PRACTICE PAPER 01 (2026-27) – ANSWER KEY WITH SOLUTIONS
CHAPTER 07 : INTEGRALS

SUBJECT: MATHEMATICS	**MAX. MARKS:** 40
CLASS: XII	**DURATION:** 1½ hrs

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Integral of $3x^2 + 4\cos x$

Ans: (a) $x^3 + 4\sin x + C$

Integrating term by term: $\int 3x^2 dx = x^3$ and $\int 4\cos x\, dx = 4\sin x$.

2. Integral of $1/(x \log x)$

Ans: (c) $\log|\log x| + C$

Put log x = t, so dx/x = dt. The integral becomes $\int \frac{dt}{t} = \log|t| + C = \log|\log x| + C$.

3. Integral of $e^x(\sin x + \cos x)$

Ans: (b) $e^x \sin x + C$

Using $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$ with f(x) = sin x, f′(x) = cos x.

4. $\int|x - 1|\,dx$ from 0 to 2

Ans: (b) 1

$$\int_0^2 |x-1|dx = \int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx = \tfrac{1}{2} + \tfrac{1}{2} = 1.$$

5. $\int 2x/(1 + x^2)\,dx$ from 0 to 1

Ans: (c) log 2

Put $1 + x^2 = t$, so $2x\,dx = dt$; limits 1 to 2. $\int_1^2 \frac{dt}{t} = \log 2$.

6. $\int dx/(x^2 + 16)$

Ans: (d) $\frac{1}{4}\tan^{-1}\frac{x}{4} + C$

Using $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ with a = 4.

7. $\int dx/(\sin^2 x \cos^2 x)$

Ans: (a) $\tan x - \cot x + C$

Write $1 = \sin^2 x + \cos^2 x$: $\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \sec^2 x + \operatorname{cosec}^2 x$.

So the integral = tan x − cot x + C.

8. Integral with √x and √(3 − x)

Ans: (b) 3/2

Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$: adding the two forms gives $2I = \int_0^3 1\,dx = 3$, so I = 3/2.

9. Assertion–Reason

Ans: (d) A is false but R is true

A is false: the formula fails for n = −1, where the integral is log|x| + C. R is true.

10. Assertion–Reason

Ans: (a) Both A and R are true and R is the correct explanation of A

sin x is an odd function, so by the property of odd functions the integral over [−π/2, π/2] is 0. R explains A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Evaluate $\int \left(x + \frac{1}{x}\right)^2 dx$

$\int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$. (1)

$= \frac{x^3}{3} + 2x - \frac{1}{x} + C$. (1)

12. Evaluate $\int \frac{\cos x}{1 + \sin^2 x}\, dx$

Put sin x = t, so cos x dx = dt. (1)

$\int \frac{dt}{1 + t^2} = \tan^{-1} t + C = \tan^{-1}(\sin x) + C$. (1)

13. Evaluate $\int_1^e \frac{\log x}{x}\, dx$

Put log x = t, so dx/x = dt; when x = 1, t = 0 and when x = e, t = 1. (1)

$\int_0^1 t\, dt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$. (1)

14. Evaluate $\int \frac{dx}{x^2 - 4}$

Using $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left|\frac{x - a}{x + a}\right| + C$ with a = 2: (1)

$\int \frac{dx}{x^2 - 4} = \frac{1}{4} \log \left|\frac{x - 2}{x + 2}\right| + C$. (1)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Evaluate $\int x \sec^2 x\, dx$

Integrate by parts, taking u = x and v = sec²x: (1)

$\int x \sec^2 x\, dx = x \tan x - \int \tan x\, dx$. (1)

$= x \tan x + \log|\cos x| + C$. (1)

16. Evaluate $\int \frac{3x + 5}{(x + 1)(x + 3)}\, dx$

Let $\frac{3x + 5}{(x + 1)(x + 3)} = \frac{A}{x + 1} + \frac{B}{x + 3}$, so 3x + 5 = A(x + 3) + B(x + 1). (1)

Put x = −1: 2 = 2A, so A = 1. Put x = −3: −4 = −2B, so B = 2. (1)

Integral = $\log|x+1| + 2\log|x+3| + C$. (1)

17. Evaluate $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

Put cos x = t, so sin x dx = −dt; when x = 0, t = 1 and when x = π/2, t = 0. (1)

I = $-\int_1^0 \frac{dt}{1+t^2} = \int_0^1 \frac{dt}{1+t^2}$. (1)

= $[\tan^{-1} t]_0^1 = \frac{\pi}{4}$. (1)

SECTION – D

Question 18 carries 5 marks.

18. Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$

Let $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$. (1)

Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$: $I = \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$. (1)

Adding: $2I = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$. (1)

Put cos x = t, so sin x dx = −dt; limits x = 0 → t = 1 and x = π → t = −1. $2I = \pi\int_{-1}^{1} \frac{dt}{1+t^2}$. (1)

$2I = \pi[\tan^{-1} t]_{-1}^{1} = \pi(\frac{\pi}{4} + \frac{\pi}{4}) = \frac{\pi^2}{2}$, so **I = π²/4**. (1)

OR

Let $\frac{1}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$, so 1 = A(x² + 4) + (Bx + C)(x + 1). (1)

Put x = −1: 1 = 5A, so A = 1/5. Comparing coefficients of x²: A + B = 0, so B = −1/5. Constant terms: 4A + C = 1, so C = 1/5. (2)

$\int \frac{dx}{(x+1)(x^2+4)} = \frac{1}{5}\int \frac{dx}{x+1} - \frac{1}{5}\int \frac{x\,dx}{x^2+4} + \frac{1}{5}\int \frac{dx}{x^2+4}$. (1)

= $\frac{1}{5}\log|x+1| - \frac{1}{10}\log(x^2+4) + \frac{1}{10}\tan^{-1}\frac{x}{2} + C$. (1)

SECTION – E (Case Study Based Questions)

Questions 19 and 20 carry 4 marks each.

19. Marginal cost dC/dx = 6x + 40, fixed cost ₹500

(i) $\int (6x+40)\,dx = 3x^2 + 40x + k$, where k is the constant of integration. (1)

(ii) C(x) = 3x² + 40x + k. C(0) = 500 gives k = 500, so **C(x) = 3x² + 40x + 500**. (1)

(iii) $\int_{10}^{20} (6x+40)\,dx = [3x^2 + 40x]_{10}^{20}$. (1)

= (1200 + 800) − (300 + 400) = 2000 − 700 = **₹1300**. (1)

OR

(iii) C(30) = 3(900) + 40(30) + 500 = 2700 + 1200 + 500. (1)

Total cost of 30 units = **₹4400**. (1)

20. Particle with velocity v(t) = 3t² − 12t + 9

(i) $s(t) = \int (3t^2 - 12t + 9)\,dt = t^3 - 6t^2 + 9t + k$; s(0) = 0 gives k = 0, so **s(t) = t³ − 6t² + 9t**. (1)

(ii) Displacement = s(3) − s(0) = 27 − 54 + 27 = **0 m**. (1)

(iii) v(t) = 3(t − 1)(t − 3). So v ≥ 0 on [0, 1] and v ≤ 0 on [1, 3]. (1)

Distance = $\int_0^1 v\,dt - \int_1^3 v\,dt$ = [s(1) − s(0)] − [s(3) − s(1)] = 4 − (0 − 4) = **8 m**. (1)

OR

(iii) Displacement = s(5) − s(0). (1)

s(5) = 125 − 150 + 45 = 20, so displacement = **20 m**. (1)

— End of Solutions —