PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 13 : Section and Mid-Point Formula

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Formulae used

Section formula: P divides $A(x_1, y_1)$, $B(x_2, y_2)$ in the ratio $m_1 : m_2 \Rightarrow P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.

To find a ratio, take it as k : 1. On the x-axis y = 0; on the y-axis x = 0.

Mid-point = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ | Centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ | Diagonals of a parallelogram bisect each other.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(a)**	((−6 + 2)/2, (4 − 10)/2) = (−2, −3).
(ii)	**(b)**	((1 × 7 + 2 × 1)/3, (1 × 10 + 2 × (−2))/3) = (3, 2).
(iii)	**(c)**	((4 − 2 + 1)/3, (−1 + 5 + 2)/3) = (1, 2).
(iv)	**(d)**	Centre is the mid-point of the diameter: ((−3 + 7)/2, (5 − 1)/2) = (2, 2).
(v)	**(c)**	Ratio k : 1, abscissa 0: (6k − 2)/(k + 1) = 0 ⇒ k = 1/3, i.e. 1 : 3.
(vi)	**(b)**	(3 + n)/2 = −1 ⇒ n = −5; (m + 2)/2 = 4 ⇒ m = 6. So m + n = 1.
(vii)	**(c)**	((−1 + 5)/2, (1 + 5)/2) = (2, 3), so (A) is true; (R) is the mid-point formula, so it explains (A).
(viii)	**(b)**	The centroid divides each median in the ratio 2 : 1 from the vertex, so (A) is false; (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

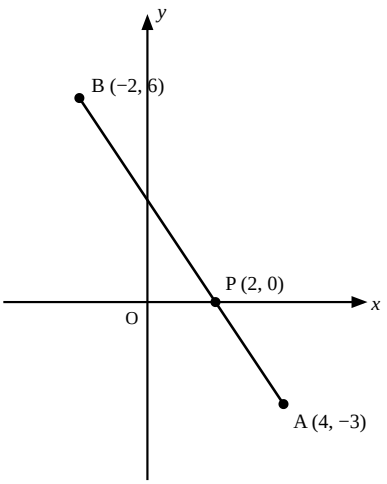
$m_1 : m_2 = 1 : 2$, A(−3, 7), B(6, −5) [1]

$x = \frac{1 \times 6 + 2 \times (-3)}{1 + 2} = 0$ [1]

$y = \frac{1 \times (-5) + 2 \times 7}{1 + 2} = \frac{9}{3} = 3$. **P = (0, 3)** [1]

Abscissa is 0, so **P lies on the y-axis** [1]

(ii)



Let the ratio be k : 1 and the point be (x, 0) [1]

$y = \frac{6k - 3}{k + 1} = 0 \Rightarrow k = \frac{1}{2}$ [1]

Ratio = ½ : 1 = 1 : 2 [1]

$x = \frac{-2k + 4}{k + 1} = \frac{-1 + 4}{3/2} = 2$. **Point = (2, 0)** [1]

(iii)

$\frac{(2a + 1) + (-3)}{2} = 5 \Rightarrow 2a - 2 = 10$ [1]

a = 6 [1]

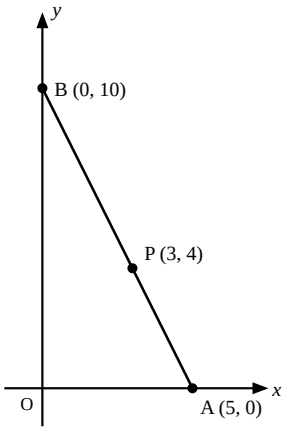
$\frac{4 + (3b - 1)}{2} = b + 1 \Rightarrow 3b + 3 = 2b + 2$ [1]

b = −1 [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)



A lies on the x-axis and B on the y-axis: let A = (a, 0) and B = (0, b). AP : PB = 2 : 3 [1]

x: $\frac{2 \times 0 + 3 \times a}{2 + 3} = 3 \Rightarrow 3a = 15 \Rightarrow$ **a = 5**, so A = (5, 0) [1]

y: $\frac{2 \times b + 3 \times 0}{2 + 3} = 4 \Rightarrow 2b = 20 \Rightarrow$ **b = 10**, so B = (0, 10) (check: 3/5 + 4/10 = 1 ✓) [1]

(ii)

Let P and Q be the points of trisection, so AP : PB = 1 : 2 and AQ : QB = 2 : 1.

