PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 11 : Geometric Progression

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Formulae used

G.P.: a, ar, ar^2, … r = (any term) ÷ (previous term) | $t_n = ar^{n-1}$ | nth term from the end = $l \div r^{n-1}$.

$S_n = \frac{a(1-r^n)}{1-r}$ $(r < 1)$ | $S_n = \frac{a(r^n-1)}{r-1} = \frac{lr-a}{r-1}$ $(r > 1)$ | $S_\infty = \frac{a}{1-r}$ $(|r| < 1)$ | Geometric mean G = $\sqrt{(ab)}$; a, b, c in G.P. $\Leftrightarrow$ $b^2 = ac$.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(a)**	Ratios 6/2 = 18/6 = 54/18 = 3, constant. The others have no constant ratio.
(ii)	**(b)** 96	$t_6 = 3 \times 2^5 = 96$.
(iii)	**(d)**	r = −27 ÷ 81 = −1/3.
(iv)	**(c)**	a = 1, r = ½: S∞ = 1/(1 − ½) = 2, so (A) is true; (R) is the formula used, so it explains (A).
(v)	**(c)** 20	G = $\sqrt{(16 \times 25)} = \sqrt{400} = 20$.
(vi)	**(b)** 728	$S_6 = 2(3^6 - 1)/(3 - 1) = 729 - 1 = 728$.
(vii)	**(a)** 5	$2 \times 3^{n-1} = 162 \Rightarrow 3^{n-1} = 81 = 3^4 \Rightarrow n = 5$.
(viii)	**(b)**	For a G.P. the correct relation is $b^2 = ac$, not 2b = a + c (that is for an A.P.): (A) is false; (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

$ar^2 = 12$ …(1) $ar^5 = 96$ …(2) [1]

(2) ÷ (1): $r^3 = 8 \Rightarrow$ **r = 2** [1]

From (1): $a \times 4 = 12 \Rightarrow$ **a = 3** [1]

$t_8 = ar^7 = 3 \times 128 =$ **384** [1]

(ii)

a = 5, r = −10 ÷ 5 = −2, n = 8 [1]

$S_8 = \frac{a(1-r^n)}{1-r}$ (use this form since r < 1) [1]

$S_8 = \frac{5[1-(-2)^8]}{1-(-2)} = \frac{5(1-256)}{3}$ [1]

$= \frac{5 \times (-255)}{3} =$ **−425** [1]

(iii)

$ar = 3$ …(1) $\quad \frac{a}{1-r} = 16 \Rightarrow a = 16(1-r)$ …(2) [1]

Put (2) in (1): $16r(1-r) = 3 \Rightarrow 16r^2 - 16r + 3 = 0$ [1]

$(4r-1)(4r-3) = 0 \Rightarrow r = \frac{1}{4}$ or $r = \frac{3}{4}$ (both satisfy $|r| < 1$) [1]

$a = 16(1 - ¼) = 12$, or $a = 16(1 - ¾) = 4$. **First term = 12 or 4** (check: $12 \times ¼ = 3$; $4 \times ¾ = 3$ ✓) [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)

$a = -3$, $r = 6 \div (-3) = -2$. $t_n = ar^{n-1} = -192$ [1]

$-3 \times (-2)^{n-1} = -192 \Rightarrow (-2)^{n-1} = 64 = (-2)^6$ [1]

$n - 1 = 6 \Rightarrow$ **n = 7**, i.e. the 7th term [1]

(ii)

$ar = 6$ …(1) $\quad ar^4 = 162$ …(2) [1]

(2) ÷ (1): $r^3 = 27 \Rightarrow r = 3$; from (1), $a = 2$ [1]

G.P.: 2, 6, 18, 54, … [1]

(iii)

Let the means be G_1, G_2, G_3. Then 5, G_1, G_2, G_3, 80 are in G.P. with 5 terms.

