

10. We have, $\lambda = \frac{hc}{\Delta E}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{hc/\Delta E_1}{hc/\Delta E_2} = \frac{\Delta E_2}{\Delta E_1}$$

$$= \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

11. $\lambda_{\text{min}} = \frac{hc}{eV}$

$$\log(\lambda_{\text{min}}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - x$$

12. According to the law of conservation of energy, i.e. Energy of a photon $(h\nu)$ = Work function (ϕ) + Kinetic energy of the photoelectron $\left(\frac{1}{2}mv^2_{\text{max}}\right)$

According to Einstein's photoelectric emission of light

i.e. $\qquad E = (\text{KE})_{\text{max}} + \phi$

As, $\qquad \frac{hc}{\lambda} = (\text{KE})_{\text{max}} + \phi$

If the wavelength of radiation is changed to $\frac{3\lambda}{4}$, then

$$\Rightarrow \quad \frac{4}{3}\frac{hc}{\lambda} = \left(\frac{4}{3}(\text{KE})_{\text{max}} + \frac{\phi}{3}\right) + \phi$$

$(\text{KE})_{\text{max}}$ for fastest emitted electron $= \frac{1}{2}mv'^2 + \phi$

$$\Rightarrow \quad \frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

i.e. $\qquad v' > v\left(\frac{4}{3}\right)^{1/2}$

13.
$$\frac{hc}{\lambda} - \phi = eV_0$$

$$\frac{hc}{0.3\times 10^{-6}} - \phi = 2e \qquad \ldots\text{(i)}$$

$$\frac{hc}{0.4\times 10^{-6}} - \phi = 1e \qquad \ldots\text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i)

$$hc\left(\frac{1}{0.3} - \frac{1}{0.4}\right)10^6 = e$$

$$hc\left(\frac{0.1}{0.12}\times 10^6\right) = e$$

$$h = 0.64\times 10^{-33} = 6.4\times 10^{-34}\,\text{J-s}$$

14. Energy corresponding to 248 nm wavelength

$$= \frac{1240}{248}\,\text{eV} = 5\,\text{eV}$$

Energy corresponding to 310 nm wavelength

$$= \frac{1240}{310}\,\text{eV} = 4\,\text{eV}$$

$$\frac{\text{KE}_1}{\text{KE}_2} = \frac{u_1^2}{u_2^2} = \frac{4}{1} = \frac{5\,\text{eV} - W}{4\,\text{eV} - W}$$

$$\Rightarrow \quad 16 - 4W = 5 - W \Rightarrow 11 = 3W$$

$$\Rightarrow \quad W = \frac{11}{3} = 3.67\,\text{eV} \cong 3.7\,\text{eV}$$

15. **Key Idea** The problem is based on frequency dependence of photoelectric emission. When incident light with certain frequency (greater than on the threshold frequency is focus on a metal surface) then some electrons are emitted from the metal with substantial initial speed.

When an electron moves in a circular path, then

$$r = \frac{mv}{eB} \quad \Rightarrow \quad \frac{r^2e^2B^2}{2} = \frac{m^2v^2}{2}$$

$$\text{KE}_{\text{max}} = \frac{(mv)^2}{2m} \quad \Rightarrow \quad \frac{r^2e^2B^2}{2m} = (\text{KE})_{\text{max}}$$

Work function of the metal (W), i.e. $W = h\nu - \text{KE}_{\text{max}}$

$$1.89 - \phi = \frac{r^2e^2B^2}{2m}\frac{1}{2}\text{eV} = \frac{r^2eB^2}{2m}\text{eV}$$

[$h\nu \to 1.89$ eV, for the transition on from third to second orbit of H-atom]

$$= \frac{100\times 10^{-6}\times 1.6\times 10^{-19}\times 9\times 10^{-8}}{2\times 9.1\times 10^{-31}}$$

$$\phi = 1.89 - \frac{1.6\times 9}{2\times 9.1} = 1.89 - 0.79 = 1.1\,\text{eV}$$

16. $E_1 = \frac{1240}{550} = 2.25\,\text{eV}, \qquad E_2 = \frac{1240}{450} = 2.75\,\text{eV}$

$E_3 = \frac{1240}{350} = 3.54\,\text{eV}$

E_1 cannot emit photoelectrons from q and r plates. E_2 can not emit photoelectrons from r.

Further, work function of p is least and it can emit photoelectrons from all three wavelengths. Hence magnitude of its stopping potential and saturation current both will be maximum.

17. Saturation current is proportional to intensity while stopping potential increases with increase in frequency.

Hence, $\qquad f_a = f_b$ while $I_a < I_b$

18. $\lambda\ (\text{in Å}) = \frac{12375}{W\ (\text{eV})} = \frac{12375}{4.0}\,\text{Å} \approx 3093\,\text{Å}$

or $\qquad \lambda \approx 309.3\,\text{nm} \approx 310\,\text{nm}$

NOTE $\lambda(\text{in Å}) = \frac{12375}{W\ (\text{eV})}$ comes from $W = \frac{hc}{\lambda}$

19. Stopping potential is the negative potential applied to stop the electrons having maximum kinetic energy. Therefore, stopping potential will be 4 V.

20. $$eV_0 = \frac{hc}{\lambda} - W$$

$$V_0 = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{W}{e}$$

V_0 *versus* $\frac{1}{\lambda}$ graph is in the form $y = mx - c$

Therefore option (c) is correct.

Clearly, V_0 *versus* λ graph is not a straight line but V_0 decreases with increase in λ and V_0 becomes zero when $\frac{hc}{\lambda} = W$.

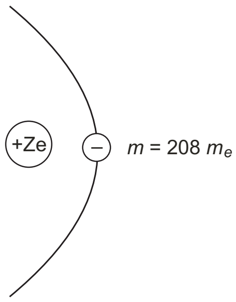
i.e. $\lambda = \lambda_0$ (Threshold wavelength)

$\therefore$ Option (a) is also correct.

21. From the relation,

$$eV = \frac{hc}{\lambda} - \phi$$

or $$V = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$$



This is equation of straight line.

Slope is $\tan\theta = \frac{hc}{e}$

$$\phi_1 : \phi_2 : \phi_3 = \frac{hc}{\lambda_{01}} : \frac{hc}{\lambda_{02}} : \frac{hc}{\lambda_{03}}$$

$$= \frac{1}{\lambda_{01}} : \frac{1}{\lambda_{02}} : \frac{1}{\lambda_{03}} = 1 : 2 : 4$$

$$\frac{1}{\lambda_{01}} = 0.001\ \text{nm}^{-1} \text{ or } \lambda_{01} = 10000\ \text{Å}$$

$$\frac{1}{\lambda_{02}} = 0.002\ \text{nm}^{-1} \text{ or } \lambda_{02} = 5000\ \text{Å}$$

$$\frac{1}{\lambda_{03}} = 0.004\ \text{nm}^{-1} \text{ or } \lambda_{03} = 2500\ \text{Å}$$

Violet colour has wavelength 4000 Å.

So, violet colour can eject photoelectrons from metal-1 and metal-2.

22. $K_{\max} = E - W$

Therefore,

$$T_A = 4.25 - W_A \qquad \text{...(i)}$$

$$T_B = (T_A - 1.50) = 4.70 - W_B \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$W_B - W_A = 1.95\ \text{eV} \qquad \text{...(iii)}$$

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2Km}} \quad \text{or} \quad \lambda \propto \frac{1}{\sqrt{K}} \qquad K = \text{KE of electron}$$

$$\therefore \quad \frac{\lambda_B}{\lambda_A} = \sqrt{\frac{K_A}{K_B}} \quad \text{or} \quad 2 = \sqrt{\frac{T_A}{T_A - 1.5}}$$

This gives, $T_A = 2\ \text{eV}$

From Eq. (i) $W_A = 4.25 - T_A = 2.25\ \text{eV}$

From Eq. (iii) $W_B = W_A + 1.95\text{eV} = (2.25 + 1.95)\text{eV}$

or $W_B = 4.20\ \text{eV}$

$$T_B = 4.70 - W_B = 4.70 - 4.20$$
$$= 0.50\ \text{eV}$$

23. (b) Stopping potential depends on two factors – one the energy of incident light and the other the work function of the metal. By increasing the distance of source from the cell, neither of the two change. Therefore, stopping potential remains the same.

(d) Saturation current is directly proportional to the intensity of light incident on cell and for a point source, intensity $I \propto 1/r^2$

When distance is increased from 0.2 m to 0.6 m (three times), the intensity and hence the saturation current will decrease 9 times, i.e. the saturation current will be reduced to 2.0 mA.

24. No solution is required.

25. For photoemission to take place, wavelength of incident light should be less than the threshold wavelength. Wavelength of ultraviolet light < 5200 Å while that of infrared radiation > 5200 Å.

26. $$eV_0 = hf - W$$

$$\therefore \quad V_0 = \left(\frac{h}{e}\right)f - \frac{W}{e}$$

V_0 *versus* f graph is a straight line with slope $= \frac{h}{e} =$ a universal constant. Therefore, the ratio of two slopes should be 1.

27. Photo emission will stop when potential on silver sphere becomes equal to the stopping potential.

$$\therefore \quad \frac{hc}{\lambda} - W = eV_0$$

Here, $$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{ze}{r}$$

$$\therefore \quad \left(\frac{1240}{200}\ \text{eV}\right) - (4.7\ \text{eV}) = \frac{9 \times 10^9 \times n \times 1.6 \times 10^{-19}}{10^{-2}}$$

$$(6.2 - 4.7) = \frac{9 \times 10^9 \times n \times 1.6 \times 10^{-19}}{10^{-2}}$$

or $$z = \frac{1.5 \times 10^{-2}}{9 \times 1.6 \times 10^{-10}}$$

$$= 1.04 \times 10^7$$

$\therefore$ Answer is 7.

28. $K_{\max} = h\nu - W$

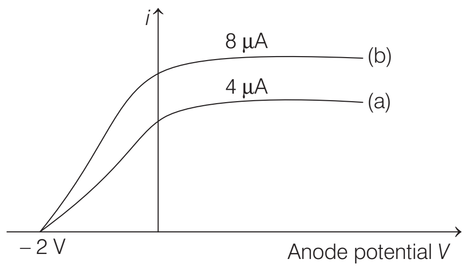
Therefore, $K_{\max}$ is linearly dependent on frequency of incident radiation.

29. Maximum energy of photoelectrons increases with increase in frequency of incident light. So, if intensity is increased by increasing frequency of incident light, maximum energy will increase. If intensity is increased merely by increasing number of photons incident per second, maximum energy of photoelectrons will not change. So, question is incomplete because it is not mentioned whether how the intensity is increased ?

30. Kinetic energy of photoelectrons depends on frequency of incident radiation.

31. Maximum kinetic energy of the photoelectrons would be

$$K_{\max} = E - W = (5 - 3)\text{eV} = 2\,\text{eV}$$



Therefore, the stopping potential is 2 V. Saturation current depends on the intensity of light incident. When the intensity is doubled the saturation current will also become two fold. The corresponding graphs are shown in above figure.

32. Area of plates $A = 5 \times 10^{-4}\ \text{m}^2$

Distance between the plates $d = 1\ \text{cm} = 10^{-2}\,\text{m}$

(a) Number of photoelectrons emitted upto $t = 10$ s are

$$n = \frac{\text{(number of photons falling in unit area in unit time)} \times \text{(area} \times \text{time)}}{10^6}$$

$$= \frac{1}{10^6}[(10)^{16} \times (5 \times 10^{-4}) \times (10)] = 5.0 \times 10^7$$

(b) At time, $t = 10\,\text{s}$

Charge on plate A, $q_A = + ne = (5.0 \times 10^7)(1.6 \times 10^{-19})$

$$= 8.0 \times 10^{-12}\ \text{C}$$

and charge on plate B,

$$q_B = (33.7 \times 10^{-12} - 8.0 \times 10^{-12})$$

$$= 25.7 \times 10^{-12}\ \text{C}$$

$\therefore$ Electric field between the plates, $E = \dfrac{(q_B - q_A)}{2A\varepsilon_0}$

or $E = \dfrac{(25.7 - 8.0) \times 10^{-12}}{2 \times (5 \times 10^{-4})(8.85 \times 10^{-12})} = 2 \times 10^3\ \text{N/C}$

(c) Energy of photoelectrons at plate A

$$= E - W = (5 - 2)\,\text{eV} = 3\,\text{eV}$$

Increase in energy of photoelectrons

$$= (eEd)\ \text{joule} = (Ed)\ \text{eV}$$

$$= (2 \times 10^3)(10^{-2})\ \text{eV} = 20\ \text{eV}$$

Energy of photoelectrons at plate B

$$= (20 + 3)\,\text{eV} = 23\ \text{eV}$$

33. Given work function, $W = 1.9\,\text{eV}$

Wavelength of incident light, $\lambda = 400\,\text{nm}$

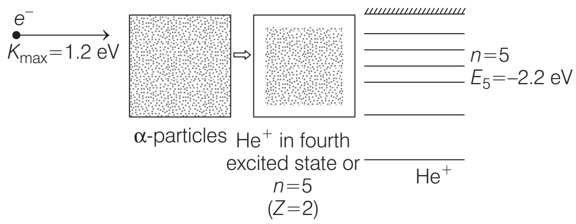
$\therefore$ Energy of incident light, $E = \dfrac{hc}{\lambda} = 3.1\,\text{eV}$

(Substituting the values of h, c and λ)

Therefore, maximum kinetic energy of photoelectrons

$$K_{\max} = E - W = (3.1 - 1.9) = 1.2\,\text{eV}$$

Now the situation is as shown below :



Energy of electron in 4[th] excited state of He$^+$ $(n = 5)$ will be

$$E_5 = -13.6\frac{Z^2}{n^2}\ \text{eV} \quad \Rightarrow \quad E_5 = -(13.6)\frac{(2)^2}{(5)^2} = -2.2\,\text{eV}$$

Therefore, energy released during the combination

$$= 1.2 - (-2.1) = 3.4\ \text{eV}$$

Similarly, energies in other energy states of He$^+$ will be

$$E_4 = -13.6\frac{(2)^2}{(4)^2} = -3.4\ \text{eV}$$

$$E_3 = -13.6\frac{(2)^2}{(3)^2} = -6.04\ \text{eV}$$

$$E_2 = -13.6\frac{(2)^2}{2^2} = -13.6\,\text{eV}$$

The possible transitions are

$\Delta E_{5\to 4} = E_5 - E_4 = 1.2\,\text{eV} < 2\ \text{eV}$

$\Delta E_{5\to 3} = E_5 - E_3 = 3.84\ \text{eV}$

$\Delta E_{5\to 2} = E_5 - E_2 = 11.4\ \text{eV} > 4\text{eV}$

$\Delta E_{4\to 3} = E_4 - E_3 = 2.64\ \text{eV}$

$\Delta E_{4\to 2} = E_4 - E_2 = 10.2\,\text{eV} > 4\ \text{eV}$

Hence, the energy of emitted photons in the range of 2 eV and 4 eV are

3.4 eV during combination and

3.84 eV and 2.64 after combination.

34. (a) Energy of emitted photons

$$E_1 = 5.0\ \text{eV} = 5.0 \times 1.6 \times 10^{-19}\,\text{J} = 8.0 \times 10^{-19}\,\text{J}$$

Power of the point source is 3.2×10^{-3} W or 3.2×10^{-3} J/s.

Therefore, energy emitted per second,

$$E_2 = 3.2 \times 10^{-3}\,\text{J}$$

Hence, number of photons emitted per second $n_1 = \dfrac{E_2}{E_1}$

or $$n_1 = \frac{3.2 \times 10^{-3}}{8.0 \times 10^{-19}}$$

$$n_1 = 4.0 \times 10^{15} \text{ photon/s}$$

Number of photons incident on unit area at a distance of 0.8 m from the source S will be

$$n_2 = \frac{n_1}{4\pi\,(0.8)^2} = \frac{4.0 \times 10^{15}}{4\pi\,(0.64)}$$

$$\approx 5 \times 10^{14} \text{photon/s -m}^2$$

The area of metallic sphere over which photons will fall is

$$A = \pi r^2 = \pi\,(8 \times 10^{-3})^2 \text{ m}^2 \approx 2.01 \times 10^{-4} \text{ m}^2$$

Therefore, number of photons incident on the sphere per second are

$$n_3 = n_2\,A = (5.0 \times 10^{14} \times 2.01 \times 10^{-4}) \approx 10^{11}/\text{s}$$

But since, one photoelectron is emitted for every 10^6 photons, hence number of photoelectrons emitted per second.

$$n = \frac{n_3}{10^6} = \frac{10^{11}}{10^6} = 10^5/\text{s} \text{ or } n = 10^5/\text{s}$$

(b) Maximum kinetic energy of photoelectrons

K_{max} = Energy of incident photons – work function

$$= (5.0 - 3.0)\,\text{eV} = 2.0\,\text{eV}$$

$$= 2.0 \times 1.6 \times 10^{-19}\,\text{J}$$

$$K_{max} = 3.2 \times 10^{-19}\,\text{J}$$

The de-Broglie wavelength of these photoelectrons will be

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2K_{max}m}}$$

Here, h = Planck's constant and m = mass of electron

$$\lambda_1 = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-19} \times 9.1 \times 10^{-31}}}$$

$$= 8.68 \times 10^{-10} = 8.68\,\text{Å}$$

Wavelength of incident light λ_2 (in Å) $= \dfrac{12375}{E_1\,(\text{in eV})}$

or $$\lambda_2 = \frac{12375}{5} = 2475\text{Å}$$

Therefore, the desired ratio is

$$\frac{\lambda_2}{\lambda_1} = \frac{2475}{8.68} = 285.1$$

(c) As soon as electrons are emitted from the metal sphere, it gets positively charged and acquires positive potential. The positive potential gradually increases as more and more photoelectrons are emitted from its surface. Emission of photoelectrons is stopped when its potential is equal to the stopping potential required for fastest moving electrons.

(d) As discussed in part (c), emission of photoelectrons is stopped when potential on the metal sphere is equal to the stopping potential of fastest moving electrons.

Since, $K_{max} = 2.0\,\text{eV}$

Therefore, stopping potential $V_0 = 2$ V. Let q be the charge required for the potential on the sphere to be equal to stopping potential or 2 V. Then

$$2 = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{q}{r} = (9.0 \times 10^9)\,\frac{q}{8.0 \times 10^{-3}}$$

$$\therefore \quad q = 1.78 \times 10^{-12}\,\text{C}$$

Photoelectrons emitted per second = 10^5 [Part (a)]

or charge emitted per second = $(1.6 \times 10^{-19}) \times 10^5$ C

$$= 1.6 \times 10^{-14}\,\text{C}$$

Therefore, time required to acquire the charge q will be

$$t = \frac{q}{1.6 \times 10^{-14}}\,\text{s} = \frac{1.78 \times 10^{-12}}{1.6 \times 10^{-14}}\,\text{s} \quad \text{or} \quad t \approx 111 \text{ s}$$

35. (a) From Einstein's equation of photoelectric effect,

Energy of photons causing the photoelectric emission = Maximum kinetic energy of emitted photons + work function

or $$E = K_{max} + W = (0.73 + 1.82) \text{ eV}$$

or $$E = 2.55\,\text{eV}$$

(b) In case of a hydrogen atom,

$$E_1 = -13.6 \text{ eV}, E_2 = -3.4 \text{ eV}, E_3 = -1.5\,\text{eV},$$
$$E_4 = -0.85\,\text{eV}$$

Since, $E_4 - E_2 = 2.55\,\text{eV}$

Therefore, quantum numbers of the two levels involved in the emission of these photons are 4 and 2($4 \to 2$).

(c) Change in angular momentum in transition from 4 to 2 will be

$$\Delta L = L_2 - L_4 = 2\left(\frac{h}{2\pi}\right) - 4\left(\frac{h}{2\pi}\right) \text{ or } \Delta L = -\frac{h}{\pi}$$

(d) From conservation of linear momentum

| Momentum of hydrogen atom | = | Momentum of emitted photon |

or $$mv = \frac{E}{c} \qquad (m = \text{mass of hydrogen atom})$$

or $$v = \frac{E}{mc} = \frac{(2.55 \times 1.6 \times 10^{-19} \text{ J})}{(1.67 \times 10^{-27}\text{kg})\,(3.0 \times 10^8\text{m/s})}$$

$$v = 0.814 \text{ m/s}$$

36. Energy of photon having wavelength 4144 Å,

$$E_1 = \frac{12375}{4144} \text{ eV} = 2.99 \text{ eV}$$

Similarly, $$E_2 = \frac{12375}{4972} \text{ eV} = 2.49 \text{ eV and}$$

$$E_3 = \frac{12375}{6216} \text{ eV} = 1.99 \text{ eV}$$

Since, only E_1 and E_2 are greater than the work function $W = 2.3$ eV, only first two wavelengths are capable for ejecting photoelectrons. Given intensity is equally distributed in all wavelengths. Therefore, intensity corresponding to each wavelength is

$$\frac{3.6 \times 10^{-3}}{3} = 1.2 \times 10^{-3} \text{ W/m}^2$$

Or energy incident per second in the given area $(A = 1.0 \text{ cm}^2 = 10^{-4} \text{ m}^2)$ is

$$\rho = 1.2 \times 10^{-3} \times 10^{-4}$$
$$= 1.2 \times 10^{-7} \text{ J/s}$$

Let n_1 be the number of photons incident per unit time in the given area corresponding to first wavelength. Then

$$n_1 = \frac{\rho}{E_1} = \frac{1.2 \times 10^{-7}}{2.99 \times 1.6 \times 10^{-19}}$$
$$= 2.5 \times 10^{11}$$

Similarly, $$n_2 = \frac{\rho}{E_2} = \frac{1.2 \times 10^{-7}}{2.49 \times 1.6 \times 10^{-19}}$$
$$= 3.0 \times 10^{11}$$

Since, each energetically capable photon ejects electron, total number of photoelectrons liberated in 2 s.

$$= 2(n_1 + n_2) = 2(2.5 + 3.0) \times 10^{11}$$
$$= 1.1 \times 10^{12}$$

Topic 3 Radioactivity

1. An α-particle decay $({}_2^4\text{He})$ reduces, mass number by 4 and atomic number by 2.

$\therefore$ Decay of 6α-particles results

$$^{232}_{90}\text{Th} \xrightarrow{6\alpha} {}^{232-24}_{90-12}\text{Y} = {}^{208}_{78}\text{Y}$$

A β-decay does not produces any change in mass number but it increases atomic number by 1.

$\therefore$ Decay of 4β-particles results

$$^{208}_{78}\text{Y} \xrightarrow{4\beta} {}^{208}_{82}\text{X}$$

$\therefore$ In the end nucleus $A = 208, Z = 82$

2. Here given, at $t = 0$, count rate or initial activity is

$$A_0 = 1600 \text{ s}^{-1}.$$

At $t = 8$ s, count rate or activity is

$$A = 100 \text{ s}^{-1}$$

So, decay scheme for given sample is

$$1600 \xrightarrow{T_{1/2}} 800 \xrightarrow{T_{1/2}} 400 \xrightarrow{T_{1/2}} 200 \xrightarrow{T_{1/2}} 100$$

So, $$8 \text{ s} = 4T_{1/2}$$

where, $T_{1/2}$ = Half-life time.

$$\Rightarrow \quad T_{1/2} = 2 \text{ s}$$

$\therefore$ From above decay scheme, we see that activity after 6 s is 200 counts per second.

3. Activity of radioactive material is given as

$$R = \lambda N$$

where, λ is the decay constant N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = \lambda_A N_{0A} \text{ (initially } N_A = N_{0A})$$

For substance B,

$$R_B = \lambda_B N_B = \lambda_B N_{0B} \text{ (initially } N_B = N_{0B})$$

At $t = 0$, activity is equal, therefore

$$\lambda_A N_{0A} = \lambda_B N_{0B} \quad \ldots\text{(i)}$$

The half-life is given by

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$$

So, for substance A,

$$(T_{1/2})_A = \frac{\ln 2}{\lambda_A} \Rightarrow \ln 2 = \frac{\ln 2}{\lambda_A}$$

$$\lambda_A = 1 \quad \ldots\text{(ii)}$$

According to the given question,

at time t,

$$\frac{R_B}{R_A} = e^{-3t} \quad \ldots\text{(iii)}$$

Using Eqs. (i), (ii) and (iii)

$$\frac{R_B}{R_A} = e^{-3t} = \frac{\lambda_B N_{0B} e^{-\lambda_B t}}{\lambda_A N_{0A} e^{-\lambda_A t}}$$

$$\Rightarrow \quad e^{-3t} = e^{(\lambda_A - \lambda_B)t}$$

$$-3 = \lambda_A - \lambda_B$$
$$\Rightarrow \quad \lambda_B = \lambda_A + 3$$
$$\lambda_B = 1 + 3 = 4 \quad \ldots\text{(iv)}$$

The half-life of substance B is

$$(T_{1/2})_B = \frac{\ln 2}{\lambda_B} = \frac{\ln 2}{4}$$

4. Activity of a radioactive material is given as

$$R = \lambda N$$

where, λ is the decay constant and N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = 10 \text{ mCi}$$

For substance B,

$$R_B = \lambda_B N_B = 20 \text{ mCi} \quad \ldots\text{(i)}$$

As given in the question,

$$N_A = 2N_B$$

$\Rightarrow \qquad R_A = \lambda_A (2N_B) = 10 \text{ mCi} \qquad ...(ii)$

$\therefore$ Dividing Eq. (ii) and Eq.(i), we get

$$\frac{R_A}{R_B} = \frac{\lambda_A (2N_B)}{\lambda_B (N_B)} = \frac{10}{20}$$

or $\qquad \dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{4} \qquad ...(iii)$

As, half-life of a radioactive material is given as

$$T_{1/2} = \frac{0.693}{\lambda}$$

$\therefore$ For material A and B, we can write

$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{\dfrac{0.693}{\lambda_A}}{\dfrac{0.693}{\lambda_B}} = \frac{\lambda_B}{\lambda_A}$$

Using Eq. (iii), we get

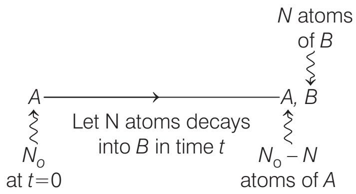
$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{4}{1}$$

Hence, from the given options, only option (d) satisfies this ratio.

Therefore, $(T_{1/2})_A = 20$ days

and $\qquad (T_{1/2})_B = 5$ days

5. Decay scheme is ,



Given, $\dfrac{N_B}{N_A} = 0.3 = \dfrac{3}{10}$

$\Rightarrow \qquad \dfrac{N_B}{N_A} = \dfrac{30}{100}$

So, $\qquad N_0 = 100 + 30 = 130$ atoms

By using $\quad N = N_0 e^{-\lambda t}$

We have, $\qquad 100 = 130 e^{-\lambda t}$

$\Rightarrow \qquad \dfrac{1}{1.3} = e^{-\lambda t} \Rightarrow \log 1.3 = \lambda t$

$\Rightarrow \qquad \log 1.3 = \dfrac{\log 2}{T} \cdot t$

$\therefore \qquad T = \dfrac{T \cdot \log (1.3)}{\log 2}$

6. **A :** Numbers left : N $\rightarrow \dfrac{N}{2} \rightarrow \dfrac{N}{4} \rightarrow \dfrac{N}{8} \rightarrow \dfrac{N}{16}$

$\therefore$ Number decayed, $N_A = N - \dfrac{N}{16} = \dfrac{15}{16}N$

B : Numbers left : $N \rightarrow \dfrac{N}{2} \rightarrow \dfrac{N}{4}$

$\therefore$ Numbers decayed, $N_B = N - \dfrac{N}{4} = \dfrac{3}{4}N$

Ratio : $\qquad \dfrac{N_A}{N_B} = \dfrac{(15/16)N}{(3/4)N} = \dfrac{5}{4}$

7. Using the relation

$$R = R_0 \left(\frac{1}{2}\right)^n$$

Here, R is activity of radioactive substance, R_0 initial activity and n is number of half lives.

$$1 = 64 \left(\frac{1}{2}\right)^n$$

Solving we get, $n = 6$

Now, $\qquad t = n(t_{12})$

$\qquad = 6(18 \text{ days})$

$\qquad = 108 \text{ days}$

8. Activity of $S_1 = \dfrac{1}{2}$(activity of S_2)

or $\qquad \lambda_1 N_1 = \dfrac{1}{2}(\lambda_2 N_2)$

or $\qquad \dfrac{\lambda_1}{\lambda_2} = \dfrac{N_2}{2N_1}$

or $\qquad \dfrac{T_1}{T_2} = \dfrac{2N_1}{N_2} \qquad \left(T = \text{half-life} = \dfrac{\ln 2}{\lambda}\right)$

Given $\qquad N_1 = 2N_2$

$\therefore \qquad \dfrac{T_1}{T_2} = 4$

9. After two half lives $\dfrac{1}{4}$th fraction of nuclei will remain undecayed. Or, $\dfrac{3}{4}$th fraction will decay. Hence, the probability that a nucleus decays in two half lives is $\dfrac{3}{4}$.