$P = (\frac{1 \times 8 + 2 \times (-4)}{3}, \frac{1 \times (-5) + 2 \times 7}{3}) =$ **(0, 3)** [1]

$Q = (\frac{2 \times 8 + 1 \times (-4)}{3}, \frac{2 \times (-5) + 1 \times 7}{3}) =$ **(4, −1)** [1]

Check: mid-point of PQ = (2, 1) = mid-point of AB ✓ [1]

(iii)

Let P divide AB in the ratio $m_1 : m_2$. $\frac{9m_1 + (-3)m_2}{m_1 + m_2} = 5$ [1]

$9m_1 - 3m_2 = 5m_1 + 5m_2 \Rightarrow 4m_1 = 8m_2 \Rightarrow m_1 : m_2 = 2 : 1$ [1]

P divides AB in the ratio 2 : 1, so P is a point of trisection (nearer to B). **Shown.** [1]

$m = \frac{2 \times (-4) + 1 \times 2}{3} = \frac{-6}{3} =$ **−2** [1]

Question 4 [3 + 3 + 4 = 10]

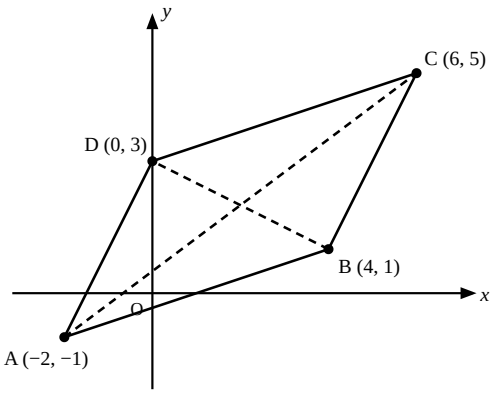
(i)

Let B = (x, y). Mid-point of AB = M(1, −2) [1]

$\frac{-3 + x}{2} = 1 \Rightarrow x = 5$; $\frac{4 + y}{2} = -2 \Rightarrow y = -8$ [1]

B = (5, −8) [1]

(ii)



Diagonals AC and BD of a parallelogram bisect each other. Mid-point of AC = ((−2 + 6)/2, (−1 + 5)/2) = (2, 2) [1]

Let D = (x, y). Mid-point of BD = ((4 + x)/2, (1 + y)/2) = (2, 2) [1]

4 + x = 4 ⇒ x = 0; 1 + y = 4 ⇒ y = 3. **D = (0, 3)** (check: AB = (6, 2) = DC ✓) [1]

(iii)

Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$. D, E, F are mid-points of BC, CA, AB.

Abscissae: $x_2 + x_3 = 6$, $x_3 + x_1 = 10$, $x_1 + x_2 = 2$. Ordinates: $y_2 + y_3 = -2$, $y_3 + y_1 = 6$, $y_1 + y_2 = 4$ [1]

Adding: $x_1 + x_2 + x_3 = 9$ and $y_1 + y_2 + y_3 = 4$ [1]

$x_1 = 9 - 6 = 3$, $x_2 = 9 - 10 = -1$, $x_3 = 9 - 2 = 7$; $y_1 = 4 - (-2) = 6$, $y_2 = 4 - 6 = -2$, $y_3 = 4 - 4 = 0$ [1]

A = (3, 6), B = (−1, −2), C = (7, 0) (check: mid-point of BC = (3, −1) = D ✓) [1]

Question 5 [3 + 3 + 4 = 10]

(i)

Centroid = $(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3})$ [1]

$x = \frac{-4 + 2 + 8}{3} = 2$ [1]

$y = \frac{3 - 5 + 5}{3} = 1$. **Centroid = (2, 1)** [1]

