

# CLASS 10 – MATHEMATICS

## Chapter: Some Applications of Trigonometry

### Answer Key with Step-by-Step Solutions

**Maximum Marks: 40**

**Time Allowed: 1 hour 30 minutes**

**Results used:** in a right triangle,  $\tan \theta = \text{opposite/adjacent}$ ,  $\sin \theta = \text{opposite/hypotenuse}$ ,  $\cos \theta = \text{adjacent/hypotenuse}$ ;  $\tan 30^\circ = \frac{1}{\sqrt{3}}$ ,  $\tan 45^\circ = 1$ ,  $\tan 60^\circ = \sqrt{3}$ ;  $\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$ ,  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ . The angle of depression from P to Q equals the angle of elevation from Q to P.

### SECTION A – MCQ Answer Key

| Q   | Option | Answer                  | Explanation / calculation                                                                                                                           |
|-----|--------|-------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| Q1  | (B)    | $45^\circ$              | $\tan \theta = \text{height/shadow} = 1 \Rightarrow \theta = 45^\circ$ .                                                                            |
| Q2  | (A)    | $5\sqrt{3}$ m           | Height = $10 \sin 60^\circ = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ m.                                                                           |
| Q3  | (C)    | $\frac{50}{\sqrt{3}}$ m | $\tan 60^\circ = \frac{50}{d} \Rightarrow d = \frac{50}{\sqrt{3}}$ m ( $\approx 28.9$ m).                                                           |
| Q4  | (D)    | 30 m                    | $h = 30\sqrt{3} \times \tan 30^\circ = 30\sqrt{3} \times \frac{1}{\sqrt{3}} = 30$ m.                                                                |
| Q5  | (B)    | $30^\circ$              | The angle of depression from the cliff top equals the angle of elevation from the boat (alternate angles, since the horizontal lines are parallel). |
| Q6  | (A)    | $30^\circ$              | $\tan \theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$ .                                                            |
| Q7  | (C)    | 50 m                    | Height = $100 \sin 30^\circ = 100 \times \frac{1}{2} = 50$ m.                                                                                       |
| Q8  | (D)    | $20\sqrt{3}$ m          | Let the nearer distance be $x$ . $h = x\sqrt{3}$ and $h = \frac{x+40}{\sqrt{3}} \Rightarrow 3x = x+40 \Rightarrow x = 20$ , $h = 20\sqrt{3}$ m.     |
| Q9  | (A)    | Both true; R explains A | Moving closer decreases the distance, so $\tan \theta$ increases, and $\theta$ increases. R gives exactly this reasoning.                           |
| Q10 | (D)    | A false, R true         | $\tan \theta = 10/10 = 1 \Rightarrow \theta = 45^\circ$ , not $60^\circ$ . A is false; R is true.                                                   |

**Marking:** 1 mark for each correct option; no part marks in Section A.

### SECTION B – Very Short Answer

**Q11. Height of the pole** [2 marks]

$$\tan 30^\circ = \frac{h}{6\sqrt{3}} \Rightarrow h = 6\sqrt{3} \times \frac{1}{\sqrt{3}} = 6 \text{ m.}$$

**Answer: 6 m**

**Marking scheme:**Correct ratio  $\tan 30^\circ$ : 1 mark $h = 6$  m: 1 mark**Q12. Height of the tower** [2 marks]

Height of the tower above eye level:  $\tan 30^\circ = \frac{x}{20\sqrt{3}} \Rightarrow x = 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 20$  m.

Height of the tower =  $20 + 1.6 = 21.6$  m.