10. Activity reduces from 6000 dps to 3000 dps in 140 days. It implies that half-life of the radioactive sample is 140 days. In 280 days (or two half-lives) activity will remain $\dfrac{1}{4}$ th of the initial activity. Hence, the initial activity of the sample is

$$4 \times 6000 \text{ dps} = 24000 \text{ dps}$$

11. During γ-decay atomic number (Z) and mass number (A) does not change. So, the correct option is (c) because in all other options either Z, A or both is/are changing.

12. $\qquad R = R_0 \left(\dfrac{1}{2}\right)^n \qquad \ldots(i)$

Here R = activity of radioactive substance after n half-lives

$= \dfrac{R_0}{16} \qquad$ (given)

Substituting in Eq. (i), we get $n = 4$

$\therefore \qquad t = (n)\, t_{1/2} = (4)\,(100\,\mu\text{s}) = 400\ \mu\text{s}$

13. The total number of atoms can neither remain constant (as in option a) nor can ever increase (as in options b and c). They will continuously decrease with time. Therefore, (d) is the appropriate option.

14. During β -decay, a neutron is transformed into a proton and an electron. This is why atomic number (Z = number of protons) increases by one and mass number (A = number of protons + neutrons) remains unchanged during beta decay.

15. $\dfrac{N_{x_1}(t)}{N_{x_2}(t)} = \dfrac{1}{e}$ or $\dfrac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}} = \dfrac{1}{e}$

(Initially, both have same number of nuclei say N_0)

or $e = e^{-\lambda t}/e^{-10\lambda t}$

or $e = e^{9\lambda t}$

or $9\lambda t = 1$

or $t = \dfrac{1}{9\lambda}$

16. $(t_{1/2})_x = (t_{\text{mean}})_y$

or $\dfrac{0.693}{\lambda_x} = \dfrac{1}{\lambda_y}$

$\therefore$ $\lambda_x = 0.693\,\lambda_y$

$\lambda_x < \lambda_y$

or Rate of decay $= \lambda N$

Initially number of atoms (N) of both are equal but since $\lambda_y > \lambda_x$, therefore, y will decay at a faster rate than x.

17. Both the beta rays and the cathode rays are made up of electrons. So, only option (a) is correct.
(b) Gamma rays are electromagnetic waves.
(c) Alpha particles are doubly ionized helium atoms and
(d) Protons and neutrons have approximately the same mass.
Therefore, (b), (c) and (d) are wrong options.

18. Number of nuclei decreases exponentially

$$N = N_0\, e^{-\lambda t}$$

and rate of decay $\left(-\dfrac{dN}{dt}\right) = \lambda N$

Therefore, decay process lasts upto $t = \infty$. Therefore, a given nucleus may decay at any time after $t = 0$.

19. In beta decay, atomic number increases by 1 whereas the mass number remains the same.

Therefore, following equation can be possible

$$^{64}_{29}\text{Cu} \longrightarrow {}^{64}_{30}\text{Zn} + {}_{-1}e^0$$

20. Penetrating power is maximum for γ-rays, then of β -particles and then α-particles because basically it depends on the velocity. However, ionization power is in reverse order.

21. As we know, $T_{y2} = \dfrac{\ln 2}{\lambda}$ and $\tau = \dfrac{1}{\lambda}$.

22. From $R = R_0\left(\dfrac{1}{2}\right)^n$

we have, $1 = 64\left(\dfrac{1}{2}\right)^n$

or $n = 6$ = number of half lives

$\therefore$ $t = n \times t_{t_{1/2}} = 6 \times 2 = 12\,\text{h}$

24. Beta particles are fast moving electrons which are emitted by the nucleus.

25. Using $N = N_0 e^{-\lambda t}$

where $\lambda = \dfrac{\ln 2}{t_{1/2}} = \dfrac{\ln(2)}{3.8}$

$\therefore$ $\dfrac{N_0}{20} = N_0 e^{-\frac{\ln(2)}{3.8}t}$

Solving this equation with the help of given data we find :

$$t = 16.5\,\text{days}$$

26. $^{232}_{90}\text{Th}$ is converting into $^{212}_{82}\text{Pb}$.

Change in mass number $(A) = 20$

$\therefore$ Number of α-particle emitted $= \dfrac{20}{4} = 5$

Due to 5 α-particles, Z will change by 10 units.

Since, given change is 8, therefore number of β-particles emitted is 2.

27. $\text{I}^{131} \xrightarrow[T_{1/2} = 8\text{ Days}]{} \text{Xe}^{131} + \beta$

$A_0 = 2.4 \times 10^5\ \text{Bq} = \lambda N_0$

Let the volume is V,

$t = 0$ $\quad A_0 = \lambda N_0$

$t = 11.5\,\text{h}$ $\quad A = \lambda N$

$$115 = \lambda\left(\frac{N}{V} \times 2.5\right)$$

$$115 = \frac{\lambda}{V} \times 2.5 \times (N_0 e^{-\lambda t})$$

$$115 = \frac{(N_0\lambda)}{V} \times (2.5) \times e^{-\frac{\ln 2}{8\text{ day}}(11.5\text{ h})}$$

$$115 = \frac{(2.4 \times 10^5)}{V} \times (2.5) \times e^{-1/24}$$

$$V = \frac{2.4 \times 10^5}{115} \times 2.5\left[1 - \frac{1}{24}\right]$$

$$= \frac{2.4 \times 10^5}{115} \times 2.5\left[\frac{23}{24}\right]$$

$$= \frac{10^5 \times 23 \times 25}{115 \times 10^2} = 5 \times 10^3\,\text{ml} = 5\,\text{L}$$

28. Let initial numbers are N_1 and N_2.

$$\frac{\lambda_1}{\lambda_2} = \frac{\tau_2}{\tau_1} = \frac{2\tau}{\tau} = 2 = \frac{T_2}{T_1} \qquad (T = \text{Half life})$$

$$A = \frac{-dN}{dt} = \lambda N$$

Initial activity is same

$\therefore\ \lambda_1 N_1 = \lambda_2 N_2$ ...(i)

Activity at time t,

$$A = \lambda N = \lambda N_0\, e^{-\lambda t}$$

$$A_1 = \lambda_1 N_1\, e^{-\lambda_1 t}$$

$$\Rightarrow \quad R_1 - = \frac{dA_1}{dt} = \lambda_1^2\, N_1\, e^{-\lambda_1 t}$$

Similarly, $R_2 = \lambda_2^2\, N_2\, e^{-\lambda_2 t}$

After $t = 2\tau$

$$\lambda_1 t = \frac{1}{\tau_1}(t) = \frac{1}{\tau}(2\tau) = 2$$

$$\lambda_2 t = \frac{1}{\tau_2}(t) = 1 = \frac{1}{2\tau}(2\tau) = 1$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1^2 N_1 e^{-\lambda_1 t}}{\lambda_2^2 N_2 e^{-\lambda_2 t}}$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1}{\lambda_2}\left(\frac{e^{-2}}{e^{-1}}\right) = \frac{2}{e}$$

29. Number of nuclei decayed in time t,

$$N_d = N_0(1 - e^{-\lambda t})$$

$$\therefore \quad \%\text{ decayed} = \left(\frac{N_d}{N_0}\right) \times 100$$

$$= (1 - e^{-\lambda t} d) \times 100 \quad \text{...(i)}$$

Here, $\lambda = \dfrac{0.693}{1386} = 5 \times 10^{-4}\ \text{s}^{-1}$

$$\therefore \quad \%\text{ decayed} \approx (\lambda t) \times 100$$

$$= (5 \times 10^{-4})(80)(100) = 4$$

30. Activity $\left(-\dfrac{dN}{dt}\right) = \lambda N = \left(\dfrac{1}{t_{\text{mean}}}\right) \times N$

$\therefore\ N = \left(-\dfrac{dN}{dt}\right) \times t_{\text{mean}}$ = Total number of atoms

Mass of one atom is 10^{-25} kg = m (say)

$\therefore$ Total mass of radioactive substance

= (number of atoms) × (mass of one atom)

$$= \left(-\frac{dN}{dt}\right)(t_{\text{mean}})(m)$$

Substituting the values, we get

Total mass of radioactive substance = 1 mg

$\therefore$ Answer is 1.

31. ${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + \beta^+ + \gamma$ (neutrino)

32. ${}_5\text{B}^{10} + {}_0n^1 \longrightarrow {}_2\text{He}^4 + {}_3\text{Li}^7$

Therefore, resulting nucleus is lithium and its mass number is 7.

33. Number of α-particles emitted, $n_1 = \dfrac{238 - 206}{4} = 8$

and number of β-particles emitted are say n_2, then

$$92 - 8 \times 2 + n_2 = 82$$

$$\therefore \quad n_2 = 6$$

34. $R = R_0\left(\dfrac{1}{2}\right)^n$

Here R_0 = initial activity = 1000 disintegration/s

and n = number of half-lives.

At $t = 1$ s, $n = 1$

$$\therefore \quad R = 10^3\left(\frac{1}{2}\right) = 500 \text{ disintegration/s}$$

At $t = 3$ s, $n = 3$

$$R = 10^3\left(\frac{1}{2}\right)^3 = 125 \text{ disintegration/s}$$

35. Let N_0 be the initial number of nuclei of ^{238}U.

After time t, $N_{\text{U}} = N_0\left(\dfrac{1}{2}\right)^n$

Here n = number of half-lives $= \dfrac{t}{t_{1/2}} = \dfrac{1.5 \times 10^9}{4.5 \times 10^9} = \dfrac{1}{3}$

$$N_{\text{U}} = N_0\left(\frac{1}{2}\right)^{\frac{1}{3}} \text{ and } N_{\text{Pb}} = N_0 - N_{\text{U}} = N_0\left[1 - \left(\frac{1}{2}\right)^{1/3}\right]$$

$$\therefore \quad \frac{N_{\text{U}}}{N_{\text{Pb}}} = \frac{\left(\frac{1}{2}\right)^{1/3}}{1 - \left(\frac{1}{2}\right)^3} = 3.861$$

36. Let n_0 be the number of radioactive nuclei at time $t = 0$. Number of nuclei decayed in time t are given by $n_0\,(1 - e^{-\lambda t})$, which is also equal to the number of beta particles emitted during the same interval of time. For the given condition,

$$n = n_0\,(1 - e^{-2\lambda}) \quad \text{...(i)}$$

$$(n + 0.75n) = n_0\,(1 - e^{-4\lambda}) \quad \text{...(ii)}$$

Dividing Eq. (ii) by (i), we get

$$1.75 = \frac{1 - e^{-4\lambda}}{1 - e^{-2\lambda}}$$

or $\quad 1.75 - 1.75\, e^{-2\lambda} = 1 - e^{-4\lambda}$

$$\therefore \quad 1.75\, e^{-2\lambda} - e^{-4\lambda} = \frac{3}{4} \quad \text{...(iii)}$$

Let us take $e^{-2\lambda} = x$

Then, the above equation is

$$x^2 - 1.75x + 0.75 = 0$$

or $$x = \frac{1.75 \pm \sqrt{(1.75)^2 - (4)(0.75)}}{2}$$

or $$x = 1 \text{ and } \frac{3}{4}$$

$\therefore$ From Eq. (iii) either

$$e^{-2\lambda} = 1 \text{ or } e^{-2\lambda} = \frac{3}{4}$$

but $e^{-2\lambda} = 1$ is not accepted because which means $\lambda = 0$. Hence, $e^{-2\lambda} = \frac{3}{4}$

or $-2\lambda \ln(e) = \ln(3) - \ln(4) = \ln(3) - 2\ln(2)$

$\therefore$ $$\lambda = \ln(2) - \frac{1}{2}\ln(3)$$

Substituting the given values,

$$\lambda = 0.6931 - \frac{1}{2} \times (1.0986) = 0.14395\ \text{s}^{-1}$$

$\therefore$ Mean-life $t_{\text{means}} = \frac{1}{\lambda} = 6.947$ s

37. (a) Let at time $t = t$, number of nuclei of Y and Z are N_Y and N_Z. Then,

Rate equations of the populations of X, Y and Z are

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \quad \text{...(i)}$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \quad \text{...(ii)}$$

and $$\left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \quad \text{...(iii)}$$

(b) Given $N_Y(t) = \dfrac{N_0 \lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$

For N_Y to be maximum

$$\frac{dN_Y(t)}{dt} = 0$$

i.e $\lambda_X N_X = \lambda_Y N_Y$ ...(iv) [from Eq. (ii)]

or $\lambda_X (N_0 e^{-\lambda_X t}) = \lambda_Y \dfrac{N_0 \lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$

or $$\frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1$$

$$\frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

or $$(\lambda_X - \lambda_y)\, t \ln(e) = \ln\left(\frac{\lambda_X}{\lambda_Y}\right)$$

or $$t = \frac{1}{\lambda_X - \lambda_Y} \ln\left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of λ_X and λ_Y, we have

$$t = \frac{1}{(0.1 - 1/30)} \ln\left(\frac{0.1}{1/30}\right) = 15 \ln(3)$$

or $t = 16.48$ s.

(c) The population of X at this moment,

$$N_X = N_0\, e^{-\lambda_X t} = (10^{20})\, e^{-(0.1)(16.48)}$$

$$N_X = 1.92 \times 10^{19}$$

$$N_Y = \frac{N_X \lambda_X}{\lambda_Y} \text{ [From Eq. (iv)]}$$

$$= (1.92 \times 10^{19})\left(\frac{0.1)}{(1/30)} = 5.76 \times 10^{19}$$

$$N_Z = N_0 - N_X - N_Y$$
$$= 10^{20} - 1.92 \times 10^{19} - 5.76 \times 10^{19}$$

or $N_Z = 2.32 \times 10^{19}$

38. (a) Let at time t, number of radioactive nuclei are N. Net rate of formation of nuclei of A

$$\frac{dN}{dt} = \alpha - \lambda N$$

or $$\frac{dN}{\alpha - \lambda N} = dt$$

or $$\int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving this equation, we get

$$N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}] \quad \text{...(i)}$$

(b) (i) Substituting $\alpha = 2\lambda N_0$ and $t = t_{1/2} = \dfrac{\ln(2)}{\lambda}$ in Eq. (i) we get, $N = \dfrac{3}{2}N_0$

(ii) Substituting $\alpha = 2\lambda N_0$ and $t \to \infty$ in Eq. (i), we get

$$N = \frac{\alpha}{\lambda} = 2N_0 \text{ or } N = 2N_0$$

39. (a) In 10 s, number of nuclei has been reduced to half (25% to 12.5%).

Therefore, its half-life is $t_{1/2} = 10$ s

Relation between half-life and mean life is

$$t_{\text{mean}} = \frac{t_{1/2}}{\ln(2)} = \frac{10}{0.693}\text{s}$$

$$t_{\text{mean}} = 14.43 \text{ s}$$

(b) From initial 100% to reduction till 6.25%, it takes four half lives.

$$100\% \xrightarrow{t_{1/2}} 50\% \xrightarrow{t_{1/2}} 25\% \xrightarrow{t_{1/2}} 12.5\% \xrightarrow{t_{1/2}} 6.25\%$$

$\therefore$ $t = 4\, t_{1/2} = 4\,(10)$ s $= 40$ s

$t = 40$ s

40. λ = Disintegration constant

$$\frac{0.693}{t_{1/2}} = \frac{0.693}{15}\,\text{h}^{-1} = 0.0462\,\text{h}^{-1}$$

Let R_0 = initial activity = 1 microcurie

$= 3.7 \times 10^4$ disintegrations per second.

r = Activity in 1 cm^3 of blood at $t = 5$h

$= \dfrac{296}{60}$ disintegration per second

$= 4.93$ disintegration per second, and

R = Activity of whole blood at time $t = 5$h

Total volume of blood should be

$$V = \frac{R}{r} = \frac{R_0 e^{-\lambda t}}{r}$$

Substituting the values, we have

$$V = \left(\frac{3.7 \times 10^4}{4.93}\right) e^{-(0.0462)(5)}\ \text{cm}^3$$

$$V = 5.95 \times 10^3\ \text{cm}^3 \quad \text{or} \quad V = 5.95\ \text{L}$$

41. Speed of neutrons $= \sqrt{\dfrac{2K}{m}} \quad \left(\text{From}\, K = \dfrac{1}{2}mv^2\right)$

or $$v = \sqrt{\frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}}$$

$$\approx 2.5 \times 10^3\ \text{m/s}$$

Time taken by the neutrons to travel a distance of 10 m :

$$t = \frac{d}{v} = \frac{10}{2.5 \times 10^3} = 4.0 \times 10^{-3}$$

Number of neutrons decayed after time

$t : N = N_0(1 - e^{-\lambda t})$

$\therefore$ Fraction of neutrons that will decay in this time interval

$$= \frac{N}{N_0} = (1 - e^{-\lambda t}) = 1 - e^{-\frac{\ln(2)}{700} \times 4.0 \times 10^{-3}}$$

$$= 3.96 \times 10^{-6}$$

42. $_{92}\text{U}^{238} \xrightarrow{\alpha\text{-decay}} {}_{90}X^{234} \xrightarrow{\beta\text{ - decay}} {}_{91}Y^{234}$

During an α-decay atomic number decreases by 2 and mass number by 4. During a β-decay, atomic number increases by 1 while mass number remains unchanged.

Topic 4 X-Rays and de-Broglie Wavelength

1. By Bohr's IInd postulate, for revolving electron,

$$\text{Angular momentum} = \frac{nh}{2\pi} \Rightarrow mvr_n = \frac{nh}{2\pi}$$

$\Rightarrow$ Momentum of electron, $p = mv = \dfrac{nh}{2\pi r_n}$

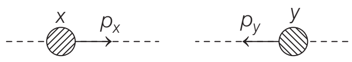
de-Broglie wavelength associated with electron is

$$\lambda_n = \frac{h}{p} = \frac{2\pi r_n}{n}$$

Given, $n = 3$, $r_n = 4.65$ Å

$$\therefore \quad \lambda_n = \frac{(2 \times \pi \times 4.65)}{3} \approx 9.7\ \text{Å}$$

2. Initially,

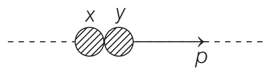


We have, de-Broglie wavelengths associated with particles are

$$\lambda_x = \frac{h}{p_x} \text{ and } \lambda_y = \frac{h}{p_y}$$

$$\Rightarrow \quad p_x = \frac{h}{\lambda_x} \text{ and } p_y = \frac{h}{\lambda_y}$$

Finally, particles collided to form a single particle.



As we know that linear momentum is conserved in collision, so

$$\mathbf{p}_p = |\,\mathbf{p}_x - \mathbf{p}_y\,| \Rightarrow \mathbf{p}_p = \left|\frac{h}{\lambda_x} - \frac{h}{\lambda_y}\right|$$

So, de-Broglie wavelength of combined particle is

$$\lambda_p = \frac{h}{|\,\mathbf{p}_p\,|} = \frac{h}{\left|\dfrac{h}{\lambda_x} - \dfrac{h}{\lambda_y}\right|} = \frac{h}{\left|\dfrac{h\lambda_y - h\lambda_x}{\lambda_x\lambda_y}\right|} = \frac{\lambda_x\lambda_y}{|\,\lambda_x - \lambda_y\,|}$$

3. Given, de-Broglie wavelengths for particles are λ_1 and λ_2.

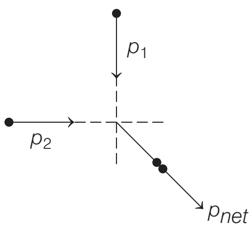
So, $\lambda_1 = \dfrac{h}{p_1}$ and $\lambda_2 = \dfrac{h}{p_2}$

and momentum of particles are

$$p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

Given that, particles are moving perpendicular to each other and collide inelastically.

So, they move as a single particle.



So, by conservation of momentum and vector addition law, net momentum after collision,

$$p_{\text{net}} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 90^\circ} = \sqrt{p_1^2 + p_2^2}$$

Since, $\quad p_1 = \dfrac{h}{\lambda_1}$ and $p_2 = \dfrac{h}{\lambda_2}$

So, $\quad p_{\text{net}} = \sqrt{\dfrac{h^2}{\lambda_1^2} + \dfrac{h^2}{\lambda_2^2}} \quad$ ...(i)

Let the de-Broglie wavelength after the collision is λ_{net}, then

$$p_{\text{net}} = \frac{h}{\lambda_{\text{net}}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{h}{\lambda_{\text{net}}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \Rightarrow \frac{1}{\lambda_{\text{net}}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

4. de-Broglie wavelength associated with a moving charged particle of charge q is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

where, $V =$ accelerating potential.

Ratio of de-Broglie wavelength for particle A and B is,

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{m_B q_B V_B}}{\sqrt{m_A q_A V_A}} = \sqrt{\frac{m_B}{m_A}} \cdot \sqrt{\frac{q_B}{q_A}} \cdot \sqrt{\frac{V_B}{V_A}}$$

Substituting the given values, we get.

$$= \sqrt{\frac{4m}{m}} \cdot \sqrt{\frac{q}{q}} \cdot \sqrt{\frac{2500}{50}}$$

$$= 2 \times 1 \times 5 \times 1.414 = 14.14$$

5. Wavelength of the given photon is given as,

$$\lambda_p = \frac{c}{\nu_p} = \frac{3 \times 10^8}{6 \times 10^{14}} \text{ m}$$

$$= 5 \times 10^{-7} \text{m} \quad \text{...(i)}$$

As, it is given that, de-Broglie wavelength of the electron is

$$\lambda_e = 10^{-3} \times \lambda_p \quad [\because \text{using Eq. (i)}]$$

$$= 5 \times 10^{-10} \text{m}$$

Also, the de-Broglie wavelength of an electron is given as,

$$\lambda_e = \frac{h}{p} = \frac{h}{mv_e} \Rightarrow v_e = \frac{h}{\lambda_e m_e}$$

Substituting the given values, we get

$$= \frac{6.63 \times 10^{-34}}{5 \times 10^{-10} \times 9.1 \times 10^{-31}} \text{ m/s}$$

$$= 1.45 \times 10^6 \text{ m/s}$$

6. $\quad 2\pi r = n\lambda_n$

$$\lambda_n = \frac{2\pi r}{n} = \frac{2\pi r_0 n^2}{n} = 2\pi r_0 n \quad \text{...(i)}$$

$$\frac{1}{\Lambda_n} = R\left\{\frac{1}{1^2} - \frac{1}{n^2}\right\}$$

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{1}{n^2 - 1}\right\}$$

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{1}{n^2}\right\} \quad (n >> 1) \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{4\pi^2 r_0^2}{\lambda_n^2}\right\} = A + \frac{B}{\lambda_n^2}$$

7. For elastic collision,

$$p_{\textbf{before collision}} = p_{\textbf{after collision}}.$$

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \quad \text{...(i)}$$

Now, coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - v_B}$$

Here, $u_B = 0$ (Particle at rest) and for elastic collision $e = 1$

$$\therefore \quad 1 = \frac{v_B - v_A}{v}$$

$$\Rightarrow \quad v = v_B - v_A \quad \text{...(ii)}$$

From Eq. (i) and Eq. (ii)

$$v_A = \frac{v}{3} \text{ and } v_B = \frac{4v}{3}$$

Hence, $\dfrac{\lambda_A}{\lambda_B} = \dfrac{\left(\dfrac{h}{mV_A}\right)}{\dfrac{h}{\dfrac{m}{2}.V_B}} = \dfrac{V_B}{2V_A} = \dfrac{4/3}{2/3} = 2$

8. According to photoelectric effect equation

$$\text{KE}_{\text{max}} = \frac{hc}{\lambda} - \phi_0$$

$$\frac{p^2}{2m} = \frac{hc}{\lambda} - \phi_0 \quad [\text{KE} = p^2/2m]$$

$$\frac{(h/\lambda_d)^2}{2m} = \frac{hc}{\lambda} - \phi_0 \quad [p = h/\lambda]$$

Assuming small changes, differentiating both sides,

$$\frac{h^2}{2m}\left(-\frac{2d\lambda_d}{\lambda_d^3}\right) = -\frac{hc}{\lambda^2}d\lambda, \quad \frac{d\lambda_d}{d\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

9. $K_{\text{max}} = \dfrac{hc}{\lambda_{\text{ph}}} - \phi$

Kinetic energy of electron reaching the anode will be

$$K = \frac{hc}{\lambda_{\text{ph}}} - \phi + eV$$

Now, $\quad \lambda_e = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{\sqrt{2m\left(\dfrac{hc}{\lambda_{\text{ph}}} - \phi + eV\right)}}$

If eV >> ϕ then, $\lambda_e = \dfrac{h}{\sqrt{2m\left(\dfrac{hc}{\lambda_{\text{ph}}} + eV\right)}}$

If $V_f = 4V_i$ then, $(\lambda_e)_f \simeq \dfrac{(\lambda_e)_i}{2}$

10. K_α transition takes place from $n_1 = 2$ to $n_2 = 1$

$$\therefore \quad \frac{1}{\lambda} = R(Z-b)^2\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$$

For K-series, $b = 1$

$$\therefore \quad \frac{1}{\lambda} \propto (Z-1)^2$$

$$\Rightarrow \quad \frac{\lambda_{\text{Cu}}}{\lambda_{\text{Mo}}} = \frac{(z_{\text{Mo}}-1)^2}{(z_{\text{Cu}}-1)^2} = \frac{(42-1)^2}{(29-1)^2}$$

$$= \frac{41\times 41}{28\times 28} = \frac{1681}{784} = 2.144$$

11. Cut-off wavelength depends on the applied voltage not on the atomic number of the target. Characteristic wavelengths depend on the atomic number of target.

12. Momentum of striking electrons

$$p = \frac{h}{\lambda}$$

$\therefore$ Kinetic energy of striking electrons

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

This is also, maximum energy of X-ray photons.

Therefore, $\dfrac{hc}{\lambda_0} = \dfrac{h^2}{2m\lambda^2}$ or $\lambda_0 = \dfrac{2m\lambda^2 c}{h}$

13. $$\frac{1}{\lambda} \propto (Z-1)^2$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \left(\frac{Z_2-1}{Z_1-1}\right)^2 \text{ or } \frac{1}{4} = \left(\frac{Z_2-1}{11-1}\right)^2$$

Solving this, we get $Z_2 = 6$

14. $$\frac{\lambda_1}{\lambda_2} = \frac{\dfrac{h}{\sqrt{2mE}}}{\dfrac{hc}{E}} \text{ or } \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$$

15. $$i = \frac{q}{t} = \frac{ne}{t} \quad \therefore \quad n = \frac{it}{e}$$

Substituting $i = 3.2\times 10^{-3}$ A,

$$e = 1.6\times 10^{-19} \text{ C and } t = 1\text{s}$$

we get, $n = 2\times 10^{16}$

16. Wavelength λ_k is independent of the accelerating voltage (V), while the minimum wavelength λ_c is inversely proportional to V. Therefore, as V is increased, λ_k remains unchanged whereas λ_c decreases or $\lambda_k - \lambda_c$ will increase.

