CLASS IX – MATHEMATICS

Chapter Test: Exploring Algebraic Identities

Step-by-Step Solutions and Marking Scheme | M. M.: 40

Identities used

$(a + b)^2 = a^2 + 2ab + b^2$ | $(a - b)^2 = a^2 - 2ab + b^2$ | $a^2 - b^2 = (a + b)(a - b)$ | $(x + a)(x + b) = x^2 + (a + b)x + ab$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ | $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ | $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ | $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Section A (1 mark each)

Q	Ans	Reason
1	(B)	$(3x + 2y)^2 = (3x)^2 + 2(3x)(2y) + (2y)^2 =$ **$9x^2 + 12xy + 4y^2$**.
2	(C)	$a^2 + b^2 = (a + b)^2 - 2ab = 7^2 - 2(12) = 49 - 24 =$ **25**.
3	(D)	$98^2 = (100 - 2)^2 = 100^2 - 2(100)(2) + 2^2 = 10000 - 400 + 4 =$ **9604**.
4	(A)	$(x + 3)(x - 7) = x^2 + (3 - 7)x + (3)(-7) =$ **$x^2 - 4x - 21$**.
5	(B)	$x^2 - 10x + 25 = x^2 - 2(x)(5) + 5^2 =$ **$(x - 5)^2$**.
6	(C)	$(x + 1/x)^2 = x^2 + 2 + 1/x^2 \Rightarrow 25 = x^2 + 1/x^2 + 2 \Rightarrow x^2 + 1/x^2 =$ **23**.
7	(A)	$101 \times 99 = (100 + 1)(100 - 1) = 100^2 - 1^2 =$ **9999**.
8	(D)	The term in bc is $2 \times (2b) \times (3c) = 12bc$, so the coefficient is **12**.
9	(B)	From the figure, the length is an x^2 tile plus 3 x-tiles = x + 3 and the breadth is an x^2 tile plus 2 x-tiles = x + 2. $(x + 2)(x + 3) = x^2 + 5x + 6$: one x^2 tile, five x-tiles, six unit tiles (verified). So **(x + 2) and (x + 3)**.
10	(C)	$x^3 + 8 = x^3 + 2^3$. Using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with a = x, b = 2: $x^3 + 8 =$ **$(x + 2)(x^2 - 2x + 4)$**.
11	(D)	A is false: for a = 2, b = −1, $(a + b)^2 = 1$ but $a^2 + b^2 = 5$, so $(a + b)^2$ is smaller. (It is greater only when ab > 0.) R is a true identity. → **(D)**.
12	(A)	A is true: $(2x + 3)^2 = 4x^2 + 12x + 9$. R is true and gives exactly this factorisation, so R explains A. → **(A)**.

Section B (2 marks each)

Q.13 (2 marks)

Use $(a - b)^2 = a^2 - 2ab + b^2$ with a = 3x, b = 5y. **[½]**

$(3x - 5y)^2 = (3x)^2 - 2(3x)(5y) + (5y)^2$ **[1]**

Ans. = $9x^2 - 30xy + 25y^2$. [½]

Q.14 (2 marks)

(A)

$49p^2 = (7p)^2$, $9q^2 = (3q)^2$ and $42pq = 2(7p)(3q)$, so the expression has the form $a^2 + 2ab + b^2$ with a = 7p, b = 3q. **[1]**

$49p^2 + 42pq + 9q^2 =$ **$(7p + 3q)^2$**. **[1]**

OR (B)

$81a^2 - 4b^2 = (9a)^2 - (2b)^2$ **[1]**

Using $a^2 - b^2 = (a + b)(a - b)$: **$(9a + 2b)(9a - 2b)$**. **[1]**

Q.15 (2 marks)

1004 = 1000 + 4. Using $(a + b)^2 = a^2 + 2ab + b^2$: **[½]**

$(1000 + 4)^2 = 1000^2 + 2(1000)(4) + 4^2 = 1\,000\,000 + 8000 + 16$ **[1]**

Ans. = 1 008 016. [½]

Q.16 (2 marks)

(A)

$a^2 + b^2 = (a + b)^2 - 2ab = 81 - 40 =$ **41**. **[1]**

$(a - b)^2 = (a + b)^2 - 4ab = 81 - 80 =$ **1**. **[1]**

OR (B)

$(x - 1/x)^2 = x^2 - 2 + 1/x^2 \Rightarrow 16 = x^2 + 1/x^2 - 2 \Rightarrow x^2 + 1/x^2 =$ **18**. **[1]**

$(x + 1/x)^2 = x^2 + 2 + 1/x^2 = 18 + 2 =$ **20**. **[1]**

Section C (3 marks each)

Q.17 (3 marks)

(A) Use $(a + b + c)^2$ with a = 2x, b = 3y, c = −z.

$(2x + 3y - z)^2 = (2x)^2 + (3y)^2 + (-z)^2 + 2(2x)(3y) + 2(3y)(-z) + 2(-z)(2x)$ **[2]**

$=$ **$4x^2 + 9y^2 + z^2 + 12xy - 6yz - 4zx$**. **[1]**

OR (B) Use $(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$ with A = 3a and B = 2b.