**Answer: 21.6 m****Marking scheme:**

Height above eye level 20 m: 1 mark

Adding eye height: 21.6 m: 1 mark

**Q13. Distance of the boat** [2 marks]

The angle of elevation of the cliff top from the boat is also  $45^\circ$ .

$$\tan 45^\circ = \frac{60}{d} \Rightarrow d = 60 \text{ m.}$$

**Answer: 60 m****Marking scheme:**Using equal angles and  $\tan 45^\circ$ : 1 mark

60 m: 1 mark

**Q14. Length of the wire** [2 marks]

$$\sin 60^\circ = \frac{12}{l} \Rightarrow l = \frac{12}{\frac{\sqrt{3}}{2}} = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m.}$$

$$8 \times 1.73 = 13.84 \text{ m.}$$

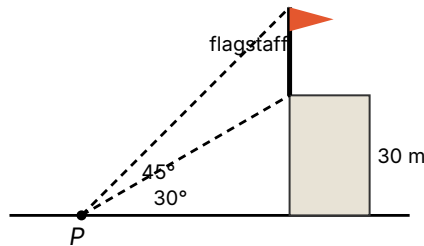
**Answer:  $8\sqrt{3}$  m  $\approx$  13.84 m****Marking scheme:**Correct ratio  $\sin 60^\circ$ : 1 mark $8\sqrt{3} \approx 13.84$  m: 1 mark**Q15. Angle of elevation of the sun** [2 marks]

$$\tan \theta = \frac{15}{5\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60^\circ.$$

**Answer:  $60^\circ$** **Marking scheme:** $\tan \theta = \sqrt{3}$ : 1 mark $\theta = 60^\circ$ : 1 mark

## SECTION C – Short Answer

### Q16. Length of the flagstaff [3 marks]



Let the foot of the building be B, its top (bottom of the flagstaff) be C, and the top of the flagstaff be D. Let  $PB = d$ .

$$\text{In } \triangle PBC: \tan 30^\circ = \frac{30}{d} \Rightarrow d = 30\sqrt{3} \text{ m.}$$

$$\text{In } \triangle PBD: \tan 45^\circ = \frac{BD}{d} \Rightarrow BD = d = 30\sqrt{3} \text{ m.}$$

$$\text{Flagstaff } CD = BD - BC = 30\sqrt{3} - 30 = 30(\sqrt{3} - 1) = 30 \times 0.732 = 21.96 \text{ m.}$$

**Answer:  $30(\sqrt{3} - 1)$  m  $\approx$  21.96 m**

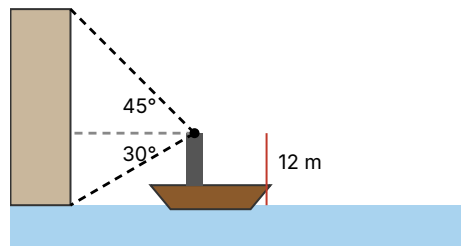
#### Marking scheme:

$d = 30\sqrt{3}$  m: 1 mark

$BD = 30\sqrt{3}$  m: 1 mark

Flagstaff  $\approx$  21.96 m: 1 mark

### Q17. Cliff and ship [3 marks]



Let the man's eye be E (12 m above water), the base of the cliff be B and its top be T. Draw  $EM \perp$  cliff, so  $MB = 12$  m and  $EM =$  horizontal distance  $d$ .

$$\text{Angle of depression of B is } 30^\circ: \tan 30^\circ = \frac{12}{d} \Rightarrow d = 12\sqrt{3} \text{ m} \approx 20.78 \text{ m.}$$

$$\text{Angle of elevation of T is } 45^\circ: \tan 45^\circ = \frac{TM}{d} \Rightarrow TM = 12\sqrt{3} \text{ m.}$$

$$\text{Height of cliff} = TM + MB = 12\sqrt{3} + 12 = 12(\sqrt{3} + 1) \text{ m} \approx 32.78 \text{ m.}$$

**Answer: Distance  $12\sqrt{3}$  m  $\approx$  20.78 m; height  $12(\sqrt{3} + 1)$  m  $\approx$  32.78 m**

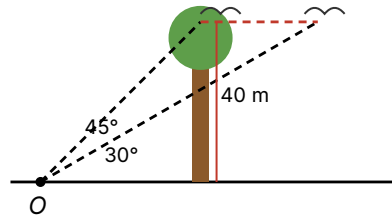
#### Marking scheme:

Distance  $d = 12\sqrt{3}$  m: 1 mark

$TM = 12\sqrt{3}$  m: 1 mark

Height  $12(\sqrt{3} + 1)$  m: 1 mark

**Q18. Speed of the bird** [3 marks]



First position:  $\tan 45^\circ = \frac{40}{x_1} \Rightarrow x_1 = 40$  m (horizontal distance from O).

