PRACTICE PAPER 01 (2026-27) – ANSWER KEY WITH SOLUTIONS
CHAPTER 02 : INVERSE TRIGONOMETRIC FUNCTIONS

SUBJECT: MATHEMATICS	**MAX. MARKS:** 40
CLASS: XII	**DURATION:** 1½ hrs

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Principal value of $\sec^{-1}(-\sqrt{2})$

Ans: (c) $3\pi/4$

Range of $\sec^{-1}$ is $[0, \pi] - \{\pi/2\}$. Let $\sec^{-1}(-\sqrt{2}) = y$, so $\sec y = -\sqrt{2}$, $\cos y = -1/\sqrt{2}$. In $[0, \pi]$, $y = \pi - \pi/4 = 3\pi/4$.

2. Domain of $f(x) = \sin^{-1}(2x - 1)$

Ans: (a) [0, 1]

Need $-1 \le 2x - 1 \le 1 \Rightarrow 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$.

3. $\cos^{-1}(\cos 5\pi/3)$

Ans: (b) $\pi/3$

$5\pi/3 \notin [0, \pi]$. $\cos(5\pi/3) = \cos(2\pi - \pi/3) = \cos(\pi/3)$, and $\pi/3 \in [0, \pi]$. So the value is $\pi/3$.

4. Graph of the principal value branch

Ans: (d) $\sec^{-1}x$

The graph has domain $\mathbb{R} - (-1, 1)$ (two branches, starting at $(1, 0)$ and $(-1, \pi)$) and range $[0, \pi] - \{\pi/2\}$, with the line $y = \pi/2$ approached but never reached. This is the graph of $\sec^{-1}x$.

5. $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$

Ans: (c) π

$\tan^{-1}1 = \pi/4$. Since $(1)(2) = 2 > 1$: $\tan^{-1}1 + \tan^{-1}2 = \pi + \tan^{-1}((1 + 2)/(1 - 2)) = \pi + \tan^{-1}(-3) = \pi - \tan^{-1}3$.

So the sum $= \pi - \tan^{-1}3 + \tan^{-1}3 = \pi$.

6. If $\cot^{-1}x = 2\pi/3$, find x

Ans: (d) $-1/\sqrt{3}$

$x = \cot(2\pi/3) = -\cot(\pi/3) = -1/\sqrt{3}$ ($2\pi/3 \in (0, \pi)$, the range of $\cot^{-1}$).

7. If $\sin^{-1}x + \sin^{-1}y = 2\pi/3$, find $\cos^{-1}x + \cos^{-1}y$

Ans: (a) $\pi/3$

Using $\cos^{-1}t = \pi/2 - \sin^{-1}t$ for $t \in [-1, 1]$: $\cos^{-1}x + \cos^{-1}y = \pi - (\sin^{-1}x + \sin^{-1}y) = \pi - 2\pi/3 = \pi/3$.

8. $\sin^{-1}(\sin 4)$

Ans: (b) $\pi - 4$

4 radians lies in $(\pi/2, 3\pi/2)$, outside $[-\pi/2, \pi/2]$. $\sin 4 = \sin(\pi - 4)$, and $\pi - 4 \approx -0.858 \in [-\pi/2, \pi/2]$. So the value is $\pi - 4$.

9. Assertion–Reason

Ans: (a) Both A and R are true and R is the correct explanation of A

$\sin(7\pi/6) = -1/2$, and $7\pi/6 \notin [-\pi/2, \pi/2]$, so $\sin^{-1}(\sin 7\pi/6) = \sin^{-1}(-1/2) = -\pi/6$. A is true. R is true: $\sin^{-1}(\sin x) = x$ holds only on the principal interval $[-\pi/2, \pi/2]$. It explains why A's value is $-\pi/6$ and not $7\pi/6$.

10. Assertion–Reason

Ans: (b) Both A and R are true but R is not the correct explanation of A

A: for all real x, $\tan^{-1}x$ and $\cot^{-1}x$ are complementary, so $\tan^{-1}x + \cot^{-1}x = \pi/2$. A is true.
R correctly states the ranges of the two principal branches, so R is true. However, the identity comes from the complementary relation ($\cot\theta = \tan(\pi/2 - \theta)$), and the ranges alone do not explain it, so R is not the correct explanation.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. $\sin^{-1}(\sin 2\pi/3) + \cos^{-1}(\cos 4\pi/3)$

