PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 22 : Heights and Distances

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used ($\sqrt{3} = 1.732$)

In a right-angled triangle: $\tan\theta$ = height ÷ base, $\sin\theta$ = height ÷ hypotenuse, $\cos\theta$ = base ÷ hypotenuse. The angle of depression from a point equals the angle of elevation of that point's observer (alternate angles).

$\tan 30° = 1/\sqrt{3}$, $\tan 45° = 1$, $\tan 60° = \sqrt{3}$, $\sin 30° = 1/2$, $\sin 60° = \sqrt{3}/2$, $\cos 60° = 1/2$.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(b)**	$\tan 45° = h/20 \Rightarrow h = 20$ m.
(ii)	**(a)**	$\tan\theta = h \div (\sqrt{3}\,h) = 1/\sqrt{3} \Rightarrow \theta = 30°$.
(iii)	**(b)**	Angle of elevation = angle of depression (alternate angles) = 30°.
(iv)	**(c)**	Height = $10 \sin 60° = 10 \times (\sqrt{3}/2) = 5\sqrt{3}$ m.
(v)	**(b)**	$\tan 60° = 30/d \Rightarrow d = 30/\sqrt{3} = 10\sqrt{3}$ m.
(vi)	**(a)**	$\tan\theta$ = height ÷ distance; as the distance decreases, $\tan\theta$ and hence θ increase.
(vii)	**(c)**	Both true: $\tan 45°$ = height ÷ distance = 1, so height = distance.
(viii)	**(b)**	(A) is false: walking away decreases the angle. (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

Let the tower be h m high. $\tan 30° = h/60$ [1]

$h = 60 \tan 30° = 60/\sqrt{3} = 20\sqrt{3}$ [1]

$= 20 \times 1.732$ [1]

h = 34.64 m [1]

(ii)

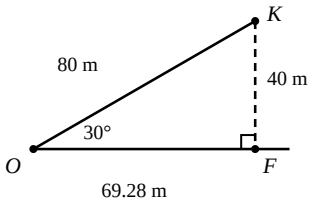
(a) Height $= 12 \sin 60° = 12 \times (\sqrt{3}/2) = 6\sqrt{3}$ [1]

$= 6 \times 1.732 =$ **10.39 m** [1]

(b) Distance of the foot from the wall $= 12 \cos 60° = 12 \times 1/2$ [1]

= **6 m** [1]

(iii)



(a) sin 30° = 40 ÷ (string) ⇒ string = 40 ÷ 1/2 [1]

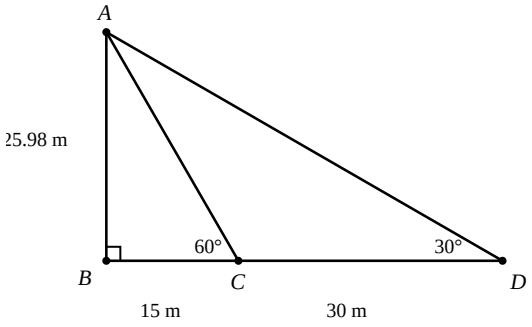
= **80 m** [1]

(b) tan 30° = 40 ÷ d ⇒ d = 40√3 = 40 × 1.732 [1]

= **69.28 m** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]



(i)

In right ΔABC: tan 60° = h/BC ⇒ √3 = h/BC [1]

BC = h/√3 [1]

In right ΔABD: tan 30° = h/BD ⇒ **BD = h√3** [1]

(ii)

BD − BC = CD ⇒ h√3 − h/√3 = 30 [1]

h (3 − 1)/√3 = 30 ⇒ 2h = 30√3 ⇒ h = 15√3 [1]

= 15 × 1.732 = **25.98 m** [1]

(iii)

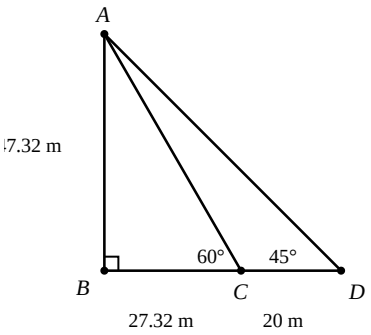
BC = h/√3 = 15√3/√3 [1]

= **15 m** [1]

BD = h√3 = 15√3 × √3 [1]

= **45 m** (check: BD − BC = 30 m ✓) [1]

Question 4 [3 + 3 + 4 = 10]



(i)

Shadow at 60°: tan 60° = h/BC ⇒ **BC = h/√3** [2]

Shadow at 45°: tan 45° = h/BD ⇒ **BD = h** [1]

(ii)

BD − BC = 20 ⇒ h − h/√3 = 20 [1]

h (√3 − 1)/√3 = 20 ⇒ h = 20√3/(√3 − 1) = 20√3 (√3 + 1)/2 = 10(3 + √3) [1]

= 10 × 4.732 = **47.32 m** [1]

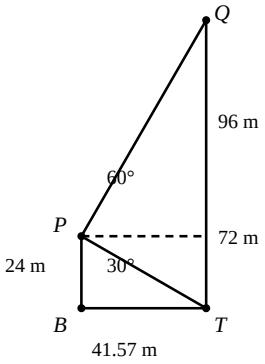
(iii)

Shadow at 45° = h = **47.32 m** [2]

Shadow at 60° = h/√3 = 10(√3 + 1) = 10 × 2.732 = **27.32 m** [1]

Check: 47.32 − 27.32 = 20 m ✓ [1]

Question 5 [3 + 3 + 4 = 10]



(i)

Angle of elevation of P from T = 30° (alternate angles); in right ΔPBT, tan 30° = PB/BT [1]

1/√3 = 24/BT ⇒ BT = 24√3 [1]

= 24 × 1.732 = **41.57 m** [1]

(ii)

Draw PE horizontal, meeting the tower at E: PE = BT = 24√3 m [1]

In right ΔPEQ: tan 60° = QE/PE ⇒ QE = 24√3 × √3 [1]

= **72 m** [1]

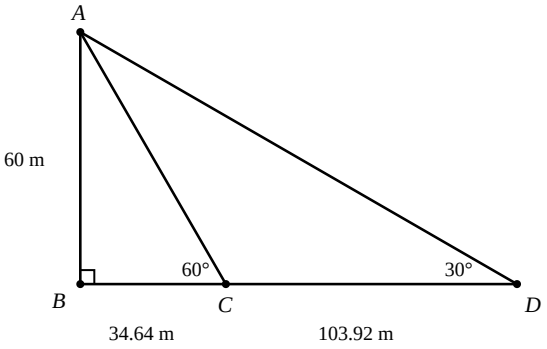
(iii)

Total height of tower QT = QE + ET = 72 + 24 = **96 m** [2]

cos 60° = PE/PQ ⇒ PQ = 24√3 ÷ (1/2) = 48√3 [1]

= 48 × 1.732 = **83.14 m** [1]

Question 6 [3 + 3 + 4 = 10]



(i)

∠ACB = 60° (alternate angles with the angle of depression) [1]

tan 60° = AB/BC ⇒ √3 = 60/BC ⇒ BC = 60/√3 = 20√3 [1]

= 20 × 1.732 = **34.64 m** [1]

(ii)

∠ADB = 30°, tan 30° = AB/BD [1]

1/√3 = 60/BD ⇒ BD = 60√3 [1]

= 60 × 1.732 = **103.92 m** [1]

(iii)

(a) Same side: CD = BD − BC = 103.92 − 34.64 [1]

= **69.28 m** [1]

(b) Opposite sides: CD = BD + BC = 103.92 + 34.64 [1]

= **138.56 m** [1]

— End of Solutions —