PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 7 : Ratio and Proportion

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

Proportion: $a : b = c : d \Leftrightarrow ad = bc$. Continued proportion: $a : b = b : c \Leftrightarrow b^2 = ac$ (b = mean proportional, c = third proportional to a and b).

Compound ratio of $a : b$ and $c : d = ac : bd$. Duplicate: $a^2 : b^2$ | Sub-duplicate: $\sqrt{a} : \sqrt{b}$ | Reciprocal: $b : a$.

If $a : b = c : d$ then: componendo $\frac{a+b}{b} = \frac{c+d}{d}$; dividendo $\frac{a-b}{b} = \frac{c-d}{d}$; componendo–dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. **k-method:** if $\frac{a}{b} = \frac{c}{d} = k$, then $a = bk$, $c = dk$.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(a)**	Take $a = 3k$, $b = 4k$: $(6k + 4k) : (3k + 8k) = 10k : 11k = 10 : 11$.
(ii)	**(c)**	Duplicate ratio of $4 : 5 = 4^2 : 5^2 = 16 : 25$.
(iii)	**(a)** 10	Mean proportional $= \sqrt{(4 \times 25)} = \sqrt{100} = 10$.
(iv)	**(c)**	(A) is the componendo–dividendo property and is true. (R) describes exactly how it is proved, so it correctly explains (A).
(v)	**(b)** 9	$4 : 6 = 6 : c \Rightarrow c = 36 \div 4 = 9$.
(vi)	**(c)** ₹ 120	Original : new $= 4 : 5 \Rightarrow$ original $= \frac{4}{5} \times 150 =$ ₹ 120.
(vii)	**(a)**	Take $x = 3k$, $y = 5k$: $(8k) : (2k) = 4 : 1$.
(viii)	**(b)**	Product of extremes $= 3 \times 9 = 27$, product of means $= 4 \times 6 = 24$; these are unequal, so (A) is false. (R) is a true statement.

Question 2 [4 + 4 + 4 = 12]

(i)

$\frac{7x + 2y}{3x + 4y} = \frac{10}{9} \Rightarrow 9(7x + 2y) = 10(3x + 4y)$ [1]

$63x + 18y = 30x + 40y \Rightarrow 33x = 22y$ [1]

$\frac{x}{y} = \frac{22}{33} = \frac{2}{3} \Rightarrow$ **x : y = 2 : 3** [1]

Put $x = 2k$, $y = 3k$: $(3x + y) : (x + 2y) = (6k + 3k) : (2k + 6k) = 9k : 8k =$ **9 : 8** [1]

(ii)

Let the present ages be 5x and 3x years.

After 6 years: $\frac{5x + 6}{3x + 6} = \frac{7}{5}$ [1]

$5(5x + 6) = 7(3x + 6) \Rightarrow 25x + 30 = 21x + 42$ [1]

$4x = 12 \Rightarrow x = 3$ [1]

Present ages: 15 years and 9 years (check: 21 : 15 = 7 : 5 ✓) [1]

(iii)

Duplicate ratio of 2 : 3 = 4 : 9 [1]

Reciprocal ratio of 5 : 12 = 12 : 5 [1]

Sub-duplicate ratio of 49 : 36 = 7 : 6 [1]

Compound ratio = (4 × 12 × 7) : (9 × 5 × 6) = 336 : 270 = **56 : 45** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)

A : B = 2 : 3 = 8 : 12 and B : C = 4 : 5 = 12 : 15 (making B equal) [1]

A : B : C = 8 : 12 : 15; total parts = 35 [1]

One part = 1750 ÷ 35 = ₹ 50 ⇒ **A = ₹ 400, B = ₹ 600, C = ₹ 750** [1]

(ii)

Let x be added to each term.

$\frac{7 + x}{11 + x} = \frac{3}{4}$ [1]

$4(7 + x) = 3(11 + x) \Rightarrow 28 + 4x = 33 + 3x$ [1]

x = 5 (check: 12 : 16 = 3 : 4 ✓) [1]

(iii)

Let the incomes be ₹ 5x and ₹ 4x, and the expenditures ₹ 3y and ₹ 2y.

Savings: $5x - 3y = 1200$ …(1) $4x - 2y = 1200$ …(2) [2]

From (2): $y = 2x - 600$. Put in (1): $5x - 6x + 1800 = 1200 \Rightarrow x = 600$ [1]

Incomes: 5 × 600 = ₹ 3,000 and 4 × 600 = ₹ 2,400 (y = 600: expenditures ₹ 1,800 and ₹ 1,200; savings ₹ 1,200 each ✓) [1]

Question 4 [3 + 3 + 4 = 10]

(i)

(a) 4 : 9 = 12 : x ⇒ 4x = 108 ⇒ **x = 27** [1]

(b) Mean proportional = $\sqrt{(0.2 \times 1.8)} = \sqrt{0.36} =$ **0.6** [1]

(c) Third proportional = $(3.5)^2 \div 2.5 = 12.25 \div 2.5 =$ **4.9** [1]

(ii)

Let x be subtracted. Then (7 − x), (11 − x), (19 − x) are in continued proportion.

