CLASS IX – MATHEMATICS

Chapter Test: Quadrilaterals

Step-by-Step Solutions and Marking Scheme | M. M.: 40

Ideas used

Parallelogram (Thm 1): opposite sides equal; opposite angles equal; adjacent angles add to 180°; diagonals bisect each other. **Converses (Thm 2–5):** opposite sides equal, or opposite angles equal, or diagonals bisecting each other, or one pair of opposite sides equal and parallel, each makes a quadrilateral a parallelogram.

Midpoint Theorem: the segment joining the midpoints of two sides of a triangle is parallel to the third side and half of it. **Centroid Theorem:** medians meet at G, with AG : GD = 2 : 1. **Varignon:** the midpoints of the sides of any quadrilateral form a parallelogram, with sides parallel and equal to half of the diagonals.

Section A (1 mark each)

Q	Ans	Reason
1	(B)	Adjacent angles of a parallelogram add to 180°: ∠B = 180° − 72° = **108°**.
2	(C)	Perimeter = 2(8 + 5) = **26 cm** (opposite sides are equal).
3	(D)	Diagonals bisect each other, so OD = ½ × BD = ½ × 18 = **9 cm**.
4	(D)	Opposite sides, opposite angles and the bisecting of diagonals are always true. Diagonals are equal only in a rectangle (or square), so **(D)**.
5	(A)	Midpoint Theorem: DE = ½ BC = ½ × 18 = **9 cm**.
6	(B)	Angle sum is 360°: 360° − (85° + 110° + 70°) = 360° − 265° = **95°**.
7	(C)	A parallelogram with equal diagonals is a **rectangle**.
8	(B)	(2x + 10) + (3x − 20) = 180 ⇒ 5x − 10 = 180 ⇒ x = **38**.
9	(C)	Each side of the midpoint triangle is half the opposite side, so its perimeter is ½(10 + 14 + 18) = **21 cm**.
10	(A)	Varignon's theorem: the midpoint quadrilateral is always a **parallelogram**.
11	(D)	A is false: diagonals of a parallelogram are equal only for a rectangle (a non-rectangular parallelogram has unequal diagonals). R is true. → **(D)**.
12	(A)	R is the Midpoint Theorem (true). By it, DE = ½ BC, so BC = 2 × 5 = 10 cm, so A is true and R explains it. → **(A)**.

Section B (2 marks each)

Q.13 (2 marks)

Opposite angles of a parallelogram are equal: ∠A = ∠C, so 4x + 5 = 6x − 25 ⇒ 2x = 30 ⇒ **x = 15**. [1]

Ans. ∠A = ∠C = 4(15) + 5 = 65°. ∠B = ∠D = 180° − 65° = 115°. [1] ***(65 + 115 + 65 + 115 = 360 (verified))***

Q.14 (2 marks)

(A) Diagonals of a parallelogram bisect each other, so PO = OR: 2x + 3 = 4x − 7 ⇒ 2x = 10 ⇒ **x = 5**. [1]

Ans. PO = 2(5) + 3 = 13 cm, so PR = 2 × 13 = 26 cm. [1]

OR (B) Yes, ABCD is a parallelogram. [1]

Both pairs of opposite sides are equal (AB = CD and BC = DA); by the converse of the opposite-sides property (SSS argument), the quadrilateral is a parallelogram. [1]

Q.15 (2 marks)

By the Midpoint Theorem in ΔDEF: DE = ½ AB = 8 cm, EF = ½ BC = 11 cm, FD = ½ CA = 7 cm. [1]

Ans. Perimeter of ΔDEF = 8 + 11 + 7 = 26 cm (half the perimeter 52 cm of ΔABC). [1]

Q.16 (2 marks)

(A) Let the adjacent angles be 2k and 3k. Then 2k + 3k = 180° ⇒ k = 36°. [1]

Ans. The angles are 72° and 108°; so ∠A = ∠C = 72° and ∠B = ∠D = 108°. [1]

OR (B) (i) **True.** If one angle is 90°, the adjacent angles are 180° − 90° = 90° and the opposite angles are also 90°. So all angles are 90°, and it is a rectangle. [1]

(ii) **False.** The diagonals of every rhombus are perpendicular, so that fact does not make it a square. A rhombus needs a right angle (or equal diagonals) to be a square. [1]

Section C (3 marks each)

Q.17 (3 marks)

(A) Since AC bisects ∠A, ∠BAC = ∠DAC. [½]

AB ∥ DC and AC is a transversal, so ∠BAC = ∠DCA (alternate angles). [1]

Therefore ∠DAC = ∠DCA, so in ΔADC, DC = DA. [1]

Opposite sides of a parallelogram are equal: AB = DC and AD = BC. So AB = BC = CD = DA, and ABCD is a **rhombus**. [½]

OR (B) Let AB = CD and AB ∥ CD in quadrilateral ABCD; join AC. AB ∥ CD gives ∠BAC = ∠DCA (alternate angles). [1]

