

CLASS 10 — MATHEMATICS

Chapter: Arithmetic Progressions

Answer Key with Step-by-Step Solutions

Maximum Marks: 40

Time Allowed: 1 hour 30 minutes

Results used: n th term $a_n = a + (n - 1)d$; sum of first n terms $S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$, where l is the last term; $a_n = S_n - S_{n-1}$; three numbers p, q, r are in AP if $2q = p + r$.

SECTION A — MCQ Answer Key

Q	Option	Answer	Explanation / calculation
Q1	(C)	-3, -1, 1, 3, ...	Differences are 2, 2, 2 — constant. (A) differences 3, 5, 7; (B) is doubling; (D) differences $-1/2, -1/6, \dots$ are not constant.
Q2	(B)	-1	$d = \frac{1}{2} - \frac{3}{2} = -1$, and every later difference is also -1.
Q3	(D)	-49	$a = 7, d = -4$: $a_{15} = 7 + 14(-4) = 7 - 56 = -49$.
Q4	(B)	22nd	$4 + (n - 1)5 = 109 \Rightarrow (n - 1)5 = 105 \Rightarrow n = 22$.
Q5	(C)	400	$1 + 3 + \dots$ (20 terms): $S_{20} = \frac{20}{2}[2 + 19 \times 2] = 10 \times 40 = 400 (= 20^2)$.
Q6	(A)	3	Middle term $\times 2 =$ sum of the other two: $2(4k - 6) = (k + 2) + (3k - 2) \Rightarrow 8k - 12 = 4k \Rightarrow k = 3$. (Terms 5, 6, 7.)
Q7	(D)	-3	$a_n - a_{n-1} = (7 - 3n) - (7 - 3n + 3) = -3$. (Terms: 4, 1, -2, ...)
Q8	(A)	62	$a_{10} = S_{10} - S_9 = (300 + 50) - (243 + 45) = 350 - 288 = 62$.
Q9	(B)	Both true; R does not explain A	$31 + (n - 1)(-3) = 0 \Rightarrow n = 34/3$, not a natural number, so A is true. R is true ($d = -3$), but a negative d does not by itself stop 0 from being a term (e.g. 30, 27, 24, ... reaches 0). So R does not explain A.
Q10	(A)	Both true; R explains A	With $a = 1, d = 1$: $S_n = \frac{n}{2}[2 + (n - 1)] = \frac{n(n + 1)}{2}$. R is exactly how A is derived.

Marking: 1 mark for each correct option; no part marks in Section A.

SECTION B – Very Short Answer

Q11. 20th term of -11, -7, -3, ... [2 marks]

$$a = -11, d = -7 - (-11) = 4.$$

$$a_{20} = a + 19d = -11 + 19 \times 4 = -11 + 76 = 65.$$

Answer: 65

Marking scheme:

Correct a , d and formula: 1 mark

Answer 65: 1 mark

Q12. Middle term of 6, 13, 20, ..., 216 [2 marks]

$$a = 6, d = 7, \text{ last term } 216. \text{ Number of terms: } 6 + (n - 1)7 = 216 \Rightarrow n - 1 = 30 \Rightarrow n = 31.$$

$$31 \text{ is odd, so the middle term is the } \frac{31 + 1}{2} = 16\text{th term: } a_{16} = 6 + 15 \times 7 = 111.$$

Answer: 111 (the 16th term)

Marking scheme:

Number of terms = 31: 1 mark

Middle term 111: 1 mark

Q13. Three-digit numbers divisible by 7 [2 marks]

Smallest three-digit multiple of 7 is 105 (7×15); largest is 994 (7×142). They form an AP 105, 112, ..., 994 with $d = 7$.

$$994 = 105 + (n - 1)7 \Rightarrow (n - 1)7 = 889 \Rightarrow n - 1 = 127 \Rightarrow n = 128.$$

Answer: 128

Marking scheme:

