PHYSICS – CLASS XI

Chapter 7 : Gravitation

SOLUTIONS AND MARKING SCHEME (Step-by-Step)

Marks for each step are shown in brackets. Alternative correct methods carry full marks.

Quick answer key – Section A

Q	1	2	3	4	5	6	7	8
Ans	(b)	(c)	(c)	(c)	zero	-2×10^9 J	5 m s^{-2}	zero

SECTION A

1. **Ans: (b) $g/4$.** **(1)**

Distance of P from the centre = $R + R = 2R$. $g' = GM/(2R)^2 = GM/4R^2 = \boldsymbol{g/4}$.

2. **Ans: (c) 27.** **(1)**

Kepler's law of periods: $T^2 \propto r^3$. $T = (9)^{3/2} \times 1$ yr = 27 yr, i.e. **27 years**.

3. **Ans: (c) F.** **(1)**

$F \propto m_1 m_2/d^2$. New force = $G(2m_1)(2m_2)/(2d)^2 = 4Gm_1m_2/4d^2 = \boldsymbol{F}$.

4. **Ans: (c).** **(1)**

A is true: the astronaut and the satellite are both in free fall, so the astronaut feels weightless. R is false: gravity at that height is still large (nearly the same as on the ground); it is not zero.

5. **Ans: zero.** **(1)**

Inside a uniform hollow spherical shell the gravitational forces due to the different parts of the shell cancel at every point, so the net force on m is **zero**.

6. **Ans: -2×10^9 J.** **(1)**

For a circular orbit $E = -K$ (since $U = -2K$ and $E = K + U$). So $E = \mathbf{-2 \times 10^9}$ **J**.

7. **Ans: 5 m s^{-2}.** **(1)**

$g(d) = g(1 - d/R) = 10(1 - \tfrac{1}{2}) = \mathbf{5\ m\ s^{-2}}$.

8. **Ans: zero.** **(1)**

The gravitational force is always directed towards the Earth's centre, i.e. perpendicular to the Moon's velocity (displacement) in a circular orbit. So $W = Fd \cos 90° = 0$ at every instant, and the net work in one revolution is **zero**.

SECTION B

9. **(i)** Gravity gives the centripetal force: $mg = mv^2/R \rightarrow v = \sqrt{(gR)} = \sqrt{(10 \times 6.4 \times 10^6)} = \sqrt{(6.4 \times 10^7)} = \mathbf{8 \times 10^3\ m\ s^{-1}}$ (8 km s^{-1}). (1) **(2)**

(ii) $T = 2\pi R/v = 2 \times 3.14 \times 6.4 \times 10^6/(8 \times 10^3) = 5024$ s ≈ **84 minutes** (5024/60 ≈ 83.7). (1)

10. **(i)** Major axis = perihelion + aphelion distance = 2 + 6 = 8 AU, so the semi-major axis $a = 8/2 = \mathbf{4\ AU}$. (1) **(2)**

(ii) $T^2 = a^3 = 4^3 = 64 \rightarrow T = \mathbf{8\ years}$. (1)

11. Weight = mg', with $g' = g/(1 + h/R)^2 = g/(1.5)^2 = 4g/9$. (1) **(2)**
Weight at height $R/2 = 90 \times 4/9 =$ **40 N**. (1)

OR

At depth d: $g' = g(1 - d/R) = g(1 - ⅓) = 2g/3$. (1)
Weight $= 90 \times 2/3 =$ **60 N**. (1)

12. **(i)** The two equal masses are at equal distances (0.5 m) from P and their fields are equal and opposite, so the net field at P is **zero**. (1) **(2)**
(ii) Potential is a scalar, so the potentials add: $V = -2GM/r = -2 \times 6.67 \times 10^{-11} \times 100/0.5 =$ **-2.67×10^{-8} J kg^{-1}**. (1)

SECTION C

13. **(i)** Force due to B (or D), at distance a: $F_1 = Gm^2/a^2$, along AB (or AD) respectively. (1) **(3)**
(ii) C is at distance $a\sqrt{2}$ along the diagonal: $F_2 = Gm^2/(2a^2)$, directed along AC. (1)
(iii) The forces due to B and D are at 90°, so their resultant is $\sqrt{2}\,F_1 = \sqrt{2}\,Gm^2/a^2$ along the diagonal AC. Adding F_2 (same direction): $F = (Gm^2/a^2)(\sqrt{2} + ½) =$ **$(2\sqrt{2} + 1)Gm^2/2a^2$**, directed along the diagonal AC towards C (the centre of the square). (1)

14. **(i)** At N the two gravitational pulls are equal and opposite: $GMm/r^2 = G(9M)m/(8 - r)^2$. (1) **(3)**
(ii) $(8 - r)^2 = 9r^2 \rightarrow 8 - r = 3r \rightarrow r =$ **2 m** from the mass M. (1)
(iii) Unstable. If the mass is displaced slightly towards either star, the pull of that star increases and exceeds the other, so it moves further away from N instead of returning. (1)

