CLASS IX – MATHEMATICS

Chapter Test: Math of Space: Surface Area and Volume

Step-by-Step Solutions and Marking Scheme | M. M.: 40

Formulas used

Cuboid: TSA = 2(lw + wh + hl), V = lwh. **Cube:** TSA = $6a^2$, V = a^3. **Cylinder:** CSA = $2\pi rh$, TSA = $2\pi r(r + h)$, V = $\pi r^2 h$. **Cone:** $l^2 = r^2 + h^2$, CSA = πrl, TSA = $\pi rl + \pi r^2$, V = $\frac{1}{3}\pi r^2 h$.

Sphere: SA = $4\pi r^2$, V = $(4/3)\pi r^3$. **Hemisphere:** CSA = $2\pi r^2$, TSA = $3\pi r^2$, V = $(2/3)\pi r^3$. Where needed, π = 22/7.

Section A (1 mark each)

Q	Ans	Reason
1	(C)	$a^3 = 343 \Rightarrow a = 7$ cm. TSA = $6a^2 = 6 \times 49 =$ **294 cm²**.
2	(B)	TSA = 2(8×5 + 5×3 + 8×3) = 2(40 + 15 + 24) = **158 cm²**.
3	(A)	CSA = $2\pi rh = 2 \times 22/7 \times 7 \times 10 =$ **440 cm²**.
4	(D)	V = $\pi r^2 h = 22/7 \times 3.5 \times 3.5 \times 12 =$ **462 cm³**.
5	(A)	$h^2 = l^2 - r^2 = 100 - 36 = 64$, so h = **8 cm**.
6	(D)	V = $\frac{1}{3} \times 22/7 \times 3 \times 3 \times 7 =$ **66 cm³**.
7	(B)	SA = $4\pi r^2 = 4 \times 22/7 \times 49 =$ **616 cm²**.
8	(C)	V = $(2/3)\pi r^3 = (2/3)\pi \times 27 =$ **18π cm³**.
9	(A)	SA = $4\pi r^2$. With radius 2r: $4\pi(2r)^2 = 16\pi r^2 =$ **4 times**.
10	(D)	Cone volume = $\frac{1}{3}\pi r^2 h$, cylinder volume = $\pi r^2 h$, so the ratio is **1 : 3**.
11	(B)	A is true (half of $4\pi r^2$). R is also true, but it is about the volume and does not explain the curved surface area. → **(B)**.
12	(C)	A is true: the original cube has TSA 6 × 16 = 96 cm², while 64 small cubes have 64 × 6 = 384 cm². R is false: the volume stays the same (64 cm³). → **(C)**.

Section B (2 marks each)

Q.13 (2 marks)

Volume = $a^3 = 9 \times 9 \times 9 =$ **729 cm³**. [1]

Ans. TSA = $6a^2 = 6 \times 81 = 486$ cm². [1]

Q.14 (2 marks)

(A) TSA = $2\pi r(h + r) = 2 \times 22/7 \times 7 \times (15 + 7)$. [1]

Ans. = $44 \times 22 = 968$ cm². [1]

OR (B) Sheet needed = curved surface area = $\pi rl = 22/7 \times 3.5 \times 12$. [1]

Ans. = $22 \times 0.5 \times 12 = 132$ cm². [1]

Q.15 (2 marks)

Volume = $(4/3)\pi r^3 = (4/3)\pi \times 216 =$ **288π cm³**. [1]

Ans. Surface area = $4\pi r^2 = 4\pi \times 36 = 144\pi$ cm². [1]

Q.16 (2 marks)

(A) Let $r_1 : r_2 = 3 : 4$ and $h_1 : h_2 = 5 : 2$. (i) $V_1 : V_2 = (3^2 \times 5) : (4^2 \times 2) = 45 : 32$. [1]

Ans. (ii) $CSA_1 : CSA_2 = (3 \times 5) : (4 \times 2) = 15 : 8$. [1] (volume ratio 45 : 32)

OR (B) Each edge gives 6 ÷ 2 = 3 pieces, so the number of cubes = 3 × 3 × 3 = **27**. [1]

Original SA = $6 \times 36 = 216$ cm². Total SA of small cubes = $27 \times 6 \times 4 = 648$ cm². Ratio = 216 : 648 = **1 : 3**. [1]

Section C (3 marks each)

Q.17 (3 marks)

(A) Volume of the godown = 30 × 20 × 12 = 7200 m^3. [1]

