PRACTICE PAPER 01 (2026-27) – ANSWER KEY WITH SOLUTIONS
CHAPTER 09 : DIFFERENTIAL EQUATIONS

SUBJECT: MATHEMATICS	**MAX. MARKS:** 40
CLASS: XII	**DURATION:** 1½ hrs

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Order and degree

Ans: (b) 2, 2

The highest order derivative is d^2y/dx^2, so order = 2. Squaring both sides to remove the fractional power: $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$, so the degree (power of d^2y/dx^2) = 2.

2. $dy/dx = e^{x-y}$

Ans: (a) $e^y = e^x + C$

Separating variables: $e^y\,dy = e^x\,dx$, so $e^y = e^x + C$.

3. Arbitrary constants

Ans: (c) 3

The general solution of an n^{th} order differential equation has n arbitrary constants, so 3.

4. Integrating factor

Ans: (d) x^2

Standard form: $\frac{dy}{dx} + \frac{2}{x}y = x$. I.F. = $e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$.

5. Family $y = Ae^{2x}$

Ans: (c) dy/dx = 2y

Differentiating: $dy/dx = 2Ae^{2x} = 2y$.

6. Substitution y = vx

Ans: (a) 1 + v

$dy/dx = v + x\,dv/dx$. So $v + x\frac{dv}{dx} = \frac{x+2vx}{x} = 1 + 2v$, giving $x\frac{dv}{dx} = 1 + v$.

7. $dy/dx = 2x$, $y(0) = 3$

Ans: (a) $y = x^2 + 3$

Integrating: $y = x^2 + C$. Using $y(0) = 3$: $C = 3$, so $y = x^2 + 3$.

8. $x\,dy + y\,dx = 0$

Ans: (d) xy = C

$x\,dy + y\,dx = d(xy) = 0$, so $xy = C$.

9. Assertion–Reason

Ans: (d) A is false but R is true

Integrating $dy/dx = 3$ gives $y = 3x + C$, which has one arbitrary constant. So A is false (it is only a particular solution). R is true.

10. Assertion–Reason

Ans: (b) Both A and R are true but R is not the correct explanation of A

Both statements are standard results of the linear equation. R gives the form of the solution once the I.F. is known; it does not explain why the I.F. is $e^{\int P\,dx}$. So R is not the correct explanation.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Order and degree of $(y - xy')^2 = 1 + (y')^2$

The highest order derivative present is dy/dx, so the order is 1. (1)

Expanding gives a polynomial in dy/dx: $y^2 - 2xyy' + x^2(y')^2 = 1 + (y')^2$, whose highest power of dy/dx is 2. So the degree is 2. (1)

12. Differential equation of $y^2 = 4ax$

Differentiating $y^2 = 4ax$: $2y\frac{dy}{dx} = 4a$. (1)

Substituting $4a = 2y\,dy/dx$ in $y^2 = 4ax$: $y^2 = 2xy\,dy/dx$, i.e. **$y = 2x\,dy/dx$**. (1)

13. Solve $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Separating variables: $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$. (1)

Integrating: **$\tan^{-1} y = \tan^{-1} x + C$**. (1)

14. Solve $\frac{dy}{dx} = y\tan x$

$\frac{dy}{y} = \tan x\,dx \Rightarrow \log|y| = \log|\sec x| + \log C$. (1)

So **$y = C \sec x$**. (1)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. $dy/dx = 3x^2e^{-y}$, $y(0) = 0$

Separating: $e^y\,dy = 3x^2\,dx$. (1)

Integrating: $e^y = x^3 + C$. (1)

Using $y = 0$ when $x = 0$: $1 = C$. So $e^y = x^3 + 1$, i.e. **$y = \log(x^3 + 1)$**. (1)

16. $(x^2 + y^2)\,dx = 2xy\,dy$

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ is homogeneous. Put $y = vx$: $v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$. (1)

$x\frac{dv}{dx} = \frac{1-v^2}{2v} \Rightarrow \int \frac{2v\,dv}{1-v^2} = \int \frac{dx}{x}$. (1)

$-\log|1 - v^2| = \log|x| + \log C \Rightarrow x(1 - y^2/x^2) = C_1$, i.e. **$x^2 - y^2 = Cx$**. (1)

17. $dy/dx + 2y = e^{-x}$

Here $P = 2$, $Q = e^{-x}$. I.F. $= e^{\int 2\,dx} = e^{2x}$. (1)

Solution: $y\,e^{2x} = \int e^{-x}e^{2x}\,dx = \int e^x\,dx = e^x + C$. (1)

So $\mathbf{y = e^{-x} + Ce^{-2x}}$. (1)

SECTION – D

Question 18 carries 5 marks.

18. x log x dy/dx + y = 2 log x, y(e) = 2

Standard form: $\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$. (1)

I.F. = $e^{\int \frac{dx}{x\log x}} = e^{\log|\log x|} = \log x$. (1)

Solution: $y\log x = \int \frac{2}{x}\log x\,dx = (\log x)^2 + C$. (1)

General solution: **y = log x + C/log x**. (1)

Using y = 2 at x = e (log e = 1): 2 = 1 + C, so C = 1. **y = log x + 1/log x**, i.e. y log x = (log x)² + 1. (1)

OR

x dy − y dx = √(x² + y²) dx gives $\frac{dy}{dx} = \frac{y + \sqrt{x^2+y^2}}{x}$, a homogeneous equation. (1)

Put y = vx: $v + x\frac{dv}{dx} = v + \sqrt{1+v^2} \Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$. (2)

Integrating: $\log\left|v + \sqrt{1+v^2}\right| = \log|x| + \log C$. (1)

So $\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = Cx$, i.e. **y + √(x² + y²) = Cx²**. (1)

SECTION – E (Case Study Based Questions)

Questions 19 and 20 carry 4 marks each.

19. Bacterial growth

(i) Rate of growth is proportional to N: **dN/dt = kN**, where k is a constant. (1)

(ii) $\frac{dN}{N} = k\,dt \Rightarrow$ log N = kt + c, so $\mathbf{N = Ce^{kt}}$. (1)

(iii) N(0) = 1000 gives C = 1000. N(2) = 2000 gives $e^{2k} = 2$, so $e^k = \sqrt{2}$. Hence N(t) = $1000 \cdot 2^{t/2}$. (1)

N(6) = $1000 \cdot 2^3$ = **8000 bacteria**. (1)

OR

(iii) As above, N(t) = $1000 \cdot 2^{t/2}$. Setting N = 16000: $2^{t/2} = 16$. (1)

So t/2 = 4, i.e. **t = 8 hours**. (1)

20. dv/dt = 10 − 0.2v, v(0) = 0

(i) $\frac{dv}{dt} + 0.2\,v = 10$, so P = 0.2 and Q = 10. I.F. = $e^{\int 0.2\,dt} = e^{0.2t}$. (1)

(ii) $v\,e^{0.2t} = \int 10\,e^{0.2t}\,dt = 50\,e^{0.2t} + C$, so $\mathbf{v = 50 + Ce^{-0.2t}}$. (1)

(iii) v(0) = 0 gives 0 = 50 + C, so C = −50. $\mathbf{v = 50(1 - e^{-0.2t})}$. (1)

As t → ∞, $e^{-0.2t} \to 0$, so the terminal velocity is **50 m/s**. (1)

OR

(iii) $50(1 - e^{-0.2t}) = 25 \Rightarrow e^{-0.2t} = 1/2$. (1)

So 0.2t = log 2, i.e. **t = 5 log 2 ≈ 3.47 s**. (1)

— End of Solutions —