PRACTICE PAPER 01 (2026-27) – ANSWER KEY WITH SOLUTIONS
CHAPTER 05 : CONTINUITY AND DIFFERENTIABILITY

SUBJECT: MATHEMATICS	**MAX. MARKS:** 40
CLASS: XII	**DURATION:** 1½ hrs

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Continuity at $x = 2$

Ans: (a) 3/4

LHL $= k(2)^2 = 4k$, RHL $= 3$, $f(2) = 4k$. Continuity needs $4k = 3$, so $k = 3/4$.

2. $d/dx \sin(\log x)$

Ans: (c) cos(log x)/x

By the chain rule, $d/dx \sin(\log x) = \cos(\log x) \cdot (1/x)$.

3. $f(x) = |x - 3|$ at $x = 3$

Ans: (b) continuous, not differentiable

f is continuous everywhere, being a modulus function. At $x = 3$, LHD $= -1$ and RHD $= +1$, which are not equal, so f is not differentiable at $x = 3$.

4. $y = 5 \cos x - 3 \sin x$

Ans: (d) 0

$dy/dx = -5 \sin x - 3 \cos x$, $d^2y/dx^2 = -5 \cos x + 3 \sin x = -y$. So $d^2y/dx^2 + y = 0$.

5. Continuity at $x = 3$

Ans: (b) 6

For $x \neq 3$, $f(x) = (x - 3)(x + 3)/(x - 3) = x + 3$, so $\lim f(x)$ as $x \to 3$ equals 6. For continuity $f(3) = 6$.

6. $d/dx (5^x + x^5)$

Ans: (c) $5^x \log 5 + 5x^4$

$d/dx(a^x) = a^x \log a$ and $d/dx(x^5) = 5x^4$.

7. $x = t^2$, $y = t^3$

Ans: (a) 3

$dy/dt = 3t^2$, $dx/dt = 2t$, so $dy/dx = 3t/2$. At $t = 2$, $dy/dx = 3$.

8. Rolle's theorem

Ans: (d) 2

$f'(x) = 2x - 4 = 0 \Rightarrow x = 2$, which lies in (1, 3). (Also $f(1) = f(3) = 0$.) So $c = 2$.

9. Assertion–Reason

Ans: (d) A is false but R is true

A is false: continuity does not imply differentiability. R gives the counter-example $|x|$ at $x = 0$, where LHD $= -1$ and RHD $= 1$, so R is true.

10. Assertion–Reason

Ans: (a) Both A and R are true and R is the correct explanation of A

For $x > 0$, $d/dx(\log x) = 1/x$. For $x < 0$, $d/dx[\log(-x)] = (1/(-x))(-1) = 1/x$. R gives this second case, so R explains A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. d/dx [log(sin e^x)]

dy/dx = (1/sin e^x) · cos e^x · e^x. (1)

= **e^x cot(e^x)**. (1)

12. y = $x^{\cos x}$

Taking logs: log y = cos x · log x. Differentiating: (1/y) dy/dx = −sin x · log x + cos x / x. (1)

dy/dx = **$x^{\cos x}$ (cos x / x − sin x · log x)**. (1)

13. $x^2y + xy^2 = 12$

Differentiating with respect to x: $2xy + x^2$ (dy/dx) + $y^2 + 2xy$ (dy/dx) = 0. (1)

($x^2 + 2xy$) dy/dx = −($2xy + y^2$), so **dy/dx = −($2xy + y^2$)/($x^2 + 2xy$)**. (1)

14. Continuity at x = π

LHL = lim (kx + 1) = kπ + 1, f(π) = kπ + 1. RHL = lim cos x = cos π = −1. (1)

For continuity, kπ + 1 = −1 ⇒ **k = −2/π**. (1)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Continuous but not differentiable at x = 1

Continuity: LHL = lim x^2 = 1; RHL = lim (3x − 2) = 1; f(1) = 1. All three are equal, so f is continuous at x = 1. (1½)

Differentiability: LHD = lim [f(x) − f(1)]/(x − 1) for x → 1^- = lim ($x^2 − 1$)/(x − 1) = lim (x + 1) = 2. (½)

RHD = lim [(3x − 2) − 1]/(x − 1) for x → 1^+ = lim 3(x − 1)/(x − 1) = 3. (½)

LHD ≠ RHD, so f is not differentiable at x = 1. (½)

16. y = $2e^{3x} + 5e^{-x}$

dy/dx = $6e^{3x} − 5e^{-x}$. (1)

$d^2y/dx^2 = 18e^{3x} + 5e^{-x}$. (1)

