PHYSICS – CLASS XI

Chapter 12 : Kinetic Theory

SOLUTIONS AND MARKING SCHEME (Step-by-Step)

Marks for each step are shown in brackets. Alternative correct methods carry full marks.

Quick answer key – Section A

Q	1	2	3	4	5	6	7	8
Ans	(a)	(b)	(b)	(a)	2 times	5×10^5 Pa	$\pi d^2 \bar{v} \Delta t$	10^{26} m^{-3}

SECTION A

1. **Ans: (a).** **(1)**

Boyle's law: PV = constant at fixed T, so the graph of PV against P is a horizontal straight line.

2. **Ans: (b) 2 : 1.** **(1)**

At the same T, $v_{rms} \propto 1/\sqrt{M}$. $v_{He}/v_{CH_4} = \sqrt{(16/4)} = \mathbf{2 : 1}$.

3. **Ans: (b) 5.** **(1)**

3 translational + 2 rotational (rotation about the molecular axis is not counted) = **5**.

4. **Ans: (a).** **(1)**

Average translational KE per molecule = $(3/2)k_B T$ for every ideal gas, so it is equal for He and O_2 at the same T; R explains A.

5. **Ans: 2 times.** **(1)**

$v_{rms} \propto \sqrt{T}$: $\sqrt{(1200/300)} = \sqrt{4} = \mathbf{2}$, so it doubles.

6. **Ans: 5×10^5 Pa.** **(1)**

Dalton's law: $P = P_A + P_B = 3 \times 10^5 + 2 \times 10^5 = \mathbf{5 \times 10^5\ Pa}$.

7. **Ans: $\pi d^2 \bar{v} \Delta t$.** **(1)**

Any molecule whose centre lies within a distance d of the path collides, so the swept volume is a cylinder of base area πd^2 and length $\bar{v}\Delta t$: $\boldsymbol{\pi d^2 \bar{v} \Delta t}$.

8. **Ans: 10^{26} m^{-3}.** **(1)**

$P = nk_B T \rightarrow n = 1.38 \times 10^5/(1.38 \times 10^{-23} \times 100) = \mathbf{10^{26}\ m^{-3}}$.

SECTION B

9. **(2)**

(i) Same V and T, so $P \propto N$: $N_{He}/N_{O_2} = P_{He}/P_{O_2} = \mathbf{4 : 1}$. (1)

(ii) $\rho \propto \mu M$, so $\rho_{He}/\rho_{O_2} = (4/1) \times (4/32) = \mathbf{1/2}$, i.e. **1 : 2**. (1)

10. **(2)**

(i) $P = (1/3)\rho v^2 = (1/3) \times 1.5 \times (400)^2 = 0.5 \times 1.6 \times 10^5 = \mathbf{8 \times 10^4\ Pa}$. (1)

(ii) KE per unit volume = $\tfrac{1}{2}\rho v^2 = \tfrac{1}{2} \times 1.5 \times 1.6 \times 10^5 = \mathbf{1.2 \times 10^5\ J}$ $(= (3/2)PV)$. (1)

11. **(2)**

$P = (1/3)nm\bar{v}^2$ and $P = nk_B T$ give $(1/3)nm\bar{v}^2 = nk_B T$. (1)

Hence $m\bar{v}^2 = 3k_B T$ and $\tfrac{1}{2}m\bar{v}^2 = \boldsymbol{(3/2)k_B T}$. (1)

OR

Law: In thermal equilibrium at absolute temperature T, the total energy is equally distributed among all energy modes, each quadratic term (degree of freedom) having average energy $\tfrac{1}{2}k_B T$. (1)

Rigid diatomic: 3 translational + 2 rotational terms → average energy = $5 \times \tfrac{1}{2}k_B T = \boldsymbol{(5/2)k_B T}$. (1)

12. **(i)** V_1 is smaller; V_2 is larger (line 1 is steeper). (1) **(2)**
(ii) At constant volume $P = (\mu R/V)T$, so the slope = $\mu R/V$. For the same μ, the steeper line (1) has the smaller volume. (1)

SECTION C

13. **(i)** $\Delta p = 2mv_x = 2 \times 5 \times 10^{-26} \times 400 = \mathbf{4 \times 10^{-23}\ kg\ m\ s^{-1}}$. (1) **(3)**
(ii) The molecule travels $2L$ between successive hits on the same wall: $t = 2L/v_x = 0.2/400 = \mathbf{5 \times 10^{-4}\ s}$. (1)
(iii) $F = \Delta p/t = 4 \times 10^{-23}/(5 \times 10^{-4}) = \mathbf{8 \times 10^{-20}\ N}$ $(= mv_x^2/L)$. (1)

