SAMPLE PAPER 2

MATHEMATICS STANDARD (041)

CLASS X (2026 – 27)

Answer Key with Step-by-Step Solutions and Marking Scheme

Maximum Marks: 80 **Time Allowed: 3 hours**

SECTION – A (20 × 1 = 20)

Q	Option	Answer	Explanation / calculation	Chapter
Q1	**(B)**	216	LCM = highest powers: $2^3 \times 3^3 = 216$.	Real Numbers
Q2	**(C)**	$x^2 - 5x + 6$	$x^2 -$ (sum)x + product.	Polynomials
Q3	**(A)**	parallel	$\frac{2}{4} = \frac{3}{6} \neq \frac{5}{7} \Rightarrow$ parallel lines.	Linear Equations
Q4	**(D)**	6	$D = k^2 - 36 = 0 \Rightarrow k = \pm 6$; positive value 6.	Quadratic Equations
Q5	**(B)**	480	$4(1 + 2 + \ldots + 15) = 4 \times 120 = 480$.	Arithmetic Progressions
Q6	**(C)**	36 cm	Ratio of perimeters = ratio of sides = 4 : 6, so perimeter = $24 \times 6/4 = 36$ cm.	Triangles
Q7	**(A)**	(4, 3)	$((1 + 7)/2, (8 - 2)/2) = (4, 3)$.	Coordinate Geometry
Q8	**(D)**	$\frac{5}{4}$	$2 \times \frac{1}{4} + 3 \times \frac{1}{4} = \frac{5}{4}$.	Trigonometry
Q9	**(B)**	4 cm	The radii to the points of contact and the two tangents form a square of side 4 cm.	Circles
Q10	**(A)**	154 cm²	15 minutes $\Rightarrow 90°$: $\frac{1}{4} \times \frac{22}{7} \times 196 = 154$ cm^2.	Areas Related to Circles
Q11	**(C)**	1848 cm²	$3\pi r^2 = 3 \times \frac{22}{7} \times 196 = 1848$ cm^2.	Surface Areas and Volumes
Q12	**(D)**	22	Mean = $a + h \times$ (mean of u) $= 25 + 10 \times (-0.3) = 22$.	Statistics
Q13	**(A)**	$\frac{2}{7}$	A leap year has 52 weeks + 2 extra days; of the 7 equally likely pairs of extra days, 2 contain a Sunday.	Probability
Q14	**(B)**	(−1, 1)	$x = (3 \times (-3) + 2 \times 2)/5 = -1$; $y = (3 \times 5 + 2 \times (-5))/5 = 1$.	Coordinate Geometry
Q15	**(C)**	$\frac{12}{5}$	$\cos\theta = \frac{12}{13}$, $\sin\theta = \frac{5}{13} \Rightarrow \cot\theta = \frac{12}{5}$.	Trigonometry

Q16	**(D)**	$30\sqrt{3}$ m	$h = 30\tan 60° = 30\sqrt{3}$ m.	Applications of Trigonometry
Q17	**(B)**	616 m²	$2\pi r = 88 \Rightarrow r = 14$ m; area $= \frac{22}{7} \times 196 = 616$ m^2.	Areas Related to Circles
Q18	**(A)**	$\frac{3}{26}$	Black face cards: 3 spades + 3 clubs = 6; $\frac{6}{52} = \frac{3}{26}$.	Probability
Q19	**(A)**	Both true; R explains A	(0, 4) has abscissa 0, so it lies on the y-axis, exactly as R states.	Coordinate Geometry
Q20	**(C)**	A true, R false	$a_{23} = 3 + 22 \times 5 = 113$, so A is true. The correct formula is $a + (n-1)d$; R is false.	Arithmetic Progressions

Marking: 1 mark for each correct option.

SECTION – B **(5 × 2 = 10)**

Q21(A). HCF and LCM of 120 and 252 [2 marks]

$120 = 2^3 \times 3 \times 5$; $252 = 2^2 \times 3^2 \times 7$.