17. Minimum wavelength of continuous X-ray spectrum is given by $\lambda_{\min}$ (in Å) $= \dfrac{12375}{E\,(\text{in eV})}$

Here E = energy of incident electrons (in eV)
= energy corresponding to minimum wavelength $\lambda_{\min}$ of X-rays.

$$E = 80\text{ keV} = 80\times 10^3 \text{ eV}$$

$$\therefore \quad \lambda_{\min} \text{ (in Å)} = \frac{12375}{80\times 10^3} \approx 0.155$$

Also the energy of the incident electrons (80 keV) is more than the ionization energy of the K-shell electrons (*i.e.*, 72.5 keV). Therefore, characteristic X-ray spectrum will also be obtained because energy of incident electron is high enough to knock out the electron from K or L-shells.

18. From law of conservation of momentum,

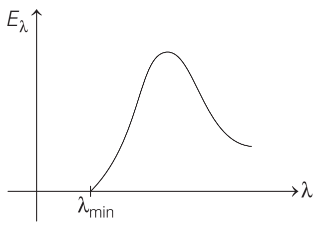
$$p_1 = p_2 \qquad \text{(in opposite directions)}$$

Now de-Broglie wavelength is given by

$$\lambda = \frac{h}{p}, \text{ where } h = \text{Planck's constant}$$

Since magnitude of momentum (p) of both the particles is equal, therefore $\lambda_1 = \lambda_2$ or $\lambda_1/\lambda_2 = 1$

19. The continuous X-ray spectrum is shown in figure.



All wavelengths $>\lambda_{\min}$ are found, where

$$\lambda_{\min} = \frac{12375}{V\text{ (in volt)}} \text{ Å}$$

Here, V is the applied voltage.

20. $\lambda_{k_\alpha} = 0.021\text{nm} = 0.21\text{Å}$

Since, λ_{k_α} corresponds to the transition of an electron from L-shell to K-shell, therefore

$$E_L - E_K = (\text{in eV}) = \frac{12375}{\lambda\text{ (in Å)}}$$

$$= \frac{12375}{0.21} \approx 58928\text{ eV}$$

or $\qquad \Delta E \approx 59$ keV

22. Shortest wavelength or cut-off wavelength depends only upon the voltage applied in the coolidge tube.

23. Cut-off wavelength depends on the accelerating voltage, not the characteristic wavelengths. Further, approximately 2% kinetic energy of the electrons is utilised in producing X-rays. Rest 98% is lost in heat.

24. $\lambda_m \text{ (in Å)} = \dfrac{12375}{V \text{ (in volt)}}$

With increase in V, λ_m will decrease. With decrease in λ_m energy of emitted photons will increase. And hence intensity will increase even if number of photons emitted per second are constant. Because intensity is basically energy per unit area per unit time.

25. Angular momentum $= n\left(\dfrac{h}{2\pi}\right) = 3\left(\dfrac{h}{2\pi}\right)$

$\therefore \quad n = 3$

Now, $r_n \propto \dfrac{n^2}{z}$

$\therefore \quad r_3 = \dfrac{(3)^2}{3}(a_0) = 3a_0$

Now, $mv_3 r_3 = 3\left(\dfrac{h}{2\pi}\right)$

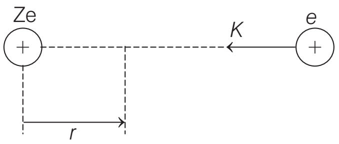
$\therefore \quad mv_3(3a_0) = 3\left(\dfrac{h}{2\pi}\right)$

or $\dfrac{h}{mv_3} = 2\pi a_0$ or $\dfrac{h}{P_3} = 2\pi a_0 \qquad (\because P = mv)$

or $\lambda_3 = 2\pi a_0 \qquad \left(\lambda = \dfrac{h}{P}\right)$

$\therefore$ Answer is 2.

26.



r = closest distance = 10 fm.

From energy conservation, we have

$$K_i + U_i = K_f + U_f$$

or $$K + 0 = 0 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

or $$K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(120\,e)(e)}{r} \qquad \text{...(i)}$$

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2Km}} \qquad \text{...(ii)}$$

Substituting the given values in above two equations, we get

$$\lambda = 7 \times 10^{-15} \text{ m} = 7 \text{ fm}$$

27. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2qqVm}}$ or $\lambda \propto \dfrac{1}{\sqrt{qm}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{q_\alpha}{q_p} \cdot \frac{m_\alpha}{m_p}} = \sqrt{\frac{(2)(4)}{(1)(1)}} = 2.828$$

The nearest integer is 3.

$\therefore$ Answer is 3.

28. $\dfrac{1}{\lambda_{K_\alpha}} = R(Z-1)^2\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) = \dfrac{3}{4}R(Z-1)^2$

$\therefore \quad (Z-1) = \dfrac{2}{\sqrt{3R\lambda_{K_\alpha}}}$

Here, R = Rydberg's constant
$= 1.097 \times 10^7 \text{ m}^{-1}$ and

$\lambda_{K_\alpha} = 0.76 \text{ Å} = 0.76 \times 10^{-10}$ m

Substituting the values, we have

or $\quad Z = 39$

29. $\lambda_{\text{min}} \text{(in Å)} = \dfrac{12375}{V \text{ (in volts)}}$

$$\lambda_{\text{min}} = \frac{12375}{20 \times 10^3} = 0.62 \text{ Å}$$

30. X-rays are electromagnetic waves and electromagnetic waves travel with the speed of light i.e. 3.0×10^8 m/s.

31. K_α corresponds to $n = 2$ to $n = 1$

and K_β corresponds to $n = 3$ to $n = 1$

Since, $E_{31} > E_{21}$

$\therefore \quad \lambda_{K_\beta} < \lambda_{K_\alpha}$ or $\lambda_{K_\beta} = \lambda_{K_\alpha}\left[\dfrac{\dfrac{1}{1^2} - \dfrac{1}{2^2}}{\dfrac{1}{1^2} - \dfrac{1}{3^2}}\right]$

$$= 0.32\left(\frac{3/4}{8/9}\right) = 0.27\text{Å}$$

32. Cut-off wavelength is given by

$$\lambda_{\text{min}} = \left\{\frac{12375}{V \text{(in volts)}}\right\} \text{(in Å)}$$

with increase in applied voltage V, speed of electrons striking the anode is increased or cut-off wavelength of the emitted X-rays decreases.

Further, with increase in number of electrons striking the anode more number of photons of X-rays will be emitted. Therefore, intensity of X-rays will increase.

33. Minimum voltage required is corresponding to $n = 1$ to $n = 2$.

Binding energy of the innermost electron is given as 40 keV i.e. ionisation potential is 40 kV. Therefore,

$$V_{\text{min}} = \frac{40 \times 10^3 \left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)}{\left(\dfrac{1}{1^2} - \dfrac{1}{\infty}\right)} = 30 \times 10^3 \text{ V.}$$

The energy of the characteristic radiation will be 30×10^3 eV.

34. nth line of Lymen series means transition from $(n+1)^{\text{th}}$ state to first state.

$$\frac{1}{\lambda} = RZ^2\left[1 - \frac{1}{(n+1)^2}\right] \qquad \text{...(i)}$$

de-Broglie wavelength in $(n+1)^{\text{th}}$ orbit :

$$\lambda = \frac{h}{mv} = \frac{hr}{mvr} = \frac{(2\pi)(hr)}{(n+1)h} = \frac{2\pi r}{(n+1)}$$

or $$\frac{1}{\lambda} = \frac{(n+1)}{2\pi r} \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\left(\frac{n+1}{2\pi r}\right) = RZ^2\left[\frac{n(n+2)}{(n+1)^2}\right] \quad \text{...(iii)}$$

Now, as $r \propto \frac{n^2}{Z}$

$\therefore$ $$r = \frac{(n+1)^2}{11} r_o$$

Substituting in Eq. (iii), we get

$$\frac{11}{2\pi r_o} = \frac{R(11)^2 (n)(n+2)}{(n+1)}$$

or $$(n+1) = (1.09\times 10^7)(11)(2\pi)\times (0.529\times 10^{-10})(n^2+2n)$$

Solving this equation,

We get, $n = 24$

35. (a) From the relation $r \propto A^{1/3}$,

we have, $$\frac{r_2}{r_1} = \left(\frac{A_2}{A_1}\right)^{1/3} \text{ or } \left(\frac{A_2}{4}\right)^{1/3} = (14)^{1/3}$$

$\therefore$ $A_2 = 56$

(b) $Z_2 = A_2 -$ number of neutrons

$= 56 - 30 = 26$

$$\therefore\ fk_\alpha = Rc(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3Rc}{4}(Z-1)^2$$

Substituting the given values of R, c and Z, we get

$$fk_\alpha = 1.55\times 10^{18}\ \text{Hz}$$

36. For $0 \le x \le 1$, PE $= E_0$

$\therefore$ Kinetic energy $K_1 =$ Total energy – PE

$= 2E_0 - E_0 = E_0$

$\therefore$ $$\lambda_1 = \frac{h}{\sqrt{2mE_0}} \quad \text{...(i)}$$

For $x > 1$, PE $= 0$

$\therefore$ Kinetic energy $K_2 =$ Total energy $= 2E_0$

$\therefore$ $$\lambda_2 = \frac{h}{\sqrt{2m(E_0)}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

37. $$\Delta E = h\nu = Rhc(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For K-series, $b = 1$

$\therefore$ $$\nu = Rc(Z-1)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Substituting the values,

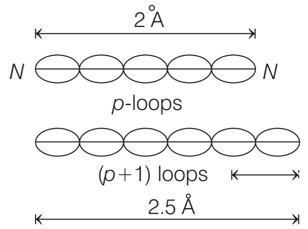
$$4.2\times 10^{18} = (1.1\times 10^7)(3\times 10^8)(Z-1)^2\left(\frac{1}{1} - \frac{1}{4}\right)$$

$\therefore$ $(Z-1)^2 = 1697$

or $Z - 1 \approx 41$

or $Z = 42$

38. From the figure it is clear that



$$p\cdot(\lambda/2) = 2\ \text{Å}$$

$$(p+1)\cdot\lambda/2 = 2.5\ \text{Å}$$

$\therefore$ $\lambda/2 = (2.5 - 2.0)\text{Å} = 0.5\ \text{Å}$

or $\lambda = 1\ \text{Å} = 10^{-10}$ m.

(a) de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}$$ Here, $K =$ kinetic energy of electron

$$\therefore\ K = \frac{h^2}{2m\lambda^2} = \frac{(6.63\times 10^{-34})^2}{2(9.1\times 10^{-31})(10^{-10})^2}$$

$$= 2.415\times 10^{-17}\ \text{J} = \left(\frac{2.415\times 10^{-17}}{1.6\times 10^{-19}}\right)\text{eV}$$

$\therefore$ $K = 150.8$ eV

(b) The least value of d will be, when only one loop is formed .

$\therefore$ $d_{\min} = \lambda/2$ or $d_{\min} = 0.5$Å

Topic 5 Nuclear Physics

1. For substance A, half-life is 10 min, so it decays as

$$\underset{\substack{\text{(initial number}\\ \text{of nuclei at } t=0)}}{N_{0_A}} \xrightarrow{10\text{ min}} \underset{\substack{\text{(Active nuclei}\\ \text{remained after}\\ \text{10 min)}}}{\frac{N_{0_A}}{2}} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{4} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{8}$$

$$\xrightarrow{10\text{ min}} \frac{N_{0_A}}{16} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{32} \xrightarrow{10\text{ min}} \frac{N_{0_A}}{64}$$

$\therefore$ For substance A, number of nuclei decayed in 60 min is

$$N_{0_A} - \frac{N_{0_A}}{64} = \frac{63N_{0_A}}{64}$$

Similarly, for substance B, half-life is 20 min, so it's decay scheme is

$$N_{0_B} \xrightarrow{20\text{ min}} \frac{N_{0_B}}{2} \xrightarrow{20\text{ min}} \frac{N_{0_B}}{4} \xrightarrow{20\text{ min}} \frac{N_{0_B}}{8}$$

So, number of nuclei of B decayed in 60 min is

$$N_{0_B} - \frac{N_{0_B}}{8} = \frac{7}{8} N_{0_B}$$

Hence, ratio of decayed nuclei of A and B in 60 min is

$$\frac{\frac{63}{64} N_{0_A}}{\frac{7}{8} N_{0_B}} = \frac{9}{8} \qquad [\because N_{0_A} = N_{0_B}]$$

Alternate Solution

Number of active nuclei remained after 60 min can also be calculated as

$$N' = \frac{N}{2^{T/t_{1/2}}}$$

where, $T = 60$ min

So, for nuclei A,

$$N'_{0_A} = \frac{N_{0_A}}{2^{\frac{60}{10}}} = \frac{N_{0_A}}{2^6} = \frac{N_{0_A}}{64}$$

Similarly, for nuclei B,

$$N'_{0_B} = \frac{N_{0_B}}{2^{\frac{60}{20}}} = \frac{N_{0_B}}{2^3} = \frac{N_{0_B}}{8}$$

2. Number of active nuclei remained after time t in a sample of radioactive substance is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = initial number of nuclei at $t = 0$ and λ = decay constant.

Here, at $t = 0$,

Number of nuclei in sample A and B are equal, i.e.

$$N_{0_A} = N_{0_B} = N_0$$

Also, $\lambda_A = 5\lambda$ and $\lambda_B = \lambda$

So, after time t, number of active nuclei of A and B are

$$N_A = N_0 e^{-5\lambda t} \text{ and } N_B = N_0 e^{-\lambda t}$$

If $\frac{N_A}{N_B} = \frac{1}{e^2}$, then

$$\frac{N_A}{N_B} = \frac{N_0 e^{-5\lambda t}}{N_0 e^{-\lambda t}} = \frac{1}{e^2} \Rightarrow e^{(\lambda - 5\lambda)t} = e^{-2}$$

Comparing the power of e on both sides, we get

$$4\lambda t = 2$$

$$\Rightarrow \qquad t = \frac{1}{2\lambda}$$

3. Given, $\lambda_A = 10\lambda$ and $\lambda_B = \lambda$

Number of nuclei (at any instant) present in material is

$$N = N_0 e^{-\lambda t}$$

So, for materials A and B, we can write

$$\frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t} \qquad \ldots \text{(i)}$$

Given,

$$\frac{N_A}{N_B} = \frac{1}{e} \qquad \ldots \text{(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{1}{e} = e^{-(\lambda_A - \lambda_B)t}$$

$$\Rightarrow \qquad e^{-1} = e^{-(\lambda_A - \lambda_B)t}$$

Comparing the power of 'e' on both sides, we get

or $\qquad (\lambda_A - \lambda_B)t = 1$

$$\Rightarrow \qquad t = \frac{1}{\lambda_A - \lambda_B}$$

By putting values of λ_A and λ_B in the above equation, we get

$$t = \frac{1}{10\lambda - \lambda} \Rightarrow t = \frac{1}{9\lambda}$$

4. Mass density of nuclear matter is a constant quantity for all elements. It does not depends on element's mass number or atomic radius.

$\therefore$ The ratio of mass densities of ^{40}Ca and ^{16}O is 1 : 1.

5. Energy absorbed or released in a nuclear reaction is given by

ΔQ = Binding energy of products − Binding energy of reactants.

If energy is absorbed, ΔQ is negative and if it is positive then energy is released. Also, Binding energy = Binding energy per nucleon $\times$ Number of nucleons.

Here, binding energy of products

$$= 2 \times (\text{B.E. of He}^4) + (\text{B.E. of C}^{12})$$

$$= 2(4 \times 7.07) + (12 \times 7.86) = 150.88 \text{ MeV}$$

and binding energy of reactants $= 20 \times 8.03 = 160.6$ MeV

So, $\Delta Q = (\text{B.E.})_{\text{Products}} - (\text{B.E.})_{\text{reactants}}$

$$= 150.88 - 160.6 = -9.72 \text{ MeV.}$$

As ΔQ is negative

$\therefore$ energy of 9.72 Mev is absorbed in the reaction.

$\therefore$ No option is correct.

6. Electrostatic energy

= Binding energy of N − Binding energy of O

$$= [[7M_H + 8M_n - M_N] - [8M_H + 7M_n - M_O]] \times C^2$$

$$= [-M_H + M_n + M_O\ M_N]\, C^2$$

$$= [-1.007825 + 1.008665 + 15.003065 - 15.000109] \times 93.15$$

$$= +3.5359 \text{ MeV}$$

$$\Delta E = \frac{3}{5} \times \frac{1.44 \times 8 \times 7}{R} = 3.5359$$

$$R = \frac{3 \times 1.44 \times 14}{5 \times 3.5359} = 3.42 \text{ fm}$$

7. From conservation laws of mass number and atomic number, we can say that $x = n,\ y = n$

$$(x = {}_0^1n,\ y = {}_0^1n)$$

$\therefore$ Only (a) and (d) options may be correct.

From conservation of momentum, $|P_{xe}| = |P_{sr}|$

From $$K = \frac{P^2}{2m} \Rightarrow K \propto \frac{1}{m}$$

$$\frac{K_{sr}}{K_{xe}} = \frac{m_{xe}}{m_{sr}}$$

$\therefore$ $$K_{sr} = 129\ \text{MeV},\ K_{xe} = 86\ \text{MeV}$$

NOTE There is no need of finding total energy released in the process.

8. Rest mass of parent nucleus should be greater than the rest mass of daughter nuclei. Therefore, option (a) will be correct.

9. $4\ ({}_2He^4) = {}_8O^{16}$

Mass defect, $\Delta m = \{4\ (4.0026) - 15.9994\} = 0.011$ amu

$\therefore$ Energy released per oxygen nuclei

$$= (0.011)\ (931.48)\ \text{MeV} = 10.24\ \text{MeV}$$

10. Given that $K_1 + K_2 = 5.5\ \text{MeV}$ ...(i)

From conservation of linear momentum,

$$p_1 = p_2$$

or $\sqrt{2K_1\ (216m)} = \sqrt{2K_2\ (4m)}$ as $p = \sqrt{2Km}$

$\therefore$ $$K_2 = 54\ K_1 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

K_2 = KE of α-particle = 5.4 MeV.

11. Nuclear density is constant hence, mass $\propto$ volume

or $$m \propto V$$

12. Radius of a nucleus is given by

$$R = R_0\ A^{1/3} \quad (\text{where } R_0 = 1.25 \times 10^{-15}\ \text{m})$$

$$= 1.25\ A^{1/3} \times 10^{-15}\ \text{m}$$

Here A is the mass number and mass of the uranium nucleus will be

$m \approx Am_p$ where m_p = mass of proton

$$= A\ (1.67 \times 10^{-27}\ \text{kg})$$

$$\therefore \text{ Density } \rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi R^3}$$

$$= \frac{A\ (1.67 \times 10^{-27}\ \text{kg})}{A\ (1.25 \times 10^{-15}\ \text{m})^3} \text{ or } \rho \approx 2.0 \times 10^{17}\ \text{kg/m}^3$$

13. Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon $\times$ number of nucleons) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in option (c), this happens.

Given : $$W \to 2Y$$

Binding energy of reactants = $120 \times 7.5 = 900$ MeV

and binding energy of products = $2\ (60 \times 8.5)$

$$= 1020\ \ \text{MeV} > 900\ \text{MeV}$$

14. Heavy water is used as moderators in nuclear reactors to slow down the neutrons.

15. The given reactions are :

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + p$$
$${}_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + n$$
$$3\,{}_1H^2 \longrightarrow {}_2He^4 + n + p$$

Mass defect

$$\Delta m = (3 \times 2.014 - 4.001 - 1.007 - 1.008)\ \text{amu}$$
$$= 0.026\ \text{amu}$$

Energy released = 0.026×931 MeV

$$= 0.026 \times 931 \times 1.6 \times 10^{-13}\ \text{J} = 3.87 \times 10^{-12}\ \text{J}$$

This is the energy produced by the consumption of three deuteron atoms.

$\therefore$ Total energy released by 10^{40} deuterons

$$= \frac{10^{40}}{3} \times 3.87 \times 10^{-12}\ \text{J} = 1.29 \times 10^{28}\ \text{J}$$

The average power radiated is $P = 10^{16}$ W or 10^{16} J/s.

Therefore, total time to exhaust all deuterons of the star will be $t = \dfrac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12}\text{s} \approx 10^{12}\text{s}$

17. During fusion process two or more lighter nuclei combine to form a heavy nucleus.

18. (a) ${}_3Li^7 \to {}_2He^4 + {}_1H^3$

$$\Delta m = [M_{Li} - M_{He} - M_{H^3}]$$
$$= [6.01513 - 4.002603 - 3.016050]$$
$$= -1.003523\ \text{u}$$

Δm is negative so reaction is not possible.

(b) ${}_{84}Po^{210} \to {}_{83}Bi^{209} + {}_1P^1$

Δm is negative so reaction is not possible.

(c) ${}_1H^2 \to {}_2He^4 + {}_3Li^6$

Δm is positive so reaction is possible.

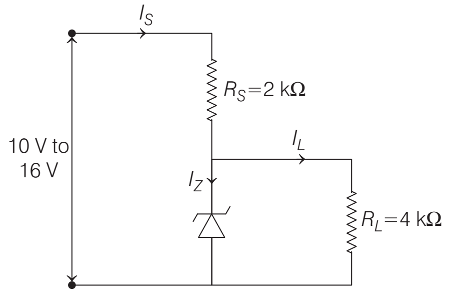
(d) ${}_{30}Zn^{70} + {}_{34}Se^{82} \to {}_{64}Gd^{152}$

Δm is positive so reaction is not possible.

19. $${}_{84}Po^{210} \longrightarrow {}_2He^4 + {}_{82}Pb^{206}$$

Mass defect $\Delta m = (m_{Po} - M_{He} - m_{Pb}) = 0.005818$ u

$\therefore$ $$Q = (\Delta m)\ (931.48)\ \text{MeV} = 5.4193\ \text{MeV}$$
$$= 5419\ \text{keV}$$



From conservation of linear momentum,

$$p_{Pb} = p_\alpha$$

$\therefore$ $\sqrt{2m_{Pb}\, k_{Pb}} = \sqrt{2m_\alpha\, k_\alpha}$ or $\dfrac{k_\alpha}{k_{Pb}} = \dfrac{m_{Pb}}{m_\alpha} = \dfrac{206}{4}$

$\therefore$ $k_\alpha = \left(\dfrac{206}{206+4}\right)(k_{total})$

$$= \left(\frac{206}{210}\right)(5419) = 5316\text{ keV}$$

21. From conservation of mechanical energy, we have

$$U_i + K_i = U_f + U_f$$

$$0 + 2(1.5kT) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(e)(e)}{d} + 0$$

Substituting the values, we get

$$T = 1.4\times 10^9\text{ K}$$

22. As given in the paragraph, a reactor is termed successful, if

$$nt_0 > 5\times 10^{14}\text{ s cm}^{-3}$$

24. In fusion, two or more lighter nuclei combine to make a comparatively heavier nucleus.

In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei.

Further, energy will be released in a nuclear process if total binding energy increases.

$\therefore$ Correct options are (b) and (d).

25. Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

${}^{20}_{10}\text{Ne}$ is made up of 10 protons plus 10 neutrons. Therefore, mass of ${}^{20}_{10}\text{Ne}$ nucleus, $M_1 < 10\,(m_p + m_n)$.

Also, heavier the nucleus, more is the mass defect.

Thus, $20(m_n + m_p) - M_2 > 10\,(m_p + m_n) - M_1$

or $10\,(m_p + m_n) > M_2 - M_1$

or $M_2 < M_1 + 10\,(m_p + m_n)$

Now since $M_1 < 10\,(m_p + m_n)$

$\therefore$ $M_2 < 2M_1$

26. In nuclear fusion, two or more lighter nuclei are combined to form a relatively heavy nucleus and thus, releasing the energy.

27. In case of ${}_1\text{H}^1$, mass number and atomic number are equal and in case of ${}_1\text{H}^2$, mass number is greater than its atomic number.

28. In fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus.

29. $Q = (\Delta\, m \text{ in atomic mass unit}) \times 931.4$ MeV

$= (2\times \text{mass of } {}_1\text{H}^2 - \text{mass of } {}_2\text{He}^4)\times 931.4$ MeV

$= (2\times 2.0141 - 4.0024)\times 931.4$ MeV

$Q \approx 24$ MeV (fusion)

30. $2\,{}_1\text{H}^2 \longrightarrow {}_2\text{He}^4$

Binding energy of two deuterons,

$$E_1 = 2\,[2\times 1.1] = 4.4\text{ MeV}$$

Binding energy of helium nucleus,

$$E_2 = 4\,(7.0) = 28.0\text{ MeV}$$

$\therefore$ Energy released $\Delta E = E_2 - E_1$

$$= (28 - 4.4)\text{ MeV} = 23.6\text{ MeV}$$

31. Order of magnitude of nuclear density is 10^{17} kg/m^3.

32. ${}^{12}_{5}B \longrightarrow {}^{13}_{6}C + {}^{0}_{-1}e + \bar{\nu}$

Mass of ${}^{12}_{6}\text{C} = 12.000$ u (by definition of 1 a.m.u.)

Q-value of reaction,

$Q = (M_B - M_C)\times c^2 = (12.014 - 12.000)\times 931.5$

$= 13.041$ MeV

4.041 MeV of energy is taken by ${}^{12}_{65}\text{C}^*$

$\Rightarrow$ Maximum KE of β-particle is $(13.041 - 4.041) = 9$ MeV

33. Let initial power available from the plant is P_0. After time $t = nT$ or n half lives, this will become $\left(\dfrac{1}{2}\right)^n P_0$. Now, it is given that, $\left(\dfrac{1}{2}\right)^n P_0 = 12.5\%$ of $P_0 = (0.125)\,P_0$

Solving this equation we get, $n = 3$

34. The reactor produces 1000 MW power or 10^9 J/s. The reactor is to function for 10 yr. Therefore, total energy which the reactor will supply in 10 yr is

E = (power) (time)

$= (10^9\text{ J/s})(10\times 365\times 24\times 3600\text{s})$

$= 3.1536\times 10^{17}$ J

But since the efficiency of the reactor is only 10%, therefore actual energy needed is 10 times of it or 3.1536×10^{18}J. One uranium atom liberates 200 MeV of energy or $200\times 1.6\times 10^{-13}$ J or 3.2×10^{-11} J of energy. So, number of uranium atoms needed are

$$\frac{3.1536\times 10^{18}}{3.2\times 10^{-11}} = 0.9855\times 10^{29}$$

or number of kg-moles of uranium needed are

$$n = \frac{0.9855\times 10^{29}}{6.02\times 10^{26}} = 163.7$$

Hence, total mass of uranium required is

$m = (n)M = (163.7)(235)$ kg

or $m \approx 38470$ kg

or $m = 3.847\times 10^4$ kg

35. The reaction involved in α-decay is

$${}^{248}_{96}\text{Cm} \rightarrow {}^{244}_{94}\text{Pu} + {}^{4}_{2}\text{He}$$

Mass defect

$\Delta m =$ mass of ${}^{248}_{96}\text{Cm} -$ mass of ${}^{244}_{94}\text{Pu} -$ mass of ${}^{4}_{2}\text{He}$

$= (248.072220 - 244.064100 - 4.002603)\text{u}$

$= 0.005517\text{u}$

Therefore, energy released in α-decay will be

$$E_\alpha = (0.005517 \times 931)\text{MeV} = 5.136 \text{ MeV}$$

Similarly, $E_{\text{fission}} = 200\text{MeV}$ (given)

Mean life is given as $t_{\text{mean}} = 10^{13}\text{ s} = 1/\lambda$

$\therefore$ Disintegration constant $\lambda = 10^{-13}\text{ s}^{-1}$

Rate of decay at the moment when number of nuclei are 10^{20}

$= \lambda N = (10^{-13})(10^{20})$

$= 10^7$ disintegration per second

Of these disintegrations, 8% are in fission and 92% are in α-decay.

