

CLASS 10 — MATHEMATICS

Chapter: Number Systems

Answer Key with Step-by-Step Solutions

Maximum Marks: 40

Time Allowed: 1 hour 30 minutes

Concepts used: Fundamental Theorem of Arithmetic; prime factorisation; HCF and LCM by prime factorisation; $\text{HCF} \times \text{LCM} = \text{product of two numbers}$; if a prime p divides a^2 then p divides a ; proof of irrationality by contradiction; sums, differences and products involving rational and irrational numbers.

SECTION A — MCQ Answer Key

Q	Option	Answer	Explanation / calculation
Q1	(A)	$2^2 \times 3 \times 13$	$156 = 2 \times 78 = 2 \times 2 \times 39 = 2 \times 2 \times 3 \times 13$. Option (C) is not a prime factorisation because $39 = 3 \times 13$ is composite.
Q2	(C)	150	$\text{HCF} \times \text{LCM} = \text{product of the two numbers}$, so $\text{LCM} = 1800 \div 12 = 150$.
Q3	(D)	36	$\text{HCF} = \text{product of the smallest powers of common primes} = 2^2 \times 3^2 = 36$. (7 is not common to both; 5 and 7 appear in only one number.)
Q4	(B)	$\frac{2\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3} = \frac{2}{3} \times \sqrt{3}$, a product of a non-zero rational and an irrational, so it is irrational. Others simplify to $\frac{4}{3}$, 2 and 5.
Q5	(D)	420	$12 = 2^2 \times 3$, $15 = 3 \times 5$, $21 = 3 \times 7$. $\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$.
Q6	(C)	4	$4^n = 2^{2n}$. Powers of 4 end in 4 (odd n) or 6 (even n): 4, 16, 64, 256, ... It cannot end in 0, as 5 is not a prime factor.
Q7	(B)	a composite number divisible by 11	$3 \times 5 \times 7 \times 11 + 11 = 11 \times (105 + 1) = 11 \times 106 = 1166$. It has factors other than 1 and itself, it is divisible by 11, and it is even.
Q8	(D)	77	Required number = $\text{LCM}(18, 24) + 5$. $18 = 2 \times 3^2$, $24 = 2^3 \times 3$, $\text{LCM} = 2^3 \times 3^2 = 72$. Number = 77. (149 = 144 + 5 also works but is not the least.)
Q9	(A)	Both true; R explains A	2 is a non-zero rational and $\sqrt{7}$ is irrational, so $2\sqrt{7}$ is irrational. R is a true general result, and A is a direct case of it, so R correctly explains A.

Q10	(C)	A true, R false	$15 \times 40 = 600$, and $\text{HCF} \times \text{LCM} = \text{product}$ holds for two numbers, so A is true. For three numbers it fails in general, e.g. $\text{HCF}(6, 72, 120) \times \text{LCM}(6, 72, 120) = 6 \times 360 = 2160 \neq 6 \times 72 \times 120 = 51840$. So R is false.
-----	-----	-----------------	---

Marking: 1 mark for each correct option; no part marks in Section A.

SECTION B — Very Short Answer

Q11. Prime factorisation of 3150 [2 marks]

Divide repeatedly by primes, smallest first:

$$3150 = 2 \times 1575; 1575 = 3 \times 525; 525 = 3 \times 175; 175 = 5 \times 35; 35 = 5 \times 7.$$

$$\text{So } 3150 = 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 2 \times 3^2 \times 5^2 \times 7.$$

$$\text{Check: } 2 \times 9 \times 25 \times 7 = 3150 \checkmark$$

Answer: $3150 = 2 \times 3^2 \times 5^2 \times 7$

Marking scheme:

Correct repeated division / factor tree: 1 mark

Correct final form $2 \times 3^2 \times 5^2 \times 7$: 1 mark

Q12. LCM(144, 198) given HCF = 18 [2 marks]

For two positive integers a and b : $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$.

$$\text{LCM}(144, 198) = \frac{144 \times 198}{18} = 144 \times 11 = 1584.$$

$$\text{Check by prime factorisation: } 144 = 2^4 \times 3^2, 198 = 2 \times 3^2 \times 11, \text{LCM} = 2^4 \times 3^2 \times 11 = 1584 \checkmark$$

Answer: $\text{LCM}(144, 198) = 1584$

Marking scheme:

Stating / using $\text{HCF} \times \text{LCM} = \text{product}$: 1 mark

Correct calculation of LCM = 1584: 1 mark

Q13. Can 15^n end with 0? [2 marks]

A number ends with the digit 0 only if it is divisible by $10 = 2 \times 5$, i.e. its prime factorisation contains both 2 and 5.

$$15^n = (3 \times 5)^n = 3^n \times 5^n. \text{ The only primes in its factorisation are 3 and 5.}$$

By the uniqueness part of the Fundamental Theorem of Arithmetic, 2 can never be a prime factor of 15^n .

Hence 15^n is never divisible by 10, so it cannot end with 0. (In fact it always ends with 5.)

Answer: No; 15^n cannot end with the digit 0 for any natural number n .