Second position (same height 40 m):  $\tan 30^\circ = \frac{40}{x_2} \Rightarrow x_2 = 40\sqrt{3}$  m.

Distance flown  $= 40\sqrt{3} - 40 = 40(\sqrt{3} - 1) = 40 \times 0.732 = 29.28$  m.

Speed  $= 29.28 \div 4 = 7.32$  m/s.

**Answer: 7.32 m/s**

**Marking scheme:**

Both horizontal distances: 1 mark

Distance flown  $40(\sqrt{3} - 1)$  m: 1 mark

Speed 7.32 m/s: 1 mark

**Q19. Height of the taller building** [3 marks]

From the top of the shorter building, the horizontal distance to the taller building is 60 m.

Height of the taller building above the shorter one:  $\tan 30^\circ = \frac{x}{60} \Rightarrow x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$  m.

Height of the taller building  $= 20 + 20\sqrt{3} = 20(1 + \sqrt{3})$  m  $\approx 54.64$  m.

**Answer:  $20(1 + \sqrt{3})$  m  $\approx 54.64$  m**

**Marking scheme:**

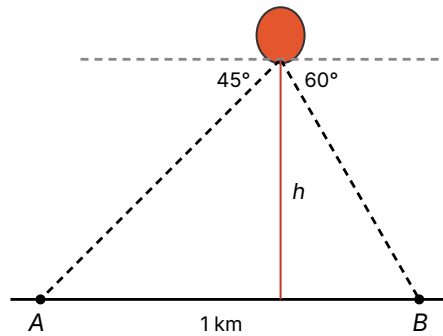
Correct figure and horizontal 60 m: 1 mark

$x = 20\sqrt{3}$  m: 1 mark

Height  $\approx 54.64$  m: 1 mark

## SECTION D — Long Answer

### Q20. Height of the balloon [4 marks]



Let the balloon be at height  $h$  km above the point P on the road directly below it. A and B are on opposite sides of P, and  $AB = 1$  km.

The angles of depression equal the angles of elevation of the balloon from A and B:  $45^\circ$  at A and  $60^\circ$  at B.

In the right triangle at A:  $\tan 45^\circ = \frac{h}{AP} \Rightarrow AP = h$ .

In the right triangle at B:  $\tan 60^\circ = \frac{h}{BP} \Rightarrow BP = \frac{h}{\sqrt{3}}$ .

$$AP + BP = 1 \Rightarrow h + \frac{h}{\sqrt{3}} = 1 \Rightarrow h\left(\frac{\sqrt{3} + 1}{\sqrt{3}}\right) = 1 \Rightarrow h = \frac{\sqrt{3}}{\sqrt{3} + 1}.$$

Rationalising:  $h = \frac{\sqrt{3}(\sqrt{3} - 1)}{3 - 1} = \frac{3 - \sqrt{3}}{2}$  km.

$$h = \frac{3 - 1.732}{2} = \frac{1.268}{2} = 0.634 \text{ km} = 634 \text{ m}.$$

