PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 5 : Quadratic Equations

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used (for $ax^2 + bx + c = 0$, $a \neq 0$):

Discriminant $D = b^2 - 4ac$. $D > 0$: real and unequal roots (rational if D is a perfect square) | $D = 0$: real and equal roots | $D < 0$: roots not real.

Formula: $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ | Zero product rule: if $pq = 0$ then $p = 0$ or $q = 0$ | Equation with roots α, β: $(x - \alpha)(x - \beta) = 0$.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(a)** 25	$D = (-7)^2 - 4 \times 3 \times 2 = 49 - 24 = 25$.
(ii)	**(b)**	$D = (-12)^2 - 4 \times 4 \times 9 = 144 - 144 = 0$, so the roots are real and equal.
(iii)	**(c)** 9	Equal roots $\Rightarrow D = 36 - 4k = 0 \Rightarrow k = 9$.
(iv)	**(b)**	$D = 16 - 4 = 12$, which is not a perfect square, so the roots are irrational; (A) is false. (R) is a true statement.
(v)	**(d)** 3	Putting $x = 2$: $4 + 2k - 10 = 0 \Rightarrow 2k = 6 \Rightarrow k = 3$.
(vi)	**(c)**	$x(x - 5) = 0 \Rightarrow x = 0$ or $x = 5$.
(vii)	**(d)**	$(x + 2)(x - 5) = 0 \Rightarrow x^2 - 3x - 10 = 0$.
(viii)	**(c)**	$2x^2 - 5x + 2 = (2x - 1)(x - 2) = 0 \Rightarrow x = ½$ or 2, so (A) is true. (R) is the zero product rule used to get these roots, so it correctly explains (A).

Question 2 [4 + 4 + 4 = 12]

(i)

$a = k + 1$, $b = -2(k - 1)$, $c = 1$

$D = b^2 - 4ac = 4(k - 1)^2 - 4(k + 1)$ [1]

Equal roots $\Rightarrow D = 0 \Rightarrow 4(k - 1)^2 - 4(k + 1) = 0 \Rightarrow (k - 1)^2 = k + 1$ [1]

$k^2 - 2k + 1 = k + 1 \Rightarrow k^2 - 3k = 0 \Rightarrow k(k - 3) = 0$ [1]

$k = 0$ or $k = 3$ (check: $k = 0$ gives $x^2 + 2x + 1 = 0$; $k = 3$ gives $4x^2 - 4x + 1 = 0$; both are perfect squares ✓) [1]

(ii)

$6x^2 + x = 2 \Rightarrow 6x^2 + x - 2 = 0$ [1]

$6x^2 + 4x - 3x - 2 = 0 \Rightarrow 2x(3x + 2) - 1(3x + 2) = 0 \Rightarrow (3x + 2)(2x - 1) = 0$ [2]

$x = -\dfrac{2}{3}$ or $x = \dfrac{1}{2}$ [1]

(iii)

$a = 1, b = -6, c = 4$ [1]

$D = 36 - 16 = 20; \sqrt{20} = 4.472$ [1]

$x = \frac{6 \pm 4.472}{2} \Rightarrow x = \frac{10.472}{2} = 5.236$ or $x = \frac{1.528}{2} = 0.764$ [1]

x = 5.24 or x = 0.76 (correct to 2 decimal places) [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)

(a) $D = 5^2 - 4(2)(-3) = 25 + 24 = 49 > 0$ (a perfect square): roots are **real, unequal (and rational)** [1]

(b) $D = (-24)^2 - 4(9)(16) = 576 - 576 = 0$: roots are **real and equal** [1]

(c) $D = (-4)^2 - 4(5)(1) = 16 - 20 = -4 < 0$: roots are **not real (imaginary)** [1]

(ii)

Putting $x = 2$: $p(4) - 7(2) + 6 = 0 \Rightarrow 4p - 8 = 0 \Rightarrow \mathbf{p = 2}$ [1]

The equation becomes $2x^2 - 7x + 6 = 0 \Rightarrow 2x^2 - 4x - 3x + 6 = 0 \Rightarrow (x - 2)(2x - 3) = 0$ [1]

Roots are 2 and $\frac{3}{2}$; the other root is $\mathbf{x = \frac{3}{2}}$ [1]

(iii)

$x = 3$ is a root: $2(9) + 3p + q = 0 \Rightarrow 3p + q = -18$ …(1) [1]

$x = -½$ is a root: $2(\frac{1}{4}) - \frac{p}{2} + q = 0 \Rightarrow p - 2q = 1$ …(2) [1]

From (1), $q = -18 - 3p$. Put in (2): $p + 36 + 6p = 1 \Rightarrow 7p = -35 \Rightarrow \mathbf{p = -5}$ [1]

$q = -18 - 3(-5) = -3 \Rightarrow \mathbf{q = -3}$ (check: $2x^2 - 5x - 3 = (2x + 1)(x - 3)$ ✓) [1]

