PRACTICE PAPER 01 (2026-27) – ANSWER KEY WITH SOLUTIONS
CHAPTER 06 : APPLICATION OF DERIVATIVES

SUBJECT: MATHEMATICS	**MAX. MARKS:** 40
CLASS: XII	**DURATION:** 1½ hrs

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Area of circle, dr/dt = 2

Ans: (b) 20π cm²/s

$A = \pi r^2$, so $dA/dt = 2\pi r \cdot dr/dt = 2\pi(5)(2) = 20\pi$ cm²/s.

2. $f(x) = x^3 - 3x^2$

Ans: (d) (0, 2)

$f'(x) = 3x^2 - 6x = 3x(x - 2)$. $f'(x) < 0$ when $0 < x < 2$, so f is strictly decreasing in (0, 2).

3. Normal to $y = x^2 - 4x$ at (3, −3)

Ans: (c) −1/2

$dy/dx = 2x - 4 = 2$ at $x = 3$. Slope of the normal = −1/(slope of tangent) = −1/2.

4. √36.1

Ans: (a) 6.0083

Let $y = \sqrt{x}$, $x = 36$, $\Delta x = 0.1$. $dy = \Delta x/(2\sqrt{x}) = 0.1/12 = 0.00833$. So $\sqrt{36.1} \approx 6 + 0.0083 = 6.0083$.

5. $f(x) = x^2 - 6x + 11$

Ans: (c) 2

$f'(x) = 2x - 6 = 0 \Rightarrow x = 3$, $f''(x) = 2 > 0$. Minimum value = $f(3) = 9 - 18 + 11 = 2$.

6. Strictly increasing function

Ans: (d) $x^3 + x$

For $f(x) = x^3 + x$, $f'(x) = 3x^2 + 1 > 0$ for all x. The other options are not increasing throughout $\mathbb{R}$ (x^2 and sin x change direction, e^{-x} is decreasing).

7. $f(x) = x + 4/x$

Ans: (a) 2

$f'(x) = 1 - 4/x^2 = 0 \Rightarrow x = 2$ ($x > 0$). $f''(x) = 8/x^3 > 0$ at $x = 2$, so $x = 2$ is a point of local minimum.

8. Volume of cube

Ans: (c) 144 cm³/s

$V = x^3$, $dV/dt = 3x^2 \cdot dx/dt = 3(16)(3) = 144$ cm³/s.

9. Assertion–Reason

Ans: (c) A is true but R is false

$f'(x) = 3x^2 = 0$ only at $x = 0$, but f' does not change sign there, so x^3 has no extremum. A is true.

R is false: $f'(x) = 0$ does not guarantee an extremum (e.g. x^3 at $x = 0$).

10. Assertion–Reason

Ans: (a) Both A and R are true and R is the correct explanation of A

$f'(x) = -\sin x < 0$ on $(0, \pi)$, so cos x is strictly decreasing there. R explains A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. $f(x) = x^2 - 4x + 6$

$f'(x) = 2x - 4 = 2(x - 2)$. (1)

$f'(x) > 0$ for $x > 2$ and $f'(x) < 0$ for $x < 2$. So f is **strictly increasing in (2, ∞)** and **strictly decreasing in (−∞, 2)**. (1)

12. Approximate value of $(65)^{1/3}$

Let $y = x^{1/3}$, $x = 64$, $\Delta x = 1$. $dy = (1/3)x^{-2/3}\,\Delta x = (1/3)(1/16)(1) = 1/48 \approx 0.0208$. (1)

$(65)^{1/3} \approx 4 + 0.0208 =$ **4.0208**. (1)

13. Spherical balloon

$S = 4\pi r^2$, $dS/dt = 8\pi r \cdot dr/dt$. (1)

At $r = 6$, $dr/dt = 0.5$: $dS/dt = 8\pi(6)(0.5) =$ **24π cm²/s**. (1)

14. $y = x^2 - 2x + 3$, tangent parallel to $4x - y + 5 = 0$

Slope of the line = 4. $dy/dx = 2x - 2 = 4 \Rightarrow x = 3$. (1)

$y = 9 - 6 + 3 = 6$. The required point is **(3, 6)**. (1)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. $f(x) = x^3 - 6x^2 + 9x + 5$

$f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3) = 0 \Rightarrow x = 1, 3$. (1)

$f''(x) = 6x - 12$. At $x = 1$, $f'' = -6 < 0$, so $x = 1$ is a point of local maximum; **local maximum value = f(1) = 1 − 6 + 9 + 5 = 9**. (1)

