PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 10 : Arithmetic Progression

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Formulae used

A.P.: a, a + d, a + 2d, … $t_n = a + (n-1)d$ | $l = a + (n-1)d$ (last term) | $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$ | $t_n = S_n - S_{n-1}$.

Three terms: a − d, a, a + d. Arithmetic means: if $A_1, \ldots, A_m$ are inserted between p and q, then $q = p + (m+1)d$. a, b, c in A.P. $\Leftrightarrow$ $2b = a + c$.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(c)**	Differences in 1, 3, 6, 10 are 2, 3, 4 — not constant. The others have d = 4, −3, 0.5.
(ii)	**(b)** 74	$t_{15} = 4 + 14 \times 5 = 74$.
(iii)	**(b)** −3	$t_1 = 4$, $t_2 = 1 \Rightarrow d = 1 - 4 = -3$.
(iv)	**(c)**	1 + 2 + … + 10 is an A.P. with a = 1, d = 1: $S_{10} = 5(2 + 9) = 55$. (R) is the formula used, so it explains (A).
(v)	**(b)** 300	$S_{12} = 6[6 + 11 \times 4] = 6 \times 50 = 300$.
(vi)	**(b)** 3	$2(2k + 1) = k + (4k - 1) \Rightarrow 4k + 2 = 5k - 1 \Rightarrow k = 3$ (terms 3, 7, 11).
(vii)	**(c)** 17	$53 = 5 + (n - 1) \times 3 \Rightarrow n - 1 = 16 \Rightarrow n = 17$.
(viii)	**(b)**	$S_1 = 3$, $S_2 = 8 \Rightarrow t_2 = S_2 - S_1 = 5$, not 8; (A) is false. (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

$a + 4d = 17$ …(1) $a + 8d = 29$ …(2) [1]

Subtracting: $4d = 12 \Rightarrow$ **d = 3**; from (1): $a = 17 - 12 =$ **5** [1]

$t_{25} = a + 24d = 5 + 24 \times 3$ [1]

t_{25} = 77 [1]

(ii)

a = 18, d = −3. $S_n = 63 \Rightarrow (n/2)[36 + (n-1)(-3)] = 63$ [1]

$n(39 - 3n) = 126 \Rightarrow 3n^2 - 39n + 126 = 0 \Rightarrow n^2 - 13n + 42 = 0$ [1]

$(n - 6)(n - 7) = 0 \Rightarrow$ **n = 6 or n = 7** [1]

Reason: the 7th term is 18 + 6(−3) = 0, so adding it does not change the sum; $S_6 = S_7 = 63$ [1]

(iii)

Let the means be A_1, A_2, A_3, A_4. Then 6, A_1, A_2, A_3, A_4, 31 are in A.P. with 6 terms.

$31 = t_6 = 6 + 5d$ [1]

$5d = 25 \Rightarrow d = 5$ [1]

$A_1 = 6 + 5 = 11,\ \ A_2 = 16,\ \ A_3 = 21,\ \ A_4 = 26$ [1]

The four arithmetic means are 11, 16, 21, 26 [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)

First multiple of 6 above 100 is 102; last below 400 is 396 (since $400 = 66 \times 6 + 4$) [1]

A.P. 102, 108, …, 396: $a = 102,\ d = 6,\ l = 396.\ \ 396 = 102 + (n - 1) \times 6$ [1]

$(n - 1) = 294 \div 6 = 49 \Rightarrow$ **n = 50** [1]

(ii)

Let the numbers be $a - d,\ a,\ a + d$. Sum: $3a = 27 \Rightarrow a = 9$ [1]

Product: $9(81 - d^2) = 693 \Rightarrow 81 - d^2 = 77 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$ [1]

The numbers are 7, 9, 11 (or 11, 9, 7) (check: $7 \times 9 \times 11 = 693$ ✓) [1]

(iii)

$t_1 = S_1 = 4(1) - 1 = 3$ [1]

$S_2 = 4(4) - 2 = 14 \Rightarrow t_2 = 14 - 3 = 11 \Rightarrow d = 11 - 3 = 8.$ **A.P.: 3, 11, 19, …** [1]

