PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 6 : Solving Simple Problems

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Method used for every word problem

1. Let the unknown quantity be x. 2. Translate the statement into an equation in x. 3. Solve the quadratic equation (factorisation). 4. Reject the root that is not admissible (negative length, age, speed, etc.). 5. Answer the question asked and check.

Time = $\frac{\text{Distance}}{\text{Speed}}$ | Upstream speed = (boat − stream) | Downstream speed = (boat + stream) |

Work: if a job takes p and q days alone, together it takes 1/p + 1/q per day.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(b)** 7	$x^2 + x - 56 = 0 \Rightarrow (x + 8)(x - 7) = 0 \Rightarrow x = 7$ (positive).
(ii)	**(b)** 7 m	$x(x + 3) = 70 \Rightarrow x^2 + 3x - 70 = 0 \Rightarrow (x + 10)(x - 7) = 0 \Rightarrow x = 7$.
(iii)	**(c)** 3	$x^2 + (x + 1)^2 = (x + 2)^2 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = 3$ (sides 3, 4, 5).
(iv)	**(c)**	$(x - 4)(x + 2) = 0 \Rightarrow x = 4$ or -2; the negative root is rejected, so the length is 4 cm. (A) is true, (R) is true and it explains (A).
(v)	**(c)** 12	$n(n + 1) = 132 \Rightarrow n^2 + n - 132 = 0 \Rightarrow (n + 12)(n - 11) = 0 \Rightarrow n = 11$; larger integer = 12.
(vi)	**(c)** 8	$n(n - 3) = 40 \Rightarrow n^2 - 3n - 40 = 0 \Rightarrow (n - 8)(n + 5) = 0 \Rightarrow n = 8$.
(vii)	**(b)**	New speed = (x + 10) km/h, so time = $\frac{120}{x + 10}$ hours.
(viii)	**(b)**	$x(x + 2) = 35 \Rightarrow x^2 + 2x - 35 = 0 \Rightarrow (x + 7)(x - 5) = 0 \Rightarrow x = 5$, so the numbers are 5 and 7 ($7 \times 9 = 63$). (A) is false; (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

Let the smaller number be x; the other number is x + 5.

$x^2 + (x + 5)^2 = 325 \Rightarrow 2x^2 + 10x + 25 = 325$ [1]

$2x^2 + 10x - 300 = 0 \Rightarrow x^2 + 5x - 150 = 0$ [1]

$(x + 15)(x - 10) = 0 \Rightarrow x = -15$ or $x = 10$; reject $x = -15$ (not natural) [1]

The numbers are 10 and 15 (check: 100 + 225 = 325 ✓) [1]

(ii)

Let the brother's present age be x years; the girl's age is (x + 4) years.

After 3 years: ages are (x + 3) and (x + 7) [1]

$(x + 3)(x + 7) = 192 \Rightarrow x^2 + 10x + 21 = 192 \Rightarrow x^2 + 10x - 171 = 0$ [1]

$(x + 19)(x - 9) = 0 \Rightarrow x = 9$ (reject $x = -19$) [1]

Brother is 9 years, girl is 13 years (check: $12 \times 16 = 192$ ✓) [1]

(iii)

Length + breadth = $46 \div 2 = 23$ m. Let the length be x m; then breadth = $(23 - x)$ m.

By Pythagoras: $x^2 + (23 - x)^2 = 17^2$ [1]

$2x^2 - 46x + 529 = 289 \Rightarrow 2x^2 - 46x + 240 = 0 \Rightarrow \mathbf{x^2 - 23x + 120 = 0}$ [1]

$(x - 8)(x - 15) = 0 \Rightarrow x = 8$ or $x = 15$ [1]

Taking the longer side as the length: **length = 15 m, breadth = 8 m** (check: $225 + 64 = 289$ ✓) [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)

Let the numbers be $3x$ and $3x + 3$ (x a natural number).

$3x(3x + 3) = 270 \Rightarrow 9x^2 + 9x = 270 \Rightarrow x^2 + x - 30 = 0$ [1]

$(x + 6)(x - 5) = 0 \Rightarrow x = 5$ (reject $x = -6$) [1]

The numbers are 15 and 18 ($15 \times 18 = 270$ ✓) [1]

(ii)

Let the numerator be x; the fraction is $\frac{x}{x+3}$. After adding 2: $\frac{x+2}{x+5}$.

$\frac{x+2}{x+5} - \frac{x}{x+3} = \frac{2}{21} \Rightarrow \frac{(x+2)(x+3) - x(x+5)}{(x+5)(x+3)} = \frac{2}{21} \Rightarrow \frac{6}{(x+3)(x+5)} = \frac{2}{21}$ [1]

$(x + 3)(x + 5) = 63 \Rightarrow x^2 + 8x - 48 = 0 \Rightarrow (x + 12)(x - 4) = 0$ [1]

$x = 4$ (reject $x = -12$). **The fraction is $\frac{4}{7}$** (check: $\frac{6}{9} - \frac{4}{7} = \frac{2}{3} - \frac{4}{7} = \frac{2}{21}$ ✓) [1]

(iii)

Let the son's age be x years; the man's age is $(5x + 3)$ years.

$x(5x + 3) = 198$ [1]

$5x^2 + 3x - 198 = 0$ [1]

$5x^2 - 30x + 33x - 198 = 0 \Rightarrow 5x(x - 6) + 33(x - 6) = 0 \Rightarrow (x - 6)(5x + 33) = 0$ [1]

$x = 6$ (reject $x = -\frac{33}{5}$). **Son is 6 years, man is 33 years** ($6 \times 33 = 198$ ✓) [1]

Question 4 [3 + 3 + 4 = 10]

(i)

Let tap B alone take x hours; tap A alone takes $(x - 5)$ hours.

