CLASS IX – MATHEMATICS

Chapter Test: Predicting What Comes Next – Exploring Sequences and Progressions

Step-by-Step Solutions and Marking Scheme | M. M.: 40

Formulas used

AP: $t_n = a + (n-1)d$ **GP:** $t_n = ar^{n-1}$ **Sum of first n natural numbers:** $S_n = n(n+1)/2$

Section A (1 mark each)

Q	Ans	Reason
1	(B)	$t_5 = 3(5) - 7 = 15 - 7 =$ **8**.
2	(A)	$d = 7 - 11 =$ **−4** (each term is 4 less than the previous one).
3	(C)	$a = 4, d = 5$: $t_{10} = 4 + (10-1) \times 5 = 4 + 45 =$ **49**.
4	(D)	$r = 27/81 =$ **1/3** (also 9/27 = 1/3, 3/9 = 1/3).
5	(C)	$S_{20} = 20 \times 21/2 =$ **210**.
6	(B)	$t_2 = 3(2) - 1 = 5$; $t_3 = 3(5) - 1 =$ **14**.
7	(A)	2, 6, 18, 54: each ratio = 3, a GP. In 1, 4, 9, 16 the ratios are 4 and 9/4; in 3, 5, 7, 9 they are 5/3 and 7/5; in 1, 1, 2, 3 they are 1 and 2 (not constant).
8	(B)	Dots are 1, 3, 6, 10, … (triangular numbers): Stage *n* has $n(n+1)/2$ dots. Stage 6: $6 \times 7/2 =$ **21**.
9	(D)	$7 + 6(n-1) = 139 \Rightarrow 6(n-1) = 132 \Rightarrow n - 1 = 22 \Rightarrow n =$ **23**, the 23rd term.
10	(A)	$a = 2, r = 3$: $t_6 = 2 \times 3^5 = 2 \times 243 =$ **486**.
11	(D)	A is false: the differences in 1, 4, 9, 16, 25 are 3, 5, 7, 9 (not constant), so it is not an AP. R is a true statement. → **(D)**.
12	(C)	A is true: $S_{100} = 100 \times 101/2 = 5050$. R is false: 1, 2, 3, … has a constant difference 1, so it is an AP; the ratios 2, 3/2, … are not constant. → **(C)**.

Section B (2 marks each)

Q.13 (2 marks)

$t_1 = 4$ **[½]**

$t_2 = 2(4) - 3 = 5$; $t_3 = 2(5) - 3 = 7$ **[1]**

$t_4 = 2(7) - 3 = 11$; $t_5 = 2(11) - 3 = 19$ **[½]**

Ans. The first five terms are 4, 5, 7, 11, 19.

Q.14 (2 marks)

(A)

For 150: $4n - 6 = 150 \Rightarrow 4n = 156 \Rightarrow n = 39$, a natural number. So 150 is the **39th term**. **[1]**

For 173: $4n - 6 = 173 \Rightarrow 4n = 179 \Rightarrow n = 44.75$, not a natural number. So 173 is **not** a term. **[1]**

OR (B)

$a = 5, r = 10/5 = 2$. $t_9 = ar^8 = 5 \times 2^8 = 5 \times 256$ **[1]**

Ans. $t_9 = 1280$. [1]

Q.15 (2 marks)

$21 + 22 + \ldots + 50 = (1 + 2 + \ldots + 50) - (1 + 2 + \ldots + 20) = S_{50} - S_{20}$ **[1]**

$= 50 \times 51/2 - 20 \times 21/2 = 1275 - 210$ **[½]**

Ans. 1065. [½]

Q.16 (2 marks)

(A) a = 9, d = 13 − 9 = 4

Explicit rule: $t_n = 9 + (n - 1) \times 4 =$ **4n + 5**. **[1]**

Recursive rule: $\mathbf{t_1 = 9,\ t_n = t_{n-1} + 4}$ **for n ≥ 2**. **[1]**

OR (B)

Fare for 1 km = 50 + 12 = ₹62; 2 km = 50 + 24 = ₹74; 3 km = 50 + 36 = ₹86. Sequence: **62, 74, 86, ...** **[1]**

It is an AP with a = 62, d = 12, so $t_n = 62 + (n - 1) \times 12 =$ **50 + 12n**. **[1]**

Section C (3 marks each)

Q.17 (3 marks)

(A)

$t_4 = a + 3d = 17$ ...(1) $t_9 = a + 8d = 42$ ...(2) **[1]**

Subtracting (1) from (2): 5d = 25 ⇒ d = 5. From (1): a = 17 − 15 = 2. **[1]**

The AP is **2, 7, 12, 17, ...** and $t_{25} = 2 + 24 \times 5 =$ **122**. **[1]**

OR (B)

Smallest three-digit multiple of 8 = 104 (8 × 13); largest = 992 (8 × 124). They form an AP with a = 104, d = 8. **[1]**

