CLASS XII – PHYSICS (042)
STEP-BY-STEP SOLUTIONS AND MARKING SCHEME

Chapter 2: Electrostatic Potential and Capacitance – Practice Paper (35 Marks)

Answer key – Section A

Q.	1	2	3	4	5	6	7	8	9
Ans.	(b)	(b)	(c)	(c)	(b)	(d)	(d)	(a)	(c)

Marks are shown in square brackets. Equivalent correct methods are acceptable. Take $k = 1/(4\pi\varepsilon_0) = 9 \times 10^9$ N m^2 C^{-2}.

SECTION A

Q.1 (b)

$E = |\Delta V|/\Delta l = 10\ V/0.02\ m = 500\ V\ m^{-1}$. The field is perpendicular to the equipotentials and points in the direction of decreasing potential, i.e. from 30 V towards 10 V: along −x. **[1]**

Q.2 (b)

The potentials of +q and −q at equal distances cancel in pairs, so V = 0. The field of the two positive charges (pointing away from them) and of the two negative charges (pointing towards them) both point from the AB side towards the DC side; they add and do not cancel, so $E \neq 0$. **[1]**

Q.3 (c)

$V = kQ\cdot(1/r)$, so the slope of the V–(1/r) line is kQ = 450/5 = 90 V m. $Q = 90/(9 \times 10^9) = 10^{-8}$ C = 10 nC. **[1]**

Q.4 (c)

The electrostatic force is conservative, so the work done depends only on the end points. A and B are fixed, so $W_1 = W_2 = W_3 = q(V_B - V_A)$. **[1]**

Q.5 (b)

3 µF and 6 µF are in series: 3×6/(3+6) = 2 µF. This is in parallel with 4 µF: 2 + 4 = 6 µF. **[1]**

Q.6 (d)

$U = \tfrac{1}{2}QV$: P = ½(12)(4) = 24 µJ; Q = ½(20)(10) = 100 µJ; R = ½(18)(3) = 27 µJ; S = ½(40)(20) = 400 µJ. Maximum for S. **[1]**

Q.7 (d)

The slab and the air gap act as two capacitors in series. $C_{air} = \varepsilon_0 A/(d/2) = 2C_0$; $C_{slab} = K\varepsilon_0 A/(d/2) = 6C_0$. $C = (2 \times 6)/(2 + 6)\ C_0 = 3C_0/2$. (Also $C = 2KC_0/(K + 1)$.) **[1]**

Q.8 (a)

Both statements are true and R explains A: a tangential component would drive surface charges, which contradicts the static situation, so E is normal to the surface. **[1]**

Q.9 (c)

A is true: the potentials of +q and −q cancel at the centre (and on the whole equatorial plane). R is false: on the axis $V = \pm p/(4\pi\varepsilon_0 r^2) \neq 0$. **[1]**

SECTION B

Q.10

$V_P = k(q_1 + q_2)/r$ with r = 3 cm = 0.03 m: $V = 9 \times 10^9(6 - 3) \times 10^{-6}/0.03 = 9 \times 10^5$ V. **[1]**
$W = qV = (2 \times 10^{-9})(9 \times 10^5) = 1.8 \times 10^{-3}$ J. **[1]**

Q.11

Potential difference $V = Ed = (9.1 \times 10^3)(0.02) = 182$ V. KE gained = eV = 182 eV = $182 \times 1.6 \times 10^{-19} = 2.91 \times 10^{-17}$ J. **[1]**
$\tfrac{1}{2}mv^2 = eV$: $v = \sqrt{(2 \times 2.912 \times 10^{-17}/9.1 \times 10^{-31})} = \sqrt{(6.4 \times 10^{13})} = 8 \times 10^6$ m s^{-1}. **[1]**

Q.12 (A)

Charge is conserved: $Q = C_1V = 12 \times 50 = 600\ \mu C$. Common potential $V' = Q/(C_1 + C_2) = 600/16 = 37.5$ V. **[1]**

$U_i = \frac{1}{2}(12\ \mu F)(50)^2 = 15$ mJ; $U_f = \frac{1}{2}(16\ \mu F)(37.5)^2 = 11.25$ mJ. Energy lost = 3.75 mJ (as heat and radiation during charge flow). **[1]**

