# CLASS IX – MATHEMATICS

## Chapter Test: Propositions and their Converses

**Step-by-Step Solutions and Marking Scheme** | M. M.: 40

### Ideas used

The converse of "if P then Q" is "if Q then P". A true proposition can have a false converse (or a true one). A **counterexample** is one instance that makes a statement false.

A number has an odd number of factors exactly when it is a perfect square. A number divisible by two numbers with no common factor (other than 1) is divisible by their product.

Converse of the Baudhāyana–Pythagoras theorem: if $a^2 + b^2 = c^2$ for the sides of a triangle, it is right-angled, with the right angle opposite the side c.

### Section A (1 mark each)

| Q | Ans | Reason |
|---|---|---|
| 1 | (B) | Swap the two parts of "if P then Q": **If a number is divisible by 5, then it is divisible by 10**. (This converse is false: 15.) |
| 2 | (C) | P: it is Sunday; Q: school is closed. The converse is "if Q then P": **If the school is closed, then it is Sunday**. |
| 3 | (D) | 9 is odd and greater than 1 but $9 = 3 \times 3$ is not prime. (3, 7 and 11 are prime.) So **9**. |
| 4 | (B) | Converse: "if a number has an odd number of factors, then it is a perfect square" is true. The converses of the other three are false (9; a rectangle that is not a square; $x = 3, y = -3$). |
| 5 | (A) | Converse: if $x^2 = y^2$, then $x = y$. For $x = 3, y = -3$: $x^2 = 9 = y^2$ but $x \neq y$. So $\mathbf{x = 3, y = -3}$. |
| 6 | (C) | Only perfect squares have an odd number of factors. $36 = 6^2$ has 9 factors; 24, 50, 60 have 8, 6, 12 factors. So **36**. |
| 7 | (A) | $77 = 7 \times 11$ has divisors 1, 7, 11, 77 = **4** divisors. |
| 8 | (D) | "Y when X" means "if X then Y": **If a number is a perfect square, then it has an odd number of factors**. |
| 9 | (B) | $9^2 + 12^2 = 81 + 144 = 225 = 15^2$, so the triangle is right-angled opposite the longest side: **15 cm**. |
| 10 | (C) | An isosceles trapezium has equal diagonals but is not a rectangle. (A square is a rectangle; the diagonals of a rhombus or a general parallelogram are unequal.) So **isosceles trapezium**. |
| 11 | (D) | A is false: the converse of a true proposition can be false. R gives such a case and is true. → **(D)**. |
| 12 | (A) | A is true. R is true and is exactly the reason: if $n = p^2$, its factors are 1, p, $p^2$, which is 3 factors. → **(A)**. |

### Section B (2 marks each)

#### Q.13 (2 marks)

**Converse:** If the alternate interior angles formed by a transversal are equal, then the two lines are parallel. [1]

**Ans. The converse is true (it is the test for parallel lines). [1]**

#### Q.14 (2 marks)

**(A)** A counterexample to a statement is one case where the "if" part is true but the "then" part is false.

Take n = 2: it is a prime number, but it is even (not odd). [1]

**Ans. So the statement is false, and n = 2 is a counterexample. [1]**

**OR (B)**

Take n = 40: $n^2 + n + 41 = 1600 + 40 + 41 = 1681$. [1]

**Ans. $1681 = 41 \times 41$ is not prime, so n = 40 is a counterexample. [1]**

### Q.15 (2 marks)

**Converse:** If a number is divisible by both 6 and 9, then it is divisible by 54. [1]

**Ans. False. Counterexample: 18 is divisible by 6 and by 9, but not by 54 (18 ÷ 54 is not a whole number). [1] *(The proposition itself is true since 54 = 6 × 9.)***

### Q.16 (2 marks)

**(A)** Check: $8^2 + 15^2 = 64 + 225 = 289 = 17^2$. [1]

**Ans. By the converse of the Baudhāyana-Pythagoras theorem, the triangle is right-angled (the right angle is opposite the 17 cm side). [1]**

**OR (B)** Check: $9^2 + 12^2 = 81 + 144 = 225$, but $16^2 = 256$. [1]

**Ans. Since $9^2 + 12^2 \neq 16^2$, the triangle is not right-angled (if it were, the sides would satisfy the theorem). [1]**

## Section C (3 marks each)

### Q.17 (3 marks)

**(A) Converse:** If n has exactly 3 factors, then n is the square of a prime number. [1]

**It is true.** [1] Let the three factors be 1, f and n ($1 < f < n$). The partner of f is n/f, which is also a factor, so $n/f = f$ (the only factor between 1 and n). Hence $n = f^2$. If f had a factor g with $1 < g < f$, then g would also be a factor of n, giving a fourth factor. So f has no factor other than 1 and f, i.e. f is prime. Hence n is the square of a prime. [1]

**OR (B) Converse:** If two triangles have equal perimeters, then they are congruent. [1]

**False.** [1] Counterexample: a triangle with sides 3, 4, 5 and a triangle with sides 2, 5, 5 both have perimeter 12 (both are valid triangles), but they are not congruent. [1]

### Q.18 (3 marks)

**(A)** *Given:* $\Delta ABC$ with $\angle B = \angle C$. *To prove:* AB = AC. Draw AD $\perp$ BC, meeting BC at D. [½]

In $\Delta ABD$ and $\Delta ACD$: $\angle ADB = \angle ADC = 90°$ [½]; $\angle B = \angle C$ (given) [½]; AD = AD (common) [½].

