

CLASS 10 — MATHEMATICS

Chapter: Polynomials

Answer Key with Step-by-Step Solutions

Maximum Marks: 40

Time Allowed: 1 hour 30 minutes

Results used: degree of a polynomial; k is a zero of $p(x)$ if $p(k) = 0$; zeroes are the x -coordinates where the graph of $y = p(x)$ meets the x -axis; for $ax^2 + bx + c$ with zeroes α, β : $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$; a quadratic with given sum S and product P of zeroes is $k(x^2 - Sx + P)$.

SECTION A — MCQ Answer Key

Q	Option	Answer	Explanation / calculation
Q1	(C)	5	$(x^2 + 1)(x^3 - 2x) = x^5 - x^3 - 2x$. Highest power of x is 5.
Q2	(B)	1	$p(2) = 0 \Rightarrow 4 + 2k - 6 = 0 \Rightarrow 2k = 2 \Rightarrow k = 1$.
Q3	(D)	3	The graph intersects the x -axis at three points ($x = -3, 0, 2$), so $p(x)$ has 3 zeroes.
Q4	(B)	4	Sum $= -b/a = -(-12)/3 = 4$.
Q5	(A)	$\frac{5}{6}$	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{5}{6}$, since $\alpha + \beta = 5$ and $\alpha\beta = 6$.
Q6	(D)	$x^2 - x - 12$	Sum $= -3 + 4 = 1$, product $= -12$. Polynomial: $x^2 - (\text{sum})x + \text{product} = x^2 - x - 12$.
Q7	(D)	$b = 0$	If the zeroes are m and $-m$, their sum is 0, so $-b/a = 0 \Rightarrow b = 0$. The other conditions are not necessary.
Q8	(A)	2	Zeroes α and $1/\alpha$: product $= 1 = k/2 \Rightarrow k = 2$. (Check: $2x^2 - 5x + 2 = (2x - 1)(x - 2)$, zeroes $\frac{1}{2}$ and 2.)
Q9	(A)	Both true; R explains A	$x^2 + 4 \geq 4 > 0$ for every real x , so the graph stays above the x -axis. Zeroes are the x -coordinates where the graph meets the x -axis, so R correctly explains why there is no zero.
Q10	(C)	A true, R false	$p(4) = 16 - 8 - 8 = 0$ and $p(-2) = 4 + 4 - 8 = 0$, so A is true. The product of zeroes is c/a , not $-c/a$, so R is false.

Marking: 1 mark for each correct option; no part marks in Section A.

SECTION B — Very Short Answer

Q11. Zeroes of $4x^2 - 4x - 3$ [2 marks]

Split the middle term: product of the end terms $= 4 \times (-3) = -12$; $-6 \times 2 = -12$ and $-6 + 2 = -4$.

$$4x^2 - 4x - 3 = 4x^2 - 6x + 2x - 3 = 2x(2x - 3) + 1(2x - 3) = (2x + 1)(2x - 3).$$

Zeroes: $x = -\frac{1}{2}$ and $x = \frac{3}{2}$.

Sum $= -\frac{1}{2} + \frac{3}{2} = 1$, and $\frac{-b}{a} = \frac{-(-4)}{4} = 1 \checkmark$

Product $= (-\frac{1}{2}) \times \frac{3}{2} = -\frac{3}{4}$, and $\frac{c}{a} = -\frac{3}{4} \checkmark$

Answer: Zeroes are $\frac{3}{2}$ and $-\frac{1}{2}$; relationship verified.

Marking scheme:

Correct factorisation and zeroes: 1 mark

Verification of both sum and product: 1 mark

Q12. Quadratic with sum $-3/2$ and product -1 [2 marks]

Required polynomial $= k[x^2 - (\text{sum})x + \text{product}] = k[x^2 + \frac{3}{2}x - 1]$.

Taking $k = 2$ to clear the fraction: $2x^2 + 3x - 2$.

Zeroes: $2x^2 + 4x - x - 2 = 2x(x + 2) - 1(x + 2) = (2x - 1)(x + 2)$, so zeroes are $\frac{1}{2}$ and -2 .

Check: $\frac{1}{2} + (-2) = -\frac{3}{2} \checkmark$, $\frac{1}{2} \times (-2) = -1 \checkmark$

Answer: $2x^2 + 3x - 2$ (or any non-zero multiple); zeroes $\frac{1}{2}$ and -2 .

Marking scheme:

Forming the polynomial correctly: 1 mark

Finding zeroes $\frac{1}{2}$ and -2 : 1 mark

Q13. Zeroes of $y^2 - 7$ [2 marks]

Using $a^2 - b^2 = (a - b)(a + b)$: $y^2 - 7 = (y - \sqrt{7})(y + \sqrt{7})$.

Zeroes: $y = \sqrt{7}$ and $y = -\sqrt{7}$.