Q.12 (B)

$V = p\cos\theta/(4\pi\varepsilon_0 r^2) = (9 \times 10^9)(4 \times 10^{-9})(0.5)/(0.2)^2 = 450$ V. **[1]**

On the equatorial plane $\theta = 90°$, $\cos\theta = 0$, so $V = 0$: the potentials of $+q$ and $-q$ cancel because the point is equidistant from them. **[1]**

Q.13

$U = k[q_Aq_B + q_Bq_C + q_Cq_A]/a = k(q^2/a)(1 - 1 - 1) = -(9 \times 10^9)(2 \times 10^{-6})^2/0.1 = -0.36$ J. **[1]**

The negative sign means that the charges are bound: 0.36 J of work would have to be supplied to separate them to infinity. (Work done in assembling them from infinity is negative.) **[1]**

SECTION C

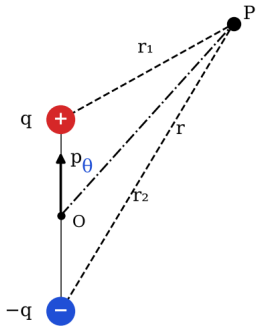
Q.14

(a) C_2 and C_3 are in parallel: $3 + 6 = 9\ \mu F$. This is in series with C_1: $C = (18 \times 9)/(18 + 9) = 6\ \mu F$. **[1]**

(b) Total charge $Q = CV = 6 \times 12 = 72\ \mu C$. This is the charge on C_1 (series). Potential across $C_1 = 72/18 = 4$ V, so the parallel pair has $12 - 4 = 8$ V. $Q_2 = 3 \times 8 = 24\ \mu C$; $Q_3 = 6 \times 8 = 48\ \mu C$. **[1]**

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6 \times 10^{-6})(12)^2 = 4.32 \times 10^{-4}$ J. **[1]**

Q.15 (A)



Let the dipole ($\pm q$ at separation $2a$) be at the origin and P at position r making angle θ with **p**. By superposition $V = [q/(4\pi\varepsilon_0)](1/r_1 - 1/r_2)$. **[½]**

$r_1^2 = r^2 + a^2 - 2ar\cos\theta$; $r_2^2 = r^2 + a^2 + 2ar\cos\theta$. For $r >> a$, $1/r_1 \approx (1/r)(1 + (a/r)\cos\theta)$ and $1/r_2 \approx (1/r)(1 - (a/r)\cos\theta)$. **[1]**

$V = [q/(4\pi\varepsilon_0)] \cdot 2a\cos\theta/r^2 = p\cos\theta/(4\pi\varepsilon_0 r^2) = \mathbf{p}\cdot\hat{\mathbf{r}}/(4\pi\varepsilon_0 r^2)$. **[1]**

On the axis ($\theta = 0$ or π): $V = \pm p/(4\pi\varepsilon_0 r^2)$; on the equatorial plane ($\theta = 90°$): $V = 0$. **[½]**

Q.15 (B)

Let charge Q' be on the capacitor, so the potential difference is $V' = Q'/C$. The work to transfer a further $\delta Q'$ is $\delta W = V'\delta Q' = (Q'/C)\delta Q'$. **[1]**

Total work in charging from 0 to Q: $W = \int (Q'/C)dQ' = Q^2/2C = \frac{1}{2}CV^2 = \frac{1}{2}QV$. This is stored as the energy U of the capacitor. **[1]**

For a parallel plate capacitor $C = \varepsilon_0 A/d$ and $V = Ed$: $U = \frac{1}{2}(\varepsilon_0 A/d)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 \times (Ad)$. Ad is the volume of the field region, so the energy density $u = U/(Ad) = \frac{1}{2}\varepsilon_0 E^2$. **[1]**

Q.16

$E = -dV/dx$. Region OA: V rises from 0 to 40 V over 2 cm: $dV/dx = 40/0.02 = 2000$ V m^{-1}, so $E = 2000$ V m^{-1} along $-x$. **[1]**

