# CLASS IX – MATHEMATICS

## Chapter Test: The World of Algorithms

(Ganita Manjari, Grade 9, Part II – Chapter 11)

**Time: 1½ Hours** **M. M.: 40**

**General Instructions:**

1. This paper contains 21 questions. All questions are compulsory.
2. This paper is divided into 5 sections: **A, B, C, D** and **E**.
3. In **Section A**, Questions no. **1** to **10** are MCQs and questions no. **11** and **12** are Assertion-Reason based questions of 1 mark each.
4. In **Section B**, Questions no. **13** to **16** are Very Short Answer type questions carrying 2 marks each.
5. In **Section C**, Questions no. **17** and **18** are Short Answer type questions carrying 3 marks each.
6. In **Section D**, Questions no. **19** and **20** are Long Answer type questions carrying 5 marks each.
7. In **Section E**, Question no. **21** is a case based question carrying 4 marks. Internal choice is provided in the 2 marks part of the case study.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 1 part of Section E.
9. Show every step of an algorithm that you execute. Figures are not drawn to scale.
10. Use of calculator is **not** allowed.

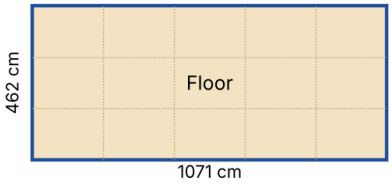
| Q. No. | Section – A | Marks |
|---|---|---|
| **1.** | A step-by-step procedure to solve a problem is called<br>(A) a theorem (B) an algorithm (C) a data type (D) an equation | **[1]** |
| **2.** | While adding 4687 and 5538 by the digit-by-digit addition algorithm, the first (units) column gives the digit written and the carry as<br>(A) 5 and 1 (B) 15 and 0 (C) 5 and 0 (D) 1 and 5 | **[1]** |
| **3.** | Executing the algorithm for the divisors of 20 gives the list<br>(A) [1, 2, 4, 5, 20] (B) [1, 2, 4, 5, 10] (C) [1, 2, 4, 5, 10, 20] (D) [2, 4, 5, 10, 20] | **[1]** |
| **4.** | The value of 95 mod 8 (the remainder when 95 is divided by 8) is<br>(A) 11 (B) 5 (C) 12 (D) 7 | **[1]** |
| **5.** | The greatest common divisor of 54 and 36 is<br>(A) 6 (B) 18 (C) 9 (D) 108 | **[1]** |
| **6.** | By Euclid's rule $\gcd(m, n) = \gcd(n, m - n)$, the first reduction of gcd(72, 30) gives<br>(A) gcd(102, 30) (B) gcd(42, 12) (C) gcd(30, 42) (D) gcd(72, 102) | **[1]** |
| **7.** | In Euclid's algorithm, $\gcd(m, 0)$ is reported as<br>(A) $m$ (B) 0 (C) 1 (D) $m + 1$ | **[1]** |
| **8.** | Using the division (remainder) version of Euclid's algorithm, the number of reduction steps needed for gcd(101, 3) is<br>(A) 2 (B) 3 (C) 4 (D) 33 | **[1]** |
| **9.** | In the algorithm that scans $k$ from 2 to $\min(m, n)$ and updates the most-recent-common-divisor (initially 1), for $m = 12$ and $n = 18$ the value of the most-recent-common-divisor after $k = 4$ has been checked is<br>(A) 2 (B) 4 (C) 3 (D) 6 | **[1]** |
| **10.** | When two numbers are added by the digit-by-digit algorithm, the largest possible sum in any column (two digits and a carry) is<br>(A) 10 (B) 18 (C) 20 (D) 19 | **[1]** |