$2\pi/3 \notin [-\pi/2, \pi/2]$; $\sin(2\pi/3) = \sin(\pi - 2\pi/3) = \sin(\pi/3)$ with $\pi/3 \in [-\pi/2, \pi/2]$, so $\sin^{-1}(\sin 2\pi/3) = \pi/3$. (1)
$4\pi/3 \notin [0, \pi]$; $\cos(4\pi/3) = \cos(2\pi - 4\pi/3) = \cos(2\pi/3)$ with $2\pi/3 \in [0, \pi]$, so $\cos^{-1}(\cos 4\pi/3) = 2\pi/3$. (½)
Sum = $\pi/3 + 2\pi/3 = \mathbf{\pi}$. (½)

12. Prove $\tan^{-1}(1/5) + \tan^{-1}(2/3) = \pi/4$

Here $xy = (1/5)(2/3) = 2/15 < 1$, so $\tan^{-1}x + \tan^{-1}y = \tan^{-1}((x + y)/(1 - xy))$. (1)
LHS = $\tan^{-1}((1/5 + 2/3)/(1 - 2/15)) = \tan^{-1}((13/15)/(13/15)) = \tan^{-1}1 = \pi/4$ = RHS. (1)

13. Simplify $\tan^{-1}(\sqrt{(1 + x^2)} - x)$

Put $x = \tan\theta$, so $\theta = \tan^{-1}x \in (-\pi/2, \pi/2)$. (½)
$\sqrt{(1 + x^2)} - x = \sec\theta - \tan\theta = (1 - \sin\theta)/\cos\theta = (\cos(\theta/2) - \sin(\theta/2))^2 / [(\cos(\theta/2) - \sin(\theta/2))(\cos(\theta/2) + \sin(\theta/2))] = (1 - \tan(\theta/2))/(1 + \tan(\theta/2)) = \tan(\pi/4 - \theta/2)$. (1)
Since $\pi/4 - \theta/2 \in (0, \pi/2)$, the expression = $\tan^{-1}[\tan(\pi/4 - \theta/2)] = \mathbf{\pi/4 - ½\ \tan^{-1}x}$. (½)

14. Find $\sin(2\tan^{-1}(1/3))$

Let $\theta = \tan^{-1}(1/3)$, so $\tan\theta = 1/3$. (1)
$\sin 2\theta = 2\tan\theta/(1 + \tan^2\theta) = (2/3)/(1 + 1/9) = (2/3)(9/10) = \mathbf{3/5}$. (1)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove $\sin^{-1}(12/13) - \sin^{-1}(3/5) = \sin^{-1}(33/65)$

Let $A = \sin^{-1}(12/13)$ and $B = \sin^{-1}(3/5)$. Both lie in $(0, \pi/2)$, so $\cos A = 5/13$ and $\cos B = 4/5$. (1)
$\sin(A - B) = \sin A\cos B - \cos A\sin B = (12/13)(4/5) - (5/13)(3/5) = 48/65 - 15/65 = 33/65$. (1)
Since $\sin A > \sin B$, $A > B$, so $A - B \in (0, \pi/2) \subset [-\pi/2, \pi/2]$. Hence $A - B = \sin^{-1}(33/65)$. (1)

16. Solve $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

If $(2x)(3x) = 6x^2 < 1$, then $\tan^{-1}(2x) + \tan^{-1}(3x) = \tan^{-1}(5x/(1 - 6x^2))$. (1)
So $5x/(1 - 6x^2) = \tan(\pi/4) = 1 \Rightarrow 6x^2 + 5x - 1 = 0 \Rightarrow (6x - 1)(x + 1) = 0 \Rightarrow x = 1/6$ or $x = -1$. (1)
For $x = -1$, $6x^2 = 6 > 1$, so the formula is not applicable; directly, LHS = $\tan^{-1}(-2) + \tan^{-1}(-3) = -3\pi/4 \neq \pi/4$, so $x = -1$ is rejected. For $x = 1/6$, $6x^2 = 1/6 < 1$, so it is valid. Hence **x = 1/6**. (1)

17. Domain of $f(x) = \sin^{-1}(2x + 1) + \cos^{-1}(3x + 2)$; value of $f(-1/2)$

$\sin^{-1}(2x + 1)$ is defined when $-1 \le 2x + 1 \le 1 \Rightarrow -1 \le x \le 0$. (1)
$\cos^{-1}(3x + 2)$ is defined when $-1 \le 3x + 2 \le 1 \Rightarrow -1 \le x \le -1/3$. (½)
Common values: **Domain = [−1, −1/3]**. (½)
$x = -1/2$ lies in the domain. $f(-1/2) = \sin^{-1}(0) + \cos^{-1}(1/2) = 0 + \pi/3 = \mathbf{\pi/3}$. (1)

SECTION – D

Question 18 carries 5 marks.

