PHYSICS – CLASS XI

Chapter 13 : Oscillations

SOLUTIONS AND MARKING SCHEME (Step-by-Step)

Marks for each step are shown in brackets. Alternative correct methods carry full marks.

Quick answer key – Section A

Q	1	2	3	4	5	6	7	8
Ans	(c)	(b)	(c)	(a)	2 cos π*t*	1 m s^{-1}	2 J	0.5 s

SECTION A

1. **Ans: (c).** **(1)**

SHM is a sinusoidal function of time with constant amplitude. (a) is non-periodic, (b) is periodic but not sinusoidal, (d) has decaying amplitude; only graph (c) is a pure sine curve.

2. **Ans: (b) π rad s^{-1}.** **(1)**

The graph repeats after 2 s, so $T = 2$ s and $\omega = 2\pi/T =$ **π rad s^{-1}**.

3. **Ans: (c) 2*T*.** **(1)**

$T = 2\pi\sqrt{(L/g)} \propto \sqrt{L}$; for $4L$ the period is $\sqrt{4} = 2$ times, i.e. **2*T***.

4. **Ans: (a).** **(1)**

$a = -\omega^2 x$ is zero at $x = 0$ (mean position), so R explains A.

5. **Ans: $x = 2\cos \pi t$.** **(1)**

$\omega = 2\pi/T = \pi$ rad s^{-1}, $A = 2$ cm, and P starts on the $+x$ axis ($\varphi = 0$): $x =$ **2 cos(π*t*) cm**.

6. **Ans: 1 m s^{-1}.** **(1)**

$v_{max} = \omega A = 5 \times 0.2 =$ **1 m s^{-1}**.

7. **Ans: 2 J.** **(1)**

$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2} \times 100 \times (0.2)^2 = \tfrac{1}{2} \times 100 \times 0.04 =$ **2 J**.

8. **Ans: 0.5 s.** **(1)**

Frequency = 120/60 = 2 Hz, so $T = 1/2 =$ **0.5 s**.

SECTION B

9. **(i)** $A = 20/2 = 10$ cm $= 0.1$ m. $v_{max} = \omega A = 20 \times 0.1 =$ **2 m s^{-1}**. (1) **(2)**
(ii) $a_{max} = \omega^2 A = 400 \times 0.1 =$ **40 m s^{-2}**. (1)

10. $\omega = 2\pi/T = 2\pi/4 = \pi/2$ rad s^{-1}. At $t = 0$, OP makes 90° (π/2) with the x-axis, so $\varphi = \pi/2$: $x =$ **4 cos(π*t*/2 + π/2) cm** ($= -4\sin(\pi t/2)$). (1) **(2)**
At $t = 1$ s: $x = 4\cos(\pi/2 + \pi/2) = 4\cos\pi =$ **−4 cm**. (1)

11. $v = dx/dt =$ **$-\omega A \sin(\omega t + \varphi)$**. (½) **(2)**
$a = dv/dt =$ **$-\omega^2 A \cos(\omega t + \varphi)$**. (½)
Since $A\cos(\omega t + \varphi) = x$, $a =$ **$-\omega^2 x$**. (1)

OR

$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2A^2\sin^2(\omega t + \varphi) = \tfrac{1}{2}kA^2\sin^2(\omega t + \varphi)$ (using $k = m\omega^2$); $U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2\cos^2(\omega t + \varphi)$. (1)
$E = K + U = \tfrac{1}{2}kA^2[\sin^2 + \cos^2] =$ **$\tfrac{1}{2}kA^2$**, a constant. (1)

12. **(i)** Curve **2**. It is maximum at $x = 0$, where the speed is maximum, and zero at $x = \pm A$, where the particle is momentarily at rest. (1) **(2)**
(ii) At $x = A/2$: $U = \frac{1}{2}k(A/2)^2 = E/4$, so $K = E - E/4 = 3E/4$. $K : U = \mathbf{3 : 1}$. (1)

SECTION C

13. **(i)** $k = F/x = 2/0.2 = \mathbf{10\ N\ m^{-1}}$ (slope of the graph). (1) **(3)**
(ii) $T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(1/10)} = 2\pi/\sqrt{10} = 2\pi/\pi = \mathbf{2\ s}$. (1)
(iii) $a_{max} = kA/m = 10 \times 0.2/1 = \mathbf{2\ m\ s^{-2}}$. (1)

14. **(i)** For a displacement x, one spring stretches and the other compresses, so both forces oppose the displacement: $F = -k_1x - k_2x = \mathbf{-(k_1 + k_2)x}$. (1) **(3)**
(ii) $k_{eff} = 20 + 30 = \mathbf{50\ N\ m^{-1}}$. (1)
(iii) $T = 2\pi\sqrt{(m/k_{eff})} = 2\pi\sqrt{(5/50)} = 2\pi/\sqrt{10} = \mathbf{2\ s}$. (1)