| Q. No. | Section – A | Marks |
|---|---|---|
| | Questions number 11 and 12 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.<br>(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).<br>(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).<br>(C) Assertion (A) is true but Reason (R) is false.<br>(D) Assertion (A) is false but Reason (R) is true. | |
| **11.** | **Assertion (A):** If the smaller of two numbers changes from a 3-digit to a 5-digit number, the scanning algorithm for gcd needs about 100 times as many checks.<br>**Reason (R):** The scanning algorithm checks every number from 1 to min($m$, $n$), so its work grows with the value of the numbers. | **[1]** |
| **12.** | **Assertion (A):** Euclid's subtraction algorithm needs only 2 reduction steps to find gcd(99, 2).<br>**Reason (R):** The division (remainder) version of Euclid's algorithm needs only 2 reduction steps for gcd(99, 2). | **[1]** |

| Q. No. | Section – B | Marks |
|---|---|---|
| **13.** | Execute the algorithm for the list of divisors of 28 (check each $j$ from 1 to 28) and write the final list. | **[2]** |
| **14.** | (A) Add 6847 and 3759 by the digit-by-digit algorithm, showing the digit written and the carry in each column.<br>**OR**<br>(B) Add 85 and 47 by the algorithm. What would go wrong if the final step ("if the carry is 1, write 1 to the left of the bottom row") were omitted? | **[2]** |
| **15.** | Find gcd(36, 60) by writing the lists of divisors of both numbers, the list of common divisors, and picking the rightmost element. | **[2]** |
| **16.** | (A) Execute the most-recent-common-divisor algorithm for $m = 8$ and $n = 12$. Show the value after each update and the final gcd.<br>**OR**<br>(B) Execute the same algorithm for $m = 18$ and $n = 24$. Show every value taken by the most-recent-common-divisor and the final gcd. | **[2]** |

| Q. No. | Section – C | Marks |
|---|---|---|
| **17.** | (A) Use Euclid's subtraction algorithm to find gcd(48, 18). Write every reduced problem in order and count the reductions.<br>**OR**<br>(B) Let $m \geq n$. Prove that every common divisor of $m$ and $n$ is a common divisor of $n$ and $m - n$, and conversely. Hence conclude $\gcd(m, n) = \gcd(n, m - n)$. | **[3]** |
| **18.** | (A) Use the division algorithm (reduce $\gcd(m, n)$ to $\gcd(n, m \bmod n)$) to find gcd(924, 168). Show each step.<br>**OR**<br>(B) Use the division algorithm to find gcd(2025, 1215). Show each step. | **[3]** |

| Q. No. | Section – D | Marks |
|---|---|---|
| **19.** | (A) Write an algorithm *prime*($n$) that uses the list divisors($n$) to decide whether $n$ is prime. [3] Execute it for $n = 15$ and for $n = 13$. [2]<br>**OR**<br>(B) (i) Find gcd(252, 105) using the division algorithm, showing each step. [3] (ii) Using lcm($m$, $n$) = $m \times n \div$ gcd($m$, $n$), find the lcm of 252 and 105. [2] | **[5]** |
| **20.** | (A) (i) How many reductions does Euclid's subtraction algorithm need for gcd(35, 2)? [2] (ii) How many reductions does the division version need for gcd(35, 2)? Show the steps. [2] (iii) Which version is better when one number is much smaller than the other? Why? [1]<br>**OR**<br>(B) (i) Find gcd(8736, 3640) with the division algorithm, showing each step. [3] (ii) How many values of $k$ would the scanning algorithm ($k$ from 2 to min($m$, $n$)) have to check for these numbers? [1] (iii) Which method is faster? [1] | **[5]** |

| Q. No. | Section – E | Marks |
|---|---|---|
| **21.** | **Case Study:** A builder is tiling a rectangular floor of size 1071 cm by 462 cm with identical square tiles. The tiles must fit exactly, with no cutting, and each tile should be as large as possible.<br>  <br><br>(i) The side of the largest tile is the gcd of which two numbers? Why? **[1]**<br>(ii) Find this gcd using the division algorithm, showing every step. **[1]**<br>(iii) (A) How many tiles are needed to cover the floor? **[2]**<br>**OR**<br>(iii) (B) A second floor measures 756 cm by 288 cm. Find the side of the largest square tile and the number of tiles needed for this floor. **[2]** | **[4]** |