PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 19 : Constructions (Circles)

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

Tangent ⊥ radius at the point of contact; tangents from an external point are equal ($PA^2 = OP^2 - OA^2$). Circumcentre = meeting point of the perpendicular bisectors of the sides (equidistant from the vertices).

Incentre = meeting point of the angle bisectors (equidistant from the sides); inradius r = Area ÷ s. Regular hexagon of side a: circumradius = a, inradius = $(\sqrt{3}/2)a$. Tangent lengths from a vertex of a triangle: BD = s − AC.

In construction answers, the figure shown is drawn to the stated measurements; accept a measured value within ± 0.1 cm (lengths) or ± 1° (angles).

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(c)**	Length² = 13² − 5² = 144, so the tangent = 12 cm.
(ii)	**(a)**	The circumcentre is where the perpendicular bisectors of the sides meet.
(iii)	**(b)**	The incentre lies on the angle bisectors and is equidistant from the three sides.
(iv)	**(d)**	For a regular hexagon the circumradius equals the side = 5 cm.
(v)	**(b)**	Angle between the tangents + angle between the radii = 180°, so 180° − 50° = 130°.
(vi)	**(d)**	The angle in a semicircle is 90°, so the hypotenuse is a diameter and its mid-point is the circumcentre.
(vii)	**(c)**	Both true: the construction is valid precisely because a tangent is perpendicular to the radius at the point of contact.
(viii)	**(b)**	(A) is false: for an obtuse triangle the circumcentre lies outside it. (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

$9^2 + 12^2 = 81 + 144 = 225 = 15^2$, so the triangle is right-angled and the hypotenuse is 15 cm [1]

Circumradius = ½ × hypotenuse = **7.5 cm** [1]

Area = $\frac{1}{2} \times 9 \times 12 = 54$ cm²; s = (9 + 12 + 15) ÷ 2 = 18 cm [1]

Inradius r = Area ÷ s = 54 ÷ 18 = **3 cm** [1]

(ii)

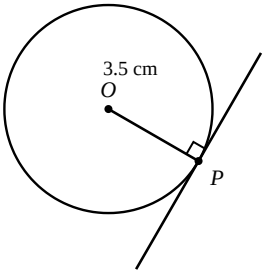
Angle between the radii = 180° − 60° = **120°** [1]

OP bisects ∠APB, so ∠OPA = 30°. In right ΔOAP: sin 30° = OA ÷ OP ⇒ OP = 4 ÷ 0.5 = **8 cm** [1]

$PA^2 = OP^2 - OA^2 = 64 - 16 = 48$ [1]

PA = PB = √48 = **4√3 cm** (≈ 6.9 cm) [1]

(iii)



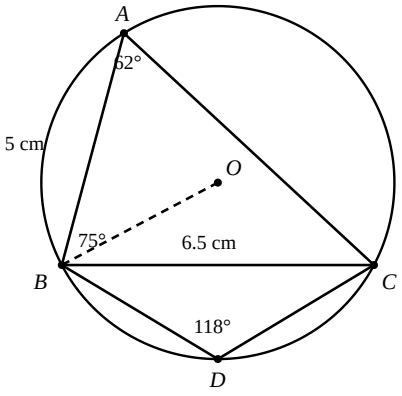
Draw a circle with centre O and radius 3.5 cm; mark a point P on it and join OP [1]

At P construct ∠OPX = 90°: with centre P cut OP produced on both sides at equal distances, then with a larger equal radius draw arcs from these two points to cross at Q [2]

Join P to Q and extend both ways: **the line through P and Q is the required tangent** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]



(i)

Draw BC = 6.5 cm; at B construct ∠CBX = 75° (e.g. 60° + 15° by bisecting the angle between 60° and 90°) or use a protractor [1]

With centre B and radius 5 cm cut BX at A [1]

Join AC to complete ΔABC (AC ≈ 7.1 cm) [1]

(ii)

Draw the perpendicular bisectors of BC and of AB; they meet at O, the circumcentre [2]

With centre O and radius OB draw the circle through A, B and C. Measured circumradius **≈ 3.7 cm** [1]

(iii)

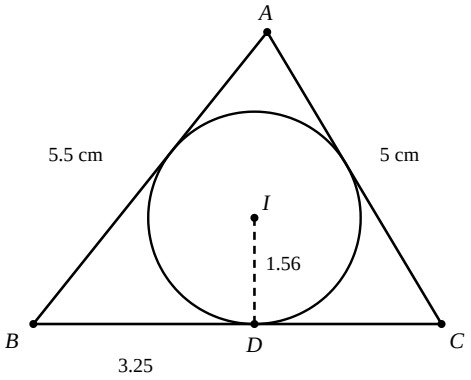
D is equidistant from B and C, so it lies on the perpendicular bisector of BC; the bisector meets the circle on the side opposite A at **D** [1]

