PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 12 : Reflection

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

$M_x(x, y) = (x, -y)$ | $M_y(x, y) = (-x, y)$ | $M_o(x, y) = (-x, -y)$.

Reflection in the line $x = a$: $(x, y) \to (2a - x, y)$ | in the line $y = a$: $(x, y) \to (x, 2a - y)$.

A point lying on the mirror line is invariant. The mirror line is the perpendicular bisector of PP′. $M_x \cdot M_y = M_y \cdot M_x = M_o$; $M_o \cdot M_x = M_y$; $M_o \cdot M_y = M_x$.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(b)**	$M_x(x, y) = (x, -y)$, so $M_x(-7, 4) = (-7, -4)$.
(ii)	**(a)**	$(2a - x, y) = (2 \times 2 - 5, -3) = (-1, -3)$.
(iii)	**(c)**	$(-a, -b) = (-6, 9) \Rightarrow a = 6, b = -9$; $a - b = 6 + 9 = 15$.
(iv)	**(d)**	$M_y \cdot M_x\,(x, y) = M_y(x, -y) = (-x, -y) = M_o(x, y)$.
(v)	**(b)**	$(x, 2a - y) = (-4, 2(-1) - 5) = (-4, -7)$.
(vi)	**(c)**	Only $(3, -5)$ lies on the line $x = 3$, so it is its own image.
(vii)	**(c)**	$(0, 6)$ lies on $x = 0$ (the y-axis), so (A) is true; (R) states exactly why, so it explains (A).
(viii)	**(a)**	(A) is true: reflecting twice in the same line returns the point. (R) is false: in the x-axis only the ordinate changes sign.

Question 2 [4 + 4 + 4 = 12]

(i)

$A' = M_y(-5, 2) =$ **(5, 2)** [1]

$B' = M_o(4, -2) =$ **(−4, 2)** [1]

A′ and B′ have the same ordinate (2), so A′B′ is horizontal [1]

$A'B' = 5 - (-4) =$ **9 units** [1]

(ii)

$P' = M_y(a, b) = (-a, b)$ [1]

$P'' = M_o(-a, b) = (a, -b)$ [1]

$(a, -b) = (-7, 3) \Rightarrow$ **$a = -7, b = -3$**, so $P = (-7, -3)$ [1]

$P(-7, -3) \to P''(-7, 3)$: only the sign of the ordinate changes. **Single transformation: reflection in the x-axis** [1]

(iii)

$P' = (2 \times 1 - (-4), 5) =$ **(6, 5)** [1]

$Q' = (6, 2(-2) - (-3)) =$ **(6, −1)** [1]

P' and Q' have the same abscissa (6), so $P'Q'$ is vertical [1]

$P'Q' = 5 - (-1) =$ **6 units** [1]

SECTION B (20 Marks) — any two questions

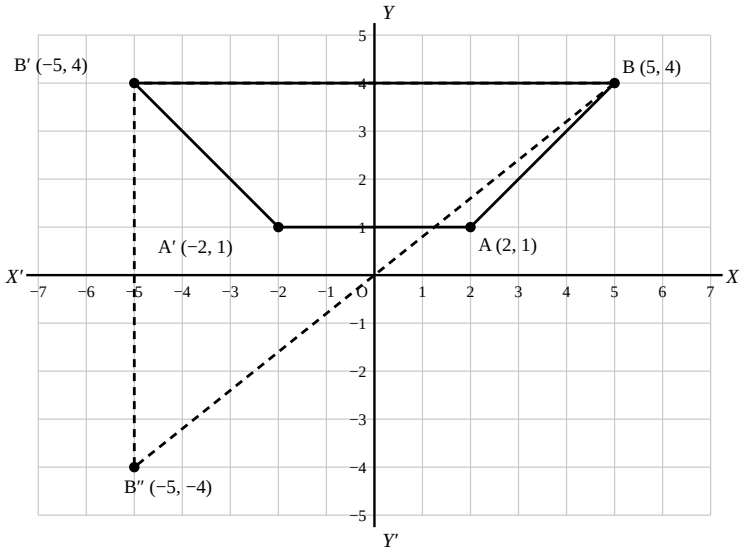
Question 3 [3 + 3 + 4 = 10]

(i)

$A' = M_y(2, 1) =$ **(−2, 1)** [1]

$B' = M_y(5, 4) =$ **(−5, 4)** [1]

A, B, A′, B′ plotted correctly (figure below) [1]



Solid: trapezium ABB′A′. Dashed: triangle BB′B″.