Correct first and last terms: 1 mark

$n = 128$: 1 mark

Q14. $5 + 9 + 13 + \dots + 101$ [2 marks]

$$a = 5, d = 4, l = 101. \text{ Number of terms: } 5 + (n - 1)4 = 101 \Rightarrow n = 25.$$

$$\text{Sum} = \frac{n}{2}(a + l) = \frac{25}{2}(5 + 101) = 25 \times 53 = 1325.$$

Answer: 1325

Marking scheme:

$n = 25$: 1 mark

Sum 1325: 1 mark

Q15. 10th term from the end of 4, 9, 14, ..., 254 [2 marks]

Reading the AP backwards from 254 gives a new AP with first term 254 and common difference -5 .

$$10\text{th term from the end} = 254 + 9(-5) = 254 - 45 = 209.$$

Check: the AP has $(254 - 4)/5 + 1 = 51$ terms; the 10th from the end is the 42nd term: $4 + 41 \times 5 = 209$ ✓

Answer: 209

Marking scheme:

Reversing the AP (first term 254, $d = -5$): 1 mark

Answer 209: 1 mark

SECTION C — Short Answer

Q16. Terms of 45, 42, 39, ... with sum 360 [3 marks]

$$a = 45, d = -3. S_n = \frac{n}{2}[90 + (n-1)(-3)] = \frac{n}{2}(93 - 3n) = 360.$$

$$93n - 3n^2 = 720 \Rightarrow 3n^2 - 93n + 720 = 0 \Rightarrow n^2 - 31n + 240 = 0 \Rightarrow (n-15)(n-16) = 0.$$

So $n = 15$ or $n = 16$.

Reason: $a_{16} = 45 + 15(-3) = 0$. Adding a zero term does not change the sum, so

$$S_{15} = S_{16} = 360.$$

Answer: 15 terms or 16 terms (the 16th term is 0)

Marking scheme:

Forming the equation in n : 1 mark

Solving: $n = 15$ or 16 : 1 mark

Explaining the 16th term is 0: 1 mark

Q17. 28th term from partial sums [3 marks]

$$S_7 = 63 \Rightarrow \frac{7}{2}(2a + 6d) = 63 \Rightarrow a + 3d = 9 \dots (1)$$

$$\text{Sum of first 14 terms} = 63 + 161 = 224 \Rightarrow \frac{14}{2}(2a + 13d) = 224 \Rightarrow 2a + 13d = 32 \dots (2)$$

From (1): $2a + 6d = 18$. Subtracting from (2): $7d = 14 \Rightarrow d = 2$, and $a = 9 - 6 = 3$.

$$a_{28} = 3 + 27 \times 2 = 57.$$

Answer: 57

Marking scheme:

Using $S_{14} = 224$ (not 161): 1 mark

Finding $a = 3, d = 2$: 1 mark

28th term 57: 1 mark

Q18. AP from the 7th and 13th terms [3 marks]

$$a + 6d = 32 \dots (1), \quad a + 12d = 62 \dots (2)$$

$$(2) - (1): 6d = 30 \Rightarrow d = 5; \text{ then } a = 32 - 30 = 2. \text{ AP: } 2, 7, 12, 17, \dots$$

$$S_{20} = \frac{20}{2}[2 \times 2 + 19 \times 5] = 10 \times 99 = 990.$$

Answer: AP 2, 7, 12, ...; sum of first 20 terms = 990

Marking scheme:

Both equations: 1 mark

$a = 2, d = 5$ and the AP: 1 mark

Sum 990: 1 mark

Q19. Two-digit numbers not divisible by 3 [3 marks]

All two-digit numbers: 10, 11, ..., 99 — 90 terms. Sum = $\frac{90}{2}(10 + 99) = 45 \times 109 = 4905$.

Two-digit multiples of 3: 12, 15, ..., 99. Number of terms: $12 + (n-1)3 = 99 \Rightarrow n = 30$. Sum = $\frac{30}{2}(12 + 99) = 15 \times 111 = 1665$.

Required sum = $4905 - 1665 = 3240$.