HCF $= 2^2 \times 3 = 12$; LCM $= 2^3 \times 3^2 \times 5 \times 7 = 2520$.

HCF × LCM $= 12 \times 2520 = 30240 = 120 \times 252$ ✓

Answer: HCF = 12, LCM = 2520

> **Marking scheme:**
> Prime factorisations, HCF and LCM: 1 mark
> Verification: 1 mark

Q21(B). 3 + 2√5 is irrational [2 marks]

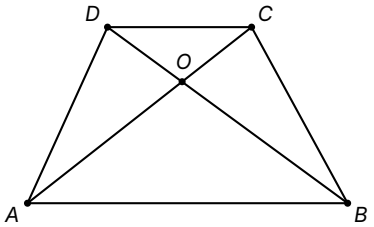
Assume $3 + 2\sqrt{5} = r$, a rational number. Then $\sqrt{5} = \frac{r-3}{2}$.

The right side is rational, so $\sqrt{5}$ would be rational — a contradiction. Hence $3 + 2\sqrt{5}$ is irrational.

Answer: Proved

> **Marking scheme:**
> Assumption and rearrangement: 1 mark
> Contradiction and conclusion: 1 mark

Q22(A). Trapezium diagonals [2 marks]



△AOB ~ △COD (alternate angles, AB ∥ DC), so OA/OC = OB/OD.

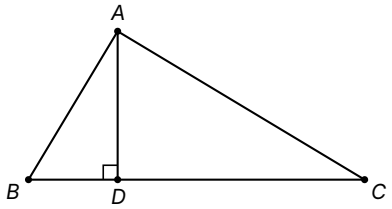
$\frac{3x-1}{5x-3} = \frac{2x+1}{6x-5} \Rightarrow 18x^2 - 21x + 5 = 10x^2 - x - 3 \Rightarrow 8x^2 - 20x + 8 = 0 \Rightarrow 2x^2 - 5x + 2 = 0$.

$(2x-1)(x-2) = 0 \Rightarrow x = \frac{1}{2}$ or 2. For $x = \frac{1}{2}$, OC = −0.5 cm (impossible). So $x = 2$ (OA = OB = 5 cm, OC = OD = 7 cm).

Answer: $x = 2$

> **Marking scheme:**
> Correct proportion: 1 mark
> x = 2 with x = 1/2 rejected: 1 mark

Q22(B). Prove AD² = BD × DC [2 marks]



In △ADB and △CDA: ∠ADB = ∠CDA = 90°, and ∠BAD = 90° − ∠DAC = ∠ACD (since ∠BAC = 90° and ∠ADC = 90°). So △ADB ~ △CDA (AA).

Hence $\frac{AD}{CD} = \frac{BD}{AD} \Rightarrow AD^2 = BD \times DC$.

Answer: Proved

> **Marking scheme:**
> Similarity with reasons: 1 mark
> Proportion and conclusion: 1 mark

Q23. tan θ + cot θ [2 marks]

Squaring: $1 + 2\sin\theta\cos\theta = 2 \Rightarrow \sin\theta\cos\theta = \frac{1}{2}$.

$\tan\theta + \cot\theta = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\frac{1}{2}} = 2$.

Answer: 2

> **Marking scheme:**
> sin θ cos θ = 1/2: 1 mark
> Answer 2: 1 mark

Q24. Area of the sector [2 marks]

Arc length = 27.2 − 2 × 5.6 = 16 cm.

Area = $\frac{1}{2}$ × arc × $r = \frac{1}{2} \times 16 \times 5.6 = 44.8$ cm^2.

Answer: 44.8 cm^2

> **Marking scheme:**
> Arc 16 cm: 1 mark
> Area 44.8 cm²: 1 mark

Q25. Defective bulbs [2 marks]

Defective bulbs = 0.04 × 600 = 24, so good bulbs = 600 − 24 = 576.

After removing 8 good bulbs: total = 592, defective = 24. P(defective) = $\frac{24}{592} = \frac{3}{74}$.

