CLASS XII – PHYSICS (042)
STEP-BY-STEP SOLUTIONS AND MARKING SCHEME

Chapter 7: Alternating Current – Practice Paper (35 Marks)

Answer key – Section A

Q.	1	2	3	4	5	6	7	8	9
Ans.	(c)	(c)	(a)	(d)	(b)	(b)	(d)	(a)	(d)

Marks are shown in square brackets. Equivalent correct methods are acceptable. Take π = 3.14 and √2 = 1.41; $\varphi = \tan^{-1}[(X_C - X_L)/R]$, so the current leads the voltage when $X_C > X_L$.

SECTION A

Q.1 (c)

The current reaches its maximum (and zero) a quarter of a cycle before the voltage, i.e. it leads the voltage by π/2. This is the behaviour of a pure capacitor (a pure inductor has i lagging v by π/2; in a resistor they are in phase). **[1]**

Q.2 (c)

From the graph v_m = 311 V, so $V = v_m/\sqrt{2} = 311/1.41 \approx 220$ V. The period is T = 20 ms, so $\nu = 1/T = 50$ Hz. **[1]**

Q.3 (a)

$Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(9 + 16)} = 5\ \Omega$. Power factor $\cos\varphi = R/Z = 3/5 = 0.6$. **[1]**

Q.4 (d)

V_L and V_C are in opposite directions, so the net reactive voltage = 150 − 90 = 60 V, perpendicular to V_R: $V = \sqrt{(80^2 + 60^2)} = 100$ V. **[1]**

Q.5 (b)

At resonance Z = R and $i_m = v_m/R$. From the graph the peak current is 2 A, so R = 100/2 = 50 Ω. **[1]**

Q.6 (b)

$V_s = V_p(N_s/N_p) = 220 \times 50/500 = 22$ V. For an ideal transformer $I_s = I_p(N_p/N_s) = 0.2 \times 10 = 2$ A (step-down: voltage falls, current rises). **[1]**

Q.7 (d)

The iron rod magnetises and increases the self-inductance L of the coil, so $X_L = \omega L$ increases. A larger fraction of the source voltage falls across the coil, leaving less across the bulb: the current falls and the bulb glows less brightly. **[1]**

Q.8 (a)

Both true. $X_C = 1/\omega C \to \infty$ for dc (ω = 0), so no steady current flows; for ac X_C is finite, so the capacitor charges and discharges every cycle and ac flows. R explains A. **[1]**

Q.9 (d)

A is false: resonance needs both L and C ($X_L = X_C$); an RC circuit has no inductor so cannot resonate. R is true. **[1]**

SECTION B

Q.10

v_m = 311 V, so V = 311/1.41 = 220 V and I = V/R = 220/100 = 2.2 A. **[1]**
$P = I^2R = (2.2)^2 \times 100 = 484$ W (equivalently VI = 220 × 2.2). **[1]**

Q.11

ω = 500 rad s^{-1}: $X_L = \omega L = 500 \times 40 \times 10^{-3} = 20\ \Omega$; $i_m = v_m/X_L = 100/20 = 5$ A. **[1]**
In an inductor the current lags the voltage by π/2: $i = 5\sin(500\,t - \pi/2)$ A. **[1]**

Q.12 (A)

$X_C = 1/\omega C = 1/(400 \times 25 \times 10^{-6}) = 1/(10^{-2}) = 100\ \Omega$; $i_m = 200/100 = 2$ A. **[1]**

In a capacitor the current leads the voltage by π/2: $i = 2 \sin(400\ t + \pi/2)$ A. **[1]**

Q.12 (B)

$v = v_m \sin \omega t$ and $i = i_m \sin(\omega t - \pi/2) = -i_m \cos \omega t$, so the instantaneous power $p = vi = -v_m i_m \sin \omega t \cos \omega t = -(v_m i_m/2) \sin 2\omega t$. **[1]**

The average of sin 2ωt over a complete cycle is zero, so the average power is zero (the energy drawn in one quarter cycle is returned in the next). **[1]**

Q.13

$Z = \sqrt{(R^2 + (X_L - X_C)^2)} = \sqrt{(30^2 + 40^2)} = 50\ \Omega$; $I = V/Z = 200/50 = 4$ A. **[1]**

$\tan \varphi = (X_C - X_L)/R = (20 - 60)/30 = -4/3$, so $\varphi = -53°$. Since $X_L > X_C$ the circuit is predominantly inductive: the current lags the source voltage by 53°. **[1]**

SECTION C

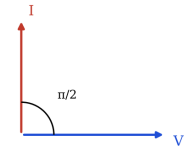
Q.14

(a) At resonance Z = R, so $R = v_m/i_m$ (peak). Circuit (i): $R = 100/2 = 50\ \Omega$. Circuit (ii): $R = 100/1 = 100\ \Omega$. **[1]**

(b) From the graph $\omega_0 = 5 \times 10^3$ rad s^{-1}. $C = 1/(\omega_0^2 L) = 1/[(25 \times 10^6)(0.02)] = 2 \times 10^{-6}$ F = 2 μF. **[1]**

(c) $Q = \omega_0 L/R = (5000 \times 0.02)/50 = 2$ for circuit (i) (and 1 for circuit (ii)). Circuit (i), having smaller R and higher Q, has the sharper resonance. **[1]**

Q.15 (A)



current leads voltage by π/2

At every instant the voltage across the capacitor equals the source voltage: $q/C = v_m \sin \omega t$, so $q = Cv_m \sin \omega t$. **[1]**

