CLASS 10 — MATHEMATICS

Chapter: Surface Areas and Volumes

Answer Key with Step-by-Step Solutions

Maximum Marks: 40 **Time Allowed: 1 hour 30 minutes**

Results used: cylinder — CSA $2\pi rh$, volume $\pi r^2 h$; cone — slant height $l = \sqrt{r^2 + h^2}$, CSA πrl, volume $\frac{1}{3}\pi r^2 h$; hemisphere — CSA $2\pi r^2$, volume $\frac{2}{3}\pi r^3$; sphere — SA $4\pi r^2$, volume $\frac{4}{3}\pi r^3$; cuboid — SA $2(lb + bh + hl)$. For a combined solid, add only the surfaces that remain exposed; volumes add (or subtract for a cavity).

SECTION A — MCQ Answer Key

Q	Option	Answer	Explanation / calculation
Q1	**(A)**	$4\pi r^2$	The flat faces are joined and hidden; the outer surface is two curved hemispheres: $2 \times 2\pi r^2 = 4\pi r^2$ (a sphere).
Q2	**(B)**	462 cm²	TSA $= 3\pi r^2 = 3 \times \frac{22}{7} \times 49 = 462$ cm^2 (curved 308 + flat base 154).
Q3	**(A)**	$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$	Volume of cone + volume of hemisphere.
Q4	**(B)**	$2\pi rh + 4\pi r^2$	CSA of cylinder $2\pi rh$ + two hemispherical surfaces $2 \times 2\pi r^2$.
Q5	**(A)**	1 : 2	Height of hemisphere = r, so $h = r$. Cone : hemisphere $= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 = 1 : 2$.
Q6	**(B)**	a cone and a cylinder	The sharpened tip is conical; the rest of the pencil is cylindrical.
Q7	**(A)**	60π cm³	Remaining $= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \pi \times 9 \times 10 = 60\pi$ cm^3.
Q8	**(A)**	$1 : \sqrt{3}$	$\pi rl = 2\pi r^2 \Rightarrow l = 2r$. Then $h = \sqrt{4r^2 - r^2} = \sqrt{3}r$, so $r : h = 1 : \sqrt{3}$.
Q9	**(D)**	A false, R true	The surfaces in contact are hidden, so the TSA of the combined solid is less than the sum of the separate TSAs. A is false; R is true.
Q10	**(A)**	Both true; R explains A	No material is lost or gained on joining, so the volumes simply add. R is the reason.

Marking: 1 mark for each correct option; no part marks in Section A.

SECTION B — Very Short Answer

Q11. Three cubes joined end to end [2 marks]

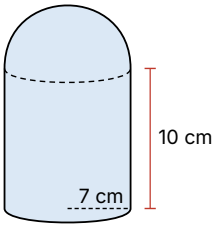
Edge of each cube = $\sqrt[3]{64} = 4$ cm. The cuboid is 12 cm × 4 cm × 4 cm.

Surface area = $2(12 \times 4 + 4 \times 4 + 4 \times 12) = 2(48 + 16 + 48) = 224$ cm^2.

Answer: 224 cm^2

> **Marking scheme:**
> Edge 4 cm and cuboid 12 × 4 × 4: 1 mark
> 224 cm²: 1 mark

Q12. Cylinder with a hemisphere on top [2 marks]



Exposed surfaces: CSA of cylinder + base of cylinder + CSA of hemisphere (the top of the cylinder is hidden).

$= 2\pi rh + \pi r^2 + 2\pi r^2 = 2 \times \frac{22}{7} \times 7 \times 10 + \frac{22}{7} \times 49 + 2 \times \frac{22}{7} \times 49 = 440 + 154 + 308 = 902$ cm^2.

Answer: 902 cm^2

> **Marking scheme:**
> Identifying the three exposed surfaces: 1 mark
> 902 cm²: 1 mark

Q13. Hemisphere scooped from a cylinder [2 marks]

Volume of cylinder = $3.14 \times 9 \times 10 = 282.6$ cm^3. Volume of hemisphere = $\frac{2}{3} \times 3.14 \times 27 = 56.52$ cm^3.

Remaining volume = $282.6 - 56.52 = 226.08$ cm^3.

Answer: 226.08 cm^3

> **Marking scheme:**
> Both volumes: 1 mark
> 226.08 cm³: 1 mark

Q14. Cube with a hemispherical depression [2 marks]

Surface area = $6a^2$ − (circular opening πr^2) + (inner hemispherical surface $2\pi r^2$) $= 6a^2 + \pi r^2$.

$= 6 \times 196 + \frac{22}{7} \times 49 = 1176 + 154 = 1330$ cm^2.