Answer: 576 good bulbs; P(defective) = $\frac{3}{74}$

> **Marking scheme:**
> 576 good bulbs: 1 mark
> Probability 3/74: 1 mark

SECTION – C (6 × 3 = 18)

Q26. Prove $\sqrt{3}$ is irrational [3 marks]

Assume $\sqrt{3} = \frac{a}{b}$, with a, b coprime integers and $b \neq 0$. Then $3b^2 = a^2$, so 3 divides a^2, and hence 3 divides a.

Let $a = 3c$: $3b^2 = 9c^2 \Rightarrow b^2 = 3c^2$, so 3 divides b.

3 is a common factor of a and b, contradicting that they are coprime. Hence $\sqrt{3}$ is irrational.

Answer: Proved

> **Marking scheme:**
> Assumption and $3b^2 = a^2$: 1 mark
> 3 | a and 3 | b: 1 mark
> Contradiction and conclusion: 1 mark

Q27. Polynomial with reciprocal zeroes [3 marks]

$2x^2 - 7x + 3 = (2x - 1)(x - 3)$, zeroes $\frac{1}{2}$ and 3.

Reciprocals: 2 and $\frac{1}{3}$. Sum = $\frac{7}{3}$, product = $\frac{2}{3}$.

Required polynomial = $k(x^2 - \frac{7}{3}x + \frac{2}{3})$; with $k = 3$: $3x^2 - 7x + 2$.

Answer: $3x^2 - 7x + 2$ (or any non-zero multiple)

> **Marking scheme:**
> Zeroes 1/2 and 3: 1 mark
> Sum and product of reciprocals: 1 mark
> Polynomial: 1 mark

Q28(A). Chairs and tables [3 marks]

Let a chair cost ₹ c and a table ₹ t: $4c + 3t = 5400$ ... (1), $3c + 2t = 3750$ ... (2).

(1) × 2 − (2) × 3: $8c - 9c = 10800 - 11250 \Rightarrow -c = -450 \Rightarrow c = 450$.

From (2): $1350 + 2t = 3750 \Rightarrow t = 1200$. Check: $4(450) + 3(1200) = 5400$ ✓

Answer: Chair ₹ 450; table ₹ 1200

> **Marking scheme:**
> Both equations: 1 mark
> Elimination: 1 mark
> Chair ₹ 450, table ₹ 1200: 1 mark

Q28(B). Angles of the triangle [3 marks]

Let $\angle B = x$. Then $\angle C = 3x$, and $3x = 2(\angle A + x) \Rightarrow \angle A = x/2$.

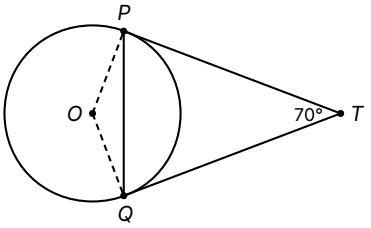
Angle sum: $x/2 + x + 3x = 180° \Rightarrow 4.5x = 180° \Rightarrow x = 40°$.

$\angle A = 20°$, $\angle B = 40°$, $\angle C = 120°$. Check: $2(20° + 40°) = 120°$ ✓

Answer: $\angle A = 20°$, $\angle B = 40°$, $\angle C = 120°$

> **Marking scheme:**
> Expressing angles in one variable: 1 mark
> Equation and x = 40°: 1 mark
> All three angles: 1 mark

Q29. Tangents at the ends of a chord [3 marks]



TP and TQ are tangents from the external point T, so TP = TQ (lengths of tangents from an external point are equal).

In $\triangle TPQ$, TP = TQ, so the angles opposite these sides are equal: $\angle TQP = \angle TPQ$. Hence proved.

$\angle TPQ = \angle TQP = \frac{180° - 70°}{2} = 55°$. Since $\angle OPT = \angle OQT = 90°$, in quadrilateral OPTQ: $\angle POQ = 360° - 90° - 90° - 70° = 110°$.