(ii)

AB = A′B′ (reflection preserves length) and AA′ ∥ BB′ (both horizontal, y = 1 and y = 4) [1]

ABB′A′ is an isosceles trapezium (one pair of parallel sides, non-parallel sides equal)

AA′ = 4, BB′ = 10, distance between them = 4 − 1 = 3 [1]

Area = $\frac{1}{2}(4 + 10) \times 3 =$ **21 sq. units** [1]

(iii)

$B'' = M_x(-5, 4) =$ **(−5, −4)** [1]

B(5, 4) → B″(−5, −4): both signs change. **Single transformation: reflection in the origin** [1]

BB′ = 10 (horizontal), B′B″ = 8 (vertical), so ∠BB′B″ = 90° [1]

Area = $\frac{1}{2} \times 10 \times 8 =$ **40 sq. units** [1]

Question 4 [3 + 3 + 4 = 10]

(i)

P and P′ have the same ordinate (4), so PP′ is horizontal and the mirror line is vertical [1]

The mirror line is the perpendicular bisector of PP′; mid-point of PP′ = ((−2 + 6)/2, 4) = (2, 4) [1]

l is the line parallel to the y-axis through (2, 4): its equation is x = 2 [1]

(ii)

$Q' = (2 \times 2 - (-5), -3)$ [1]

Q′ = (9, −3) [1]

The line x = 2 meets the x-axis at (2, 0); it lies on l, so **(2, 0)** is invariant [1]

(iii)

A′ = (2 × 2 − (−3), 2) = **(7, 2)** [1]

A″ = (7, 2(−1) − 2) = **(7, −4)** [1]

AA′ = 7 − (−3) = 10 (horizontal); A′A″ = 2 − (−4) = 6 (vertical), so ∠AA′A″ = 90° [1]

Area = $\frac{1}{2}$ × 10 × 6 = **30 sq. units** [1]

Question 5 [3 + 3 + 4 = 10]

(i)

(a) (2(−1) − 5, −4) = **(−7, −4)** [1]

(b) (−3, 2 × 4 − 2) = **(−3, 6)** [1]

(c) (2 × 3 − 0, 0) = **(6, 0)** [1]

(ii)

Image of (m, n) in x = 3 is (6 − m, n) = (−1, 7) [1]

6 − m = −1 ⇒ **m = 7**; **n = 7** [1]

Image of P(7, 7) in y = 1: (7, 2 × 1 − 7) = **(7, −5)** [1]

(iii)

P and P′ have the same abscissa (3), so PP′ is vertical and the mirror line is horizontal, through the mid-point (3, (−5 + 9)/2) = (3, 2) [1]

(a) **Equation of the line: y = 2** [1]

(b) Image of Q(−6, −1) = (−6, 2 × 2 − (−1)) = **(−6, 5)** [1]

(c) The line PP′ is x = 3 and it meets y = 2 at **(3, 2)** [1]

Question 6 [3 + 3 + 4 = 10]

(i)

$M_x \cdot M_o(-3, 7) = M_x[M_o(-3, 7)] = M_x(3, -7) =$ **(3, 7)** [1]

(−3, 7) → (3, 7): only the abscissa changes sign. $\mathbf{M_x \cdot M_o = M_y}$ **(reflection in the y-axis)** [1]

$M_o \cdot M_x(-3, 7) = M_o(-3, -7) = (3, 7)$ as well. **Yes,** $\mathbf{M_x \cdot M_o = M_o \cdot M_x}$ (reflections in the axes and origin always commute) [1]

(ii)

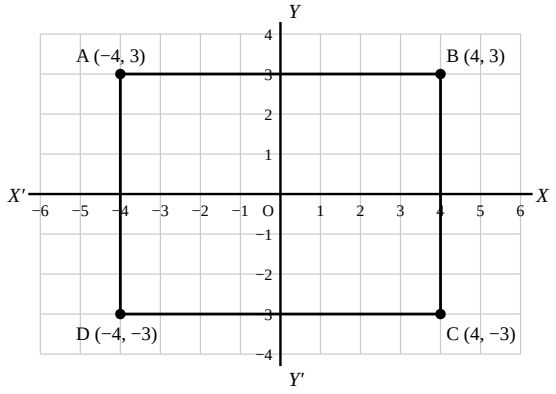
Invariant in the y-axis ⇒ lies on it ⇒ abscissa 0: 2t − 4 = 0 ⇒ t = 2; **P = (0, 4)** [1]

Invariant in the x-axis ⇒ ordinate 0: 3s − 9 = 0 ⇒ s = 3; **Q = (8, 0)** [1]

Q(8, 0) lies on the line x = 8, so it is its own image: **image = (8, 0)** [1]

(iii)

B = $M_y(-4, 3)$ = **(4, 3)**; C = $M_o(-4, 3)$ = **(4, −3)**; D = $M_x(-4, 3)$ = **(−4, −3)** [2]



AB = 8 (horizontal), BC = 6 (vertical), CD = 8, DA = 6, all angles 90°. **ABCD is a rectangle** [1]

Area = 8 × 6 = **48 sq. units** [1]

— End of Solutions —