$t_{20} = a + 19d = 3 + 19 \times 8$ [1]

t_{20} = 155 (check: $S_{20} - S_{19} = 1580 - 1425 = 155$ ✓) [1]

Question 4 [3 + 3 + 4 = 10]

(i)

$a = 7,\ d = 6.\ t_{12} = 7 + 11 \times 6 = 73$ [1]

Required term $= 73 + 60 = 133.\ \ 7 + (n - 1) \times 6 = 133$ [1]

$(n - 1) = 21 \Rightarrow$ **n = 22**, i.e. the 22nd term [1]

(ii)

$t_9 = 0 \Rightarrow a + 8d = 0 \Rightarrow a = -8d$ [1]

$t_{29} = a + 28d = -8d + 28d = 20d;\ \ t_{19} = a + 18d = -8d + 18d = 10d$ [1]

$t_{29} = 20d = 2 \times 10d = 2 \times t_{19}.$ **Hence proved.** [1]

(iii)

First multiple of 7 above 100 is 105 (7×15); last below 500 is 497 (7×71) [1]

$a = 105,\ d = 7,\ l = 497: 497 = 105 + (n - 1) \times 7 \Rightarrow n - 1 = 56$ [1]

n = 57 [1]

$S = (57/2)(105 + 497) = 57 \times 301 =$ **17,157** [1]

Question 5 [3 + 3 + 4 = 10]

(i)

$t_1 = 7,\ t_2 = 10,\ t_3 = 13, \ldots;\ t_{n+1} - t_n = 3(n + 1) + 4 - (3n + 4) = 3$, a constant, so it is an A.P. with $a = 7,\ d = 3$ [1]

$S_{20} = 10[2 \times 7 + 19 \times 3]$ [1]

$= 10 \times 71 =$ **710** [1]

(ii)

$S = \frac{n}{2}(a + l) \Rightarrow 510 = \frac{n}{2}(6 + 62) = 34n$ [1]

n = 15 [1]

$62 = 6 + 14d \Rightarrow 14d = 56 \Rightarrow$ **d = 4** [1]

(iii)

$S_7 = 63 \Rightarrow (7/2)(2a + 6d) = 63 \Rightarrow a + 3d = 9$ …(1) [1]

Sum of 14 terms = 63 + 161 = 224 $\Rightarrow 7(2a + 13d) = 224 \Rightarrow 2a + 13d = 32$ …(2) [1]

From (1), a = 9 − 3d. Put in (2): 18 − 6d + 13d = 32 $\Rightarrow$ 7d = 14 $\Rightarrow$ **d = 2**, a = 3 [1]

A.P.: 3, 5, 7, 9, … (check: $S_7 = 63$; $S_{14} = 224$ ✓) [1]

Question 6 [3 + 3 + 4 = 10]

(i)

Monthly savings 500, 550, 600, … is an A.P. with a = 500, d = 50 [1]

$t_{24} = 500 + 23 \times 50 =$ **₹ 1,650** [1]

$S_{24} = 12[2 \times 500 + 23 \times 50] = 12 \times 2150 =$ **₹ 25,800** [1]

(ii)

a, b, c in A.P. $\Rightarrow 2b = a + c \Rightarrow 4b^2 = (a + c)^2$ [1]

$4(b^2 - ac) = 4b^2 - 4ac = (a + c)^2 - 4ac$ [1]

$= a^2 + 2ac + c^2 - 4ac = (a - c)^2$. **Hence proved.** [1]

(iii)

A.P. with a = 2, d = 3, n terms, last term x. Sum: $(n/2)[4 + 3(n - 1)] = 155$ [1]

$n(3n + 1) = 310 \Rightarrow 3n^2 + n - 310 = 0$ [1]

$(3n + 31)(n - 10) = 0 \Rightarrow n = 10$ (reject $n = -31/3$) [1]

$x = t_{10} = 2 + 9 \times 3 =$ **29** (check: $S_{10} = 5(2 + 29) = 155$ ✓) [1]

— End of Solutions —