

CLASS 10 – MATHEMATICS

Chapter: Introduction to Trigonometry

Answer Key with Step-by-Step Solutions

Maximum Marks: 40

Time Allowed: 1 hour 30 minutes

Results used: definitions of the six ratios in a right triangle; values at 0° , 30° , 45° , 60° , 90° ; identities $\sin^2 A + \cos^2 A = 1$, $1 + \tan^2 A = \sec^2 A$, $1 + \cot^2 A = \operatorname{cosec}^2 A$; $\tan A = \sin A / \cos A$, $\sec A = 1 / \cos A$.

SECTION A – MCQ Answer Key

Q	Option	Answer	Explanation / calculation
Q1	(A)	$\frac{4}{5}$	Opposite = $3k$, hypotenuse = $5k \Rightarrow$ adjacent = $\sqrt{25 - 9}k = 4k$. $\cos A = \frac{4}{5}$.
Q2	(C)	1	$\frac{1}{2} + \frac{1}{2} = 1$.
Q3	(B)	60°	$\tan 60^\circ = \sqrt{3}$.
Q4	(D)	5	$5(\operatorname{cosec}^2 \theta - \cot^2 \theta) = 5 \times 1 = 5$.
Q5	(B)	$\cos 60^\circ$	$\frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \cos 60^\circ$. ($\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$.)
Q6	(C)	1	$\sqrt{1 - \sin^2 A} = \cos A$ (A acute, so $\cos A > 0$); $\sec A \times \cos A = 1$.
Q7	(D)	$\frac{5}{12}$	$BC = \sqrt{13^2 - 5^2} = 12$ cm. $\tan C = \frac{AB}{BC} = \frac{5}{12}$.
Q8	(A)	$\sqrt{2} - 1$	$\sin \theta = (\sqrt{2} - 1) \cos \theta$; dividing by $\cos \theta$: $\tan \theta = \sqrt{2} - 1$.
Q9	(D)	A false, R true	$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ and the hypotenuse is longest, so $\sin A < 1$. Since $4/3 > 1$, A is false; R is true.
Q10	(A)	Both true; R explains A	The product is $\sec^2 \theta - \tan^2 \theta$ (difference of squares), which equals 1 by R.

Marking: 1 mark for each correct option; no part marks in Section A.

SECTION B – Very Short Answer

Q11. $\sin A$ and $\cos A$ from 8 $\tan A = 15$ [2 marks]

$\tan A = \frac{15}{8} = \frac{\text{opposite}}{\text{adjacent}}$. Let opposite = $15k$, adjacent = $8k$.

Hypotenuse = $\sqrt{225k^2 + 64k^2} = \sqrt{289k} = 17k$.

$\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$.

Answer: $\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$

Marking scheme:

Hypotenuse 17k: 1 mark

Both ratios: 1 mark

Q12. Evaluate [2 marks]

$$4 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2 = 4 \times \frac{1}{4} + 4 \times \frac{1}{2} - \frac{1}{4}$$

$$= 1 + 2 - \frac{1}{4} = \frac{11}{4}.$$

Answer: $\frac{11}{4}$ **Marking scheme:**

Correct values substituted: 1 mark

Answer 11/4: 1 mark

Q13. Find θ [2 marks]

$$\sqrt{3} \tan 2\theta = 3 \Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

Since $0^\circ < \theta < 45^\circ$, $0^\circ < 2\theta < 90^\circ$, so $2\theta = 60^\circ \Rightarrow \theta = 30^\circ$.**Answer: $\theta = 30^\circ$** **Marking scheme:** $\tan 2\theta = \sqrt{3}$: 1 mark $\theta = 30^\circ$: 1 mark**Q14. Prove $\sin A \tan A + \cos A = \sec A$** [2 marks]

$$\text{LHS} = \sin A \times \frac{\sin A}{\cos A} + \cos A = \frac{\sin^2 A + \cos^2 A}{\cos A}$$

$$= \frac{1}{\cos A} \text{ (since } \sin^2 A + \cos^2 A = 1) = \sec A = \text{RHS.}$$

Answer: Proved**Marking scheme:**Writing $\tan A = \sin A / \cos A$ and combining: 1 mark