Answer: Proved; $\angle TPQ = 55°$, $\angle POQ = 110°$

> **Marking scheme:**
> TP = TQ with reason: 1 mark
> Isosceles triangle ⇒ equal angles: 1 mark
> $\angle TPQ = 55°$ and $\angle POQ = 110°$: 1 mark

Q30. Collinear points [3 marks]

$AB = \sqrt{(4-1)^2 + (3+1)^2} = 5$; $BC = \sqrt{(7-4)^2 + (7-3)^2} = 5$; $AC = \sqrt{(7-1)^2 + (7+1)^2} = 10$.

AB + BC = 5 + 5 = 10 = AC, so A, B and C are collinear.

Since AB = BC, B divides AC in the ratio 1 : 1 (B is the mid-point of AC; check: ((1 + 7)/2, (−1 + 7)/2) = (4, 3) ✓).

Answer: Collinear; B divides AC in the ratio 1 : 1

> **Marking scheme:**
> All three distances: 1 mark
> AB + BC = AC ⇒ collinear: 1 mark
> Ratio 1 : 1: 1 mark

Q31(A). Prove $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$ [3 marks]

$$\text{LHS} = \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \sin^2\theta\left(\frac{1 - \cos^2\theta}{\cos^2\theta}\right)$$

$$= \sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta} = \sin^2\theta \times \tan^2\theta = \text{RHS}.$$

Answer: Proved

> **Marking scheme:**
> Writing tan θ = sin θ/cos θ and taking $\sin^2\theta$ common: 1 mark
> Using $1 - \cos^2\theta = \sin^2\theta$: 1 mark
> Conclusion: 1 mark

Q31(B). Prove $\cos^2\theta + \cos^4\theta = 1$ [3 marks]

$\sin\theta = 1 - \sin^2\theta = \cos^2\theta$.

So $\cos^4\theta = (\cos^2\theta)^2 = \sin^2\theta$.

$\cos^2\theta + \cos^4\theta = \cos^2\theta + \sin^2\theta = 1$. Proved.

Answer: Proved

> **Marking scheme:**
> $\sin\theta = \cos^2\theta$: 1 mark
> $\cos^4\theta = \sin^2\theta$: 1 mark
> Conclusion: 1 mark

SECTION – D (4 × 5 = 20)

Q32. Books bought by the shopkeeper [5 marks]

Let the number of books be x; the cost of each is ₹ $1200/x$.

$\frac{1200}{x} - \frac{1200}{x+10} = 20 \Rightarrow 1200 \times 10 = 20x(x+10) \Rightarrow x^2 + 10x - 600 = 0.$

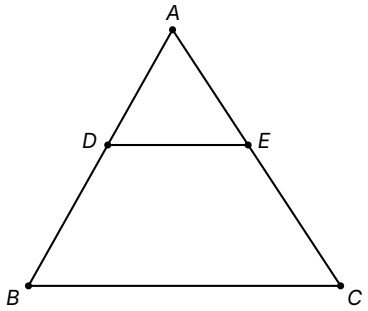
$(x + 30)(x - 20) = 0 \Rightarrow x = 20$ (the number of books cannot be negative).

Cost of each book = 1200 ÷ 20 = ₹ 60. Check: 1200 ÷ 30 = ₹ 40, which is ₹ 20 less ✓

Answer: 20 books; ₹ 60 each

> **Marking scheme:**
> Forming the equation: 1 mark
> Quadratic $x^2 + 10x - 600 = 0$: 1 mark
> Solving: 2 marks
> 20 books at ₹ 60 each: 1 mark

Q33. Converse of the Basic Proportionality Theorem [5 marks]



Given: In △ABC, D and E lie on AB and AC such that AD/DB = AE/EC. **To prove:** DE ∥ BC.

Proof: Suppose DE is not parallel to BC. Draw DE′ ∥ BC, meeting AC at E′.

By BPT, AD/DB = AE′/E′C. With the given ratio, AE/EC = AE′/E′C.

