CLASS XII – PHYSICS (042)
STEP-BY-STEP SOLUTIONS AND MARKING SCHEME

Chapter 10: Wave Optics – Practice Paper (35 Marks)

Answer key – Section A

Q.	1	2	3	4	5	6	7	8	9
Ans.	(b)	(c)	(c)	(b)	(b)	(c)	(c)	(a)	(c)

Marks are shown in square brackets. Equivalent correct methods are acceptable. Fringe width $\beta = \lambda D/d$; single-slit minima $a \sin\theta = n\lambda$; Malus' law $I = I_0 \cos^2\theta$; $I = 4I_0 \cos^2(\varphi/2)$.

SECTION A

Q.1 (b)

The part of the plane wavefront that passes through the thick central portion of the lens travels the longest path in glass, where the speed is less, and so is delayed the most. The emerging wavefront is therefore depressed at the centre, i.e. spherical, and converges to the focus F. **[1]**

Q.2 (c)

Frequency does not change on refraction: $\nu = c/\lambda = 3 \times 10^8/(600 \times 10^{-9}) = 5 \times 10^{14}$ Hz. Speed in the medium $v = \nu\lambda' = 5 \times 10^{14} \times 400 \times 10^{-9} = 2 \times 10^8$ m s^{-1} (so $n = 1.5$). **[1]**

Q.3 (c)

Path difference $\lambda/6$ gives phase difference $\varphi = (2\pi/\lambda)(\lambda/6) = \pi/3$. $I = I_{max} \cos^2(\varphi/2) = 80 \cos^2(\pi/6) = 80 \times (3/4) = 60$ units. **[1]**

Q.4 (b)

$\beta = \lambda D/d = (500 \times 10^{-9} \times 1.0)/(0.5 \times 10^{-3}) = 1$ mm. Dark fringes lie at $x = (n + ½)\beta$; the second dark fringe is $n = 1$: $x = 1.5\,\beta = 1.5$ mm. **[1]**

Q.5 (b)

The first minima are at $\sin\theta \approx \theta = \lambda/a = (6 \times 10^{-7})/(2 \times 10^{-4}) = 3 \times 10^{-3}$ rad on each side. Width of the central maximum $= 2\lambda D/a = 2 \times 3 \times 10^{-3} \times 1 = 6 \times 10^{-3}$ m $= 6$ mm. **[1]**

Q.6 (c)

After P_1 the intensity is $I_0/2$. By Malus' law, after P_2: $I = (I_0/2)\cos^2 60° = (I_0/2)(1/4) = I_0/8$. **[1]**

Q.7 (c)

Fringe width $\beta \propto \lambda$. Strip A has 6 fringes and strip B has 8 in the same width, so $\beta_A/\beta_B = 8/6 = 4/3$. Hence $\lambda_A = (4/3)(450\text{ nm}) = 600$ nm. **[1]**

Q.8 (a)

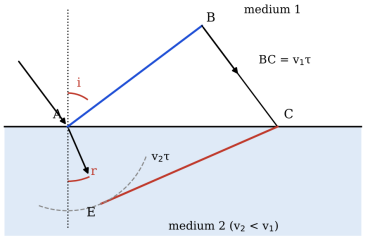
Both true. The phase of light emitted by an ordinary source changes abruptly every $\sim 10^{-10}$ s, so two independent sources are incoherent; the phase difference changes rapidly and the intensities just add. R explains A. **[1]**

Q.9 (c)

A is true. R is false: the wavelength of light (about 0.5 µm) is very small compared with the size of everyday obstacles, which is why diffraction effects are not noticed. **[1]**

SECTION B

Q.10



(a) In the time τ the incident wavefront advances BC = $v_1\tau$ in medium 1 and the wavelet from A reaches E, AE = $v_2\tau$, in medium 2. From the right-angled triangles ABC and AEC: sin i = BC/AC = $v_1\tau$/AC and sin r = AE/AC = $v_2\tau$/AC. Dividing, sin i/sin r = v_1/v_2. **[1]**
(b) sin r = sin i × (v_2/v_1) = 0.6 × (2/3) = 0.4. **[1]**