18. $\tan^{-1}((x-2)/(x-3)) + \tan^{-1}((x+2)/(x+3)) = \pi/4$

Let $a = (x-2)/(x-3)$ and $b = (x+2)/(x+3)$, with $x \neq \pm 3$. If $ab < 1$, then $\tan^{-1}a + \tan^{-1}b = \tan^{-1}((a+b)/(1-ab))$. (1)

$a + b = [(x-2)(x+3) + (x+2)(x-3)]/(x^2-9) = (2x^2-12)/(x^2-9)$. (1)

$1 - ab = 1 - (x^2-4)/(x^2-9) = [(x^2-9) - (x^2-4)]/(x^2-9) = -5/(x^2-9)$. (1)

So $(a+b)/(1-ab) = (2x^2-12)/(-5) = (12-2x^2)/5$.

Equation: $(12-2x^2)/5 = \tan(\pi/4) = 1 \Rightarrow 12 - 2x^2 = 5 \Rightarrow x^2 = 7/2 \Rightarrow x = \pm\sqrt{(7/2)} = \pm\sqrt{14}/2$. (1)

Check: for $x^2 = 7/2$, $ab = (x^2-4)/(x^2-9) = (-1/2)/(-11/2) = 1/11 < 1$, so the formula used is valid for both values. Hence **$x = \pm\sqrt{14}/2$**. (1)

OR

Step 1: $(1/2)(1/5) = 1/10 < 1$, so $\tan^{-1}(1/2) + \tan^{-1}(1/5) = \tan^{-1}((1/2 + 1/5)/(1 - 1/10)) = \tan^{-1}((7/10)/(9/10)) = \tan^{-1}(7/9)$. (2)

Step 2: $(7/9)(1/8) = 7/72 < 1$, so $\tan^{-1}(7/9) + \tan^{-1}(1/8) = \tan^{-1}((7/9 + 1/8)/(1 - 7/72)) = \tan^{-1}((65/72)/(65/72)) = \tan^{-1}1$. (2)

Hence LHS $= \tan^{-1}1 = \pi/4 =$ RHS. (1)

SECTION – E (Case Study Based Questions)

Questions 19 and 20 carry 4 marks each.

19. Art gallery case study

(i) $\tan\alpha = 1/x$ and $\tan\beta = 3/x$, so $\alpha = \tan^{-1}(1/x)$ and $\beta = \tan^{-1}(3/x)$. Hence $\theta = \beta - \alpha =$ **$\tan^{-1}(3/x) - \tan^{-1}(1/x)$**. (1)

(ii) $\tan\theta = (\tan\beta - \tan\alpha)/(1 + \tan\beta\tan\alpha) = (3/x - 1/x)/(1 + 3/x^2) = (2/x)\cdot x^2/(x^2+3) = 2x/(x^2+3)$. Since $0 < \alpha < \beta < \pi/2$, $\theta \in (0, \pi/2)$, so $\theta = \tan^{-1}(2x/(x^2+3))$. (1)

(iii) For $x = \sqrt{3}$: $\tan\theta = 2\sqrt{3}/(3+3) = 1/\sqrt{3}$, so $\theta = \tan^{-1}(1/\sqrt{3}) =$ **$\pi/6$**. (2)

OR

(iii) For $x = 1$: $\tan\theta = 2(1)/(1+3) = 1/2$. For $x = 3$: $\tan\theta = 2(3)/(9+3) = 6/12 = 1/2$. (1)

Both give $\theta = \tan^{-1}(1/2)$, so the two viewers see the painting at the same angle. (1)

20. Radar case study

(i) $\tan\theta = 5/12$, so the slant distance is $\sqrt{(12^2+5^2)} = 13$ km and $\sin\theta = 5/13$. As $\theta \in (0, \pi/2)$, $\theta =$ **$\sin^{-1}(5/13)$**. (1)

(ii) $\tan\varphi = 3/4$, so the slant distance is 5 km and $\cos\varphi = 4/5$. As $\varphi \in (0, \pi/2)$, $\varphi =$ **$\cos^{-1}(4/5)$**. (1)

(iii) $\tan\theta = 5/12$, $\tan\varphi = 3/4$, $\tan\theta\cdot\tan\varphi = 5/16 < 1$, so $\theta + \varphi < \pi/2$. (1)

$\tan(\theta+\varphi) = (5/12 + 3/4)/(1 - 5/16) = (7/6)/(11/16) = 56/33$. Hence $\theta + \varphi = \tan^{-1}(56/33)$. (1)

OR

(iii) Since $\tan\varphi > \tan\theta$, $\varphi > \theta$, so $\varphi - \theta \in (0, \pi/2)$. (1)

$\tan(\varphi - \theta) = (3/4 - 5/12)/(1 + 5/16) = (1/3)/(21/16) = 16/63$. Hence $\varphi - \theta = \tan^{-1}(16/63)$. (1)

— End of Solutions —