CLASS IX – MATHEMATICS

Chapter Test: I'm Up and Down, and Round and Round (Circles)

Step-by-Step Solutions and Marking Scheme | M. M.: 40

Results used

The perpendicular from the centre to a chord bisects the chord, so (radius)² = (distance)² + (half-chord)². | Equal chords are equidistant from the centre and conversely; a longer chord is nearer the centre.

Angle at the centre = 2 × angle at the remaining part of the circle. | Angle in a semicircle = 90°. | Opposite angles of a cyclic quadrilateral add to 180°; an exterior angle equals the interior opposite angle.

Section A (1 mark each)

Q	Ans	Reason
1	(B)	Radius = 25 cm, half-chord = 24 cm. Distance = $\sqrt{(25^2 - 24^2)} = \sqrt{49}$ = **7 cm**.
2	(C)	The longest chord is the diameter = 2 × 6.5 = **13 cm**.
3	(A)	Angle at the circumference = ½ × 130° = **65°**.
4	(B)	PQ is a diameter, so the angle in a semicircle ∠PRQ = **90°**.
5	(D)	Opposite angles of a cyclic quadrilateral are supplementary: ∠D = 180° − 110° = **70°**.
6	(C)	The angle in a semicircle is 90°, so the hypotenuse is a diameter of the circumcircle; the circumcentre is the **midpoint of the hypotenuse**.
7	(D)	Centres lie on the perpendicular bisector of the two points, which has infinitely many points: **infinitely many** circles.
8	(B)	The perpendicular from O bisects AB, so AM = 8 cm. OM = $\sqrt{(10^2 - 8^2)} = \sqrt{36}$ = **6 cm**.
9	(D)	C is on the major arc, so ∠ACB = ½ ∠AOB = ½ × 100° = **50°**.
10	(A)	A longer chord is **nearer to the centre** (the diameter, the longest chord, passes through the centre).
11	(D)	A is false: three collinear points do not lie on any circle. R is true (exactly one circle through three non-collinear points). → **(D)**.
12	(B)	A is true: equal chords are equidistant from the centre. R is true (the diameter is the longest chord) but it does not explain A. → **(B)**.

Section B (2 marks each)

Q.13 (2 marks)

Let the chord be AB, with M its foot from the centre O. OA = 29 cm, OM = 20 cm. In right ΔOMA, AM² = 29² − 20² = 841 − 400 = 441. **[1]**

AM = 21 cm. The perpendicular from the centre bisects the chord, so AB = 2 × 21 =

Ans. 42 cm. [1]

Q.14 (2 marks)

(A)

The angle at the centre is twice the angle at the remaining part of the circle. **[1]**

Required angle = ½ × 124° = **62°**. **[1]**

OR (B)

OA = OB = 9 cm (radii) and ∠AOB = 60°, so ∠OAB = ∠OBA = (180° − 60°)/2 = 60°. ΔOAB is equilateral. **[1]**

Hence AB = OA = **9 cm**. **[1]**

Q.15 (2 marks)

Opposite angles of a cyclic quadrilateral add to 180°: (3x + 5) + (2x + 15) = 180 ⇒ 5x + 20 = 180 ⇒ x = 32. **[1]**

Ans. x = 32, ∠A = 3(32) + 5 = 101°, ∠C = 2(32) + 15 = 79°. [1] ***(101 + 79 = 180 (verified))***

Q.16 (2 marks)

(A)

MN is a diameter, so ∠MPN is an angle in a semicircle: **∠MPN = 90°**. **[1]**

In ΔMPN, ∠PNM = 180° − 90° − 35° = **55°**. **[1]**

OR (B)

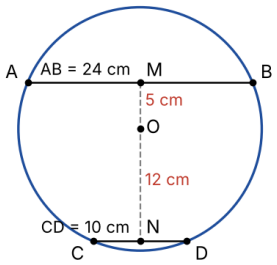
Half-chord = 9 cm, distance = 12 cm. Radius² = 9² + 12² = 81 + 144 = 225. **[1]**

Radius = **15 cm**. **[1]**

Section C (3 marks each)

Q.17 (3 marks)

(A) Draw OM ⊥ AB and ON ⊥ CD. The perpendiculars from the centre bisect the chords, so AM = 12 cm and CN = 5 cm. (M, O, N lie on one line since AB ∥ CD.)



OM = √(13² − 12²) = √25 = 5 cm. **[1]**

ON = √(13² − 5²) = √144 = 12 cm. **[1]**

The chords are on opposite sides of O, so the distance between them = OM + ON = 5 + 12 =

Ans. 17 cm. [1]

OR (B)

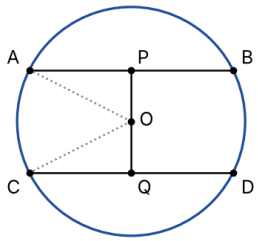
Opposite angles of a cyclic quadrilateral add to 180°: ∠C = 180° − ∠A = 180° − 65° = **115°**. **[1]**

∠D = 180° − ∠B = 180° − 85° = **95°**. **[1]**

∠BCE and ∠BCD form a linear pair: ∠BCE = 180° − 115° = **65°** (equal to the interior opposite angle ∠A). [1]

Q.18 (3 marks)

(A) *Given:* chords AB and CD of a circle with centre O, with OP ⊥ AB, OQ ⊥ CD and OP = OQ. *To prove:* AB = CD.