Therefore, energy released per second

$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136)$ MeV

$= 2.074 \times 10^8$ MeV

$\therefore$ Power output (in watt)

= energy released per second (J/s)

$= (2.074 \times 10^8)(1.6 \times 10^{-13})$

$\therefore$ Power output $= 3.32 \times 10^{-5}$ W

36. (a) $A - 4 = 228$

$\therefore \quad A = 232$

$92 - 2 = Z \quad \text{or} \quad Z = 90$

(b) From the relation, $r = \dfrac{\sqrt{2Km}}{Bq}$

$$K_\alpha = \frac{r^2B^2q^2}{2m} = \frac{(0.11)^2(3)^2\,(2 \times 1.6 \times 10^{-19})^2}{2 \times 4.003 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}}$$

$= 5.21$ MeV

From the conservation of momentum,

$$p_\gamma = p_\alpha \quad \text{or} \quad \sqrt{2K_\gamma m_\gamma} = \sqrt{2K_\alpha m_\alpha}$$

$$\therefore \quad K_\gamma = \left(\frac{m_\alpha}{m_\gamma}\right)K_\alpha = \frac{4.003}{228.03} \times 5.21$$

$= 0.09\text{MeV}$

$\therefore$ Total energy released $= K_\alpha + K_\gamma = 5.3\text{MeV}$

Total binding energy of daugther products

$= [92 \times (\text{mass of proton}) + (232 - 92)(\text{mass of neutron}) - (m_\gamma) - (m_\alpha)] \times 931.48$ MeV

$= [(92 \times 1.008) + (140)(1.009) - 228.03 - 4.003]\, 931.48$ MeV

$= 1828.5$ MeV

$\therefore$ Binding energy of parent nucleus

= binding energy of daughter products − energy released

$= (1828.5 - 5.3)$ MeV $= 1823.2$ MeV

37. Mass defect in the given nuclear reaction :

$\Delta m = 2(\text{mass of deuterium}) - (\text{mass of helium})$

$= 2\,(2.0141) - (4.0026) = 0.0256$

Therefore, energy released

$\Delta E = (\Delta m)\,(931.48)$ MeV $= 23.85$ MeV

$= 23.85 \times 1.6 \times 10^{-13}\text{ J} = 3.82 \times 10^{-12}$ J

Efficiency is only 25%, therefore,

$$25\% \text{ of } \Delta E = \left(\frac{25}{100}\right)(3.82 \times 10^{-12})\text{ J}$$

$= 9.55 \times 10^{-13}$ J

i.e, by the fusion of two deuterium nuclei, 9.55×10^{-13} J energy is available to the nuclear reactor.

Total energy required in one day to run the reactor with a given power of 200 MW :

$$E_{\text{Total}} = 200 \times 10^6 \times 24 \times 3600 = 1.728 \times 10^{13}\text{J}$$

$\therefore$ Total number of deuterium nuclei required for this purpose

$$n = \frac{E_{\text{Total}}}{\Delta E/2} = \frac{2 \times 1.728 \times 10^{13}}{9.55 \times 10^{-13}}$$

$= 0.362 \times 10^{26}$

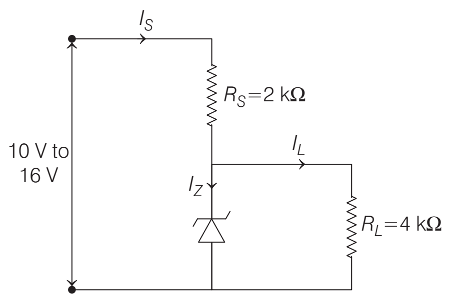
$\therefore$ Mass of deuterium required

= (Number of g-moles of deuterium required) × 2 g

$$= \left(\frac{0.362 \times 10^{26}}{6.02 \times 10^{23}}\right) \times 2 = 120.26\text{ g.}$$

Topic 6 Semiconductor Devices, Diodes and Triodes

1. In given voltage regulator circuit breakdown of Zener occurs at 6 V.



After breakdown, voltage across load resistance ($R_L = 4\text{k}\Omega$) is,

$$V = V_Z = 6\text{V}$$

$\therefore$ Load current after breakdown,

$$I_L = \frac{V_Z}{R_L} = \frac{6}{4000} = 1.5 \times 10^{-3} \text{ A}$$

When unregulated supply is of 16 V, potential drop occurring across series resistance ($R_S = 2\text{ k}\Omega$) is;

$$V_S = V - V_Z = 16 - 6 = 10\text{ V}$$

So, current across series resistance is

$$I_S = \frac{V_S}{R_S} = \frac{10}{2 \times 10^3} = 5 \times 10^{-3} \text{ A}$$

So, current across Zener diode is

$$I_Z = I_S - I_L = 5 \times 10^{-3} - 1.5 \times 10^{-3} = 3.5 \times 10^{-3} \text{ A}$$

2. Given curve is between I_c and I_b as output and input currents, respectively.

So, it is transfer characteristics curve of a common emitter (CE) configuration.

In CE configuration,

Current gain, $\beta = \dfrac{I_{\text{out}}}{I_{\text{in}}} = \dfrac{I_c}{I_b}$ ...(i)

Voltage gain,

$$A_V = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{I_c \times R_{\text{out}}}{I_b \times R_{\text{in}}} = \beta \times \frac{R_{\text{out}}}{R_{\text{in}}} \quad \text{...(ii)}$$

and power gain,

$$A_P = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{I_c^2 \times R_{\text{out}}}{I_b^2 \times R_{\text{in}}} = \beta^2 \times \frac{R_{\text{out}}}{R_{\text{in}}} \quad \text{...(iii)}$$

Given, $R_{\text{in}} = 100\ \Omega$ and $R_{\text{out}} = 100 \times 10^3\ \Omega$

From Eq. (i), we get

$$\beta = \frac{5\text{ mA}}{100\,\mu\text{A}}\left(\text{or } \frac{10\text{ mA}}{200\,\mu\text{A}} \text{ or } \frac{15\text{ mA}}{300\,\mu\text{A}} \text{ or } \frac{20\text{ mA}}{400\,\mu\text{A}}\right)$$

$$\Rightarrow \quad \beta = \frac{5 \times 10^{-3}}{100 \times 10^{-6}} = 50 \quad \text{...(iv)}$$

From Eqs. (ii) and (iv), we get

Voltage gain, $A_V = \beta \times \dfrac{R_{\text{out}}}{R_{\text{in}}} = 50 \times \dfrac{100 \times 10^3}{100}$

$$\Rightarrow \quad A_V = 50000 = 5 \times 10^4 \quad \text{...(v)}$$

From Eqs. (iii) and (iv), we get

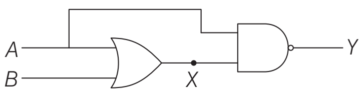
Power gain, $A_P = \beta^2 \times \dfrac{R_{\text{out}}}{R_{\text{in}}}$

$$= (50)^2 \times \frac{100 \times 10^3}{100}$$

$$= 2500 \times 1000$$

$$\Rightarrow \quad A_P = 2.5 \times 10^6$$

3. Given circuit is



Let the intermediate state X of OR gate is shown in figure.

Clearly, $Y = \overline{AX}$ ...(i)

Here, $X = A + B$ ...(ii)

$$\therefore \quad Y = \overline{A\,(A + B)} = \overline{AA + AB}$$

$$= \overline{A + AB} \quad (\because AA = A)$$

$$= \overline{A\,(1 + B)}$$

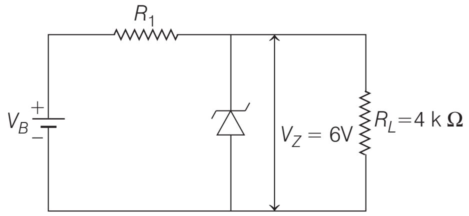
$$\Rightarrow \quad Y = \overline{A} \quad (\because 1 + B = 1)$$

So, truth table shown in option (c) is correct.

Alternate Solution We can solve it using truth table

A	B	$X = A + B$	$Y = \overline{AX}$
0	0	0	1
0	1	1	1
1	0	1	0
1	1	1	0

4. In given voltage regulator circuit,



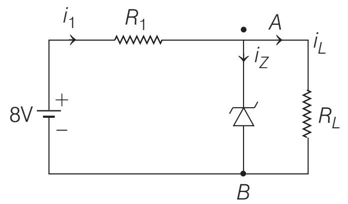
Zener breakdown voltage, $V_Z = 6$ V

So, across R_L, potential drop is always 6 V.

So, current through load resistance is

$$i_L = \frac{V_Z}{R_L} = \frac{6}{4 \times 10^3} = 1.5 \times 10^{-3} \text{ A}$$

Now, when $V_B = 8$ V



Potential drop across $R_1 = 8 - 6 = 2$ V

So, current through R_1 is $i_1 = \dfrac{V}{R_1} = \dfrac{2}{1 \times 10^3} = 2 \times 10^{-3}$ A

So, current through Zener diode is

$$i_Z = i_1 - i_L = 2 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= 0.5 \times 10^{-3} \text{ A} = 0.5\text{ mA}$$

Similarly, when $V_B = 16$ V

$$V_{R_1} = 16 - 6 = 10\text{ V}$$

$$\therefore \quad i_1 = \frac{10}{1 \times 10^3} = 10 \times 10^{-3} \text{ A}$$

Hence,

$$i_Z = i_1 - i_L = 10 \times 10^{-3} - 1.5 \times 10^{-3}$$
$$= 8.5 \times 10^{-3}\ \text{A} = 8.5\,\text{mA}$$

5. Given, $A_P = 60\,\text{dB}$ (in decibel)

Power gain in decibel can be given as

$$A_P = 10 \log_{10}\left(\frac{\text{Output power}}{\text{Input power}}\right)$$

$$\Rightarrow \quad 60 = 10 \log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \Rightarrow \log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) = 6$$

$$\Rightarrow \quad \frac{P_{\text{out}}}{P_{\text{in}}} = 10^6 = A_P \qquad \ldots \text{(i)}$$

Also, given $R_{\text{out}} = 10\,\text{k}\Omega$, $R_{\text{in}} = 100\,\Omega$

$\therefore$ Power gain of a transistor is given by

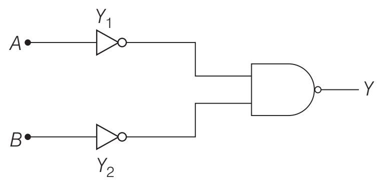
$$A_P = \beta^2 \left(\frac{R_{\text{out}}}{R_{\text{in}}}\right)$$

where, β is current gain.

$$\Rightarrow \beta^2 = A_P \times \frac{R_{\text{in}}}{R_{\text{out}}} = 10^6 \times \frac{100}{10 \times 10^3}$$

$$\Rightarrow \quad \beta^2 = 10^4 \text{ or } \beta = 10^2$$

6. Truth table for given combination of logic gates is

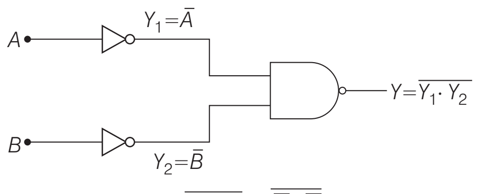


A	B	$Y_1 = \overline{A}$	$Y_2 = \overline{B}$	$Y = \overline{Y_1 \cdot Y_2}$
0	0	1	1	0
1	0	0	1	1
0	1	1	0	1
1	1	0	0	1

Output Y resembles output of an OR gate. So, given combination acts like an OR gate.

Alternate Solution

The given logic gate circuit can be drawn as shown below



Here, $\quad Y = \overline{Y_1 \cdot Y_2} = \overline{\overline{A} \cdot \overline{B}}$

Using de-Morgan's theorem, i.e.

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\therefore \quad Y = \overline{\overline{A}} + \overline{\overline{B}} = A + B \qquad [\because \overline{\overline{x}} = x]$$

This represents the boolean expression for OR gate.

7. Given, load resistance, $R_L = 1\,\text{k}\Omega$

Input voltage, $V_{\text{in}} = 10\ \text{mV} = 10 \times 10^{-3}\ \text{V}$

Base current, $\Delta I_B = 15\,\mu\text{A} = 15 \times 10^{-6}\ \text{A}$

Collector current, $\Delta I_C = 3\,\text{mA}$

Input resistance,

$$R_{\text{in}} = \frac{V_{\text{in}}}{\Delta I_B} = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67\,\text{k}\Omega$$

and voltage gain $= \beta \times \dfrac{R_L}{R_{\text{in}}} = \dfrac{\Delta I_C \times R_L}{\Delta I_B \times R_{\text{in}}}$

$$= \left(\frac{3\ \text{mA}}{15\,\mu\text{A}}\right) \times \left(\frac{1\,\text{k}\Omega}{0.67\ \text{k}\Omega}\right)$$

$$= \left(\frac{3 \times 10^{-3}}{15 \times 10^{-6}}\right)\left(\frac{1 \times 10^3}{0.67 \times 10^3}\right)$$

$$= \frac{1000 \times 3 \times 3}{15 \times 2} = 300 \qquad (\because 0.67 \cong 2/3)$$

Alternate Solution

$$\therefore \text{Voltage gain} = \frac{V_{\text{output}}}{V_{\text{input}}} = \frac{R_L \times \Delta I_C}{V_{\text{in}}}$$

$$= \frac{1 \times 10^3 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$$

8. For a common emitter *n-p-n* transistor, DC current gain is

$$\beta_{\text{DC}} = \frac{I_C}{I_B}$$

At saturation state, V_{CE} becomes zero.

$$\therefore \quad V_{CC} - I_C R_C = 0$$

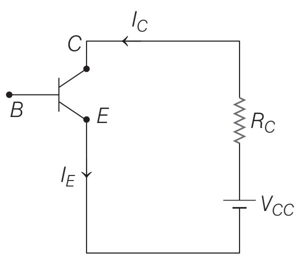
$$I_C \approx \frac{V_{CC}}{R_C} = \frac{10}{1000} = 10^{-2}\text{A}$$

Hence, saturation base current,

$$I_B = \frac{I_C}{\beta_{\text{DC}}} = \frac{10^{-2}}{250} = 40\,\mu\text{A}$$

9. Transistor saturation occurs when $V_{CE} = 0$.

Now, for closed loop of collector and emitter by Kirchhoff's voltage rule, we have



$$V_{CE} = V_{CC} - I_C R_C \Rightarrow 0 = V_{CC} - I_C R_c$$

$$\Rightarrow \quad I_C = \frac{V_{CC}}{R_C} = \frac{5}{1 \times 10^3} = 5 \times 10^{-3}\ \text{A}$$

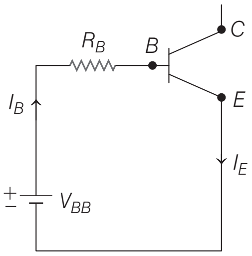
Now, $\beta_{DC} = 200\,(\text{given}) \Rightarrow \dfrac{I_C}{I_B} = \beta_{DC} = 200$

$$\Rightarrow \quad I_B = \frac{I_C}{200} = \frac{5\times 10^{-3}}{200}$$

$$\Rightarrow \quad I_B = 2.5\times 10^{-5} = 25\,\mu\text{A}$$

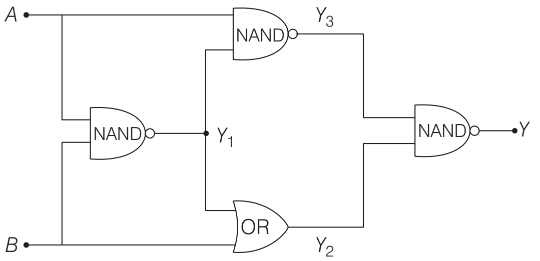
Now, we apply Kirchhoff's voltage rule in base-emitter closed loop, we get

$$V_{BB} = I_B R_B + V_{BE}$$



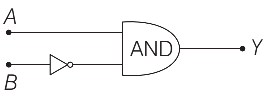
$$\Rightarrow V_{BB} = (25\times 10^{-6} \times 100\times 1000) + 1 = 3.5\text{ V}$$

10. Truth table for given circuit is



A	B	Y_1	Y_2	Y_3	Y
0	0	1	1	1	0
1	0	1	1	0	1
0	1	1	1	1	0
1	1	0	1	1	0

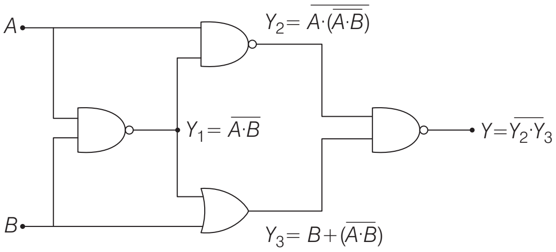
This is the same output produced by $A\cdot\overline{B}$ gate or



So, given circuit is equivalent to Boolean expression $A\cdot\overline{B}$.

Alternate Method

Using the Boolean algebra, output of the given logic circuit can be given as



Here $\quad Y_2 = \overline{A\cdot(\overline{A\cdot B})}$

Using de-Morgan's principle,

$$\overline{x\cdot y} = \bar{x} + \bar{y} \text{ and } \overline{x+y} = \bar{x}\cdot\bar{y}$$

$\Rightarrow \quad Y_2 = \bar{A} + \overline{\overline{(A\cdot B)}} \qquad [\because \bar{\bar{x}} = x]$

$\qquad = \bar{A} + (A.B) \qquad \ldots\text{(i)}$

Similarly, $Y_3 = B + \bar{A} + \bar{B} = 1 + \bar{A} \qquad [\because x + \bar{x} = 1]$

$\Rightarrow \quad Y_3 = 1 \qquad \text{(ii)} \; [\because \bar{x} + 1 = 1]$

As, $\quad Y = \overline{Y_2\cdot Y_3}$

Using Eqs. (i) and (ii), we get

$Y = \overline{(\bar{A} + A\cdot B)(1)}$

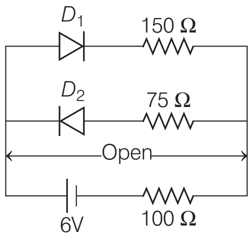
$\quad = \overline{(\bar{A} + A\cdot B)} + \bar{1} = \bar{\bar{A}}\cdot(\overline{A\cdot B}) + 0$

$\quad = A\cdot(\bar{A} + \bar{B}) \qquad [\because x + 0 = x]$

$\quad = A\cdot\bar{A} + A\cdot\bar{B} \qquad [\because x\bar{x} = 0]$

$\quad = A\cdot\bar{B}$

11. In this circuit, D_1 is forward biased and D_2 is reversed biased.



Resistance of D_1 is 50Ω.

∴ Net resistance of the circuit,

$$R_{net} = 50 + 150 + 100 = 300\Omega$$

∴ Current through the 100Ω resistance

$$= \frac{V}{R_{net}} = \frac{6}{300} = 0.020\text{ A}$$

12. **Key Idea** When the applied reverse voltage (V) reaches the breakdown voltage of the Zener diode, then only a large amount of current is flown through it, otherwise it is approximately zero.

In the given situation, if we consider that Zener diode is at breakdown. Then, potential drop across 1500 Ω resistances will be 10 V. So potential drop at 500 Ω resistor will be 2 V.

∴ Current in $R_1 = \dfrac{2}{500} = 4\text{mA} = I_1$ (say)

Current in each

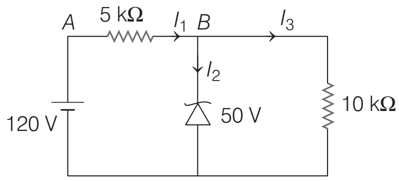
$$R_2 = \frac{10}{750} = \frac{2}{150} = 13.33\text{ mA} = I_2 \text{ (say)}$$

$\Rightarrow \quad I_1 < I_2$ which is not possible.

So, Zener diode will never reach to its breakdown.

∴ Current flowing through a reverse biased Zener diode = 0.

13. In the circuit, let the current in branches is as shown in figure below



By Kirchhoff's node law,

$$I_1 = I_2 + I_3 \quad \ldots \text{(i)}$$

Now, when diode conducts, voltage difference between points A and B will be

$$V_{AB} = 120 - 50 = 70 \text{ V}$$

So, current $I_1 = \dfrac{V_{AB}}{5 \text{ k}\Omega} = \dfrac{70}{5 \times 10^3}$

$$I_1 = 14 \text{ mA} \quad \ldots \text{(ii)}$$

Since, diode and 10 kΩ resistor are in parallel combination, so voltage across 10 k Ω resistor will be 50 V only.

$$\Rightarrow \quad I_3 = \frac{50}{10 \text{ k}\Omega} = \frac{50}{10 \times 10^3}$$

$$\Rightarrow \quad I_3 = 5 \text{ mA} \quad \ldots \text{(iii)}$$

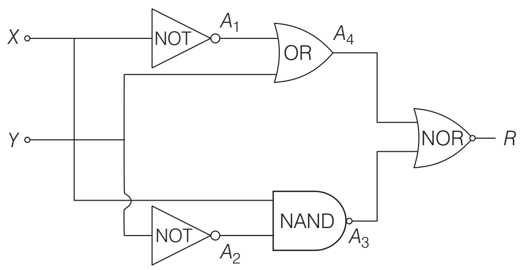
∴ From Eqs. (i), (ii) and (iii), we get

$$14 \text{ mA} = I_2 + 5\text{mA}$$

or current through diode,

$$I_2 = 14 \text{ mA} - 5\text{mA} = 9 \text{ mA}$$

14. The given circuit can be drawn as shown in the figure given below

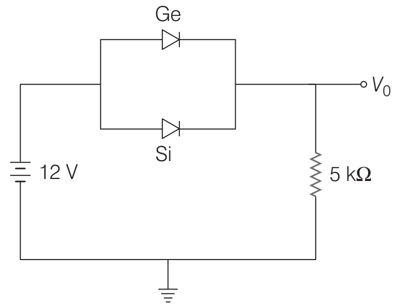


Truth table for this given logic gate is

Given inputs in the options		A_1	A_2	A_3	A_4	R
X	Y					
0	0	1	1	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	1	0
0	1	1	0	1	1	0

So to get output $R = 1$, inputs must be $X = 1$ and $Y = 0$.

15. Initially Ge and Si are both forward biased. So, current will effectively pass through Ge diode with a voltage drop of 0.3 V.

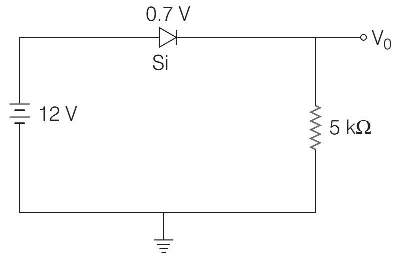


∴ Initial output voltage, $V_0 = 12 - 0.3 = 11.7$ V

If Ge is reversed biased, then only Si diode will work.

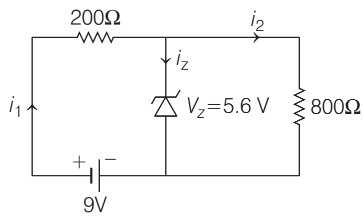
In this condition, output voltage

$$V_0 = 12 - 0.7 = 11.3 \text{ V}$$



∴ Change in output voltage = 11.7 − 11.3 = 0.4 V

16. Given circuit is Zener diode circuit



where,potential drop across 800 Ω resistance = potential drop across Zener diode = 5.6 V

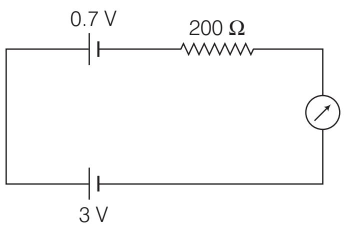
So, current, $i_2 = \dfrac{V}{R} = \dfrac{5.6}{800} = 7\text{mA}$

Now, potential drop across 200 Ω resistance
= 9 − 5.6 = 3.4 V

Current, $i_1 = \dfrac{V}{R} = \dfrac{3.4}{200} = 17$ mA

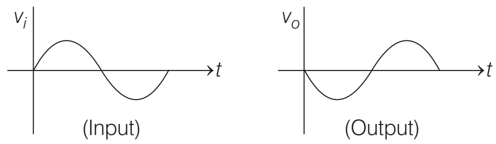
So, current, $i_z = i_1 - i_2 = 17 - 7 = 10$ mA

17. For silicon diode, potential barrier is 0.7 V. Therefore, the given circuit is as shown in figure.



$$I = \frac{\text{net emf}}{\text{net resistance}} = \frac{3-0.7}{200}\text{A} = 11.5\,\text{mA}$$

18.

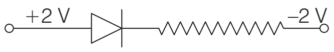


In a CE n-p-n transistor amplifier, output is 180° out of phase with input.

19. As, we know Cu is conductor, so increase in temperature, resistance will increase. Then, Si is semiconductor, so with increase in temperature, resistance will decrease.

20. Theoretical question. Therefore, no solution is required.

21. For forward bias, *p*-side must be a higher potential than *n*-side.

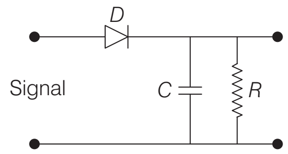


So, is forward biased.

22. For same value of current higher value of voltage is required for higher frequency.

23. As λ is increased, there will be a value of λ above which photoelectron will cease to come out. So, photocurrent will be zero.

24.



$$\tau = RC = 100 \times 10^3 \times 250 \times 10^{-12}\text{ s}$$
$$= 2.5 \times 10^{-5}\text{ s}$$

The higher frequency which can be detected with tolerable distortion is

$$f = \frac{1}{2\pi m_a RC} = \frac{1}{2\pi \times 0.6 \times 2.5 \times 10^{-5}}\text{ Hz}$$
$$= \frac{100 \times 10^4}{25 \times 1.2\,\pi}\text{ Hz} = \frac{4}{1.2\,\pi} \times 10^4\text{ Hz} = 10.61\,\text{kHz}$$

25. At junction a potential barrier/depletion layer is formed, with *n*-side at higher potential and *p*-side at lower potential. Therefore, there is an electric field at the junction directed from the *n*-side to *p*-side.

26. In *n*-type semiconductors, electrons are the majority charge carriers.

27. In the circuit, diode D_1 is forward biased, while D_2 is reverse biased. Therefore, current i (through D_1 and 100 Ω resistance) will be

$$i = \frac{6}{50 + 100 + 150} = 0.02\,\text{A}$$

Here 50 Ω is the resistance of D_1 in forward biasing.

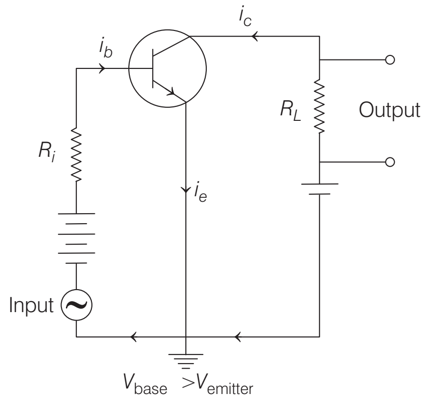
29. $\lambda_{\min} = 2480\text{nm} = 24800\text{Å}$

$$\text{Energy (in eV)} = \frac{12375}{\lambda}(\text{in Å})$$
$$E = \frac{12375}{24800}\text{eV}, E \approx 0.5\text{ eV}$$

30. In circuit 2 both the diodes are forward biased and in circuit 3 both the diodes are reverse biased.

33.
$$I_b + I_c = I_e$$
$$\Rightarrow \quad \frac{I_b}{I_c} + 1 = \frac{I_e}{I_c}$$
$$\Rightarrow \quad \frac{1}{\beta} + 1 = \frac{1}{\alpha}$$
$$\Rightarrow \quad \frac{1}{\alpha} = \frac{1+\beta}{\beta} \quad \Rightarrow \alpha = \frac{\beta}{1+\beta}$$

34. The circuit of a common emitter amplifier is as shown.



This has been shown an *n-p-n* transistor. Therefore, base emitter are forward biased and input signal is connected between base and emitter.

