PHYSICS – CLASS XI

Chapter 6 : Systems of Particles and Rotational Motion

SOLUTIONS AND MARKING SCHEME (Step-by-Step)

Marks for each step are shown in brackets. Alternative correct methods carry full marks.

Quick answer key – Section A

Q	1	2	3	4	5	6	7	8
Ans	(c)	(b)	(a)	(d)	6 m s^{-1}	2 rad s^{-2}	1 m	10

SECTION A

1. **Ans: (c) x = 3 m.** **(1)**

$X = (m_1x_1 + m_2x_2)/(m_1 + m_2) = (2 \times 0 + 3 \times 5)/5 =$ **3 m**.

2. **Ans: (b) 2.5 N m.** **(1)**

$\tau = rF \sin\theta = 0.5 \times 10 \times \sin 30° = 0.5 \times 10 \times ½ =$ **2.5 N m**.

3. **Ans: (a) 0.1 kg m².** **(1)**

$I = ML^2/12 = 1.2 \times 1^2/12 =$ **0.1 kg m²**.

4. **Ans: (d).** **(1)**

A is false: the centre of mass can lie outside the body (e.g. a ring, where no mass is present at the centre). R is true: by symmetry the centre of mass of a uniform ring is at its centre.

5. **Ans: 6 m s⁻¹, tangential.** **(1)**

$v = \omega r = 20 \times 0.3 =$ **6 m s⁻¹**, directed along the tangent to the circle at P (perpendicular to OP, in the sense of rotation).

6. **Ans: 2 rad s⁻².** **(1)**

$\tau = FR = 10 \times 0.4 = 4$ N m; $\alpha = \tau/I = 4/2 =$ **2 rad s⁻²**.

7. **Ans: 1 m.** **(1)**

Principle of moments: $40 \times 1.5 = 60 \times d \rightarrow d = 60/60 =$ **1 m**.

8. **Ans: 10, out of the page.** **(1)**

$|\mathbf{a} \times \mathbf{b}| = ab \sin 30° = 4 \times 5 \times ½ =$ **10**. By the right-hand rule (rotating from **a** to **b** is anticlockwise) it is perpendicular to the plane of the page and points **out of the page**.

SECTION B

9. $X = (1 \times 0 + 2 \times 3 + 3 \times 0)/(1 + 2 + 3) = 6/6 =$ **1 m**. (1) **(2)**

$Y = (1 \times 0 + 2 \times 0 + 3 \times 2)/6 = 6/6 =$ **1 m**. Centre of mass = **(1 m, 1 m)**. (1)

10. $\omega_0 = 2\pi \times 120/60 = 4\pi$ rad s^{-1}; $\omega = 2\pi \times 300/60 = 10\pi$ rad s^{-1}. (½) **(2)**

(i) $\alpha = (\omega - \omega_0)/t = (10\pi - 4\pi)/6 =$ **π rad s⁻²** (≈ 3.14 rad s^{-2}). (1)

(ii) $\theta = \omega_0 t + ½\alpha t^2 = 4\pi \times 6 + ½ \times \pi \times 36 = 24\pi + 18\pi = 42\pi$ rad = **21 revolutions**. (½)

11. **(i)** Axis through the centre, perpendicular to the plane: $I = MR^2 = 2 \times (0.5)^2 =$ **0.5 kg m²**. (1) **(2)**

(ii) About a diameter: $I = MR^2/2 = 2 \times 0.25/2 =$ **0.25 kg m²**. (1)

OR

(i) The forces are equal and opposite, so the net force = 5 − 5 = **0**. (1)

(ii) Moment of the couple = force × perpendicular distance between the lines of action = 5 × 0.4 = **2 N m**. (1)

12. **(i)** $L = mvr_\perp = 2 \times 3 \times 4 =$ **24 kg m² s⁻¹** (directed into the page). (1) **(2)**

(ii) No. The perpendicular distance $r_\perp = 4$ m and v stay constant, and no external torque acts ($dL/dt = \tau = 0$), so L is conserved. (1)

SECTION C

13. **(i)** Translational equilibrium: $\Sigma F = 0$ (net force zero). Rotational equilibrium: $\Sigma\tau = 0$ (net torque about any point zero). (1) **(3)**

(ii) Forces: $R_A + R_B = 40 + 60 = 100$ N. Moments about A: $R_B \times 2 = 60 \times 0.5 + 40 \times 1$. (1)

$R_B = 70/2 =$ **35 N** and $R_A = 100 - 35 =$ **65 N**. (Check by moments about B: $65 \times 2 = 40 \times 1 + 60 \times 1.5 = 130$.) (1)

