PHYSICS – CLASS XI

Chapter 3 : Motion in a Plane

SOLUTIONS AND MARKING SCHEME (Step-by-Step)

Marks for each step are shown in brackets. Alternative correct methods carry full marks.

Quick answer key – Section A

Q	1	2	3	4	5	6	7	8
Ans	(b)	(a)	(b)	(a)	(b)	5√3, 5	22 m, 14 m	(c)

SECTION A

1. **Ans: (b) 5 N.** **(1)**

For perpendicular vectors, $R = \sqrt{(A^2 + B^2)} = \sqrt{(9 + 16)} = \sqrt{25} = \mathbf{5\ N}$.

2. **Ans: (a) a null vector.** **(1)**

A + B = PQ + QR = PR (triangle law). C = RP = −PR. Hence A + B + C = PR + RP = **0**: vectors forming a closed polygon in order add to zero.

3. **Ans: (b).** **(1)**

The horizontal component $u \cos\theta$ stays constant throughout the flight, and at the top $v_y = 0$, so the velocity is $u \cos\theta$ horizontally. The acceleration is always g downwards.

4. **Ans: (a).** **(1)**

Both are true. Although the speed is constant, the direction of velocity changes continuously, so the velocity vector changes; hence there is an acceleration (v^2/R, towards the centre). R explains A.

5. **Ans: (b) 1 : 1.** **(1)**

$R = u^2 \sin 2\theta / g$. For 30°, $\sin 2\theta = \sin 60°$; for 60°, $\sin 2\theta = \sin 120° = \sin(180° - 60°) = \sin 60°$. The ranges are equal (complementary angles), ratio **1 : 1**.

6. **Ans: $A_x = 5\sqrt{3} \approx 8.7$ units, $A_y = 5$ units.** **(1)**

$A_x = A \cos 30° = 10 \times \sqrt{3}/2 = \mathbf{5\sqrt{3}}$ (½); $A_y = A \sin 30° = 10 \times ½ = \mathbf{5}$. (½)

7. **Ans: 22 m and 14 m.** **(1)**

(i) Distance = semicircle = $\pi r = (22/7) \times 7 = \mathbf{22\ m}$. (½)

(ii) Displacement = straight line PQ = diameter = $2 \times 7 = \mathbf{14\ m}$. (½)

8. **Ans: (c).** **(1)**

The vertical motion of B is independent of its horizontal motion: both start with zero vertical velocity and fall through the same h under g, so $t = \sqrt{(2h/g)}$ is the same for both.

SECTION B

9. **(i)** $R = \sqrt{(A^2 + B^2 + 2AB\cos\theta)} = \sqrt{(9 + 25 + 2 \times 3 \times 5 \times ½)} = \sqrt{49} = \mathbf{7\ units}$. (1) **(2)**

(ii) $\tan\alpha = B \sin\theta / (A + B\cos\theta) = (5 \times \sqrt{3}/2) / (3 + 5 \times ½) = (5\sqrt{3}/2)/(11/2) = \mathbf{5\sqrt{3}/11}$. (1)

10. **(i)** Velocity of rain with respect to man, $\mathbf{v}_{rm} = \mathbf{v}_r - \mathbf{v}_m$. Its components are 8 m s^{-1} downwards and 6 m s^{-1} backwards, so $|\mathbf{v}_{rm}| = \sqrt{(8^2 + 6^2)} = \mathbf{10\ m\ s^{-1}}$. (1) **(2)**

(ii) $\tan\theta = 6/8 = 3/4 \rightarrow \theta \approx \mathbf{37°}$ **with the vertical**, the umbrella being tilted forward, i.e. in the direction in which the man walks. (1)

11. Take the ground as $y = 0$. Vertical motion: $y = (u \sin\theta)t - \tfrac{1}{2}gt^2$. (1) **(2)**
At the end of the flight $y = 0$: $t(u \sin\theta - \tfrac{1}{2}gt) = 0 \rightarrow t = 0$ (start) or $\boldsymbol{T = 2u \sin\theta / g}$. (1)

OR

$R = u^2 \sin 2\theta / g$. (½)
For $\theta = 45° + \alpha$: $\sin 2\theta = \sin(90° + 2\alpha) = \cos 2\alpha$. For $\theta = 45° - \alpha$: $\sin 2\theta = \sin(90° - 2\alpha) = \cos 2\alpha$. (1)
Both give the same value of $R = u^2 \cos 2\alpha / g$, so the ranges are equal. (½)

12. **(i)** Distance = 12 + 5 = **17 m**. (½) **(2)**
(ii) Displacement OP = $\sqrt{(12^2 + 5^2)} = \sqrt{169} =$ **13 m**. (1)
(iii) $\tan\theta = 5/12$, so the displacement is directed at $\theta = \tan^{-1}(5/12)$ ($\approx 22.6°$) **north of east**. (½)

SECTION C

13. **(i)** $\mathbf{v} = d\mathbf{r}/dt = \boldsymbol{6\hat{i} + 8t\hat{j}}$ m s^{-1}. (1) $\mathbf{a} = d\mathbf{v}/dt = \boldsymbol{8\hat{j}}$ m s^{-2} (constant, along y). (½) **(3)**
(ii) At $t = 1$ s: $\mathbf{v} = 6\hat{i} + 8\hat{j}$, so $v = \sqrt{(6^2 + 8^2)} =$ **10 m s^{-1}**. (1)
$\tan\theta = v_y/v_x = 8/6 = 4/3 \rightarrow \theta \approx$ **53° with the *x*-axis**. (½)