$(11 - x)^2 = (7 - x)(19 - x)$ [1]

$121 - 22x + x^2 = 133 - 26x + x^2 \Rightarrow 4x = 12$ [1]

x = 3 (remainders 4, 8, 16; 4 : 8 = 8 : 16 ✓) [1]

(iii)

Let the numbers be x and y. Mean proportional: x : 10 = 10 : y. Third proportional: x : y = y : 80.

$xy = 100$ …(1) [1]

$y^2 = 80x$ …(2) [1]

From (1), $x = 100/y$. Put in (2): $y^2 = 8000/y \Rightarrow y^3 = 8000 \Rightarrow y = 20$ [1]

$x = 100 \div 20 = 5$. **The numbers are 5 and 20** (mean $\sqrt{100} = 10$; third proportional $400 \div 5 = 80$ ✓) [1]

Question 5 [3 + 3 + 4 = 10]

(i)

Let $\frac{a}{b} = \frac{c}{d} = k$, so $a = bk$ and $c = dk$ [1]

$\text{LHS} = \frac{5a + 2c}{5b + 2d} = \frac{5bk + 2dk}{5b + 2d} = \frac{k(5b + 2d)}{5b + 2d} = k$ [1]

$\text{RHS} = \frac{a - 4c}{b - 4d} = \frac{bk - 4dk}{b - 4d} = \frac{k(b - 4d)}{b - 4d} = k$. LHS = RHS. **Hence proved.** [1]

(ii)

a, b, c are in continued proportion $\Rightarrow b^2 = ac$ …(1) [1]

$\text{LHS} = (a + b + c)(a - b + c) = (a + c)^2 - b^2 = a^2 + 2ac + c^2 - b^2$ [1]

Using (1), $2ac = 2b^2$: $\text{LHS} = a^2 + 2b^2 + c^2 - b^2 = a^2 + b^2 + c^2 = \text{RHS}$. **Hence proved.** [1]

(iii)

Let $\frac{a}{b} = \frac{c}{d} = k$, so $a = bk$ and $c = dk$ [1]

$\text{LHS} = \frac{a^2 + ab + b^2}{c^2 + cd + d^2} = \frac{b^2k^2 + b^2k + b^2}{d^2k^2 + d^2k + d^2} = \frac{b^2(k^2 + k + 1)}{d^2(k^2 + k + 1)} = \frac{b^2}{d^2}$ [1.5]

$\text{RHS} = \frac{a^2 - ab + b^2}{c^2 - cd + d^2} = \frac{b^2(k^2 - k + 1)}{d^2(k^2 - k + 1)} = \frac{b^2}{d^2}$. LHS = RHS. **Hence proved.** [1.5]

Question 6 [3 + 3 + 4 = 10]

(i)

$\frac{6x + 5y}{6x - 5y} = \frac{3}{1}$ [1]

By componendo and dividendo: $\frac{(6x + 5y) + (6x - 5y)}{(6x + 5y) - (6x - 5y)} = \frac{3 + 1}{3 - 1} \Rightarrow \frac{12x}{10y} = \frac{4}{2} = 2$ [1]

$12x = 20y \Rightarrow \frac{x}{y} = \frac{5}{3}$. **x : y = 5 : 3** [1]

(ii)

By componendo and dividendo: $\frac{2\sqrt{(2x + 3)}}{2\sqrt{(2x - 3)}} = \frac{3 + 1}{3 - 1} = 2$ [1]

$\frac{\sqrt{(2x + 3)}}{\sqrt{(2x - 3)}} = 2 \Rightarrow$ squaring: $2x + 3 = 4(2x - 3)$ [1]

$2x + 3 = 8x - 12 \Rightarrow 6x = 15 \Rightarrow \mathbf{x = \frac{5}{2}}$ (check: $\frac{\sqrt{8} + \sqrt{2}}{\sqrt{8} - \sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}} = 3$ ✓) [1]

(iii)

Let $p = (x + 2)^3$ and $q = (x - 2)^3$.

By componendo and dividendo: $\frac{p + q + p - q}{p + q - p + q} = \frac{9 + 7}{9 - 7} \Rightarrow \frac{2p}{2q} = 8$ [1]

$\frac{(x + 2)^3}{(x - 2)^3} = 8 = 2^3 \Rightarrow \frac{x + 2}{x - 2} = 2$ [1]

Again by componendo and dividendo: $\frac{(x + 2) + (x - 2)}{(x + 2) - (x - 2)} = \frac{2 + 1}{2 - 1} \Rightarrow \frac{2x}{4} = 3$ [1]

x = 6 (check: $8^3 = 512$ and $4^3 = 64$; $(512 + 64)/(512 - 64) = 576/448 = 9/7$ ✓) [1]

— End of Solutions —