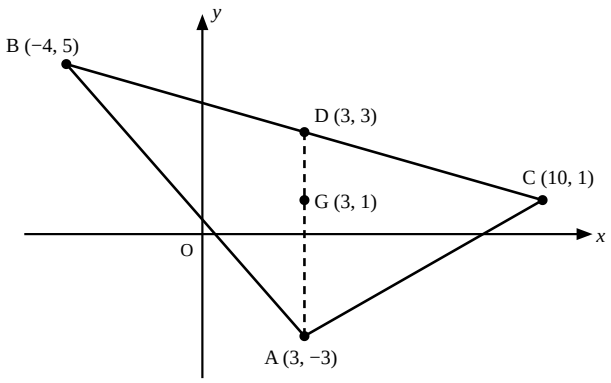
(ii)

$\frac{1 + (a + 2) + 5}{3} = 3 \Rightarrow a + 8 = 9$ [1]

a = 1 [1]

$\frac{4 + 3 + (b - 1)}{3} = 1 \Rightarrow b + 6 = 3 \Rightarrow$ **b = −3** [1]

(iii)



(a) $D = (\frac{-4 + 10}{2}, \frac{5 + 1}{2})$ = **(3, 3)** [1]

(b) A(3, −3) and D(3, 3) have the same abscissa, so AD is vertical: **AD = 3 − (−3) = 6 units** [1]

(c) $G = (\frac{3 - 4 + 10}{3}, \frac{-3 + 5 + 1}{3})$ = **(3, 1)** [1]

(d) Point dividing AD in 2 : 1 = $(\frac{2 \times 3 + 1 \times 3}{3}, \frac{2 \times 3 + 1 \times (-3)}{3})$ = (3, 1) = G. Hence verified (AG = 4, GD = 2) [1]

Question 6 [3 + 3 + 4 = 10]

(i)

Let P divide AB in the ratio k : 1 [1]

$y = \frac{9k + 3}{k + 1} = 5 \Rightarrow 9k + 3 = 5k + 5 \Rightarrow k = \frac{1}{2}$. **Ratio = 1 : 2** [1]

$a = \frac{7k - 2}{k + 1} = \frac{3.5 - 2}{1.5} =$ **1** (check: $(1 \times 7 + 2 \times (-2))/3 = 1$ ✓) [1]

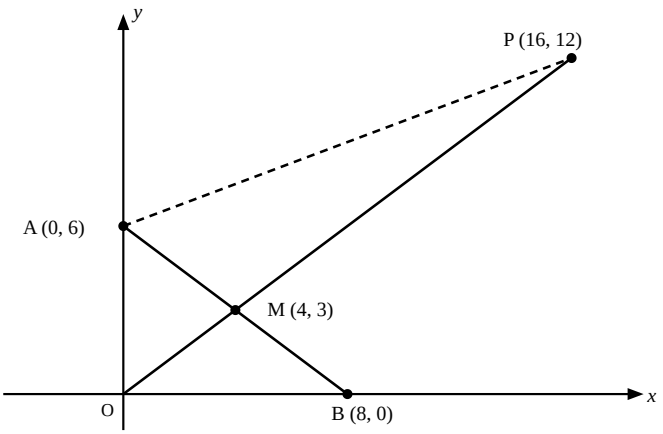
(ii)

Q is the mid-point of PR, so (a, 5) = ((4 + 10)/2, (2 + b)/2) [1]

a = 7 [1]

(2 + b)/2 = 5 ⇒ **b = 8** [1]

(iii)



(a) M = ((0 + 8)/2, (6 + 0)/2) = **(4, 3)** [1]

(b) Let P = (x, y). OM : MP = 1 : 3, so M = $(\frac{1 \times x + 3 \times 0}{4}, \frac{1 \times y + 3 \times 0}{4})$ [1]

x/4 = 4 and y/4 = 3 ⇒ **P = (16, 12)** [1]

(c) O(0, 0) and A(0, 6) lie on the y-axis, so base OA = 6 and the height = abscissa of P = 16. Area = $\frac{1}{2}$ × 6 × 16 = **48 sq. units** [1]

— End of Solutions —