Marking scheme:

Writing $15^n = 3^n \times 5^n$ and the condition for ending in 0: 1 mark

Conclusion using uniqueness (no factor 2): 1 mark

Q14. Prove $4 - 3\sqrt{2}$ is irrational [2 marks]

Assume, to the contrary, that $4 - 3\sqrt{2}$ is rational. Then $4 - 3\sqrt{2} = \frac{a}{b}$, where a, b are integers and $b \neq 0$.

Rearranging: $3\sqrt{2} = 4 - \frac{a}{b}$, so $\sqrt{2} = \frac{4b - a}{3b}$.

Since a, b are integers and $3b \neq 0$, $\frac{4b - a}{3b}$ is rational, so $\sqrt{2}$ would be rational.

This contradicts the given fact that $\sqrt{2}$ is irrational. Hence our assumption is wrong.

Answer: $4 - 3\sqrt{2}$ is irrational.

Marking scheme:

Correct assumption and expressing $\sqrt{2}$ in terms of a, b : 1 mark

Identifying the contradiction and concluding: 1 mark

Q15. Largest number dividing 245 and 1029 leaving remainder 5 [2 marks]

If a number leaves remainder 5, it exactly divides $(245 - 5) = 240$ and $(1029 - 5) = 1024$. So the required number = HCF(240, 1024).

$$240 = 2^4 \times 3 \times 5, \quad 1024 = 2^{10}.$$

HCF = smallest power of the common prime $2 = 2^4 = 16$.

Check: $245 = 16 \times 15 + 5$ ✓, $1029 = 16 \times 64 + 5$ ✓, and $16 > 5$ so remainder 5 is possible.

Answer: 16

Marking scheme:

Subtracting remainders and forming HCF(240, 1024): 1 mark

Correct factorisation and HCF = 16: 1 mark

SECTION C – Short Answer**Q16. Prove that $\sqrt{7}$ is irrational** [3 marks]

Assume, to the contrary, that $\sqrt{7}$ is rational. Then $\sqrt{7} = \frac{a}{b}$, where a, b are integers, $b \neq 0$, and a, b are coprime (any common factor can be cancelled).

So $b\sqrt{7} = a$. Squaring: $7b^2 = a^2$. Hence 7 divides a^2 .

Since 7 is prime, by the theorem “if a prime p divides a^2 , then p divides a ”, 7 divides a . Write $a = 7c$ for some integer c .

Substituting: $7b^2 = 49c^2$, so $b^2 = 7c^2$. Hence 7 divides b^2 , and so 7 divides b .

So 7 is a common factor of a and b . This contradicts the fact that a and b are coprime.

The contradiction arose from our assumption. Therefore $\sqrt{7}$ is irrational.

Answer: $\sqrt{7}$ is irrational.

Marking scheme:

Assumption with coprime a, b and obtaining $7b^2 = a^2$: 1 mark

Showing 7 divides a and then 7 divides b (with reason): 1 mark

Stating the contradiction and conclusion: 1 mark

Q17. HCF and LCM of 72, 120, 180 [3 marks]

$$72 = 2^3 \times 3^2, \quad 120 = 2^3 \times 3 \times 5, \quad 180 = 2^2 \times 3^2 \times 5.$$

HCF = product of smallest powers of common primes (2 and 3) = $2^2 \times 3^1 = 12$.

LCM = product of greatest powers of all primes involved = $2^3 \times 3^2 \times 5 = 360$.

HCF $\times$ LCM = $12 \times 360 = 4320$. Product of numbers = $72 \times 120 \times 180 = 15,55,200$.

Since $4320 \neq 15,55,200$, HCF $\times$ LCM is not equal to the product of the three numbers.

Answer: HCF = 12, LCM = 360; HCF $\times$ LCM $\neq$ product of the three numbers.

Marking scheme:

Correct prime factorisation of all three numbers: 1 mark

HCF = 12 and LCM = 360: 1 mark

Correct comparison and conclusion (not equal): 1 mark

Q18. Three bells [3 marks]

The bells ring together again after a time equal to LCM(9, 12, 15) minutes.

$$9 = 3^2, \quad 12 = 2^2 \times 3, \quad 15 = 3 \times 5. \quad \text{LCM} = 2^2 \times 3^2 \times 5 = 180 \text{ minutes} = 3 \text{ hours.}$$

(a) Next time together: 8:00 a.m. + 3 hours = 11:00 a.m.

(b) They ring together every 3 hours: 11:00 a.m., 2:00 p.m., 5:00 p.m. The next would be 8:00 p.m., which is after 6:00 p.m.

So they ring together 3 times after 8:00 a.m. up to 6:00 p.m.