$80 = t_5 = 5r^4 \Rightarrow r^4 = 16$ [1]

$r = 2$ (all terms positive) [1]

$G_1 = 10$, $G_2 = 20$, $G_3 = 40$. **The geometric means are 10, 20, 40** [1]

$S_5 = \frac{5(2^5 - 1)}{2 - 1} = 155$ (check: 5 + 10 + 20 + 40 + 80 = 155 ✓) [1]

Question 4 [3 + 3 + 4 = 10]

(i)

$a = 1$, $r = 3$, last term $l = 729$ [1]

4th term from the end $= \frac{l}{r^{4-1}} = \frac{729}{3^3}$ [1]

$= 729 \div 27 =$ **27** [1]

(ii)

$a = 2$, $r = 3$, $l = 1458$ $(r > 1)$ [1]

$S = \frac{lr - a}{r - 1} = \frac{1458 \times 3 - 2}{3 - 1}$ [1]

$= \frac{4372}{2} =$ **2186** [1]

(iii)

$a = 3$, $r = 2$. $S_n = 381 \Rightarrow 3(2^n - 1) \div (2 - 1) = 381$ [1]

$2^n - 1 = 127$ [1]

$2^n = 128 = 2^7$ [1]

n = 7 (check: 3 + 6 + 12 + 24 + 48 + 96 + 192 = 381 ✓) [1]

Question 5 [3 + 3 + 4 = 10]

(i)

Let the numbers be $\frac{a}{r}$, a, ar. Product: $a^3 = 64 \Rightarrow a = 4$ [1]

Sum: $\frac{4}{r} + 4 + 4r = 14 \Rightarrow 4 + 4r + 4r^2 = 14r \Rightarrow 2r^2 - 5r + 2 = 0$ [1]

$(2r - 1)(r - 2) = 0 \Rightarrow r = 2$ or $r = ½$. **The numbers are 2, 4, 8** (or 8, 4, 2) [1]

(ii)

a, b, c in G.P. $\Rightarrow b^2 = ac$. Let $b = ar$ and $c = ar^2$ (a = first term, r = common ratio) [1]

LHS $= (a^2 + a^2r^2)(a^2r^2 + a^2r^4) = a^2(1 + r^2) \times a^2r^2(1 + r^2) = a^4r^2(1 + r^2)^2$ [1]

RHS $= (ab + bc)^2 = (a^2r + a^2r^3)^2 = a^4r^2(1 + r^2)^2$. LHS = RHS. **Hence proved.** [1]

(iii)

$0.3545454\ldots = 0.3 + (0.054 + 0.00054 + 0.0000054 + \ldots)$ [1]

The bracket is an infinite G.P. with $a = 0.054$ and $r = 0.01$ ($|r| < 1$) [1]

Its sum $= \frac{0.054}{1 - 0.01} = \frac{0.054}{0.99} = \frac{54}{990} = \frac{3}{55}$ [1]

Total $= \frac{3}{10} + \frac{3}{55} = \frac{33 + 6}{110} = \mathbf{\frac{39}{110}}$ ($39 \div 110 = 0.35454\ldots$ ✓) [1]

Question 6 [3 + 3 + 4 = 10]

(i)

Savings 1, 2, 4, 8, … form a G.P. with $a = 1$, $r = 2$ [1]

$S_n = \frac{1 \times (2^n - 1)}{2 - 1} = 1023 \Rightarrow 2^n = 1024 = 2^{10}$ [1]

n = 10 days [1]

(ii)

$\frac{a}{1 - r} = 15$ …(1). The squares $a^2, a^2r^2, a^2r^4, \ldots$ form a G.P. with ratio r^2: $\frac{a^2}{1 - r^2} = 45$ …(2) [1]

Squaring (1): $\frac{a^2}{(1 - r)^2} = 225$. Divide this by (2): $\frac{1 - r^2}{(1 - r)^2} = \frac{1 + r}{1 - r} = 5$ [1]

$1 + r = 5 - 5r \Rightarrow \mathbf{r = \frac{2}{3}}$; from (1), $a = 15 \times ⅓ = \mathbf{5}$ (check: $25 \div (1 - 4/9) = 45$ ✓) [1]

(iii)

Let the terms be a, ar, ar^2, ar^3 (so $b = ar$, $c = ar^2$, $d = ar^3$) [1]

$b - c = ar(1 - r)$; $c - a = a(r^2 - 1)$; $d - b = ar(r^2 - 1)$ [1]

LHS $= a^2[r^2(1 - r)^2 + (r^2 - 1)^2 + r^2(r^2 - 1)^2] = a^2[(r^2 - 2r^3 + r^4) + (r^4 - 2r^2 + 1) + (r^6 - 2r^4 + r^2)]$ [1]

$= a^2(1 - 2r^3 + r^6) = a^2(1 - r^3)^2 = (a - ar^3)^2 = (a - d)^2$ = RHS. **Hence proved.** [1]

— End of Solutions —