Using the identity to conclude: 1 mark

Q15. Angles of $\triangle PQR$ [2 marks]

$$\cos P = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2} \Rightarrow \angle QPR = 60^\circ.$$

$$\angle PRQ = 180^\circ - 90^\circ - 60^\circ = 30^\circ.$$

Answer: $\angle QPR = 60^\circ$, $\angle PRQ = 30^\circ$ **Marking scheme:** $\angle QPR = 60^\circ$ with ratio: 1 mark $\angle PRQ = 30^\circ$: 1 mark**SECTION C – Short Answer****Q16. Evaluate using $5 \cot \theta = 12$** [3 marks]

$$\cot \theta = \frac{12}{5} = \frac{\text{adjacent}}{\text{opposite}}. \text{ Hypotenuse} = \sqrt{144 + 25} = 13.$$

$$\sin \theta = \frac{5}{13}, \cos \theta = \frac{12}{13}.$$

$$\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{\frac{17}{13}}{-\frac{7}{13}} = -\frac{17}{7}.$$

(Shortcut: divide numerator and denominator by $\sin \theta$: $\frac{1 + \cot \theta}{1 - \cot \theta} = \frac{\frac{5}{7}}{-\frac{7}{5}} = -\frac{17}{7}$.)

Answer: $-\frac{17}{7}$

Marking scheme:

Hypotenuse 13 and sin, cos: 1 mark

Correct substitution: 1 mark

Answer $-17/7$: 1 mark

Q17. Find A and B [3 marks]

$$\sin(A + B) = 1 = \sin 90^\circ \Rightarrow A + B = 90^\circ \dots (1)$$

$$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ \Rightarrow A - B = 30^\circ \dots (2)$$

Adding: $2A = 120^\circ \Rightarrow A = 60^\circ$. Then $B = 30^\circ$.

Answer: A = 60°, B = 30°

Marking scheme:

$A + B = 90^\circ$: 1 mark

$A - B = 30^\circ$: 1 mark

$A = 60^\circ, B = 30^\circ$: 1 mark

Q18. Prove the identity [3 marks]

Since $\sec^2 A - \tan^2 A = 1$, $(\sec A - \tan A)(\sec A + \tan A) = 1$. So $\frac{1}{\sec A - \tan A} = \sec A + \tan A$ and $\frac{1}{\sec A + \tan A} = \sec A - \tan A$.

$$\text{LHS} = (\sec A + \tan A) - \frac{1}{\cos A} = \sec A + \tan A - \sec A = \tan A.$$

$$\text{RHS} = \frac{1}{\cos A} - (\sec A - \tan A) = \sec A - \sec A + \tan A = \tan A.$$

LHS = RHS. Hence proved.

Answer: Proved

Marking scheme:

Using $\sec^2 A - \tan^2 A = 1$ to rewrite both reciprocals: 1 mark

LHS = $\tan A$: 1 mark

RHS = $\tan A$ and conclusion: 1 mark

Q19. Show $m^2 - n^2 = 4\sqrt{mn}$ [3 marks]

$$m^2 - n^2 = (m + n)(m - n) = (2 \tan \theta)(2 \sin \theta) = 4 \tan \theta \sin \theta.$$

$$mn = \tan^2 \theta - \sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta = \sin^2 \theta \left(\frac{1 - \cos^2 \theta}{\cos^2 \theta} \right) = \sin^2 \theta \times \frac{\sin^2 \theta}{\cos^2 \theta} = \sin^2 \theta \tan^2 \theta.$$

Since θ is acute, $\sin \theta$ and $\tan \theta$ are positive, so $\sqrt{mn} = \sin \theta \tan \theta$.

Hence $4\sqrt{mn} = 4 \tan \theta \sin \theta = m^2 - n^2$. Proved.

Answer: Proved

Marking scheme:

$$m^2 - n^2 = 4 \tan \theta \sin \theta: 1 \text{ mark}$$

$$mn = \sin^2 \theta \tan^2 \theta: 1 \text{ mark}$$

Square root and conclusion: 1 mark

SECTION D – Long Answer**Q20. Identity and its value at 30° [4 marks]**

(a) Replace the 1 in the numerator by $\sec^2 \theta - \tan^2 \theta$:

$$\begin{aligned} \text{Numerator} &= (\tan \theta + \sec \theta) - (\sec^2 \theta - \tan^2 \theta) = (\tan \theta + \sec \theta) - (\sec \theta - \tan \theta) \\ &= (\tan \theta + \sec \theta)(\sec \theta + \tan \theta) \end{aligned}$$

$$= (\tan \theta + \sec \theta)[1 - \sec \theta + \tan \theta] = (\tan \theta + \sec \theta)(\tan \theta - \sec \theta + 1).$$

$$\text{So LHS} = \frac{(\tan \theta + \sec \theta)(\tan \theta - \sec \theta + 1)}{\tan \theta - \sec \theta + 1} = \tan \theta + \sec \theta.$$

$$\tan \theta + \sec \theta = \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} = \frac{1 + \sin \theta}{\cos \theta} = \text{RHS. Hence proved.}$$

$$\text{(b) At } \theta = 30^\circ: \frac{1 + \sin 30^\circ}{\cos 30^\circ} = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