Answer: 1330 cm^2

> **Marking scheme:**
> Correct expression $6a^2 + \pi r^2$: 1 mark
> 1330 cm²: 1 mark

Q15. Cost of painting the toy [2 marks]

Slant height $l = \sqrt{3.5^2 + 12^2} = \sqrt{156.25} = 12.5$ cm. CSA of cone $= \frac{22}{7} \times 3.5 \times 12.5 = 137.5$ cm^2; CSA of hemisphere $= 2 \times \frac{22}{7} \times 12.25 = 77$ cm^2.

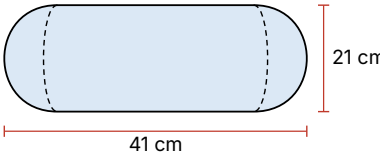
Total $= 214.5$ cm^2. Cost $= 214.5 \times 5 = ₹\ 1072.50$.

Answer: ₹ 1072.50

> **Marking scheme:**
> Slant height and both surface areas: 1 mark
> Cost ₹ 1072.50: 1 mark

SECTION C — Short Answer

Q16. Cylinder with hemispherical ends [3 marks]



$r = 10.5$ cm. Length of the cylindrical part $= 41 - 2 \times 10.5 = 20$ cm.

Volume $= \pi r^2 h + \frac{4}{3}\pi r^3 = \frac{22}{7} \times 110.25 \times 20 + \frac{4}{3} \times \frac{22}{7} \times 1157.625 = 6930 + 4851 = 11781$ cm^3.

Surface area $= 2\pi rh + 4\pi r^2 = 2 \times \frac{22}{7} \times 10.5 \times 20 + 4 \times \frac{22}{7} \times 110.25 = 1320 + 1386 = 2706$ cm^2.

Answer: 11781 cm^3; 2706 cm^2

> **Marking scheme:**
> Cylinder length 20 cm: 1 mark
> Volume 11781 cm³: 1 mark
> Surface area 2706 cm²: 1 mark

Q17. Tent [3 marks]

Slant height of the cone $= \sqrt{7^2 + 24^2} = 25$ m.

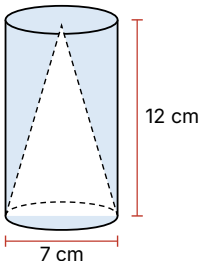
Canvas = CSA of cylinder + CSA of cone $= 2 \times \frac{22}{7} \times 7 \times 4 + \frac{22}{7} \times 7 \times 25 = 176 + 550 = 726$ m^2.

Air $= \pi r^2(4) + \frac{1}{3}\pi r^2(24) = 154 \times 4 + \frac{1}{3} \times 154 \times 24 = 616 + 1232 = 1848$ m^3.

Answer: 726 m^2; 1848 m^3

> **Marking scheme:**
> Slant height 25 m: 1 mark
> Canvas 726 m²: 1 mark
> Volume 1848 m³: 1 mark

Q18. Cylinder with a conical cavity [3 marks]



$r = 3.5$ cm, $h = 12$ cm, slant height $l = \sqrt{12.25 + 144} = 12.5$ cm.

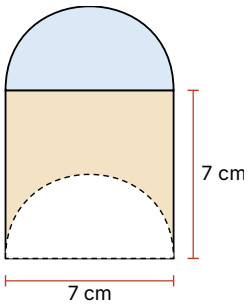
TSA = CSA of cylinder + top circle + inner CSA of cone $= 2 \times \frac{22}{7} \times 3.5 \times 12 + \frac{22}{7} \times 12.25 + \frac{22}{7} \times 3.5 \times 12.5 = 264 + 38.5 + 137.5 = 440$ cm^2.

Volume $= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{22}{7} \times 12.25 \times 12 = 308$ cm^3.

Answer: 440 cm^2; 308 cm^3

> **Marking scheme:**
> Slant height and correct surfaces: 1 mark
> TSA 440 cm²: 1 mark
> Volume 308 cm³: 1 mark

Q19. Cube with a dome and a bowl [3 marks]



$r = 3.5$ cm. On top, the hemisphere covers a circle (πr^2) but adds its curved surface ($2\pi r^2$): net $+\pi r^2$.

Underneath, a circle (πr^2) is removed and the bowl's inner surface ($2\pi r^2$) is added: net $+\pi r^2$.

TSA $= 6 \times 49 + 2\pi r^2 = 294 + 2 \times 38.5 = 371$ cm^2.

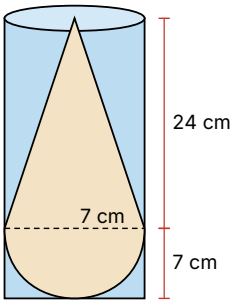
Volume = cube + hemisphere − hemisphere $= 7^3 = 343$ cm^3.