35. In intrinsic semiconductors, number of holes = number of free electrons while in case of a *p*-type semiconductor, number of holes > number of free electrons.

36. For half cycle diode 1 is forward biased and for the rest half it is reverse biased. Therefore, it will conduct only for one half cycle. Therefore, choice (b) or (c) is correct. (Not both)

37. Given : $i_c = 10\text{mA} = (0.9)\,i_e$ [Given that i_c is 90% of i_e]

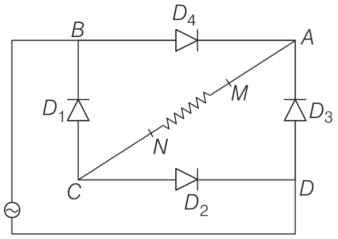
$$\therefore \quad i_e = \frac{10}{0.9}\text{ mA} \approx 11\text{mA}$$

and $\quad i_b = i_e - i_c = (11 - 10)\text{ mA} = 1\text{ mA}$

38. To make a *p*-type semiconductor, a trivalent impurity should be added to pure tetravalent compounds.

39. Reverse

40. In one half cycle when B is at higher potential then, D_4 and D_2 are forward biased and current follows the path BD_4MND_2DB.



In the second half cycle when D is at higher potential, D_3 and D_1 are forward biased. Hence, current follows the path DD_3MND_1BD. In both the cycles we see that current through the resistance is from M to N, *i.e.,* the given circuit behaves as full wave rectifier.

41. No solution is required.

42. No solution is required.

43. With increase in temperature saturation current in diode valve gets increased. Hence, $T_2 > T_1$.

44. Given, at $V_g = -1\text{V}$

$$I_p = (0.125V_p - 7.5) \times 10^{-3}\text{ A}$$

$$\left.\frac{dI_p}{dV_p}\right|_{V_g = -1\text{ volt}} = 0.125 \times 10^{-3}\text{ A/V}$$

$$\therefore \quad r_p = \left.\frac{dV_p}{dI_p}\right|_{V_g = -1\text{ volt}} = \frac{1}{0.125 \times 10^{-3}}\ \Omega$$

$$= 8 \times 10^3\ \Omega = 8\text{ k}\Omega$$

From the given equation,

$$I_p = (0.125 \times 300 - 7.5)\text{ mA}$$

$$(\text{at } V_g = -1\text{V and } V_p = 300\text{V}) = 30\text{ mA}$$

Now, $$g_m = \left.\frac{\Delta I_p}{\Delta V_g}\right|_{V_p = \text{constant}}$$

$$= \left.\frac{(30-5)}{(-1)-(-3)}\right|_{V_p = 300\text{ volt}}$$

$$= 12.5 \times 10^{-3}\text{ A/V}$$

Amplification factor $\mu = r_p \times g_m = 100$

Topic 7 Miscellaneous Problems

1. In an electromagnetic wave, magnetic field and electric field are perpendicular to each other and both are also perpendicular to the direction of propagation of wave.

Now, given direction of propagation is along z-direction. So, magnetic field is in either x or y direction.

Also, angular wave number for wave is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{2\pi \times 23.9 \times 10^9}{3 \times 10^8} \approx 0.5 \times 10^3\text{ m}^{-1}$$

and angular frequency ω for wave is

$$\omega = 2\pi\nu = 2\pi \times 23.9 \times 10^9\text{ Hz} = 1.5 \times 10^{11}\text{ Hz}$$

Magnitude of magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{60}{3 \times 10^8} = 2 \times 10^{-7}\text{T}$$

As the general equation of magnetic field of an electromagnetic wave propagating in $+z$- direction is given as,

$$\mathbf{B} = B_0 \sin(kz - \omega t)\hat{\mathbf{i}} \text{ or } \hat{\mathbf{j}}$$

Thus, substituting the values of B_0, k and ω, we get

$$\Rightarrow \mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\hat{\mathbf{i}} \text{ or } \hat{\mathbf{j}}$$

2. Given, carrier wave,

$$C(t) = 4\sin(20000\,\pi t)$$

Modulating signal,

$$m(t) = 2\sin(2000\,\pi t)$$

So, carrier wave's amplitude and frequency are

$$A_c = 4\text{ V},\ \omega_c = 20000\pi = 2\pi \times 10^4\text{ rad/s}$$

$$\Rightarrow \quad f_c = \frac{\omega_c}{2\pi} = 10^4\text{ Hz} = 10\text{ kHz}$$

and modulating signal's amplitude and frequency are

$$A_m = 2\text{V}, \omega_m = 2000\pi = 2\pi \times 10^3\text{ rad/s}$$

$$\Rightarrow f_m = \frac{\omega_m}{2\pi} = 10^3\text{ Hz} = 1\text{ kHz}$$

So, modulating index is $m = \dfrac{A_m}{A_c} = \dfrac{2}{4} = 0.5$

and lower side band frequency is,

$$f_{\text{LSB}} = f_c - f_m = 10 - 1 = 9\text{ kHz}$$

3. Standard expression of electromagnetic wave is given by

$$\mathbf{E} = E_0\hat{\mathbf{n}}\,[\sin(\omega t - \mathbf{k}\cdot\hat{\mathbf{r}})] \quad \ldots\text{(i)}$$

Here, $\mathbf{k}$ is the propagation vector.

Direction of propagation in this case is $\hat{\mathbf{k}}$.

Given expression of electromagnetic wave,

$$\mathbf{E} = E_0\hat{\mathbf{n}}\sin[\omega t + (6y - 8z)]$$

$$\Rightarrow \quad \mathbf{E} = E_0\hat{\mathbf{n}}\sin[\omega t - (8z - 6y)] \quad \ldots\text{(ii)}$$

Comparing Eq. (ii) with Eq. (i), we get

$$\mathbf{k}\cdot\hat{\mathbf{r}} = 8z - 6y \quad \ldots\text{(iii)}$$

Here, $$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

and $$\mathbf{k} = k_x\hat{\mathbf{i}} + k_y\hat{\mathbf{j}} + k_z\hat{\mathbf{k}}$$

$$\therefore \quad \mathbf{k}\cdot\hat{\mathbf{r}} = xk_x + yk_y + zk_z \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$xk_x = \text{zero} \Rightarrow k_x = 0$$

$$yk_y = -6y \Rightarrow k_y = -6$$

$$zk_z = 8z \Rightarrow k_z = 8$$

Hence, $\mathbf{k} = -6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

So, direction of propagation,

$$\hat{\mathbf{s}} = \hat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|} = \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{\sqrt{6^2 + 8^2}} = \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{10} = \frac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5}$$

4. Radiation pressure over an absorbing surface is, $p = \frac{I}{c}$

where, I = intensity or energy flux

and c = speed of light.

If A = area of surface, then force due to radiation on the surface is

$$F = p \times A = \frac{IA}{c}$$

If force F acts for a duration of Δt seconds, then momentum transferred to the surface is

$$\Delta p = F \times \Delta t = \frac{IA}{c} \times \Delta t$$

Here, $I = 25\,\text{W cm}^{-2}$, $A = 25\,\text{cm}^2$,

$c = 3 \times 10^8\,\text{ms}^{-1}$, $\Delta t = 40\,\text{min} = 2400\,\text{s}$

So, momentum transferred to the surface,

$$\Delta p = \frac{25 \times 25 \times 2400}{3 \times 10^8} = 5 \times 10^{-3}\ \text{N-s}$$

5. In optical fibre communication, infrared light is used to transmit information from one point to another.

RADAR (Radio detection and ranging) is a detection system that uses radio waves to determine range, angle or velocity of objects.

SONAR (Sound navigation and ranging) is also a detection system that uses ultrasound to detect under water objects, submarines, etc.

Mobile phone is a portable telephone that can make and receive calls, which make use of microwave.

$\therefore$ Correct sequence is

$$\text{A} \rightarrow \text{Q},\ \text{B} \rightarrow \text{S},\ \text{C} \rightarrow \text{P and D} \rightarrow \text{R}$$

6. **Key Idea** For an electromagnetic wave, its electric field vector (**E**) and magnetic field vector (**B**) is mutually perpendicular to each other and also to its direction of propagation.

We know that, $\mathbf{E} \times \mathbf{B}$ represents direction of propagation of an electromagnetic wave

$$\Rightarrow \quad (\mathbf{E} \times \mathbf{B}) \parallel v$$

$\therefore$ From the given electric field, we can state that direction of propagation is along Z-axis and direction of $\mathbf{E}$ is along X-axis.

Thus, from the above discussion, direction of $\mathbf{B}$ must be Y-axis.

From Maxwell's equation,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Here, $$\frac{\partial \mathbf{E}}{\partial Z} = -\frac{\partial B}{\partial t} \quad \ldots \text{(i)}$$

and $$B_0 = E_0 / C \quad \ldots \text{(ii)}$$

Given, $$\mathbf{E} = E_0\,\hat{\mathbf{i}} \cos kz \cos \omega t$$

$$\Rightarrow \quad \frac{-\partial \mathbf{E}}{\partial Z} = kE_0 \sin kz \cos \omega t$$

$\therefore$ Using Eq. (i), we get

$$\frac{\partial \mathbf{B}}{\partial t} = kE_0 \sin kz \cos \omega t$$

Integrating both sides of the above equation w.r.t. t, we get

$$\Rightarrow \quad \mathbf{B} = \frac{k}{\omega} E_0 \sin kz \sin \omega t = \frac{E_0}{C} \sin kz \sin \omega t$$

$$\Rightarrow \quad \mathbf{B} = \frac{E_0}{C} \sin(kz) \sin(\omega t)\,\hat{\mathbf{j}}$$

7. Given, $f_m = 100\,\text{MHz} = 100 \times 10^6\ \text{Hz}$,

$$A_m = 100\,\text{V},$$
$$A_c = 400\,\text{V}$$

Range of frequency in case of amplitude modulation is $(f_c - f_m)$ to $(f_c + f_m)$.

$\therefore$ Bandwidth $= 2f_m = 2 \times 100 \times 10^6\ \text{Hz}$

$$= 2 \times 10^8\,\text{Hz}$$

and modulation index,

$$\text{MI} = \frac{A_m}{A_c} = \frac{100}{400} = 0.25$$

8. Size of antenna is directly proportional to the wavelength of the signal.

Also, the speed at which signal moves = carrier frequency $\times$ wavelength

$$\Rightarrow \quad f\lambda = c \Rightarrow \lambda \propto \frac{1}{f}$$

$\therefore$ Size of antenna $\propto \frac{1}{f}$.

NOTE Minimum size of the antenna is $\lambda / 4$.

9. Radiation pressure or momentum imparted per second per unit area when light falls is

$$p = \begin{cases} \dfrac{2I}{c} & ; \quad \text{for reflection of radiation} \\[2mm] \dfrac{I}{c} & ; \quad \text{for absorption of radiation} \end{cases}$$

where, I is the intensity of the light.

In given case, there is 25% reflection and 75% absorption, so radiation pressure = force per unit area

$$= \frac{25}{100} \times \frac{2I}{c} + \frac{75}{100} \times \frac{I}{c}$$

$$= \frac{1}{2} \times \frac{I}{c} + \frac{3}{4} \times \frac{I}{c} = \frac{5}{4} \times \frac{I}{c} = \frac{5}{4} \times \frac{50}{3 \times 10^8}$$

$$= 20.83 \times 10^{-8}\,\text{N/m}^2 \approx 20 \times 10^{-8}\ \text{N/m}^2$$

10. Given, modulating signal,

$$A_m = A \cos \omega t$$

Carrier wave, $A_c = v_0 \sin \omega_0 t$

In amplitude modulation, modulated wave is given by

$$Y_m = [A_0 + A_m] \sin \omega_0 t$$

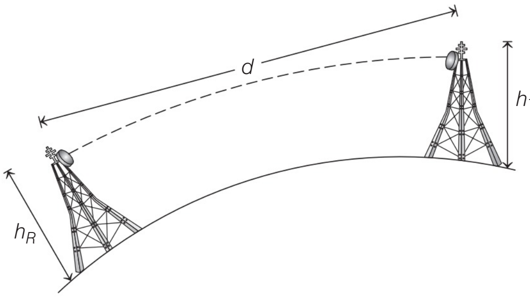
where, A_0 is amplitude of the carrier wave (given as v_0)

$$\therefore\ Y_m = [v_0 + A\cos\omega t]\sin\omega_0 t$$
$$= v_0\sin\omega_0 t + A\sin\omega_0 t\cos\omega t$$
$$= v_0\sin\omega_0 t + \frac{A}{2}[\sin(\omega_0+\omega)t + \sin(\omega_0-\omega)t]$$
$$= v_0\sin\omega_0 t + \frac{A}{2}\sin(\omega_0-\omega)t + \frac{A}{2}\sin(\omega_0+\omega)t$$

11. **Key Idea** In line of sight communication, distance d between transmitting antenna and receiving antenna is given by

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Here in figure, h_R and h_T is the height of receiving and transmitting antenna, respectively.



Given, $d = 50\,\text{km} = 50\times10^3$ m

$h_R = 70\,\text{m},\ R = 6.4\times10^6$ m

Then, distance between transmitting and receiving antenna, i.e. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$50\times10^3 = \sqrt{2R}\,(\sqrt{h_T} + \sqrt{h_R})$$
$$= \sqrt{2\times6.4\times10^6}\,(\sqrt{h_T} + \sqrt{70})$$
$$\Rightarrow\quad \sqrt{h_T} \approx \frac{50\times10^3}{3577.7} - 8.37$$
$$= 13.98 - 8.37 = 5.61$$
or $\qquad h_T = 31.5 \approx 32$ m

12. Given,

$$\mathbf{B} = 1.6\times10^{-6}\cos(2\times10^7 z + 6\times10^{15} t)(2\hat{\mathbf{i}} + \hat{\mathbf{j}})\,\text{Wbm}^{-2}$$

From the given equation, it can be said that the electromagnetic wave is propagating negative z-direction, i.e. $-\hat{\mathbf{k}}$.

Equation of associated electric field will be

$$\mathbf{E} = (|\mathbf{B}|c)\cos(kz+\omega t)\cdot\hat{\mathbf{n}}$$

where, $\hat{\mathbf{n}}$ = a vector perpendicular to $\mathbf{B}$.

So, $|\mathbf{E}| = |\mathbf{B}|c$

$$= 1.6\times10^{-6}\times3\times10^8 = 4.8\times10^2\ \text{V/m}$$

Since, we know that for an electromagnetic wave, $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular to each other.

So, $\qquad \mathbf{E}\cdot\mathbf{B} = 0$

From the given options, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E}\cdot\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = 0$$

Also, when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\mathbf{E}\cdot\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = 0$$

But, we also know that the direction of propagation of electromagnetic wave is perpendicular to both $\mathbf{E}$ and $\mathbf{B}$, i.e. it is in the direction of $\mathbf{E}\times\mathbf{B}$.

Again, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E}\times\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\times(\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = -\hat{\mathbf{k}}$$

and when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\mathbf{E}\times\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\times(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = \hat{\mathbf{k}}$$

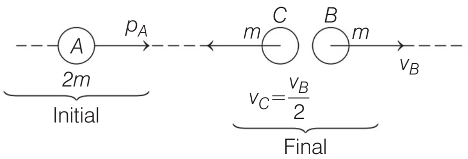
But, it is been given in the question that the direction of propagation of wave is in $-\hat{\mathbf{k}}$.

Thus, associated electric field will be

$$\mathbf{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15} t)(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\,\text{Vm}^{-1}$$

13. Let m be the mass of nuclei B and C.

So, the given situation can be shown in the figure below



Now, according to the conservation of linear momentum,

Initial momentum = Final momentum

$$\Rightarrow\quad p_A = p_B + p_C \quad\text{or}\quad 2mv_A = mv_B + mv_C$$
$$2mv_A = mv_B - \frac{mv_B}{2}$$
$$\Rightarrow\quad 2v_A = \frac{1}{2}v_B \Rightarrow v_B = 4v_A \qquad \ldots\text{(i)}$$
and $\qquad v_C = \dfrac{v_B}{2} = 2v_A \qquad \ldots$(ii)

So, momentum of B and C respectively, can now be given as

$$p_B = m_B v_B = m4v_A = 2(2mv_A) \qquad \text{[using Eq. (i)]}$$
or $\quad p_B = 2p_A \qquad \ldots$(iii)

and $\quad p_C = m_C v_C = m2v_A$ [using Eq. (ii)]

or $\qquad p_C = p_A \qquad \ldots$(iv)

From the relation of de-Broglie wavelength, i.e. $\lambda = \dfrac{h}{p}$

where, p is momentum and h is Planck's constant.

So, for A, $\lambda_A = \dfrac{h}{p_A}$ or $p_A = \dfrac{h}{\lambda_A} \qquad \ldots$(v)

From Eq. (v), λ_B can be written as

$$\lambda_B = \frac{h}{2\times\frac{h}{\lambda_A}} = \frac{\lambda_A}{2}$$

Similarly, for C, $\lambda_C = \dfrac{h}{p_C} = \dfrac{h}{p_A} \qquad$ [using Eq. (iv)]

Similarly, from Eq. (v), λ_C can be written as

$$\lambda_C = \frac{h}{\frac{h}{\lambda_A}} = \lambda_A$$

14. In optical fibre communication network, the signals are transmitted by laser light operating in range of 1310 nm-1550 nm.

So, the closest value is 1500 nm.

15. **Key Idea** For an electromagnetic wave, ratio of magnitudes of electric and magnetic field is

$$\frac{E}{B} = c$$

where, c is the speed of electromagnetic wave in vacuum.

Given, $E = 6\,\text{V/m}, c = 3 \times 10^8\ \text{ms}^{-1}$

So, $B = \frac{E}{c} = \frac{6}{3 \times 10^8} = 2 \times 10^{-8}$ T

Also, direction of propagation of electromagnetic wave is given by

$$\hat{\mathbf{n}} = \mathbf{E} \times \mathbf{B}$$

Here, $\hat{\mathbf{n}} = \hat{\mathbf{i}}$ and $\mathbf{E}$ = Unit vector of electric field $(\hat{\mathbf{j}})$

$\mathbf{B}$ = unit vector of magnetic field.

$\Rightarrow \quad \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \mathbf{B} \Rightarrow \mathbf{B} = \hat{\mathbf{k}}$

Hence, magnetic field components,

$$\mathbf{B} = 2 \times 10^{-8}\,\hat{\mathbf{k}}\text{T} = 2 \times 10^{-8}\text{T} \qquad \text{(along } z\text{-direction)}$$

16. Range of TV transmitting tower $d = \sqrt{2hR}$

where, h is the height of the transmission tower. when range is doubled,

$$\therefore \quad 2d = 2\sqrt{2hR}$$

$$= \sqrt{2(4h)R}$$

So, height must be multiplied with 4.

17. Minimum wavelength occurs when mercury atom deexcites from highest energy level.

$\therefore$ Maximum possible energy absorbed by mercury atom

$$= \Delta E = 5.6 - 0.7 = 4.9\,\text{eV}$$

Wavelength of photon emitted in deexcitation is

$$\lambda = \frac{hc}{E} \approx \frac{1240\ \text{eVnm}}{4.9\ \text{eV}} \approx 250\,\text{nm}$$

NOTE

Frank-Hertz experiment was the first electrical measurement to show quantum nature of atoms. In a vacuum tube energatic electrons are passed through thin mercury vapour film. It was discovered that when an electron collided with a mercury atom, it loses only a specific quantity (4.9 eV) of it's kinetic energy. This experiment shows existence of quantum energy levels.

18. Modulation index is given by

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

$$= \frac{160 - 40}{160 + 40} = \frac{120}{200} = 0.6$$

19. Given,

Power of laser beam $(P) = 27\text{mW} = 27 \times 10^{-3}$ W

Area of cros- section $(A) = 10\text{m m}^2 = 10 \times 10^{-6}\text{m}^2$

Permittivity of free space $(\varepsilon_0) = 9 \times 10^{-12}$ SI unit

Speed of light $(c) = 3 \times 10^8$ m/s

Intensity of electromagnetic wave is given by the relation

$$I = \frac{1}{2} nc\varepsilon_0 E^2$$

where, n is refractive index, for air $n = 1$.

$$\therefore \quad I = \frac{1}{2} c \cdot \varepsilon_0 E^2 \qquad \text{...(i)}$$

$$\text{Also,} \quad I = \frac{P}{A} \qquad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$\frac{1}{2} c\varepsilon_0 E^2 = \frac{P}{A} \text{ or } E^2 = \frac{2P}{Ac\varepsilon_0}$$

$$\text{or} \quad E = \sqrt{\frac{2 \times 27 \times 10^{-3}}{10 \times 10^{-6} \times 3 \times 10^8 \times 9 \times 10^{-12}}}$$

$$\simeq 1.4 \times 10^3\ \text{V/m} = 1.4\ \text{kV/m}$$

20 Equation of an amplitude modulated wave is given by the relation,

$$C_m = (A_c + A_m \sin \omega_m t) \cdot \sin \omega_c \cdot t \qquad \text{...(i)}$$

For the given graph, maximum amplitude,

$$A_c + A_m = 10 \qquad \text{...(ii)}$$

and minimum amplitude, $A_c - A_m = 8$ ...(iii)

From Eqs. (ii) and (iii), we get

$$A_c = 9\text{V}$$

$$\text{and} \quad A_m = 1\,\text{V} \qquad \text{...(iv)}$$

$\because$ For angular frequency of message signal and carrier wave, we use a relation

$$\omega_c = \frac{2\pi}{T_c} = \frac{2\pi}{8 \times 10^{-6}}$$

(as from given graph, $T_c = 8 \times 10^{-6}$ s)

$$= 2.5\pi \times 10^5\,\text{s}^{-1} \qquad \text{...(v)}$$

$$\text{and} \quad \omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{100 \times 10^{-6}}$$

(as from given graph, $T_m = 100 \times 10^{-6}$s)

$$= 2\pi \times 10^4\,\text{s}^{-1}$$

When we put values of A_c, A_m, ω_c and ω_m in Eq. (i), we get

$$C_m = [9 + \sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)\,\text{V}$$

21. $v(t) = 10\,[1 + 0.3\cos(2.2 \times 10^4 t)] \qquad [\sin(5.5 \times 10^5 t)]$

Upper band angular frequency

$$\omega_v = (2.2 \times 10^4 + 5.5 \times 10^5)\ \text{rad / s}$$

$$= 572 \times 10^3\ \text{rad / s}$$

Similarly, lower band angular frequency.

$$\omega_L = (5.5\times 10^5 - 2.2\times 10^4)\ \text{rad / s}$$
$$= 528\times 10^3\ \text{rad / s}$$

$\therefore$ Side band frequency are,

$$f_u = \frac{\omega_u}{2\pi} = \frac{572}{2\pi}\ \text{kHz} \simeq 91\ \text{kHz}$$

and $$f_L = \frac{\omega_L}{2\pi} = \frac{528}{2\pi}\ \text{kHz} \simeq 84\ \text{kHz}$$

22. In the free space, the speed of electromagnetic wave is given as,

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E_0}{B_0} \quad \ldots\text{(i)}$$

where, E_0 and B_0 are the amplitudes of varying electric and magnetic fields, respectively.

Now, when it enters in a medium of refractive index 'n', its speed is given as,

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{K\varepsilon_0\mu}} = \frac{E}{B} \quad \ldots\text{(ii)}$$

where, K is dielectric strength of the medium.

Using Eqs. (i) and (ii), we get

$$\frac{v}{c} = \frac{1}{\sqrt{K}} \quad \ldots\text{(iii)}$$

($\because$ For a transparent medium, $\mu_0 \approx \mu$)

Also, refractive index of medium is 'n' and is given as

$$\frac{c}{v} = n \quad \text{or} \quad \frac{v}{c} = \frac{1}{n} \quad \ldots\text{(iv)}$$

$\therefore$ From Eqs. (iii) and (iv), we get

$$n = \sqrt{K} \quad \text{or} \quad K = n^2 \quad \text{...(v)}$$

The intensity of a EM wave is given as,

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

and in the medium, it is given as $I' = \frac{1}{2}K\varepsilon_0 E^2 v$

It is given that, $I = I'$

$$\Rightarrow \quad \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}K\varepsilon_0 E^2 v \quad \text{or} \quad \left(\frac{E_0}{E}\right)^2 = \frac{Kv}{c} \quad \ldots\text{(vi)}$$

From Eqs. (iv), (v) and (vi), we get

$$\left(\frac{E_0}{E}\right)^2 = \frac{n^2}{n} = n \quad \text{or} \quad E_0 / E = \sqrt{n}$$

Similarly, $\frac{1}{2}\cdot\frac{B_0^2}{\mu_0}c = \frac{1}{2}\cdot\frac{B^2}{\mu_0}v \Rightarrow \frac{B_0}{B} = \frac{1}{\sqrt{n}}$

Alternate method

We know that,

$$\left(\frac{E_0}{B_0}\right)_{\text{air/ vacuum}} = c \quad \text{and} \quad \left(\frac{E}{B}\right)_{\text{medium}} = v$$

Also, $$n = \frac{c}{v}$$

$$\frac{E_0 / B_0}{E / B} = \frac{c}{v} = n$$

$$\Rightarrow \quad \frac{E_0 / E}{B_0 / B} = n$$

This is possible only if $\frac{E_0}{E} = \sqrt{n}$ and $\frac{B_0}{B} = \frac{1}{\sqrt{n}}$.

23. We are given electric field as

$$\mathbf{E} = 10\,\hat{\mathbf{j}}\cos(6x + 8z) \quad \ldots\text{(i)}$$

where, phase angle is independent of time,

i.e., phase angle at $t = 0$ is $\phi = 6x + 8z$.

Phase angle for $\mathbf{B}$ will also be $6x + 8z$ because for an electromagnetic wave $\mathbf{E}$ and $\mathbf{B}$ oscillate in same phase.

Thus, direction of wave propagation

$$= \frac{6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}}{\sqrt{6^2 + 8^2}} = \frac{6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}}{10} \quad \ldots\text{(ii)}$$

Let magnetic field vector,

$\mathbf{B} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + d\hat{\mathbf{k}}$, then direction of wave propagation is given by

$$\frac{\mathbf{E}\times\mathbf{B}}{|\mathbf{E}'||\mathbf{B}|} = \frac{10\hat{\mathbf{j}}\times(a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + d\hat{\mathbf{k}})}{10\times(a^2 + b^2 + d^2)^{1/2}}$$

$$= \frac{-10a\hat{\mathbf{k}} + 10d\hat{\mathbf{i}}}{10(a^2 + b^2 + d^2)^{1/2}} \quad \ldots\text{(iii)}$$

As, $$|\mathbf{B}| = \frac{|\mathbf{E}|}{c} = \frac{10}{c}$$

We get, $$|\mathbf{B}| = \sqrt{a^2 + b^2 + d^2} = 10/c$$

By putting this value in Eqs. (iii) and (ii), we get direction of propagation

$$\Rightarrow \quad \frac{c10(d\,\hat{\mathbf{i}} - a\hat{\mathbf{k}})}{10\times 10} = \frac{6\hat{\mathbf{i}}\times 8\hat{\mathbf{k}}}{10}$$

$$\Rightarrow \quad d = 6/c \text{ and } a = -8/c$$

Hence, $\mathbf{B} = \frac{6}{c}\hat{\mathbf{k}} - \frac{8}{c}\hat{\mathbf{i}} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})$

As the general equation of magnetic field of an EM wave propagating in positive y-direction is given as,

$$B = B_0\cos(Ry - \omega t))$$

$$\therefore \quad \mathbf{B} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos(6x + 8z - 10ct)$$

Alternate method Given, electric field is $E(x, y)$, i.e. electric field is in xy-plane which is given as

$\because \quad \mathbf{E} = 10\hat{\mathbf{j}}\cos(6x + 8z)$

Since, the magnetic field given is $\mathbf{B}(x, z, t)$, this means $\mathbf{B}$ is in xz-plane.