Answer: 3240

Marking scheme:

Sum of all two-digit numbers 4905: 1 mark

Sum of two-digit multiples of 3 = 1665: 1 mark

Final answer 3240: 1 mark

SECTION D – Long Answer

Q20. Sum of mn terms is $\frac{mn+1}{2}$ [4 marks]

$$a_m = a + (m-1)d = \frac{1}{n} \dots (1), \quad a_n = a + (n-1)d = \frac{1}{m} \dots (2)$$

$$(1) - (2): (m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}. \text{ Since } m \neq n, \text{ divide by } (m-n): d = \frac{1}{mn}.$$

$$\text{From (1): } a = \frac{1}{n} - \frac{m-1}{mn} = \frac{m-(m-1)}{mn} = \frac{1}{mn}.$$

$$S_{mn} = \frac{mn}{2}[2a + (mn-1)d] = \frac{mn}{2}\left[\frac{2}{mn} + \frac{mn-1}{mn}\right] = \frac{mn}{2} \times \frac{mn+1}{mn} = \frac{mn+1}{2}.$$

Hence proved.

Answer: $S_{mn} = \frac{mn+1}{2}$ (proved)

Marking scheme:

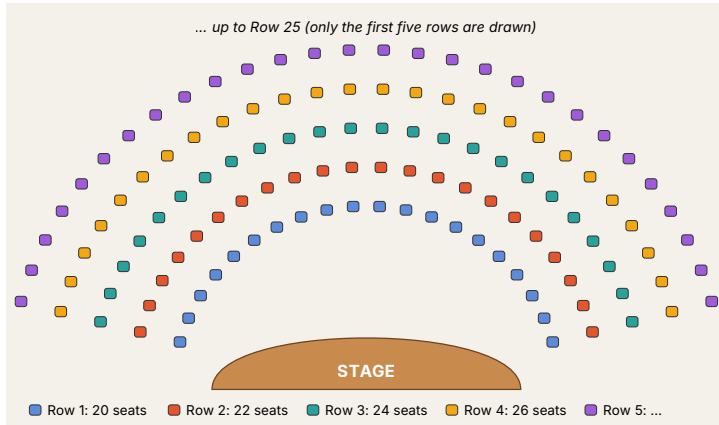
Writing both n th-term equations: 1 mark

Finding $d = 1/(mn)$ (using $m \neq n$): 1 mark

Finding $a = 1/(mn)$: 1 mark

Correct sum and conclusion: 1 mark

SECTION E — Case Study



Q21. Open-air theatre [4 marks]

Seats in successive rows: 20, 22, 24, ... form an AP with $a = 20$, $d = 2$.

(i) $a_{10} = 20 + 9 \times 2 = \mathbf{38 \text{ seats}}$.

(ii) $20 + (n - 1)2 = 60 \Rightarrow (n - 1)2 = 40 \Rightarrow n = 21$. The **21st row** has 60 seats.

(iii) Total: $S_{25} = \frac{25}{2}[40 + 24 \times 2] = \frac{25}{2} \times 88 = \mathbf{1100 \text{ seats}}$.

Booked: $S_{15} = \frac{15}{2}[40 + 14 \times 2] = \frac{15}{2} \times 68 = 510$. Available = $1100 - 510 = \mathbf{590 \text{ seats}}$.

Answer: (i) 38 (ii) 21st row (iii) 1100 seats in all; 590 available

Marking scheme:

(i) 38 seats: 1 mark

(ii) 21st row: 1 mark

(iii) Total 1100: 1 mark; available 590: 1 mark

Final Answer Summary

Q	Final answer	Q	Final answer
Q1	(C)	Q12	111
Q2	(B) -1	Q13	128
Q3	(D) -49	Q14	1325
Q4	(B) 22nd	Q15	209
Q5	(C) 400	Q16	15 or 16 (16th term is 0)
Q6	(A) 3	Q17	57
Q7	(D) -3	Q18	2, 7, 12, ...; 990
Q8	(A) 62	Q19	3240
Q9	(B)	Q20	Proof: $S_{mn} = (mn + 1)/2$
Q10	(A)	Q21	(i) 38 (ii) 21st (iii) 1100; 590
Q11	65		

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

— End of Solutions —