CLASS IX – MATHEMATICS

Chapter Test: The World of Algorithms

Step-by-Step Solutions and Marking Scheme | M. M.: 40

Ideas used

Euclid (subtraction): 1. If m < n, reverse. 2. If n = 0, answer is m. 3. Otherwise reduce gcd(m, n) to gcd(n, m − n). | **Āryabhaṭa (division):** same, but reduce to gcd(n, m mod n).

m mod n is the remainder when m is divided by n. The scanning algorithm checks every k from 1 (or 2) to min(m, n), so its work grows with the value of the numbers, not with the number of digits.

Section A (1 mark each)

Q	Ans	Reason
1	(B)	A step-by-step procedure to solve a problem is an **algorithm**.
2	(A)	Units column: 7 + 8 = 15, which is 10 or more, so write the units digit **5** and set the carry to **1**.
3	(C)	The numbers from 1 to 20 that divide 20 are 1, 2, 4, 5, 10, 20: **[1, 2, 4, 5, 10, 20]**.
4	(D)	95 = 8 × 11 + 7, so 95 mod 8 = **7** (11 is the quotient).
5	(B)	Divisors of 54: 1, 2, 3, 6, 9, 18, 27, 54; of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36. The greatest common one is **18**.
6	(C)	m = 72, n = 30, so gcd(72, 30) reduces to gcd(n, m − n) = gcd(30, 72 − 30) = **gcd(30, 42)**.
7	(A)	When n = 0, Step 2 reports the answer as **m**.
8	(B)	gcd(101, 3) → gcd(3, 2) [101 mod 3 = 2] → gcd(2, 1) → gcd(1, 0): **3** reductions.
9	(C)	Start at 1. k = 2 divides both → 2. k = 3 divides both → 3. k = 4 divides 12 but not 18, so no update. The value is **3**.
10	(D)	The largest digit sum with a carry is 9 + 9 + 1 = **19**, so the carry is never more than 1.
11	(A)	A is true: the checks go from 1 to min(m, n), and 5-digit numbers are about 100 times larger than 3-digit numbers. R is true and explains it. → **(A)**.
12	(D)	A is false: the subtraction version needs about 50 reductions for gcd(99, 2) (99 → 97 → 95 → … → 1). R is true: gcd(99, 2) → gcd(2, 1) → gcd(1, 0). → **(D)**.

Section B (2 marks each)

Q.13 (2 marks)

Start with the empty list [ ]. Check j = 1, 2, …, 28: j divides 28 for j = 1, 2, 4, 7, 14, 28 and not for the other numbers. [1]

Ans. divisors(28) = [1, 2, 4, 7, 14, 28]. [1]

Q.14 (2 marks)

(A) Units: 7 + 9 = 16: write 6, carry 1. Tens: 4 + 5 + 1 = 10: write 0, carry 1. Hundreds: 8 + 7 + 1 = 16: write 6, carry 1. Thousands: 6 + 3 + 1 = 10: write 0, carry 1. [1]

Ans. The carry is 1, so write 1 to the left: 10606. [1] ***(6847 + 3759 = 10606 (verified))***

OR (B) Units: 5 + 7 = 12: write 2, carry 1. Tens: 8 + 4 + 1 = 13: write 3, carry 1. Final step: write 1, giving **132**. [1]

Without the final step the answer would be 32, which is wrong because the last carry of 1 (a hundred) would be lost. [1]

Q.15 (2 marks)

divisors(36) = [1, 2, 3, 4, 6, 9, 12, 18, 36]; divisors(60) = [1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60]. [½ + ½]

Common divisors = [1, 2, 3, 4, 6, 12]. [½]

Ans. The rightmost element gives gcd(36, 60) = 12. [½]

Q.16 (2 marks)

(A) min(8, 12) = 8. Start: most-recent-common-divisor = 1. k = 2 divides 8 and 12 → 2. k = 3: 3 does not divide 8. k = 4 divides both → 4. k = 5, 6, 7: none divides 8. k = 8: 8 does not divide 12. [1]

Ans. Values: 1 → 2 → 4; gcd(8, 12) = 4. [1]

OR (B) min(18, 24) = 18. Start: 1. k = 2 divides both → 2. k = 3 divides both → 3. k = 4: does not divide 18. k = 6 divides both → 6. No k from 7 to 18 divides both. [1]

Ans. Values: 1 → 2 → 3 → 6; gcd(18, 24) = 6. [1]

Section C (3 marks each)

Q.17 (3 marks)

(A) Apply the three steps:

gcd(48, 18) → gcd(18, 30) → (18 < 30, reverse) gcd(30, 18) → gcd(18, 12) → gcd(12, 6) → gcd(6, 6) → gcd(6, 0). [2]