Answer: 371 cm^2; 343 cm^3

> **Marking scheme:**
> Net change on each face (+πr²): 1 mark
> TSA 371 cm²: 1 mark
> Volume 343 cm³ (the hemispheres cancel): 1 mark

SECTION D — Long Answer

Q20. Toy in a cylinder of water [4 marks]



(i) Cone = $\frac{1}{3} \times \frac{22}{7} \times 49 \times 24 = 1232$ cm^3; hemisphere = $\frac{2}{3} \times \frac{22}{7} \times 343 =$ 718.67 cm^3. Toy = 1232 + 718.67 = **1950.67 cm^3**.

(ii) Cylinder = $\frac{22}{7} \times 49 \times 31 = 4774$ cm^3 (its height 31 = 24 + 7 equals the toy's height). Water left = 4774 − 1950.67 = **2823.33 cm^3**.

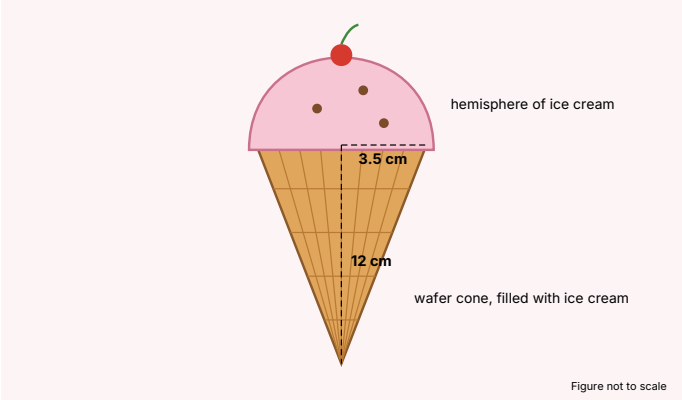
(iii) Slant height = $\sqrt{49 + 576} = 25$ cm. CSA of cone = $\frac{22}{7} \times 7 \times 25 = 550$ cm^2; CSA of hemisphere = $2 \times \frac{22}{7} \times 49 = 308$ cm^2.

TSA of toy = 550 + 308 = **858 cm^2**.

Answer: (i) 1950.67 cm^3 (ii) 2823.33 cm^3 (iii) 858 cm^2

> **Marking scheme:**
> (i) Cone and hemisphere volumes: 1 mark
> (i) Toy 1950.67 cm³: 1 mark
> (ii) Water left 2823.33 cm³: 1 mark
> (iii) TSA 858 cm²: 1 mark

SECTION E — Case Study



Q21. Ice cream cone [4 marks]

(i) $l = \sqrt{3.5^2 + 12^2} = \sqrt{156.25} =$ **12.5 cm**.

(ii) Wafer = $\pi rl = \frac{22}{7} \times 3.5 \times 12.5 =$ **137.5 cm^2**.

(iii) Ice cream = cone + hemisphere = $\frac{1}{3} \times \frac{22}{7} \times 12.25 \times 12 + \frac{2}{3} \times \frac{22}{7} \times$ 42.875 = 154 + 89.83 = **243.83 cm^3**.

5 litres = 5000 cm^3. 5000 ÷ 243.83 ≈ 20.5, so **20 full cones** can be made.

Answer: (i) 12.5 cm (ii) 137.5 cm^2 (iii) 243.83 cm^3; 20 cones

> **Marking scheme:**
> (i) 12.5 cm: 1 mark
> (ii) 137.5 cm^2: 1 mark
> (iii) Volume 243.83 cm^3: 1 mark; 20 cones: 1 mark

Final Answer Summary

Q	Final answer	Q	Final answer
Q1	(A) $4\pi r^2$	**Q12**	902 cm^2
Q2	(B) 462 cm^2	**Q13**	226.08 cm^3
Q3	(A)	**Q14**	1330 cm^2
Q4	(B) $2\pi rh + 4\pi r^2$	**Q15**	₹ 1072.50
Q5	(A) 1 : 2	**Q16**	11781 cm^3; 2706 cm^2
Q6	(B)	**Q17**	726 m^2; 1848 m^3
Q7	(A) 60π cm^3	**Q18**	440 cm^2; 308 cm^3
Q8	(A) 1 : $\sqrt{3}$	**Q19**	371 cm^2; 343 cm^3
Q9	(D)	**Q20**	1950.67 cm^3; 2823.33 cm^3; 858 cm^2
Q10	(A)	**Q21**	12.5 cm; 137.5 cm^2; 243.83 cm^3; 20
Q11	224 cm^2		

Marks: Section A 10 + Section B 10 + Section C 12 + Section D 4 + Section E 4 = **40**

— End of Solutions —