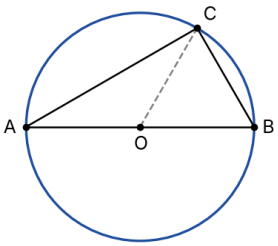
Join OA and OC. Then OA = OC (radii). [½]

∠OPA = ∠OQC = 90°, OP = OQ (given), and the hypotenuses OA = OC. So ΔOPA is congruent to ΔOQC (RHS). [1]

Hence AP = CQ (CPCT). [½]

The perpendicular from the centre bisects the chord: AB = 2AP and CD = 2CQ. Therefore **AB = CD**. [1]

OR (B) *Given:* AB is a diameter of a circle with centre O and C is any point on the circle. *To prove:* ∠ACB = 90°.



Join OC. Arc AB subtends the straight angle ∠AOB = 180° at the centre. [1]

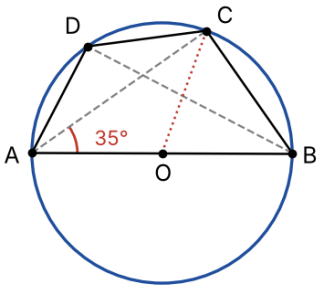
The angle at the centre is twice the angle at the remaining part of the circle, so ∠ACB = ½ × 180°. [1]

Hence **∠ACB = 90°**. (Alternatively: OA = OB = OC, so ∠OAC = ∠OCA and ∠OBC = ∠OCB; the angle sum of ΔABC gives 2(∠OCA + ∠OCB) = 180°.) [1]

Section D (5 marks each)

Q.19 (5 marks)

(A) AB is a diameter and ∠CAB = 35°.



(i) ∠ACB is an angle in a semicircle: **∠ACB = 90°**. **[1]**

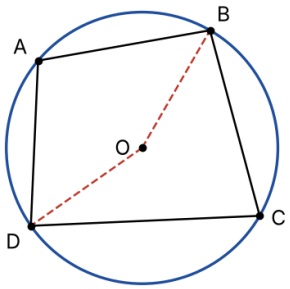
(ii) In ΔABC: ∠ABC = 180° − 90° − 35° = **55°**. **[1]**

(iii) ABCD is cyclic, so ∠ADC = 180° − ∠ABC = 180° − 55° = **125°**. **[1]**

(iv) ∠BDC and ∠BAC stand on the same arc BC, so **∠BDC = ∠CAB = 35°**. **[1]**

(v) ∠AOC is the angle at the centre on arc AC and ∠ABC = 55° is the angle on the remaining part, so **∠AOC = 2 × 55° = 110°**. **[1]**

OR (B) *Given:* cyclic quadrilateral ABCD with centre O. *To prove:* ∠A + ∠C = 180° and ∠B + ∠D = 180°.



Join OB and OD. Arc BCD subtends the reflex angle ∠BOD (reflex) at the centre and the angle ∠BAD = ∠A at the remaining part. So ∠A = ½ × (reflex ∠BOD). [1½]

Arc BAD subtends ∠BOD (non-reflex) at the centre and ∠BCD = ∠C at the remaining part. So ∠C = ½ × ∠BOD. [1½]

Adding: ∠A + ∠C = ½ (reflex ∠BOD + ∠BOD) = ½ × 360° = **180°**. [1]

In the same way, ∠B + ∠D = 360° − 180° = **180°** (angle sum of the quadrilateral). [1]

Q.20 (5 marks)

(A) Radius OA = OC = 17 m. Let OM ⊥ AB and ON ⊥ CD.

(i) AM = 30/2 = 15 m. OM = √(17² − 15²) = √(289 − 225) = √64 = **8 m**. **[2]**

(ii) CN = 16/2 = 8 m. ON = √(17² − 8²) = √(289 − 64) = √225 = **15 m**. **[1]**

(iii) **AB is nearer** (8 m < 15 m). The longer chord is nearer to the centre. [1]

(iv) The paths are on opposite sides of O: distance = 8 + 15 = **23 m**. **[1]**

OR (B)

(i) $AC^2 = 6^2 + 8^2 = 100 \Rightarrow AC = 10$ cm. The angle in a semicircle is 90°, so the hypotenuse AC is a diameter of the circumcircle. The circumcentre is the **midpoint of the hypotenuse** and the circumradius = 10/2 = **5 cm**. **[2]**

(ii) Draw PM ⊥ QR. In an isosceles triangle M is the midpoint of QR, so QM = 6 cm and PM = √(10² − 6²) = 8 cm. [1]

The circumcentre O lies on PM. Let OP = OQ = R, so OM = 8 − R. In right ΔOMQ: $R^2 = (8 - R)^2 + 6^2$ [1]

$R^2 = 64 - 16R + R^2 + 36 \Rightarrow 16R = 100 \Rightarrow R = 6.25$.

Ans. Circumradius = 6.25 cm. [1] ***(Check: abc/4×Area = 1200/192 = 6.25 (verified))***

Section E (4 marks)

Q.21 (4 marks)

(i) Half of AB = $\sqrt{(25^2 - 15^2)} = \sqrt{400} = 20$ m. AB = $2 \times 20 =$ **40 m**. **[1]**

(ii) Half of CD = 15 m. Distance from O = $\sqrt{(25^2 - 15^2)} = \sqrt{400} =$ **20 m**. **[1]**

(iii) (A) Opposite sides of the centre: distance = 15 + 20 = **35 m**. [2]

OR (iii) (B)

Same side of the centre: distance = 20 − 15 = **5 m**. [2]

— End of Solutions —