Here $a = 1$, $b = 0$, $c = -7$. Sum $= \sqrt{7} + (-\sqrt{7}) = 0 = -b/a \checkmark$

Product $= \sqrt{7} \times (-\sqrt{7}) = -7 = c/a \checkmark$

Answer: Zeroes are $\sqrt{7}$ and $-\sqrt{7}$; relationship verified.

Marking scheme:

Zeroes $\pm\sqrt{7}$: 1 mark

Verification of sum and product: 1 mark

Q14. Value of $\alpha^2 + \beta^2$ [2 marks]

For $x^2 - 6x + 4$: $\alpha + \beta = 6$ and $\alpha\beta = 4$.

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 6^2 - 2 \times 4 = 36 - 8 = 28$.

Answer: 28

Marking scheme:

Correct sum and product of zeroes: 1 mark

Correct identity and value 28: 1 mark

Q15. Values of $x^3 - 9x$ [2 marks]

$p(-3) = (-3)^3 - 9(-3) = -27 + 27 = 0$; $p(0) = 0$; $p(3) = 27 - 27 = 0$.

So -3 , 0 and 3 are zeroes of $p(x)$.

$p(x)$ is a cubic polynomial, and a polynomial of degree 3 has at most 3 zeroes.

Hence -3 , 0 and 3 are the only zeroes.

Answer: -3, 0 and 3 are the (only) zeroes of $p(x)$.

Marking scheme:

All three values correct: 1 mark

Conclusion with reason (at most 3 zeroes): 1 mark

SECTION C – Short Answer

Q16. Polynomial with zeroes 2α and 2β [3 marks]

For $2x^2 + 5x - 3$: $\alpha + \beta = -\frac{5}{2}$ and $\alpha\beta = -\frac{3}{2}$.

New sum $= 2\alpha + 2\beta = 2(\alpha + \beta) = -5$. New product $= 2\alpha \times 2\beta = 4\alpha\beta = -6$.

Required polynomial $= x^2 - (-5)x + (-6) = x^2 + 5x - 6$.

Check: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$ has zeroes $\frac{1}{2}$ and -3 ; doubled they are 1 and -6 , and $(x - 1)(x + 6) = x^2 + 5x - 6$ ✓

Answer: $x^2 + 5x - 6$ (or any non-zero multiple)

Marking scheme:

$\alpha + \beta$ and $\alpha\beta$ correct: 1 mark

New sum -5 and new product -6 : 1 mark

Correct polynomial: 1 mark

Q17. One zero seven times the other [3 marks]

Let the zeroes be α and 7α . Here $a = 3$, $b = -8$, $c = 2k + 1$.

Sum: $\alpha + 7\alpha = \frac{8}{3} \Rightarrow 8\alpha = \frac{8}{3} \Rightarrow \alpha = \frac{1}{3}$. So the zeroes are $\frac{1}{3}$ and $\frac{7}{3}$.

Product: $\alpha \times 7\alpha = 7\alpha^2 = 7 \times \frac{1}{9} = \frac{7}{9}$. Also product $= \frac{2k + 1}{3}$.

$\frac{2k + 1}{3} = \frac{7}{9} \Rightarrow 2k + 1 = \frac{7}{3} \Rightarrow 2k = \frac{4}{3} \Rightarrow k = \frac{2}{3}$.

Answer: Zeroes $\frac{1}{3}$ and $\frac{7}{3}$; $k = \frac{2}{3}$

Marking scheme:

Using sum to get $\alpha = 1/3$ and both zeroes: 1 mark

Forming the product equation: 1 mark

Correct value $k = 2/3$: 1 mark

Q18. Find k when $\alpha - \beta = 1$ [3 marks]

For $x^2 - 5x + k$: $\alpha + \beta = 5$ and $\alpha\beta = k$.

Use the identity $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$.

$1^2 = 5^2 - 4k \Rightarrow 1 = 25 - 4k \Rightarrow 4k = 24 \Rightarrow k = 6$.

Check: $x^2 - 5x + 6 = (x - 2)(x - 3)$, zeroes 3 and 2, difference 1 ✓

Answer: $k = 6$

Marking scheme:

Sum and product of zeroes: 1 mark

Correct identity: 1 mark

Correct value $k = 6$: 1 mark

Q19. Zeroes $3 + \sqrt{2}$ and $3 - \sqrt{2}$ [3 marks]

Sum $= (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$.

$$\text{Product} = (3 + \sqrt{2})(3 - \sqrt{2}) = 3^2 - (\sqrt{2})^2 = 9 - 2 = 7.$$

$$\text{Required polynomial} = x^2 - 6x + 7.$$

$$\text{Value at } x = 3: 3^2 - 6 \times 3 + 7 = 9 - 18 + 7 = -2.$$

Answer: $x^2 - 6x + 7$; its value at $x = 3$ is -2 .

