PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 18 : Tangents and Intersecting Chords

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

A tangent is perpendicular to the radius at the point of contact. Tangents from an external point are equal in length. Touching circles: $d = r_1 + r_2$ (externally), $d = r_1 - r_2$ (internally).

Intersecting chords: $PA \times PB = PC \times PD$. Tangent and secant: $PT^2 = PA \times PB$. Angle between a tangent and a chord = angle in the alternate segment.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(d)**	Tangent ⊥ radius: length² = 10² − 6² = 64, so length = 8 cm.
(ii)	**(b)**	∠AOB + ∠APB = 180° (quadrilateral OAPB has two right angles), so ∠AOB = 110°.
(iii)	**(a)**	PA × PB = PC × PD ⇒ 6 × 4 = 3 × PD ⇒ PD = 8 cm.
(iv)	**(c)**	$PT^2 = PB \times PA \Rightarrow 144 = 8 \times PA \Rightarrow PA = 18$ cm, so AB = 18 − 8 = 10 cm.
(v)	**(a)**	Internal contact: d = 9 − 4 = 5 cm.
(vi)	**(b)**	Angle between tangent and chord = angle in the alternate segment = 52°.
(vii)	**(b)**	(A) is false: no tangent can be drawn from an inside point. (R) is true.
(viii)	**(c)**	Both true: ΔPAC ~ ΔPDB (AA) gives PA/PD = PC/PB, hence PA × PB = PC × PD.

Question 2 [4 + 4 + 4 = 12]

(i)

OA ⊥ PA and OB ⊥ PB, so in quadrilateral OAPB: ∠AOB = 360° − 90° − 90° − 50° = **130°** [1]

PA = PB (tangents from P), so ∠PAB = ∠PBA = (180° − 50°) ÷ 2 = **65°** [2]

∠OAB = ∠OAP − ∠PAB = 90° − 65° = **25°** [1]

(ii)

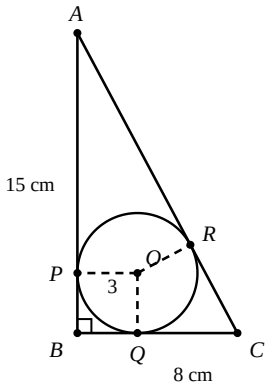
(a) AP × PB = CP × PD ⇒ 9 × 4 = 3 × PD [1]

PD = 36 ÷ 3 = **12 cm** [1]

(b) PB = PA + AB = 5 + 15 = 20 cm; $PT^2 = PA \times PB = 5 \times 20 = 100$ [1]

PT = **10 cm** [1]

(iii)



In right ΔABC: $AC^2 = 15^2 + 8^2 = 289 \Rightarrow AC = 17$ cm [1]

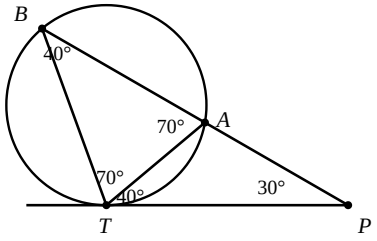
Let the radius be r. The circle touches AB at P and BC at Q; OPBQ is a square, so BP = BQ = r [1]

AR = AP = 15 − r and CR = CQ = 8 − r (tangents from a point are equal). AC = AR + RC ⇒ 17 = (15 − r) + (8 − r) [1]

2r = 6 ⇒ **r = 3 cm** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]



(i)

The angle between the tangent PT and the chord TA equals the angle in the alternate segment [1]

∠TBA = ∠PTA (alternate segment theorem) [1]

∠TBA = **40°** [1]

(ii)

In ΔPAT: ∠PAT = 180° − 30° − 40° = 110° [1]

A lies on PB, so ∠PAT + ∠TAB = 180° (linear pair) [1]

∠TAB = 180° − 110° = **70°** [1]

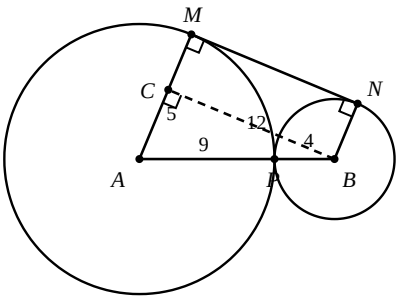
(iii)