Answer: (a) 11:00 a.m. (b) 3 times

Marking scheme:

Prime factorisation and LCM = 180 minutes: 1 mark

(a) 11:00 a.m.: 1 mark

(b) 3 times, with listed times: 1 mark

Q19. Smallest perfect square divisible by 8, 12 and 20 [3 marks]

Any number divisible by 8, 12 and 20 is a multiple of LCM(8, 12, 20).

$$8 = 2^3, \quad 12 = 2^2 \times 3, \quad 20 = 2^2 \times 5. \quad \text{LCM} = 2^3 \times 3 \times 5 = 120.$$

In a perfect square, every prime in the prime factorisation has an even power. In

$$120 = 2^3 \times 3^1 \times 5^1, \text{ all powers are odd.}$$

$$\text{Multiply by } 2 \times 3 \times 5 = 30 \text{ to make each power even: } 120 \times 30 = 2^4 \times 3^2 \times 5^2 = 3600 = 60^2.$$

$$\text{Check: } 3600 \div 8 = 450, \quad 3600 \div 12 = 300, \quad 3600 \div 20 = 180 \checkmark$$

Answer: 3600

Marking scheme:

LCM = 120 by prime factorisation: 1 mark

Reasoning that each prime needs an even power (multiply by 30): 1 mark

Correct answer 3600: 1 mark

SECTION D – Long Answer

Q20. Prove that $\sqrt{2} + \sqrt{3}$ is irrational [4 marks]

Assume, to the contrary, that $\sqrt{2} + \sqrt{3}$ is rational. Let $\sqrt{2} + \sqrt{3} = x$, where x is rational. Since $\sqrt{2} + \sqrt{3} > 0$, $x \neq 0$.

Then $\sqrt{3} = x - \sqrt{2}$. Squaring both sides: $3 = x^2 - 2x\sqrt{2} + 2$.

Rearranging: $2x\sqrt{2} = x^2 - 1$, so $\sqrt{2} = \frac{x^2 - 1}{2x}$.

Since x is rational and non-zero, $x^2 - 1$ and $2x$ are rational with $2x \neq 0$, so $\frac{x^2 - 1}{2x}$ is rational. Hence $\sqrt{2}$ would be rational.

This contradicts the given fact that $\sqrt{2}$ is irrational. So our assumption is wrong, and $\sqrt{2} + \sqrt{3}$ is irrational.

Answer: $\sqrt{2} + \sqrt{3}$ is irrational.

Marking scheme:

Assuming $\sqrt{2} + \sqrt{3} = x$ (rational, non-zero) and isolating $\sqrt{3}$: 1 mark

Squaring correctly to get $3 = x^2 - 2x\sqrt{2} + 2$: 1 mark

Expressing $\sqrt{2} = (x^2 - 1)/(2x)$ and stating it is rational: 1 mark

Stating the contradiction and final conclusion: 1 mark

SECTION E – Case Study

Prime factorisations used: $144 = 2^4 \times 3^2$, $180 = 2^2 \times 3^2 \times 5$, $240 = 2^4 \times 3 \times 5$.

Q21. Stationery kits [4 marks]

(i) $180 = 2 \times 90 = 2 \times 2 \times 45 = 2 \times 2 \times 3 \times 15 = 2 \times 2 \times 3 \times 3 \times 5$, so **$180 = 2^2 \times 3^2 \times 5$** .

(ii) $\text{LCM}(144, 180) = \text{greatest powers of all primes} = 2^4 \times 3^2 \times 5 = \mathbf{720}$.

(iii) Each item must be shared equally among the kits with nothing left over, so the number of kits must divide 144, 180 and 240. The maximum number of kits = $\text{HCF}(144, 180, 240)$.

Common primes are 2 and 3; smallest powers are 2^2 and 3^1 . $\text{HCF} = 2^2 \times 3 = \mathbf{12 \text{ kits}}$.

Per kit: notebooks = $144 \div 12 = \mathbf{12}$; pens = $180 \div 12 = \mathbf{15}$; pencils = $240 \div 12 = \mathbf{20}$.

Answer: (i) $2^2 \times 3^2 \times 5$ (ii) 720 (iii) 12 kits; each with 12 notebooks, 15 pens, 20 pencils

Marking scheme:

(i) Correct prime factorisation: 1 mark

(ii) $\text{LCM} = 720$: 1 mark

(iii) Recognising HCF and finding 12 kits: 1 mark; correct contents 12, 15, 20: 1 mark

Final Answer Summary

Q	Final answer	Q	Final answer
Q1	(A) $2^2 \times 3 \times 13$	Q12	1584
Q2	(C) 150	Q13	No (no prime factor 2)
Q3	(D) 36	Q14	$4 - 3\sqrt{2}$ is irrational (proof)
Q4	(B) $\frac{2\sqrt{3}}{3}$	Q15	16
Q5	(D) 420	Q16	$\sqrt{7}$ is irrational (proof)
Q6	(C) 4	Q17	HCF = 12, LCM = 360; not equal to product
Q7	(B) composite, divisible by 11	Q18	(a) 11:00 a.m. (b) 3 times
Q8	(D) 77	Q19	3600
Q9	(A)	Q20	$\sqrt{2} + \sqrt{3}$ is irrational (proof)
Q10	(C)	Q21	(i) $2^2 \times 3^2 \times 5$ (ii) 720 (iii) 12 kits: 12, 15, 20
Q11	$2 \times 3^2 \times 5^2 \times 7$		

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

— End of Solutions —