CLASS VIII – MATHEMATICS

Chapter Test: Number Play – Step-by-Step Solutions & Marking Scheme

(Ganita Prakash, Grade 8, Part I – Chapter 5)

M. M.: 40 | Marks for each step are shown in square brackets.

Section A – Multiple Choice Questions (1 mark each)

Q.	Ans.	Solution
1	**(B)**	Let the numbers be n, n + 1, n + 2, n + 3. Sum = 4n + 6 = 42, so n = 9. Smallest number = **9** (numbers 9, 10, 11, 12).
2	**(A)**	6m − 4n = 2(3m − 2n) has 2 as a factor, so it is always even. The others can be odd (e.g. 3m + 5n with m = 1, n = 0 gives 3).
3	**(C)**	Multiples of 7 have the form 7k. Numbers that are 4 more than a multiple of 7 are **7k + 4**.
4	**(D)**	5 + 3 + 8 + 2 = 18, which is divisible by 9, so **5,382** is divisible by 9. (The other digit sums are 16, 10 and 14.)
5	**(D)**	Digit sum of 4,287 = 4 + 2 + 8 + 7 = 21, and 2 + 1 = 3. The remainder on division by 9 is **3**.
6	**(C)**	For 3,729, from the units digit: 9 − 2 + 7 − 3 = 11, a multiple of 11, so **3,729** is divisible by 11. (The others give 10, 9 and 1.)
7	**(B)**	A number divisible by 8 and 12 is divisible by their LCM. $8 = 2^3$ and $12 = 2^2 \times 3$, so LCM = $2^3 \times 3$ = **24**.
8	**(A)**	7 + 8 + 6 + 5 = 26, and 2 + 6 = 8. Digital root = **8**.
9	**(B)**	p full rows of 4 dots give 4p dots, and the extra row has 2 dots. Total = **4p + 2**.
10	**(C)**	Every multiple of 30 is a multiple of 10, and every multiple of 10 is a multiple of 5. So the circles are nested: the outermost A = multiples of 5, B = multiples of 10 and the innermost C = multiples of 30.

Section B – Very Short Answer Questions (2 marks each)

Q11. Blocks (Fig. 3)

(i) Each '99 + 1' block is 99 + 1 = 100, each '9 + 1' block is 9 + 1 = 10, and the loose tags are 1 each. Group P: 4 × 100 = 400. Group Q: 7 × 10 = 70. Group R: 8. Number = 400 + 70 + 8 = **478**. [1]

(ii) The 99 and 9 parts are multiples of 9. Only the unit tags remain: 4 + 7 + 8 = 19, and 19 = 2 × 9 + 1, so the remainder is **1**. (Check: 478 = 9 × 53 + 1.) [1]

Q12.

(A) (i) odd + odd = **even**. [½]

(ii) even × odd = **even** (36 is even, so 2 is a factor of the product). [½]

(iii) even − odd = **odd**. [1]

(B) 6m + 4n = 2(3m + 2n), which has a factor 2, so it is always even. [1]

For x = 3 (any odd x), $x^2 + 4 = 9 + 4 = 13$, which is odd. [1]

Q13. Remainders 3 and 3

The number is 3 more than a multiple of 5 and 3 more than a multiple of 6, so (number − 3) is a multiple of both 5 and 6, i.e. of LCM(5, 6) = 30. The numbers are **30k + 3**. [1]

For k = 1, 2: **33 and 63**. (33 = 5 × 6 + 3 = 6 × 5 + 3; 63 = 5 × 12 + 3 = 6 × 10 + 3.) [1]

Section C – Short Answer Questions (3 marks each)

Q14. Tree of expressions (Fig. 4)

(i) p = 6 + 9 + 4 = **19**; q = 6 + 9 − 4 = **11**; r = 6 − 9 + 4 = **1**; s = 6 − 9 − 4 = **−7**. [½ each = 2]

(ii) All four values (19, 11, 1, −7) are **odd**. [½]

Switching a sign changes the value by twice that number (2 × 9 or 2 × 4), which is even. A number that changes by an even amount keeps its parity, so all the expressions have the same parity as 6 + 9 + 4 = 19. [½]

Q15.

(A) Alternating sum of 4,719 from the units digit: 9 − 1 + 7 − 4 = 11. [½]

11 is a multiple of 11, so 4,719 is **divisible by 11**. (4,719 = 11 × 429.) [1]

For 9,245: 5 − 4 + 2 − 9 = −6, so the number is 6 less than a multiple of 11. [½]

Remainder = 11 − 6 = **5**. (Check: 11 × 840 = 9,240 and 9,245 − 9,240 = 5.) [1]

(B) Digit sum of 8,000 is 8, so 8,000 leaves a remainder of 8 when divided by 9. [1]

The multiple of 9 just below is 8,000 − 8 = 7,992 (8 away); the next multiple is 7,992 + 9 = 8,001 (1 away). [1]

The closest multiple of 9 is **8,001**. (Digit sum 9 ✓.) [1]

Q16.