In ΔTAB: ∠ATB = 180° − ∠TAB − ∠TBA = 180° − 70° − 40° [2]

∠ATB = **70°** [1]

∠ATB = ∠TAB = 70°, so the sides opposite them are equal: **BA = BT** [1]

Question 4 [3 + 3 + 4 = 10]



(i)

The circles touch externally, so AB = AP + PB = 9 + 4 = **13 cm** [1]

AM ⊥ MN and BN ⊥ MN, so BC ∥ MN and BCMN is a rectangle: CM = BN = 4 cm [1]

AC = AM − CM = 9 − 4 = **5 cm** [1]

(ii)

In right ΔACB: $BC^2 = AB^2 - AC^2 = 13^2 - 5^2$ [1]

$BC^2 = 169 - 25 = 144 \Rightarrow$ **BC = 12 cm** [1]

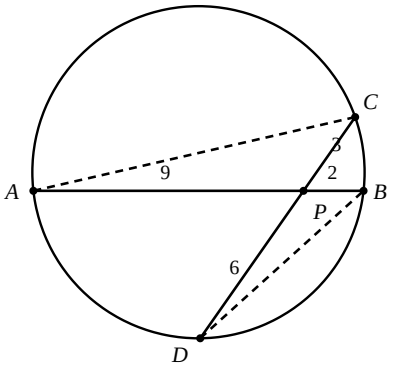
BCMN is a rectangle, so **MN = BC = 12 cm** [1]

(iii)

AM ∥ BN (both perpendicular to MN), so AMNB is a trapezium with parallel sides AM = 9 cm and BN = 4 cm [2]

Area = $\frac{1}{2} \times (AM + BN) \times MN = \frac{1}{2} \times (9 + 4) \times 12 =$ **78 cm²** [2]

Question 5 [3 + 3 + 4 = 10]



(i)

Chords AB and CD intersect inside the circle, so AP × PB = CP × PD [1]

9 × 2 = x (x + 3) [1]

$x^2 + 3x - 18 = 0$ [1]

(ii)

$x^2 + 6x - 3x - 18 = 0 \Rightarrow (x + 6)(x - 3) = 0$ [1]

x = −6 (rejected, a length cannot be negative) or **x = 3** [1]

CP = **3 cm**, PD = 3 + 3 = **6 cm**, CD = 3 + 6 = **9 cm** [1]

(iii)

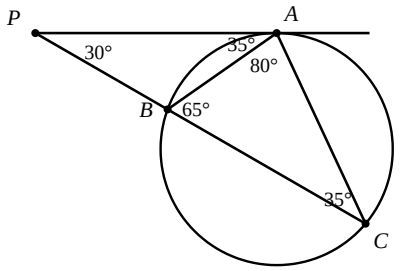
In ΔAPC and ΔDPB: ∠CAP = ∠BDP (angles in the same segment) and ∠ACP = ∠DBP (angles in the same segment) [1]

ΔAPC ~ ΔDPB (AA similarity) [1]

Ratio of corresponding sides AP : DP = 9 : 6 = 3 : 2 [1]

Ratio of areas = $(3 : 2)^2$ = **9 : 4** [1]

Question 6 [3 + 3 + 4 = 10]



(i)

The angle between the tangent AP and the chord AB equals the angle in the alternate segment [1]

∠ACB = ∠PAB (alternate segment theorem) [1]

∠ACB = **35°** [1]

(ii)

P, B, C are collinear, so ∠ABC is an exterior angle of ΔPAB [1]

∠ABC = ∠APB + ∠PAB = 30° + 35° [1]

∠ABC = **65°** [1]

(iii)

In ΔABC: ∠BAC = 180° − ∠ABC − ∠ACB = 180° − 65° − 35° [1]

∠BAC = **80°** [1]

∠PAC = ∠PAB + ∠BAC = 35° + 80° [1]

∠PAC = **115°** (check in ΔPAC: 30° + 115° + 35° = 180° ✓) [1]

— End of Solutions —