At $x = 3$, $f'' = 6 > 0$, so $x = 3$ is a point of local minimum; **local minimum value = f(3) = 27 − 54 + 27 + 5 = 5**. (1)

16. $f(x) = x^3 - 3x^2 + 6x - 5$ is strictly increasing

$f'(x) = 3x^2 - 6x + 6$. (1)

$= 3(x^2 - 2x + 2) = 3[(x - 1)^2 + 1]$. (1)

Since $(x - 1)^2 \geq 0$, $f'(x) \geq 3 > 0$ for all $x \in \mathbb{R}$. Hence f is strictly increasing on $\mathbb{R}$. (1)

17. Tangent and normal to $y = x^3 - 2x + 1$ at (2, 5)

$dy/dx = 3x^2 - 2$. Slope of the tangent at (2, 5) is $m = 3(4) - 2 = 10$. (1)

Tangent: $y - 5 = 10(x - 2) \Rightarrow$ **10x − y − 15 = 0**. (1)

Slope of the normal = −1/10. Normal: $y - 5 = -(1/10)(x - 2) \Rightarrow$ **x + 10y − 52 = 0**. (1)

SECTION – D

Question 18 carries 5 marks.

18. Open tank, volume 32 m³

Let the side of the square base be x m and the height h m. Volume: $x^2h = 32$, so $h = 32/x^2$. (1)

Area of sheet metal $S = x^2 + 4xh = x^2 + 4x(32/x^2) = x^2 + 128/x$. (1)

$dS/dx = 2x - 128/x^2 = 0 \Rightarrow x^3 = 64 \Rightarrow x = 4$. (1)

$d^2S/dx^2 = 2 + 256/x^3$. At $x = 4$: $2 + 4 = 6 > 0$, so S is minimum at $x = 4$. (1)

Then $h = 32/16 = 2$. **Dimensions: base 4 m × 4 m, height 2 m. Minimum area = 16 + 32 = 48 m².** (1)

OR

$f'(x) = 6x^2 - 30x + 36 = 6(x - 2)(x - 3) = 0 \Rightarrow x = 2, 3$, both in [1, 5]. (1)

$f(1) = 2 - 15 + 36 + 1 = 24$. (1)

$f(2) = 16 - 60 + 72 + 1 = 29$. (1)

$f(3) = 54 - 135 + 108 + 1 = 28$; $f(5) = 250 - 375 + 180 + 1 = 56$. (1)

Absolute maximum value = 56 (at x = 5); absolute minimum value = 24 (at x = 1). (1)

SECTION – E (Case Study Based Questions)

Questions 19 and 20 carry 4 marks each.

19. Profit $P(x) = -x^2 + 40x - 150$

(i) $dP/dx = \mathbf{-2x + 40}$. (1)

(ii) $dP/dx = 0 \Rightarrow x = 20$. The profit is maximum for **20 hundred units** (2000 units). (1)

(iii) $d^2P/dx^2 = -2 < 0$, so $x = 20$ gives a maximum. (1)

Maximum profit = $P(20) = -400 + 800 - 150 =$ **₹250 thousand**. (1)

OR

(iii) $P'(x) = -2(x - 20)$. $P'(x) > 0$ for $x < 20$ and $P'(x) < 0$ for $x > 20$. (1)

So P is **increasing on (0, 20)** and **decreasing on (20, ∞)**. (1)

20. Sliding ladder

(i) By Pythagoras, $\mathbf{x^2 + y^2 = 25}$. (1)

(ii) When $x = 3$: $y^2 = 25 - 9 = 16$, so $\mathbf{y = 4}$ **m**. (1)

(iii) Differentiating: $2x\,dx/dt + 2y\,dy/dt = 0 \Rightarrow dy/dt = -(x/y)\,dx/dt$. (1)

At $x = 3$, $y = 4$, $dx/dt = 0.5$: $dy/dt = -(3/4)(0.5) = -3/8$. The top slides down at **3/8 m/s**. (1)

OR

(iii) Area $A = \frac{1}{2}xy$, so $dA/dt = \frac{1}{2}(x\,dy/dt + y\,dx/dt)$. (1)

$= \frac{1}{2}[3(-3/8) + 4(0.5)] = \frac{1}{2}(-9/8 + 2) = \frac{1}{2}(7/8) =$ **7/16 m²/s** (the area is increasing). (1)

— End of Solutions —