

CLASS 10 — MATHEMATICS

Chapter: Quadratic Equations

Answer Key with Step-by-Step Solutions

Maximum Marks: 40

Time Allowed: 1 hour 30 minutes

Results used: standard form $ax^2 + bx + c = 0$, $a \neq 0$; factorisation by splitting the middle term; quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; discriminant $D = b^2 - 4ac$: $D > 0$ two distinct real roots, $D = 0$ two equal real roots, $D < 0$ no real roots.

SECTION A — MCQ Answer Key

Q	Option	Answer	Explanation / calculation
Q1	(C)	$x^3 + 2x = (x + 1)^3$	Expanding: $x^3 + 2x = x^3 + 3x^2 + 3x + 1 \Rightarrow 3x^2 + x + 1 = 0$, degree 2. (A) gives $4x + 1 = 0$, (B) gives $x + 12 = 0$, (D) is linear.
Q2	(B)	5, -2	$x^2 - 3x - 10 = (x - 5)(x + 2) = 0 \Rightarrow x = 5$ or $x = -2$.
Q3	(D)	1	$D = b^2 - 4ac = (-5)^2 - 4 \times 3 \times 2 = 25 - 24 = 1$.
Q4	(B)	two equal real roots	$D = 144 - 4 \times 4 \times 9 = 0$, so the roots are real and equal (in fact $4x^2 - 12x + 9 = (2x - 3)^2$).
Q5	(D)	2	$4k - 10 + 2 = 0 \Rightarrow 4k = 8 \Rightarrow k = 2$.
Q6	(A)	± 8	Equal roots $\Rightarrow D = k^2 - 64 = 0 \Rightarrow k = \pm 8$.
Q7	(A)	$x^2 - 4x + 3\sqrt{2} = 0$	$D = 16 - 12\sqrt{2}$. Since $\sqrt{2} > 1.4$, $12\sqrt{2} > 16.8 > 16$, so $D < 0$: no real roots. (B): $D = 16 + 12\sqrt{2} > 0$; (C): $D = 17 > 0$; (D): $D = 0$.
Q8	(C)	0 or 3	$D = 4(k - 1)^2 - 4(k + 1) = 4(k^2 - 3k) = 0 \Rightarrow k = 0$ or 3 . Both keep $a = k + 1 \neq 0$.
Q9	(A)	Both true; R explains A	$D = 4 - 20 = -16 < 0$, so A is true. R is the general rule, and it is exactly why A holds.
Q10	(D)	A false, R true	$(x - 3)^2 = x^2 + 2 = x^2 - 6x + 9 = x^2 + 2 \Rightarrow -6x + 7 = 0$, which is linear. So A is false; R is the correct definition.

Marking: 1 mark for each correct option; no part marks in Section A.

SECTION B — Very Short Answer

Q11. Factorise $2x^2 + x - 6 = 0$ [2 marks]

Product $ac = 2 \times (-6) = -12$. Two numbers with product -12 and sum 1 are 4 and -3 .

$$2x^2 + 4x - 3x - 6 = 2x(x + 2) - 3(x + 2) = (2x - 3)(x + 2) = 0.$$

$$\text{So } 2x - 3 = 0 \text{ or } x + 2 = 0 \Rightarrow x = \frac{3}{2} \text{ or } x = -2.$$

Answer: $\frac{3}{2}$ and -2

Marking scheme:

Correct splitting and factorisation: 1 mark

Both roots: 1 mark

Q12. Quadratic formula: $x^2 - 4x + 1 = 0$ [2 marks]

$$a = 1, b = -4, c = 1. D = 16 - 4 = 12 > 0.$$

$$x = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}.$$

Answer: $2 + \sqrt{3}$ and $2 - \sqrt{3}$

Marking scheme:

Correct D and substitution in the formula: 1 mark

Simplified roots $2 \pm \sqrt{3}$: 1 mark

Q13. Nature of roots of $2x^2 - 2\sqrt{2}x + 1 = 0$ [2 marks]

$$a = 2, b = -2\sqrt{2}, c = 1. D = (-2\sqrt{2})^2 - 4 \times 2 \times 1 = 8 - 8 = 0.$$

Since $D = 0$, the equation has two equal real roots.

$$\text{Each root} = \frac{-b}{2a} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} \left(= \frac{1}{\sqrt{2}} \right).$$

Answer: Two equal real roots, each $\frac{\sqrt{2}}{2}$

Marking scheme:

$D = 0$ and conclusion (equal real roots): 1 mark

Roots $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$: 1 mark

Q14. Equal roots of $kx(x - 3) + 9 = 0$ [2 marks]

$$\text{Standard form: } kx^2 - 3kx + 9 = 0, \text{ so } a = k, b = -3k, c = 9.$$

$$\text{Equal roots} \Rightarrow D = 9k^2 - 36k = 9k(k - 4) = 0 \Rightarrow k = 0 \text{ or } k = 4.$$

$k = 0$ is rejected: the equation would become $9 = 0$, which is not a quadratic equation. So $k = 4$.