$\therefore$ Propagation of wave is in y-direction.

[$\because$ for an electromagnetic wave, $\mathbf{E}\perp\mathbf{B}\perp$ propagation direction]

As Poynting vector suggests that $\mathbf{E} \times \mathbf{B}$ is parallel to $(6\hat{\mathbf{i}} + 8\hat{\mathbf{k}})$.

Let $\mathbf{B} = (x\hat{\mathbf{i}} + z\hat{\mathbf{k}})$,

then $\mathbf{E} \times \mathbf{B} = \hat{\mathbf{j}} \times (x\hat{\mathbf{i}} + z\hat{\mathbf{k}}) = 6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}$

or $-x\hat{\mathbf{k}} + z\hat{\mathbf{i}} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}$

or $x = -8$ and $z = 6$

$$\therefore \quad \mathbf{B} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos(6x + 8z - 10ct) \qquad \left[\because \frac{|\mathbf{E}|}{|\mathbf{B}|} = c\right]$$

24. For a given carrier wave of frequency f_c with modulation frequency f_m, the bandwidth is calculated by

$$f_{\text{upper}} = f_c + f_m$$
$$f_{\text{lower}} = f_c - f_m \qquad \dots \text{(i)}$$

To avoid overlapping of bandwidths, next broadcast frequencies can be

$$f_1 = f_c \pm 2f_m,\ f_2 = f_c \pm 3f_m$$

So, next immediate available broadcast frequency is

$$f_1 = f_c + 2f_m \text{ and } f_1' = f_c - 2f_m$$

Given, $f_m = 250$ kHz

and also that f_m is 10% of f_c, i.e. $f_c = 2500$ kHz.

So, $f_1 = 2500 + (2 \times 250) = 3000$ kHz

and $f'_1 = 2500 - (2 \times 250) = 2000$ kHz

25. Given, instantaneous value of magnetic field

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$$

and speed of light, $c = 3 \times 10^8 \text{ ms}^{-1}$

For an electromagnetic wave,

$$E_{\max} = B_{\max} \times c$$

where, $E_{\max}$ = maximum value of the electric field.

We get, $E_{\max} = 100 \times 10^{-6} \times 3 \times 10^8 = 3 \times 10^4 \ \frac{\text{N}}{\text{C}}$

26. Maximum distance of transmission is given by

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where, h_T = height of transmitter, = 140 m,

h_R = height of receiver = 40 m and

R = radius of earth = 6.4×10^6 m.

Substituting values, we get

$$d = \sqrt{2 \times 6.4 \times 10^6}\ (\sqrt{140} + \sqrt{40}) = 65 \text{ km}$$

27. Given, resolution achieved in electron microscope is of the order of wavelength.

So, to resolve 7.5×10^{-12} m seperation wavelength associated with electrons is

$$\lambda = 7.5 \times 10^{-12} \text{ m}$$

$\therefore$ Momentum of electrons required is

$$p = \frac{h}{\lambda}$$

or kinetic energy of electron must be

$$\text{KE} = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m}$$

Substituting the given values, we get

$$= \frac{\left(\dfrac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right)^2}{2 \times 9.1 \times 10^{-31}} \text{ J}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (7.5 \times 10^{-12})^2 \times (1.6 \times 10^{-19})} \text{eV}$$

$$(\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

$= 26593.4 \simeq 26.6 \times 10^3 \text{ eV} \simeq 26$ keV

which is nearest to 25 keV.

28. Energy density of an electomagnetic wave in electric field,

$$U_E = \frac{1}{2}\varepsilon_0 \cdot E^2 \qquad \dots\text{(i)}$$

Energy density of an electromagnetic wave in magnetic field,

$$U_B = \frac{B^2}{2\mu_0} \qquad \dots\text{(ii)}$$

where, E = electric field,

B = magnetic field,

ε_0 = permittivity of medium and

μ_0 = magnetic permeability of medium.

From the theory of electro-magnetic waves, the relation between μ_0 and ε_0 is

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \dots\text{(iii)}$$

where, c = velocity of light $= 3 \times 10^8$ m/s

and $$\frac{E}{B} = c \qquad \dots\text{(iv)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{U_E}{U_B} = \frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{1}{2}B^2 \times \frac{1}{\mu_0}} = \frac{\mu_0\varepsilon_0 E^2}{B^2} \qquad \dots\text{(v)}$$

Using Eqs. (iii), (iv) and (v), we get

$$\frac{U_E}{U_B} = \frac{c^2}{c^2} = 1$$

Therefore, $U_E = U_B$

29. Here,

Signal wavelength, $\lambda = 800$ n-m $= 8 \times 10^{-7}$ m

Frequency of source is

As, $$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-7}}$$
$$= 3.75 \times 10^{14} \text{ Hz}$$

$\therefore$ Total bandwidth used for communication

$$= 1\% \text{ of } 3.75 \times 10^{14}$$
$$= 3.75 \times 10^{12} \text{ Hz} \quad \text{...(i)}$$

So, number of channel for signals

$$= \frac{\text{total bandwidth available for communication}}{\text{bandwidth of TV signal}}$$

$$= \frac{3.75 \times 10^{12}}{6 \times 10^6} = 0.625 \times 10^6 = 6.25 \times 10^5$$

30. $v = \dfrac{\text{coefficient of } t}{\text{coefficient of } z} = \dfrac{1}{\sqrt{\varepsilon\mu}}$

where, $\varepsilon = \varepsilon_0 \varepsilon_r$ and $\mu = \mu_0 \mu_r$.

$$\frac{v_{\text{air}}}{v_{\text{med}}} = \frac{c}{c/2} = 2 = \frac{\sqrt{\mu_0 \varepsilon_0 \mu_{r_2} \varepsilon_{r_2}}}{\sqrt{\mu_0 \varepsilon_0 \mu_{r_1} \varepsilon_{r_1}}} \Rightarrow \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

Note Medium is non-magnetic.

$\therefore \quad \mu_{r_1} = \mu_{r_2}$

31. $N = \dfrac{1}{10} \cdot \dfrac{(10 \text{ kHz})}{(5 \text{kHz})} = \dfrac{10^9}{5 \times 10^3} = \dfrac{10^6}{5} = 2 \times 10^5$

32. $f_c = 2 \text{ MHz} = 2000 \text{ kHz}, \quad f_m = 5 \text{ kHz}$

Resultant frequencies are,

$$f_c + f_m,\ f_c \text{ and } f_c - f_m$$

or, 2005 kHz, 2000 kHz and 1995 kHz

33. Intensity at a distance r from a point source of power P is given by

$$I = \frac{P}{4\pi r^2} \quad \text{...(i)}$$

Also, $$I = \frac{1}{2} \varepsilon_0 E_0^2 c \quad \text{...(ii)}$$

where, E_0 is amplitude of electric field and c the speed of light. Eqs. (i) and (ii) we get

$$E_0 = \sqrt{\frac{2P}{4\pi\varepsilon_0 r^2 c}}$$
$$= \sqrt{\frac{2 \times 9 \times 10^9 \times 0.1}{(1)^2 \times 3 \times 10^8}}$$
$$= \sqrt{6} = 2.45 \text{ V/m}$$

34. Both the energy densities are equal i.e. energy is equally divided between electric and magnetic fields.

35. Peak value of electric field

$$E_0 = B_0 c = 20 \times 10^{-9} \times 3 \times 10^8 = 6 \text{ V/m}$$

36. As velocity (or momentum) of electron is increased, the wavelength $\left(\lambda = \dfrac{h}{p}\right)$ will decrease. Hence, fringe width will decrease ($\omega \propto \lambda$).

37. Atomic number of neon is 10.

By the emission of two α-particles, atomic number will be reduced by 4. Therefore, atomic number of the unknown element will be $Z = 10 - 4 = 6$

Similarly, mass number of the unknown element will be

$$A = 22 - 2 \times 4 = 14$$

Unknown nucleus is carbon ($A = 14, Z = 6$).

39. The maximum number of electrons in an orbit are $2n^2$. If $n > 4$, is not allowed, then the number of maximum electrons that can be in first four orbit are :

$$2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 = 2 + 8 + 18 + 32 = 60$$

Therefore, possible elements can be 60.

40. Given, $r_p = 3 \times 10^3\ \Omega,\ g_m = 1.5 \times 10^{-3}$ A/V

$\therefore$ Amplification factor, $\mu = g_m \times r_p = 4.5$

41. $\therefore$ Power $= nhf$

(where, n = number of photons incident per second)

Since, KE = 0, hf = work-funcition W

$$200 = nW = n\,[6.25 \times 1.6 \times 10^{-19}]$$

$$\Rightarrow \quad n = \frac{200}{1.6 \times 10^{-19} \times 6.25}$$

As photon is just above threshold frequency KE_{max} is zero and they are accelerated by potential difference of 500 V.

$\therefore$ $$\text{KE}_f = q\Delta V$$
$$\frac{P^2}{2m} = q\Delta V \Rightarrow P = \sqrt{2\,mq\,\Delta V}$$

Since, efficiency is 100%, number of electrons emitted per second = number of photons incident per second.

As, photon is completely absorbed, force exerted

$$= n(mV) = nP = n\sqrt{2mq\Delta V}$$
$$= \frac{200}{6.25 \times 1.6 \times 10^{-19}} \times \sqrt{2\,(9 \times 10^{-31}) \times 1.6 \times 10^{-19} \times 500}$$
$$= 24$$

42-43. Maximum kinetic energy of anti-neutrino is nearly (0.8×10^6) eV.

44. (A) → (R or RT); (B) → (P, S); (C) → (Q, T); (D) → (R,)

No solution is required.

45. No Solution is required

46. (a) Infrared rays are used to treat muscular strain.

(b) Radiowaves are used for broadcasting purposes.

(c) X-rays are used to detect fracture of bones.

(d) Ultraviolet rays are absorbed by ozone.

47. (p) In α-decay mass number decreases by 4 and atomic number decreases by 2.

(q) In β^+-decay mass number remains unchanged while atomic number decreases by 1.

(r) In fission, parent nucleus breaks into all most two equal fragments.

(s) In proton emission both mass number and atomic number decreases by 1.

49. $\left|\frac{dN}{dt}\right| = $ |Activity of radioactive substance|

$$= \lambda N = \lambda N_0 \, e^{-\lambda t}$$

Taking log both sides

$$\ln\left|\frac{dN}{dt}\right| = \ln\,(\lambda N_0) - \lambda t$$

Hence, $\ln\left|\frac{dN}{dt}\right|$ *versus* t graph is a straight line with slope $-\lambda$.

From the graph we can see that, $\lambda = \frac{1}{2} = 0.5\ \text{yr}^{-1}$

Now applying the equation,

$$N = N_0 e^{-\lambda t} = N_0\, e^{-0.5\times 4.16}$$

$$= N_0\, e^{-2.08} = 0.125\, N_0 = \frac{N_0}{8}$$

i.e, nuclei decreases by a factor of 8.

Hence, the answer is 8.

50. No solution is required.

51. (a) Given mass of α-particle, $m = 4.002$ amu and mass of daughter nucleus,

$$M = 223.610\ \text{amu},$$

de-Broglie wavelength of α-particle,

$$\lambda = 5.76\times 10^{-15}\text{m}$$

So, momentum of α-particle would be

$$p = \frac{h}{\lambda} = \frac{6.63\times 10^{-34}}{5.76\times 10^{-15}}\ \text{kg-m/s}$$

or $\quad p = 1.151\times 10^{-19}$ kg-m/s

From law of conservation of linear momentum, this should also be equal to the linear momentum of the daughter nucleus (in opposite direction).

Let K_1 and K_2 be the kinetic energies of α-particle and daughter nucleus. Then total kinetic energy in the final state is

$$K = K_1 + K_2 = \frac{p^2}{2m} + \frac{p^2}{2M} = \frac{p^2}{2}\left(\frac{1}{m} + \frac{1}{M}\right)$$

$$K = \frac{p^2}{2}\left(\frac{M+m}{Mm}\right)$$

$$1\ \text{amu} = 1.67\times 10^{-27}\ \text{kg}$$

Substituting the values, we get

$$K = 10^{-12}\ \text{J}$$

$$K = \frac{10^{-12}}{1.6\times 10^{-13}} = 6.25\ \text{MeV}$$

or $\quad K = 6.25$ MeV.

(b) Mass defect, $\Delta m = \frac{6.25}{931.470} = 0.0067$ amu

Therefore, mass of parent nucleus = mass of α-particle + mass of daughter nucleus + mass defect (Δm)

$$= (4.002 + 223.610 + 0.0067)\ \text{amu}$$

$$= 227.62\ \text{amu}$$

Hence, mass of parent nucleus is 227.62 amu.

52. Energy of incident photons,

$$E_i = 10.6\ \text{eV} = 10.6\times 1.6\times 10^{-19}\ \text{J}$$

$$= 16.96\times 10^{-19}\ \text{J}$$

Energy incident per unit area per unit time (intensity) = 2 J

$\therefore$ Number of photons incident on unit area in unit time

$$= \frac{2}{16.96\times 10^{-19}} = 1.18\times 10^{18}$$

Therefore, number of photons incident per unit time on given area $(1.0\times 10^{-4}\ \text{m}^2)$

$$= (1.18\times 10^{18})\,(1.0\times 10^{-4}) = 1.18\times 10^{14}$$

But only 0.53% of incident photons emit photoelectrons

$\therefore$ Number of photoelectrons emitted per second (n)

$$n = \left(\frac{0.53}{100}\right)(1.18\times 10^{14})$$

$$n = 6.25\times 10^{11}$$

$$K_{\text{min}} = 0$$

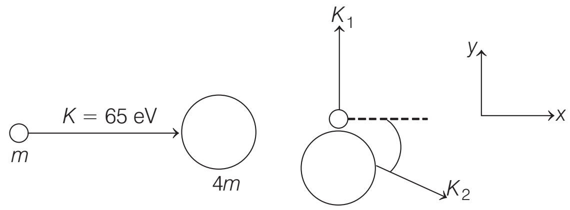
and $\quad K_{\text{max}} = E_i -$ work function

$$= (10.6 - 5.6)\text{eV} = 5.0\ \text{eV}$$

$\therefore \quad K_{\text{max}} = 5.0$ eV

and $\quad K_{\text{min}} = 0$

54. (a) Let K_1 and K_2 be the kinetic energies of neutron and helium atom after collision and ΔE be the excitation energy.



From conservation of linear momentum along x-direction.

$$p_i = p_f$$

$\Rightarrow \quad \sqrt{2Km} = \sqrt{2(4m)K_2}\cos\theta \qquad$ ....(i)

Similarly, applying conservation of linear momentum in y-direction, we have

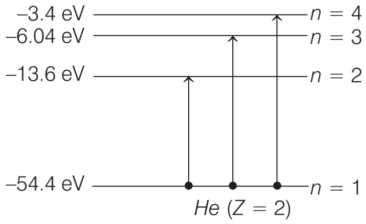
$\sqrt{2K_1 m} = \sqrt{2(4m)K_2}\sin\theta \qquad$ ...(ii)

Squaring and adding Eqs. (i) and (ii), we get

$K + K_1 = 4K_2 \qquad$ ...(iii)

or $\quad 4K_2 - K_1 = K = 65$ eV $\qquad$ ...(iv)

Now, during collision, electron can be excited to any higher energy state. Applying conservation of energy, we get $K = K_1 + K_2 + \Delta E$



or $$65 = K_1 + K_2 + \Delta E \quad \ldots(v)$$

ΔE can have the following values,

$$\Delta E_1 = \{-13.6 - (-54.4)\} \text{ eV} = 40.8 \text{ eV}$$

Substituting in (v), we get

$$K_1 + K_2 = 24.2 \text{ eV} \quad \ldots(vi)$$

Solving (iv) and (vi), we get

$$K_1 = 6.36 \text{ eV}$$

and $$K_2 = 17.84 \text{ eV}$$

Similarly, when we put $\Delta E = \Delta E_2$

$$= \{-6.04 - (-54.4)\}\text{eV}$$
$$= 48.36 \text{ eV}$$

Put in Eq. (v), we get

$$K_1 + K_2 = 16.64 \text{ eV} \quad \ldots(vii)$$

Solving Eqs. (iv) and (vii), we get

$$K_1 = 0.312 \text{ eV} \text{ and } K_2 = 16.328 \text{ eV}$$

Similarly, when we put

$$\Delta E = \Delta E_3 = \{-3.4 - (-54.4)\} = 51 \text{ eV}$$

Put in Eq. (v), we get

$$K_1 + K_2 = 14 \text{ eV} \quad \ldots(viii)$$

Now, solving Eqs. (iv) and (viii), we get

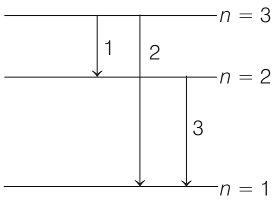
$$K_1 = -1.8 \text{ eV and } K_2 = 15.8 \text{ eV}$$

But since the kinetic energy cannot have the negative values, the electron will not jump to third excited state or $n = 4$.

Therefore, the allowed values of K_1(KE of neutron) are 6.36 eV and 0.312 eV and of K_2 (KE of the atom) are 17.84 eV and 16.328 eV and the electron can jump upto second excited state only ($n=3$).

(b) Possible emission lines are only three as shown in figure. The corresponding frequencies are

$$\nu_1 = \frac{(E_3 - E_2)}{h}$$



$$= \frac{\{-6.04 - (-13.6)\} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 1.82 \times 10^{15} \text{ Hz}$$

$$\nu_2 = \frac{E_3 - E_1}{h} = \frac{\{-6.04 - (-54.4)\} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 11.67 \times 10^{15} \text{ Hz}$$

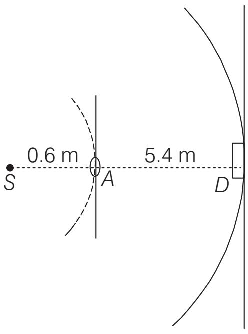
and $$\nu_3 = \frac{E_2 - E_1}{h}$$

$$= \frac{\{-13.6 - (-54.4)\} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 9.84 \times 10^{15} \text{ Hz}$$

Hence, the frequencies of emitted radiations are 1.82×10^{15} Hz, 11.67×10^{15} Hz and 9.84×10^{15} Hz.

55. (a) Energy of one photon,



$$E = \frac{hc}{\lambda}$$

$$= \frac{(6.6 \times 10^{-34})(3.0 \times 10^8)}{6000 \times 10^{-10}}$$

$$= 3.3 \times 10^{-19} \text{ J}$$

Power of the source is 2 W or 2 J/s. Therefore, number of photons emitting per second,

$$n_1 = \frac{2}{3.3 \times 10^{-19}} = 6.06 \times 10^{18}\text{/s}$$

At distance 0.6 m, number of photons incident per unit area per unit time :

$$n_2 = \frac{n_1}{4\pi(0.6)^2} = 1.34 \times 10^{18}\text{/m}^2\text{/s}$$

Area of aperture is,

$$S_1 = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.1)^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$\therefore$Total number of photons incident per unit time on the aperture,

$$n_3 = n_2 S_1$$
$$= (1.34 \times 10^{18})(7.85 \times 10^{-3})\text{/s}$$
$$= 1.052 \times 10^{16}\text{/s}$$

The aperture will become new source of light.

Now, these photons are further distributed in all directions. Hence, at the location of detector, photons incident per unit area per unit time

$$n_4 = \frac{n_3}{4\pi(6-0.6)^2} = \frac{1.052\times 10^{16}}{4\pi\,(5.4)^2}$$

$$= 2.87\times 10^{13}\ \text{s}^{-1}\text{m}^{-2}$$

This is the photon flux at the centre of the screen. Area of detector is 0.5 cm^{2}or $0.5\times 10^{-4}\,\text{m}^2$. Therefore, total number of photons incident on the detector per unit time

$$n_5 = (0.5\times 10^{-4})\,(2.87\times 10^{13} d)$$

$$= 1.435\times 10^{9}\ \text{s}^{-1}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time

$$n_6 = 0.9 n_5$$

$$= 1.2915\times 10^{9}\text{s}^{-1}$$

or, photocurrent in the detector

$$i = (e) n_6 = (1.6\times 10^{-19})(1.2915\times 10^{9})$$

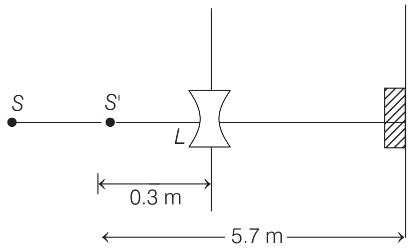
$$= 2.07\times 10^{-10}\ \text{A}$$

(b) Using the lens formula :

$$\frac{1}{v} - \frac{1}{-0.6} = \frac{1}{-0.6}$$

or $\qquad v = -\,0.3\,\text{m}$

i.e. image of source (say S', is formed at 0.3 m) from the lens.



Total number of photons incident per unit time on the lens are still n_3 or 1.052×10^{16}/s. 80% of it transmits to second medium. Therefore, at a distance of 5.7 m from S' number of photons incident per unit area per unit time will be

$$n_7 = \frac{(80/100)\,(1.052\times 10^{16})}{(4\pi)\,(5.7)^2}$$

$$= 2.06\times 10^{13}\ \text{s}^{-1}\text{m}^{-2}$$

This is the photon flux at the detector

New, value of photocurrent is

$$i' = (2.06\times 10^{13})\,(0.5\times 10^{-4})\,(0.9)\,(1.6\times 10^{-19})$$

$$= 1.483\times 10^{-10}\,\text{A}$$

(c) Energy of incident photons (in both the cases) :

$$E = \frac{12375\,\text{eV-Å}}{6000\ \text{Å}} = 2.06\,\text{eV}$$

Work function $W = 1.0\,\text{eV}$

$\therefore$ Maximum kinetic energy of photoelectrons in both cases,

$$K_{\max} = E - W = 1.06\,\text{eV}$$

or the stopping potential will be 1.06 V.

56. The stopping potential for shorter wavelength is 3.95 V i.e. maximum kinetic energy of photoelectrons corresponding to shorter wavelength will be 3.95 eV. Further energy of incident photons corresponding to shorter wavelength will be in transition from $n = 4$ to $n = 3$.

$$E_{4-3} = E_4 - E_3 = \frac{-\,(13.6)\,(3)^2}{(4)^2} - \left[\frac{-(13.6)\,(3)^2}{(3)^2}\right]$$

$$= 5.95\ \text{eV}$$

Now, from the equation,

$$K_{\max} = E - W$$

we have $\quad W = E - K_{\max} = E_{4-3} - K_{\max}$

$$= (5.95 - 3.95)\,\text{eV} = 2\,\text{eV}$$

Longer wavelength will correspond to transition from $n = 5$ to $n = 4$. From the relation,

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{N_f^{\,2}} - \frac{1}{N_i^{\,2}}\right)$$

The longer wavelength,

$$\frac{1}{\lambda} = (1.094\times 10^{7})\,(3)^3\left(\frac{1}{16} - \frac{1}{25}\right)$$

or $\qquad \lambda = 4.514\times 10^{-7}\ \text{m} = 4514\ \text{Å}$

Energy corresponding to this wavelength,

$$E = \frac{12375\,\text{eV-Å}}{4514\,\text{Å}} = 2.74\ \text{eV}$$

$\therefore$ Maximum kinetic energy of photo-electrons

$$K_{\max} = E - W = (2.74 - 2)\,\text{eV}$$

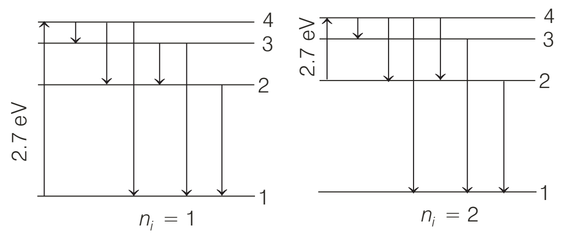
$$= 0.74\ \text{eV}$$

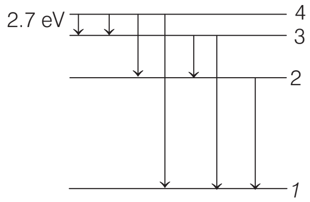
or the stoping potential is 0.74 V.

57. (a) In emission spectrum total six lines are obtained. Hence, after excitation if n_f be the final principal quantum number then,

$$\frac{n_f\,(n_f - 1)}{2} = 6 \quad \text{or}\ \ n_f = 4\text{z}$$

i.e. after excitation atom goes to 4$^{\text{th}}$ energy state. Hence, n_i can be either 1, 2 or 3.





$n_i = 3$

Absorption and emission spectrum corresponding to $n_i = 1, n_i = 2$ and $n_i = 3$ are shown in figure.

For $n_i = 1$, energy of emitted photons ≤ 2.7 eV

For $n_i = 2$, energy of emitted photons $>=<$ 2.7 eV and

For $n_i = 3$, energy of emitted photons ≥ 2.7 eV.

As per the given condition $n_i = 2$.

(b) $$E_4 - E_2 = 2.7 \text{ eV}$$

or $$\frac{E_1}{16} - \frac{E_1}{4} = 2.7$$

or $$-\frac{3}{16}E_1 = 2.7$$

$$E_1 = -14.4 \text{ eV}$$

Therefore, ionization energy for the gas atoms is $|E_1|$ or 14.4 eV.

(c) Maximum energy of the emitted photons is corresponding to transition from $n = 4$ to $n = 1$.

$$\therefore \quad E_{max} = E_4 - E_1$$

$$= \frac{E_1}{16} - E_1 = \frac{-15E_1}{16}$$

$$= \left(-\frac{15}{16}\right)(14.4) = 13.5 \text{ eV}$$

Similarly, minimum energy of the emitted photons is corresponding to transition from $n = 4$ to $n = 3$.

$$\therefore \quad E_{min} = E_4 - E_3$$

$$= \frac{E_1}{16} - \frac{E_1}{9} = -\frac{7E_1}{16 \times 9}$$

$$= \frac{7 \times 14.4}{16 \times 9} = 0.7 \text{ eV}$$

58. Number of proton = atomic number = 11

Number of neutron = mass number – atomic number = 13

But note that in the nucleus number of electrons will be zero.

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JEE ADVANCED

Solved Paper 2019

Paper 1

Section 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : + 3 If ONLY the correct options is chosen.
  Zero Marks : **0** If none of the options is chosen. (i.e., the question is unanswered)
  Negative Marks : −1 In all other cases.

1. In a radioactive sample, ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t \times 10^{9}$ years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be

[Given: $\ln 10 = 2.3$]

(a) 9.2 (b) 1.15
(c) 4.6 (d) 2.3

2. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as $T(t) = T_0\left(1 + \beta t^{\frac{1}{4}}\right)$,

where, β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is

(a) $\dfrac{4P(T(t) - T_0)^4}{\beta^4 T_0^5}$ (b) $\dfrac{4P(T(t) - T_0)^3}{\beta^4 T_0^4}$

(c) $\dfrac{4P(T(t) - T_0)}{\beta^4 T_0^2}$ (d) $\dfrac{4P(T(t) - T_0)^2}{\beta^4 T_0^3}$

3. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2 (\alpha << 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statements is correct?

(a) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$.

(b) The potential at the center of the shell is reduced by $2\alpha V_0$.

(c) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$.

(d) The magnitude of electric field at a point, located on a line passing through the hole and shell's centre, on a distance $2R$ from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$.

4. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is [G is universal gravitational constant]

(a) $\dfrac{K}{6\pi r^2 m^2 G}$ (b) $\dfrac{K}{\pi r^2 m^2 G}$

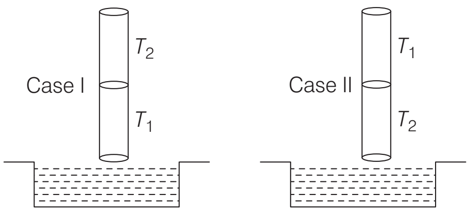
(c) $\dfrac{3K}{\pi r^2 m^2 G}$ (d) $\dfrac{K}{2\pi r^2 m^2 G}$

Section 2 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct options(s).
- For each question, choose the options(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : + 4 If only (all) the correct option(s) is (are) chosen.
  Partial Marks : + 3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen, both of which are correct.
  Partial Marks : + 1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the option is chosen (i.e., the question is unanswered);
  Negative Marks : – 1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get + 4 marks
  choosing ONLY (A) and (B) will get + 2 marks
  choosing ONLY (A) and (D) will get + 2 marks
  choosing ONLY (B) and (D) will get + 2 marks
  choosing ONLY (A) will get + 1 marks
  choosing ONLY (B) will get + 1 marks
  choosing ONLY (D) will get + 1 marks
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get – 1 mark.

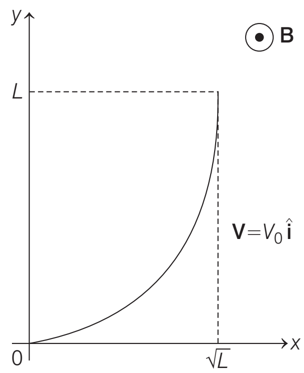
5. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0^o and 60^o, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct? [Surface tension of water = 0.075 N / m, density of water = 1000 kg / m^3, take $g = 10 \text{ m / s}^2$]



(a) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus).
(b) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus).
(c) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.
(d) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus).

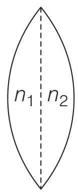
6. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $v = v_0\hat{i}$ in a non-uniform magnetic field $B = B_0\left(1 + \left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure.

If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are



(a) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$
(b) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$
(c) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$
(d) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

7. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in the figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n - 1)$ and $1 < n < 2$, the correct statement(s) is/are



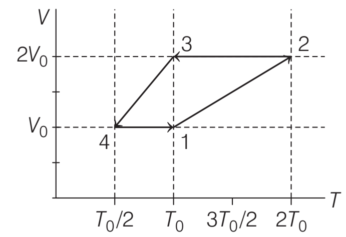
(a) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

(b) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2^{nd} decimal place).

(c) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(d) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

8. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V - T) diagram. The correct statement(s) is/are [R is the gas constant]



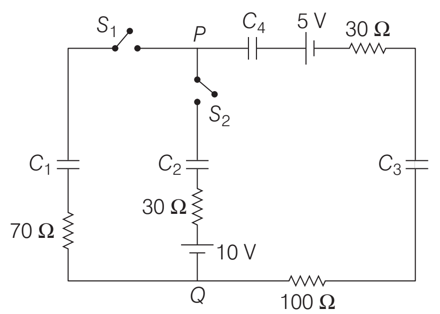
(a) Work done in this thermodynamic cycle ($1 \to 2 \to 3 \to 4 \to 1$) is $|W| = \frac{1}{2}RT_0$.

(b) The ratio of heat transfer during processes $1 \to 2$ and $2 \to 3$ is $\left|\frac{Q_{1 \to 2}}{Q_{2 \to 3}}\right| = \frac{5}{3}$.

(c) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(d) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1 \to 2}}{Q_{3 \to 4}}\right| = \frac{1}{2}$.

9. In the circuit shown, initially there is no charge on the capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\ \mu F$, $C_2 = 30\ \mu F$ and $C_3 = C_4 = 80\ \mu F$.



Which of the statement(s) is/are correct?

(a) The key S_1 is kept closed for long time such that capacitors are fully charged. Now, key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1^{st} decimal place).

(b) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be 4V.

(c) At time $t = 0$, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.

(d) If key S_1 is kept closed for long time such that the capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

10. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?

[$\in_0$ is the permittivity of free space]

(a) If $h > 2R$ and $r = 4R/5$ then $\phi = Q/5\in_0$

(b) If $h > 2R$ and $r = 3R/5$ then $\phi = Q/5\in_0$

(c) If $h < 8R/5$ and $r = 3R/5$ then $\phi = 0$

(d) If $h > 2R$ and $r > R$ then $\phi = Q/\in_0$

11. Two identical moving coil galvanometers have 10 Ω resistance and full scale deflection at 2μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct?

(a) The resistance of the voltmeter will be 100 kΩ.

(b) The resistance of the ammeter will be 0.02 Ω. (round off to 2^{nd} decimal place).

(c) If the ideal cell is replaced by a cell having internal resistance of 5Ω then the measured value of R will be more than 1000 Ω.

(d) The measured value of R will be $978\ \Omega < R < 982\ \Omega$.

12. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

(a) The dimension of force is $[L]^{-3}$.

(b) The dimension of power is $[L]^{-5}$.

(c) The dimension of energy is $[L]^{-2}$.

(d) The dimension of linear momentum is $[L]^{-1}$.

Section 3 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- Four each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

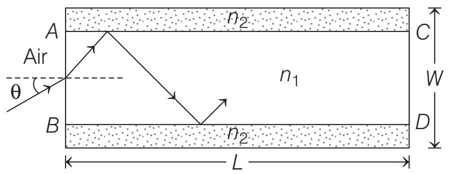
Full Marks : + 3 If ONLY the correct numerical value is entered.
Zero Marks : **0** In all other cases.

13. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness, $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1 + \frac{m}{N}\right)$. For a very large $N(> 10^3)$, the capacitance C is $\alpha\left(\frac{K\varepsilon_0 A}{d \ln 2}\right)$.
The value of α will be ....

[$\in_0$ is the permittivity of free space.]

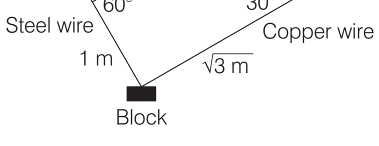
14. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where, t is ....

[Speed of light, $c = 3 \times 10^8$ m / s]



15. A block of weight 100 N is suspended by copper and steel wires of same cross-sectional area 0.5 cm^2 and length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is (Δl_C) and elongation in steel wire is (Δl_S), then the ratio $\frac{\Delta l_C}{\Delta l_S}$ is ……… .

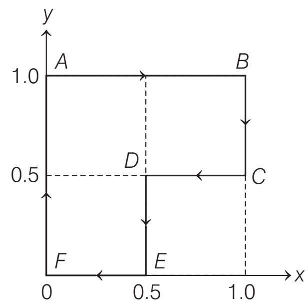
[Young's modulus for copper and steel are 1×10^{11} N / m^2 and 2×10^{11} N/m^2 respectively.]



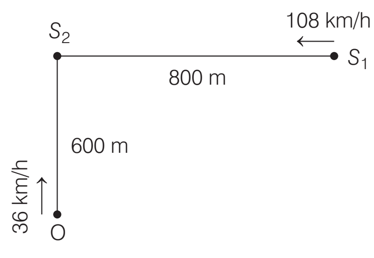
16. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the latent heat of the liquid to its specific heat will be .... °C.

[Neglect the heat exchange with surrounding]

17. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $F = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N, where x and y are in meter and $\alpha = -1\,Nm^{-1}$. The work done on the particle by this force F will be .... Joule.



18. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure.



Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ……… .

[Speed of the sound = 330 m / s ……… .]

Paper 2

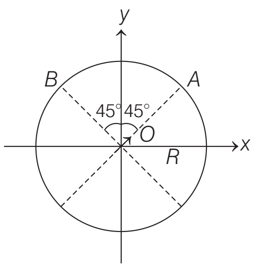
Section 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct options(s).
- For each question, choose the correct options(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : + 4 If only (all) the correct option(s) is (are) chosen.
  Partial Marks : + 3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : + 1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : −2 In all other cases.

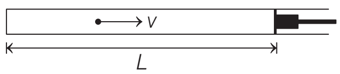
- **For example:** in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answer, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will −1 mark.

1. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of a uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are,
(ϵ_0 is the permittivity of the free space, R >> dipole size)



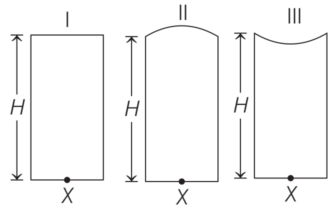
(a) The magnitude of total electric field on any two points of the circle will be same.
(b) Total electric field at point B is $\vec{E}_B = 0$
(c) $R = \left(\frac{p_0}{4\pi \epsilon_0 E_0}\right)^{1/3}$
(d) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

2. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$, the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?



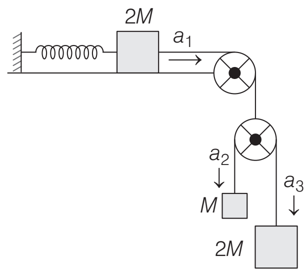
(a) After each collision with the piston, the particle speed increases by $2V$.
(b) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$.
(c) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $1/2\ L_0$.
(d) The rate at which the particle strikes the piston is v/L.

3. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has s convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3 m). If H_1, H_2, and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are



(a) $H_2 > H_1$
(b) $H_3 > H_1$
(c) $0.85\text{ cm} < (H_2 - H_1) < 0.9\text{ cm}$
(d) $H_2 > H_3$

4. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations



of the blocks are a_1, a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct?

[g is the acceleration due to gravity. Neglect friction]

(a) $a_2 - a_1 = a_1 - a_3$

(b) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$.

(c) $x_0 = \frac{4Mg}{k}$

(d) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$.

5. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical?
[g is the acceleration due to gravity]

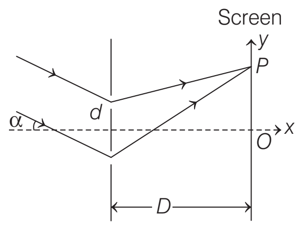
(a) The angular acceleration of the rod will be $\frac{2g}{L}$.

(b) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$.

(c) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$.

(d) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$.

6. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct?



(a) For $\alpha = 0$, there will be constructive interference at point P.

(b) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.

(c) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

(d) Fringe spacing depends on α.

7. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission be Δp_a and Δp_e, respectively. If $\frac{\lambda_a}{\lambda_e} = \frac{1}{5}$, which of the option(s) is/are correct?

[Use $hc = 1242$ eVnm; 1 nm $= 10^{-9}$ m, h and c are Planck's constant and speed of light in vacuum, respectively]

(a) The ratio of kinetic energy of the electron in the state $n = m$ to the state, $n = 1$ is $\frac{1}{4}$

(b) $m = 2$

(c) $\frac{\Delta p_a}{\Delta p_e} = \frac{1}{2}$

(d) $\lambda_e = 418$ nm

8. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $\frac{V_0}{4}$, then the correct statement(s) is/are

(Given, $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is a gas constant)

(a) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$.

(b) The average kinetic energy of the gas mixture after compression is in between $18\,RT_0$ and $19\,RT_0$.

(c) Adiabatic constant of the gas mixture is 1.6.

(d) The work $|W|$ done during the process is $13RT_0$.

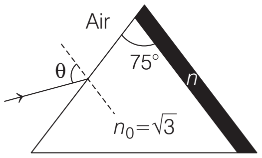
Section 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : + 3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : **0** In all other cases.

9. Suppose a $^{226}_{88}$Ra nucleus at rest and in ground state undergoes α-decay to a $^{222}_{86}$Rn nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. $^{222}_{86}$Rn nucleus then goes to its ground state by γ-decay. The energy of the emitted γ photon is ……… keV.

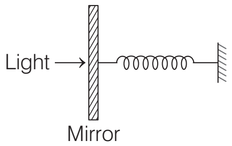
[Given : atomic mass of $^{226}_{88}Ra = 226.005$ u, atomic mass of $^{222}_{86}Rn = 222.000$ u, atomic mass of α particle $= 4.000$ u, 1 u $= 931$ MeV/e^2, c is speed of the light]

10. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is ....



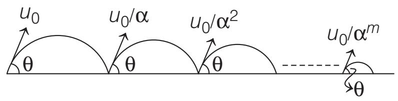
11. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24}\ m^{-2}$ with h as Planck's constant. Nphotons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μm. If the value of N is $x \times 10^{12}$, then the value of x is ....

[Consider the spring as massless]

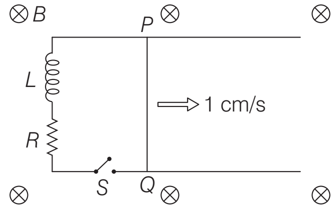


12. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is ....

13. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of $\frac{u_0}{\alpha}$. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is ....



14. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1 Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is ....
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given, $e^{-1} = 0.37$, where e is base of the natural logarithm]



Section 3 (Maximum Marks : 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists : List-I and List-II
- List-I has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Questions.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : + 3 If ONLY the option corresponding to the correct combination is chosen:
Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : −1 In all other cases.

Answer the following by appropriately matching the lists based on the information given in the paragraph

A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I		List-II	
(I)	String-1 (μ)	(P)	1
(II)	String-2 (2μ)	(Q)	1/2
(III)	String-3 (3μ)	(R)	$1/\sqrt{2}$
(IV)	String-4 (4μ)	(S)	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

15. If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be

(a) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ S
(b) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ Q
(c) I $\rightarrow$ Q, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ P
(d) I $\rightarrow$ Q, II $\rightarrow$ P, III $\rightarrow$ R, IV $\rightarrow$ T

16. The length of the strings 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2} \cdot \frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3 and 4 are vibrated at their 1st, 3rd, 5th and 14th harmonics, respectively such that all the strings have same frequency.

The correct match for the tension in the four strings in the units of T_0 will be

(a) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ U (b) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ R, IV $\rightarrow$ T
(c) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ U (d) I $\rightarrow$ T, II $\rightarrow$ Q, III $\rightarrow$ R, IV $\rightarrow$ U

Answer the following by appropriately matching the lists based on the information given in the paragraph

In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$. where, T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas,

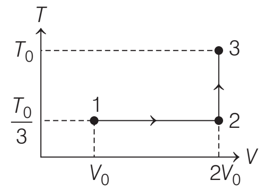
$$X = \frac{3}{2} R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$$

Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

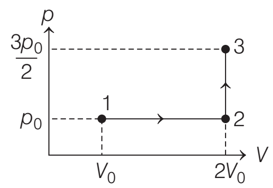
List-I		List-II	
(I)	Work done by the system in process $1 \rightarrow 2 \rightarrow 3$	(P)	$\frac{1}{3} RT_0 \ln 2$
(II)	Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$	(Q)	$\frac{1}{3} RT_0$
(III)	Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	(R)	RT_0
(IV)	Heat absorbed by the system in process $1 \rightarrow 2$	(S)	$\frac{4}{3} RT_0$
		(T)	$\frac{1}{3} RT_0 (3 + \ln 2)$
		(U)	$\frac{5}{6} RT_0$

17. If the process on one mole of monatomic ideal gas is as shown in the TV-diagram with $P_0 V_0 = \frac{1}{3} RT_0$, the correct match is,



(a) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ S
(b) I $\rightarrow$ P, II $\rightarrow$ T, III $\rightarrow$ Q, IV $\rightarrow$ T
(c) I $\rightarrow$ S, II $\rightarrow$ T, III $\rightarrow$ Q, IV $\rightarrow$ U
(d) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ P

18. If the process carried out on one mole of monoatomic ideal gas is as shown in the PV-diagram with $p_0 V_0 = \frac{1}{3} RT_0$, the correct match is,

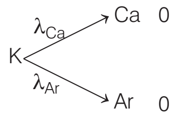


(a) I $\rightarrow$ S, II $\rightarrow$ R, III $\rightarrow$ Q, IV $\rightarrow$ T
(b) I $\rightarrow$ Q, II $\rightarrow$ R, III $\rightarrow$ P, IV $\rightarrow$ U
(c) I $\rightarrow$ Q, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ U
(d) I $\rightarrow$ Q, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ U

Answer *with* Explanations

Paper 1

1. *(a)*



at t = 0 dissipated energy,

$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2) \times N \Rightarrow \frac{dN}{N} = -(\lambda_1 + \lambda_2)\, dt$$

Integration on both sides, we get

$$\log_e\left(\frac{N}{N_0}\right) = -(\lambda_1 + \lambda_2)t$$

$$2.3 \times \log_{10}\left(\frac{N_0}{N_0/100}\right) = 5 \times 10^{-10}\, t$$

$$\frac{2.303 \times 2}{5 \times 10^{-10}} = t$$

$$2.303 \times 0.4 \times 10^{10} = t \Rightarrow t = 9.2 \times 10^9 \text{ Yr}$$

2. *(b)* Heat capacity, $\frac{dQ}{dt} = H\frac{dT}{dt}$

Power of the rod, $P = H.T_0.\beta.\frac{1}{4}.t^{-3/4}$

$$\frac{4P}{T_0.\beta} = t^{-3/4}.H$$

$$\Rightarrow \quad H = \left(\frac{4P}{T_0\beta}\right)t^{3/4} \quad \ldots(i)$$

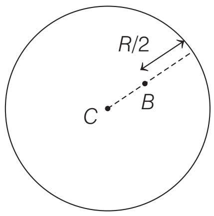
Now, $T - T_0 = T_0\beta t^{1/4}$

So, $t^{3/4} = \left(\frac{T - T_0}{T_0\beta}\right)^3$

Substituting this value of $t^{3/4}$ in equation (i) we get,

$$H = \frac{4P(T - T_0)^3}{T_0^4\beta^4}$$

3. *(a)*



Given, V at surface of the sphere

$$V_0 = \frac{KQ}{R}$$

Here, $K = \frac{1}{4\pi\varepsilon_0} = \text{constant}$

V at point C,

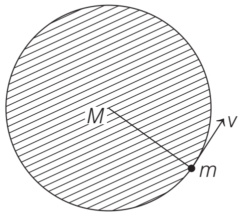
$$V_C = \frac{KQ}{R} - \frac{K\alpha Q}{R} = V_0(1 - \alpha)$$

V at point B,

$$V_B = \frac{KQ}{R} - \frac{K(\alpha Q)}{R/2} = V_0(1 - 2\alpha)$$

$$\therefore \quad \frac{V_C}{V_B} = \frac{1 - \alpha}{1 - 2\alpha}$$

4. *(d)*



Gravitational force = Centripetal force of the earth

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad (\because M = \text{total mass from 0 to r})$$

$$= \frac{2}{r}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad \frac{GMm}{r^2} = \frac{2K}{r} \Rightarrow M = \frac{2Kr}{Gm} \quad \left(\because \frac{1}{2}mv^2 = K\right)$$

Differentiate on both sides, we get

$$\Rightarrow \quad dM = \frac{2K}{Gm}dr \Rightarrow 4\pi r^2 dr\rho = \frac{2K}{Gm}dr$$

$$(\because \text{volume} = \text{mass} \times \text{density})$$

$$\therefore \quad \rho = \frac{K}{2\pi Gmr^2}$$

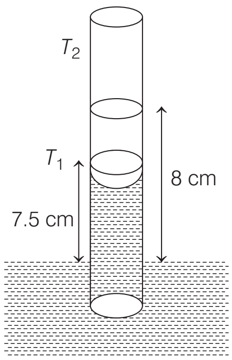
$$\therefore \quad \frac{\rho}{m} = \frac{K}{2\pi r^2 Gm^2} \quad \left(\because \frac{\rho}{m} = \text{volume}\right)$$

5. *(a, c, d)* Heights if only single material tubes are used of sufficient length,

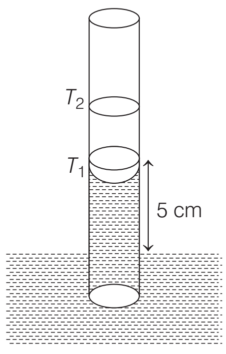
$$h_1 = \frac{2R\cos\theta}{\rho rg} = \frac{2 \times 0.075 \times \cos 0°}{1000 \times 2 \times 10^{-4} \times 10} = 7.5\text{cm}$$

$$h_2 = \frac{2T\cos\theta'}{\rho rg} = \frac{2 \times 0.075 \times \cos 60°}{1000 \times 2 \times 10^{-4} \times 10} = 3.75\text{cm}$$

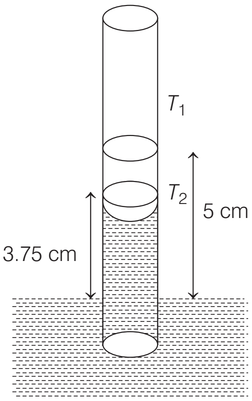
Case- I



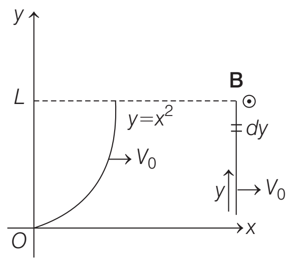
Case-II



2. Liquid will rise only upto height of 5 cm and meniscus will adjust by changing is radius of curvature. If the liquid goes up in tube 2 then it will not be able to support the weight of the liquid.
3. Weight of water in meniscus will be different in two cases because angle of contact is different.



6. *(a, b, d)*



Motional emf across the length dy is,

$$d\varepsilon = BV_0\ dy = B_0\left[1+\left(\frac{y}{L}\right)^{\beta}\right]V_0 dy$$

$$\therefore \qquad \varepsilon = \int_0^L B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)V_0 dy$$

$$= B_0V_0\left[1+\frac{1}{\beta+1}\right]$$

emf in loop is proportional to L for given value of β,

$$\beta = 0,\ \varepsilon = 2B_0V_0L$$

$$\beta = 2,\ \varepsilon = B_0V_0L\left[1+\frac{1}{3}\right] = \frac{4}{3}B_0V_0L$$

The length of projection of the wire $Y = X$ of length $\sqrt{2}L$ on the y-axis is L thus, the answer remain unchanged.

Therefore, answers are a, b, d.

7. *(a, b, d)* $\frac{1}{f} = (n-1)\left(\frac{1}{R}-\frac{1}{\infty}\right) \Rightarrow \frac{1}{f_0} = \frac{2(n-1)}{R}$ ...(1)

$$\frac{1}{\Delta f} = (n+\Delta n-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$$

$$\frac{1}{f_0+\Delta f_0} = \frac{(n-1)}{R} + (n+\Delta n-1)\left(\frac{1}{R}\right)$$

$$\frac{1}{f_0+\Delta f_0} = \frac{2n+\Delta n-2}{R} \qquad ...(2)$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \frac{f_0+\Delta f_0}{f_0} = \frac{\frac{2(n-1)}{R}}{\frac{2n+\Delta n-2}{R}}$$

$$1+\frac{\Delta f_0}{f_0} = \frac{2(n-1)}{(2n+\Delta n-2)}$$

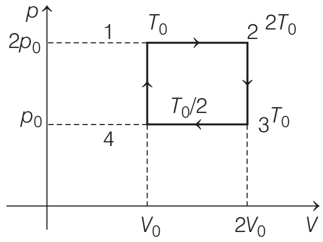
$$\frac{\Delta f_0}{f_0} = \frac{-\Delta n}{(2n+\Delta n-2)}$$

$$\frac{\Delta f_0}{20} = \frac{10^{-3}}{3+10^{-3}-2}$$

$$\Rightarrow \qquad \Delta f_0 = -2\times10^{-2}$$

$$|\Delta f_0| = 0.02\text{ cm}$$

8. *(a, b)* P-V graph of the given V-T graph is shown below.



(d) $\left|\frac{\Delta Q_{1\to2}}{\Delta Q_{3\to4}}\right| = \left|\frac{NC_p\Delta T_{1\to2}}{NC_p\Delta T_{3\to4}}\right| = \frac{T_0}{T_0/2} = 2$

(b) $\left|\frac{\Delta Q_{1\to2}}{\Delta Q_{2\to3}}\right| = \left|\frac{NC_p\Delta T_{1\to2}}{NC_p\Delta T_{3\to4}}\right| = \frac{C_p}{C_V} = \frac{5}{3}$

(a) $W_{cycle} = p_0V_0 = nR\left[\frac{T_0}{2}\right]$

Note *For ideal gas equation,*

$$pV = nRT$$

$\therefore \qquad (pV)_4 = (nRT)_4$

or $\qquad p_0V_0 = nR\frac{T_0}{2} = R\frac{T_0}{2}$

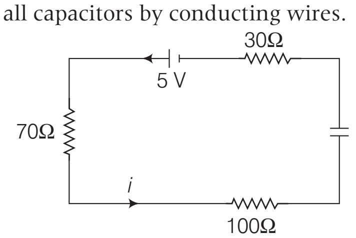
as $\qquad n = 1$

(c) Wrong as no adiabatic process is involved.

9. *(b, c)*

Just after closing of switch charge on any capacitor is zero.

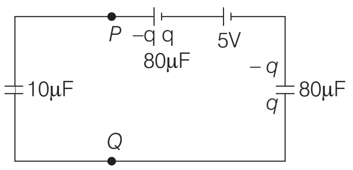
$\therefore$ Replace all capacitors by conducting wires.



Current flow in the circuit,

$$i = \frac{5}{70+100+30} = \frac{5}{200} = 25\text{ mA}$$

Now S_1 is kept closed for long time circuit is in steady state.

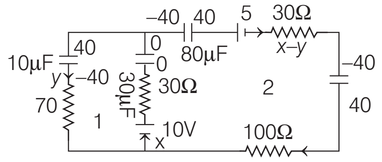


$$\frac{q}{10}+\frac{q}{80}+\frac{q}{80}-5 = 0 \quad \Rightarrow \quad \frac{10q}{80} = 5$$

$\therefore \quad q = 40\,\mu C$

$\therefore \quad V\text{across } C_1 = \dfrac{40}{10} = 4V$

Now just after closing S_2 charge on each capacitor remains same.



Applying KVL,

$$-10 + x \times 30 + \frac{40}{10} + y \times 70 = 0$$

$$30x + 70y = 6 \qquad \ldots(1)$$

$$-\frac{40}{80} + 5 + (x-y)\,30 - \frac{40}{80} + (x+y)\times 100 - 10 + x \times 30 = 0$$

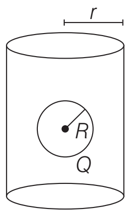
$$160x - 130y - 6 = 0 \qquad \ldots(2)$$

$$y = \frac{96}{1510}$$

$$x = 0.05\text{A}$$

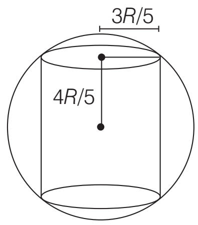
Note Charges shown in

10. ***(b, c, d)*** (a) $h > 2R$ and $r > R$



$\phi = \dfrac{Q}{\varepsilon_0}$, clearly from Gauss' Law

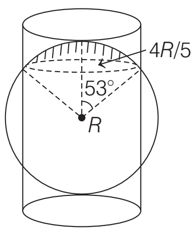
(b) suppose $h = \dfrac{8R}{5}$ and $r = \dfrac{3R}{5}$



$\therefore \quad \phi = 0$

so, for $h < \dfrac{8R}{5}$ then $\phi = 0$.