Question 4 [3 + 3 + 4 = 10]

(i)

$2x^2 - 7x - 15 = 0 \Rightarrow 2x^2 - 10x + 3x - 15 = 0 \Rightarrow 2x(x - 5) + 3(x - 5) = 0 \Rightarrow (x - 5)(2x + 3) = 0$ [2]

$x = 5$ or $x = -\frac{3}{2}$. (a) $x \in Z$: solution set = **{5}** (b) $x \in Q$: solution set = $\mathbf{\{5, -\frac{3}{2}\}}$ [1]

(ii)

$\frac{2(x + 2) + 3(x + 1)}{(x + 1)(x + 2)} = 2 \Rightarrow 5x + 7 = 2(x^2 + 3x + 2)$ [1]

$2x^2 + 6x + 4 - 5x - 7 = 0 \Rightarrow 2x^2 + x - 3 = 0$ [1]

$(2x + 3)(x - 1) = 0 \Rightarrow \mathbf{x = 1}$ **or** $\mathbf{x = -\frac{3}{2}}$ (both are admissible, since neither is −1 or −2) [1]

(iii)

With $y = x^2 + 3x$: $y^2 - 2y - 8 = 0$ [1]

$(y - 4)(y + 2) = 0 \Rightarrow y = 4$ or $y = -2$ [1]

$y = 4$: $x^2 + 3x - 4 = 0 \Rightarrow (x + 4)(x - 1) = 0 \Rightarrow x = -4$ or 1 [1]

$y = -2$: $x^2 + 3x + 2 = 0 \Rightarrow (x + 1)(x + 2) = 0 \Rightarrow x = -1$ or -2. **Solutions: x = −4, −2, −1, 1** [1]

Question 5 [3 + 3 + 4 = 10]

(i)

$a = 5, b = -8, c = 2$ [1]

$D = 64 - 40 = 24; \sqrt{24} = 2\sqrt{6}$ [1]

$x = \frac{8 \pm 2\sqrt{6}}{10} = \frac{\mathbf{4 \pm \sqrt{6}}}{\mathbf{5}}$, i.e. $x = \frac{4 + \sqrt{6}}{5}$ or $x = \frac{4 - \sqrt{6}}{5}$ [1]

(ii)

$a = 3, b = -7, c = 1$; $D = 49 - 12 = 37$; $\sqrt{37} = 6.083$ [1]

$x = \frac{7 \pm 6.083}{6}$ [1]

$x = \frac{13.083}{6} = 2.1805$ or $x = \frac{0.917}{6} = 0.1528$. **x = 2.18 or x = 0.153** (3 significant figures) [1]

(iii)

Let $x^2 = y$: $2y^2 - 9y + 4 = 0$ [1]

$2y^2 - 8y - y + 4 = 0 \Rightarrow (2y - 1)(y - 4) = 0 \Rightarrow y = \frac{1}{2}$ or $y = 4$ [1]

$y = 4 \Rightarrow x^2 = 4 \Rightarrow \mathbf{x = \pm 2}$ [1]

$y = \frac{1}{2} \Rightarrow x^2 = \frac{1}{2} \Rightarrow \mathbf{x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}}$. Real solutions: $2, -2, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$ [1]

Question 6 [3 + 3 + 4 = 10]

(i)

Roots: $x = -3$ or $x = \frac{5}{2} \Rightarrow x + 3 = 0$ or $2x - 5 = 0$ [1]

$(x + 3)(2x - 5) = 0$ [1]

$2x^2 - 5x + 6x - 15 = 0 \Rightarrow \mathbf{2x^2 + x - 15 = 0}$ [1]

(ii)

$a = m + 1, b = 2(m + 3), c = m + 8$; $D = 4(m + 3)^2 - 4(m + 1)(m + 8)$ [1]

$D = 0 \Rightarrow (m^2 + 6m + 9) - (m^2 + 9m + 8) = 0 \Rightarrow 1 - 3m = 0 \Rightarrow \mathbf{m = \frac{1}{3}}$ [1]

The equation becomes $\frac{4}{3}x^2 + \frac{20}{3}x + \frac{25}{3} = 0 \Rightarrow 4x^2 + 20x + 25 = 0 \Rightarrow (2x + 5)^2 = 0 \Rightarrow \mathbf{x = -\frac{5}{2}}$ (twice) [1]

(iii)

$D = (-6)^2 - 4(2)(k) = 36 - 8k$ [1]

Real roots $\Rightarrow D \geq 0 \Rightarrow 36 - 8k \geq 0 \Rightarrow k \leq 4.5$ [1]

The greatest integral value is **k = 4** [1]

$2x^2 - 6x + 4 = 0 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow$ **x = 1 or x = 2** [1]

— End of Solutions —