

CLASS 10 — MATHEMATICS

Chapter: Circles

Answer Key with Step-by-Step Solutions

Maximum Marks: 40

Time Allowed: 1 hour 30 minutes

Results used: Theorem 10.1 — the tangent at any point of a circle is perpendicular to the radius through the point of contact; Theorem 10.2 — the lengths of tangents drawn from an external point to a circle are equal; Pythagoras theorem; the angle sum of a triangle and of a quadrilateral.

SECTION A — MCQ Answer Key

Q	Option	Answer	Explanation / calculation
Q1	(A)	0	Every line through an interior point cuts the circle at two points, so no tangent is possible.
Q2	(C)	12 cm	Radius $\perp$ tangent: tangent length $= \sqrt{13^2 - 5^2} = \sqrt{144} = 12$ cm.
Q3	(B)	90°	Theorem 10.1: the tangent at any point is perpendicular to the radius through the point of contact.
Q4	(D)	110°	In quadrilateral OAPB, $\angle OAP = \angle OBP = 90^\circ$, so $\angle AOB = 360^\circ - 90^\circ - 90^\circ - 70^\circ = 110^\circ$.
Q5	(B)	2	A line parallel to the given line can touch the circle only at the two ends of the diameter perpendicular to it.
Q6	(C)	16 cm	The chord touches the smaller circle at its mid-point, so half-chord $= \sqrt{10^2 - 6^2} = 8$ cm; chord = 16 cm.
Q7	(A)	3 cm	For a circumscribed quadrilateral, $AB + CD = AD + BC$ (equal tangents): $6 + 4 = AD + 7 \Rightarrow AD = 3$ cm.
Q8	(D)	5 cm	OP bisects $\angle APB$, so $\angle APO = 30^\circ$. In right $\triangle OAP$, $OA = OP \sin 30^\circ = 10 \times \frac{1}{2} = 5$ cm.
Q9	(A)	Both true; R explains A	Two lines perpendicular to the same line are parallel. R gives exactly this reason.
Q10	(D)	A false, R true	No tangent can be drawn from an interior point, so A is false. R is a true property of tangents.

Marking: 1 mark for each correct option; no part marks in Section A.

SECTION B — Very Short Answer

Q11. Radius of the circle [2 marks]

Let the point be Q and the point of contact T. $OT \perp QT$ (Theorem 10.1), so $\triangle OTQ$ is right-angled at T.

$$OT = \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8 \text{ cm.}$$

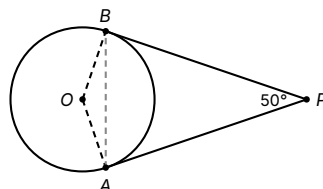
Answer: 8 cm

Marking scheme:

Right angle at the point of contact: 1 mark

Radius 8 cm: 1 mark

Q12. Find $\angle OAB$ [2 marks]



$\angle OAP = \angle OBP = 90^\circ$ (Theorem 10.1). In quadrilateral OAPB: $\angle AOB = 360^\circ - 90^\circ - 90^\circ - 50^\circ = 130^\circ$.

$OA = OB$ (radii), so $\triangle OAB$ is isosceles: $\angle OAB = \angle OBA = \frac{180^\circ - 130^\circ}{2} = 25^\circ$.

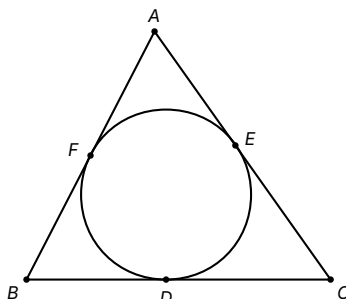
Answer: 25°

Marking scheme:

$\angle AOB = 130^\circ$: 1 mark

$\angle OAB = 25^\circ$: 1 mark

Q13. Perimeter of $\triangle ABC$ [2 marks]



Tangents from an external point are equal: $AE = AF = 4 \text{ cm}$, $BF = BD = 6 \text{ cm}$, $CD = CE = 5 \text{ cm}$.

Perimeter = $AB + BC + CA = (AF + FB) + (BD + DC) + (CE + EA) = 2(4 + 6 + 5) = 30 \text{ cm}$.