LHS = ($18e^{3x} + 5e^{-x}$) − 2($6e^{3x} − 5e^{-x}$) − 3($2e^{3x} + 5e^{-x}$) = (18 − 12 − 6)e^{3x} + (5 + 10 − 15)e^{-x} = 0. Hence proved. (1)

17. x = a(θ − sin θ), y = a(1 − cos θ)

dx/dθ = a(1 − cos θ), dy/dθ = a sin θ. (1)

dy/dx = sin θ/(1 − cos θ) = 2 sin(θ/2) cos(θ/2) / [2 $\sin^2$(θ/2)] = **cot(θ/2)**. (1)

At θ = 2π/3: dy/dx = cot(π/3) = **1/√3**. (1)

SECTION – D

Question 18 carries 5 marks.

18. y = $(\tan x)^x + x^{\tan x}$

Let u = $(\tan x)^x$ and v = $x^{\tan x}$, so y = u + v and dy/dx = du/dx + dv/dx. (1)

For u: log u = x log tan x. Differentiating: (1/u) du/dx = log tan x + x · $\sec^2x$ / tan x. (1)

du/dx = $(\tan x)^x$ [log tan x + x $\sec^2x$ / tan x]. (1)

For v: log v = tan x · log x. Differentiating: (1/v) dv/dx = $\sec^2x$ · log x + tan x / x. (1)

dv/dx = $x^{\tan x}$ [$\sec^2x$ · log x + tan x / x]. Therefore

dy/dx = $(\tan x)^x$ [log tan x + x $\sec^2x$ / tan x] + $x^{\tan x}$ [$\sec^2x$ log x + tan x / x]. (1)

OR

y = $(\sin^{-1} x)^2$. Differentiating: dy/dx = 2 $\sin^{-1}$ x · 1/√($1 − x^2$). (1)

So √($1 − x^2$) · dy/dx = 2 $\sin^{-1}$ x. (1)

Differentiating again: $\sqrt{(1-x^2)} \cdot d^2y/dx^2 + (-x/\sqrt{(1-x^2)}) \cdot dy/dx = 2/\sqrt{(1-x^2)}$. (2)

Multiplying by $\sqrt{(1-x^2)}$: **$(1-x^2)\, d^2y/dx^2 - x\, dy/dx = 2$**. (1)

SECTION – E (Case Study Based Questions)

Questions 19 and 20 carry 4 marks each.

19. Cab fare function

(i) LHL at $x = 3 = \lim (20x + 10) = 20(3) + 10 =$ **70**. (1)

(ii) $f(3) = 70$ and RHL $= 15(3) + k = 45 + k$. Continuity needs $45 + k = 70$, so **$k = 25$**. (1)

(iii) For $x < 3$, rate of change = 20 ₹/km. For $x > 3$, rate of change = 15 ₹/km. (1)

LHD = 20 and RHD = 15 are not equal, so f is continuous but **not differentiable** at $x = 3$. (1)

OR

(iii) Since $10 > 3$, $f(10) = 15(10) + 25 =$ **₹175**. (1)

For $x > 3$, $f'(x) = 15$, so the rate of change at $x = 10$ is **₹15 per km**. (1)

20. Medicine concentration $C(t) = 20t\, e^{-t/2}$

(i) $dC/dt = 20[e^{-t/2} + t \cdot (-½)e^{-t/2}] =$ **$20e^{-t/2}(1 - t/2)$** $= 10(2 - t)e^{-t/2}$. (1)

(ii) $dC/dt = 0 \Rightarrow 2 - t = 0$ (as $e^{-t/2} \neq 0$) $\Rightarrow$ **$t = 2$ hours**. (1)

(iii) $d^2C/dt^2 = 10[-e^{-t/2} + (2 - t)(-½)e^{-t/2}] = 10e^{-t/2}(-1 - 1 + t/2) = 5(t - 4)e^{-t/2}$. (1)

At $t = 2$: $d^2C/dt^2 = 5(-2)e^{-1} = -10/e < 0$, so the concentration is maximum at $t = 2$ hours ($C = 40/e$ mg/L). (1)

OR

(iii) $C = 20t\, e^{-t/2}$, $C' = 10(2 - t)e^{-t/2}$, $C'' = 5(t - 4)e^{-t/2}$. (1)

$4C'' + 4C' + C = e^{-t/2}[20(t - 4) + 40(2 - t) + 20t] = e^{-t/2}[20t - 80 + 80 - 40t + 20t] = 0$.

Hence proved. (1)

— End of Solutions —