So $\Delta ABD$ is congruent to $\Delta ACD$ (AAS). [½]

Hence AB = AC (corresponding parts of congruent triangles). [½] This proves the converse of "equal sides have equal opposite angles".

**OR (B) Justification:** A number ending in 5 can be written as $10k + 5 = 5(2k + 1)$, which is a multiple of 5. [1]

**Converse:** If a number is divisible by 5, then it ends in 5. [1]

**False.** Counterexample: 10 is divisible by 5 but ends in 0. [1]

## Section D (5 marks each)

### Q.19 (5 marks)

**(A)**

(i) **Converse:** If a triangle is isosceles, then it is equilateral. **False.** Counterexample: a triangle with sides 5 cm, 5 cm, 8 cm is isosceles ($5 + 5 > 8$, so it exists) but not equilateral. [2]

(ii) **Converse:** If a quadrilateral is a rhombus, then it is a square. **False.** Counterexample: a rhombus of side 5 cm with one angle 60° has unequal angles, so it is not a square. [2]

(iii) Example: "If AB = AC in $\Delta ABC$, then $\angle B = \angle C$". Its converse "if $\angle B = \angle C$, then AB = AC" is also true (Q.18(A)). (Any correct pair is accepted.) [1]

**OR (B) Converse:** If $a^2 + b^2 = c^2$ for the sides of a triangle, then the triangle is right-angled. [1]

*Proof.* In $\Delta ABC$, BC = a, AC = b, AB = c and $a^2 + b^2 = c^2$. Construct a right triangle XYZ with $\angle Z = 90°$, YZ = a and XZ = b. [1]

By the Baudhāyana–Pythagoras theorem in $\Delta XYZ$: $XY^2 = a^2 + b^2$. [1]

Since $a^2 + b^2 = c^2$, $XY^2 = c^2$, so XY = c = AB. [1]

In $\Delta ABC$ and $\Delta XYZ$: BC = YZ = a, AC = XZ = b, AB = XY = c, so $\Delta ABC$ is congruent to $\Delta XYZ$ (SSS). [½]

Hence $\angle C = \angle Z = 90°$, so $\Delta ABC$ is right-angled. [½]

## Q.20 (5 marks)

**(A)**

(i) Factor-partner pairs of 36: (1, 36), (2, 18), (3, 12), (4, 9), (6, 6). [1] 36 has 9 factors; the special pair is **(6, 6)**, where a factor is its own partner. [1]

(ii) **P:** If n is a perfect square, then it has an odd number of factors. **Converse Q:** If n has an odd number of factors, then it is a perfect square. [1]

$49 = 7^2$ has the 3 factors 1, 7, 49 (odd). 50 is not a square and has 6 factors 1, 2, 5, 10, 25, 50 (even). So P and Q both hold in these cases (both are true statements). [1]

(iii) $14^2 = 196$ and $15^2 = 225$, so 200 is not a perfect square. By Q, a number with an odd number of factors must be a square, so 200 has an **even** number of factors (it has 12). [1]

**OR (B)**

(i) $n = 4$: $6(4) + 1 = 25 = 5 \times 5$ is not prime. [2] *($n = 1, 2, 3$ give 7, 13, 19, which are prime, so the claim fails first at $n = 4$.)*

(ii) $n = 11$ is prime, but $2^{11} - 1 = 2047 = 23 \times 89$ is not prime. [2]

(iii) $n = 6$ is even, but $2^6 + 1 = 65 = 5 \times 13$ is not prime. [1]

## Section E (4 marks)

## Q.21 (4 marks)

**(i)** If $a^2 + b^2 = c^2$ for the sides a, b, c of a triangle, then the triangle is right-angled (the right angle is opposite the side c). **[1]**

**(ii)** $24^2 + 7^2 = 576 + 49 = 625 = 25^2$. So by the converse the bed is right-angled, with the right angle opposite the longest side AC (25 m), i.e. at vertex **B**. **[1]**

**(iii) (A)** The longest side is 15: $10^2 + 11^2 = 100 + 121 = 221$, but $15^2 = 225$. [1]

Since $221 \neq 225$, the bed is **not right-angled**. [1]

**OR (iii) (B)**

$6^2 + 8^2 = 100 = 10^2$ and $9^2 + 12^2 = 225 = 15^2$, so both beds are right-angled. [1]

But their sides are different (6, 8, 10 and 9, 12, 15), so the triangles are not congruent. The statement is **false**, and these two beds are a counterexample. [1]

**— End of Solutions —**