Adding 1 to both sides: (AE + EC)/EC = (AE′ + E′C)/E′C ⇒ AC/EC = AC/E′C ⇒ EC = E′C.

So E and E′ coincide, i.e. DE ∥ BC. Hence proved.

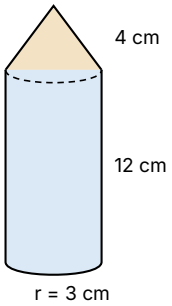
Application: $\frac{PS}{SQ} = \frac{1.5}{3} = \frac{1}{2}$ and $\frac{PT}{TR} = \frac{2}{4} = \frac{1}{2}$. Equal ratios ⇒ ST ∥ QR (converse of BPT).

△PST ~ △PQR (AA), so $\frac{ST}{QR} = \frac{PS}{PQ} = \frac{1.5}{4.5} = \frac{1}{3} \Rightarrow$ ST = 3 cm.

Answer: Proved; ST = 3 cm

> **Marking scheme:**
> Given, to prove and construction: 1 mark
> Using BPT and equating ratios: 1 mark
> EC = E′C ⇒ conclusion: 1 mark
> ST ∥ QR: 1 mark
> ST = 3 cm: 1 mark

Q34(A). Toy rocket [5 marks]



Slant height of cone = $\sqrt{3^2 + 4^2}$ = 5 cm.

Volume = $\pi r^2 h + \frac{1}{3}\pi r^2 h' = \pi(9)(12) + \frac{1}{3}\pi(9)(4) = 108\pi + 12\pi = 120\pi = 120 \times 3.14 = 376.8$ cm^3.

TSA = base + CSA of cylinder + CSA of cone = $\pi r^2 + 2\pi rh + \pi rl = 9\pi + 72\pi + 15\pi = 96\pi = 301.44$ cm^2.

Answer: Volume 376.8 cm^3; TSA 301.44 cm^2

> **Marking scheme:**
> Slant height 5 cm: 1 mark
> Volume 376.8 cm³: 2 marks
> TSA 301.44 cm²: 2 marks

Q34(B). Cone + cylinder + hemisphere [5 marks]

r = 6 cm. Cone: $\frac{1}{3}\pi(36)(8) = 96\pi$; cylinder: $\pi(36)(15) = 540\pi$; hemisphere: $\frac{2}{3}\pi(216) = 144\pi$.

Volume = $96\pi + 540\pi + 144\pi = 780\pi = 780 \times 3.14 = 2449.2$ cm^3.

Slant height = $\sqrt{36 + 64}$ = 10 cm. TSA = $\pi rl + 2\pi rh + 2\pi r^2 = 60\pi + 180\pi + 72\pi = 312\pi = 979.68$ cm^2.

Answer: Volume = 2449.2 cm^3; TSA = 979.68 cm^2

> **Marking scheme:**
> Three volumes: 2 marks
> Total volume: 1 mark
> TSA 979.68 cm²: 2 marks

Q35(A). Two cars from a tower [5 marks]

Let the tower be AB = 50 m, and the cars C (nearer) and D, with angles of depression 60° and 30°.

BC = 50/tan 60° = $\frac{50}{\sqrt{3}}$ m; BD = 50/tan 30° = $50\sqrt{3}$ m.

CD = $50\sqrt{3} - \frac{50}{\sqrt{3}} = \frac{150 - 50}{\sqrt{3}} = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$ m $\approx \frac{100 \times 1.73}{3} = 57.67$ m.

Answer: $\frac{100\sqrt{3}}{3}$ m ≈ 57.7 m

> **Marking scheme:**
> Correct figure: 1 mark
> BC: 1 mark
> BD: 1 mark
> Distance ≈ 57.7 m: 2 marks

Q35(B). Speed of the aeroplane [5 marks]

First position: horizontal distance = 1200/tan 60° = $\frac{1200}{\sqrt{3}} = 400\sqrt{3}$ m.

Second position: horizontal distance = 1200/tan 30° = $1200\sqrt{3}$ m.