$i = dq/dt = \omega C v_m \cos \omega t = i_m \sin(\omega t + \pi/2)$, where $i_m = \omega C v_m = v_m/(1/\omega C)$. Hence the capacitive reactance $X_C = 1/\omega C$ (unit ohm), and $i_m = v_m/X_C$. **[1]**

The current leads the voltage by π/2 (a quarter of a cycle). **[1]**

Q.15 (B)

$p = vi = v_m i_m \sin \omega t \sin(\omega t + \varphi) = (v_m i_m/2)[\cos \varphi - \cos(2\omega t + \varphi)]$. **[1]**

The average of the second term over a cycle is zero, so $P = (v_m i_m/2) \cos \varphi = (v_m/\sqrt{2})(i_m/\sqrt{2}) \cos \varphi = VI \cos \varphi$. **[1]**

Purely inductive: $\varphi = -90°$, so $\cos \varphi = 0$ and P = 0 (wattless current). At resonance $X_L = X_C$, $\varphi = 0$, $\cos \varphi = 1$ and $P = VI = I^2R$, the maximum. **[1]**

Q.16

(a) $V_s = V_p(N_s/N_p) = 220 \times 50/1000 = 11$ V. **[1]**

(b) $I_s = V_s/R = 11/5.5 = 2$ A; power delivered $P_{out} = V_s I_s = 11 \times 2 = 22$ W. **[1]**

(c) $P_{in} = P_{out}/0.9 = 22/0.9 \approx 24.4$ W, so $I_p = 24.4/220 \approx 0.11$ A. A cause of loss: eddy currents in the core; remedy: use a laminated core. (Also acceptable: flux leakage – wind coils one over the other; resistance of windings – thick wire; hysteresis – soft iron core with low hysteresis loss.) **[1]**

SECTION D

Q.17 (Case study)

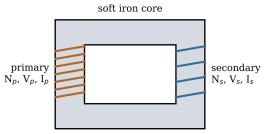
(i) **(c)** Resonance requires both an inductor and a capacitor, so that X_L and X_C can cancel. **[1]**

(ii) **(c)** $\omega_0 = 1/\sqrt{(LC)} = 1/\sqrt{(10^{-2} \times 10^{-6})} = 1/\sqrt{(10^{-8})} = 10^4$ rad s^{-1}. **[1]**

(iii) **(b)** At resonance Z = R = 10 Ω, so I = V/R = 20/10 = 2 A. **[1]**
(iv) **(c)** Q = $\omega_0 L/R = (10^4 \times 10 \times 10^{-3})/10 = 100/10 = 10$. **[1]**
OR (iv) (d) $X_L = \omega_0 L = 100\ \Omega$, so $V_L = IX_L = 2 \times 100 = 200$ V (larger than the source voltage; V_L and V_C cancel). **[1]**

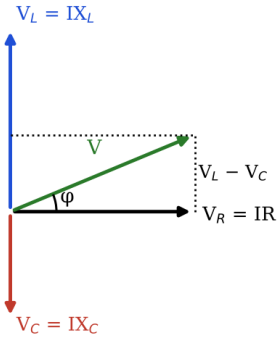
SECTION E

Q.18 (A)



Construction and working: two insulated coils, primary (N_p turns) and secondary (N_s turns), are wound on a laminated soft-iron core. An ac voltage applied to the primary produces an alternating flux in the core which links the secondary and induces an emf in it (mutual induction). **[1]**
For an ideal transformer the same flux φ links both coils: $v_s = -N_s\, d\varphi/dt$ and $v_p = -N_p\, d\varphi/dt$, so $V_s/V_p = N_s/N_p$. **[1]**
With no losses, input power = output power: $I_p V_p = I_s V_s$, so $I_p/I_s = V_s/V_p = N_s/N_p$. **[1]**
Numerical: $N_p/N_s = V_p/V_s = 11000/220 = 50$. **[1]**
$I_s = P/V_s = 22000/220 = 100$ A and $I_p = P/V_p = 22000/11000 = 2$ A. **[1]**

Q.18 (B)



The current $i = i_m \sin(\omega t + \varphi)$ is the same in R, L and C. Phasors: V_R is along I ($v_{Rm} = i_m R$); V_L leads I by π/2 ($v_{Lm} = i_m X_L$); V_C lags I by π/2 ($v_{Cm} = i_m X_C$). The source voltage phasor $V = V_R + V_L + V_C$. **[1]**
V_L and V_C are opposite, so $v_m^2 = v_{Rm}^2 + (v_{Cm} - v_{Lm})^2 = i_m^2[R^2 + (X_C - X_L)^2]$. Hence $i_m = v_m/Z$ with $Z = \sqrt{[R^2 + (X_C - X_L)^2]}$. **[1]**
$\tan\varphi = (v_{Cm} - v_{Lm})/v_{Rm} = (X_C - X_L)/R$. The current leads the voltage when $X_C > X_L$ (φ positive) and lags when $X_L > X_C$. **[1]**
Numerical: $Z = \sqrt{(12^2 + (25 - 20)^2)} = \sqrt{169} = 13\ \Omega$; $I = V/Z = 130/13 = 10$ A. **[1]**
$P = I^2R = 100 \times 12 = 1200$ W; power factor $\cos\varphi = R/Z = 12/13 \approx 0.92$. **[1]**

— End of Solutions —