PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 15 : Similarity

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

AA: two triangles with two pairs of equal angles are similar. Corresponding sides of similar triangles are proportional.

Basic Proportionality Theorem: DE ∥ BC ⇒ $\frac{AD}{DB} = \frac{AE}{EC}$ and ΔADE ~ ΔABC. | Ratio of areas of similar triangles = (ratio of corresponding sides)².

Scale factor k: lengths × k, areas × k², volumes × k³. Right triangle with altitude: ΔABD ~ ΔCBD ~ ΔABC. Triangles with a common vertex and bases on one line: areas are in the ratio of the bases.

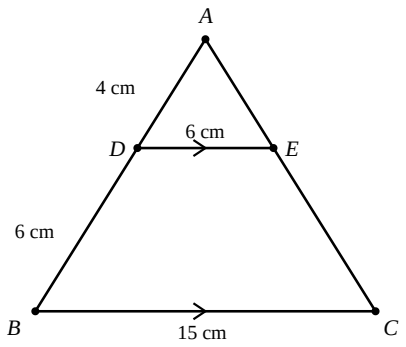
SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(b)**	AB/PQ = BC/QR ⇒ 6/9 = 8/QR ⇒ QR = 12 cm.
(ii)	**(c)**	Ratio of sides = √(4/9) = 2 : 3.
(iii)	**(a)**	AD/DB = AE/EC ⇒ 4/6 = 6/EC ⇒ EC = 9 cm.
(iv)	**(b)**	4 × 50,000 cm = 200,000 cm = 2 km.
(v)	**(c)**	Volume of model = k³ × 640 = (1/64) × 640 = 10 cm³.
(vi)	**(a)**	Ratio of sides = 5 : 6, so perimeter of ΔDEF = 30 × 6/5 = 36 cm.
(vii)	**(b)**	(A) is false: similar triangles have the same shape but not necessarily the same size. (R) is true.
(viii)	**(c)**	ΔABD ~ ΔBCD gives AD/BD = BD/DC ⇒ BD² = AD × DC, so (A) is true; (R) is the property used, so it explains (A).

Question 2 [4 + 4 + 4 = 12]

(i)



(a) DE ∥ BC ⇒ ∠ADE = ∠ABC (corresponding angles) and ∠A is common. **ΔADE ~ ΔABC** (AA) [1]

(b) AB = 4 + 6 = 10 cm. $\frac{DE}{BC} = \frac{AD}{AB} \Rightarrow \frac{6}{BC} = \frac{4}{10} \Rightarrow$ **BC = 15 cm** [1]

(c) Area(ΔADE) : Area(ΔABC) = $(AD/AB)^2 = (\frac{4}{10})^2 = \frac{4}{25}$, i.e. **4 : 25** [1]

Area(trapezium DBCE) = 25 − 4 = 21 parts, so Area(ΔADE) : Area(trapezium) = **4 : 21** [1]

(ii)

Scale factor k = 1/25,000, so actual length = 25,000 × map length [1]

Length = 8 × 25,000 = 200,000 cm = **2 km**; breadth = 5 × 25,000 = 125,000 cm = **1.25 km** [1]

Area = 2 × 1.25 = **2.5 sq. km** [2]

(iii)

Area ratio = $\frac{54}{96} = \frac{9}{16}$ [1]

Ratio of corresponding sides = $\sqrt{(\frac{9}{16})} = 3 : 4$ [1]

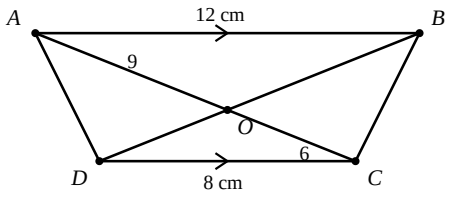
$\frac{9}{x} = \frac{3}{4} \Rightarrow$ **x = 12 cm** [1]

Ratio of perimeters = ratio of corresponding sides = **3 : 4** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)



In ΔAOB and ΔCOD: ∠AOB = ∠COD (vertically opposite angles) [1]

∠OAB = ∠OCD (alternate angles, AB ∥ DC with transversal AC) [1]

ΔAOB ~ ΔCOD (AA) [1]

(ii)

$\frac{OA}{OC} = \frac{AB}{CD} = \frac{12}{8} = \frac{3}{2}$ [1]

OA : OC = 3 : 2 and OA + OC = 15 [1]

OA = 9 cm, OC = 6 cm [1]

(iii)

Area(ΔAOB) : Area(ΔCOD) = $(AB/CD)^2 = (\frac{12}{8})^2 = 9 : 4$ [1]

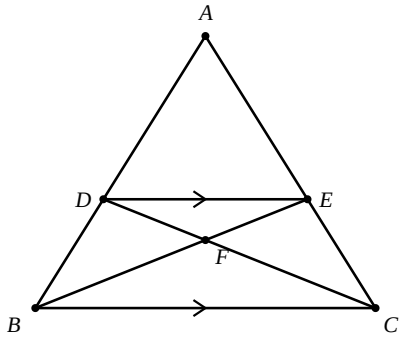
Area(ΔAOB) = $\frac{9}{4} \times 16 =$ **36 cm²** [1]

ΔAOB and ΔAOD have the common vertex A and bases OB, OD on one line: $\frac{\text{Area}(\Delta AOD)}{\text{Area}(\Delta AOB)} = \frac{OD}{OB} = \frac{CD}{AB}$ $= \frac{2}{3}$ [1]

Area(ΔAOD) = $\frac{2}{3} \times 36 =$ **24 cm²** [1]