Distance flown = $1200\sqrt{3} - 400\sqrt{3} = 800\sqrt{3}$ = 800 × 1.732 = 1385.6 m in 10 s.

Speed = 138.56 m/s = 138.56 × 18/5 ≈ 498.8 km/h.

Answer: ≈ 138.56 m/s ≈ 498.8 km/h

> **Marking scheme:**
> Correct figure: 1 mark
> Both horizontal distances: 2 marks
> Distance 800√3 m: 1 mark
> Speed in m/s and km/h: 1 mark

SECTION – E (3 × 4 = 12)

Q36. Neha's salary (AP) [4 marks]



Monthly salaries 30000, 31500, 33000, … form an AP with $a = 30000$, $d = 1500$.

(i) $a_8 = 30000 + 7 \times 1500 =$ **₹ 40,500**.

(ii) $30000 + (n-1)1500 = 48000 \Rightarrow n - 1 = 12 \Rightarrow$ **13th year**.

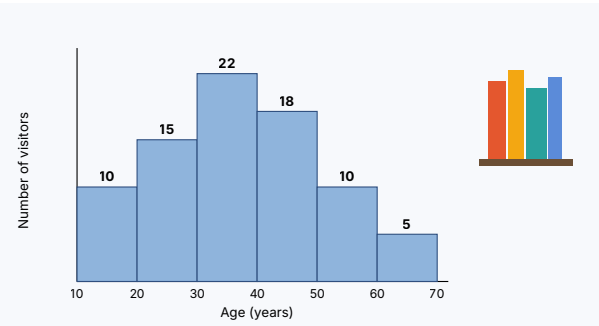
(iii)(A) Yearly salaries form an AP with $a = 12 \times 30000 = 360000$ and $d = 12 \times 1500 = 18000$. $S_{10} = \frac{10}{2}[720000 + 9 \times 18000] = 5 \times 882000 =$ **₹ 44,10,000**.

(iii)(B) $30000 + 1500(n-1) = 36000 + 1000(n-1) \Rightarrow 500(n-1) = 6000 \Rightarrow n = 13$. Both earn ₹ 48,000 in the **13th year**.

Answer: (i) ₹ 40,500 (ii) 13th year (iii)(A) ₹ 44,10,000 OR (B) 13th year

> **Marking scheme:**
> (i) ₹ 40,500: 1 mark
> (ii) 13th year: 1 mark
> (iii)(A) correct AP 1 mark + ₹ 44,10,000 1 mark; OR (iii)(B) equation 1 mark + 13th year 1 mark

Q37. Ages of library visitors [4 marks]



(i) Highest frequency 22 ⇒ modal class **30 – 40 years**.

(ii) $n = 80$, $n/2 = 40$; cumulative frequencies 10, 25, 47, … ⇒ median class **30 – 40 years**.

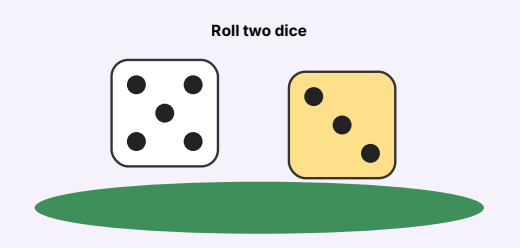
(iii)(A) Median $= 30 + \frac{40 - 25}{22} \times 10 = 30 + 6.82 =$ **36.82 years** (approx.).

(iii)(B) Class marks 15, 25, 35, 45, 55, 65; $\Sigma fx = 150 + 375 + 770 + 810 + 550 + 325 = 2980$. Mean $= 2980 \div 80 =$ **37.25 years**.

Answer: (i) 30 – 40 (ii) 30 – 40 (iii)(A) ≈ 36.82 years OR (B) 37.25 years

> **Marking scheme:**
> (i) 30 – 40: 1 mark
> (ii) 30 – 40: 1 mark
> (iii)(A) formula 1 mark + 36.82 1 mark; OR (iii)(B) Σfx = 2980 1 mark + 37.25 1 mark

Q38. Two dice game [4 marks]



There are 36 equally likely outcomes.

