PRACTICE PAPER 01 (2026-27) – ANSWER KEY WITH SOLUTIONS
CHAPTER 08 : APPLICATION OF INTEGRALS

SUBJECT: MATHEMATICS	**MAX. MARKS:** 40
CLASS: XII	**DURATION:** 1½ hrs

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. $y = x^2$, x-axis, x = 1 to 3

Ans: (a) 26/3 sq. units

Area = $\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{27-1}{3} = \frac{26}{3}$.

2. Area under $y = \sin x$, 0 to π/2

Ans: (b) 1 sq. unit

Area = $\int_0^{\pi/2} \sin x\,dx = [-\cos x]_0^{\pi/2} = 0 + 1 = 1$.

3. Circle $x^2 + y^2 = 16$

Ans: (c) 16π sq. units

Radius = 4, so area = $\pi(4)^2 = 16\pi$. (By integration: $4 \times \int_0^4 \sqrt{16 - x^2}\,dx = 4 \times 4\pi$.)

4. $y^2 = 4x$ and x = 1

Ans: (b) 8/3 sq. units

Area = $2 \times \int_0^1 2\sqrt{x}\,dx = 4\left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{8}{3}$.

5. Between $y = x$ and $y = x^2$

Ans: (c) 1/6 sq. unit

The curves meet at x = 0 and x = 1. Area = $\int_0^1 (x - x^2)\,dx = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

6. $y = \cos x$, 0 to π

Ans: (c) 2 sq. units

$\cos x \geq 0$ on $[0, \pi/2]$ and ≤ 0 on $[\pi/2, \pi]$. Area =

$$\int_0^{\pi/2} \cos x\,dx + \left|\int_{\pi/2}^{\pi} \cos x\,dx\right| = 1 + 1 = 2.$$

7. $y = |x|$ and $y = 2$

Ans: (b) 4 sq. units

The lines meet at $x = \pm 2$. Area = $\int_{-2}^{2} (2 - |x|)\,dx = 2\int_0^2 (2 - x)\,dx = 2 \times 2 = 4$.

8. $y = x^2 - 9$ and the x-axis

Ans: (d) 36 sq. units

The curve meets the x-axis at $x = \pm 3$ and lies below it in between. Area =

$$\int_{-3}^{3}(9 - x^2)\,dx = 2\left[9x - \tfrac{x^3}{3}\right]_0^3 = 2(27 - 9) = 36 \text{ sq. units.}$$

9. Assertion–Reason

Ans: (d) A is false but R is true

R is true since x^3 is odd. But the curve lies below the x-axis for $x < 0$, so the area is

$2\int_0^1 x^3\,dx = \tfrac{1}{2}$, not 0. A is false.

10. Assertion–Reason

Ans: (a) Both A and R are true and R is the correct explanation of A

Area = $4 \cdot \tfrac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx = 4 \cdot \tfrac{b}{a} \cdot \tfrac{\pi a^2}{4} = \pi ab$. R gives the area, so it explains A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. $y = 3x + 2$, x-axis, $x = 1$ and $x = 3$

The line lies above the x-axis for $1 \le x \le 3$. Area = $\int_1^3 (3x + 2)\,dx$. (1)

$= \left[\tfrac{3x^2}{2} + 2x\right]_1^3 = \left(\tfrac{27}{2} + 6\right) - \left(\tfrac{3}{2} + 2\right) = 16$ sq. units. (1)

12. $y = 1/x$, $x = 1$ to $x = e$

Area = $\int_1^e \tfrac{dx}{x}$. (1)

$= [\log x]_1^e = 1 - 0 = 1$ sq. unit. (1)

13. Ellipse $\tfrac{x^2}{9} + \tfrac{y^2}{4} = 1$

Here $y = \tfrac{2}{3}\sqrt{9 - x^2}$ in the first quadrant. Area = $4\int_0^3 \tfrac{2}{3}\sqrt{9 - x^2}\,dx$. (1)

$= \tfrac{8}{3}\left[\tfrac{x}{2}\sqrt{9 - x^2} + \tfrac{9}{2}\sin^{-1}\tfrac{x}{3}\right]_0^3 = \tfrac{8}{3} \cdot \tfrac{9}{2} \cdot \tfrac{\pi}{2} = 6\pi$ sq. units. (1)

14. $y = x^2$ and $y = 2x$

$x^2 = 2x \Rightarrow x = 0$ or $x = 2$. On $[0, 2]$ the line is above the parabola. (1)

Area = $\int_0^2 (2x - x^2)\,dx = \left[x^2 - \tfrac{x^3}{3}\right]_0^2 = 4 - \tfrac{8}{3} = \tfrac{4}{3}$ sq. units. (1)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. $y^2 = 4x$ and $y = x$