14. $I = \tfrac{1}{2}MR^2 = \tfrac{1}{2} \times 10 \times (0.2)^2 = 0.2$ kg m²; $\tau = FR = 20 \times 0.2 = 4$ N m. **(3)**

(i) $\alpha = \tau/I = 4/0.2 =$ **20 rad s⁻²**. (1)

(ii) Work = $F \times$ distance = $20 \times 2 =$ **40 J**. (1)

(iii) $\theta = 2/0.2 = 10$ rad; $\omega^2 = 2\alpha\theta = 2 \times 20 \times 10 = 400$; $K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2} \times 0.2 \times 400 =$ **40 J** (equal to the work done). (1)

OR

(i) $X = (60 \times 0 + 40 \times 10)/(60 + 40) =$ **4 m** from A. (1)

(ii) They meet at the centre of mass, **4 m from A** (A moves 4 m, B moves 6 m). (1)

(iii) The rope force is internal and the net external force on the system is zero (smooth ice), so the centre of mass, initially at rest, stays at rest. (1)

15. **(i)** $I_i = \tfrac{1}{2}MR^2 + mR^2 = \tfrac{1}{2} \times 100 \times 4 + 50 \times 4 = 200 + 200 =$ **400 kg m²**. (1) **(3)**

(ii) No external torque, so angular momentum is conserved. Finally the man is at the centre, $I_f = 200$ kg m². $I_i\omega_i = I_f\omega_f \rightarrow 400 \times 3 = 200 \times \omega_f \rightarrow \omega_f =$ **6 rad s⁻¹**. (1)

(iii) $K_i = \tfrac{1}{2} \times 400 \times 9 =$ **1800 J**; $K_f = \tfrac{1}{2} \times 200 \times 36 =$ **3600 J**. The increase of 1800 J comes from the work done by the man in walking inwards. (1)

SECTION D

16. **(5)**

(a) (i) The vector sum of all external forces is zero (translational equilibrium): $\Sigma F = 0$. (ii) The vector sum of all external torques about any point is zero (rotational equilibrium): $\Sigma\tau = 0$. (1)

(b) Vertical forces: $N = W = 20 \times 10 = \mathbf{200\ N}$. (1)

(c) Height of the top of the ladder $= \sqrt{(5^2 - 3^2)} = 4$ m. Moments about the foot: $F_1 \times 4 = W \times 1.5$ (horizontal distance of the mid-point from the foot is 3/2 = 1.5 m). (1)

$F_1 = 200 \times 1.5/4 = \mathbf{75\ N}$. (1)

(d) Horizontal forces: friction $f = F_1 = 75$ N. $\mu_{min} = f/N = 75/200 = \mathbf{0.375}$. (1)

OR

(a) The centre of gravity is the point where the total gravitational torque on the body is zero (the point at which its whole weight may be taken to act). (1)

(b) The stick is uniform, so its weight acts at its centre, the **50 cm** mark. (1)

(c) Clockwise moment = anticlockwise moment about the pivot: $W \times 40 = 200 \times (50 - 40)$. (1)

(d) $W = 200 \times 10/40 = \mathbf{50\ g}$. (1)

(e) The pivot supports the total weight: $R = (200 + 50)$ g $= 0.25$ kg $\times 10 = \mathbf{2.5\ N}$. (1)

17. **(5)**

(a) $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 2 \times (0.2)^2 = \mathbf{0.04\ kg\ m^2}$. (1)

(b) Block: $mg - T = ma \rightarrow 10 - T = 1 \times a$. (1)

(c) Pulley: $\tau = TR = I\alpha$ with $\alpha = a/R \rightarrow T = Ia/R^2 = \frac{1}{2}Ma = a$. (1)

(d) Substituting: $10 - a = a \rightarrow a = \mathbf{5\ m\ s^{-2}}$. (1)

(e) $T = a = \mathbf{5\ N}$; $\alpha = a/R = 5/0.2 = \mathbf{25\ rad\ s^{-2}}$. (Check: $I\alpha = 0.04 \times 25 = 1$ N m $= TR = 5 \times 0.2$.) (1)

OR

(a) $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 10 \times (0.4)^2 = \mathbf{0.8\ kg\ m^2}$. (1)

(b) $\alpha = \tau/I = 16/0.8 = \mathbf{20\ rad\ s^{-2}}$. (1)

(c) $\omega = \alpha t \rightarrow t = 100/20 = \mathbf{5\ s}$. (1)

(d) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 20 \times 25 = 250$ rad; $W = \tau\theta = 16 \times 250 = \mathbf{4000\ J}$. $K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.8 \times 10^4 = 4000$ J, equal to W. (1)

(e) $P = \tau\omega = 16 \times 100 = \mathbf{1600\ W}$. (1)

— END OF SOLUTIONS —