Answer: $k = 4$

Marking scheme:

Correct standard form and $D = 0$: 1 mark

$k = 4$ with $k = 0$ rejected: 1 mark

Q15. Consecutive odd integers [2 marks]

$$\text{Let the integers be } n \text{ and } n + 2. \text{ Then } n(n + 2) = 255 \Rightarrow n^2 + 2n - 255 = 0.$$

$$(n + 17)(n - 15) = 0 \Rightarrow n = 15 \text{ or } n = -17. \text{ Since the integers are positive, } n = 15.$$

$$\text{Check: } 15 \times 17 = 255 \checkmark$$

Answer: 15 and 17

Marking scheme:

Forming the quadratic equation: 1 mark

Solving and choosing 15, 17: 1 mark

SECTION C – Short Answer

Q16. Rational equation [3 marks]

$$\text{LHS} = \frac{(x-7) - (x+4)}{(x+4)(x-7)} = \frac{-11}{(x+4)(x-7)}.$$

$$\text{So } \frac{-11}{(x+4)(x-7)} = \frac{11}{30} \Rightarrow (x+4)(x-7) = -30.$$

$$x^2 - 3x - 28 = -30 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1 \text{ or } 2.$$

Both values are allowed (neither is -4 or 7). Check $x = 1$: $\frac{1}{5} - \frac{1}{-6} = \frac{11}{30}$ ✓

Answer: $x = 1$ or $x = 2$

Marking scheme:

Combining the fractions correctly: 1 mark

Reducing to $x^2 - 3x + 2 = 0$: 1 mark

Both roots 1 and 2: 1 mark

Q17. Garden with perimeter 60 m and area 250 m² [3 marks]

Let the length be l m. Since $2(l + b) = 60$, breadth = $(30 - l)$ m.

$$\text{Area: } l(30 - l) = 250 \Rightarrow l^2 - 30l + 250 = 0.$$

$$D = (-30)^2 - 4 \times 1 \times 250 = 900 - 1000 = -100 < 0.$$

So the equation has no real roots: no real length is possible. The garden cannot be designed.

Answer: Not possible, because the discriminant is negative

Marking scheme:

Forming $l^2 - 30l + 250 = 0$: 1 mark

$D = -100$: 1 mark

Conclusion: not possible: 1 mark

Q18. Speed of the bus [3 marks]

Let the original speed be v km/h. Time taken = $480/v$ hours; at $v + 8$ km/h it is $480/(v + 8)$ hours.

$$\frac{480}{v} - \frac{480}{v+8} = 2 \Rightarrow 480 \times 8 = 2v(v+8) \Rightarrow v^2 + 8v - 1920 = 0.$$

$$(v+48)(v-40) = 0 \Rightarrow v = 40 \text{ (speed cannot be negative).}$$

Check: $480/40 = 12$ h and $480/48 = 10$ h; difference 2 h ✓

Answer: 40 km/h

Marking scheme:

Forming the time equation: 1 mark

Reducing to $v^2 + 8v - 1920 = 0$: 1 mark

$v = 40$ km/h: 1 mark

Q19. Find p , then k [3 marks]

$$-5 \text{ is a root of } 2x^2 + px - 15 = 0: 2(25) - 5p - 15 = 0 \Rightarrow 35 = 5p \Rightarrow p = 7.$$

$$\text{Second equation: } 7(x^2 + x) + k = 0 \Rightarrow 7x^2 + 7x + k = 0.$$

$$\text{Equal roots } \Rightarrow D = 49 - 28k = 0 \Rightarrow k = \frac{49}{28} = \frac{7}{4}.$$

Answer: $k = \frac{7}{4}$

Marking scheme: $p = 7$: 1 markCorrect equation and $D = 0$: 1 mark $k = 7/4$: 1 mark**SECTION D – Long Answer****Q20. Values of p for equal roots** [4 marks] $a = 2p + 1$, $b = -(7p + 2)$, $c = 7p - 3$. Equal roots $\Rightarrow b^2 - 4ac = 0$.