14. **(i)** $Q_1 = \mu C_v \Delta T = 3 \times (5R/2) \times 100 = 750R = 750 \times 8.3 = \mathbf{6225\ J}$. (1) **(3)**
(ii) $Q_2 = \mu C_p \Delta T = 3 \times (7R/2) \times 100 = 1050R = \mathbf{8715\ J}$. (1)
(iii) $W = Q_2 - \Delta U = 8715 - 6225 = \mathbf{2490\ J}$ $(= \mu R \Delta T = 300R)$. (1)

OR

(i) $U = 2 \times (3/2)RT + 2 \times (5/2)RT = 3RT + 5RT = \mathbf{8RT}$. (1)
(ii) Total $C_v = dU/dT = 8R$ for 4 mol, so per mole $C_v = \mathbf{2R}$. (1)
(iii) $C_p = C_v + R = 3R$, so $\gamma = 3R/2R = \mathbf{3/2}$. (1)

15. **(i)** $\tau = l/\bar{v} = 4 \times 10^{-7}/500 = \mathbf{8 \times 10^{-10}\ s}$. (1) **(3)**
(ii) Collision frequency $= 1/\tau = 1/(8 \times 10^{-10}) = \mathbf{1.25 \times 10^{9}\ s^{-1}}$. (1)
(iii) At fixed T, $n \propto P$ and $l \propto 1/n$. The pressure becomes 1/1000, so $l = 1000 \times 4 \times 10^{-7} = \mathbf{4 \times 10^{-4}\ m}$. (1)

SECTION D

16. **(a)** $PV = \mu RT$. (1) **(5)**
(b) $\mu = PV/(RT)$. With V and R fixed, $\mu \propto P/T$. (1)
(c) $\mu_2/\mu_1 = (P_2/T_2)/(P_1/T_1) = (6/270)/(10/300) = (1/45)/(1/30) = \mathbf{2/3}$. (1)
(d) $\mu_2 = (2/3) \times 12 = 8$ mol, so moles withdrawn $= 12 - 8 = \mathbf{4\ mol}$. (1)
(e) Mass $= 4 \times 32 = 128$ g $= \mathbf{0.128\ kg}$. (1)

OR

(a) The dotted line represents the ideal gas behaviour ($PV/T = \mu R$, a constant). (1)
(b) A real gas approaches ideal behaviour better at higher temperature, and the T_1 curve deviates less from the dotted line: $\boldsymbol{T_1 > T_2}$. (1)
(c) At the y-axis ($P \to 0$) the gas is ideal: $PV/T = \mu R$; $\mu = 320/32 = 10$ mol, so $PV/T = 10 \times 8.3 = \mathbf{83\ J\ K^{-1}}$. (1)
(d) **No.** 0.32 kg of H_2 is 160 mol, giving $\mu R = 160 \times 8.3 = 1328$ J K^{-1}, which is different. (1)
(e) Same value needs $\mu = 10$ mol: mass $= 10 \times 2$ g $= \mathbf{20\ g = 0.02\ kg}$. (1)

17. **(a)** 3 translational + 2 rotational + 2 vibrational (kinetic and potential) = **7** terms. **(5)**
(1)

(b) Each term gives $\tfrac{1}{2}k_B T$: energy = $\mathbf{(7/2)k_B T}$. (1)

(c) $U = (7/2)k_B T \times N_A = \mathbf{(7/2)RT}$. (1)

(d) $C_v = dU/dT = \mathbf{7R/2}$; $C_p = C_v + R = \mathbf{9R/2}$. (1)

(e) $\gamma = (9R/2)/(7R/2) = \mathbf{9/7}$. (1)

OR

(a) At the same T all molecules have the same average KE, $\tfrac{1}{2}m\bar{v}^2 = (3/2)k_B T$; the lighter molecule (smaller m) therefore has the greater speed. (1)

(b) $v_{H_2}/v_{O_2} = \sqrt{(32/2)} = \sqrt{16} = \mathbf{4 : 1}$. (1)

(c) Both have $(3/2)k_B T$, so the ratio is **1 : 1**. (1)

(d) **Hydrogen**, because its faster molecules reach and pass through the pores more often. (1)

(e) $v_{rms} \propto \sqrt{(T/M)}$: equal speeds need $T_{O_2}/32 = 100/2 \rightarrow T_{O_2} = \mathbf{1600\ K}$. (1)

— END OF SOLUTIONS —