Join BD and CD [1]

Measured **∠BDC ≈ 118°** (∠BAC ≈ 62°) [1]

ABDC is a cyclic quadrilateral, so ∠BAC + ∠BDC = 180° (opposite angles are supplementary) [1]

Question 4 [3 + 3 + 4 = 10]



(i)

Draw BC = 6 cm [1]

With centre B radius 5.5 cm and centre C radius 5 cm draw arcs intersecting at A [1]

Join AB and AC to complete ΔABC [1]

(ii)

Bisect ∠B and ∠C; the bisectors meet at the incentre I [2]

From I draw ID ⊥ BC; with centre I and radius ID draw the circle, which touches all three sides [1]

(iii)

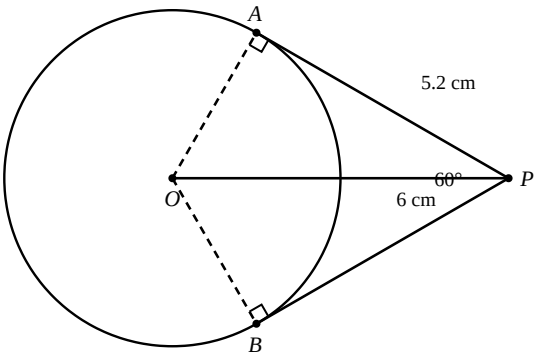
Measured inradius **≈ 1.6 cm** [1]

Measured **BD ≈ 3.25 cm** (3.2 – 3.3 cm accepted) [1]

s = (5.5 + 6 + 5) ÷ 2 = 8.25 cm, so s − AC = 8.25 − 5 = 3.25 cm [1]

BD = s − AC, so the measurement is verified. (Check: Area = √(8.25 × 2.75 × 2.25 × 3.25) ≈ 12.88 cm², r = 12.88 ÷ 8.25 ≈ 1.56 cm) [1]

Question 5 [3 + 3 + 4 = 10]



(i)

Draw a circle with centre O and radius 3 cm [2]

Along a line through O mark P with OP = 6 cm [1]

(ii)

Join OP and bisect it at M [1]

With centre M and radius MO draw a circle meeting the given circle at A and B [1]

Join PA and PB: **PA and PB are the required tangents** (∠OAP = ∠OBP = 90°, angle in a semicircle) [1]

(iii)

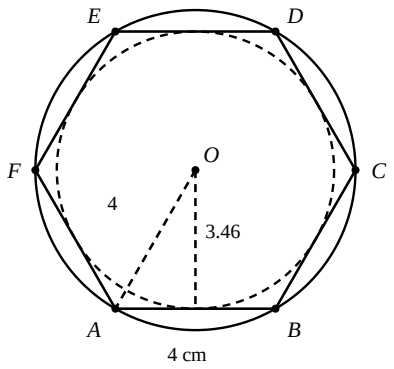
Measured **PA ≈ 5.2 cm** (calculation: PA = √(6² − 3²) = √27 = 5.196 cm) [1]

In right ΔOAP: sin ∠APO = OA ÷ OP = 3 ÷ 6 = ½ ⇒ ∠APO = 30° [1]

∠APB = 2 × 30° = **60°** [1]

PA = PB and the angle between them is 60°, so the other two angles are also 60°: **ΔPAB is equilateral** [1]

Question 6 [3 + 3 + 4 = 10]



(i)

Each interior angle of a regular hexagon = (2 × 6 − 4) × 90° ÷ 6 = 120° [1]

Draw AB = 4 cm; at A and B construct angles of 120° and mark AF = BC = 4 cm [1]

With centres F and C and radius 4 cm complete D and E (or construct the angles at F and C); join to get ABCDEF [1]

(ii)

Draw the perpendicular bisectors of AB and AF; they meet at O [1]

With centre O and radius OA draw the circle through all six vertices [1]

Circumradius = side of the hexagon = **4 cm** [1]

(iii)

Bisect ∠A and ∠B; the bisectors meet at O. Draw OM ⊥ AB [1]

With centre O and radius OM draw the circle touching all six sides [1]

Measured inradius **≈ 3.5 cm** [1]

Calculation: OM² = OA² − AM² = 4² − 2² = 12 ⇒ OM = 2√3 = 3.46 cm ≈ 3.5 cm ✓ [1]

— End of Solutions —