Answer: 30 cm

Marking scheme:

Using equal tangents from A, B and C: 1 mark

Perimeter 30 cm: 1 mark

Q14. Prove $\angle POT = \angle QOT$ [2 marks]

In $\triangle OPT$ and $\triangle OQT$: $OP = OQ$ (radii), OT is common, $\angle OPT = \angle OQT = 90^\circ$ (Theorem 10.1).

So $\triangle OPT \cong \triangle OQT$ (RHS), and hence $\angle POT = \angle QOT$ (CPCT).

Answer: Proved

Marking scheme:

Correct three conditions: 1 mark

RHS congruence and conclusion: 1 mark

Q15. Perimeter of OAPB [2 marks]

$\angle OAP = 90^\circ$, so $PA = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24$ cm. Also $PB = PA = 24$ cm (Theorem 10.2).

Perimeter of OAPB = $OA + AP + PB + BO = 7 + 24 + 24 + 7 = 62$ cm.

Answer: 62 cm

Marking scheme:

$PA = PB = 24$ cm: 1 mark

Perimeter 62 cm: 1 mark

SECTION C – Short Answer**Q16. Tangents from an external point are equal** [3 marks]

Given: A circle with centre O, and tangents PQ and PR from an external point P, touching the circle at Q and R. **To prove:** $PQ = PR$.

Construction: Join OP, OQ and OR.

Proof: $\angle OQP = \angle ORP = 90^\circ$ (radius $\perp$ tangent, Theorem 10.1). In right $\triangle OQP$ and $\triangle ORP$: $OQ = OR$ (radii), $OP = OP$ (common hypotenuse).

So $\triangle OQP \cong \triangle ORP$ (RHS), and hence $PQ = PR$ (CPCT).

(Alternatively, by Pythagoras: $PQ^2 = OP^2 - OQ^2 = OP^2 - OR^2 = PR^2$.)

Answer: Proved

Marking scheme:

Given, to prove, construction and figure: 1 mark

Right angles and equal parts: 1 mark

Congruence and conclusion: 1 mark

Q17. Tangent lengths from the vertices [3 marks]

Let $AP = AR = x$, $BP = BQ = y$, $CQ = CR = z$ (equal tangents from each vertex).

$AB: x + y = 10$; $BC: y + z = 8$; $CA: z + x = 12$.

Adding: $2(x + y + z) = 30 \Rightarrow x + y + z = 15$.

So $x = 15 - 8 = 7$, $y = 15 - 12 = 3$, $z = 15 - 10 = 5$.

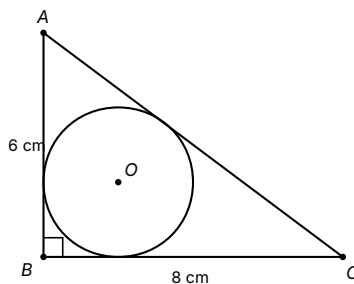
Answer: AP = 7 cm, BQ = 3 cm, CR = 5 cm

Marking scheme:

Setting up equal tangents: 1 mark

Three equations and the sum 15: 1 mark

$AP = 7$, $BQ = 3$, $CR = 5$: 1 mark

Q18. Radius of the incircle [3 marks]

$AC = \sqrt{6^2 + 8^2} = 10$ cm. Let the circle touch AB, BC, CA at D, E, F, and let the radius be r .

$OD \perp AB$ and $OE \perp BC$, and $\angle B = 90^\circ$, so $ODBE$ is a square: $BD = BE = r$.
 Then $AD = 6 - r$ and $CE = 8 - r$. Equal tangents: $AF = AD = 6 - r$ and $CF = CE = 8 - r$.

$$AF + CF = AC \Rightarrow (6 - r) + (8 - r) = 10 \Rightarrow 14 - 2r = 10 \Rightarrow r = 2 \text{ cm.}$$

Answer: 2 cm

Marking scheme:

$AC = 10$ cm and $ODBE$ a square: 1 mark

Expressing tangent lengths in r : 1 mark

$r = 2$ cm: 1 mark

Q19. Show $\triangle APB$ is equilateral [3 marks]

$$\angle OAP = 90^\circ. \text{ In right } \triangle OAP: \sin \angle APO = \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2} \Rightarrow \angle APO = 30^\circ.$$

Similarly $\angle BPO = 30^\circ$ (by $\triangle OAP \cong \triangle OBP$). So $\angle APB = 60^\circ$.

$$PA = PB \text{ (Theorem 10.2), so } \angle PAB = \angle PBA = \frac{180^\circ - 60^\circ}{2} = 60^\circ.$$

All angles are 60° , so $\triangle APB$ is equilateral.