In ΔBAC and ΔDCA: AB = CD, ∠BAC = ∠DCA, AC = CA. So ΔBAC is congruent to ΔDCA (SAS), hence BC = DA and ∠BCA = ∠DAC. [1]

∠BCA = ∠DAC are alternate angles for lines BC and AD, so BC ∥ AD. Both pairs of opposite sides are parallel: ABCD is a **parallelogram**. [1]

Q.18 (3 marks)

(A) Centroid Theorem: G divides each median in the ratio 2 : 1 (longer part from the vertex). [½]

AG : GD = 2 : 1, so GD = ½ × 14 = **7 cm**. [1]

BG : GE = 2 : 1 and BE = BG + GE = 21, so BG = ⅔ × 21 = **14 cm**. [1]

Ans. GE = 21 − 14 = 7 cm. [½]

OR (B) PQRS is a **parallelogram** (Varignon's theorem). [1]

In ΔABC, PQ = ½ AC = 5 cm, and in ΔADC, SR = ½ AC = 5 cm. In ΔBCD, QR = ½ BD = 7 cm, and in ΔABD, PS = ½ BD = 7 cm. [1]

Ans. Perimeter = 2(5 + 7) = 24 cm. [1]

Section D (5 marks each)

Q.19 (5 marks)

(A) Given: In ΔABC, P and Q are midpoints of AB and AC. **To prove:** PQ ∥ BC and PQ = ½ BC. [1]

Construction: Draw the line through C parallel to BA, meeting PQ produced at R. [½]

Proof: In ΔAPQ and ΔCRQ: AQ = CQ (Q is the midpoint), ∠AQP = ∠CQR (vertically opposite angles), ∠APQ = ∠CRQ (alternate angles, AP ∥ RC). So ΔAPQ is congruent to ΔCRQ (AAS). [1½]

Hence CR = AP and PQ = QR. Since AP = BP, CR = BP. [½]

BP and CR are equal and parallel (BP is along BA, CR ∥ BA), so BCRP is a parallelogram (one pair of opposite sides equal and parallel). [1]

So PR ∥ BC, which means PQ ∥ BC, and PR = BC. Since PQ = QR = ½ PR, **PQ = ½ BC**. [½]

OR (B) Given: P, Q, R, S are the midpoints of AB, BC, CD, DA. Join AC (and BD). [1]

In ΔABC, P and Q are midpoints of AB and BC, so PQ ∥ AC and PQ = ½ AC. [1]

In ΔADC, S and R are midpoints of AD and DC, so SR ∥ AC and SR = ½ AC. [1]

So PQ ∥ SR and PQ = SR (one pair of opposite sides equal and parallel), hence **PQRS is a parallelogram**. [1]

If AC = BD: PQ = ½ AC and QR = ½ BD (Midpoint Theorem in ΔBCD), so PQ = QR. A parallelogram with equal adjacent sides is a **rhombus**. [1]

Q.20 (5 marks)

(A) (i) Let the diagonals AC and BD meet at O. Since ABCD is a parallelogram, OA = OC and OB = OD. [1]

OE = OB − BE and OF = OD − DF. As OB = OD and BE = DF, we get OE = OF. [1]

So in quadrilateral AECF, the diagonals AC and EF bisect each other (OA = OC and OE = OF), so **AECF is a parallelogram** (converse of the diagonals property). [1]

(ii) EF = BD − BE − DF = 20 − 4 − 4 = **12 cm**. [1]

(iii) The diagonals AC = EF = 12 cm are equal, so AECF is a parallelogram with equal diagonals, i.e. a **rectangle**. [1]

OR (B) Let diagonal AC meet EF at G. In ΔADC, E is the midpoint of AD and EG ∥ DC, so G is the midpoint of AC (converse of the Midpoint Theorem), and EG = ½ DC. [1½]

In ΔCAB, G is the midpoint of AC and GF ∥ AB, so F is the midpoint of BC, and GF = ½ AB. [1½]

EF = EG + GF = ½ DC + ½ AB = **½(AB + CD)**. [1]

For AB = 18 cm and CD = 12 cm: EF = ½(18 + 12) = **15 cm**. [1]

Section E (4 marks)

Q.21 (4 marks)

(i) PQRS joins the midpoints of the sides of a quadrilateral, so it is a **parallelogram** (Varignon). **[1]**

(ii) PQ = RS = ½ AC = 40 m and QR = SP = ½ BD = 30 m. Wire = 2(40 + 30) = **140 m**. **[1]**

(iii) (A) PQ ∥ AC and QR ∥ BD (Midpoint Theorem). PQRS is a parallelogram, so it is a rectangle exactly when ∠PQR = 90°. [1]

∠PQR = 90° exactly when the lines AC and BD are perpendicular: the diagonals **AC and BD must be perpendicular**. [1]

OR (iii) (B) PQRS is a parallelogram, so its diagonals bisect each other at T. [1]

PT = ½ × 52 = **26 m**, and TS = ½ × 46 = **23 m**. [1]

— End of Solutions —