$(3a - 2b)^3 = (3a)^3 - 3(3a)^2(2b) + 3(3a)(2b)^2 - (2b)^3$ **[2]**

$= 27a^3 - 3(9a^2)(2b) + 3(3a)(4b^2) - 8b^3 =$ **$27a^3 - 54a^2b + 36ab^2 - 8b^3$**. **[1]**

Q.18 (3 marks)

(A) $2x^2 + 11x + 12$. Product of the first and last coefficients = 2 × 12 = 24. We need two numbers with product 24 and sum 11: 8 and 3.

$2x^2 + 11x + 12 = 2x^2 + 8x + 3x + 12$ **[1]**

$= 2x(x + 4) + 3(x + 4)$ **[1]**

$=$ **$(x + 4)(2x + 3)$**. **[1]**

OR (B) $64m^3 + 27n^3 = (4m)^3 + (3n)^3$. Using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with a = 4m, b = 3n:

$= (4m + 3n)[(4m)^2 - (4m)(3n) + (3n)^2]$ **[2]**

$=$ **$(4m + 3n)(16m^2 - 12mn + 9n^2)$**. **[1]**

Section D (5 marks each)

Q.19 (5 marks)

(A)

Numerator: $x^3 - 8 = x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$. **[2]**

Denominator: $x^2 + x - 6$. We need a + b = 1 and ab = −6, so a = 3, b = −2: $x^2 + 3x - 2x - 6 = x(x + 3) - 2(x + 3) = (x + 3)(x - 2)$. **[2]**

$(x^3 - 8)/(x^2 + x - 6) = (x - 2)(x^2 + 2x + 4) / [(x + 3)(x - 2)]$. Cancelling the common factor $(x - 2)$ $(x \neq 2)$:

Ans. $(x^2 + 2x + 4)/(x + 3)$, for $x \neq 2$ and $x \neq -3$. [1]

OR (B)

Numerator: $4x^2 - 12xy + 9y^2 = (2x)^2 - 2(2x)(3y) + (3y)^2 = (2x - 3y)^2$. **[2]**

Denominator: $8x^3 - 27y^3 = (2x)^3 - (3y)^3 = (2x - 3y)[(2x)^2 + (2x)(3y) + (3y)^2] = (2x - 3y)(4x^2 + 6xy + 9y^2)$. **[2]**

Cancelling the common factor $(2x - 3y)$, which is not zero:

Ans. $(2x - 3y)/(4x^2 + 6xy + 9y^2)$. [1]

Q.20 (5 marks)

(A) Let the breadth be x m. Then the length is (x + 7) m.

Area = length × breadth ⇒ $x(x + 7) = 330 \Rightarrow x^2 + 7x - 330 = 0$. **[1]**

We need two numbers with sum 7 and product −330: 22 and −15. So $7x = 22x - 15x$. **[1]**

$x^2 + 22x - 15x - 330 = x(x + 22) - 15(x + 22) = (x + 22)(x - 15) = 0$. **[1]**

$x = -22$ or $x = 15$. A length cannot be negative, so $x = 15$. **[1]**

Ans. Breadth = 15 m and length = 15 + 7 = 22 m. [1] ***(Check: 22 × 15 = 330 (verified))***

OR (B)

(i) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx) \Rightarrow 81 = x^2 + y^2 + z^2 + 2(26)$ **[1]**

$x^2 + y^2 + z^2 = 81 - 52 =$ **29**. **[1]**

(ii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ **[1]**

$= 9 \times (29 - 26) = 9 \times 3$ **[1]**

Ans. = 27. [1]

Section E (4 marks)

Q.21 (4 marks)

(i) Red square: $a \times a = a^2$. Yellow square: $b \times b = b^2$. Each green rectangle: $a \times b = ab$ (two of them: 2ab). Sum of the parts = $a^2 + 2ab + b^2$. The whole rangoli is a square of side (a + b), so its area is $(a + b)^2$. Hence **$(a + b)^2 = a^2 + 2ab + b^2$**. **[1]**

(ii) Area = $(a + b)^2 = (7 + 3)^2 = 10^2 =$ **100 m²**. *(Check: 49 + 42 + 9 = 100 (verified).)* **[1]**

(iii) (A) Red + yellow = $a^2 + b^2 = (a + b)^2 - 2ab$ [1] $= 12^2 - 2(35) = 144 - 70 =$ **74 m²**. [1]

OR (iii) (B)

$(a - b)^2 = a^2 + b^2 - 2ab \Rightarrow 3^2 = 29 - 2ab \Rightarrow 2ab = 20 \Rightarrow ab =$ **10 m²** (area of one green rectangle). [1]

$(a + b)^2 = a^2 + b^2 + 2ab = 29 + 20 = 49 \Rightarrow a + b = 7$ (a length is positive). Side of the whole rangoli = **7 m**. [1] *(Check: $a = 5, b = 2$ gives $a - b = 3$, $a^2 + b^2 = 29$ (verified).)*

— End of Solutions —