$\frac{1}{x-5} + \frac{1}{x} = \frac{1}{6}$ [1]

$6(2x - 5) = x(x - 5) \Rightarrow x^2 - 17x + 30 = 0 \Rightarrow (x - 15)(x - 2) = 0$ [1]

$x = 15$ ($x = 2$ is rejected, since A's time $x - 5$ would be negative). **B takes 15 hours, A takes 10 hours** [1]

(ii)

Let the original speed be x km/h; the new speed is $(x + 10)$ km/h.

$\frac{200}{x} - \frac{200}{x+10} = 1$ [1]

$200(x + 10) - 200x = x(x + 10) \Rightarrow x^2 + 10x - 2000 = 0$ [1]

$(x + 50)(x - 40) = 0 \Rightarrow x = 40$ (reject $x = -50$). **Original speed = 40 km/h** (5 h vs 4 h ✓) [1]

(iii)

Let the speed of the stream be x km/h. Upstream speed = (10 − x) km/h; downstream speed = (10 + x) km/h.

Time upstream − time downstream = 1 $\Rightarrow \frac{24}{10-x} - \frac{24}{10+x} = 1$ [1]

$24(10 + x) - 24(10 - x) = (10 - x)(10 + x) \Rightarrow 48x = 100 - x^2$ [1]

$x^2 + 48x - 100 = 0 \Rightarrow (x + 50)(x - 2) = 0$ [1]

x = 2 (reject x = −50). **Speed of the stream = 2 km/h** (check: 24 ÷ 8 = 3 h, 24 ÷ 12 = 2 h ✓) [1]

Question 5 [3 + 3 + 4 = 10]

(i)

Let the longer side be x cm. Then hypotenuse = (x + 1) cm and shorter side = (x − 7) cm.

$(x - 7)^2 + x^2 = (x + 1)^2 \Rightarrow x^2 - 14x + 49 + x^2 = x^2 + 2x + 1$ [1]

$x^2 - 16x + 48 = 0 \Rightarrow (x - 12)(x - 4) = 0$ [1]

x = 12 (x = 4 is rejected as the shorter side would be negative). **Sides: 5 cm, 12 cm, 13 cm** [1]

(ii)

Let the width of the path be x m. The outer rectangle measures (30 + 2x) m by (20 + 2x) m.

Area of park = 30 × 20 = 600 m², so outer area = 600 + 336 = 936 m² [1]

$(30 + 2x)(20 + 2x) = 936 \Rightarrow 4x^2 + 100x + 600 = 936 \Rightarrow x^2 + 25x - 84 = 0$ [1]

$(x + 28)(x - 3) = 0 \Rightarrow x = 3$ (reject x = −28). **Width of the path = 3 m** (36 × 26 = 936 ✓) [1]

(iii)

Let the length be x m; breadth = $\frac{120}{x}$ m.

New length = (x − 3) m, new breadth = $(\frac{120}{x} + 2)$ m [1]

$(x - 3)(\frac{120}{x} + 2) = 120 \Rightarrow 120 + 2x - \frac{360}{x} - 6 = 120$ [1]

Multiplying by x: $2x^2 - 6x - 360 = 0 \Rightarrow \mathbf{x^2 - 3x - 180 = 0}$ [1]

$(x - 15)(x + 12) = 0 \Rightarrow x = 15$ (reject x = −12). **Length = 15 m, breadth = 120 ÷ 15 = 8 m** (check: 12 × 10 = 120 ✓) [1]

Question 6 [3 + 3 + 4 = 10]

(i)

Let the cost price be ₹ x. Gain = x % of x.

Gain = ₹ $\frac{x^2}{100}$ [1]

S.P. = C.P. + Gain $\Rightarrow x + \frac{x^2}{100} = 96 \Rightarrow x^2 + 100x - 9600 = 0$ [1]

$(x + 160)(x - 60) = 0 \Rightarrow x = 60$ (reject x = −160). **Cost price = ₹ 60** (gain 60% of 60 = 36; 60 + 36 = 96 ✓) [1]

(ii)

Let the units digit be y and the tens digit be x, so the number is 10x + y and xy = 15.

$10x + y - 18 = 10y + x \Rightarrow 9x - 9y = 18 \Rightarrow x = y + 2$ [1]

$y(y + 2) = 15 \Rightarrow y^2 + 2y - 15 = 0 \Rightarrow (y + 5)(y - 3) = 0$ [1]

y = 3 (reject y = −5, not a digit); x = 5. **The number is 53** (53 − 18 = 35 ✓) [1]

(iii)

Original share of each friend = ₹ $\frac{3000}{x}$. After 5 did not come, there are $(x - 5)$ friends and each pays ₹ $\frac{3000}{x - 5}$.

Increase in share: $\frac{3000}{x - 5} - \frac{3000}{x} = 20$ [1]

$3000x - 3000(x - 5) = 20x(x - 5) \Rightarrow 15000 = 20x(x - 5)$ [1]

$\mathbf{x^2 - 5x - 750 = 0} \Rightarrow (x - 30)(x + 25) = 0$ [1]

$x = 30$ (reject $x = -25$). **Original number of friends = 30** (₹ 100 each → 25 friends pay ₹ 120 each ✓) [1]

— End of Solutions —