PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 21 : Trigonometrical Identities

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

$\sin\theta = 1/\operatorname{cosec}\theta$, $\cos\theta = 1/\sec\theta$, $\tan\theta = \sin\theta/\cos\theta = 1/\cot\theta$. $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$.

Complementary angles: $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$, $\tan(90° - \theta) = \cot\theta$, $\operatorname{cosec}(90° - \theta) = \sec\theta$. Angle sum of a triangle = 180°.

$\sin 30° = \cos 60° = 1/2$, $\cos 30° = \sin 60° = \sqrt{3}/2$, $\tan 30° = 1/\sqrt{3}$, $\tan 45° = 1$.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(c)**	$\sec\theta = \sqrt{(1 + \tan^2\theta)} = \sqrt{(1 + 25/144)} = \sqrt{(169/144)} = 13/12$.
(ii)	**(a)**	$(1 - \sin^2\theta)\sec^2\theta = \cos^2\theta \times (1/\cos^2\theta) = 1$.
(iii)	**(d)**	$\operatorname{cosec}(90° - \theta) = \sec\theta$.
(iv)	**(b)**	$\cos 65° = \cos(90° - 25°) = \sin 25°$, so the ratio is 1.
(v)	**(d)**	$1/\sec^2\theta + 1/\operatorname{cosec}^2\theta = \cos^2\theta + \sin^2\theta = 1$.
(vi)	**(b)**	$\cos^2\theta = 1/2 \Rightarrow \cos\theta = 1/\sqrt{2} \Rightarrow \theta = 45°$.
(vii)	**(c)**	Both true; dividing (perpendicular)² + (base)² = (hypotenuse)² by (hypotenuse)² gives $\sin^2\theta + \cos^2\theta = 1$.
(viii)	**(b)**	(A) is false: sin θ = perpendicular ÷ hypotenuse is less than 1. (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

L.H.S. = $[\tan^2 A + (\sec A + 1)^2] \div [\tan A (\sec A + 1)]$ (taking the L.C.M.) [1]

= $[(\sec^2 A - 1) + \sec^2 A + 2 \sec A + 1] \div [\tan A (\sec A + 1)]$ (using $\tan^2 A = \sec^2 A - 1$) [1]

= $[2 \sec A (\sec A + 1)] \div [\tan A (\sec A + 1)] = 2 \sec A \div \tan A$ [1]

= $2 \times (1/\cos A) \times (\cos A/\sin A) = 2 \operatorname{cosec} A$ = **R.H.S.** [1]

(ii)

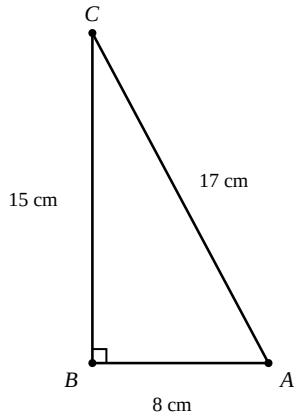
$\sin 18° \div \cos 72° = \sin 18° \div \cos(90° - 18°) = \sin 18° \div \sin 18° = 1$ [1]

$\tan 23° \div \cot 67° = \tan 23° \div \cot(90° - 23°) = \tan 23° \div \tan 23° = 1$ [1]

$2 \cos 60° \tan 45° = 2 \times (1/2) \times 1 = 1$ [1]

Value = 1 + 1 − 1 = **1** [1]

(iii)



$AC^2 = AB^2 + BC^2 = 64 + 225 = 289 \Rightarrow$ **AC = 17 cm** [1]

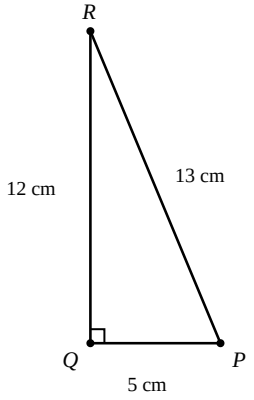
cosec A = AC/BC = 17/15 [1]

cot A = AB/BC = 8/15 [1]

(cosec A − cot A) ÷ (cosec A + cot A) = (17 − 8)/15 ÷ (17 + 8)/15 = 9/25 = **9/25** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]



(i)

$PR^2 = PQ^2 + QR^2 = 25 + 144 = 169 \Rightarrow$ **PR = 13 cm** [1]

sin P = QR/PR = **12/13**, cos P = PQ/PR = **5/13** [1]

tan P = QR/PQ = **12/5** (and sec P = 13/5) [1]

(ii)

$\sin^2P + \cos^2P = 144/169 + 25/169$ [1]

= 169/169 = **1** ✓ [1]

$1 + \tan^2P = 1 + 144/25 = 169/25 = (13/5)^2 = \sec^2P$ ✓ [1]

(iii)