Marking scheme:

Sum = 6 and product = 7: 1 mark

Polynomial $x^2 - 6x + 7$: 1 mark

Value -2 at $x = 3$: 1 mark

SECTION D – Long Answer

Q20. Zeroes of $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ [4 marks]

Here $a = \sqrt{3}$, $b = -8$, $c = 4\sqrt{3}$. Product of end terms = $\sqrt{3}x^2 \times 4\sqrt{3} = 12x^2$; we need two terms with sum $-8x$ and product $12x^2$: $-6x$ and $-2x$.

$$\begin{aligned} \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3} &= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3}) \quad [\text{since } 6x = \sqrt{3} \cdot 2\sqrt{3} \cdot x] \\ &= (\sqrt{3}x - 2)(x - 2\sqrt{3}). \end{aligned}$$

$$\text{Zeroes: } x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \text{ and } x = 2\sqrt{3}.$$

$$\text{Sum} = \frac{2\sqrt{3}}{3} + 2\sqrt{3} = \frac{2\sqrt{3} + 6\sqrt{3}}{3} = \frac{8\sqrt{3}}{3}; \quad -b/a = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3} \checkmark$$

$$\text{Product} = \frac{2}{\sqrt{3}} \times 2\sqrt{3} = 4; \quad c/a = \frac{4\sqrt{3}}{\sqrt{3}} = 4 \checkmark$$

Answer: Zeroes are $\frac{2\sqrt{3}}{3}$ and $2\sqrt{3}$; relationship verified.

Marking scheme:

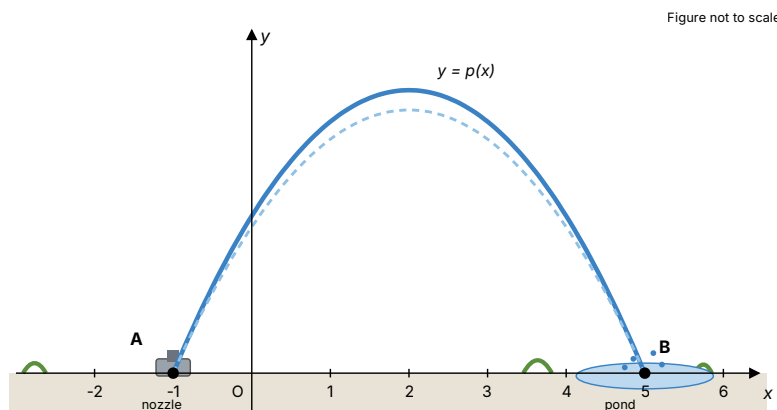
Correct splitting of the middle term: 1 mark

Correct factorisation: 1 mark

Both zeroes $\frac{2\sqrt{3}}{3}$ and $2\sqrt{3}$: 1 mark

Verification of sum and product: 1 mark

SECTION E — Case Study



Q21. Garden fountain [4 marks]

(i) The jet meets the ground (the x -axis) at $A(-1, 0)$ and $B(5, 0)$. Zeroes are the x -coordinates of these points: **-1 and 5** .

(ii) Sum = $-1 + 5 = 4$; Product = $(-1) \times 5 = -5$.

(iii) A quadratic with zeroes -1 and 5 is $k(x + 1)(x - 5) = k(x^2 - 4x - 5)$. The coefficient of x^2 is -1 , so $k = -1$.

$$p(x) = -x^2 + 4x + 5.$$

Height at $x = 2$: $p(2) = -4 + 8 + 5 = 9$ m.

Answer: (i) $-1, 5$ (ii) sum 4 , product -5 (iii) $p(x) = -x^2 + 4x + 5$; height = 9 m

Marking scheme:

(i) Both zeroes: 1 mark

(ii) Sum 4 and product -5 : 1 mark

(iii) Polynomial $-x^2 + 4x + 5$: 1 mark; height 9 m: 1 mark

Final Answer Summary

Q	Final answer	Q	Final answer
Q1	(C) 5	Q12	$2x^2 + 3x - 2$; zeroes $\frac{1}{2}, -2$
Q2	(B) 1	Q13	$\sqrt{7}, -\sqrt{7}$
Q3	(D) 3	Q14	28
Q4	(B) 4	Q15	-3, 0, 3 are the only zeroes
Q5	(A) $\frac{5}{6}$	Q16	$x^2 + 5x - 6$
Q6	(D) $x^2 - x - 12$	Q17	zeroes $\frac{1}{3}, \frac{7}{3}$; $k = \frac{2}{3}$
Q7	(D) $b = 0$	Q18	$k = 6$
Q8	(A) 2	Q19	$x^2 - 6x + 7$; value -2
Q9	(A)	Q20	$\frac{2\sqrt{3}}{3}, 2\sqrt{3}$
Q10	(C)	Q21	(i) -1, 5 (ii) 4, -5 (iii) $-x^2 + 4x + 5$; 9 m
Q11	$\frac{3}{2}, -\frac{1}{2}$		

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

— End of Solutions —