OR

Using $GM = gR^2$: $K = GMm/2r$ with $r = 2R$ gives $K = gR^2m/4R = gRm/4$. **(i)** $K = 10 \times 6.4 \times 10^6 \times 500/4 =$ **8×10^9 J**. (1)
(ii) $U = -2K =$ **-1.6×10^{10} J**. (1)
(iii) $E = K + U = 8 \times 10^9 - 16 \times 10^9 =$ **-8×10^9 J** $(= -K)$. (1)

15. **(i)** $T^2 \propto r^3 \rightarrow T_B/T_A = (4r/r)^{3/2} = 4^{3/2} =$ **8**. (1) **(3)**
(ii) $v = \sqrt{(GM/r)} \propto 1/\sqrt{r} \rightarrow v_A/v_B = \sqrt{(4r/r)} =$ **2**. (1)
(iii) $a = GM/r^2 \propto 1/r^2 \rightarrow a_A/a_B = (4r/r)^2 =$ **16**. (1)

SECTION D

16. **(5)**

(a) Escape speed is the minimum speed with which a body must be projected from the surface of a planet so that it just escapes to infinity (never returns). (1)

(b) For the body to just reach infinity, total energy = 0: $\frac{1}{2}mv_e^2 - GMm/R = 0 \rightarrow v_e = \sqrt{(2GM/R)} = \sqrt{(2gR)}$ using $GM = gR^2$. (1)

(c) $v_e = \sqrt{(2 \times 10 \times 5 \times 10^6)} = \sqrt{(10^8)} = 10^4$ m s^{-1} = **10 km s^{-1}**. (1)

(d) Orbital speed close to the surface $v_o = \sqrt{(gR)}$. So $v_e/v_o = \sqrt{(2gR)}/\sqrt{(gR)} = \sqrt{2}$. (1)

(e) Energy conservation: $\frac{1}{2}mv^2 - GMm/R = \frac{1}{2}mv_\infty^2 \rightarrow v_\infty^2 = v^2 - v_e^2 = 400 - 100 = 300$ (km s^{-1})$^2 \rightarrow v_\infty = 10\sqrt{3} \approx$ **17.3 km s^{-1}**. (1)

OR

(a) $v = \sqrt{(GM/r)} = R\sqrt{(g/r)}$ (since $GM = gR^2$). (1)

(b) Distance from the centre $r = 2R$: $g' = g/4 =$ **2.5 m s^{-2}**. (1)

(c) $v^2 = gR^2/r = 10 \times (5 \times 10^6)^2/(10^7) = 2.5 \times 10^7 \rightarrow v =$ **5000 m s^{-1}** (5 km s^{-1}). (1)

(d) $T = 2\pi r/v = 2 \times 3.14 \times 10^7/5000 = 4\pi \times 10^3 \approx$ **1.26 × 10^4 s** ($\approx$ 3.5 hours). (1)

(e) The astronaut and the satellite are both in free fall with the same acceleration towards the planet, so there is no normal reaction between them; the apparent weight is zero though gravity still acts. (1)

17. **(5)**

(a) Inside a uniform sphere, $g(r) = gr/R$ (it increases linearly with r). (1)

(b) At $r = R/2$: $g = 10 \times \frac{1}{2} =$ **5 m s^{-2}**. (1)

(c) Outside, $g(r) = g(R/r)^2$. At $r = 2R$: $g = 10/4 =$ **2.5 m s^{-2}**. (1)

(d) Inside: $10r/R = 5 \rightarrow r =$ ***R*/2**. (1) Outside: $10R^2/r^2 = 5 \rightarrow r^2 = 2R^2 \rightarrow r =$ **$\sqrt{2}$ *R*** ($\approx 1.41R$). (1)

OR

(a) To determine the universal gravitational constant G (and hence the mass of the Earth); the experiment was done by Henry Cavendish in 1798. (1)

(b) $F = GMm/d^2 = 6.67 \times 10^{-11} \times 10 \times 0.1/(0.1)^2 =$ **6.67 × 10^{-9} N**. (1)

(c) The two forces form a couple: $\tau = FL = 6.67 \times 10^{-9} \times 0.2 =$ **1.33 × 10^{-9} N m**. (1)

(d) At equilibrium $\tau = k\theta \rightarrow \theta = 1.33 \times 10^{-9}/(4 \times 10^{-7}) =$ **3.3 × 10^{-3} rad**. (1)

(e) $GMmL/d^2 = k\theta \rightarrow$ ***G* = *kθd*^{2}/(*MmL*)**. (1)

— END OF SOLUTIONS —