(i) Sum 9: (3,6), (4,5), (5,4), (6,3) — P = $\frac{4}{36} = \mathbf{\frac{1}{9}}$.

(ii) Sum less than 5 (sum 2, 3 or 4): (1,1), (1,2), (2,1), (1,3), (2,2), (3,1) — P = $\frac{6}{36} = \mathbf{\frac{1}{6}}$.

(iii)(A) Sum 4: (1,3), (2,2), (3,1); sum 9: 4 outcomes. Total 7 — P = $\mathbf{\frac{7}{36}}$.

(iii)(B) Product ≥ 20: (4,5), (5,4), (4,6), (6,4), (5,5), (5,6), (6,5), (6,6) — P = $\frac{8}{36} = \mathbf{\frac{2}{9}}$.

Answer: (i) $\mathbf{\frac{1}{9}}$ (ii) $\mathbf{\frac{1}{6}}$ (iii)(A) $\mathbf{\frac{7}{36}}$ OR (B) $\mathbf{\frac{2}{9}}$

> **Marking scheme:**
> (i) 1/9: 1 mark
> (ii) 1/6: 1 mark
> (iii)(A) listing 1 mark + 7/36 1 mark; OR (iii)(B) listing 1 mark + 2/9 1 mark

Final Answer Summary

Q	Ans	Q	Ans	Q	Ans	Q	Ans
Q1	(B)	**Q6**	(C)	**Q11**	(C)	**Q16**	(D)
Q2	(C)	**Q7**	(A)	**Q12**	(D)	**Q17**	(B)
Q3	(A)	**Q8**	(D)	**Q13**	(A)	**Q18**	(A)
Q4	(D)	**Q9**	(B)	**Q14**	(B)	**Q19**	(A)
Q5	(B)	**Q10**	(A)	**Q15**	(C)	**Q20**	(C)

Q	Final answer
Q21	(A) 12, 2520 / (B) Proof
Q22	(A) x = 2 / (B) Proof
Q23	2
Q24	44.8 cm^2
Q25	576; $\frac{3}{74}$
Q26	Proof
Q27	$3x^2 - 7x + 2$
Q28	(A) ₹ 450, ₹ 1200 / (B) 20°, 40°, 120°
Q29	Proof; 55°, 110°
Q30	Collinear; 1 : 1
Q31	Proofs
Q32	20 books, ₹ 60 each
Q33	Proof; ST = 3 cm
Q34	(A) 376.8 cm^3, 301.44 cm^2 / (B) 2449.2 cm^3, 979.68 cm^2
Q35	(A) ≈ 57.7 m / (B) ≈ 138.56 m/s ≈ 498.8 km/h
Q36	₹ 40,500; 13th year; ₹ 44,10,000 / 13th year
Q37	30–40; 30–40; 36.82 / 37.25 years
Q38	$\frac{1}{9}$; $\frac{1}{6}$; $\frac{7}{36}$ / $\frac{2}{9}$

Chapter-wise Distribution of Marks

Chapter	Questions	Marks
Real Numbers	1, 21, 26	6
Polynomials	2, 27	4
Pair of Linear Equations	3, 28	4

Quadratic Equations	4, 32	6
Arithmetic Progressions	5, 20, 36	6
Triangles	6, 22, 33	8
Circles	9, 29	4
Coordinate Geometry	7, 14, 19, 30	6
Introduction to Trigonometry	8, 15, 23, 31	7
Some Applications of Trigonometry	16, 35	6
Areas Related to Circles	10, 17, 24	4
Surface Areas and Volumes	11, 34	6
Statistics	12, 37	5
Probability	13, 18, 25, 38	8
Total		**80**

Section totals: A 20 + B 10 + C 18 + D 20 + E 12 = **80**. Marks for questions with internal choice are counted once.

— End of Solutions —