Solving, $x^2 = 4x \Rightarrow x = 0, 4$, so the points of intersection are (0, 0) and (4, 4). (1)

In the first quadrant the parabola $y = 2\sqrt{x}$ lies above the line. Area = $\int_0^4 (2\sqrt{x} - x)\,dx$. (1)

$= \left[\frac{4}{3}x^{3/2} - \frac{x^2}{2}\right]_0^4 = \frac{32}{3} - 8 = \frac{8}{3}$ sq. units. (1)

16. $y = x^2$ and $y = x + 2$

$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = -1, 2$. The points are $(-1, 1)$ and $(2, 4)$. (1)

The line lies above the parabola on $[-1, 2]$. Area = $\int_{-1}^2 (x + 2 - x^2)\,dx$. (1)

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^2 = \frac{10}{3} - \left(-\frac{7}{6}\right) = \frac{9}{2}$ sq. units. (1)

17. $y = |x + 1|$, $x = -4$ to $x = 2$

$y = -(x + 1)$ for $x < -1$ and $y = x + 1$ for $x \geq -1$. (1)

Area = $\int_{-4}^{-1} -(x+1)\,dx + \int_{-1}^2 (x+1)\,dx$. (1)

$= \frac{9}{2} + \frac{9}{2} = 9$ sq. units. (1)

SECTION – D

Question 18 carries 5 marks.

18. Circle $x^2 + y^2 = 4$ and line $x + y = 2$

Solving: $x^2 + (2 - x)^2 = 4 \Rightarrow 2x^2 - 4x = 0 \Rightarrow x = 0$ or 2. The points of intersection are $(0, 2)$ and $(2, 0)$. (1)

The smaller region lies between the circle (above) and the line $y = 2 - x$ (below) for $0 \leq x \leq 2$. (1)

Area = $\int_0^2 \sqrt{4 - x^2}\,dx - \int_0^2 (2 - x)\,dx$. (1)

$\int_0^2 \sqrt{4 - x^2}\,dx = \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_0^2 = \pi$ and $\int_0^2 (2 - x)\,dx = \left[2x - \frac{x^2}{2}\right]_0^2 = 2$. (2)

Area = (π − 2) sq. units.

OR

Let A(1, 0), B(2, 2), C(3, 1). Equation of AB: $y = 2(x - 1)$. Equation of BC: $y = 4 - x$. Equation of AC: $y = (x - 1)/2$. (2)

Area = $\int_1^2 \left[2(x-1) - \frac{x-1}{2}\right]dx + \int_2^3 \left[(4 - x) - \frac{x-1}{2}\right]dx$. (1)

$= \frac{3}{2}\left[\frac{(x-1)^2}{2}\right]_1^2 + \left[\frac{9x}{2} - \frac{3x^2}{4}\right]_2^3 = \frac{3}{4} + \frac{3}{4}$. (1)

Area = 3/2 sq. units. (1)

SECTION – E (Case Study Based Questions)

Questions 19 and 20 carry 4 marks each.

19. Field bounded by $y = 4x - x^2$ and the x-axis

(i) $y = 0 \Rightarrow x(4 - x) = 0 \Rightarrow x = 0$ or $x = 4$. The curve meets the x-axis at **(0, 0) and (4, 0)**. (1)

(ii) Area = $\int_0^4 (4x - x^2)\,dx$. (1)

(iii) $\int_0^4 (4x - x^2)\,dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$ sq. units. (1)

Area = (32/3) × 300 = **3200 m²**; cost of sowing = 3200 × 5 = **₹16 000**. (1)

OR

(iii) $4x - x^2 = x \Rightarrow x^2 - 3x = 0 \Rightarrow x = 0, 3$. On [0, 3] the curve is above the line y = x. (1)

Area = $\int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{2} - 9 = \frac{9}{2}$ sq. units. (1)

20. Lawn between $y = x^2$ and $y = x + 6$

(i) $x^2 = x + 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0$, so x = 3 or x = −2. The points are **(−2, 4) and (3, 9)**. (1)

(ii) The path is above the curve between the points. Area = $\int_{-2}^{3} (x + 6 - x^2)\,dx$. (1)

(iii) $\left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_{-2}^{3} = \left(\frac{9}{2} + 18 - 9\right) - \left(2 - 12 + \frac{8}{3}\right)$. (1)

= $\frac{27}{2} + \frac{22}{3} = \frac{125}{6}$, so the area is **125/6 m²**. (1)

OR

(iii) Area = $\int_0^3 (x + 6 - x^2)\,dx = \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_0^3$. (1)

= 9/2 + 18 − 9 = **27/2 m²**. (1)

— End of Solutions —