CLASS IX – MATHEMATICS

Chapter Test: The World of Numbers

Step-by-Step Solutions and Marking Scheme | M. M.: 40

Results used

p/q (lowest terms) has a terminating decimal exactly when the prime factors of q are only 2 and/or 5; otherwise it is non-terminating repeating.

Repeating decimals: multiply by a power of 10 to line up the repeating blocks and subtract. | Distance between a and b on the number line = |a − b|.

Between rational numbers a and b, (a + b)/2 lies between them. | Brahmagupta: debt × debt = fortune, debt × fortune = debt.

Section A (1 mark each)

Q	Ans	Reason
1	(C)	√225 = 15 and 3.14 are rational; 0.1212… repeats, so it is rational. 50 is not a perfect square, so **√50** is irrational.
2	(B)	40 = $2^3 \times 5$, so it terminates. 7/40 = 0.175, which has **3** decimal places.
3	(A)	Let x = 0.777…; 10x = 7.777…; subtracting, 9x = 7, so x = **7/9**.
4	(D)	1/3 = 0.333… and 1/2 = 0.5. 2/5 = 0.4 lies between them. (1/4 = 0.25, 7/12 ≈ 0.583, 3/5 = 0.6 do not.) So **2/5**.
5	(B)	(−8)(−5) = 40 (debt × debt = fortune) and (−3)(4) = −12. 40 − 12 = **28**.
6	(D)	|−4/3 − 5/3| = |−9/3| = |−3| = **3**.
7	(C)	30 = 2 × 3 × 5 contains the prime 3, so 7/30 is non-terminating repeating. 16 = 2^4, 40 = $2^3 \times 5$ and 125 = 5^3 give terminating decimals. **7/30**.
8	(A)	$OB^2 = OA^2 + AB^2 = 2^2 + 1^2 = 5$, so OB = √5. The arc gives OP = OB, so P represents **√5**.
9	(D)	Each part is 1/4. The third mark is 2 + 3/4 = **11/4**.
10	(B)	If a + √2 (say) were rational, then √2 = (a + √2) − a would be rational, which is false. So the sum is **always irrational**.
11	(C)	A is true (√3 is irrational). R is false: √4 = 2 is rational. → **(C)**.
12	(A)	A is true. R is true: the average lies between a and b and is rational; repeating the averaging again and again gives infinitely many rational numbers between them, so R explains A. → **(A)**.

Section B (2 marks each)

Q.13 (2 marks)

Let x = 0.3636… Two digits repeat, so multiply by $10^2 = 100$: 100x = 36.3636… **[1]**

Subtracting: 100x − x = 36 ⇒ 99x = 36 ⇒ x = 36/99 =

Ans. 4/11. [1]

Q.14 (2 marks)

(A)

80 = $2^4 \times 5$. The prime factors of the denominator are only 2 and 5, so the decimal **terminates**. **[1]**

23/80 = $23 \times 5^3/(2^4 \times 5^4)$ = 2875/10000 = 0.2875, which terminates after **4 decimal places**. **[1]**

OR (B)

Make the denominators larger: 2/7 = 8/28 and 3/7 = 12/28. **[1]**

Three rational numbers between them: **9/28, 10/28 (= 5/14), 11/28**. [1]

Q.15 (2 marks)

Debts are negative, so 6 loans of ₹150 = 6 × (−150) = −900; receiving ₹1,000 is +1000. **[1]**

Expression: (−150) × 6 + 1000 = −900 + 1000 =

Ans. 100. He is left with a fortune of ₹100. [1]

Q.16 (2 marks)

(A)

(i) √0.09 = 0.3 = 3/10 is a ratio of integers, so it is **rational**. **[1]**

(ii) 0.1010010001… never ends and never repeats (the number of zeros keeps increasing), so it is **irrational**. **[1]**

OR (B)

Distance = |−7/4 − 5/6|. LCM of 4 and 6 is 12: −7/4 = −21/12 and 5/6 = 10/12. **[1]**

|−21/12 − 10/12| = |−31/12| = **31/12** units. **[1]**

Section C (3 marks each)

Q.17 (3 marks)

(A) Assume, if possible, that √3 is rational. Then √3 = p/q, where p and q are co-prime integers and q ≠ 0. [½]

Squaring: 3 = p^2/q^2, so $p^2 = 3q^2$. [½]

So 3 divides p^2. Hence 3 divides p (if the square of a number is divisible by 3, so is the number). Let p = 3k. [½]

Then $9k^2 = 3q^2$, so $q^2 = 3k^2$. So 3 divides q^2 and hence 3 divides q. [½]

Now 3 divides both p and q, which contradicts that p and q are co-prime. [½]

Hence our assumption is wrong and **√3 is irrational**. [½]

OR (B) Assume 4 + 5√3 is rational, say 4 + 5√3 = p/q with integers p, q and q ≠ 0.