Ans. n = 0, so the answer is 6, after 5 reductions. [1]

OR (B) Let d divide m and n: m = ad, n = bd. Then m − n = (a − b)d, so d divides n and m − n. [1]

Conversely, let d divide n and m − n: n = xd and m − n = yd. Then m = n + (m − n) = (x + y)d, so d divides m (and n). [1]

So the common divisors of (m, n) and of (n, m − n) are the same; hence their greatest is the same: **gcd(m, n) = gcd(n, m − n)**. [1]

Q.18 (3 marks)

(A) 924 = 5 × 168 + 84, so 924 mod 168 = 84: gcd(924, 168) → gcd(168, 84). [1]

168 = 2 × 84 + 0, so 168 mod 84 = 0: gcd(168, 84) → gcd(84, 0). [1]

Ans. n = 0, so the answer is 84. [1]

OR (B) 2025 = 1 × 1215 + 810: gcd(2025, 1215) → gcd(1215, 810). [1]

1215 = 1 × 810 + 405: gcd(1215, 810) → gcd(810, 405); 810 = 2 × 405 + 0: gcd(810, 405) → gcd(405, 0). [1]

Ans. The answer is 405. [1]

Section D (5 marks each)

Q.19 (5 marks)

(A) Algorithm prime(n):

1. Compute the list divisors(n) using the divisor algorithm (start with an empty list; for each j from 1 to n, if j divides n, add j to the list). [1]

2. Count the numbers in the list. [1]

3. If the list has exactly 2 numbers (1 and n), report that n is prime; otherwise report that n is not prime. [1]

n = 15: divisors(15) = [1, 3, 5, 15], which has 4 numbers, so 15 is **not prime**. [1] **n = 13:** divisors(13) = [1, 13], which has 2 numbers, so 13 is **prime**. [1]

OR (B) (i) 252 = 2 × 105 + 42: gcd(252, 105) → gcd(105, 42). [1]

105 = 2 × 42 + 21: gcd(105, 42) → gcd(42, 21). [1]

42 = 2 × 21 + 0: gcd(42, 21) → gcd(21, 0), so gcd = **21**. [1]

(ii) lcm = (252 × 105) ÷ 21 = 26460 ÷ 21 = **1260**. [2] *(1260 = 252 × 5 = 105 × 12 (verified))*

Q.20 (5 marks)

(A)

(i) The subtraction version takes gcd(35, 2) → gcd(2, 33) → (reverse) gcd(33, 2) → gcd(2, 31) → … Each odd number is reduced by 2 per reduction: 35 → 33 → … → 3 takes 17 reductions to reach gcd(2, 1), then gcd(1, 1) and gcd(1, 0): **19 reductions**. [2]

(ii) Division version: 35 mod 2 = 1, so gcd(35, 2) → gcd(2, 1); 2 mod 1 = 0, so gcd(2, 1) → gcd(1, 0). Only **2 reductions**; the answer is 1. [2]

(iii) The **division version** is better: repeated subtraction of the small number is replaced by one step, so the work depends on the number of digits, not on the size of the number. [1]

OR (B) (i) 8736 = 2 × 3640 + 1456: gcd(8736, 3640) → gcd(3640, 1456). 3640 = 2 × 1456 + 728: → gcd(1456, 728). 1456 = 2 × 728 + 0: → gcd(728, 0). [2]

So gcd(8736, 3640) = **728** (3 reductions). [1]

(ii) The scan goes over k = 2, 3, …, 3640, which is **3639 values**. [1]

(iii) The **division algorithm** (3 steps against 3639 checks). [1]

Section E (4 marks)

Q.21 (4 marks)

(i) The tile side must divide both 1071 and 462 exactly, and it should be the largest such number, so it is **gcd(1071, 462)**. **[1]**

(ii) 1071 = 2 × 462 + 147: gcd(1071, 462) → gcd(462, 147); 462 = 3 × 147 + 21: → gcd(147, 21); 147 = 7 × 21 + 0: → gcd(21, 0). So the gcd is **21 cm**. **[1]**

(iii) (A) Tiles along the length = 1071/21 = 51 and along the width = 462/21 = 22. [1]

Number of tiles = 51 × 22 = **1122**. [1]

OR (iii) (B)

756 = 2 × 288 + 180; 288 = 1 × 180 + 108; 180 = 1 × 108 + 72; 108 = 1 × 72 + 36; 72 = 2 × 36 + 0, so gcd(756, 288) = **36 cm**. [1]

Tiles = (756/36) × (288/36) = 21 × 8 = **168**. [1]

— End of Solutions —