(c) For $h = 2R$ and $r = \dfrac{4R}{5}$



Shaded charge $= 2\pi\,(1 - \cos 53^\circ) \times \dfrac{Q}{4\pi}$

$\therefore \quad = \dfrac{Q}{5}$

$\therefore \quad q_{\text{enclosed}} = \dfrac{2Q}{5}$

$\therefore \quad \phi = \dfrac{2Q}{5\varepsilon_0}$

$\therefore$ for $h > 2R$ and $r = \dfrac{4R}{5}$

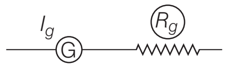
$\therefore \quad \phi = \dfrac{2Q}{5\varepsilon_0}$

(d) like option *c* for $h = 2R$ and $r = \dfrac{3R}{5}$

$$q_{\text{enclosed}} = 2 \times 2\pi\,(1 - \cos 37^\circ)\,\frac{Q}{4\pi} = \frac{Q}{5}$$

$\therefore$ Electric flux, $\phi = \dfrac{Q}{5\varepsilon_0}$

11. ***(b, d)***

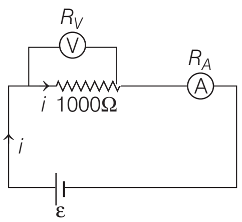


$$V = 100 \times 10^{-3} = 10^{-1}\text{ V}$$

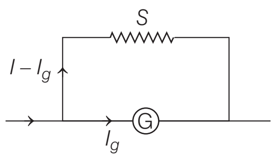
$$V = I_g(R_g + R_V)$$

$$\frac{10^{-1}}{2 \times 10^{-6}} = R_g + R_V$$

$$5 \times 10^4\,\Omega \approx R_V \qquad (\because R_V < 10^5\,\Omega)$$



$$I_g R_g = (I - I_g)S$$



$$S = \frac{2 \times 10^{-6} \times 10}{10^{-3} - 2 \times 10^{-6}}$$

$$S = 2 \times 10^{-5} \times 10^3$$

$$= 2 \times 10^{-2} = 20\,\text{m}\Omega$$

$$R_A = \frac{SR_g}{S + R_g} = \frac{20 \times 10^{-3} \times 10}{10 + 20 \times 10^{-3}}$$

$$\approx 20 \times 10^{-3}\,\Omega$$

$$i = \frac{\varepsilon}{\left(\dfrac{1000 \times 50 \times 10^3}{51 \times 10^3}\right)} = \frac{51\varepsilon}{5 \times 10^4} \qquad (\because R_A \to 0)$$

$$i' = i\left(\frac{R_V}{51 \times 10^3}\right) = \frac{\varepsilon}{1000}$$

Measured resistance,

$$\therefore \quad R_m = \frac{i' \times 1000}{i} = \frac{\varepsilon}{51\varepsilon} \times 5 \times 10 = 980.4\,\Omega$$

If the voltmeter shows full scale deflection then

$$\frac{\varepsilon}{980}\times\left(\frac{1000}{51\times10^3}\right)\times5\times10^4=10^{-1}$$

$\therefore\ \varepsilon = 999.6\,\text{mV}$

Since, $i_A = 1$ mA so maximum reading of R can be

$$\frac{999.6\text{ mV}}{1\text{ mA}} = 999.6\,\Omega$$

12. *(a,c,d)* $[M]=[\text{Mass}]=[M^0L^0T^0]$

$[J]=[\text{Angular momentum}] = [ML^2T^{-1}]$

$[L]=[\text{Length}]$

Now, $[ML^2T^{-1}]=[M^0L^0T^0]$

$\therefore\quad [L^2]=[T]$

Power $[P]=[MLT^{-2}.LT^{-1}]=[ML^2T^{-3}]=[L^2\,L^{-6}]$

$[P]=[L^{-4}]$

Energy/Work $[W]=[MLT^{-2}.L]$

$=[L^2\,L^{-4}]=[L^{-2}]$

Force $[F]=[MLT^{-2}]=[L.L^{-4}]=[L^{-3}]$

Linear momentum $[p]=[MLT^{-1}]=[L.L^{-2}]$

$[p]=[L^{-1}]$

13. *(1.00)* Parallel plate capacitor,

$$\frac{x}{m}=\frac{d}{N}$$

$$d\left(\frac{1}{C}\right)=\frac{dx}{K_m\varepsilon_0A}=\frac{dx}{K\varepsilon_0A\left(1+\frac{m}{N}\right)}=\frac{dx}{K\varepsilon_0A\left(1+\frac{x}{d}\right)}$$

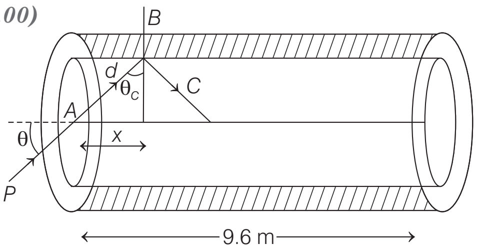
Integration on both sides, we get

$$\frac{1}{C_{eq}}=\int d\left(\frac{1}{C}\right)=\int_0^D\frac{d\,dx}{K\varepsilon_0A(d+x)}$$

$$\frac{1}{C_{eq}}=\int\frac{d}{K\varepsilon_0A}\ln2\quad\Rightarrow\quad C_{eq}=\frac{K\varepsilon_0A}{d\ln2}$$

Therefore, $\alpha=1$.

14. *(50.00)*



According to total internal reflection (TIR),

$$1.5\sin\theta_c = 1.44\sin90^\circ$$

$$\sin\theta_c=\frac{1.44}{1.50}=\frac{24}{25}$$

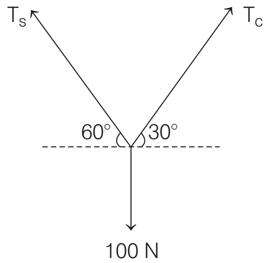
$$\therefore\quad \sin\theta_c=\frac{x}{d}=\frac{24}{25}\quad\Rightarrow\quad d=\frac{25x}{24}$$

$\therefore$ Total length travelled by light,

$$\therefore\quad t=\frac{S}{\left(\frac{c}{n_1}\right)}=\frac{10}{\frac{3\times10^8}{1.5}}=\frac{1}{2}\times10^{-7}=5\times10^{-8}$$

$$t=50\text{ ns}\Rightarrow t=50\times10^{-9}$$

15. *(2.00)*



$$\frac{T_S}{2}=T_C\frac{\sqrt3}{2}$$

$$T_S=\sqrt3\,T_C$$

$$\Delta l=\frac{Tl}{Ay}$$

$$\therefore\quad \frac{\Delta l_C}{\Delta l_S}=\left(\frac{T_C}{T_S}\right)\left(\frac{l_C}{l_S}\right)\left(\frac{Y_S}{Y_C}\right)$$

$$=\left(\frac{1}{\sqrt3}\right)\left(\frac{\sqrt3}{1}\right)\left(\frac{2\times10^{11}}{1\times10^{11}}\right)=2.00$$

16. *(270.00)* **Case-I** $5C\times50+5L=C_2\times30$ ...(i)

Case-II $80C[50-30]=C_2[80-50]$ ...(ii)

By Eq. (i) and (ii)

$$1600C=250+5L$$

$$\therefore\quad \frac{L}{C}=\frac{1350}{5}=270°C$$

17. *(0.75)* $d=F\cdot dr$

$d=\alpha ydx+2\alpha xdy$

$A\to B$, $y=1$, $dy=0$ then $W_{A\to B}=\int\alpha ydx=\alpha1\int_0^1dx=\alpha$

$B\to C$, $x=1$, $dx=0$ then $W_{B\to C}=2\alpha1\int_1^{0.5}dy=-2\alpha(0.5)=-\alpha$

$C\to D$, $y=0.5$, $dy=0$ then $W_{C\to D}=\int_1^{0.5}\alpha ydx=\alpha.\frac{1}{2}\int_1^{0.5}dx=-\frac{\alpha}{4}$

$D\to E$, $x=0.5$, $dx=0$ then $W_{D\to E}=2\alpha\int xdy=2\alpha.\frac{1}{2}\int_{0.5}^{0}dy=-\frac{\alpha}{2}$

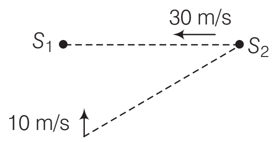
$E\to F$, $y=0$, $dy=0$ then $W_{EF}=0$

$F\to A$, $x=0$, $dx=0$ then $W_{F\to A}=0$

$$\therefore\quad W=\alpha-\alpha-\frac{\alpha}{4}-\frac{\alpha}{2}=-\frac{3\alpha}{4}$$

Given, $\alpha=-1\Rightarrow W=\frac{3}{4}J=0.75\,J$

18. *(8.13)* $V_{sound}=330$ m/s



$$f_1=120\left[\frac{330+10\cos53°}{330-30\cos37°}\right]\text{Hz}$$

$$f_2=120\left[\frac{330+10}{330}\right]\text{Hz}$$

$$\Delta f=(f_2-f_1)=120\times\left[\frac{336}{306}-\frac{34}{33}\right]=8.13\,\text{Hz}$$

Paper 2

1. *(c, d)* $R >>$ Dipole size.

Circle is equipotential.

So, E_{net} should be perpendicular to surface hence, $\dfrac{kp_0}{r^3} = E_0$

$$\Rightarrow \qquad R = \left(\frac{kp_0}{E_0}\right)^{1/3}$$

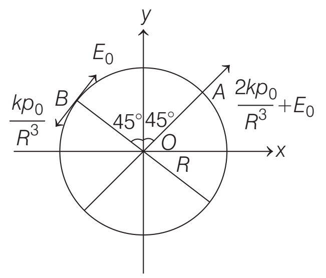
At point B, net electric field will be zero.

$$E_B = 0$$

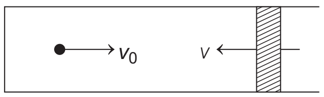
$$(E_A)_{net} = \frac{2kp_0}{R^3} + E_0 = 3E_0$$

Electric field at point A, $E_A = \dfrac{3}{\sqrt{2}} E_0[\hat{i} + \hat{j}]$

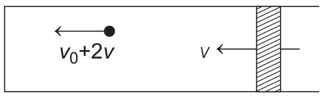
$$(E_B)_{net} = 0$$



2. *(a, c)*



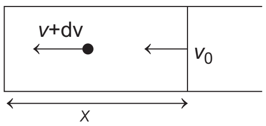
Before collision



After Collision

Change in speed = $(2v + v_0 - v_0) = 2v$

In every collision it acquires $2v$,



$$f = \frac{v}{2x}$$

$$\frac{dv}{dt} = f \times 2v$$

$$dv = \frac{v}{2x} 2v dt$$

$$dv = \frac{v}{2x} 2(-dx)$$

Integration on both sides limits v_0 to v, we get

$$\int_{v_0}^{v} \frac{dv}{v} = \int_{\ell}^{x} \frac{-dx}{x}$$

$$\Rightarrow \qquad \ln \frac{v}{v_0} = -\ln \frac{x}{\ell}$$

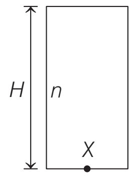
$$\Rightarrow \qquad v = \frac{v_0 \ell}{x}$$

where, $\qquad x = \dfrac{\ell}{2},\ v = 2v_0$

so, $\qquad f = \dfrac{2v_0}{2\frac{\ell}{2}} = \dfrac{2v_0}{\ell}$

$\therefore \qquad \dfrac{k_f}{k_i} = 4$

3. *(a, d)* **Case-I**

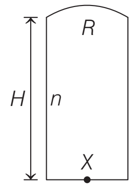


$$H = 30\text{cm}$$

$$n = \frac{3}{2}$$

$$H_1 = \frac{H}{n} \Rightarrow \frac{30 \times 2}{3} = 20\text{ cm}$$

Case-II



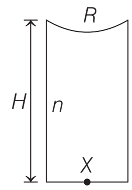
$$R = 300\text{cm}$$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{-H_2} - \frac{3}{-2 \times 30} = \frac{1 - \frac{3}{2}}{-300}$$

$$H_2 = \frac{600}{29} = 20.684\text{ cm}$$

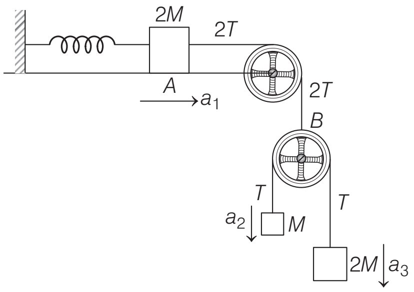
Case-III



$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R};\ \frac{1}{-H_3} - \frac{3}{-2 \times 30} = \frac{1 - \frac{3}{2}}{300}$$

$$H_3 = \frac{600}{31} = 19.354\text{ cm}$$

4. *(a)*



In the frame of pulley *B*,

the hanging masses have accelerations :

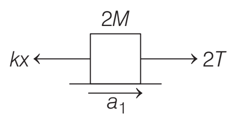
$M \to (a_2 - a_1)$, $2M \to (a_3 - a_1)$: downward.

$\therefore \quad (a_2 - a_1) = -(a_3 - a_1)$ [constant]

Assuming that the extension of the spring is *x*

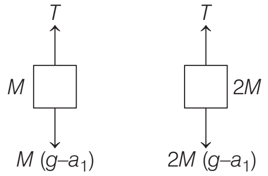
We consider the FBD of A :

$$2M \cdot \frac{d^2x}{dt^2} = 2T - kx$$



where, $\qquad a_1 \equiv \dfrac{d^2x}{dt^2} \qquad \ldots(i)$

and the FBD of the rest of the system in the frame of pulley B :



Upward acceleration of block M w.r.t. the pulley B = Downward acceleration of block 2M w.r.t the pulley,

$$\frac{T - M(g - a_1)}{M} = \frac{2M(g - a_1) - T}{2M}$$

$$\Rightarrow \qquad T = \frac{4M}{3}(g - a_1) \qquad \ldots\text{(ii)}$$

Substituting in Eq. (i), we get

$$2M \cdot a_1 = \frac{8M}{3}(g - a_1) - kx$$

or $$\frac{14M}{3} a_1 = \frac{8Mg}{3} - kx \qquad \ldots\text{(iii)}$$

This is equation of SHM

Maximum extension = 2 × amplitude

i.e., $$x_0 = 2 \times \frac{8Mg}{3k}$$

Amplitude $= \frac{x_0}{2} = \frac{3Mg}{3k}$ and $\omega = \sqrt{\frac{3k}{14M}}$

At $\frac{x_0}{4}$, acceleration is easily found from Eq. (iii) ,

$$\frac{14M}{3} a_1 = \frac{8Mg}{3} - \frac{4Mg}{3}$$

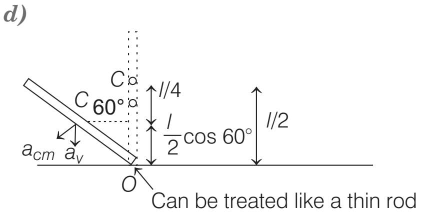
$$a_1 = \frac{2g}{7}$$

At $\frac{x_0}{2}$, speed of the block (2M) = ω × amplitude

$$= \sqrt{\frac{3k}{14M}} \times \frac{8Mg}{3k}$$

∴ option (a) is correct

5. ***(b, c, d)***



$$\Delta K + \Delta U = 0$$

$$\frac{1}{2} I_0 \omega^2 = -\Delta U$$

$$\frac{1}{2}\frac{m\ell^2}{3}\omega^2 = -\left(-mg\frac{\ell}{4}\right)$$

$$\omega = \sqrt{\frac{3g}{2\ell}}$$

$$\Rightarrow \qquad a_{radial} = \omega^2 \frac{\ell}{2} = \frac{3g}{2\ell}\frac{\ell}{2} = \frac{3g}{4} \Rightarrow \tau = I_0 \alpha$$

$$\alpha = \frac{mg\frac{\ell}{2}\sin 60°}{m\frac{\ell^2}{3}} = \frac{3\sqrt{3}g}{4\ell}$$

$$\Rightarrow \qquad a_v = \left(\alpha \frac{\ell}{2}\right)\sin 60° + \omega^2 \frac{\ell}{2}\cos 60°$$

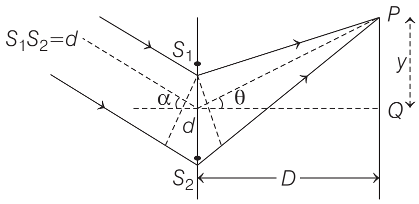
$$a_v = \frac{3\sqrt{3}g}{8}\frac{\sqrt{3}}{2} + \frac{3g}{8}$$

$$a_v = \frac{9g}{16} + \frac{6g}{16}$$

$$mg - N = ma_v$$

$$N = \frac{mg}{16}$$

6. ***(b)***



$$\Delta x = d\sin\alpha + d\sin\theta$$

θ and α are small angles

$$\therefore \qquad \Delta x = d\alpha + \frac{dy}{D}$$

(a) $\alpha = 0$

$\therefore \ \Delta x = \frac{dy}{D} = \frac{0.3 \times 11}{1000} = 33 \times 10^{-4}$ mm

Δx in terms of $\lambda = \frac{33 \times 10^{-4}}{600 \times 10^{-6}}\lambda = \frac{11\lambda}{2}$

as $$\Delta x = (2n - 1)\frac{\lambda}{2}$$

There will be destructive interference.

(b) $\Delta x = 0.3\,\text{mm} \times \frac{0.36}{\pi} \times \frac{\pi}{180} + \frac{0.3\text{ mm} \times 11\text{ mm}}{1000}$

$= 39 \times 10^{-4}$ mm

$$39 \times 10^{-4} = (2n - 1) \times \frac{600 \times 10^{-9} \times 10^3}{2}$$

$n = 7$

So, there will be destruction interference.

(c) $\Delta x = 3\,\text{mm} \times \frac{0.36}{\pi} \times \frac{\pi}{180} + 0 = 600$ nm

$600\text{ nm} = n\lambda$

$n = 1$

So, there will be construction interference.

(d) Fringe width does not depend on α.

7. ***(a, b)*** $\frac{\lambda_a}{\lambda_e} = \frac{E_4 - E_1}{E_4 - E_m} = \frac{1 - \frac{1}{16}}{\frac{1}{m^2} - \frac{1}{16}} = \frac{1}{5}$

On solving we get,

$$m = 2$$

$$\lambda_e = \frac{12400 \times 4}{13.6} = 3647$$

$\frac{K_2}{K_1} = \frac{1^2}{2^2} = \frac{1}{4}$ as kinetic energy is proportional to $\frac{1}{n^2}$.

8. ***(a, c, d)*** $\gamma_{mix} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 C_{V1} + n_2 C_{V2}} = \frac{8}{5}$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$P_0V_0^{8/5} = P_2\left(\frac{V_0}{4}\right)^{8/5}$$

$$P_2 = 9.2P_0$$

$$W = \frac{P_0V_0 - 9.2P_0\dfrac{V_0}{4}}{3/5} = -13RT_0$$

$\therefore$ $|W| = 13RT_0$

(b) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_2 = T_1(2)^{6/5} = 2.3T_0$$

Average kinetic energy of gas mixture $= nC_{V_{mix}}T_2$

$$= 23RT_0$$

9. ***(135.00)*** Mass defect $\Delta m = 226.005 - 222.000 - 4.000$

$$= 0.005 \text{amu}$$

$\therefore$Q value $= 0.005 \times 931.5 = 4.655$ MeV

Momentum $P = \sqrt{2km}$ = constant

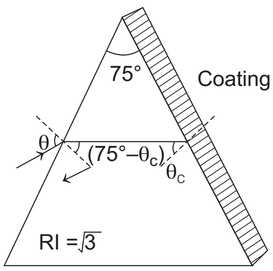
Also $\dfrac{KE_\alpha}{KE_{Rn}} = \dfrac{m_{Rn}}{m_\alpha}$

$$\Rightarrow KE_{Rn} = \frac{m_\alpha}{m_{Rn}} \cdot KE_\alpha = \frac{4}{222} \times 4.44 = 0.08 \text{ MeV}$$

$\therefore$Energy of γ – Photon $= 4.655 - (4.44 + 0.08)$

$$= 0.135 \text{ MeV} = 135 \text{ keV}$$

10. ***(1.50)*** For TIR at coating,



$$\sin\theta_c = \frac{n}{\sqrt{3}}$$

Applying snell's law at first surface

$\sin\theta = \sqrt{3}\sin(75° - \theta_c)$

For limiting condition, at $\theta = 60°$

$$\sin 60° = \sqrt{3}\sin(75° - \theta_c)$$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin(75° - \theta_c)$$

$$\frac{1}{2} = \sin(75° - \theta_c) \Rightarrow \sin 30° = \sin(75° - \theta_c)$$

$$30° = 75° - \theta_c \Rightarrow \theta_c = 45°$$

$$\frac{n}{\sqrt{3}} = \frac{1}{\sqrt{2}} \Rightarrow n^2 = \frac{3}{2} = 1.50$$

11. ***(1.00)*** Momentum transferred on mirror $= \dfrac{2Nh}{\lambda}$

$$\frac{2Nh}{\lambda} = MV_{(\text{mean position})}$$

$V_{(\text{mean position})} = \Omega A$ (where, $A = 1\,\mu m$)

$\dfrac{2Nh}{\lambda} = M\Omega A$ (where, $\lambda = 8\pi \times 10^{-6}$)

$$N = \frac{M\Omega(10^{-6})\lambda}{2h} = \frac{M\Omega 8\pi \times 10^{-6} \times 10^{-6}}{2h}$$

$$N = \frac{4\pi M\Omega}{h} \times 10^{-12} = 10^{24} \times 10^{-12}$$

$$N = 1 \times 10^{12}$$

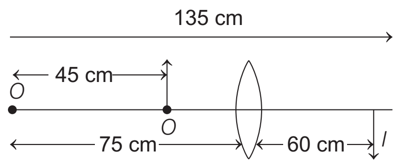
$$x = 1$$

12. ***(1.38 & 1.39)*** $u = (x_2 - x_1) = 75 - 45 = 30$ cm

$$\Delta u = \Delta x_2 + \Delta x_1 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ cm}$$

$$v = (x_3 - x_2) = 135 - 75 = 60 \text{ cm}$$

$$\Delta v = \Delta x_3 + \Delta x_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ cm}$$



$\therefore$ $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{60} + \dfrac{1}{30} = \dfrac{1}{f}$

$\therefore$ $f = 20$ cm Also, $\dfrac{-dv}{v^2} + \dfrac{-du}{u^2} = \dfrac{-df}{f^2}$

$\Rightarrow$ $\dfrac{df}{f} = f\left[\dfrac{dv}{v^2} + \dfrac{du}{u^2}\right] = 20\left[\dfrac{1}{60^2} + \dfrac{1}{30^2}\right]\dfrac{1}{2}$

$\therefore$ $\dfrac{df}{f} \times 100 = 10\left[\dfrac{1}{36} + \dfrac{1}{9}\right] = \dfrac{50}{36} = 1.38$ and 1.39 (both)

13. ***(4.00)*** For first projectile,

$$\langle V \rangle = \frac{R}{T} = U_x = v_1$$

For journey,

$$\langle v \rangle_{1 \text{ to } n} = \frac{R_1 + R_2 + \ldots + R_n}{T_1 + T_2 + \ldots + T_n}$$

$$= \frac{\dfrac{2u_{x_1}u_{y_1}}{g} + \dfrac{2u_{x_2}u_{y_2}}{g} + \ldots + \dfrac{2u_{x_n}u_{y_n}}{g}}{\dfrac{2u_{y_1}}{g} + \dfrac{2u_{y_2}}{g} + \ldots \dfrac{2u_{y_n}}{g}}$$

$$u_x\left[\frac{1 + \dfrac{1}{\alpha^2} + \dfrac{1}{\alpha^4} + \ldots \dfrac{1}{\alpha^{2n}}}{1 + \dfrac{1}{\alpha} + \dfrac{1}{\alpha^2} + \ldots + \dfrac{1}{\alpha^n}}\right] = 0.8v_1$$

$$\frac{v_0\left[\dfrac{1}{1 - \dfrac{1}{\alpha^2}}\right]}{\left[\dfrac{1}{1 - \dfrac{1}{\alpha}}\right]} = 0.8v_1 \Rightarrow \frac{\alpha}{1 + \alpha} = 0.8 \Rightarrow \alpha = 4$$

14. ***(0.63)*** Motional emf, $e = (v \times B)dl = 10^{-2} \times 1 \times 10^{-1}$

$$e = 10^{-3} \text{ V}$$

$$\tau_L = LR = (10^{-3})(1) = 10^{-3} \text{ s} = 1 \text{ ms}$$

$$i = i_0(1 - e^{-t/\tau_L}) = \frac{10^{-3}}{1}(1 - e^{-1})$$

$$i = 10^{-3}(1 - 0.37)$$

$$I = 0.63 \text{ mA}$$

15. *(b)* Fundamental frequency is maximum when length is minimum i.e. L_0,

Case 1. $L = L_0, T = T_0, f = f_0; f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

Case 2. $f_2 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{f_0}{\sqrt{2}}$

Case 3. $f_3 = \frac{1}{L_0}\sqrt{\frac{T_2}{3\mu}} = \frac{f_0}{\sqrt{3}}$

Case 4. $f_4 = \frac{1}{L_0}\sqrt{\frac{T_2}{4\mu}} = \frac{f_0}{2}$

16. *(c)*

Case 1. $L = L_0, T = T_0, f = f_0$

$$f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

Case 2. $L = \frac{3L_0}{2}$

$$f_2 = \frac{3}{2\times\frac{3L_0}{2}}\sqrt{\frac{T_2}{2\mu}} = f_0 \Rightarrow f_0 = \frac{1}{2L_0}\sqrt{\frac{T_2}{\mu}} \Rightarrow T_2 = \frac{T_0}{2}$$

Case 3. $L = \frac{5L_0}{4}$

$$f_3 = \frac{5}{2\times\frac{5L_0}{4}}\sqrt{\frac{T_3}{3\mu}} = f_0 \Rightarrow f_0 = \frac{2}{\sqrt{3}L_0}\sqrt{\frac{T_3}{\mu}} \Rightarrow T_3 = \frac{3T_0}{16}$$

Case 4. $L = \frac{7L_0}{4} \Rightarrow f_4 = \frac{14}{2\times\frac{7L_0}{4}}\sqrt{\frac{T_4}{4\mu}} = f_0$

$$\Rightarrow \quad f_0 = \frac{2}{L_0}\sqrt{\frac{T_4}{\mu}} \Rightarrow T_4 = \frac{T_0}{16}$$

17. *(d)* 1-2 process is isothermal and 2-3 process is isochoric.

(I) $W_{1\to 2} = nRI\ln\frac{V_f}{V_i} = 1\times R\frac{T_0}{3}\ln\frac{V_2}{V_1} = \frac{RT_0}{3}\ln\frac{2V_0}{V_0} = \frac{RT_0}{3}\ln 2$

$W_{2\to 3} = 0$ (Isochoric process)

$W_{1\to 2\to 3} = W_{1\to 2} + W_{2\to 3} = \frac{RT_0}{3}\ln 2$ (I → P)

(II) $\Delta U = \frac{f}{2}nR(T_f - T_i)$

$$\Delta U_{1\to 2\to 3} = \frac{3}{2}\left[1\times R\left(T_0 - \frac{T_0}{3}\right)\right]$$

$= RT_0$ (II → R)

(III) $Q_{1\to 2\to 3} = \Delta U_{1\to 2\to 3} + W_{1\to 2\to 3}$

(First law of thermodynamics)

$= RT_0 + \frac{RT_0}{3}\ln 2$

$= \frac{RT_0}{3}[3 + \ln 2]$ (III → T)

(IV) $Q_{1\to 2} = \Delta U_{1\to 2} + W_{1\to 2}$

$= 0 + \frac{RT_0}{3}\ln 2 = \frac{RT_0}{3}\ln 2$ (IV → P)

18. *(d)* (I) $W_{1\to 2\to 3} = W_{1\to 2} + W_{2\to 3}$

$= P_0[2V_0 - V_0] + 0 = P_0V_0$

$W_{1\to 2\to 3} = P_0V_0 = \frac{RT_0}{3}$ (I → Q)

(II) $U_{1\to 2\to 3} = \frac{3}{2}\left[\frac{3P_0}{2}\times 2V_0 - P_0V_0\right]$

$= \frac{3}{2}\times 2P_0V_0 = 3P_0V_0 = RT_0$ (II → R)

(III) $Q_{1\to 2\to 3} = U_{1\to 2\to 3} + W_{1\to 2\to 3}$

$= RT_0 + \frac{RT_0}{3} = \frac{4RT_0}{3}$ (III → S)

(IV) $Q_{1\to 2} = nC_P\Delta T = n\frac{5}{2}R(T_2 - T_1)$

$= \frac{5}{2}[P_0 2V_0 - P_0V_0]$

$= \frac{5}{2}P_0V_0 = \frac{5}{2}\frac{RT_0}{3}$

$= \frac{5}{6}R_0T_0$ (IV → U)