14. The velocities are perpendicular: $\mathbf{v}_b = 8$ km h^{-1} (north), $\mathbf{v}_c = 6$ km h^{-1} (east). **(3)**
(i) $R = \sqrt{(8^2 + 6^2)} =$ **10 km h^{-1}**. (1)
(ii) $\tan\theta = 6/8 = 3/4 \rightarrow \theta =$ **37° east of north**. (1)
(iii) Distance = 10 × ½ h = **5 km**. (1)

OR

Let θ be the angle between $\mathbf{r}$ and $\mathbf{r}'$ ($\Delta\theta$). Since $\mathbf{v} \perp \mathbf{r}$ and $\mathbf{v}' \perp \mathbf{r}'$, the angle between $\mathbf{v}$ and $\mathbf{v}'$ is also $\Delta\theta$; the triangle formed by $\mathbf{v}$, $\mathbf{v}'$, $\Delta\mathbf{v}$ is similar to triangle CPP′. (1)
Hence $|\Delta\mathbf{v}| / v = |\Delta\mathbf{r}| / R$, so $|\Delta\mathbf{v}| = v|\Delta\mathbf{r}| / R$. (½)
$a = \lim_{\Delta t \to 0} |\Delta\mathbf{v}|/\Delta t = (v/R) \lim |\Delta\mathbf{r}|/\Delta t = (v/R) \times v =$ $\boldsymbol{v^2/R}$, since the chord tends to the arc and $\lim |\Delta\mathbf{r}|/\Delta t = v$. (1)
As $\Delta t \rightarrow 0$, $\Delta\mathbf{v}$ becomes perpendicular to $\mathbf{v}$ and points towards the centre C. (½)

15. $u_x = 50 \times 0.8 = 40$ m s^{-1}; $u_y = 50 \times 0.6 = 30$ m s^{-1}. **(3)**
(i) $t_m = u_y/g = 30/10 =$ **3 s**. (1)
(ii) $H = u_y^2/2g = 900/20 =$ **45 m**. (1)
(iii) Time of flight = 2 × 3 = 6 s; $R = u_x \times 6 = 40 \times 6 =$ **240 m**. (1)

SECTION D

16. **(a)** $u_x = 20 \cos 53° = 20 \times 0.6 =$ **12 m s^{-1}**; $u_y = 20 \sin 53° = 20 \times 0.8 =$ **16 m s^{-1}**. (1) **(5)**
(b) Horizontal motion is uniform: $t = 24/12 =$ **2 s**. (1)
(c) $y = u_y t - \tfrac{1}{2}gt^2 = 16 \times 2 - 5 \times 4 = 32 - 20 =$ **12 m**. (1)
(d) $v_y = u_y - gt = 16 - 20 =$ **−4 m s^{-1}**. It is negative, so the ball is **falling** (already past its highest point). (1)
(e) $H = u_y^2/2g = 256/20 =$ **12.8 m**. (1)

OR

(a) $x = (u \cos\theta)t$ and $y = (u \sin\theta)t - \tfrac{1}{2}gt^2$. Eliminating $t = x/(u \cos\theta)$: (1)
$\boldsymbol{y = (\tan\theta)x - gx^2 / [2u^2 \cos^2\theta]}$, of the form $y = ax - bx^2$ with a, b constants, i.e. a parabola. (1)
(b) Time of flight $T = 2u \sin\theta/g$; $R = (u \cos\theta)T =$ $\boldsymbol{u^2 \sin 2\theta / g}$. (1)
(c) R is maximum when $\sin 2\theta = 1$, i.e. $\theta =$ **45°**; $R_{max} =$ $\boldsymbol{u^2/g}$. (1)
(d) $u^2/g = 100$ m. Thrown vertically, $H = u^2/2g = 100/2 =$ **50 m**. (1)

17. **(a)** From the grid: $\mathbf{A} = 5\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}$, $\mathbf{B} = -2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}$. (1) **(5)**

(b) $\mathbf{R} = (5 - 2)\hat{\imath} + (2 + 2)\hat{\jmath} = \mathbf{3\hat{\imath} + 4\hat{\jmath}}$. (1)

(c) $|\mathbf{R}| = \sqrt{(3^2 + 4^2)} = \mathbf{5\ units}$; $\tan\theta = 4/3 \rightarrow \theta = \mathbf{53°}$ **with the *x*-axis**. (1)

(d) $\mathbf{A} - \mathbf{B} = (5 + 2)\hat{\imath} + (2 - 2)\hat{\jmath} = \mathbf{7\hat{\imath}}$; magnitude = **7 units**. (1)

(e) $\hat{n} = \mathbf{R}/|\mathbf{R}| = (3\hat{\imath} + 4\hat{\jmath})/5 = \mathbf{0.6\hat{\imath} + 0.8\hat{\jmath}}$. (1)

OR

(a) Always directed **towards the centre** of the circle (centripetal). (1)

(b) $T = 20/5 = \mathbf{4\ s}$; $f = 1/T = \mathbf{0.25\ Hz}$. (1)

(c) $\omega = 2\pi/T = 2\pi/4 = \mathbf{\pi/2\ rad\ s^{-1}}$ (≈ 1.57 rad s^{-1}). (1)

(d) $v = \omega R = (\pi/2) \times 0.2 = 0.1\pi = \mathbf{0.314\ m\ s^{-1}}$. (1)

(e) $a_c = \omega^2 R = (\pi^2/4) \times 0.2 = 0.05\pi^2 = \mathbf{0.5\ m\ s^{-2}}$. (½) It is **not constant** as a vector: its magnitude is constant but its direction (towards the centre) keeps changing. (½)

— END OF SOLUTIONS —