**Answer:**  $\frac{3 - \sqrt{3}}{2}$  km  $\approx$  634 m

#### Marking scheme:

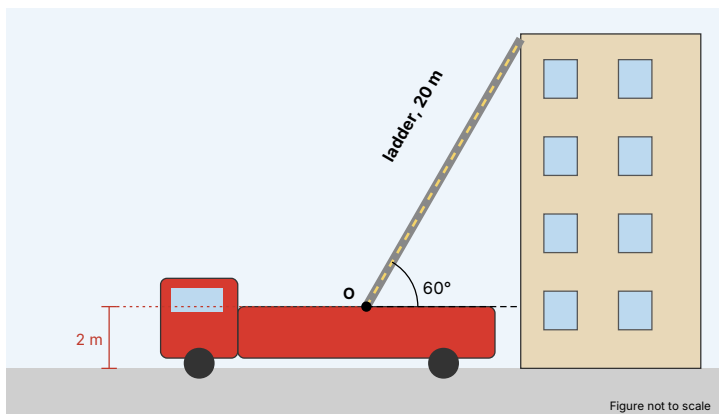
$AP = h$  and  $BP = h/\sqrt{3}$ : 1 mark

Equation  $h + h/\sqrt{3} = 1$ : 1 mark

$h = (3 - \sqrt{3})/2$  km: 1 mark

$h = 634$  m: 1 mark

## SECTION E — Case Study



### Q21. Fire engine ladder [4 marks]

(i) Height of the ladder top above O =  $20 \sin 60^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$  m. Height above ground =  $2 + 10\sqrt{3} = 2 + 17.32 = \mathbf{19.32 \text{ m}}$ .

**(ii)** Horizontal distance =  $20 \cos 60^\circ = 20 \times \frac{1}{2} = \mathbf{10 \text{ m}}$ .

**(iii)** The window is  $12 - 2 = 10 \text{ m}$  above O, and the wall is still  $10 \text{ m}$  away horizontally.

$\tan \theta = \frac{10}{10} = 1 \Rightarrow \theta = \mathbf{45^\circ}$ . Length needed =  $\sqrt{10^2 + 10^2} = 10\sqrt{2} \approx \mathbf{14.14 \text{ m}}$  (or  $10/\sin 45^\circ$ ).

**Answer: (i)  $2 + 10\sqrt{3} \approx 19.32 \text{ m}$  (ii)  $10 \text{ m}$  (iii)  $45^\circ$ ;  $10\sqrt{2} \approx 14.14 \text{ m}$**

**Marking scheme:**

(i)  $19.32 \text{ m}$ : 1 mark

(ii)  $10 \text{ m}$ : 1 mark

(iii)  $\theta = 45^\circ$ : 1 mark; length  $10\sqrt{2} \approx 14.14 \text{ m}$ : 1 mark

### **Final Answer Summary**

| <b>Q</b>   | <b>Final answer</b>         | <b>Q</b>   | <b>Final answer</b>                              |
|------------|-----------------------------|------------|--------------------------------------------------|
| <b>Q1</b>  | (B) $45^\circ$              | <b>Q12</b> | 21.6 m                                           |
| <b>Q2</b>  | (A) $5\sqrt{3}$ m           | <b>Q13</b> | 60 m                                             |
| <b>Q3</b>  | (C) $\frac{50}{\sqrt{3}}$ m | <b>Q14</b> | $8\sqrt{3} \approx 13.84$ m                      |
| <b>Q4</b>  | (D) 30 m                    | <b>Q15</b> | $60^\circ$                                       |
| <b>Q5</b>  | (B) $30^\circ$              | <b>Q16</b> | $\approx 21.96$ m                                |
| <b>Q6</b>  | (A) $30^\circ$              | <b>Q17</b> | $12\sqrt{3}$ m; $12(\sqrt{3} + 1)$ m             |
| <b>Q7</b>  | (C) 50 m                    | <b>Q18</b> | 7.32 m/s                                         |
| <b>Q8</b>  | (D) $20\sqrt{3}$ m          | <b>Q19</b> | $\approx 54.64$ m                                |
| <b>Q9</b>  | (A)                         | <b>Q20</b> | $\approx 634$ m                                  |
| <b>Q10</b> | (D)                         | <b>Q21</b> | (i) 19.32 m (ii) 10 m (iii) $45^\circ$ , 14.14 m |
| <b>Q11</b> | 6 m                         |            |                                                  |

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

**— End of Solutions —**