Question 4 [3 + 3 + 4 = 10]

(i)



$DE \parallel BC \Rightarrow \Delta ADE \sim \Delta ABC$. $AD : AB = 3 : 5$ [1]

$\frac{DE}{BC} = \frac{AD}{AB} = \frac{3}{5}$. **DE : BC = 3 : 5** [1]

By the Basic Proportionality Theorem $\frac{AE}{EC} = \frac{AD}{DB} = \frac{3}{2}$. **AE : EC = 3 : 2** [1]

(ii)

In ΔDEF and ΔCBF: $\angle DFE = \angle CFB$ (vertically opposite) and $\angle FDE = \angle FCB$ (alternate angles, $DE \parallel BC$) [1]

$\Delta DEF \sim \Delta CBF$ (AA) [1]

$\frac{EF}{FB} = \frac{DE}{CB} = \frac{3}{5}$. **EF : FB = 3 : 5** [1]

(iii)

$\text{Area}(\Delta DEF) : \text{Area}(\Delta CBF) = (DE/CB)^2 = 9 : 25$ [1]

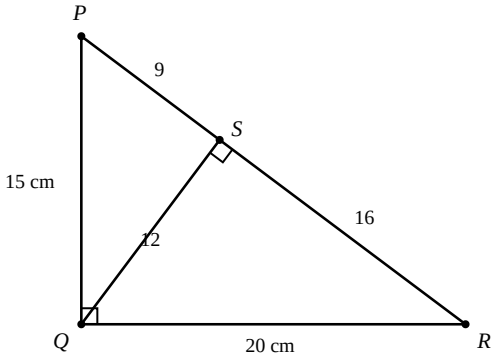
$\text{Area}(\Delta CBF) = \frac{25}{9} \times 27 =$ **75 cm²** [1]

From the similarity, $DF : FC = 3 : 5$, so $DF : DC = 3 : 8$ [1]

ΔDEF and ΔDEC have the common vertex E and bases DF, DC on one line: $\text{Area}(\Delta DEC) = \frac{8}{3} \times 27 =$ **72 cm²** [1]

Question 5 [3 + 3 + 4 = 10]

(i)



In ΔPQS and ΔPRQ: $\angle P$ is common [1]

$\angle PSQ = \angle PQR = 90°$ [1]

$\Delta PQS \sim \Delta PRQ$ (AA) [1]

(ii)

$PR^2 = PQ^2 + QR^2 = 225 + 400 = 625 \Rightarrow$ **PR = 25 cm** [1]

From (i): $\frac{PS}{PQ} = \frac{PQ}{PR} \Rightarrow PS = \frac{15 \times 15}{25} =$ **9 cm** [1]

$\frac{QS}{RQ} = \frac{PQ}{PR} \Rightarrow QS = \frac{15 \times 20}{25} =$ **12 cm** [1]

(iii)

$\text{Area}(\Delta PQR) = \frac{1}{2} \times 15 \times 20 =$ **150 cm²** [1]

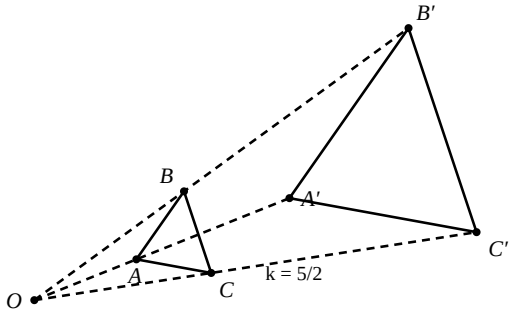
$\text{Area}(\Delta PQS) : \text{Area}(\Delta PRQ) = (PQ/PR)^2 = (\frac{15}{25})^2 = \frac{9}{25}$ [1]

$\text{Area}(\Delta PQS) = \frac{9}{25} \times 150 =$ **54 cm²** [1]

$\text{Area}(\Delta QSR) = 150 - 54 =$ **96 cm²** (check: SR = 16, $\frac{1}{2} \times 16 \times 12 = 96$ ✓) [1]

Question 6 [3 + 3 + 4 = 10]

(i)



$OA' = OA + AA' = 4 + 6 = 10$ cm. Scale factor $k = \frac{OA'}{OA} = \frac{10}{4} =$ **2.5** [1]

$A'B' = k \times AB = 2.5 \times 7 =$ **17.5 cm** [1]

$\text{Area}(\Delta A'B'C') = k^2 \times 12 = 6.25 \times 12 =$ **75 cm²** [1]

(ii)

k = 1/40,000. Longest side on the map = 5 cm ⇒ actual = 5 × 40,000 = 200,000 cm = **2 km** [1]

Area on the map = $\frac{1}{2} \times 3 \times 4 = 6$ cm² [1]

Actual area = $6 \times (40{,}000)^2$ cm² = 9.6×10^9 cm² = **0.96 sq. km** (1 sq. km = 10^{10} cm²) [1]

(iii)

$k = \frac{1}{50}$. (a) Model height = $\frac{1}{50} \times 24$ m = 0.48 m = **48 cm** [1]

(b) Area of model = $k^2 \times$ actual area ⇒ actual area = $40 \times 50^2 = 100{,}000$ cm² = **10 m²** [1]

(c) Volume of model = $k^3 \times$ actual volume ⇒ actual volume = $72 \times 50^3 = 9{,}000{,}000$ cm³ [1]

= **9 m³** ($1 \text{ m}^3 = 10^6 \text{ cm}^3$) [1]

— End of Solutions —