Answer: Proved

Marking scheme:

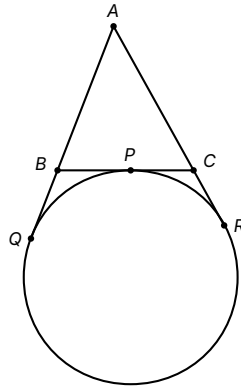
$\angle APO = 30^\circ$: 1 mark

$\angle APB = 60^\circ$: 1 mark

$PA = PB$ and conclusion: 1 mark

SECTION D — Long Answer

Q20. AQ is half the perimeter [4 marks]



Tangents from an external point are equal (Theorem 10.2):

From A: $AQ = AR \dots (1)$; from B: $BQ = BP \dots (2)$; from C: $CP = CR \dots (3)$.

Perimeter of $\triangle ABC = AB + BC + CA = AB + (BP + PC) + CA$

$= AB + BQ + CR + CA$ (using (2) and (3)) $= (AB + BQ) + (AC + CR) = AQ + AR$.

$= AQ + AQ = 2AQ$ (using (1)). Hence $AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$). Proved.

Hence: perimeter $= 7 + 6 + 5 = 18$ cm, so $AQ = \frac{18}{2} = 9$ cm.

Answer: Proved; AQ = 9 cm

Marking scheme:

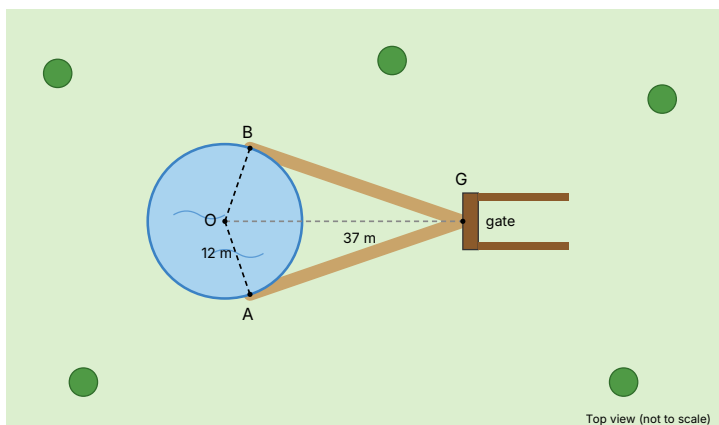
Three pairs of equal tangents: 1 mark

Splitting $BC = BP + PC$ and substituting: 1 mark

Showing perimeter $= 2AQ$: 1 mark

$AQ = 9$ cm: 1 mark

SECTION E — Case Study



Q21. Paths to the pond [4 marks]

(i) $\angle OAG = 90^\circ$, because GA is a tangent at A and the tangent is perpendicular to the radius OA (Theorem 10.1).

(ii) $GA = \sqrt{37^2 - 12^2} = \sqrt{1369 - 144} = \sqrt{1225} = 35$ m.

(iii) $GB = GA = 35$ m (equal tangents). Fence $= OA + AG + GB + BO = 12 + 35 + 35 + 12 = 94$ m.

$$\text{Area OAGB} = 2 \times \text{area of } \triangle \text{OAG} = 2 \times \frac{1}{2} \times 12 \times 35 = \mathbf{420 \text{ m}^2}.$$

Answer: (i) 90° (ii) 35 m (iii) 94 m; 420 m²

Marking scheme:

(i) 90° with reason: 1 mark

(ii) 35 m: 1 mark

(iii) Fence 94 m: 1 mark; area 420 m²: 1 mark

Final Answer Summary

Q	Final answer	Q	Final answer
Q1	(A) 0	Q12	25°
Q2	(C) 12 cm	Q13	30 cm
Q3	(B) 90°	Q14	Proof
Q4	(D) 110°	Q15	62 cm
Q5	(B) 2	Q16	Proof
Q6	(C) 16 cm	Q17	7 cm, 3 cm, 5 cm
Q7	(A) 3 cm	Q18	2 cm
Q8	(D) 5 cm	Q19	Proof
Q9	(A)	Q20	Proof; $AQ = 9$ cm
Q10	(D)	Q21	(i) 90° (ii) 35 m (iii) 94 m; 420 m^2
Q11	8 cm		

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

— End of Solutions —