Region AB: V is constant, so $E = 0$. Region BC: V falls from 40 V to 10 V over 2 cm: $dV/dx = -1500$ V m^{-1}, so $E = 1500$ V m^{-1} along $+x$. **[1]**

W (by field) = $q(V_O - V_C) = (2 \times 10^{-6})(0 - 10) = -2 \times 10^{-5}$ J. (The external agent does $+2 \times 10^{-5}$ J.) **[1]**

SECTION D

Q.17 (Case study)

(i) **(b)** $C_0 = \varepsilon_0 A/d = (8.85 \times 10^{-12})(10^{-2})/10^{-3} = 88.5$ pF; $Q = C_0 V = 88.5$ pF $\times$ 100 V = 8.85 nC. **[1]**

(ii) **(c)** The battery is disconnected, so Q is unchanged. $C = KC_0$, so $V' = Q/C = V/K = 100/5 = 20$ V. **[1]**

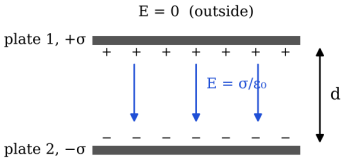
(iii) **(d)** $C = KC_0 = 5 \times 88.5 = 442.5$ pF. **[1]**

(iv) **(c)** With Q constant, $U = Q^2/2C$, so $U' = U/K = U/5$ (1/5 times). The slab is pulled into the capacitor, which lowers the energy. **[1]**

OR (iv) (c) With V = 100 V constant, $Q' = KC_0 V = 5 \times 8.85$ nC = 44.25 nC. **[1]**

SECTION E

Q.18 (A)



Plates carry $\pm Q$, giving $\sigma = Q/A$. Each sheet gives a field $\sigma/2\varepsilon_0$. Outside the plates the fields cancel (E = 0); between the plates they add, $E = \sigma/\varepsilon_0 = Q/(\varepsilon_0 A)$, directed from the positive to the negative plate. **[1]**

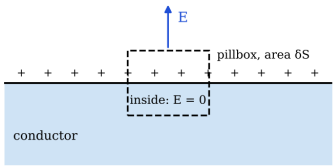
$V = Ed = Qd/(\varepsilon_0 A)$, so $C = Q/V = \varepsilon_0 A/d$. **[1]**

With a dielectric the induced surface charge $\pm\sigma_p$ reduces the net field to $E = (\sigma - \sigma_p)/\varepsilon_0 = \sigma/(K\varepsilon_0)$. Thus $V = Qd/(K\varepsilon_0 A)$ and $C = K\varepsilon_0 A/d = KC_0$: it increases K times. **[1]**

Numerical: $C_0 = (8.85 \times 10^{-12})(10^{-2})/(2 \times 10^{-3}) = 44.25$ pF; $C = 6C_0 = 265.5$ pF. **[1]**

With the battery connected V = 300 V: $Q = CV = 265.5 \times 10^{-12} \times 300 = 7.97 \times 10^{-8}$ C; $U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(265.5 \times 10^{-12})(300)^2 = 1.19 \times 10^{-5}$ J. **[1]**

Q.18 (B)



Take a small pillbox across the surface, with cross-section δS and negligible height. Inside the conductor E = 0; on the curved side the field is tangential (or zero), so no flux passes. The flux comes only from the outer face: $E\,\delta S$ (E being normal to the surface). **[1]**

The enclosed charge is $\sigma\delta S$. By Gauss's law $E\,\delta S = \sigma\delta S/\varepsilon_0$, so $E = \sigma/\varepsilon_0$, normal to the surface (outward for $\sigma > 0$, inward for $\sigma < 0$). **[1]**

Any two properties: E = 0 inside; E is normal to the surface; excess charge resides on the surface only; potential is constant throughout and on the surface; the field in a cavity is zero (shielding). **[1]**

Numerical: $E = kQ/R^2 = (9 \times 10^9)(10^{-6})/(0.1)^2 = 9 \times 10^5$ N C^{-1}, normal to the surface (outward). **[1]**

The potential inside equals the surface value: $V = kQ/R = (9 \times 10^9)(10^{-6})/0.1 = 9 \times 10^4$ V. **[1]**

— End of Solutions —