(A) QQQ = 111 × Q. Writing PQ = 10P + Q: 6(10P + Q) = 111Q. [1]

60P + 6Q = 111Q, so 60P = 105Q, which simplifies to 4P = 7Q. [1]

P and Q are digits and 4P = 7Q means Q is a multiple of 4. Q = 4 gives P = 7 (Q = 8 gives P = 14, not a digit; Q = 0 gives P = 0, not allowed). So **P = 7, Q = 4**: 74 × 6 = 444 ✓. [1]

(B) 5 + 2 + x + 4 = 11 + x must be a multiple of 9. The only digit that works is x = 7 (11 + 7 = 18). [2]

The number is 5,274. From the units digit: 4 − 7 + 2 − 5 = −6, so the remainder is 11 − 6 = **5**. (5,274 = 11 × 479 + 5.) [1]

Q17.

(A) Let the numbers be 4a and 6b. Sum = 4a + 6b = 2(2a + 3b). [1]

This is a multiple of 4 only when 2a + 3b is even, i.e. when 3b is even, i.e. when b is even. [1]

Example: 8 + 12 = 20 (b = 2) is a multiple of 4. Non-example: 8 + 6 = 14 (b = 1) is not. So the statement is **sometimes true**. [1]

(B) A number divisible by both 6 and 8 is divisible by LCM(6, 8) = 24. [1]

24 is divisible by 6 and 8 but not by 48; 48 is divisible by 6, 8 and 48. [1]

So the statement is **sometimes true**. [1]

Section D – Long Answer / Case-based Questions (4 marks each)

Q18.

(A) (i) 2 − 8 + 3 − 4 + 7 = **0**. [1]

(ii) Since 0 is a multiple of 11, 74,382 is **divisible by 11**. (74,382 = 11 × 6,762.) [1]

Why it works: 1 = 11 × 0 + 1 (one more than a multiple of 11), 10 = 11 − 1 (one less), 100 = 99 + 1 (one more), 1000 = 1001 − 1 (one less), and so on, alternately. So digits at units, hundreds, … add an excess, and digits at tens, thousands, … add a shortfall. The difference of the two sums is the remainder (or 0 / a multiple of 11 means divisible). [1]

(iii) 74,385: 5 − 8 + 3 − 4 + 7 = 3, so the remainder is **3**. (74,385 − 74,382 = 3.) [1]

(B) Let the number be 9k + 7 (digital root 7 means remainder 7 on division by 9). [½]

(i) 9k + 7 + 20 = 9k + 27 = 9(k + 3), a multiple of 9, so its digital root is **9**. [1]

(ii) 2(9k + 7) = 18k + 14 = 9(2k + 1) + 5, so the digital root is **5**. [1]

(iii) 9k + 7 = 3(3k + 2) + 1 leaves remainder 1 on division by 3, so the number is **not** divisible by 3. [1]

(iv) For example **106** (1 + 0 + 6 = 7). Any 3-digit number with digit sum 7 or 16 is acceptable. [½]

Q19.

(A) (i) Each remainder is 1 less than the divisor (4 − 1 = 3, 6 − 1 = 5, 9 − 1 = 8), so adding 1 gives a multiple of 4, 6 and 9. [1]

LCM(4, 6, 9): $4 = 2^2$, $6 = 2 \times 3$, $9 = 3^2$, so LCM = $2^2 \times 3^2 = 36$. Smallest number = 36 − 1 = **35**. (Check: 35 = 4 × 8 + 3 = 6 × 5 + 5 = 9 × 3 + 8.) [1]

(ii) All such numbers are **36k − 1** (k = 1, 2, 3, …). [1]

(iii) $36k - 1 \geq 100$ gives $k \geq 3$ (since $36 \times 2 - 1 = 71$ and $36 \times 3 - 1 = 107$). Smallest 3-digit number = **107**. [1]

(B) (i) Let the three consecutive even numbers be $2n$, $2n + 2$, $2n + 4$. Sum $= 6n + 6 = 6(n + 1)$. [1]

It has 6 as a factor, so the sum is always divisible by 6. [1]

(ii) Take consecutive integers n and $n + 1$. If n is even, $n = 2m$ and the product $2m(2m + 1)$ has factor 2. [1]

If n is odd, $n = 2m + 1$ and $n + 1 = 2m + 2$, so the product $(2m + 1) \times 2(m + 1)$ has factor 2. In both cases the product is **even**. [1]

Q20. Case Study: digital roots on the shop bill

(i) 3,486: $3 + 4 + 8 + 6 = 21$, $2 + 1 =$ **3**. 5,279: $5 + 2 + 7 + 9 = 23$, $2 + 3 =$ **5**. 8,765: $8 + 7 + 6 + 5 = 26$, $2 + 6 =$ **8**. [1]

(ii) Digital roots of the addends add up to $3 + 5 = 8$, which equals the digital root of the sum 8,765. So the first bill passes the check (and indeed $3,486 + 5,279 = 8,765$). [1]

(iii) 4,728: $4 + 7 + 2 + 8 = 21 \rightarrow 3$. Product of digital roots: $3 \times 6 = 18 \rightarrow 1 + 8 = 9$. But 28,378: $2 + 8 + 3 + 7 + 8 = 28 \rightarrow 2 + 8 = 10 \rightarrow 1$. Since $1 \neq 9$, the second bill has a **mistake**. (Correct value: 28,368, digital root 9.) [1]

(iv) For example **28,386**: digits add to 27, digital root 9, but $28,386 \neq 28,368$. Interchanging digits never changes the digital root. So the check can catch errors but cannot prove an answer correct. [1]

Note for teachers: Award full marks for any correct alternative method. In Q17 and Q18 a correct conclusion without an example or a reason earns only part of the marks. In Q18(B)(iv) and Q20(iv) any valid example is acceptable.