Q.11

Path difference = 47.5 − 40 = 7.5 cm = 1.5λ. Phase difference φ = (2π/λ) × path difference = 2π × 1.5 = 3π rad. **[1]**
The path difference is an odd multiple of λ/2 (φ = 3π, an odd multiple of π), so the interference is destructive. The two waves have equal amplitude and cancel, so the resultant intensity is zero (I = $4I_0\cos^2(3\pi/2) = 0$). **[1]**

Q.12 (A)

The bright fringes are at $x_n = n\lambda D/d$, so the fourth bright fringe is at $x_4 = 4\lambda D/d = 4\beta$ = 5 mm, giving β = 1.25 mm. **[1]**
λ = βd/D = $(1.25 \times 10^{-3} \times 0.4 \times 10^{-3})/1.0 = 5 \times 10^{-7}$ m = 500 nm. **[1]**

Q.12 (B)

For a point P at distance x from O, the path difference $S_2P - S_1P \approx xd/D$ (for x, d << D). Bright fringe when xd/D = nλ, i.e. $x_n = n\lambda D/d$. **[1]**
The separation of two consecutive bright (or dark) fringes is $\beta = x_{n+1} - x_n = \lambda D/d$. **[1]**

Q.13

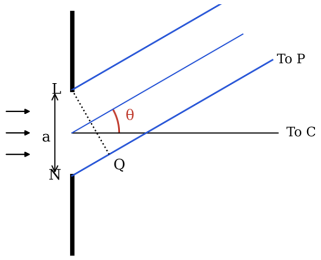
(a) The maximum transmitted intensity (θ = 0) is 40 W m^{-2} and equals the intensity I_1 of the light emerging from P_1. A polaroid transmits half of the unpolarised light incident on it, so the incident intensity is 2 × 40 = 80 W m^{-2}. **[1]**
(b) $I = I_1\cos^2\theta$: 10 = 40 $\cos^2\theta$, so cos θ = 1/2 and θ = 60° (or 120°). **[1]**

SECTION C

Q.14

(a) Successive maxima in the graph are at 0, 2, 4 mm, so the fringe width β = 2 mm. **[1]**
(b) λ = βd/D = $(2 \times 10^{-3} \times 0.3 \times 10^{-3})/1.2 = 5 \times 10^{-7}$ m = 500 nm. **[1]**
(c) In water λ′ = λ/n = 500/(4/3) = 375 nm, so β′ = λ′D/d = β/n = 2 mm/(4/3) = 1.5 mm. **[1]**

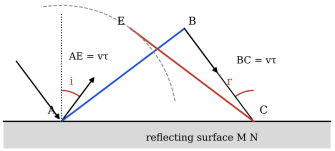
Q.15 (A)



Consider the diffracted rays making an angle θ with the normal. The path difference between the rays from the two edges L and N is NQ = a sin θ. **[1]**
For the first minimum, divide the slit into two halves; each point of one half has a partner in the other half at a distance a/2 whose path difference is (a/2) sin θ. When this equals λ/2, i.e. a sin θ

= λ, every pair cancels and the intensity is zero: sin θ = λ/a. (Minima occur at a sin θ = nλ, n = ±1, ±2, …) **[1]**
The secondary maxima are much weaker than the central maximum and become weaker as n increases. The angular width of the central maximum = 2λ/a. **[1]**

Q.15 (B)



Huygens' principle: each point of a wavefront is a source of secondary wavelets which spread out with the speed of the wave; the new position of the wavefront at a later time is the forward envelope (common tangent) of these wavelets. **[1]**
Let AB be a plane wavefront incident at angle i on the reflecting surface MN. In time τ the point B of the wavefront reaches C: BC = vτ. A secondary wavelet from A spreads a distance vτ, so AE = vτ, and the tangent CE from C to this wavelet is the reflected wavefront. **[1]**
In the right-angled triangles ABC and AEC: AE = BC = vτ and AC is common, so the triangles are congruent and ∠BAC = ∠ECA, i.e. i = r. This is the law of reflection (the incident ray, reflected ray and normal also lie in one plane). **[1]**