Volume of one box = 2 × 1.5 × 1 = 3 m^3. [1]

Ans. Number of boxes = 7200 ÷ 3 = 2400. [1]

OR (B) Volume = 6 × 5 × 4.5 = 135 m^3. [1]

Capacity = 135 × 1000 = **135000 litres**. [1]

Ans. Time = 135000 ÷ 900 = 150 minutes. [1]

Q.18 (3 marks)

(A) $l = \sqrt{(r^2 + h^2)} = \sqrt{(49 + 576)} = \sqrt{625} =$ **25 cm**. [1]

CSA = $\pi rl = 22/7 \times 7 \times 25 =$ **550 cm^2**. [1]

Ans. TSA = $\pi rl + \pi r^2 = 550 + 22/7 \times 49 = 550 + 154 = 704\ cm^2$. [1]

OR (B) $9^2 + 12^2 = 81 + 144 = 225 = 15^2$, so it is a right triangle. Rotating about the side of 12 cm forms a **cone** with h = 12 cm, r = 9 cm and l = 15 cm. [1]

Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 81 \times 12 =$ **324π cm^3**. [1]

Ans. CSA = $\pi rl = \pi \times 9 \times 15 = 135\pi\ cm^2$. [1]

Section D (5 marks each)

Q.19 (5 marks)

(A) (i) Volume of cylinder = $\pi r^2 h = \pi \times 36 \times 28 =$ **1008π cm^3**. [1]

(ii) Volume of one sphere = $(4/3)\pi \times 3^3 = (4/3)\pi \times 27 =$ **36π cm^3**. [1]

(iii) Melting and recasting does not change the volume, so the number of spheres = 1008π ÷ 36π = **28**. [2]

(iv) TSA of cylinder = $2\pi r(r + h) = 2\pi \times 6 \times 34 = 408\pi\ cm^2$. Total SA of 28 spheres = 28 × 4π × 9 = 1008π cm^2. [½]

Ans. 1008π > 408π, so the total surface area of the spheres is more. [½]

OR (B) (i) $V = (2/3)\pi r^3 = (2/3) \times 22/7 \times 10.5 \times 10.5 \times 10.5 = (2/3) \times 22/7 \times 1157.625 =$ **2425.5 cm^3**. [2]

(ii) Inner curved surface area = $2\pi r^2 = 2 \times 22/7 \times 110.25 =$ **693 cm^2**. [2]

(iii) Cost = 693 × 0.50 = **₹346.50**. [1]

Q.20 (5 marks)

(A) (i) $l = \sqrt{(3.5^2 + 12^2)} = \sqrt{(12.25 + 144)} = \sqrt{156.25} =$ **12.5 m**. [1]

(ii) Canvas = CSA = $\pi rl = 22/7 \times 3.5 \times 12.5 =$ **137.5 m^2**. [1]

(iii) Length = area ÷ width = 137.5 ÷ 2.5 = **55 m**. [1]

(iv) Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 22/7 \times 12.25 \times 12 =$ **154 m^3**. [1]

Ans. (v) Cost = 137.5 × 120 = ₹16,500. [1]

OR (B) (i) Volume = 25 × 10 × 2 = **500 m^3**. [1]

(ii) 500 × 1000 = **5,00,000 litres**. [1]

(iii) Floor = 25 × 10 = 250 m^2. Four walls = 2(25 + 10) × 2 = 140 m^2. Area to be tiled = 250 + 140 = **390 m^2**. [2]

(iv) Cost = 390 × 150 = **₹58,500**. [1]

Section E (4 marks)

Q.21 (4 marks)

(i) $l = \sqrt{(r^2 + h^2)} = \sqrt{(12.25 + 144)} = \sqrt{156.25} =$ **12.5 cm**. **[1]**

(ii) CSA $= \pi rl = 22/7 \times 3.5 \times 12.5 =$ **137.5 cm²**. **[1]**

(iii) (A) Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 22/7 \times 12.25 \times 12 =$ **154 cm³**. [1]

The tub has volume $\pi r^2 h = 462$ cm³, so the number of cones $= 462 \div 154 =$ **3** (a cone is one third of the cylinder with the same base and height). [1]

OR (iii) (B) Volume of larger cone $= \frac{1}{3} \times 22/7 \times 49 \times 24 =$ **1232 cm³**. [1]

Ratio $= 1232 \div 154 =$ **8 times** (both radius and height are doubled, so volume is $2^3 = 8$ times). [1]

— End of Solutions —