OR

(i) $v = 10 \times \sqrt{(25 - 9)} = 10 \times 4 = \mathbf{40\ cm\ s^{-1}}$. (1)
(ii) At O ($x = 0$): $v = \omega A = 10 \times 5 = \mathbf{50\ cm\ s^{-1}}$. (1)
(iii) $a = \omega^2x = 100 \times 3 = \mathbf{300\ cm\ s^{-2}}$ ($= 3\ m\ s^{-2}$), directed towards O. (1)

15. **(i)** 50 kg corresponds to a force $50 \times 10 = 500$ N and an extension of 0.2 m: $k = 500/0.2 = \mathbf{2500\ N\ m^{-1}}$. (1) **(3)**
(ii) $T = 2\pi\sqrt{(m/k)} \rightarrow m = kT^2/4\pi^2 = 2500 \times 0.16/40 = \mathbf{10\ kg}$. (1)
(iii) Weight $= mg = 10 \times 10 = \mathbf{100\ N}$. (1)

SECTION D

16. **(a)** Amplitude $A = \mathbf{5\ cm}$. (1) **(5)**
(b) The graph repeats at 4 s: $T = \mathbf{4\ s}$, frequency $= 1/T = \mathbf{0.25\ Hz}$. (1)
(c) $\omega = 2\pi/T = 2\pi/4 = \mathbf{\pi/2\ rad\ s^{-1}}$. (1)
(d) The graph starts at the maximum ($\varphi = 0$): $x = \mathbf{5\ cos(\pi t/2)\ cm}$. (1)
(e) $x = 5\ \cos(\pi/2 \times 2/3) = 5\ \cos(\pi/3) = 5 \times \frac{1}{2} = \mathbf{2.5\ cm}$. (1)

OR

(a) The level in the other limb rises by y (the volume is conserved), so the difference in levels is $\mathbf{2y}$. (1)
(b) Restoring force = weight of the unbalanced column of height $2y$: $F = -(2yA\rho)g = \mathbf{-2A\rho g\ y}$. (1)
(c) $F \propto -y$: the force is proportional to the displacement and directed towards the mean position, so the motion is SHM. (1)
(d) Mass of liquid $= AL\rho$; $AL\rho\ (d^2y/dt^2) = -2A\rho g\ y \rightarrow \omega^2 = 2g/L$; $T = 2\pi\sqrt{(L/2g)}$. (1)
(e) $T = 2\pi\sqrt{(2/20)} = 2\pi\sqrt{(1/10)} = 2\pi/\pi = \mathbf{2\ s}$. (1)

17. **(a)** Curve **2**: the block is released from rest at $x = A$, so at $t = 0$ the kinetic energy is zero and the potential energy is maximum. (1) **(5)**

(b) Kinetic energy repeats twice in each cycle: its period is $\boldsymbol{T/2}$. (1)

(c) $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 50 \times (0.2)^2 = \frac{1}{2} \times 50 \times 0.04 = \mathbf{1\ J}$. (1)

(d) At the mean position all energy is kinetic: $\frac{1}{2}mv^2 = 1 \rightarrow \frac{1}{2} \times 2 \times v^2 = 1 \rightarrow v_{max} = \mathbf{1\ m\ s^{-1}}$. (1)

(e) $U = \frac{1}{2} \times 50 \times (0.1)^2 = 0.25$ J, so $K = E - U = 1 - 0.25 = \mathbf{0.75\ J}$. (1)

OR

(a) The tension T along the string and the weight mg (components: $mg\cos\theta$ along the string, $mg\sin\theta$ perpendicular to it). (1)

(b) Only $mg\sin\theta$ gives a torque about the support: $\tau = \boldsymbol{-mgL\sin\theta}$ (negative: restoring). (1)

(c) $\tau = I\alpha$ with $I = mL^2$ and $\sin\theta \approx \theta$: $\alpha = \boldsymbol{-(g/L)\,\theta}$. (1)

(d) This is of the form $\alpha = -\omega^2\theta$ (SHM) with $\omega^2 = g/L$, so $T = 2\pi/\omega = \boldsymbol{2\pi\sqrt{(L/g)}}$. (1)

(e) $T = 2\pi\sqrt{(0.25/10)} = 2\pi \times 0.5/\sqrt{10} = \pi/\pi = \mathbf{1\ s}$ (since $\sqrt{10} = \pi$). (1)

— END OF SOLUTIONS —