104 + (n − 1) × 8 = 992 ⇒ (n − 1) × 8 = 888 ⇒ n − 1 = 111 **[1]**

Ans. n = 112, so there are 112 such numbers. [1]

Q.18 (3 marks)

(A)

$t_3 = ar^2 = 12$ ...(1) $t_6 = ar^5 = 96$ ...(2) **[1]**

(2) ÷ (1): $r^3 = 8 \Rightarrow$ **r = 2**. From (1): a × 4 = 12 ⇒ **a = 3**. **[1]**

$t_8 = 3 \times 2^7 = 3 \times 128 =$ **384**. **[1]**

OR (B) The numbers 500, 1000, 2000, ... form a GP with a = 500, r = 2.

After 3 hours: $500 \times 2^3 =$ **4000**. **[1]**

After 6 hours: $500 \times 2^6 = 500 \times 64 =$ **32 000**. **[1]**

After n hours: $\mathbf{500 \times 2^n}$. **[1]**

Section D (5 marks each)

Q.19 (5 marks)

(A)

$t_5 = a + 4d = 23$ ...(1) $t_{40} = a + 39d = 198$ ...(2) **[1]**

(2) − (1): 35d = 175 ⇒ **d = 5** [1]. From (1): a = 23 − 20 ⇒ **a = 3** [1]. **[2]**

$t_{27} = 3 + 26 \times 5 = 3 + 130 =$ **133**. **[1]**

3 + (n − 1) × 5 = 108 ⇒ (n − 1) × 5 = 105 ⇒ n − 1 = 21 ⇒ n = 22. So 108 is the $\mathbf{22^{nd}}$ **term**. **[1]**

OR (B) The number of logs per row from the bottom: 24, 23, 22, ..., 1.

(i) Rows are 24, 23, ..., 1: there are **24 rows**. **[1]**

(ii) Total = 1 + 2 + ... + 24 = $S_{24} = 24 \times 25/2 =$ **300 logs**. **[2]**

(iii) Bottom 10 rows have 24 + 23 + ... + 15 logs = (1 + 2 + ... + 24) − (1 + 2 + ... + 14) = $S_{24} - S_{14} = 300 - 14 \times 15/2 = 300 - 105 =$ **195 logs**. **[2]**

Q.20 (5 marks)

(A) Heights after bounces: 64, 32, 16, 8, 4, ... is a GP with a = 64 (first bounce), r = ½.

(i) Height after the 5^{th} bounce = $128 \times (½)^5 = 128/32 =$ **4 m**. **[2]**

(ii) Explicit rule: $h_n = 128 \times (½)^n$. Recursive rule: $h_1 = 64$, $h_n = ½ \times h_{n-1}$ for n ≥ 2. **[1]**

(iii) We need $128 \times (½)^n < 1$, that is $2^n > 128 = 2^7$. For n = 7 the height is exactly 1 m (not below); for n = 8 it is 0.5 m. **[1]**

Ans. 8 bounces. [1]

OR (B)

(i) Each triangle is replaced by 3: Stage 0 → 1, Stage 1 → **3**, Stage 2 → **9**, Stage 3 → **27**. Number at Stage n = $\mathbf{3^n}$. **[2]**

(ii) Each triangle at Stage n has area $64/4^n$. Total area = number × area of one: Stage 1: 3 × 16 = **48**; Stage 2: 9 × 4 = **36**; Stage 3: 27 × 1 = **27** square units. **[1½]**

(iii) Total area = $64 \times (¾)^n$ [½]. Stage 4: 64 × 81/256 = **20.25 square units** [½]. Each stage's area is ¾ of the previous (a GP with r = ¾ < 1), so it keeps **decreasing** and gets closer and closer to 0 [½]. **[1½]**

Section E (4 marks)

Q.21 (4 marks)

AP: a = 20, d = 2 (20, 22, 24, 26, ...)

(i) $t_6 = 20 + (6 - 1) \times 2 = 20 + 10 =$ **30 seats**. **[1]**

(ii) $20 + (n - 1) \times 2 = 50 \Rightarrow (n - 1) \times 2 = 30 \Rightarrow n - 1 = 15 \Rightarrow n = 16$. The **$16^{th}$ row** has 50 seats. **[1]**

(iii) (A) $t_{25} = 20 + (25 - 1) \times 2$ [1] $= 20 + 48 =$ **68 seats**. [1]

OR (iii) (B)

$20 + (n - 1) \times 2 = 75 \Rightarrow (n - 1) \times 2 = 55 \Rightarrow n - 1 = 27.5 \Rightarrow n = 28.5$ [1]

n must be a natural number, so no row has 75 seats: **the manager is not correct**. (Every row has an even number of seats, since 20 and 2 are even.) [1]

— End of Solutions —