Then 5√3 = p/q − 4 = (p − 4q)/q, so √3 = (p − 4q)/(5q). [1½]

The right side is a ratio of integers, so √3 would be rational. This contradicts that √3 is irrational. [1]

Hence **4 + 5√3 is irrational**. [½]

Q.18 (3 marks)

(A) Let x = 2.1666… One digit is non-repeating, so 10x = 21.666…

Move one full cycle: 100x = 216.666… [1]

Subtract: 100x − 10x = 216.666… − 21.666… ⇒ 90x = 195. [1]

x = 195/90 =

Ans. 13/6. [1] ***(Check: 13 ÷ 6 = 2.1666… (verified))***

OR (B) Let x = 0.2454545… One digit (2) is non-repeating, so 10x = 2.454545…

The block 45 has 2 digits: multiply by 100 again, 1000x = 245.4545… [1]

Subtract: 1000x − 10x = 245.4545… − 2.4545… ⇒ 990x = 243. [1]

x = 243/990 =

Ans. 27/110. [1] ***(Check: 27 ÷ 110 = 0.24545… (verified))***

Section D (5 marks each)

Q.19 (5 marks)

(A)

(i) 3/5 = 54/90 and 2/3 = 60/90 (denominator 90 leaves room for four numbers). [1]

Four rational numbers between them: **55/90, 56/90, 57/90, 58/90**, i.e. 11/18, 28/45, 19/30, 29/45. [2]

(ii) 3/5 = 0.6 and 2/3 = 0.666… An irrational number between them is **0.6101001000100001…** [1]

It lies between 0.6 and 0.666…, and it never ends and never repeats (the zeros keep increasing), so it is irrational. [1]

OR (B)

(i) Let x = 0.999… Then 10x = 9.999… [½]

Subtracting: 10x − x = 9 ⇒ 9x = 9 ⇒ **x = 1**. So 0.999… = 1. [1½]

(ii) Let x = 0.0363636… One digit (0) is non-repeating: 10x = 0.363636… [1]

The block 36 has 2 digits: 1000x = 36.3636… [1]

Subtract: 1000x − 10x = 36 ⇒ 990x = 36 ⇒ x = 36/990 = **2/55**. [1] *(Check: 2 ÷ 55 = 0.03636… (verified))*

Q.20 (5 marks)

(A) Cloth = 18¾ = 75/4 m; one shirt needs 2½ = 5/2 m.

(i) Shirts = (75/4) ÷ (5/2) = (75/4) × (2/5) = 150/20 = 15/2 = 7½. [2]

So he can make **7 complete shirts**. [1]

(ii) Cloth used = 7 × 5/2 = 35/2 m. Cloth left = 75/4 − 35/2 = 75/4 − 70/4 = 5/4. [1]

Ans. He has 1¼ m of cloth left over. [1]

OR (B)

(i) 33/500: 500 = $2^2 \times 5^3$, only 2 and 5, so **terminating**. [1]

(ii) 13/30: 30 = 2 × 3 × 5 has the prime factor 3, so **non-terminating repeating**. [1]

(iii) 49/7000 = 7/1000 (lowest terms); 1000 = $2^3 \times 5^3$, so **terminating**. [1]

(iv) 33/500 = 33 × 2/(500 × 2) = 66/1000 = **0.066**. [1] And 7/1000 = **0.007**. [1]

Section E (4 marks)

Q.21 (4 marks)

(i) Each loan is a debt of ₹75 = −75; 4 loans give (−75) × 4 = **−300**. (A debt times a fortune is a debt.) **[1]**

(ii) New balance = −250 + (−300) = **−550**, a debt of ₹550. **[1]**

(iii) (A) Removing 3 debts of ₹75 changes the balance by (−3) × (−75) = +225 (debt × debt = fortune). [1]

New balance = −550 + 225 = **−325** (a debt of ₹325). [1]

OR (iii) (B)

New balance = −550 + 900 = **350**. [1]

It is positive, so it is a **fortune (dhana) of ₹350**. [1]

— End of Solutions —