$$(7p + 2)^2 - 4(2p + 1)(7p - 3) = 0$$

$$49p^2 + 28p + 4 - 4(14p^2 + p - 3) = 0 \Rightarrow 49p^2 + 28p + 4 - 56p^2 - 4p + 12 = 0 \\ -7p^2 + 24p + 16 = 0 \Rightarrow 7p^2 - 24p - 16 = 0 \Rightarrow 7p^2 - 28p + 4p - 16 = 0 \Rightarrow (7p + 4)(p - 4) = 0.$$

So $p = 4$ or $p = -\frac{4}{7}$. In both cases $2p + 1 \neq 0$, so the equation stays quadratic.

$$\text{Equal root} = \frac{-b}{2a} = \frac{7p + 2}{2(2p + 1)}.$$

$$\text{For } p = 4: \text{root} = \frac{30}{18} = \frac{5}{3}. \text{ (Equation: } 9x^2 - 30x + 25 = (3x - 5)^2 \checkmark)$$

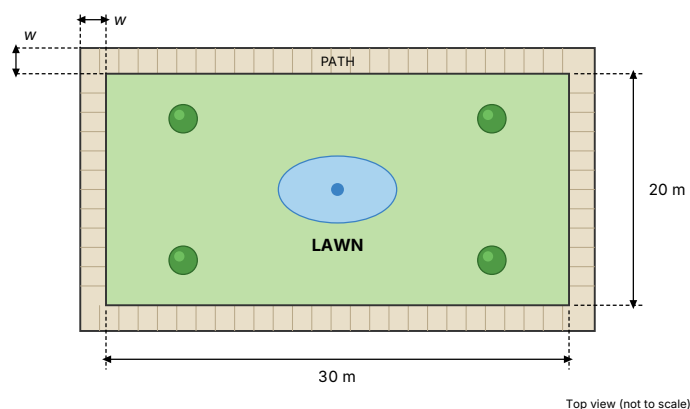
$$\text{For } p = -\frac{4}{7}: 7p + 2 = -2 \text{ and } 2p + 1 = -\frac{1}{7}, \text{ so root} = \frac{-2}{2 \times (-1/7)} = 7.$$

Answer: $p = 4$ (equal roots $\frac{5}{3}, \frac{5}{3}$) or $p = -\frac{4}{7}$ (equal roots 7, 7)**Marking scheme:**

Correct discriminant set to zero: 1 mark

Simplifying to $7p^2 - 24p - 16 = 0$: 1 mark $p = 4$ and $p = -4/7$: 1 markEqual roots $5/3$ and 7: 1 mark

SECTION E — Case Study



Q21. Path around the lawn [4 marks]

(i) The path adds w m on each side: outer length = $(30 + 2w)$ m, outer breadth = $(20 + 2w)$ m.

(ii) Area of path = outer area – lawn area: $(30 + 2w)(20 + 2w) - 600 = 216$.

$$600 + 100w + 4w^2 - 600 = 216 \Rightarrow 4w^2 + 100w - 216 = 0 \Rightarrow w^2 + 25w - 54 = 0.$$

(iii) $(w + 27)(w - 2) = 0 \Rightarrow w = 2$ (width cannot be negative). So the path is **2 m** wide.

Outer rectangle: $34 \text{ m} \times 24 \text{ m}$. Perimeter = $2(34 + 24) = \mathbf{116 \text{ m}}$.

Check: $34 \times 24 - 600 = 816 - 600 = 216 \checkmark$

Answer: (i) $(30 + 2w)$ m, $(20 + 2w)$ m (ii) $w^2 + 25w - 54 = 0$ (iii) $w = 2$ m; perimeter 116 m

Marking scheme:

(i) Both outer dimensions: 1 mark

(ii) Correct standard form: 1 mark

(iii) $w = 2$ m: 1 mark; perimeter 116 m: 1 mark

Final Answer Summary

Q	Final answer	Q	Final answer
Q1	(C)	Q12	$2 \pm \sqrt{3}$
Q2	(B) 5, -2	Q13	equal roots, $\frac{\sqrt{2}}{2}$
Q3	(D) 1	Q14	$k = 4$
Q4	(B) two equal real roots	Q15	15, 17
Q5	(D) 2	Q16	1, 2
Q6	(A) ± 8	Q17	Not possible ($D = -100$)
Q7	(A)	Q18	40 km/h
Q8	(C) 0 or 3	Q19	$k = \frac{7}{4}$
Q9	(A)	Q20	$p = 4 \rightarrow \frac{5}{3}; p = -\frac{4}{7} \rightarrow 7$
Q10	(D)	Q21	(ii) $w^2 + 25w - 54 = 0$ (iii) 2 m; 116 m
Q11	$\frac{3}{2}, -2$		

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

— End of Solutions —