Q.16

(a) A polaroid transmits half the intensity of unpolarised light: $I_1 = I_0/2$. **[1]**
(b) After P_2 (angle 45° with P_1): $I_2 = I_1 \cos^2 45° = (I_0/2)(1/2) = I_0/4$. After P_3 (45° with P_2): $I_3 = I_2 \cos^2 45° = I_0/8$. **[1]**
(c) Without P_2, P_1 and P_3 are crossed (90°): $I = I_1 \cos^2 90° = 0$. No light is transmitted, since the light polarised by P_1 has no component along the pass axis of P_3; the middle polaroid rotates the plane of polarisation so that light gets through. **[1]**

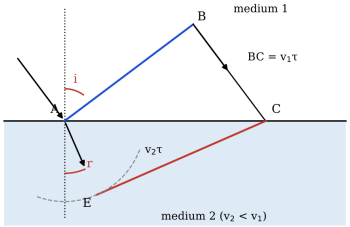
SECTION D

Q.17 (Case study)

(i) **(b)** $\beta = \lambda D/d = (6 \times 10^{-7} \times 1.5)/(0.3 \times 10^{-3}) = 3 \times 10^{-3}$ m = 3 mm. **[1]**
(ii) **(c)** $x_3 = 3\beta = 3 \times 3 = 9$ mm. **[1]**
(iii) **(d)** β ∝ D: doubling D (1.5 m → 3.0 m) doubles β to 6 mm. **[1]**
(iv) **(b)** β ∝ λ: β = 3 mm × (450/600) = 2.25 mm. **[1]**
OR (iv) (c) In the liquid λ′ = λ/n, so β′ = β/n = 3/1.5 = 2.0 mm. **[1]**

SECTION E

Q.18 (A)



Huygens' principle: every point on a wavefront acts as a source of secondary wavelets which spread in all directions with the speed of the wave in that medium; the envelope (common tangent) of the wavelets at a later time gives the new wavefront. **[1]**
Let a plane wavefront AB be incident at angle i on the surface between medium 1 (speed v_1) and medium 2 (speed v_2). In time τ the point B reaches C: BC = $v_1\tau$, while the wavelet from A travels AE = $v_2\tau$ into medium 2; CE, the tangent from C to this wavelet, is the refracted wavefront. **[1]**
From the triangles ABC and AEC: sin i = BC/AC = $v_1\tau$/AC and sin r = AE/AC = $v_2\tau$/AC, so sin i/sin r = v_1/v_2. With $n_1 = c/v_1$ and $n_2 = c/v_2$: $n_1 \sin i = n_2 \sin r$ (Snell's law). **[1]**
Numerical: $\lambda' = \lambda/n = 600/(4/3) = 450$ nm; $v = c/n = 3 \times 10^8/(4/3) = 2.25 \times 10^8$ m s^{-1}. **[1]**
The frequency does not change: $\nu = c/\lambda = 3 \times 10^8/(600 \times 10^{-9}) = 5 \times 10^{14}$ Hz. **[1]**

Q.18 (B)

Let the displacement due to one source be $y_1 = a\cos\omega t$ and that due to the other $y_2 = a\cos(\omega t + \varphi)$. By superposition $y = y_1 + y_2 = a[\cos\omega t + \cos(\omega t + \varphi)] = 2a\cos(\varphi/2)\cos(\omega t + \varphi/2)$. **[1]**

The amplitude is $2a\cos(\varphi/2)$, and the intensity is proportional to the square of the amplitude: $I = 4I_0\cos^2(\varphi/2)$, where $I_0 \propto a^2$ is the intensity of each source. **[1]**

Constructive interference ($I = 4I_0$) when $\varphi = 0, \pm 2\pi, \pm 4\pi, \ldots$, i.e. path difference $= n\lambda$ ($n = 0, 1, 2, \ldots$). Destructive interference ($I = 0$) when $\varphi = \pm\pi, \pm 3\pi, \ldots$, i.e. path difference $= (n + \tfrac{1}{2})\lambda$. **[1]**

Numerical: the central maximum has $\varphi = 0$, so $I = I_{max}$. For path difference $\lambda/4$: $\varphi = (2\pi/\lambda)(\lambda/4) = \pi/2$ and $I' = I\cos^2(\pi/4) = I/2$. **[1]**

(ii) For path difference $\lambda/3$: $\varphi = 2\pi/3$ and $I'' = I\cos^2(\pi/3) = I \times (1/4) = I/4$. **[1]**

— End of Solutions —