Answer: (a) Proved; (b) $\sqrt{3}$

Marking scheme:

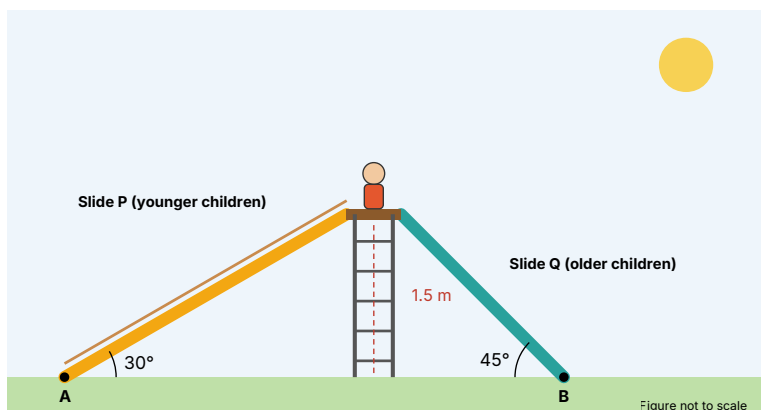
(a) Replacing 1 by $\sec^2 \theta - \tan^2 \theta$: 1 mark

(a) Factorising and cancelling: 1 mark

(a) Converting to $(1 + \sin \theta)/\cos \theta$: 1 mark

(b) Value $\sqrt{3}$: 1 mark

SECTION E — Case Study



Q21. Slides in the park [4 marks]

Let O be the foot of the platform, so the height of the platform is 1.5 m and $\angle O = 90^\circ$ in both triangles.

(i) Slide P: $\sin 30^\circ = \frac{1.5}{PA} = \frac{1}{2} = \frac{1.5}{PA} \Rightarrow PA = 3 \text{ m}.$

(ii) Slide Q: $\sin 45^\circ = \frac{1.5}{QB} = \frac{1}{\sqrt{2}} = \frac{1.5}{QB} \Rightarrow QB = 1.5\sqrt{2} \text{ m} \approx 2.12 \text{ m}.$

(iii) OA: $\tan 30^\circ = \frac{1.5}{OA} \Rightarrow OA = 1.5\sqrt{3} \approx 1.5 \times 1.73 = 2.595 \text{ m}.$ OB: $\tan 45^\circ = \frac{1.5}{OB} \Rightarrow OB = 1.5 \text{ m}.$

$AB = OA + OB = 1.5\sqrt{3} + 1.5 = 1.5(\sqrt{3} + 1) \approx 4.10 \text{ m} (4.095 \text{ m}).$

Answer: (i) 3 m (ii) $1.5\sqrt{2} \approx 2.12 \text{ m}$ (iii) $OA = 1.5\sqrt{3} \approx 2.6 \text{ m}$, $OB = 1.5 \text{ m}$, $AB \approx 4.1 \text{ m}$

Marking scheme:

(i) 3 m: 1 mark

(ii) $1.5\sqrt{2} \text{ m}$: 1 mark

(iii) OA and OB: 1 mark; $AB \approx 4.1 \text{ m}$: 1 mark

Final Answer Summary

Q	Final answer	Q	Final answer
Q1	(A) $\frac{4}{5}$	Q12	$\frac{11}{4}$
Q2	(C) 1	Q13	30°
Q3	(B) 60°	Q14	Proof
Q4	(D) 5	Q15	$60^\circ, 30^\circ$
Q5	(B) $\cos 60^\circ$	Q16	$-\frac{17}{7}$
Q6	(C) 1	Q17	$A = 60^\circ, B = 30^\circ$
Q7	(D) $\frac{5}{12}$	Q18	Proof
Q8	(A) $\sqrt{2} - 1$	Q19	Proof
Q9	(D)	Q20	(a) Proof (b) $\sqrt{3}$
Q10	(A)	Q21	(i) 3 m (ii) $1.5\sqrt{2}$ m (iii) ≈ 4.1 m
Q11	$\frac{15}{17}, \frac{8}{17}$		

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

— End of Solutions —