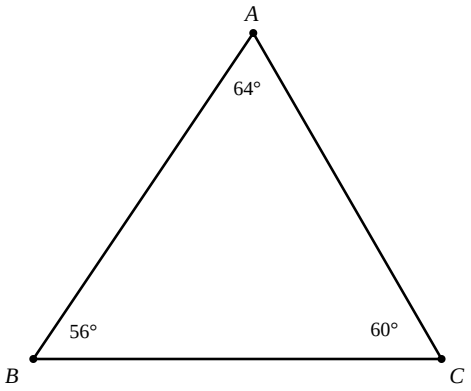
L.H.S. = (1/cos P − sin P/cos P) ÷ (1/cos P + sin P/cos P) [1]

Multiplying numerator and denominator by cos P: L.H.S. = (1 − sin P) ÷ (1 + sin P) = **R.H.S.** [1]

Value = (1 − 12/13) ÷ (1 + 12/13) = (1/13) ÷ (25/13) [1]

= **1/25** (check: (sec P − tan P) ÷ (sec P + tan P) = (1/5) ÷ (25/5) = 1/25 ✓) [1]

Question 4 [3 + 3 + 4 = 10]



(i)

In ΔABC: A + B + C = 180° ⇒ A + B = 180° − C [1]

½(A + B) = 90° − ½C [1]

sin ½(A + B) = sin (90° − ½C) = **cos ½C** [1]

(ii)

B + C = 180° − A ⇒ ½(B + C) = 90° − ½A [1]

tan ½(B + C) = tan (90° − ½A) [1]

= **cot ½A** [1]

(iii)

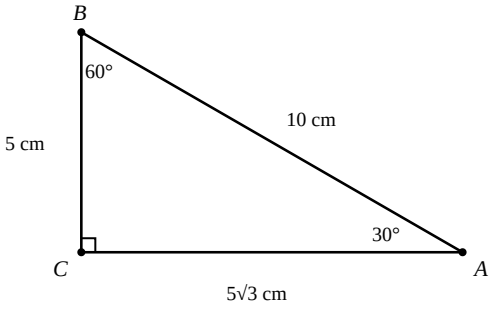
∠C = 180° − 64° − 56° = **60°** [1]

½(A + B) = ½(120°) = 60° and ½C = 30° [1]

sin 60° ÷ cos 30° = (√3/2) ÷ (√3/2) [1]

= **1** (as proved in (i), sin ½(A + B) = cos ½C) [1]

Question 5 [3 + 3 + 4 = 10]



(i)

sin A = BC/AB ⇒ BC = 10 × sin 30° = 10 × 1/2 [1]

BC = 5 cm [1]

AC² = AB² − BC² = 100 − 25 = 75 ⇒ **AC = 5√3 cm** [1]

(ii)

tan A = BC/AC = 5/(5√3) = **1/√3** [1]

cos B = BC/AB = 5/10 = **1/2** [1]

cot B = BC/AC = **1**/√**3**. Observation: tan A = cot B, because A and B are complementary (A + B = 90°) [1]

(iii)

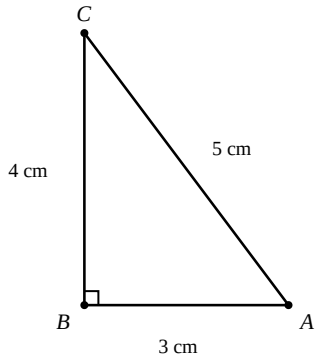
sin B = AC/AB = 5√3/10 = √**3/2** [1]

B = 90° − 30° = 60° = 2A, so sin 2A = sin 60° = √3/2 [1]

2 sin A cos A = 2 × (1/2) × (√3/2) = √3/2 (cos A = AC/AB = √3/2) [1]

sin 2A = 2 sin A cos A for A = 30° **verified** [1]

Question 6 [3 + 3 + 4 = 10]



(i)

AC² = 3² + 4² = 25 ⇒ **AC = 5 cm** [1]

sin A = BC/AC = **4/5** [1]

tan A = BC/AB = **4/3** [1]

(ii)

m = tan A + sin A = 4/3 + 4/5 = (20 + 12)/15 = **32/15** [2]

n = tan A − sin A = 4/3 − 4/5 = (20 − 12)/15 = **8/15** [1]

(iii)

m² − n² = (m + n)(m − n) = (2 tan A)(2 sin A) = 4 tan A sin A [2]

4√(mn) = 4√(tan²A − sin²A) = 4√[sin²A (sec²A − 1)] = 4√(sin²A tan²A) = 4 sin A tan A. Hence m² − n² = 4√(mn) [1]

Verification: m² − n² = (1024 − 64)/225 = 64/15; 4√(mn) = 4√(256/225) = 4 × 16/15 = 64/15 ✓ [1]

— End of Solutions —